THE COPERNICAN REVOLUTION

Shattering geocentric, anthrocentric worldviews since 1543.

SASS 2009-02-11

Aidan Randle-Conde

Overview

- Introduction – why talk about Copernicus?
- Summary – the short version of the Copernican revolution (for the lazy)
- Background – from ancient astronomy to 16th century Europe
- Copernicus to Newton
- After the revolution – what we've learned since Newton

Introduction

- Why am I giving a talk?
  - I work on BaBar studying particle physics.
  - We're in the middle of a very busy Collaboration Meeting.
  - I've not studied astrophysics or cosmology for 5 years.

Everything here is strictly extra-curricular!

- Why bother talking about Copernicus?
  - In the last organization meeting I agreed to give a talk.
  - One of the suggestions was "The History of Physics"
  - I'm pretty sure I was the person who suggested that (Oops!)

The Copernican revolution is one of the most important intellectual shifts in human history.

Summary

Today (nearly*) everyone knows the punchline to the Copernican revolution:

"The Earth goes around the sun"
(Or "We're not at the center of the universe.")

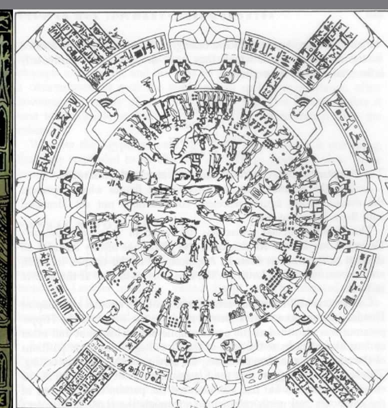
Anyone who doesn't want a nuanced understanding of how we reached this conclusion can now leave the room.

I'm more interested in taking an historical approach and seeing the revolution from the point of view of the scientists at the time.

*(72.6±1.3)% of Americans, according to the National Science Foundation.

Astronomy's origins

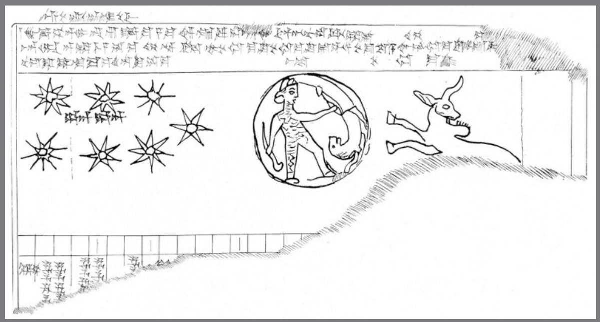
- Pretty much all the ancient civilizations recorded the passage of the sun, stars, planets and moon across the sky.

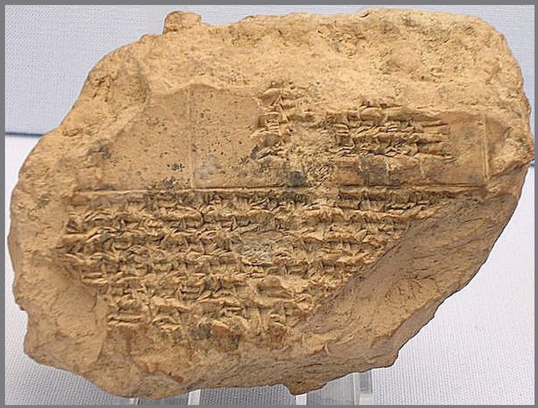




- The mediaeval European models of astronomy trace their roots back to the ancient Babylonians.
- The Babylonians observed visible heavenly bodies, believing them to influence fate. There wasn't much separation of astronomy and astrology.

Babylonian astronomy

- The Babylonians didn't seek to interpret the movements of the heavenly bodies, they just observed them.
- This is a good thing- precise astronomical observations can't be reconciled with their flat-Earth model!
- After about 2000 years they could make pretty good predictions of eclipses.



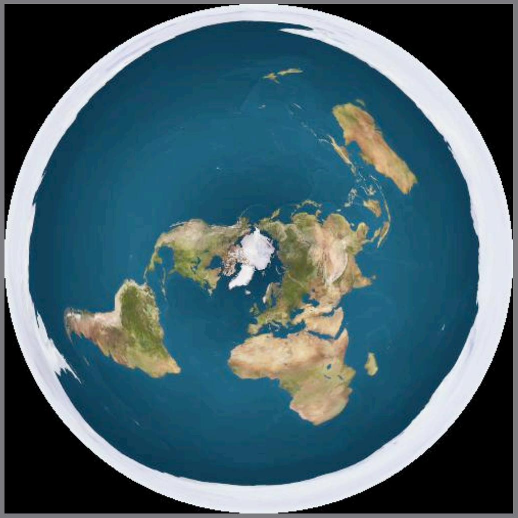


Babylonian tablet giving the dates of eclipses from 518 to 465 BC

- They discovered the Saros cycle. After 223 synodic months the cycle of solar and lunar eclipses repeats!

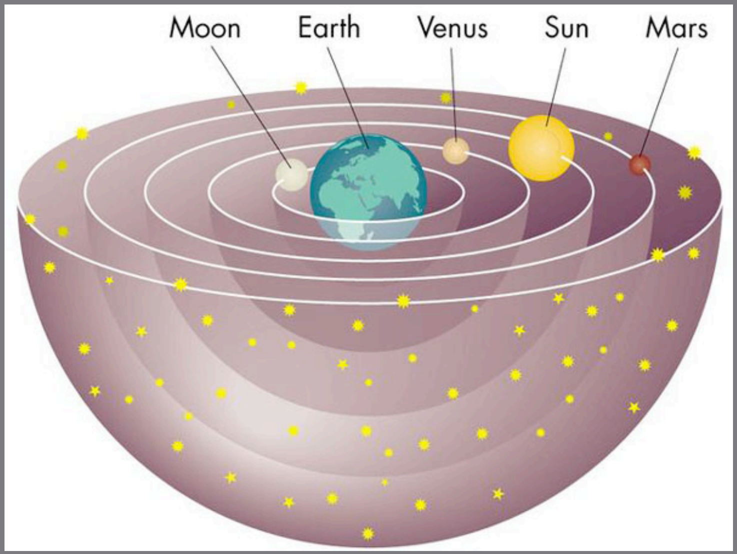
The Greeks and the end of the flat Earthism

- Initially the Greeks had some pretty strange ideas about the shape of the Earth. Many still believed the Earth was flat. Anaxiamander thought the Earth was cylindrical and we were on the top. Thales thought the Earth floated on water.
- Eventually it was realized that the Earth is round:
  - When viewing ships approaching over the horizon sailors saw the mast before the hull.
  - People at different latitudes saw constellations rise at different times.
  - Aristotle pointed out that only a spherical Earth could cast a circular shadow over the moon. (The only time I've known the guy to be right about anything to do with physics!)
- Without the round Earth hypothesis astronomy makes no sense!


Flat-Earthism didn't reemerge until the 20th century.

The Eudoxan model

- The Greeks weren't content with astronomical bookkeeping- their astronomy and mythology were intertwined and they wanted a geometric model of the universe.



- Eudoxus (~410-347 BC), a student of Plato came up with one of the first geocentric models of the universe. The Earth was surrounded by concentric spheres that held the moon, the sun, Venus, Mercury, Mars, Jupiter, Saturn and the stars in that order.

- Celestial spheres hold the visible bodies (Sun, Mercury, Venus, Moon, Mars, Jupiter and Saturn) in place. The stars had their own sphere on the outside. Eventually the spheres were thought to be real, tangible objects.

