If I only had a Brane

- A Story about Gravity and QCD.

on 20 slides and in 40 minutes.

AdS/CFT correspondence = Anti-de-Sitter / Conformal field theory correspondence.

Chapter 1: String Theory in a nutshell.
Chapter 2: The AdS/CFT conjecture
Chapter 3: A glance at AdS/QCD.

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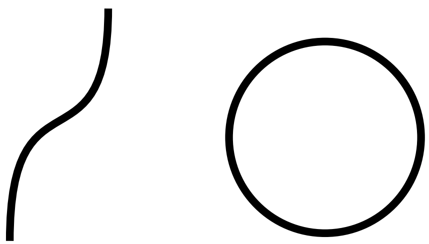
Chapter 1: String Theory in a nutshell.

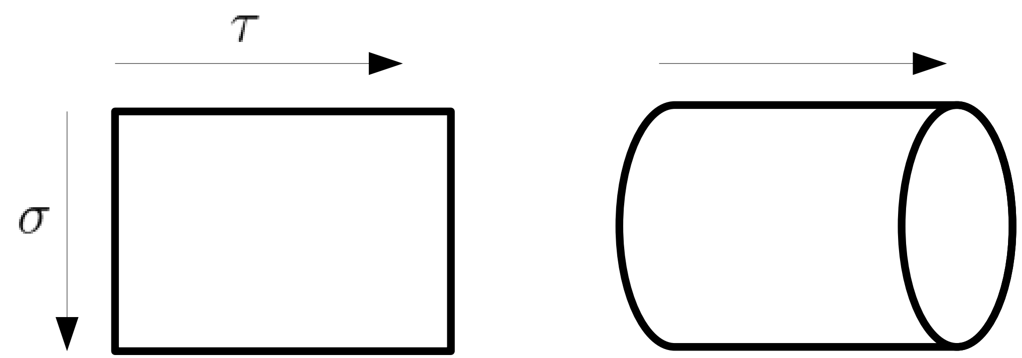
Open and closed strings.
Supersymmetry and the Bulk.
Particles vs. Strings and Dp Branes.
Quick Summary.

Anti-de-Sitter space and Black hole solutions.
Conformal invariance and field theories.

Open and closed Strings.



classical string: 1 dimensional object.



Propagation: sweeps out a 2 dimensional world sheet in d dimensional space-time. (σ, τ)

Space-time position in d dimensions: $X^\mu(\sigma,\tau)(\mu = 0, \ldots, \mu = d-1)$

Extracting classical dynamics a matter of some calculus, ask your cat.

When quantized, classical properties of the string (e.g. tension, length, ...) become parameters of the quantized theory.

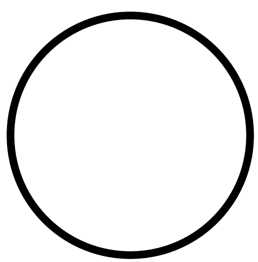
$$\{X^\mu(\sigma), \Pi^{\tau\mu}(\sigma')\} \rightarrow [X^\mu(\sigma), \Pi^{\tau\mu}(\sigma')] \propto \delta(\sigma - \sigma') \qquad \Pi^{\tau\mu} \backsim \frac{\partial X^\mu(\sigma,\tau)}{\partial \tau}$$

Lorentz invariant formulation only possible in **26** dimensions. Only **bosonic** states.

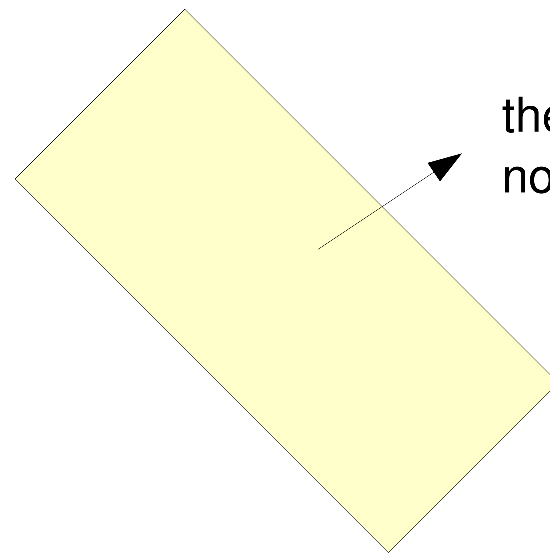
Supersymmetry, branes and the Bulk.

