Diffraction model of a step-out transition for a sheet beam in planar geometry

F. Zimmermann and A. W. Chao
Stanford Linear Accelerator, Stanford University, Stanford, California 94300

ABSTRACT

Using a diffraction model, we derive the longitudinal high-frequency impedance of a small step-out transition for a sheet beam in planar geometry.

I. Sheet Beam and Planar Impedance

Consider an infinitely wide 'sheet beam' between two perfectly conducting planar surfaces, each having a discontinuous step outward. Denoting the current and the power loss per horizontal unit length by $dJ/dx = d\hat{J}/dx\ \exp(-i\omega(t-s/c))$ and dP/dx, respectively, we define the real part of the longitudinal planar impedance $\tilde{Z}_0^{\|}$ by the equation:

$$\frac{dP}{dx} = \mathrm{Re}\tilde{Z}_0^{\|}\ \left(\frac{dJ}{dx}\right)^2 \tag{1}$$

In our convention the units of the longitudinal *planar impedance*, $\tilde{Z}_0^{\|}$, are Ωm. (The tilde over $\tilde{Z}_0^{\|}$ emphasizes these peculiar units.) The electromagnetic fields accompanying the beam are: $B_{0x} = -E_{0y} = \pm 2(\pi/c)\ dJ/dx$, where the two signs correspond to the regions above and below the sheet beam, respectively. The fields are independent of the vertical beam size, and are identical to those of a plane wave. The incident energy flux is $F_0 = c(B_{0x}^2 + E_{0y}^2)/(8\pi) = \pi/c\ (dJ/dx)^2$.

II. Planar Impedance for a Small Step Out

Let b be the initial transverse distance between the sheet beam and the upper (or lower) boundary surface. Suppose that at location $s = 0$ both surfaces undergo a step-out transition of size d. If d is small ($d \ll b$), we can ignore multiple reflections and the interference of waves diffracted at the upper and lower edge, and we can directly apply the results of Ref. [1]. There, we extended the conventional diffraction model for a round beam passing a cavity in a circular pipe [2, 3] to the case of a small step-out, by introducing a single image current of opposite polarity at a transverse distance $\Delta = (2b+2d)$ from the beam. From Ref. [1] we infer that the beam loss power for our sheet beam is $dP/dx \approx 4dF_0$. Comparison with Eq. (1) yields

$$\mathrm{Re}\tilde{Z}_0^{\|} = Z_0 d \tag{2}$$

where $Z_0 = 4\pi/c\ (= 377\Omega)$ denotes the vacuum impedance. This may be compared with the high-frequency impedance of a small transition step for a circular beam in a cylindrical beam pipe of radius b, which is $\mathrm{Re}Z_0^{\|} \approx Z_0 d/(\pi b)$ [1].

III. Outlook

If the step d is not small, we must include the interference of the two waves diffracted at the lower and upper edge and also the multiple reflections. In this case, the boundary conditions can still be satisfied in a diffraction model, namely by introducing an infinite set of image currents of alternating polarity, which are spaced a distance $\Delta \equiv (2b+2d)$ apart. The calculation is simplified by assuming that the step-out is symmetric and that the beam is centered between the two conducting surfaces. Note that, according to the planar wake theorem [4], the energy loss of the beam is independent of its vertical offset.

Instead of the situation just described, Babinet's principle allows us to consider the complementary problem of an infinite number of plane waves of alternating phase impinging on an infinite grid of slits of width $2d$ separated by opaque screens of size $2b$.

The amplitude of the diffracted wave is now proportional to

$$a(y,s) \propto \sum_{n=-\infty}^{\infty} (-1)^n \int_b^{b+2d} dy'\ e^{i\omega D_n/c} \tag{3}$$

where

$$D_n \equiv \sqrt{s^2 + (n\Delta + y - y')^2}. \tag{4}$$

Noting that at sufficiently large distances s behind the step we have $|y-y'| \ll \sqrt{s^2+n^2\Delta^2}$, the function D_n in Eq. (3) can be approximated as

$$D_n \approx \sqrt{s^2+n^2\Delta^2} + \frac{2n\Delta(y-y') + (y-y')^2}{2\sqrt{s^2+n^2\Delta^2}}. \tag{5}$$

It is not obvious how to further simplify the resulting expressions.

Finally, it is interesting to notice that the conventional diffraction model for a cylindrical geometry [2, 3] gives correct results for the high-frequency impedance of a deep cavity, although this model does include neither multiple reflections nor interference of waves diffracted at different azimuthal locations.

REFERENCES

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