SASS Seminar 2016.11.02

Extra Dimensions and the Hierarchy Problem

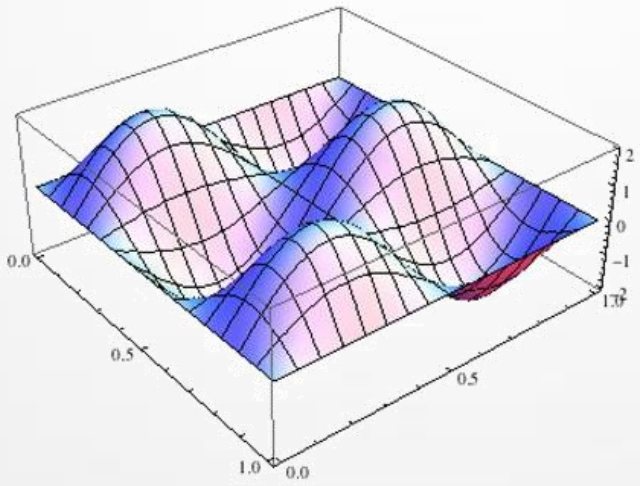
Jong Min Yoon
SLAC / Stanford University

Prelude : 2D Infinite Square Well

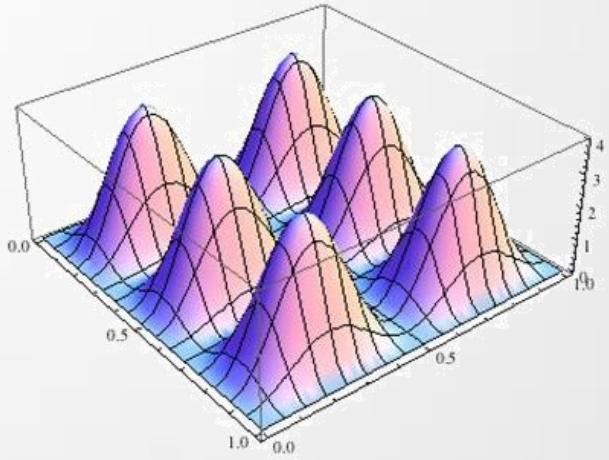
$$\psi(x,y) = \frac{2}{\sqrt{L_x L_y}} \sin\left(\frac{\pi n_x x}{L_x}\right) \sin\left(\frac{\pi n_y y}{L_y}\right)$$

$$E = \frac{\hbar^3 \pi^3}{2m}\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right)$$

$\psi_{nx=3,ny=2}(x,y)$



$|\psi_{nx=3,ny=2}|^2$



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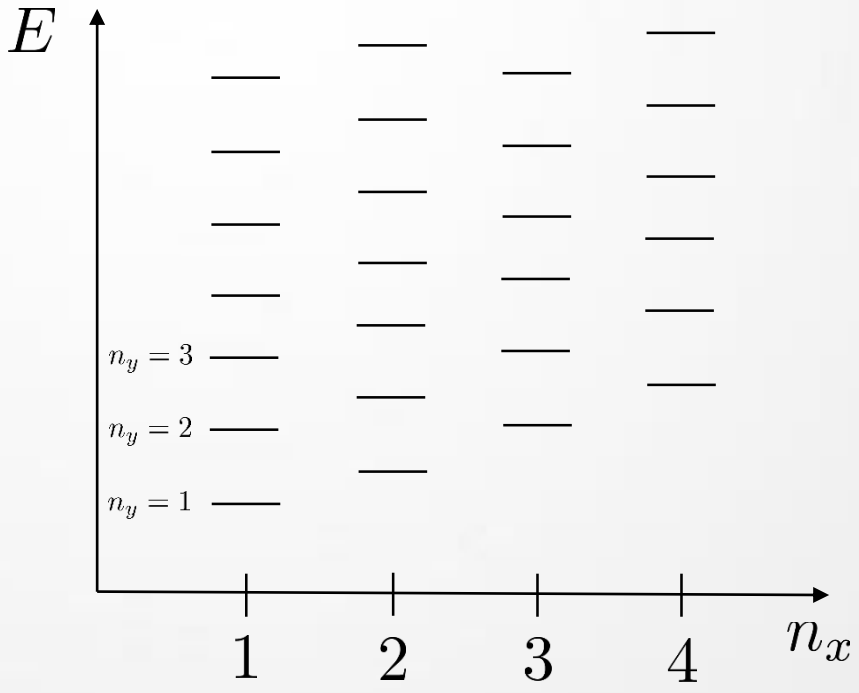