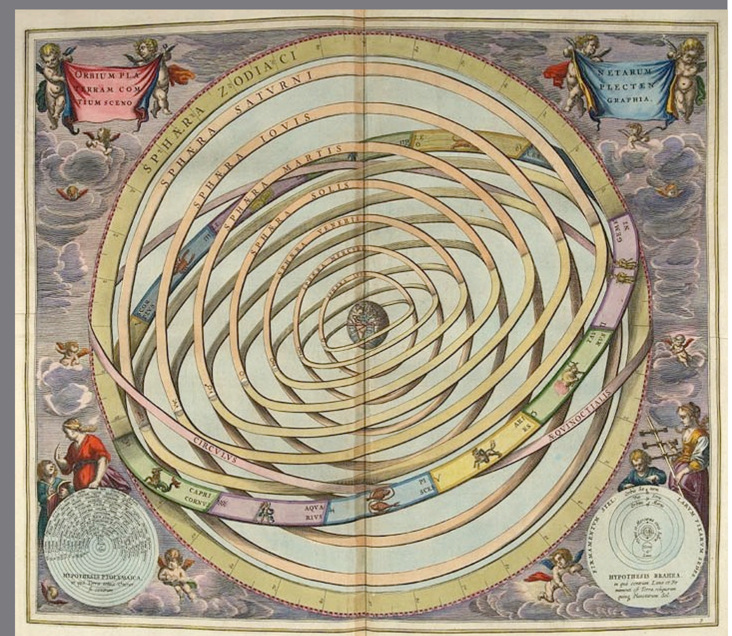
Failures of the Eudoxon model

- Eudoxus' model wasn't very good at making predictions, but it was aesthetically pleasing. (These kinds of considerations were important to the ancient Greeks.)
- The main problem was the paths of the planets:
  - While the sun, moon and backdrop of stars seemed to rotate at constant speeds, the planets changed speed and even direction as they crossed the sky. (The word planet literally means "wanderer" in Greek.)
  - People tried to add various spheres within spheres in the Eudoxon model, but nothing seemed to make the mode fit observations.
- Then in the 2nd century Ptolemy came up with a workable alternative which remained in place for the next 1200 years.

The Ptolemiac model

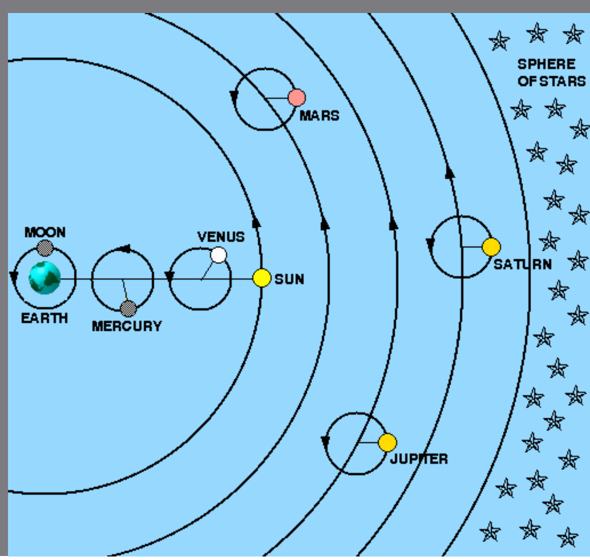
- Ptolemy outlined his model in his book Almagest:

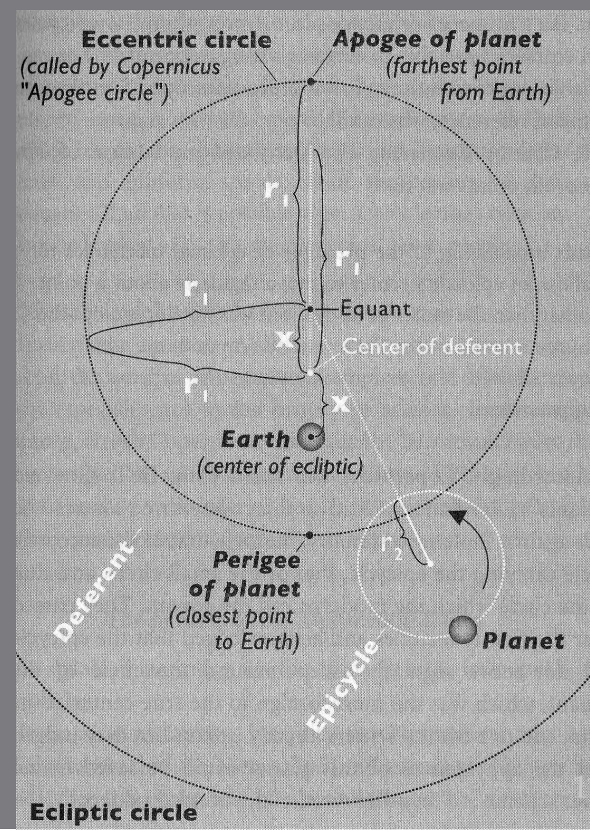
  - The planets moved around in circular epicycles, which in turn moved around circular deferents.
  - The deferents were not centered on the Earth, but a point halfway between the Earth and the "equant."
  - An observer at the equant would see a planet's epicycle move at constant speed.



The Ptolemiac model

- The center of the Earth is balanced by the equant.
- It is the epicycles, rather than the planets that orbit the center of the solar system.
- Eventually astronomers had to put epicycles with epicycles to get the model to work properly.





Add a little Greek philosophy

- In addition to the model Ptolemy also proposed a few aesthetic assumptions:
  - The planets, sun and moon were perfect spheres. (The Earth was allowed to be imperfect.)
  - The heavens were unchanging. Nothing new was created and nothing was destroyed.
  - Everything moves in circles. If something doesn't work then the model just needs more circles.
- In the coming centuries more and more epicycles were added to the model as the observations of the planets became more precise.
- Not much else happened until the 15^{th} century.

Peurbach and Müller

- Georg Peurbach (1423-1461) was a humanities professor in the University of Vienna. In 1454 he met a student, Johannes Müller (later Regiomontanus).
- The two worked together for many years. Peurbach was a particularly prolific astronomer. He:
  - built several astrolabes, sundials and other instruments.
  - kept astronomical yearbooks, charting the position of the sun, moon and planets throughout the year.
  - made several corrections to the Gerard of Cremona's interpretation of Ptolemy's model (A Theory of Planets) one of the most influential texts of the day.
  - wrote the Tables of Eclipses (over 100 pages of accurate predictions!)
- He died aged 38 before translating Ptolemy's Almagest.

Peurbach and Müller

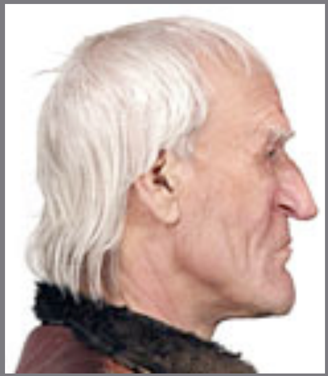
- Following Peurbach's death Müller carried on his work:
  - He learned Greek, making him one of the few people in Europe to master mathematics, astronomy, and instrument building while having access to ancient Greek knowledge.
  - He published Epitome to Almagest in 1496 adding many new observations. (Probably written in 1462.)
  - By 1470 he was convinced that the Ptolemaic model was seriously flawed and that a new model was needed.
  - In 1474 he published Ephemerides, which predicted the position of the celestial bodies every day from 1475 to 1507. (Columbus used it to scare the Jamaicans by predicting a lunar eclipse on 19th February 1504!)
  - He published On Every Kind of Triangle, which outlined the use of trigonometry in astronomy.

- He died (probably of plague) aged 40 before he had a chance to develop his own astronomical model.

Copernicus

- Nicolaus Copernicus (1473-1543) didn't have the makings of a great astronomer:
  - He started university aged 19 (most of his peers started at 14.)
  - He was an undergraduate for 12 years.
  - Unlike other astronomers at the time, he did all of his work from home, not at a university observatory.
  - He worked alone (for practical reasons, most astronomers worked in groups.)
  - Living in Poland there were many cloudy nights.
  - His instruments were not even close to state of the art.
  - He even miscalculated where he lived!

According to Archeological findings this is what Copernicus may have looked like at his death, aged 70.



Copernicus

NICOLAI COPERNICI

net, in quo terram cum orbe lunari tanquam epicyclo contineri diximus. Quinto loco Venus nono mense reducitur. Sextum denique locum Mercurius tenet, octuaginta dierum spacio circū currens. In medio uero omnium residet Sol. Quis enim in hoc

pulcherrimo templo lampadem hanc in alio uel meliori loco poneret, quàm unde totum simul possit illuminare? Siquidem non inepte quidam lucernam mundi, alij mentem, alij rectorem uocant. Trimegistus uisibilem Deum, Sophoclis Electra intuentē omnia. Ita profecto tanquam in solio regali Sol residens circum agentem gubernat Astrorum familiam. Tellus quoque minime fraudatur lunari ministerio, sed ut Aristoteles de animalibus ait, maximam Luna cum terra cognationē habet. Cōcipit interea à Sole terra, & impregnatur annno partu. Inuenimus igitur sub hac

