Ultracold Plasmas

( Zafar Yasin)
Outline

- Creation and why considered important?
- Characterization.
- Modeling.
- My Past Research.
- Current Research Trends.
Why this topic?

- Ultracold Plasmas of astrophysical interest--- so matches with SASS interest.

My Plasma Physics Background:

- Laser-produced Plasma Theory ---------
  (1995 QA Univ., Islamabad, Pakistan).

- Laser-produced Plasma, and Plasma based accelerators theory, as junior faculty with main task of teaching some undergraduate Physics ---------


(Offcourse now it is SM Physics with BaBar !. Thanks to UCR, I am at SLAC!)
Plasma?

- Freely moving positively charged ions or nuclei and negatively charged electrons, for not necessarily completely ionized gas.

- Long range electromagnetic interactions dominate the short range interatomic or intermolecular forces among large number of particles.

Main Characterising Parameters:

Thermal speed:

\[ v_\tau = \sqrt{2 kT_e / m_e} \]

(Most probable speed in a Maxwellian distribution)

Plasma Frequency:

Harmonic electron oscillations from equilibrium (homogeneous, unmagnetized plasma).

\[ \omega_{pe} = \left( n_0 e^2 / \varepsilon_0 m_e \right)^{1/2} \]

\[ \omega_{pe} \tau > 1 \]

where \( \tau \) is the collision time.

(Most fundamental time-scale in Plasma Physics)

Debye Length:

Distance a thermal particle travels during a plasma period.

\[ \lambda_D = \left( \varepsilon_0 kT_e / n_e e^2 \right)^{1/2} = \left( v_\tau / 2 \right)^{1/2} \omega_{pe} \]

\[ \lambda_D << L \]

\( N_D >> 1 \) for to be a plasma, (\( N_D \) is number density in a Debye sphere).

Degree of Ionization:

From Saha equation for ionized gas in thermal equilibrium;

\[ n^i / n^n = C T^{3/2} / n^i \exp(U_i / kT) \]

\( n^i / n^n >> 1 \) for gas to be a plasma.
Ultracold Physics

- Doppler Cooling (temperatures from $m\,K$ to $\mu\,K$).
- Bose Einstein Condensate: (Laser + evaporative) cooling.
- Fermionic condensate
- Atomic clock for precision measurement.
- Superfluidity.
- Intense atomic beams.
- Laser manipulation micron sized particles.
- Cold atom collisions.
- Nuclear Physics.
- Cold molecules.
- Cavity QED.
- Ultracold Plasmas!
Why Ultracold Plasmas are Interesting?

- Controllable density and temperatures.
- Lab for some exotic Astrophysical Plasmas.
- Produce conditions of Laser–produced plasmas.
- Challenging Modeling.

\[ \Gamma = \frac{\left( e^2 / 4 \pi \varepsilon_0 \right)}{(\kappa T_e)} \]

where, \( a \) is interparticle spacing.

(T.C. Killian et al. / Physics Reports 449 (2007) 77–130)

Various Plasmas in density-temperature parameter plot
Ultracold Plasma Creation

- Laser cool atoms to slightly above sub-milikelvin temperatures.
- Trap the laser-cooled atoms using Magneto Optical trap.
- Photoionize, using laser light near the ionization threshold.
- Diagnostics with applied electric fields.

_Schematics of a typical Experiment to create Ultracold Plasmas_

(T.C. Killian et al. / Physics Reports 449 (2007) 77–130)
What Early Experiments tell?

- *Non uniform spatial distribution,  \( \sim \) Gaussian*.
- *Spherically symmetric cloud \( \sim 200 \mu m \).*
- *Stabilizes to equilibrium after its characteristic thermalization time.*
- Some electrons leave the system on a time scale given by radius/velocity\(= 6ns \),
- for electrons with energy \( \sim 100K \).
Some Early Robust Modeling

- Shows rapid heating due to three body recombination, which prevents development of strong correlation, producing plasma which is not ultracold, but much colder then 1 eV (10,000K).
  
  \[ \frac{1}{\Gamma_e} = kT_e/(e^2/a) \]
  
  \( f/\Omega_p \)

  Scaled temperature Vs scaled time

- Three body recombination is important at very low temperatures.
- Collisions between electrons and Rydberg atoms is source of heating.
- Coulomb coupling parameter does not become larger then \( \sim 1/5 \).