"Tower" of states

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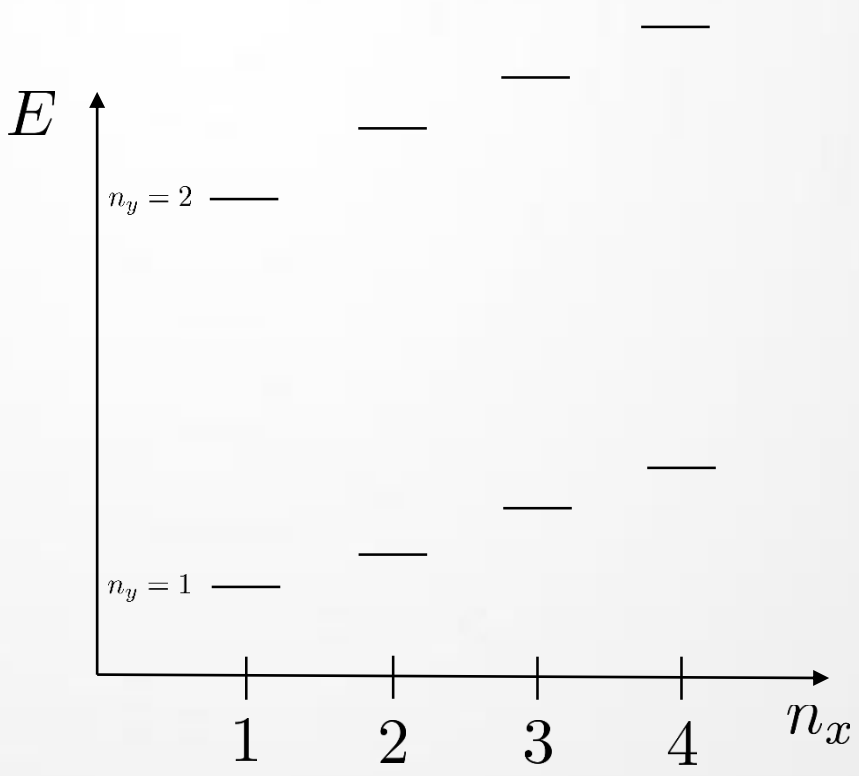

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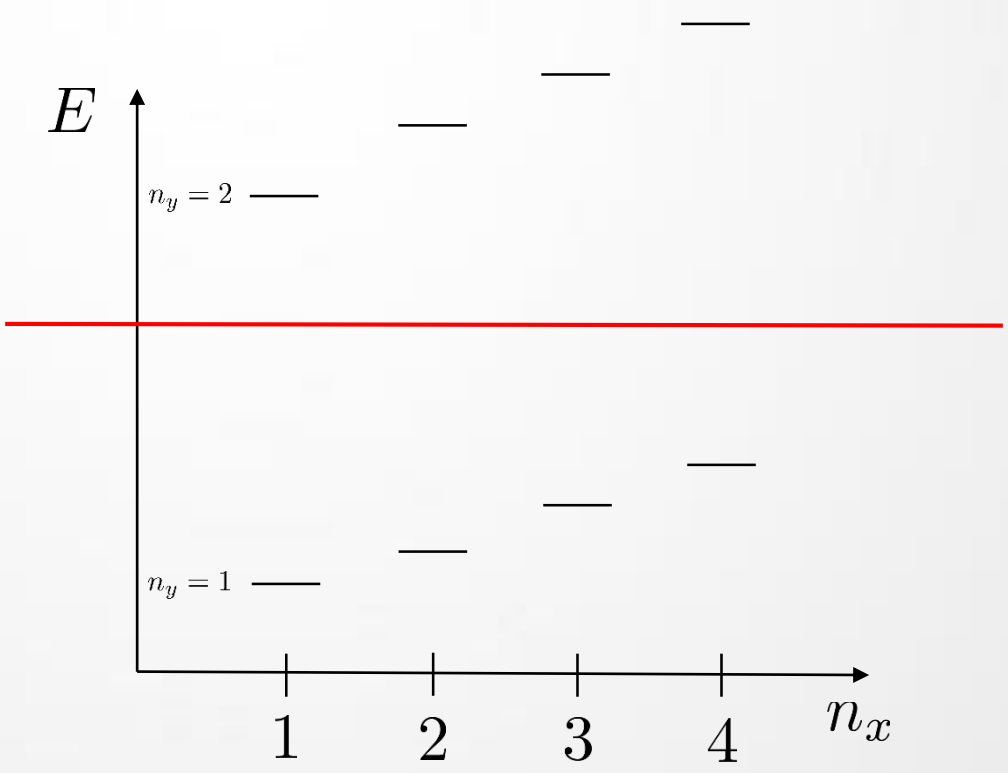