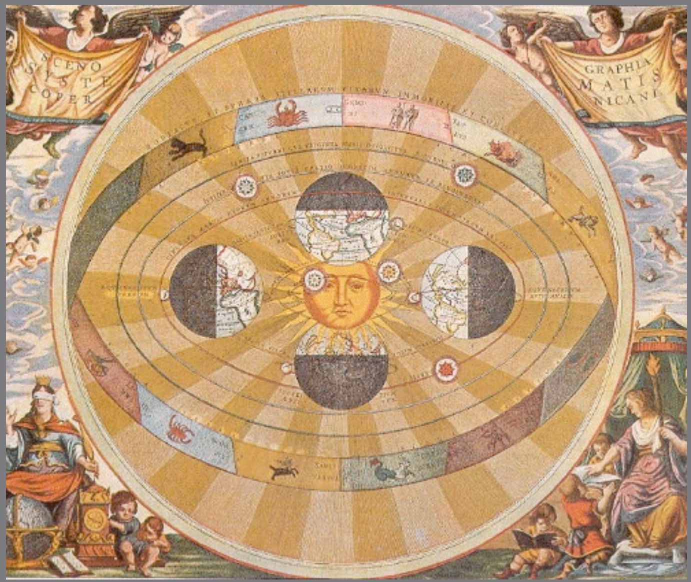
- In spite of these obstacles Copernicus made the leap to the heliocentric model in 1514. Armed with recent texts he handwrote an informal, anonymous untitled manuscript outlining his heliocentric model.
- In his manuscript he remarked

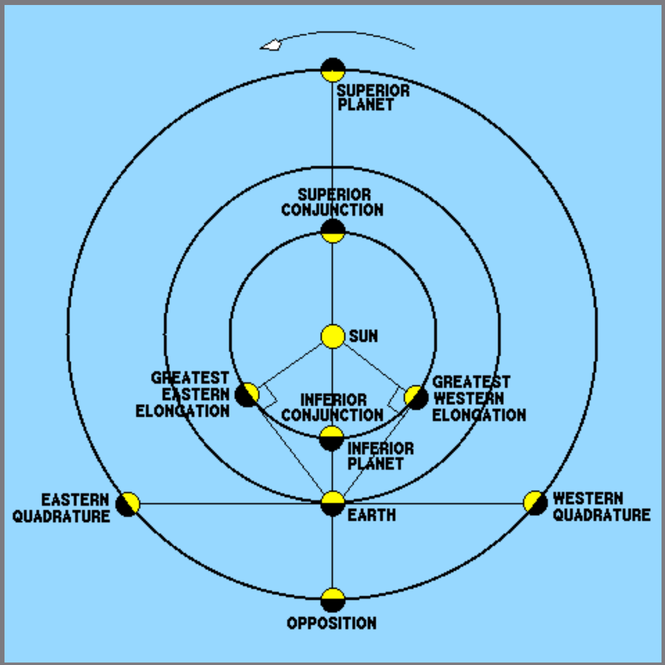
  "I have thought it well, for the sake of brevity, to omit from this sketch the mathematical demonstrations, reserving these for my larger works"

- Nearly 30 years passed before he published "On the Revolutions of the Heavenly Spheres". He died two months later.

The Copernican model

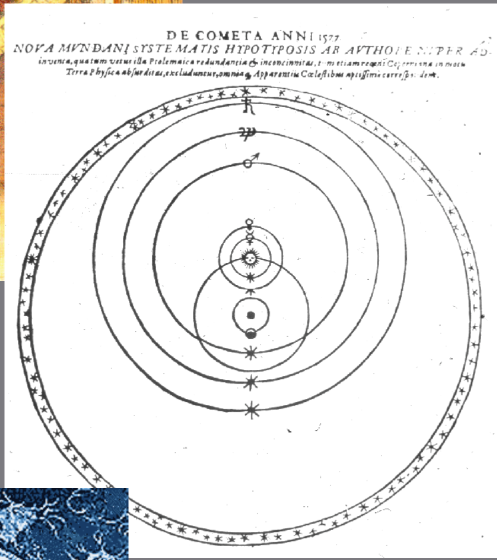
- In Copernicus' model the sun was center of the universe and all the planets revolved around the sun. The moon was the only object which (seemingly arbitrarily) orbited the Earth.
- As before, the stars rested on some external celestial sphere on the outside.





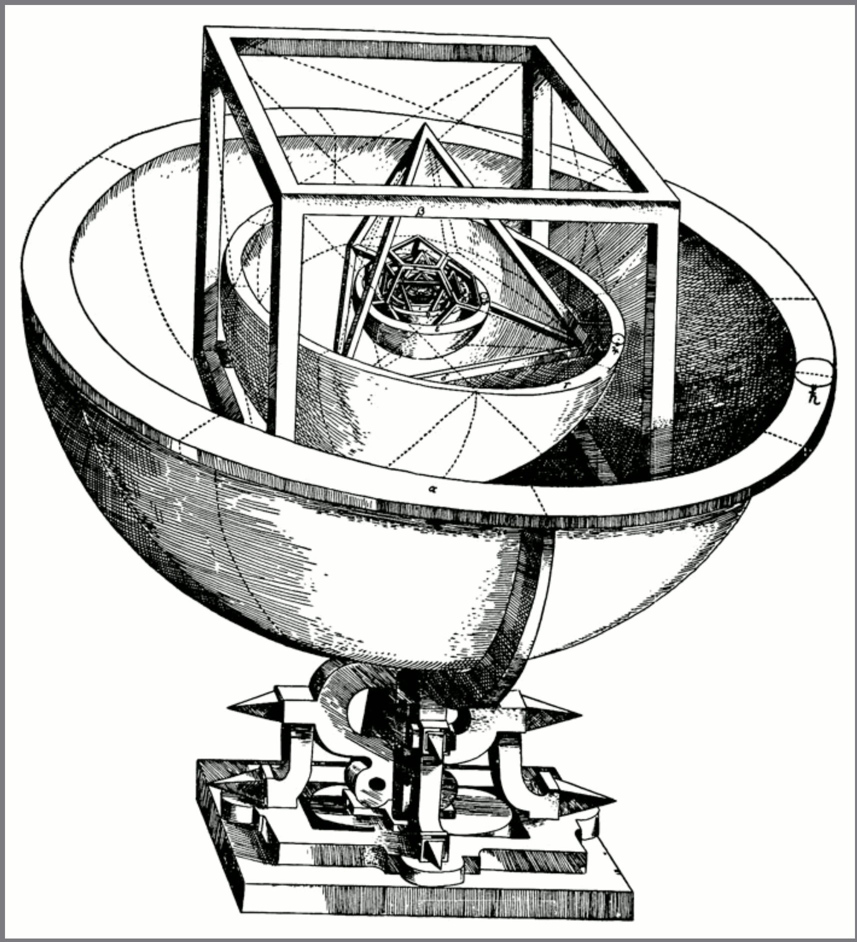
The "unchanging" heavens change

- Tycho Brahe (1546-1601) was one of the most influential astronomers of his time:
  - In 1572 he observed a supernova, showing that the heavens were not unchanging and contradicting Ptolemy!
  - In 1577 he observed a comet, demonstrating that there weren't really any celestial "shells".
  - The King of Denmark donated an observatory in Hven, a small island near Copenhagen. It was the best observatory in the world.
- Eventually he came up with a "geoheliocentric model" in which the planets orbited the sun which orbited the Earth.



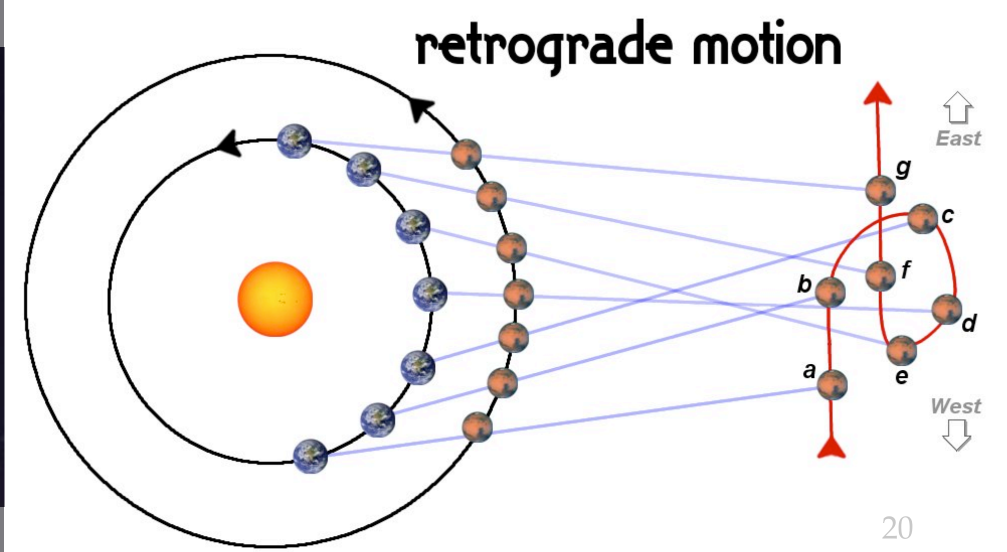
Kepler's first model

- Johannes Kepler (1571-1630) was a staunch heliocentrist. He looked at the data available to him and came to the conclusion that the ratios of the radii of the orbits can be "explained" by placing Platonic solids between the celestial shells.
- This model "explained" why there were only six visible heavenly bodies. (There are only five Platonic solids.)
- This was just a coincidence and once again the model put aesthetics above observation.
- But it got the attention of Brahe.



