A Comparative Study of RF and Induction Linac Approaches to Phase Rotation of a Muon Bunch in the Production Region of a μ^+-μ^- Collider*

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ABSTRACT

RF and induction linac approaches to phase rotation of the muon bunch have been studied. Shorter accelerator length, lower power consumption and shorter bunch length favor the RF linac approach. An important outstanding research issue is the operation of RF cavities within 2 - 3 meters of the production target and the accompanying very high radiation background.

I. INTRODUCTION

The motivation for phase rotation in the production region of a μ^+-μ^- collider is illustrated in Fig. 1. Muons captured in the solenoid capture and transport channel beginning at the production target have a large energy spread $\Delta E/E \sim 100\%$ that must be reduced to ~ 10% before entering the phase space cooling section. The muons are allowed to drift in order to reduce the instantaneous energy spread and to introduce a correlation between mean instantaneous energy and arrival time. A differential accelerating pulse is then applied to reduce the head-to-tail energy sweep. Longitudinal phase space is conserved so the reduced energy spread is exactly compensated by an increase in pulse length. The easiest way to think of the phase rotation is in the time domain (Fig. 2); a voltage pulse that complements the head-to-tail variation of mean energy is applied to the accelerator cavities. This is the language of the induction accelerator; the waveforms required turn out to be closely exponential in shape and are natural to provide with pulse lines driving the primarily resistive and capacitive loads of induction cells. Although not as easy to think about, the time variation of the voltage pulse can be Fourier transformed and an entirely equivalent frequency dependent pulse applied to accelerating cavities (Fig. 3). In practice one hopes to be able to synthesize the frequency waveshape to the accuracy needed with a small number of well chosen discrete frequencies applied to resonant cavities as indicated in Fig. 3 at $\omega T = 3$, 5 and 9. This is the RF approach.

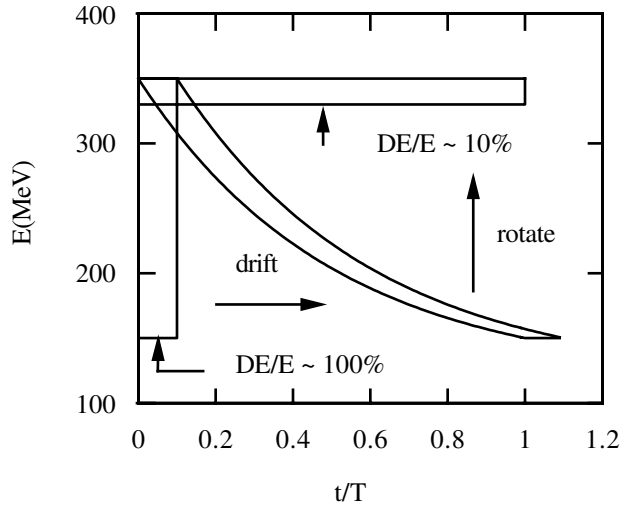

Fig.1: Muon energy versus arrival time.

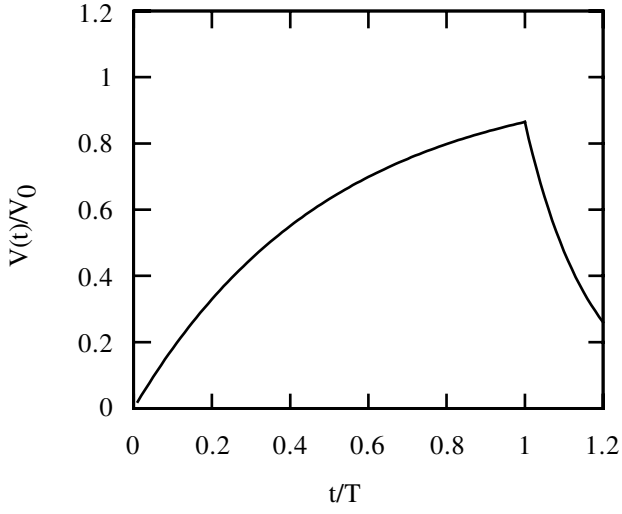

Fig. 2: Normalized cavity voltage versus arrival time.

Fig. 1 illustrates the main ideas of phase rotation although the actual problem is a little more complicated than just described. The beam, at least initially, consists of a mixture of muons and pions turning into muons so until pion decay is

completed the energy spread is partially regenerating as the beam moves along. Due to the head-to-tail energy variation, the length of the beam pulse changes as it propagates through the phase rotation linac. The transverse energy is a significant fraction of the total energy and needs to be taken into account when analyzing the longitudinal propagation of the pulse.

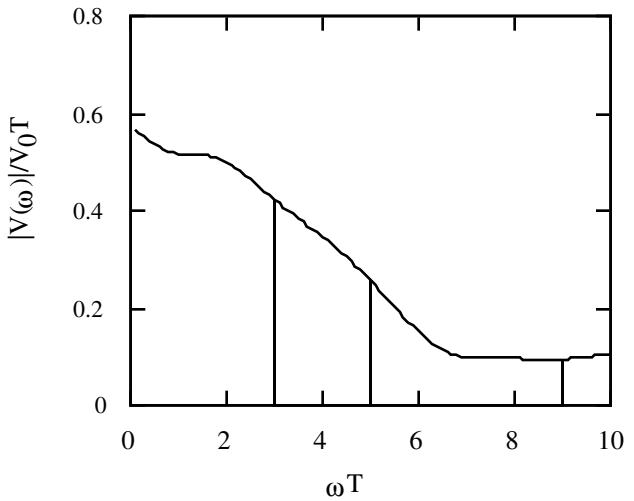

Fig. 3: Normalized Fourier transform of the cavity voltage.