Extra Dimensions

- First proposal : Kaluza and Klein in 1920's
  - An attempt to unify EM and Einstein gravity
  - Kaluza-Klein states : the "tower" of states
- Most notably used in string theory for a consistent quantum gravity
  - Compactified extra dimensions
    with size Planck scale 10^{19} GeV or 10^{-35} m
- In late 1990s, a different approach was introduced
  - Mainly focused on solving the Hierarchy Problem
  - Most notably, ADD and RS
- AdS/CFT correspondence brought an alternative interpretation
  - Dual to the strongly interacting 4D theories

Kaluza-Klein Reduction

- Consider that the 5th dimension y is compact with the topology of a circle with radius R
- Consider a ‘massless’ scalar theory

$$S_5 = \int d^4x dy \; \partial^M \phi^\dagger \partial_M \phi \qquad M = 0, 1, 2, 3, 5$$

- Since the 5th dimension is a circle, we can expand the scalar field in a Fourier series

$$\phi(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=-\infty}^{\infty} e^{iny/R} \phi^{(n)}(x)$$

Kaluza-Klein Reduction

- Inserting the Fourier expansion in the Action and integrate out the y coordinate, we get

$$S_5 = \int d^4x \left|\partial_\mu \phi^{(0)}\right|^2 + \sum_{n\neq 0}\left[\left|\partial_\mu \phi^{(n)}\right|^2 - \left(\frac{n}{R}\right)^2 \left|\phi^{(n)}\right|^2\right]$$

- One massless mode,
  and “tower” of massive modes with mass n/R
- Below the energy ~1/R , i.e. in low energy effective theory, we can neglect the KK modes
  - So we don’t ‘see’ the extra dimension in lower energies

Hierarchy Problem

- Theoretical physicists expect that the fundamental dimensionful scales should be ‘naturally’ of a similar order, if there is no reason for them to be different.
  - e.g. symmetry protects one parameter much smaller than others
- Gravity is much weaker than the other forces
  - Gravity 10^{-24} weaker than the weak force
- Planck scale 10^{19} GeV vs Higgs boson mass 125 GeV
  - Quantum field theory expects quantum corrections of the mass of Higgs boson to be of order Planck scale
  - somehow, different quantum corrections cancel each other – with the first 17 digits exactly same
  - fine-tuning problems, problem of naturalness

ADD Model

- Arkani-Hamed, Dimopoulos, Dvali in 1998
  - “The Hierarchy problem and new dimensions at a millimeter”
  - Current scitation : 5867
- Assume a $D = 4 + \delta$ dimensional spacetime, with δ compactified spatial dimensions
- Gravity can propagate in the extra dimensions, but the other forces are localized in 4D subspace
- Then the ‘real’ Planck scale in D dimension can be much smaller than the ‘effective’ Planck scale in 4d