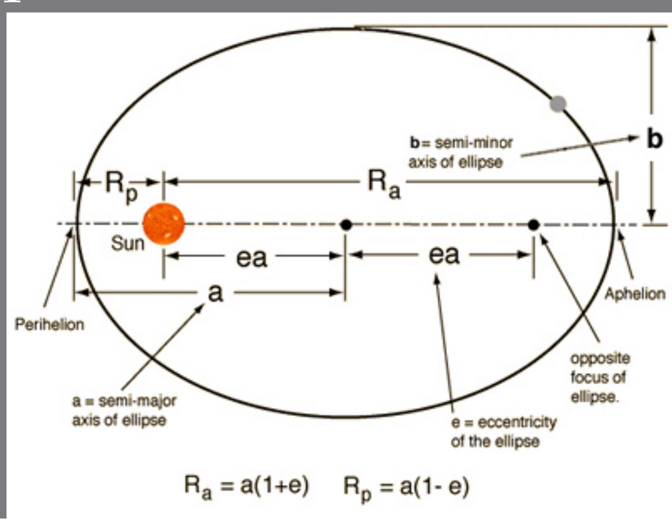
Retrograde motion of Mars

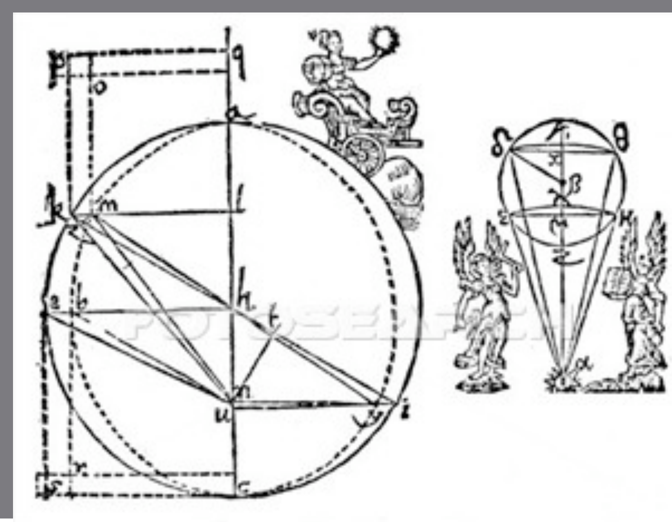
- Johannes Kepler worked with Brahe, observing the orbit of Mars.
  - He discovered that the retrograde motion of Mars was inconsistent with the Ptolemaic and Copernican models.
  - This was only noticed as a problem since the resolution of the measurements had improved.



Kepler's second model

- After Brahe's death Kepler took over the observatory and pursued a new heliocentric mode.
- The first step was to determine the Earth's orbit.
  - Kepler started off with a circular orbit with the sun just off-center.
  - This is a fair approximation (minor axis/major axis = 98%)
- After much struggling he realized that the orbit of Mars was elliptic!

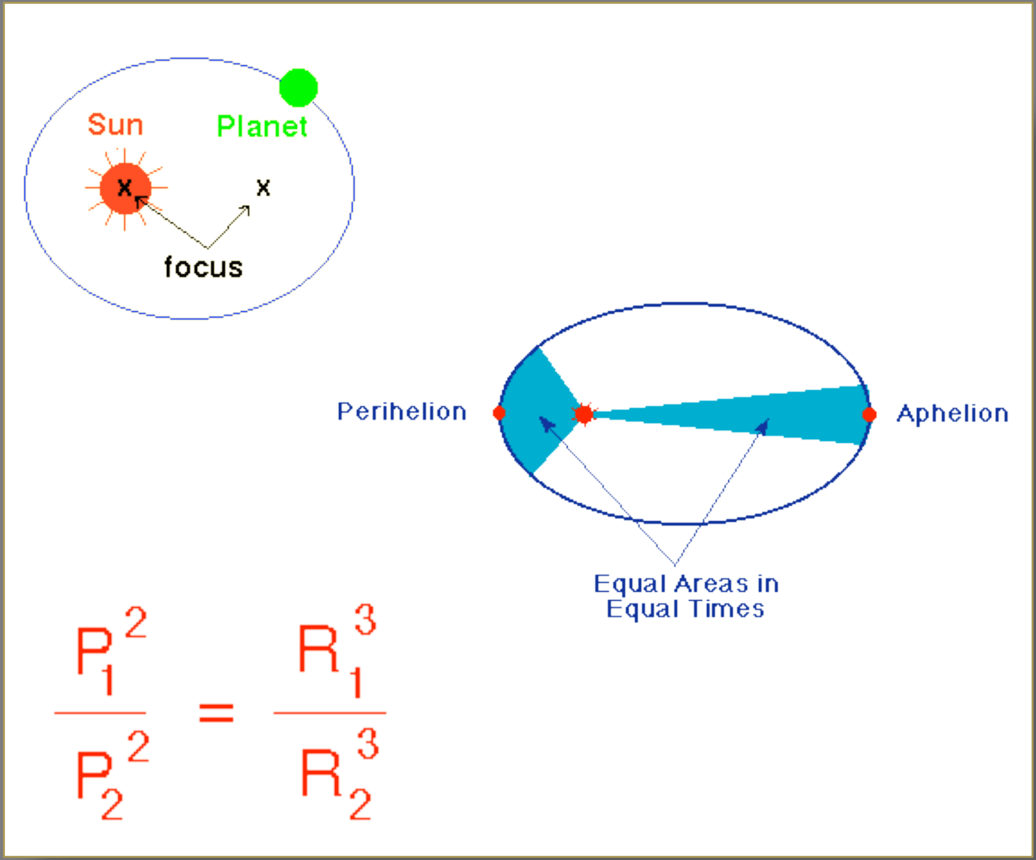




1157953 images.google.com

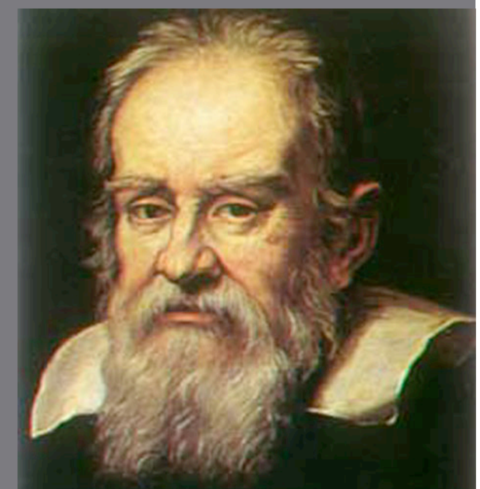
Kepler's three laws

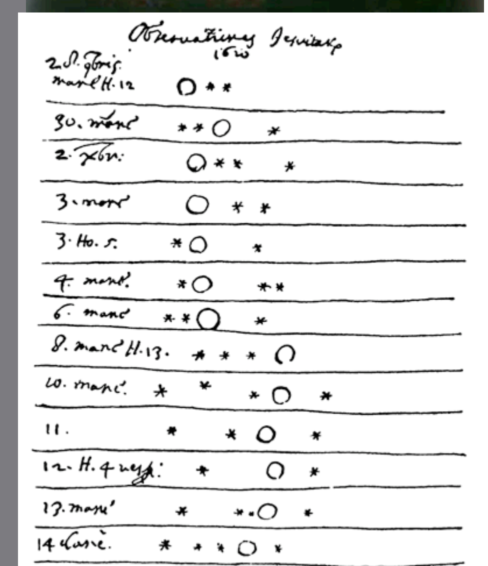
- After much computation Kepler came up with his three laws:
  - The planets orbit the sun in ellipses, with the sun at one focus.
  - Planets sweep out equal areas in equal time.
  - The ratios of the squares of orbits to the length of semi-major axes are the same for all planets.