26 dimensions, that's a lot. Only having the possibility to describe bosonic particles is bad as well. Introducing **supersymmetry** helps: fermionic states drop out, and we end up having Lorentz invariant theories with 'only' **10** space-time dimensions (Type I,**II**a/**b**,E8xE8 & SO(32)). Many different background spaces possible, ...

extra dimensions can be 'wrapped up'...



or infinitely large. Many possible topologies.



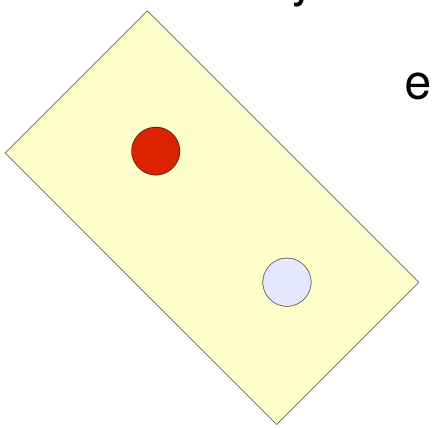
the **bulk** is everything not on a brane.

A p+1 dimensional hyperplane is called **p brane**. The SM interactions can be though of being confined on a 4 dimensional brane. Gravity however can penetrate all spatial dimensions. **Hierarchy problem**: Gravity is – compared with all other fundamental forces - very weak. Barely a 4 dimensional projection of what's going on?

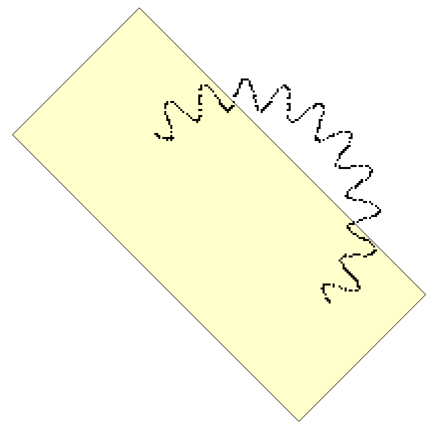
$$\frac{F_{em}}{F_{gr}} \backsim 10^{42}$$

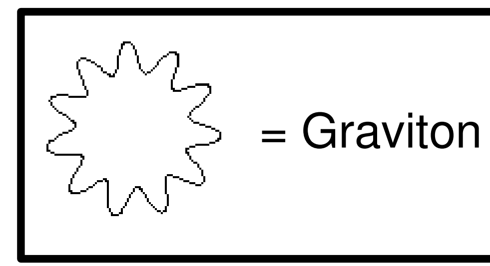
Particles vs. Strings and Dp branes (in type IIB String Theory).

So how do common particle look like in the string picture? They are resonances of open strings. A D meson yields another oscillation.

