Why Supersymmetry is Super

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Outline

- Introduce Quantum Field Theory
- Incorporating Supersymmetry in a Quantum Field Theory
- The MSSM
- Motivations for Supersymmetry in the “Real World”

Definition of a Quantum Field Theory

Quantum Mechanics and Relativity

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$$

- Not Lorentz invariant
  - Unequal number of time and space derivatives
  - Explicit mass in denominator
  - Potential usually depends explicitly on position

Quantum Mechanics and Relativity

- Different approach:

  Make a relativistic theory quantum mechanical!
- A Classical Field Theory is Lorentz invariant
- How to make it quantum mechanical:
  - Promote classical fields to operators!

$$[p_i, x_j] = i\hbar\delta_{ij}$$

$$[\pi(\vec{x}, t), \phi(\vec{x}', t)] = i\hbar\delta(\vec{x} - \vec{x}')$$

Quantum Mechanics and Relativity

- Moral:

$$\text{Quantum Mechanics} + \text{Lorentz Invariance}$$

$$=$$

$$\text{Quantum Field Theory}$$

Quantum Field Theory

- Spin defines transformations under rotations and boosts
- Quantum Field Theory naturally explains spin
  - Not added ad hoc as in non-relativistic qm

0	π	2π
Scalar	Scalar	Scalar
Fermion	Fermion	Fermion
$\overrightarrow{\text{Vector}}$	$\overleftarrow{\text{Vector}}$	$\overrightarrow{\text{Vector}}$

Example: Scalar (spin 0) theory

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 = \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}(\boldsymbol{\nabla}\phi)^2 - \frac{1}{2}m^2\phi^2$$

Kinetic energy Shear energy Mass energy

Potential Energy

Euler-Lagrange Equation: $(\partial^2 + m^2)\phi = 0$

Quantum Mechanical identification: $\partial_\mu = ip_\mu$

Einstein's relation:

$$-p^2 + m^2 = -E^2 + |\vec{p}|^2 + m^2 = 0$$

Example: Fermion (spin 1/2) theory

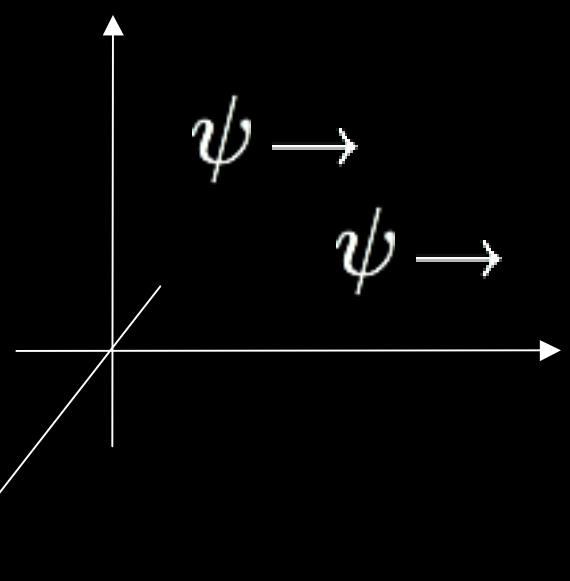
$$\mathcal{L} = i\bar{\psi}\sigma^{\mu}\partial_{\mu}\psi - m\bar{\psi}\psi$$

Aligns spins

Displaces fermions

Mass

Spins anti-aligned



$$\mathcal{L} = \frac{1}{2}\partial_{\mu}\phi\partial^{\mu}\phi - \frac{1}{2}m^2\phi^2$$

"$\psi^2 = \phi$"

Supersymmetry

Familiar Quantum Operators

- Example: Momentum

$$P_\mu|\text{boson}\rangle = p_\mu|\text{boson}\rangle$$

$$P_\mu|\text{fermion}\rangle = p_\mu|\text{fermion}\rangle$$

$$P_\mu \xrightarrow{2\pi \text{ rotation}} P_\mu$$

- Momentum is a vector (a boson)
- Momentum is Hermitian
- Momentum satisfies commutation relations

Unfamiliar Quantum Operators

$$Q|\text{fermion}\rangle = |\text{boson}\rangle$$

$$Q|\text{boson}\rangle = |\text{fermion}\rangle$$

$$Q \xrightarrow{2\pi \text{ rotation}} \mathcal{O}$$

- Q is a fermion! (spin 1/2)
- Q is not Hermitian!
  - Eigenstates of Q do not have well-defined spin

Unfamiliar Quantum Operators

- Q satisfies *anti-commutation* relations:

$$\{Q, Q^\dagger\} = 4E \quad \left(E = P_0 = -i\frac{\partial}{\partial t}\right)$$

$$\{Q, Q\} = 0$$

$$\{Q^\dagger, Q^\dagger\} = 0$$

$$Q^\dagger = \partial \neq Q$$

- Look familiar?