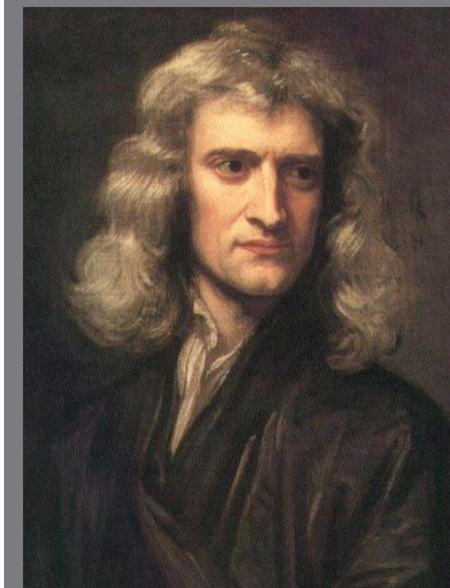
Galileo and his telescope

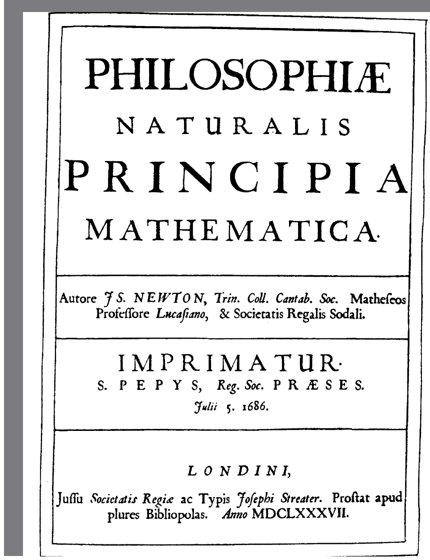
- Galileo (1564-1642) was one of the first scientists to use a telescope for astronomy. With it he made several important discoveries:
  - He found the "circumjovial planets" (the moons of Jupiter). The Earth's moon wasn't exceptional after all.
  - He showed the surface of the moon to be rough. He saw sunspots. The heavenly bodies aren't perfectly smooth and the sun rotates.
- His work on mechanics dispelled objections to a moving Earth.
- The harder people looked with the new telescopes the more bodies they discovered.





Newton's (almost) complete picture





PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore J S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

- Although Kepler's laws gave an accurate description, they provided no explanation for the obrits.
- Using his laws, Newton (1643-1727) derived the orbits:
  - The original derivation was much more thorough than what we're used to. (He used obscure properties of conic sections that were common knowledge to mathematicians at the time. Feynman attempted to follow this method, but even he couldn't manage!)
  - The laws were symmetric- the planets acted on the sun and the sun was no longer at the exact center of the solar system.
- Newton's laws also predicted paths which are hyperbolic and parabolic (the other two conic sections) which have since been seen.
- The model he provided was self-consistent, simple and matched observations of planets, moons, comets…
- The only major discrepancy was the perihelion of Mercury- the orbit shifted slightly over time.

Religious opposition

- The religious aspects of the Copernican Revolution alone could easily fill a talk. Here I'll just mention them in passing.
- Ptolemy's model was accepted as Church doctrine. Heliocentrism was nothing less than heresy:
  - Giordano Bruno was burned at the stake in 1600 for his heliocentric views.
  - Galileo was forced to recant and placed under house arrest.
  - Many others picked their words carefully to avoid a similar fate
- It was only around the time of Newton that it became safe (even respectable) to express such heretical views.

After Newton

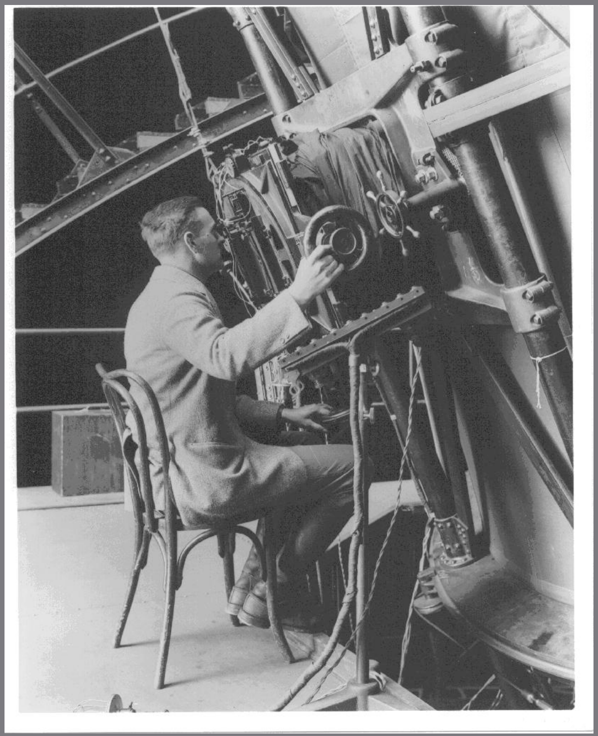
- With Newton's work published the Copernican Revolution came to a close. However there were still a number of important discoveries that continued in the same spirit:
  - 1781: Herschel discovers Uranus, the first planet not visible to the naked eye. The solar system is much bigger than previously thought.
  - 1838: Bessel measures parallax of the star 61 Cygni. The stars are at different distances from the Earth and the sun is just a close star.
- Newton's laws also raise new questions:
  - A universe infinite in age and extent isn't stable. What is the age and extent of the universe?
  - The solar system sits in an inertial frame. Are we at rest with respect to the universe?
  - Since the sun is no longer the center of the solar system there's no reason it should be the center of the universe. Where is the center?

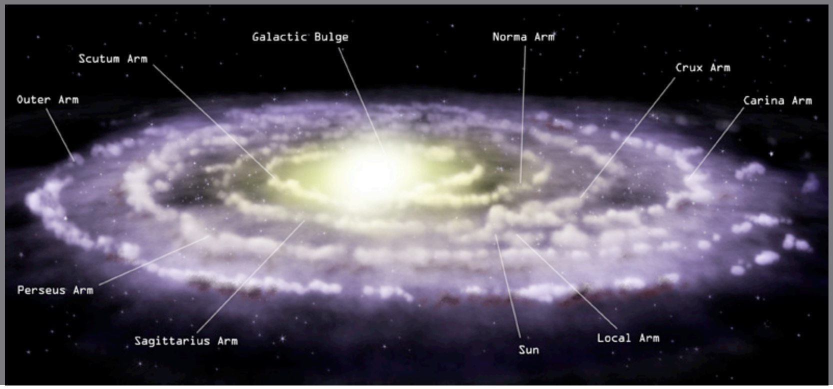
How big? How old?

- The development of general relativity in 1915 lead to many cosmological discoveries:
  - The solutions lead to a non-static universe.
  - In 1926 Hubble showed the universe is expanding.
  - Projecting the expansion back gives an age of ~14 billion years.
  - The universe has no well-defined "expiration date".
  - The edge of the visible universe is at least 46.5 billions light years away.
  - Human history accounts for one millionth of the age of the universe.
  - Our solar system extends to one part in 10^{14} of the visible universe's size.
- Relativity also answered the Mercury question. The perihelion shifted because of frame-dragging, a relativistic effect.
- I don't have time to go into details- ask a cosmologist!

Where's the center?

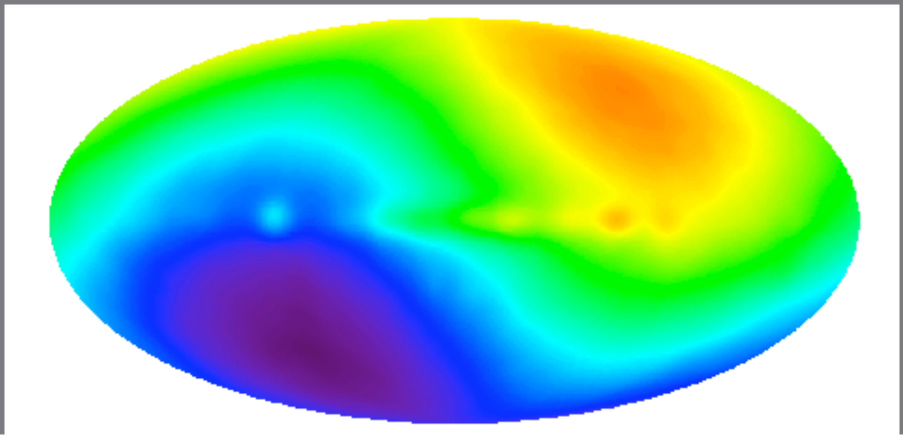
- It turns out we're not at the center of the solar system.
- The solar system isn't at the center of our galaxy.
- If the universe is expanding does that mean we're at the center? No.
- If the universe has a center we're not there.