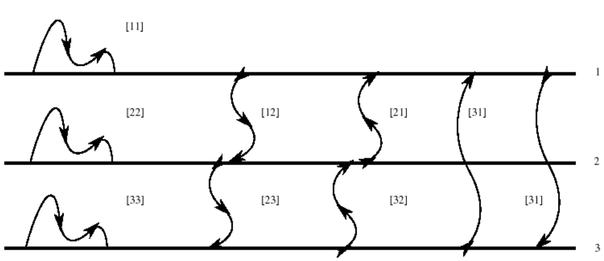


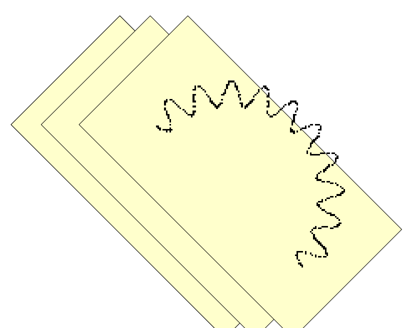
e.g. B meson





What is a **Dp brane**? A Dp brane is a p brane, that has a special property: **Open strings** are allowed to end on them. Qualitatively speaking an open string cannot detach itself from a Dp brane. Closed strings can travel through the bulk and represent gravitons.



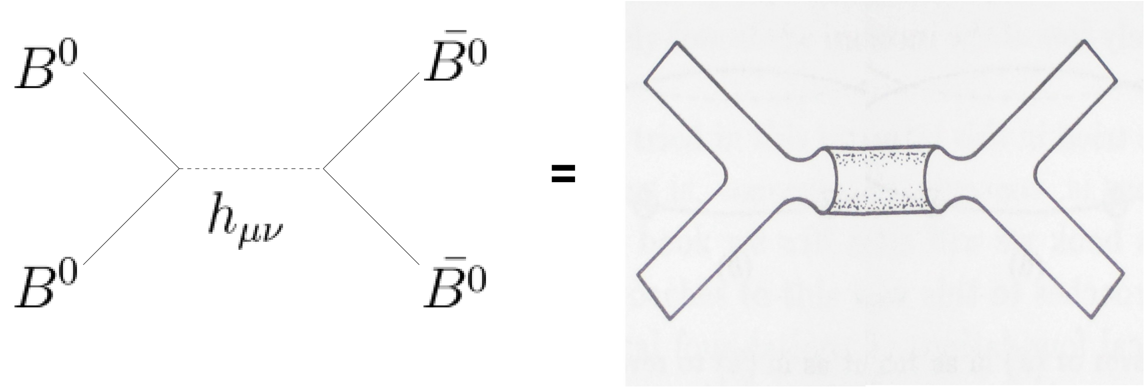


Massless SM gauge fields can be introduced by spatially **coincident** Dp branes. They arise from the additional freedom of choice in the Lagrangian of a string on which brane to reside. Entire SM complicated! D6+D? AdS/CFT only models QCD/large U(N).

N coincident Dp branes = **U(n)** gauge symmetry

Feynman graphs vs. String picture. Low energy limit of type IIb string theory.

e.g. gravitational scattering of two B mesons.



The low energy limit of type IIb string theory yields a super symmetrical **gravity** theory. Strings become point like particles in a 10 dimensional background space and dynamics can be described (up to the super symmetric isometries of the space) by General Relativity in 10 space-time dimensions.

Quick summary.

- not one string theories but many. We will only be concerned about **type IIB**, which introduces chiral fermions and has open and closed strings. Open strings represent matter and are trapped on a Dp brane, closed strings are spin 2 gravitions and can travel through the bulk.

- Introduction of U(n) gauge fields = Placing N coincident Dp branes.
All massless SM gauge fields can be reproduced by adding higher dimensional Dp branes that intersect in 4 dimensions. But **not convenient way** how to introduce certain aspects of the SM (e.g. electroweak symmetry breaking for the fields to aquire mass) without messing up the theory with extra-particles and tachyons.

- certain freedom of choice of the background space (compact or non-compact extra dimensions, cosmological constant or not). As we will see shortly AdS/CFT naturally acts in $AdS_5 \times S^5$ background space.

- the low energy limit of type IIB string theory is a supergravity theory. Its dynamics can be described by the Einstein-field equations.

next : Anti-de-Sitter space and black holes.

What is Anti-de-Sitter Space?

n+1 dimensional Anti-de-Sitter space is given by the defining constraint (defining a hyperboloid).