Unfamiliar Quantum Operators

- Rescale Q:

$$a = \frac{Q}{\sqrt{4E}} \qquad a^\dagger = \frac{Q^\dagger}{\sqrt{4E}}$$

$$\{a, a^\dagger\} = 1$$

$$\{a, a\} = 0$$

$$\{a^\dagger, a^\dagger\} = 0$$

- Spin 1/2 or two-state raising and lowering operators!

Summary

- Q satisfies the algebra:

$$\{Q, Q^\dagger\} = 4E$$
$$\{Q, Q\} = 0$$
$$\{Q^\dagger, Q^\dagger\} = 0$$

- Q interpolates between two states:

$$Q^\dagger|f\rangle = |b\rangle$$
$$Q|b\rangle = |f\rangle$$

- Q called the *supercharge*
- f and b are *superpartners* and form a *supermultiplet*

Simplest Supersymmetric Model

- Massless, noninteracting Wess-Zumino model:

$$\mathcal{L}_{\text{W-Z}} = \partial_\mu \phi \partial^\mu \phi^* + i\bar{\psi}\sigma^\mu \partial_\mu \psi$$

- ϕ is a complex scalar field:

$$\phi = \phi_1 + i\phi_2$$

- ψ is a spin 1/2 fermion field:

$$\psi = \begin{pmatrix} a \\ b \end{pmatrix}$$

Wess-Zumino Model

- Transformations of fields under supersymmetry:

$$Q\phi = \psi$$

$$Q^\dagger \psi = i\sigma^\mu \partial_\mu \phi$$

- Leave the Wess-Zumino Lagrangian invariant:

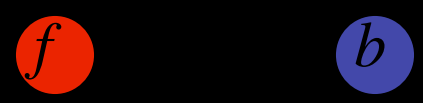
$$\mathcal{L}_{\text{W-Z}} = \partial_\mu \phi \partial^\mu \phi^* + i\bar{\psi}\sigma^\mu \partial_\mu \psi$$

- Wess-Zumino Lagrangian is *supersymmetric*!

What does Supersymmetry mean?

- Consider two electrons; one at rest, one with velocity v:

- Lorentz transformations can change velocities
  - Doesn't change laws of physics
  - Consequence: velocity is not a quantum number
- Consider a fermion and boson:



- Supersymmetry doesn't change laws of physics
  - Does change spin!
  - Consequence: spin is not a quantum number!

What does Supersymmetry mean? Summary

- Maps boson degrees of freedom to fermion degrees of freedom:

$$\text{Bosons} \overset{Q,\,Q^\dagger}{\longleftrightarrow} \text{Fermions}$$

- Theory must have equal numbers of fermion and boson d.o.f.s
- Boson and fermion superpartners must have same interactions
  - Same mass, same charges
- Moral:

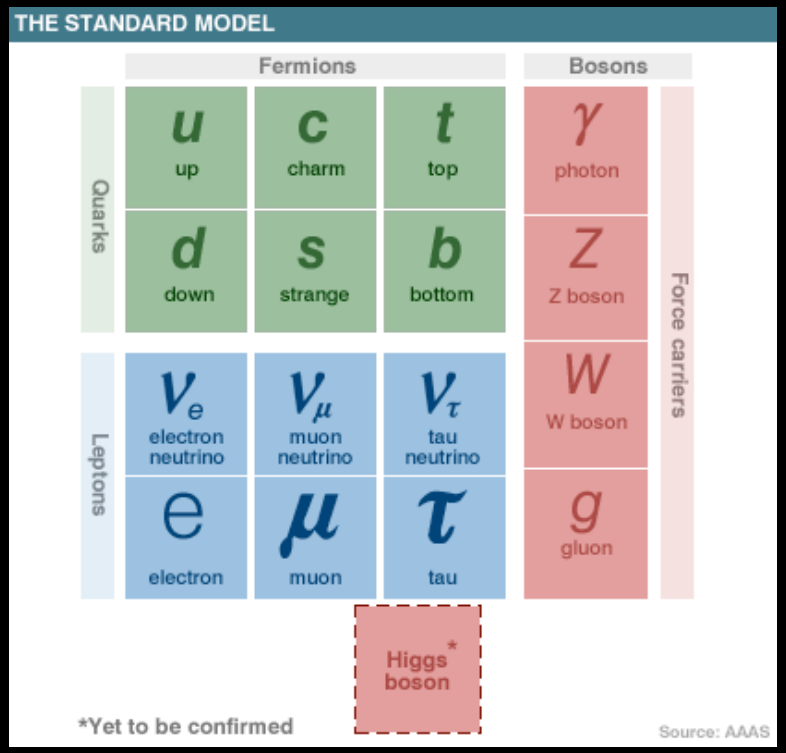
$$\text{Supersymmetry:}$$

$$\text{Bosons} \simeq \text{Fermions}$$

The Minimal Supersymmetric Standard Model

Minimal Supersymmetric Standard Model

- "Minimal Supersymmetric": The absolute minimum number of additional particles to make the Standard Model supersymmetric
- Can particles in the Standard Model be superpartners?



Minimal Supersymmetric Standard Model

- "Minimal Supersymmetric": The absolute minimum number of additional particles to make the Standard Model supersymmetric
- Can particles in the Standard Model be superpartners?
  - No! No particles have same charges or mass and spin differing by 1/2
  - Need to double particle content!

Minimal Supersymmetric Standard Model

- Superpartners in the MSSM:
  - Scalar partner to a fermion = sfermion
    - Partner to electron = selectron
    - Partner to top quark = stop
  - Fermion partner to a boson = bosino
    - Partner to photon = photino
    - Partner to gluon = gluino

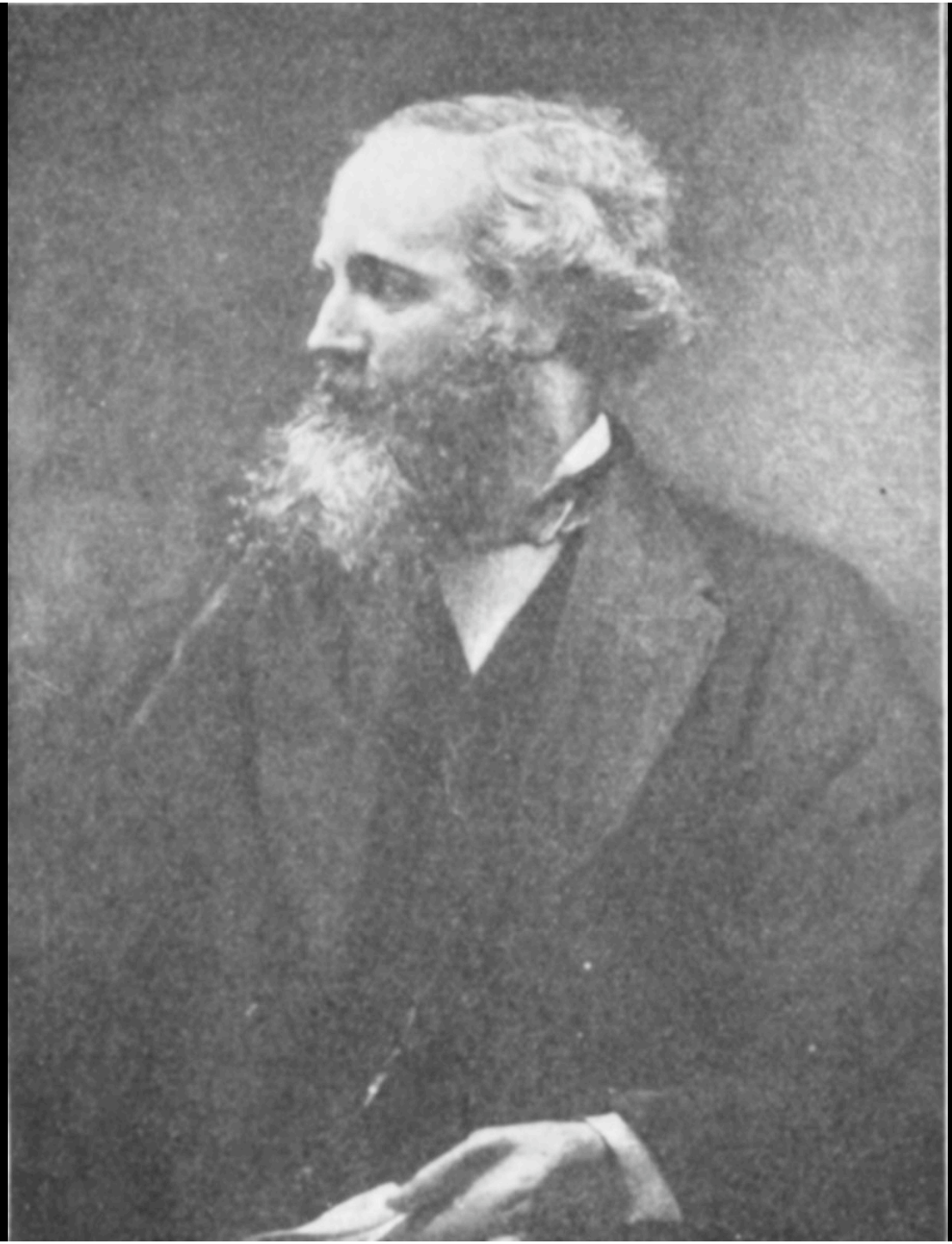
- I make no apology for the nomenclature; it is terribly silly