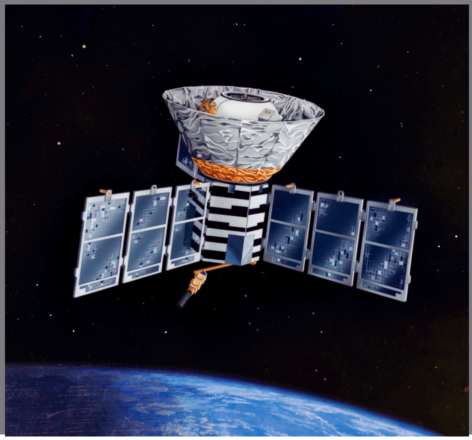




Are we at rest?

- Galileo and Newton showed we can't tell if we're moving uniformly. Every previous model assumed either the Earth or sun was at rest.
- Scientists searched for movement with respect to luminiferous aether and found none.
- It wasn't until the 1990s that COBE saw a dipole anisotropy in the cosmic microwave background- the Earth is moving with respect to rest of the matter in the universe!





Conclusion

- The Copernican Revolution was one of the most important revolutions in the history of human thought.
- The transition from ancient superstition and dogma of the 15th century to Newton's laws marked the start of the scientific age.
- Moving to the heliocentric model was just the first (and most difficult) step in a series of beautiful and mind-blowing discoveries about the universe.
- If you want to know more, go read up on it! There is so much more that I had to leave out.

Backup

NSF questions

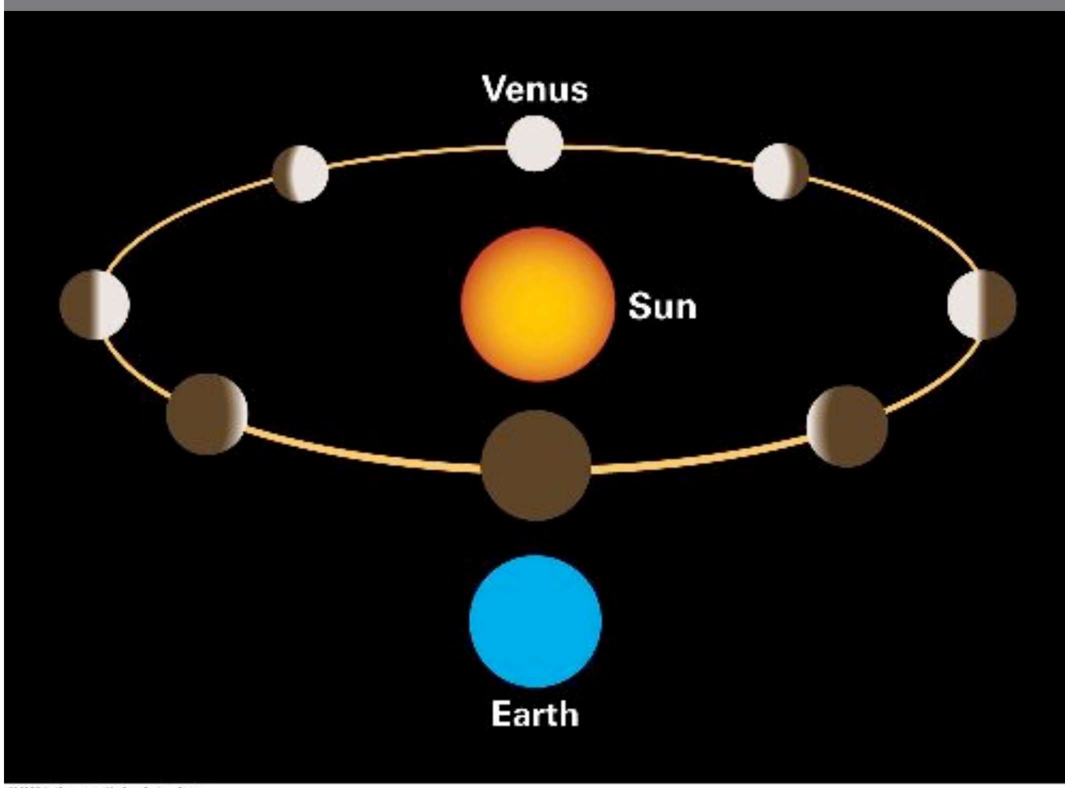
Correct answers to scientific literacy questions: 1985–2006

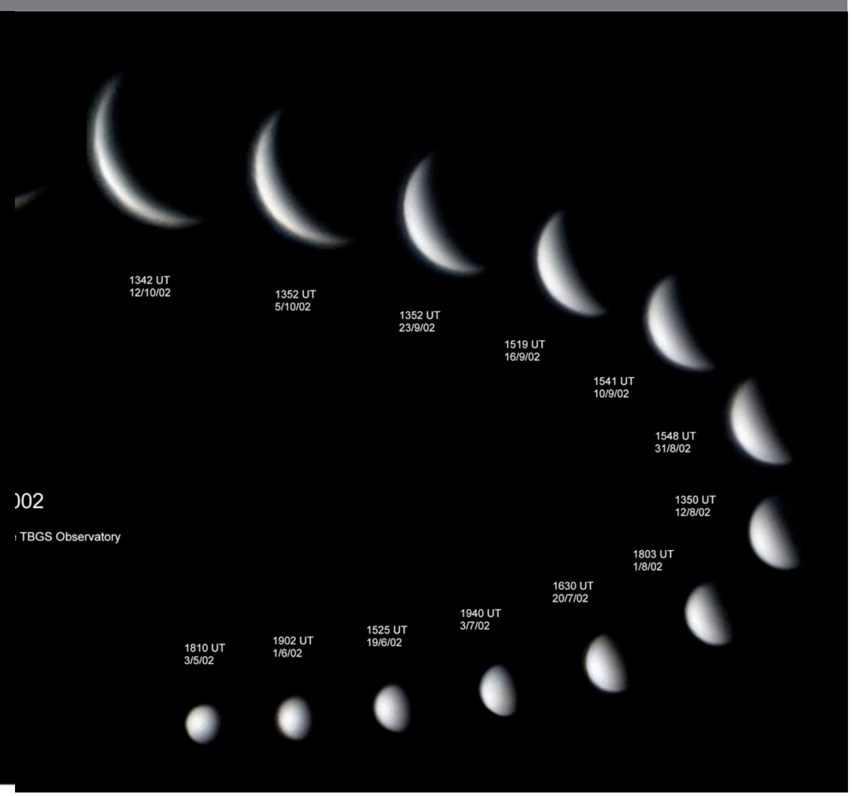
(Percent)

Question	1985 (*n* = 2,003)	1988 (*n* = 2,041)	1990 (*n* = 2,005)	1992 (*n* = 1,995)	1995 (*n* = 2,006)	1997 (*n* = 2,000)	1999 (*n* = 1,882)	2001 (*n* = 1,574)	2004 (*n* = 2,025)	2006 (*n* = 1,864)
The center of the Earth is very hot. (True)	NA	80	79	81	78	82	80	80	78	80
All radioactivity is man-made. (False)	NA	65	63	73	72	71	71	76	73	70
Lasers work by focusing sound waves. (False)	NA	36	37	37	40	39	43	45	42	45
Electrons are smaller than atoms. (True)	NA	43	41	46	44	43	46	48	45	53
The universe began with a huge explosion. (True)	NA	54	32	38	35	32	33	33	33[a]	33
The continents on which we live have been moving their location for millions of years and will continue to move in the future. (True)	79	80	77	79	78	78	80	79	77	80
Does the Earth go around the Sun, or does the Sun go around the Earth? (Earth around Sun)	NA	73	73	71	73	73	72	75	71	76
How long does it take for the Earth to go around the Sun? (One year)	NA	45	48	46	47	48	49	54	NA	55
It is the father's gene that decides whether the baby is a boy or a girl. (True)	NA	NA	NA	65	64	62	66	65	62	64
Antibiotics kill viruses as well as bacteria. (False)	NA	26	30	35	40	43	45	51	54	56
Human beings, as we know them today, developed from earlier species of animals. (True)	45	46	45	45	44	44	45	53	42[a]	43

Phases of Venus

- The phases of Venus can only be explained in a heliocentric model.
- The apparent size changes with the apparent shape.





The first heliocentrist

- Although Copernicus is credited as being the father of the heliocentric model he wasn't the first to propose it. (He was only the first to be taken seriously and start a revolution in the way we think about the universe.)
- Aristarchus of Samos proposed a heliocentric model back the 3rd century BC, but is largely ignored.
- It seems his model was just another failed attempt of the Greek era.