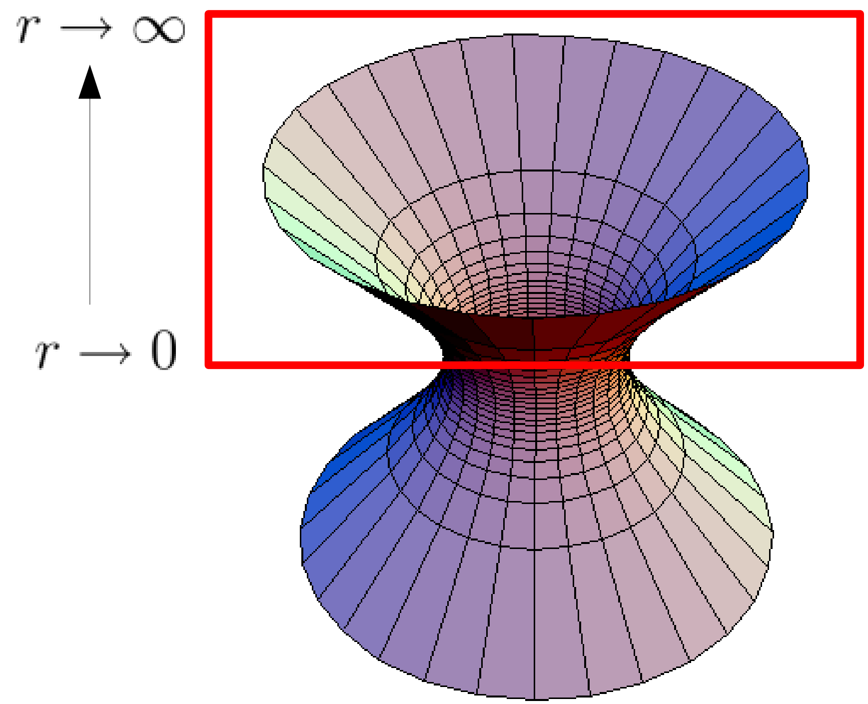
$$X_0^2 + X_{n+1}^2 - \sum_{i=0}^{n} X_i^2 = R^2$$

what yields a metric

$$\mathrm{d}s^2 = -\mathrm{d}X_0^2 - \mathrm{d}X_{n+1}^2 + \sum_{i=0}^{n} \mathrm{d}X_i^2$$

that is usually parametrized as something like

$$\mathrm{d}s^2 = -\mathrm{d}\tau^2 + \mathrm{d}\theta^2 + \sin^2(\theta)\mathrm{d}\Omega^2$$



In 1+1 dimensions embedded in 3 dimensions.

Solutions produced by GR with neg. cosmological constant.

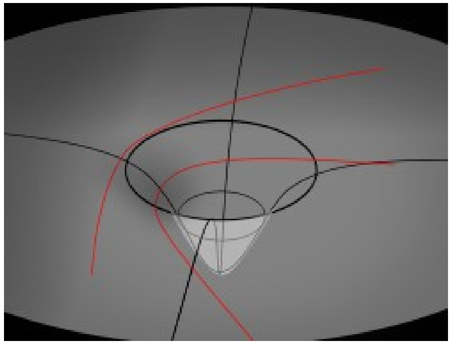
Poincaré wedge: valid in regions far away from the center. (Makes calculations less messy).

$$\mathrm{d}s^2 = \left(1 + \frac{r^2}{R^2}\right)\mathrm{d}t^2 + \left(1 + \frac{r^2}{R^2}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

R = AdS radius.

Some black holes in Anti-de-Sitter space.

Black hole solutions are regions in space that contain two or more singularities (one in the center of the black hole, one at the boundary). Once behind the first singularity nothing can escape and no information can pass through.



$$R_{\mu\nu} - \frac{1}{2} R \cdot g_{\mu\nu} = \kappa T_{\mu\nu}$$

Solutions of the Einstein-field equations.