  \( (\text{F.Robicheaux and James D.Hanson, Phys.rev.Lett.88,55002(2002)}) \)
Simulation Results
(from my graduate thesis)

- The initial physical conditions and geometry from early experiments.
- Spherical symmetry assumed.
- Ions treated as static in the background with a Gaussian distribution.
- The initial electron density Gaussian.
- Number densities ranging from \(10^{14}\) to \(10^{16}/m^3\).
- Initial electron temperatures in range from 100K to 1000K.
- Cartesian space to model radial expansion.
- Total number of electrons considered = \(10^5\).

How code works?

- Charge density on radial grid is calculated.
- Electric field at any point is estimated.
- Acceleration calculated, particle is now moved, using leapfrog algorithm
- The process is repeated sufficient number of time until steady state is established.
Temperature Vs Radius during early time
(average local temperature is calculated over small incremental radial lengths)
Temperature Vs Radius during later time (indicating steady state reached)
Volume averaged value of temperature plotted against time
The net electric field after plasma reaches steady state
( for various temperatures and number densities)
Electron density Vs, radius for various initial temperatures and number densities (dotted lines hows profiles at start of run)
Electron temperature Vs. radius for various initial temperatures and number densities
Viscous Electron Fluid Model

Continuity:

\[ \frac{\partial n}{\partial t} + \vec{v} \cdot \vec{\nabla} n + n \vec{\nabla} \cdot \vec{v} = 0 \]

Momentum:

\[ mn \left[ \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} \right] = - \vec{\nabla} p - \vec{\nabla} \cdot \vec{P} - enE + \vec{R} \]

Energy:

\[ \frac{3}{2} n \left[ \frac{\partial (k_b T)}{\partial t} + \vec{v} \cdot \vec{\nabla} (k_b T) \right] + p \vec{v} \cdot \vec{v} = - \vec{\nabla} \cdot \vec{q} - \vec{P} : \vec{\nabla} \vec{v} + Q \]
\[ Q = -3 \frac{m \, nk_b T}{m_i \, \tau} \]

where \( m_i \) is the mass of an ion, and \( \tau \) is the collisional time given by:

\[ \tau = \left[ \frac{(6\sqrt{2}(\pi)^{3/2} \, n_{00} \lambda_d^3)}{n_0 (\lambda - 0.5 \ln(n_0)) \omega_p} \right] \]

where

\[ \omega_p = \sqrt{n_0 e^2 / m \varepsilon_o} \]

is the plasma frequency,

\( n_{00} \) is the number density of electrons at centre for a Gaussian profile,

\( n_0 \) is the equilibrium electron number density scaled with \( n_{oo} \),

\( \lambda \) is the Coulomb logarithm at \( r = 0 \), given by:

\[ \lambda = 23 - \log \left[ \frac{n_{00}^{1/2}}{10^3} \left( \frac{11600}{T_{ev}} \right)^{3/2} \right] \]
\( \lambda_d \) is the Debye length at \( r = 0 \), given by:

\[
\lambda_d = \frac{k_b T_0 \varepsilon_o}{\sqrt{e^2 n_{00}}}.
\]

The momentum gained by electrons on collisions with ions per unit volume per unit time is given by:

\[
\bar{R} = -\frac{n \bar{v} m}{1.96\tau} - 0.71n \bar{\nabla}(k_B T),
\]
heat flow:

\[ \bar{q} = 0.71 \, nk_b \, T \bar{v} - (\kappa \vec{V} k_b T) \]

electron thermal conductivity:

\[ \kappa = 3.2 \, \frac{nk_b T \tau}{m} \]

viscous pressure tensor:

\[ (\vec{V} \cdot \vec{P})_r = \frac{4}{3} \frac{\partial \eta}{\partial r} \left[ \left( \frac{\partial v}{\partial r} - \frac{v}{r} \right) \right] + \frac{4}{3} \eta \left[ \frac{\partial^2 v}{\partial r^2} + \frac{2}{r} \frac{\partial v}{\partial r} - 2 \frac{v}{r^2} \right] \]

viscosity:

\[ \eta = 0.73 nk_b \tau T \]
After dimensional analysis electron fluid equations

continuity:

\[ \frac{\partial n}{\partial t} + \bar{v} \cdot \nabla n + n \nabla \cdot \bar{v} = 0 \]

momentum:

\[ mn \left[ \frac{\partial \bar{v}}{\partial t} + \bar{v} \cdot \nabla \bar{v} \right] = -\nabla p - \nabla \cdot \bar{P} - en\bar{E} + \bar{R} \]

energy:

\[ \frac{3}{2} n \left[ \frac{\partial (k_b T)}{\partial t} + \bar{v} \cdot \nabla (k_b T) \right] + p \nabla \cdot \bar{v} = -\nabla \cdot \bar{q} \]
Scaled Equations