Minimal Supersymmetric Standard Model

- A (major) problem
  - Supersymmetry demands superpartners have same mass
    - We don't observe a negatively charged, scalar with mass of 511 keV (the selectron)
  - Supersymmetry cannot be an exact symmetry of our world!
  - Sparticle masses must be > 100 GeV to escape detection
  - Many models predict masses in LHC range
    - Very exciting! (pending LHC problems)

Motivations for Supersymmetry in the Real World

Unification of Coupling Constants



Unification of Electricity and Magnetism

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} + \frac{\partial}{\partial t}\mathbf{B} = 0$$

$$\nabla \times \mathbf{B} - \mu_0 \epsilon_0 \frac{\partial}{\partial t}\mathbf{E} = \mu_0 \mathbf{J}$$

Unification of Electricity and Magnetism

- Maxwell's equations unify *E* and *B* fields
- Relates electric and magnetic couplings:

$$\boxed{\mu_0 \epsilon_0 = \frac{1}{c^2}}$$

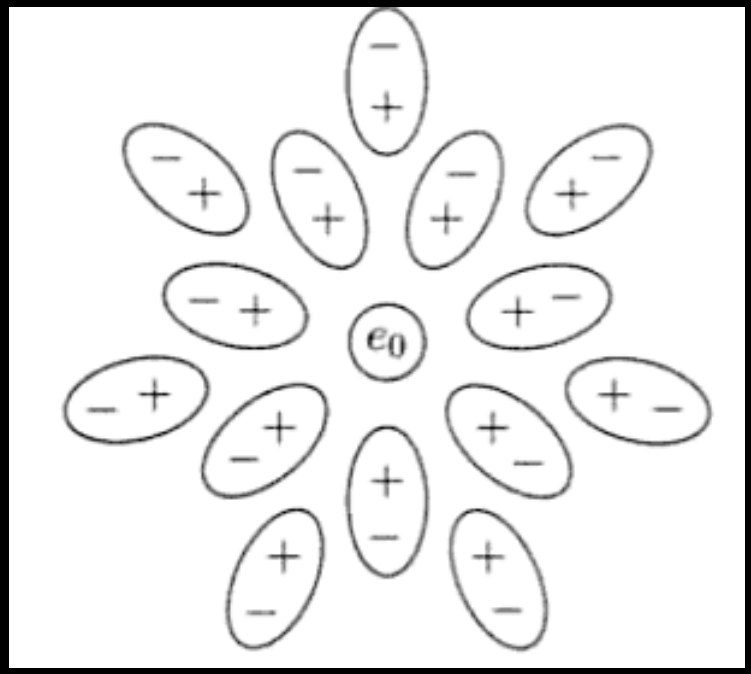
- Conversely, if couplings can be related, fields can be unified!
  - (At least a very strong indication of unification)