Schwarzschild AdS black hole solution (**regular black hole**):

$$\mathrm{d}s^2 = \left(1 + \frac{r^2}{R^2} - \frac{r_s^2}{r^2}\right)\mathrm{d}t^2 + \left(1 + \frac{r^2}{R^2} - \frac{r_s^2}{r^2}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

Reissner-Nordstrøm AdS black hole solution (**charged black hole**):

$$\mathrm{d}s^2 = \left(1 + \frac{r^2}{R^2} - \frac{r_s^2}{r^2} + \frac{r_q^4}{r^4}\right)\mathrm{d}t^2 + \left(1 + \frac{r^2}{R^2} - \frac{r_s^2}{r^2} + \frac{r_q^4}{r^4}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2$$

Connection to string theory: Black branes and D3 branes I.

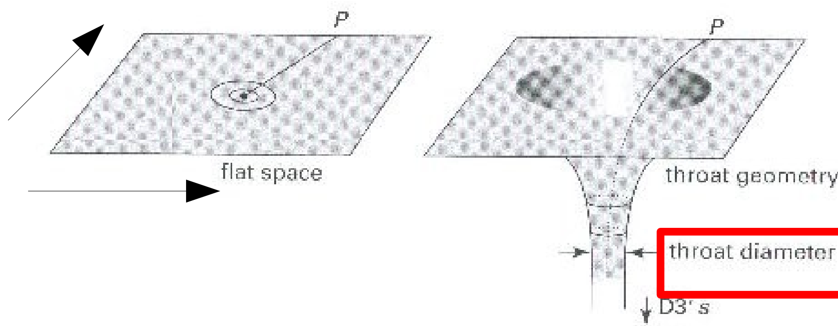
Since matter fields are **confined** on branes in the string picture, black holes only can form there and spread over the brane (the horizon has to stay on the brane!). However it is possible for the brane to be bended and twitched. Best think of it as a black pancake in 10 dimensional space-time.

Such objects are called **black branes** and are solutions of the Einstein-field equations. If one writes down the 10 dimensional metric of such a black D3 brane in **10 dimensional flat space** one finds out something astonishing:

1) in the limit of a **vanishing** black hole on the brane (if one reduces the energy density below the critical point that a black hole would form), the metric separates into an Anti-de-Sitter part that can be described by the Poincaré wedge and a 5-Sphere metric. The space in the neighborhood of our brane is naturally described by $AdS_5 \times S^5$!

no matter on brane / or far away.

spatial dimensions not on brane: the D3 brane appears only as a point in space-time.



turn on matter on brane / get closer.

determines radius of compact dimensions and AdS.

Connection to string theory: Black branes and D3 branes II. / Super-Yang-Mills theory.

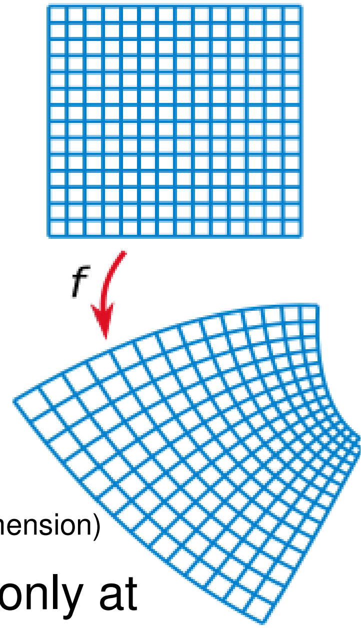
2) If we cross the energy-density threshold and a **black brane** forms, and don't go far away from this threshold, the forming black brane can be described with the metric of 5 dimensional black holes in Anti-de-Sitter space. Exactly the ones we saw before.

We discussed what will happen if we introduce a massive brane into flat 10 dimensional space in type IIB string theory from the string theory point of view. But there is another perspective: we said that introducing matter fields on N coincident branes yields a **U(N) gauge theory** with the same supersymmetries of the string theory present.