Continuity:
\[
\frac{\partial n}{\partial t} + \vec{\nabla} \cdot (n\vec{v}) = 0
\]

Poisson's Equation:
\[
\vec{\nabla} \cdot \vec{E} = -n_i + n
\]

\[
 n \left[ \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \vec{\nabla} \vec{v} \right] = -\frac{\lambda_d^2}{\sigma^2} \vec{\nabla} (nT) + \frac{\lambda_d^2}{\sigma^2} \vec{\nabla} \cdot \vec{P} - n\vec{E} - \frac{m\vec{v}}{1.96\tau} - 0.71\frac{\lambda_d^2}{\sigma^2} n\vec{\nabla}T
\]

Energy:
\[
\frac{3}{2} n \left[ \frac{\partial (k_bT)}{\partial t} + \vec{v} \cdot \vec{\nabla} T \right] + nT\vec{\nabla} \cdot \vec{v} = -0.71\vec{\nabla} \cdot (nTV) + \frac{\lambda_d^2}{\sigma^2} \vec{\nabla} \cdot (\kappa\vec{\nabla}T)
\]
Linearization gives following coupled equations for Velocity and Temperature. Subscript 1 denote perturbed part and 0 the equilibrium part. (These needs to be solved for mode frequencies!)

\[
\lambda_D^2 \left[ \left( \frac{\partial}{\partial r} \left( n_0(r) T_1(r) \right) + \frac{\partial}{\partial r} \left[ \frac{\partial}{\partial r} \left( r^2 n_0(r) v_1(r) \right) \frac{1}{i \omega r^2} \right] \right) + 0.71 n_0(r) \left( \frac{\partial}{\partial r} T_1(r) \right) \right] + \frac{\sigma^2}{\sigma^2} = 0
\]

\[
\frac{\lambda_D^2}{4} \left[ \frac{\partial}{\partial r} \eta(r) \left( \frac{\partial}{\partial r} v_1(r) - \frac{v_1(r)}{r} \right) + \eta(r) \left( \frac{\partial^2}{\partial r^2} v_1(r) \right) + \frac{2 \frac{\partial}{\partial r} v_1(r)}{r} - \frac{2 v_1(r)}{r^2} \right] = 0
\]

\[
- \frac{3}{2} i \omega n_0(r) T_1(r) + \frac{n_0(r) \left( 2 r v_1(r) + r^2 \frac{\partial}{\partial r} v_1(r) \right)}{r^2} + \frac{1.42 r n_0(r) v_1(r) + 0.71 r^2 \left( \frac{\partial}{\partial r} n_0(r) v_1(r) + 0.71 r^2 n_0(r) \frac{\partial}{\partial r} v_1(r) \right)}{r^2}
\]

\[
\lambda_D^2 \left[ 6.4 r n_0(r) \eta \left( \frac{\partial}{\partial r} T_1(r) \right) + 3.2 r^2 \frac{\partial}{\partial r} n_0(r) \eta \left( \frac{\partial}{\partial r} T_1(r) \right) + 3.2 r^2 n_0(r) \eta \left( \frac{\partial^2}{\partial r^2} T_1(r) \right) \right] = 0
\]
Plots showing the boundary conditions for mode analysis
Square of mode frequency Vs. Temperature at number density $10^{14} / m^3$

(Linear behaviour until 300 K, and then strong damping)
Square of mode frequency Vs. Temperature at number density $10^{15} / \text{m}^3$

(At very large values of temperature the mode frequency does not vary linearly with $T$)
Square of mode frequency Vs. Temperature at number density $10^{16} / \text{m}^3$

(Linear behaviour for temperature greater than 300K)
Conclusions

Going over last few years of experimental and theoretical research;

- Excellent control over lab conditions and diagnostics, is expected to create some of the plasmas of astrophysical interests.

- Some of the lab experiments indicate conditions similar to laser-produced plasmas.

- Phase transition between gas, liquid and solid states is expected to be studied in future.

- Application of strong magnetic field of several teslas, expect to open a new regime.

- Theory work needed to more fully understand how plasma evolve after creation.