Unification in the Standard Model

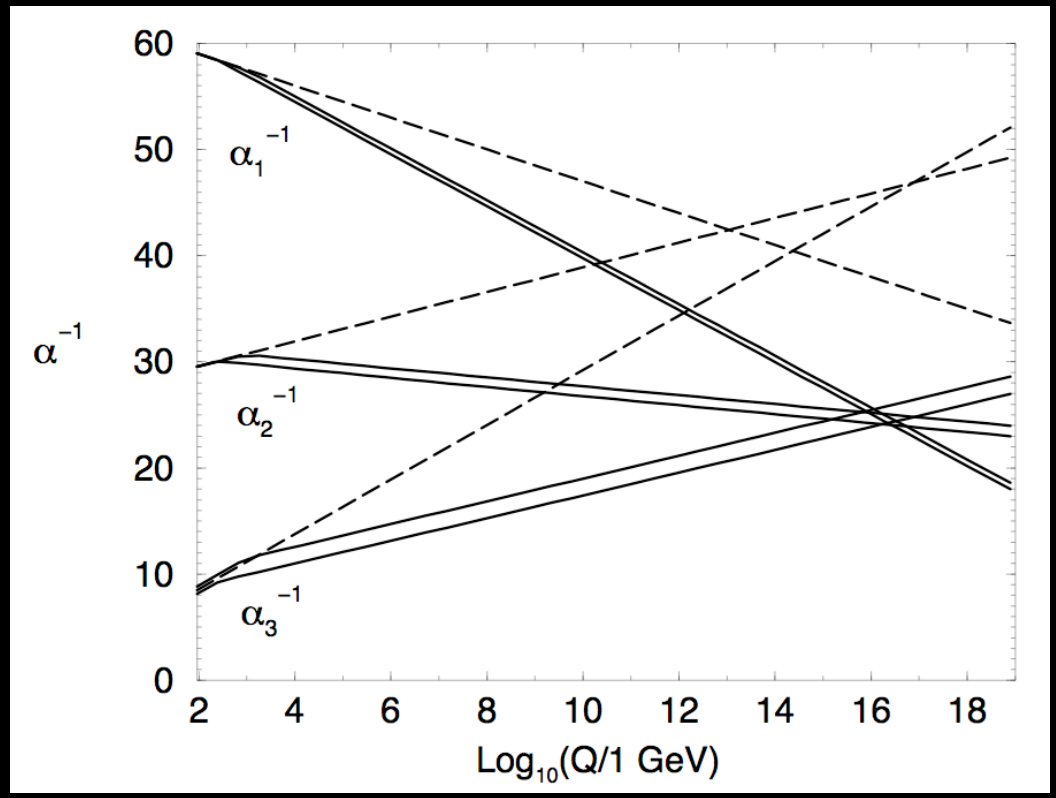
- Three forces in Standard Model
  - Electromagnetism
  - Weak
  - Strong
- Three couplings in Standard Model relating strengths of forces
- If these couplings can be related, forces could be unified!
  - Could “explain” forces and their strengths

Unification in the Standard Model

- How to relate them?
  - Couplings depend on distance (energy)!
  - An electron polarizes the region of space around it
  - Dipoles screen bare electron charge
  - Closer to electron, charge looks larger!



Unification in the Standard Model



- Standard Model: dashed
  - No unification
- Standard Model + Supersymmetry: solid
  - Unification!

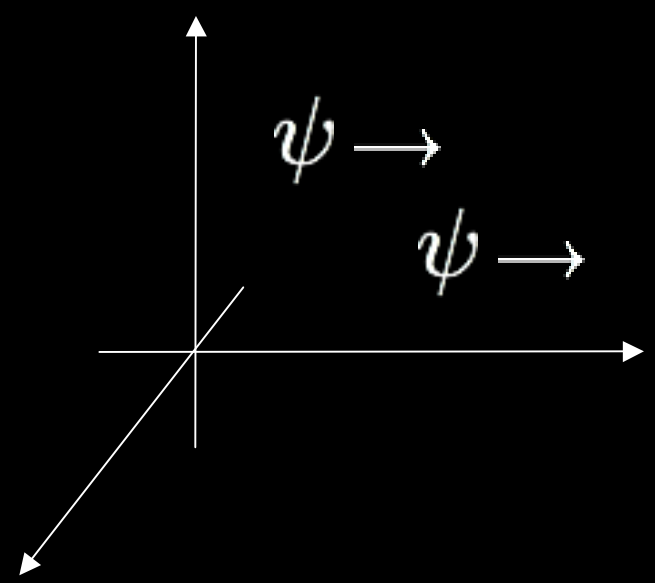
Hierarchy Problem

A Problem with Fundamental Scalars

- Or, why a massless fermion stays massless

- Recall massless fermion Lagrangian:

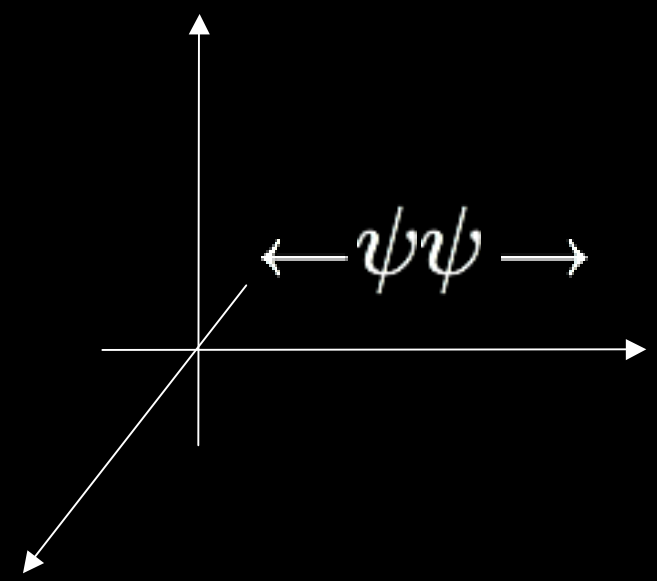
$$\mathcal{L} = i\bar{\psi}\sigma^{\mu}\partial_{\mu}\psi$$



A Problem with Fundamental Scalars

- Mass term requires anti-aligned spins:

$$\mathcal{L}_{\text{mass}} = -m\bar{\psi}\psi$$



- Quantum Mechanics respects Lorentz invariance
  - Velocity and spin are locked in place
  - Such a term can never be generated!
  - Massless fermion stays massless!

A Problem with Fundamental Scalars

- Consider a massless, fundamental scalar:

$$s \xrightarrow{v = c} \xrightarrow{\text{Lorentz Transformation}} s \xrightarrow{v = c}$$

- Scalar has no intrinsic direction or vector
- Mass term of Lagrangian is not disallowed by any symmetries:

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2$$

- Anything that is not forbidden will happen in QM
  - Mass term is generated quantum mechanically!
- (Composite scalars do not suffer this problem as they are made of fermions and inherit chiral symmetry)

Standard Model Higgs Scalar Boson

- In the Standard Model, Higgs boson is responsible for mass
  - All particles have mass proportional to Higgs mass
- How does quantum mechanics affect Higgs mass?
  - Interactions with bosons: increases mass
  - Interactions with fermions: decreases mass
- In Standard Model, these contributions are unrelated
  - Quantum mechanic shifts can be (essentially) unbounded!
  - Obviously a bad thing
- Requiring supersymmetry:
  - Fermions and bosons have same interactions with Higgs
  - Contributions to Higgs mass exactly cancel!
  - A good thing

Standard Model Higgs mass

- Resolves the hierarchy problem:
  Supersymmetry resolves the hierarchy problem by demanding that fermion and boson contributions to the higgs mass exactly cancel.

Things I Didn't Discuss

- Other solutions to the Hierarchy problem:
  - Extra Dimensions
    - Randall-Sundrum Models, etc.
  - Composite Higgs models
    - Technicolor, etc.
  - None, there is no Hierarchy problem
    - Split Supersymmetry, etc.

Things I Didn't Discuss

- Other connections of Supersymmetry
  - Adding more supersymmetries
    - Have only considered $N = 1$
    - Can add more supercharges to consider larger supermultiplets
    - Conformal theories; AdS/CFT
  - Gravity + Supersymmetry = Supergravity
    - Could be a way to control the divergences that trouble a quantum theory of gravity
  - String Theory
    - Requires supersymmetry
    - Currently the best quantum theory of gravity

An Explanation of Dark Matter

An Explanation of Dark Matter

- I won't motivate the existence of dark matter here
- Necessary to explain the rotation curves of galaxies
- Necessary to explain the distribution of mass and the gravitational potential well in the Bullet Cluster
- There are many reasons that Standard Model particles could not be dark matter
    - Dark matter is dark, must be neutral and stable
    - Could possibly be neutrinos but are much to light to remain contained in galaxies

An Explanation of Dark Matter

- Supersymmetry “naturally” includes heavy, neutral Weakly interacting particles!
  - Called WIMPs
  - Constraints on cosmological parameters demand its mass be in the 100 GeV-1 TeV range