ADD Model

- With three spatial dimensions,

$$F = -\frac{G_N M m}{r^2} = -\frac{\hbar c}{M_4^2}\frac{M m}{r^2}$$

- With δ extra dimensions, each with size R,

$$F = -\frac{\hbar c}{M_D^{2+\delta}}\frac{M m}{r^{2+\delta}} \quad \text{for } r < R$$

$$F = -\frac{\hbar c}{M_D^{2+\delta}}\frac{M m}{r^2 R^{\delta}} \quad \text{for } r > R$$

$$\Rightarrow \quad M_{4,\text{eff}}^2 = M_D^{2+\delta} R^{\delta}$$

- Setting M_D around the electroweak scale TeV,

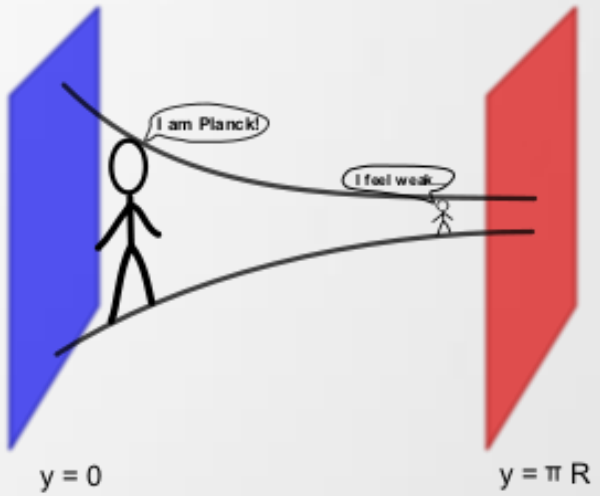
$$\delta = 1, 2, \ldots, 6 \;\rightarrow\; R \sim 10^9 \text{ km}\,, 0.5 \text{ mm}\,, \ldots\,, 0.1 \text{ MeV}^{-1}$$

ADD Model ; current constraints

- Indeed, no one tested gravity at mm scale at that time!
- Direct measurements of gravity at sub-mm distances
  - R < 37 μm for δ=2, corresponding to $M_D > 3.6\,\mathrm{TeV}$
- Astrophysical and Cosmological constraints
  - Very energetic settings can produce KK gravitons
  - From supernova SN1987A, $M_D > 27\ (2.4)\ \mathrm{TeV}\ \text{for}\ \delta = 2\ (3)$
  - Other constraints from neutron star reheating, relic KK graviton, collider signals, etc

RS Model

- Randall and Sundrum in 1999
  - “A Large mass hierarchy from a small extra dimension”
  - Current scitation : 7290
- Assume a 4+1 dimensional spacetime, where the fifth dimension is an ‘warped’ interval $y \in [0, \pi R]$
  - y = 0 : UV boundary
  - y = πR : IR boundary
- The original RS model assume all Standard model particles reside in the IR boundary



RS Model

- The metric of the system

$$ds^2 = e^{-2ky} dx^\mu dx_\mu - dy^2 \qquad y \in [0, \pi R]$$

  - The length ‘1’ in y is equivalent to the length $e^{k\pi y}$ in x
  - 4D Energy scales M_{Pl} at y = 0 → $M_{Pl} e^{-k\pi R}$ at y = πR
  - Note $e^{-33} \sim 10^{-16}$
- Therefore, if we confine the higgs field in the IR brane, the difference between the Planck scale and the weak scale is naturally explained with a number of order ~30
- Gravity in the bulk – KK gravitons
  - Current experimental limit ~ 2.7 TeV