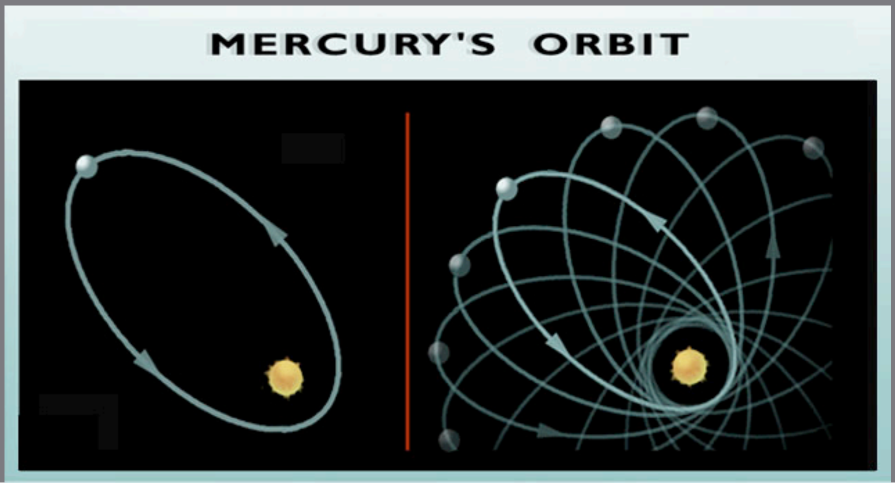
Diameter caluculations

- Here's what Ptolemy and Copernicus calculated for ratio of a few diameters:

Model	Diameter of moon	Diameter of Earth	Diameter of sun
Ptolemy	1	3.4	18.8
Copernicus	1	1.35	24.3
Modern estimate	1	1.84	400.02

Perihelion of Mercury

- The orbit of Mercury advances as Mercury moves around the sun. (That is, the perihelion, the point in the orbit closest to the sun does not stay at the same point in space as it does in the cases of the other planets.)
- This effect is predicted by Newtonian physics, but at a slower rate than observed.
- The following slides give the details of the derivation of the precession.



Perihelion of Mercury

To determine the relativistic precession of Mercury it's useful to start from the Newtonian orbit and apply corrections. We start with the Euler-Lagrange condition for a body of mass m orbiting another body of mass M ($M >> m$):

$$\begin{aligned} L &= T - V && (1)\\ &= \frac{m}{2}\dot{\underline{r}}^2 - \frac{GMm}{|\underline{r}|} && (2)\\ 0 &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} && (3)\end{aligned}$$

The solution is best expressed in spherical polar coordinates. Without loss of generatlity the solutions can be chosen to lie in the plane $\theta = \frac{\pi}{2}$. This gives $\frac{\partial}{\partial\theta} = 0$. Converting $\dot{\underline{r}}$ to spherical polar coordinates ($|\dot{\underline{r}}|^2 = \dot{r}^2 + r^2\dot{\phi}^2$), the Euler-Lagrange condition (eqn 3) for θ becomes:

$$\begin{aligned} 0 &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial}{\partial q}\left[\frac{m}{2}(\dot{r}^2 + r^2\dot{\theta}^2) - \frac{GMm}{|r|}\right]\right) && (4)\\ 0 &= \frac{\mathrm{d}}{\mathrm{d}t}\left(mr^2\dot{\phi}\right) && (5)\\ \rightarrow r^2\dot{\phi} &= h && (6)\end{aligned}$$

where h is a constant.

Conservation of energy gives:

$$\frac{m}{2}(\dot{r}^2 + r^2\dot{\phi}^2) - \frac{GMm}{r} = E \quad (7)$$

Substituting for $\dot{\phi}$ from eqn 6 and rearranging gives:

$$\dot{r}^2 = \frac{2E}{m} - \frac{1}{r^2}\left(h^2 - 2GMr\right) \quad (8)$$

Transforming to $u = 1/r$ gives:

$$\frac{1}{u^4}\left(\frac{\mathrm{d}u}{\mathrm{d}t}\right)^2 = \frac{2E}{m} - u^2\left(h^2 - \frac{2GM}{u}\right) \quad (9)$$

We can eliminate $\mathrm{d}t$ using $\frac{\mathrm{d}\phi}{\mathrm{d}t} = hu^2$:

$$\left(\frac{\mathrm{d}u}{\mathrm{d}\phi}\right)^2 + u^2 = \frac{2E}{mh^2} + \frac{2GMu}{h^2} \quad (10)$$

Differentiating with respect to ϕ gives:

$$\begin{aligned} 2\frac{\mathrm{d}u}{\mathrm{d}\phi}\frac{\mathrm{d}^2u}{\mathrm{d}\phi^2} + 2u\frac{\mathrm{d}u}{\mathrm{d}\phi} &= \frac{2GM}{h^2}\frac{\mathrm{d}u}{\mathrm{d}\phi} && (11)\\ \frac{\mathrm{d}^2u}{\mathrm{d}\phi^2} + u &= \frac{GM}{h^2} && (12)\end{aligned}$$

Perihelion of Mercury

This gives an ellipse as the general solution:

$$u = \frac{GM}{h^2}(1 + e\cos\phi) \tag{13}$$

(Note that if $h^2 < \frac{2GM}{u}$ then the general solution is a hyperbola. A few comets have been observed that have hyperbolic orbits.)

The same procedure can be repeated using the Scharzchild metric:

$$\begin{aligned} \mathrm{d}|\underline{s}|^2 &= \alpha c^2 \mathrm{d}t^2 - \frac{1}{\alpha}\mathrm{d}r^2 - r^2\left(\mathrm{d}\theta^2 + \sin\theta^2 \mathrm{d}\phi^2\right) && (14) \\ \alpha &= 1 - \frac{2GM}{c^2 r} && (15) \end{aligned}$$

where s is the path of the orbit.

In terms of the metric, the square of the Lagrangian, $\mathcal{L}$, is:

$$\mathcal{L} = \alpha c^2 \left(\frac{dt}{ds}\right)^2 - \frac{1}{\alpha}\left(\frac{\mathrm{d}r}{\mathrm{d}s}\right)^2 - r^2\left(\frac{\mathrm{d}\phi}{\mathrm{d}s}\right)^2 \tag{16}$$

The Euler-Lagrange equations for t and ϕ yield:

$$\begin{aligned} \alpha c^2 \frac{\mathrm{d}t}{\mathrm{d}s} &= K && (17) \\ r^2 \frac{\mathrm{d}\phi}{\mathrm{d}s} &= H && (18) \end{aligned}$$

where K and H are constants.

For a timelike orbit $\mathrm{d}|\underline{s}|^2 = \mathrm{d}s^2$ and $\mathcal{L} = 1$. Setting $\mathcal{L}$ to 1 in eqn 16 and multiplying by α gives:

$$\alpha = \alpha^2 c^2 \left(\frac{\mathrm{d}t}{\mathrm{d}s}\right)^2 - \left(\frac{\mathrm{d}r}{\mathrm{d}s}\right)^2 - \alpha r^2 \left(\frac{\mathrm{d}\phi}{\mathrm{d}s}\right)^2 \tag{19}$$

Substituting for $\frac{\mathrm{d}t}{\mathrm{d}s}$ and $\frac{\mathrm{d}\phi}{\mathrm{d}s}$:

$$\alpha = \frac{K^2}{c^2} - \left(\frac{\mathrm{d}r}{\mathrm{d}s}\right)^2 - \frac{H^2\alpha}{r^2} \tag{20}$$

Substituting for α and rearranging:

$$\left(\frac{\mathrm{d}r}{\mathrm{d}s}\right)^2 = \frac{K^2}{c^2} - 1 + \frac{2GMH^2}{c^2 r^3} - \frac{H^2}{r^2} + \frac{2GM}{c^2 r} \tag{21}$$

Expressed in terms of $u = 1/r$ and substituting for $\mathrm{d}s$ from eqn 18 (as we did for the Newtonian case) gives:

$$H^2\left(\frac{\mathrm{d}u}{\mathrm{d}\phi}\right)^2 = \frac{K^2}{c^2} - 1 + \frac{2GMH^2u^3}{c^2} - H^2u^2 + \frac{2GMu}{c^2} \tag{22}$$