If we tweak the amount of supersymmetries right, we gain a special gauge theory, that has an interesting extra symmetry: conformal invariance.

The key of the AdS/CFT conjecture is the connection of this two pictures: the gravity picture and the gauge theory picture. But we will come back to that.

Conformal Invariance and Field Theories.



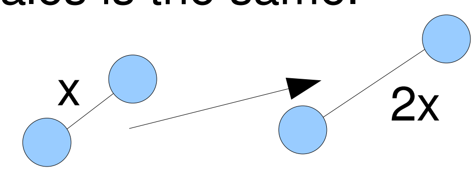
Conformal symmetry preserve the metric up to an arbitrary scale factor:

$$g_{\mu\nu}(x) \rightarrow \Omega(x) g_{\mu\nu}(x)$$

Poincaré + scale + special conf. transform. = Conformal group.

In a field theory at a glance: the physics at all scales is the same.

$$x^\mu \rightarrow \lambda x^\mu \qquad x^\mu \rightarrow \frac{x^\mu + a^\mu x^2}{1 + 2x^\nu a_\nu + a^2 x^2}$$



conformal world = “distance times 2 yields force times 2” (with the choice of a specific scaling dimension)

examples: **classical ED** without charges. **QCD as an effective theory** (valid only at one given energy scale) describing asymptotic freedom and confinement.

$$\beta(g) = \mu \frac{\partial g}{\partial \mu}$$

Scale invariance implies a vanishing beta function, e.g. the coupling g of the theory is exactly the same at all energy scales μ .

Our example is a conformal theory is super symmetric Yang-Mills theory with $\mathcal{N} = 4$ that indeed possess a vanishing beta function. $\mathcal{N}$ refers to the 'amount' of super symmetries present in the model and Yang-Mills is just a fancy way to say it is a theory of non-Abelian gauge fields (e.g. like QCD). Most theorist just write this as SYM with $\mathcal{N} = 4$.

Chapter 2: The AdS/CFT conjecture.

Maldacena's paper was published 1997. It generated a follow up of over 5000 papers and AdS/CFT has become an active playground for theoretical physicists. (“On the shoulder of giants”, based on work of 't Hooft, Witten, etc.).

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The Large N Limit of Superconformal field theories and supergravity

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Abstract

We show that the large N limit of certain conformal field theories in various dimensions include in their Hilbert space a sector describing supergravity on the product of Anti-deSitter spacetimes, spheres and other compact manifolds. This is shown by taking some branes in the full M/string theory and then taking a low energy limit where the field

But before we come to the conjecture...

We will have a look on two things: first the entropy of a black 3 brane embedded in flat 10 dimensional space and the entropy of finite temperature SYM. Second the Froissart bound.

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1) Entropy of a **black brane** vs. entropy of a finite temperature **SYM** with large N in the free field limit:

$$S_{\mathrm{bh}} = \frac{\pi^2}{2} N^2 V_3 T^3 \qquad S_{\mathrm{YM}} = \frac{2\pi^2}{3} N^2 V_3 T^3$$

Pretty amazing! The gravity theory captures the T^3 scaling of the SYM! The difference of $\frac{4}{3}$ goes away if one calculates the correction to the non-free theory (what is a non-trivial).

2) Reproduction of the Froissart bound:

The Froissart bound describes the behavior of the total cross section at high center of mass energies $s \longrightarrow \infty$. It yields a saturation of the form

$$\sigma_{tot} \backsim \ln^2 s$$

which is a feature of high energy SYM scattering implied by unitarity. The classical geometric cross section reads $\sigma_{tot} = \pi r_h^2$

It can be shown that $r_h \backsim \ln(s)$ what yields the desired behavior!