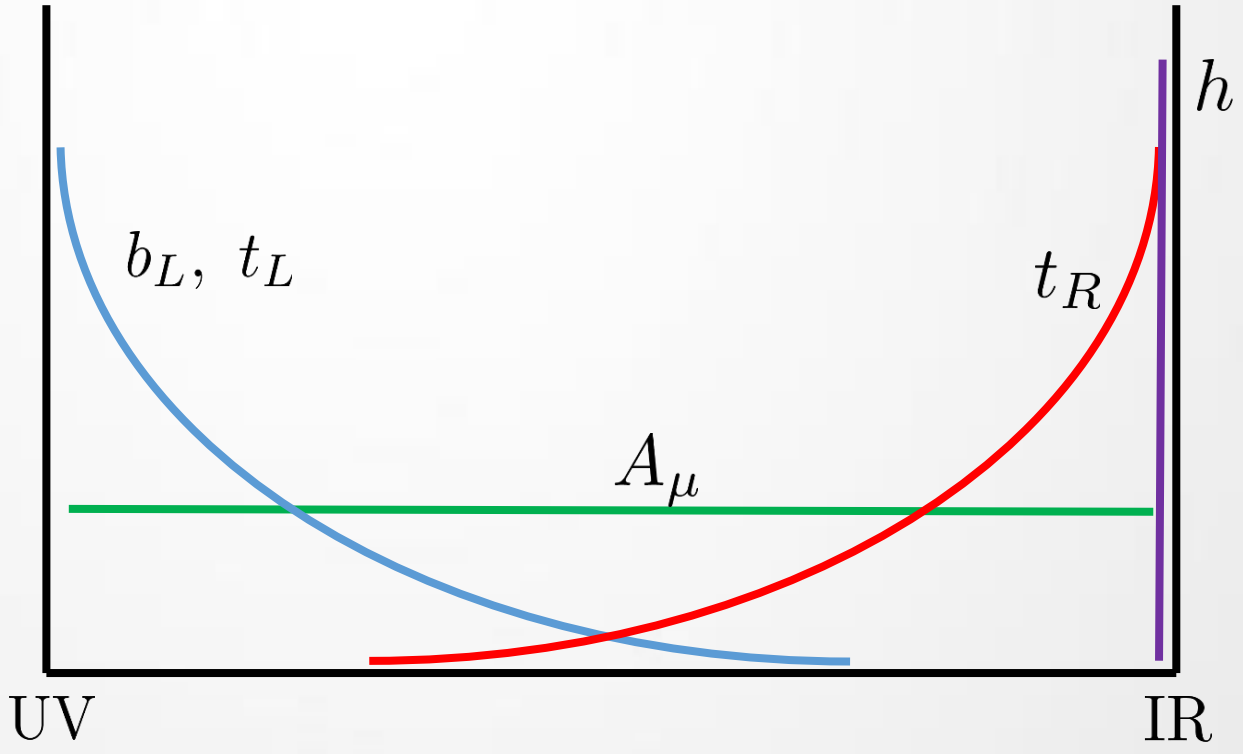
Extension of RS model: Fields in the 5D bulk

- The original RS model assumes that all Standard model particles reside in the IR boundary
- To solve the Hierarchy problem, only the Higgs field has to be localized there
- Putting other fermion and gauge boson fields in the bulk will give us a massless mode, and a tower of KK modes for each field.
- We recognize the massless mode to be the Standard Model particles, and other KK states are so heavy that we have not detected them yet.
  - Current limit on KK gluon – 2.8 TeV, KK fermion – 900 GeV

Fields in the 5D bulk: Models of Flavor Hierarchy

- At this stage, the standard model particles are massless modes of the 5D bulk fields.
- They obtain the mass (smaller than KK states) from the Higgs field vacuum expectation value
- Their shape in the y direction determines how much they overlap with the Higgs field → their mass
- The y-profile is dependent exponentially on a parameter in 5D Lagrangian
  - Natural explanation of the Flavor Hierarchy,
    i.e. mass of top quark ~ 10^4 times mass of up quark

Fields in the 5D bulk: Models of Flavor Hierarchy



Holographic Interpretation of RS

- The AdS/CFT correspondence provides the connection between the warped extra-dimensional models and strongly coupled theories in 4D.
- There, we recognize
  - KK states and IR localized states – composite fields
  - UV localized states – elementary fields
- Model-building in the RS geometry as a dual theory to the composite higgs theories.

Summary

- Extra dimensions can bring a bold solution to various problems in physics
- Hierarchy problem
- ADD – large extra dimensions dilutes gravity
- RS – Warping the 5^{th} dimension changes the hierarchy to its log
- To those who still think extra dimension is too speculative
  - as a mathematical tool to study previously incalculable 4D theories
- We might detect KK states in LHC!

THANK YOU