Again, differentiating with respect to ϕ and dividing through by $2\frac{\mathrm{d}u}{\mathrm{d}\phi}$ gives:

$$\frac{\mathrm{d}^2u}{\mathrm{d}\phi^2} + u = \frac{GM}{c^2H^2} + \frac{3GMu^2}{c^2} \tag{23}$$

Perihelion of Mercury

The new term on the right hand side leads to relativistic effects not seen in the Newtonian model. The ratio of the two terms on the right hand side is:

$$\frac{3GMu^2}{c^2} / \frac{GM}{c^2H^2} = 3u^2H^2 \tag{24}$$

$$= \frac{3H^2}{r^2} \tag{25}$$

H can be approximated as $H \sim rv/c$ (to see this note that $H = h/c$, $H = h/c = \frac{r^2}{c}\frac{d\phi}{dt} \sim r \cdot r\omega/c = rv/c$). The second term is then of order v^2/c^2. Of all the planets, Mercury has the highest $3v^2/c^2$ ($3v^2/c^2 \sim 10^{-7}$). We can find an approximate solution by recalling the Newtonian solution:

$$u = \frac{GM}{h^2}(1 + e\cos\phi) \tag{26}$$

Substituting this into eqn 23 gives:

$$\frac{d^2u}{d\phi^2} + u = \frac{GM}{h^2} + \frac{3G^3M^3}{c^2h^4}\left(1 + 2e\cos\phi + e^2\cos^2\phi\right) \tag{27}$$

The solution to this equation differs from the Newtonian equation by the addition of three extra terms (all coming from the particular integrals associated with the term in u^2.):

$$u = \frac{GM}{h^2}(1 + e\cos\phi) + \frac{3G^3M^3}{2c^2h^4} + \frac{3G^3M^3e}{c^2h^4}\phi\sin\phi - \frac{G^3M^3e^2}{2c^2h^4}\cos 2\phi \tag{28}$$

The constant term does not contribute any time dependent shift to the orbit. The final term adds a slight wiggle to the orbit. Both of these are terms are tiny and can be ignored. The term in $\phi\sin\phi$ does not have a period of 2π so we would expect to see some precession in the orbit of Mercury. Writing out our approximate solution:

$$u = \frac{GM}{h^2}\left(1 + e\cos\phi + \frac{3G^2M^2}{c^2h^2}e\phi\sin\phi\right) \tag{29}$$

$$\simeq \frac{GM}{h^2}\left(1 + e\cos\left(1 - \frac{3G^2M^2}{c^2h^2}\right)\phi\right) \tag{30}$$

(using a small angle approximation for $3G^2M^2\phi/c^2h^2$ and the identity $\cos(A-B) \equiv \cos A\cos B + \sin A\sin B$.)

The period of this extra term is:

$$\frac{2\pi}{1 - \frac{3GM^2}{h^2}} > 2\pi \tag{31}$$

This means that by the time the orbit has shifted due to this extra term, Mercury has already traced out an almost elliptic orbit. In other words the orbit appears as an ellipse that precesses about one of its foci by an amount:

$$\Delta\phi = \frac{2\pi}{1 - \frac{3GM^2}{h^2}} - 2\pi \tag{32}$$

$$\simeq \frac{6\pi G^2M^2}{h^2} \tag{33}$$

$$\simeq \frac{6\pi G^2M}{a(1-e^2)} \tag{34}$$

where a is the Newtonian semi-major axis and using the Newtonian relation $\frac{M}{h^2} = \frac{1}{a(1-e^2)}$.

This derivation is outlined (rather more quickly) in Chapter 11, Section 9 of Relativity: Special, General, and Cosmological (Rindler.)

Images, bibliography etc

Some good books

There are plenty of good books on the subject. Here are a handful of the more obscure ones I consulted:

Feynman's lost lecture, The Motion of the Planets Around the Sun. (D & J Goodstein)

A great lecture that was far too detailed to include! Great insight into Newton's original methods.

Copernicus' Secret (Jack Repcheck)

Interesting book about Copernicus' private life.

The View from the Center of the Universe (J Primack & N Abrams)

Slightly eccentric book about all sorts of concepts. Not very scientific, but amusing.

Uncentering the Earth (W Vollmann)

Detailed book with great diagrams and data. It has a useful timeline and glossary to stop you getting lost.

Relativity: Special, General and Cosmological (Rindler)

What I consult whenever I need to think about general relativity and cosmology.

Principia (Newton)

This gives you a wonderful idea of how obscure and opaque physics was at the time! In places it's quite poetic, but it's not light reading.

The web is a pretty good resource for filling in the gaps if you ever want to know more.

Images, bibliography etc

Images

Title slide image: NASA Astrophysics Picture of the Day (http://apod.nasa.gov/apod/ap051220.html)

Slide 3: Copernicus portrait (http://en.wikipedia.org/wiki/File:Copernicus.jpg)

Slide 5: Aztec Calendar Stone (http://wikis.lib.ncsu.edu/index.php/Image:Calendar_stone2.JPG), An interpretation of Aristotle's model (http://en.wikipedia.org/wiki/File:Universum.jpg), Egyptian map of the skies (http://www.astronomy.pomona.edu/archeo/egypt/egypt.html), Stonehenge (http://www.knowth.com/wallpaper/stonehenge_1024.jpg)

Slide 6: Babylonian cuneiform astrology (http://www.phys.uu.nl/~vgent/babylon/babybibl_fixedstars.htm), Babylonian eclipse tablet (http://www.livius.org/k/kidinnu/kidinnu.htm)

Slide 7: Flat Earth diagram (http://commons.wikimedia.org/wiki/File:Flat_earth.jpg)

Slide 8: Eudoxan model (http://www.redorbit.com/education/reference_library/universe/geocentric_model/204/)

Slide 10: Ptolemy (http://en.wikipedia.org/wiki/File:Ptolemaeus.jpg), Ptolemaic model (http://www.laputanlogic.com/articles/2006/05/index.html)

Slide 14: Muller, from Wikipedia (http://en.wikipedia.org/wiki/File:Johannes_Regiomontanus.jpg)

Slide 15: Reconstruction of Copernicus, Sky and Telescope (http://www.skyandtelescope.com/news/3310851.html)

Slide 16: Plate from Copernicus's Revolution (http://www.library.rochester.edu/IN/RBSCP/IMAGES/copernicus7.jpg)

Slide 17: Schematic diagram (http://www.vikdhillon.staff.shef.ac.uk/teaching/phy105/celsphere/phy105_ptolemy.html), Cartoon (http://www.law.umkc.edu/faculty/projects/ftrials/galileo/galileoimages.html)

Slide 18: Tycho Brahe, Brahe's model (http://zebu.uoregon.edu/~imamura/121/oct11/brahe.html), 1577 comet (http://www.geocities.com/Athens/Oracle/9941/comet1577.gif)

Images, bibliography etc

Images

Slide 19: Kepler's Platonic solids model (http://en.wikipedia.org/wiki/File:Kepler-solar-system-1.png)

Slide 20: Retrograde motion of Mars (http://www.splung.com/content/sid/7/page/earlymodels), Schematic diagram (http://ephemeris.sjaa.net/0711/retrograde-motion.jpg)

Slide 21: Kepler's ellipse (http://www.relativitycalculator.com/Kepler_1st_Law.shtml), Kepler's sketch (http://comps.fotosearch.com/comp/IST/IST503/kepler-s-illustration_~1157953.jpg)

Slide 22: Kepler's Laws (http://csep10.phys.utk.edu/astr161/lect/history/kepler.html)

Slide 23: Galileo (http://apod.nasa.gov/apod/image/0110/galileo_sustermans.jpg), Moons of Jupiter (http://www.splung.com/content/sid/7/page/earlymodels)

Slide 24: Newton (http://www.gwleibniz.com/britannica_pages/newton/newton_1689_gif.html), Principia (http://nsm1.nsm.iup.edu/gsstoudt/history/images/principia.html)

Slide 25: Bruno (http://www.durand-gallery.com/exhibitions/giordanobruno.html)

Slide 27: Hubble deep field (http://www.firstpr.com.au/astrophysics/hubble-deep-field/Hubble-Deep-Field-1024-wide.bmp)

Slide 28: Milky Way (http://www.galactica-science.com/battlestar/blog/astronomy/the-big-picture/), Hubble (http://www.astro.rug.nl/~weygaert/InleidingStk2/Curriculum/Colleges/College6/indexcoll6.htm)

Slide 29: COBE dipole (http://apod.nasa.gov/apod/ap970308.html), COBE sattelite (http://www.lbl.gov/Publications/Nobel/)

Slide 30: Eta Carinae (http://www.astronomy-pictures.net/hubble_telescope_images.html)