The AdS/CFT conjecture:

On the previous slide we saw two astonishing things, that somehow links under certain conditions gauge theories and string theory on a fundamental level. This is the AdS/CFT conjecture:

> $\mathcal{N}=4$ U(N) SYM in 3+1 dimensions are dual to type IIB supergravity theories in $AdS_5 \times S^5$.

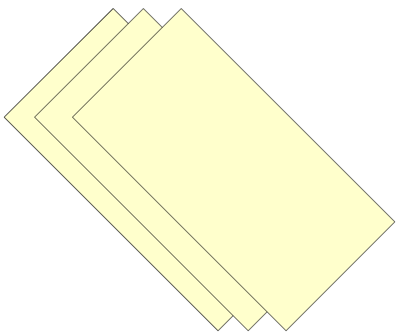
This conjecture was put to the test in the last ten years to reproduce many aspects of the dual gauge partner. Up to now it never failed. Nevertheless one should point out, it is only a conjecture and not a proven theorem. Just that no one expects it to be wrong, doesn't mean it is necessarily correct. But it looks damn correct.

There are in fact two versions of the conjecture above:

> 1.) weak conjecture: this gauge/gravity dualism only holds at large N.
> 2.) strong conjecture: it holds for any N.

So how does it work?

time to play:

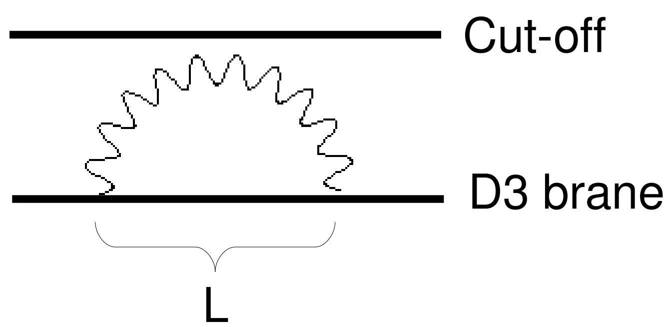
$AdS_5 \times S^5$ +  + Matter = predictions for gauge theory.

By **deforming** the original setup one can introduce new **features** to the dual gauge theory and for instance make it more QCD like. The collection of theories using this approach are called **AdS/QCD.**

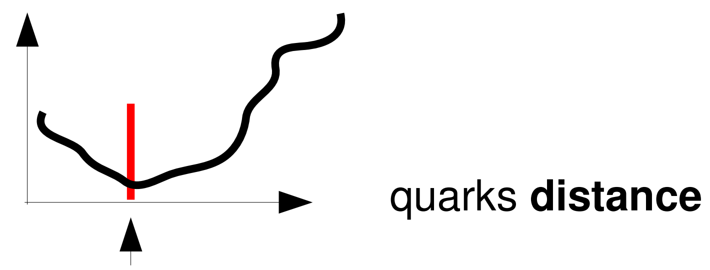
In the following we will discuss *qualitatively* how one can alter the gravity background to introduce two key features of QCD: **asymptotic freedom and confinement.**

Chapter 3: A glance at AdS/QCD

So how do we have alter the gravity background to obtain **confining** behavior of the matter fields sitting on the brane, connected with an open string? The simplest approach is **introducing** an impenetrable **cut-off** (**Hard-Wall-Model**').



potential energy of quark/anti-quark pair



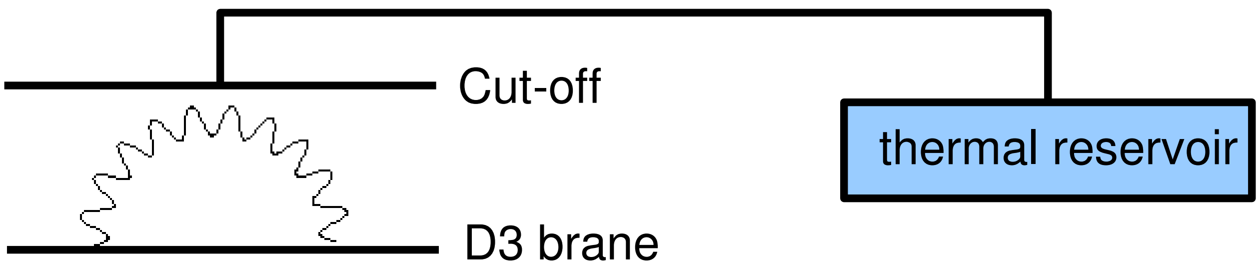
string reaches cut off.

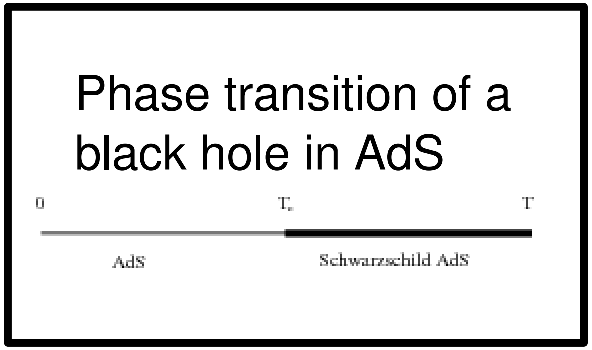
Potential energy laws between two quarks before and after reaching cut-off.

$$V(L) \backsim \frac{1}{L} \qquad V(L) \backsim -\log|\lambda - L|$$

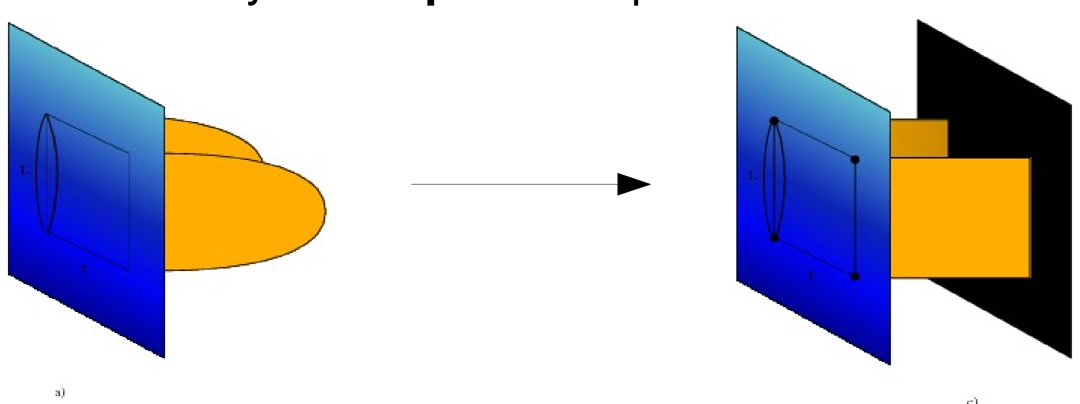
Deconfinement.

By **connecting** the cut-off to a **thermal bath**, one can introduce **deconfining** behavior in the theory. By changing the temperature of the thermal reservoir manipulates the temperature on the cut-off brane.





When the Temperature reaches a certain critical point it is **favorable** for the cut-off brane to become a **black brane**. This is a first order **phase transition**. What does this mean for the strings touching it? For them it's favorable to **split** and touch the horizon instead of holding together. This effectively **decouples** the quarks and introduces **deconfinement**.



Summary: If I only had a brane ...

The Scarecrow had no brain!

.. then one could reproduce QCD like theories and access it at energies where it couples strongly (e.g. around Λ_{QCD}).

Setup: Type IIB string theory (open strings = matter, closed strings gravity)

Introducing N coincident massive D3 brane into 10 dimensional space

↓

yields $AdS_5 \times S^5$ background geometry

↓

Conjecture: Gravity theory behaves like the gauge theory.

↓

Manipulating the Gravity background = manipulating the field theory.