The details of the two approaches to phase rotation have been previously described [1,2] and for the most part will not be repeated here. Instead we will concentrate on comparison of the RF and induction linac solutions that have been worked out and on why they turned out the way they did.

II. DESIGN CONSTRAINTS

Pions are produced by σ_t = 1 nsec 10 to 30 GeV proton pulses impinging on a target and have kinetic energy spectra peaking in the range 100 to 200 MeV. For purposes of design study we therefore chose a pion/muon kinetic energy interval 50 to 250 MeV for phase rotation. As the pions/muons in this energy interval propagate a distance L from the target the spread in their arrival times increases roughly according to $c\Delta t$ = 0.5*L. For the RF approach $c\Delta t$ should not exceed about half a wavelength so the highest frequency cavities should be within a wavelength L ~ λ of the target, or within 2 - 3 meters of the target if the size of cavities is to be reasonable (say f ~ 100 MHz). The corresponding beam pulse lengths are ~ 5 nsec. For the induction linac, $c\Delta t$ is determined by the flux swing ΔB and the maximum $\dot{B}$ that can be tolerated for hysteresis loss in the induction cores; $c\Delta t = c\frac{\Delta B}{\dot{B}_{\max}}$. For Metglas 2605SC, which has high saturation flux (ΔB = 2.5 T) and hysteresis losses measured up to $\dot{B}_{max}$ ~ 20 T/μsec [3], $c\Delta t$ ~ 40 m so the induction cavities should be placed about 80 m from the production target and the corresponding pulse length is ~ 125 nsec.

The accelerating gradient in the RF linac is limited by the maximum electric field strength before breakdown. This is determined by the Kilpatrick criterion relating E_{max} in the accelerating gap to the RF frequency f [4],

$$f(MHz) = 1.64E_K^2(MV/m)e^{-\frac{8.5}{E_K(MV/m)}} \quad (1)$$

and the enhancement factor by which the Kilpatrick field can be safely exceeded. The enhancement factor is a function of pulse length and is ~ 2 for pulse lengths 50 to 200 μsec [5] which spans the cavity fill times of interest for this study. For the three frequencies f = 30, 50 and 90 MHz chosen for the RF linac solution, the corresponding two times Kilpatrick fields are 15, 19.8 and 21.8 MV/m. The acceleration gradients averaged over the full cavity length were then calculated by SFISH giving 2.1, 3.3 and 4.2 MV/m for the particular cavity geometries that were specified. The practical gradient for the lowest frequency case doesn't simply scale from the other two and the Kilpatrick fields because in this case a folded cavity geometry was chosen to reduce the cavity diameter, and this causes some enhancement of surface field relative to average field strength in the gap.

The accelerating gradient in the induction linac is limited by consideration of the properties of the induction core, dielectric breakdown, vacuum insulator surface flashover and vacuum breakdown. Details of the analysis are in ref. [2]. Since voltage rise time requirements are relatively relaxed, τ ~ 50 nsec, the induction cores and the vacuum insulators were stacked vertically to maximize accelerating gradient. The gradient limiting factors were then insulator flashover and vacuum breakdown. For pulse duration ~ 100 nsec, the insulator surface flashover field strength was taken to be 50 kV/cm and the vacuum breakdown field 100 kV/cm. For induction cells occupying 40% of the axial length of the accelerator the practical gradient was found to be ~ 1 MV/m. If the breakdown limiting field strengths are increased by 50% then the flux core would limit the practical gradient to ~ 1.5 MV/m. In summary the realizable accelerating gradients are two to three times higher in the RF linac than in the induction linac approach to phase rotation.

For $2x10^{13}$ muons per pulse, the peak beam current in the induction linac is less than 100 A and far less than the induction core leakage current ~ 3.5 kA so waveform distortion due to beam loading is negligible and does not introduce any design constraints. Furthermore because of the relatively low beam current and a large beam tube radius of 15 cm, beam breakup instability is not a concern in the induction linac.

III. LINAC PARAMETERS

Parameters for the RF and induction linacs for phase rotation are given in Tables I and II below. For schematics and discussion of muon spectra we refer to ref. [1]. For both linacs the input muon spectrum that was effectively phase rotated had total energy from ~ 150 to ~ 350 MeV. For the RF linac three frequencies 30, 50 and 90 MHz were found to be sufficient to produce an output spectrum with the required flatness. The frequencies were chosen to be odd multiples of a fundamental frequency (10 MHz in this case) so that positive and negative bunches may be phase rotated in the same structure if they are spaced apart by an odd multiple of the fundamental period. The accelerator is arranged in three sections according to cavity frequency with the highest frequency section closest to the production target. For the case given in Table I the first 90 MHz cavity is 2.4 m from the target.

Table I: Parameters of the RF phase rotation linac.

Parameter	Value		
RF frequency (MHz)	90	50	30
Cavity length (cm)	120	120	120
Full gap length (cm)	36	36	36
Cavity radius (cm)	90	206	126
Beam tube radius (cm)	15	15	15
Q/1000, SFISH	53.4	71.1	16.8
Shunt imped. (MOhm/m), SFISH	12.2	11.7	1.1
Avg gradient (MV/m)	4.2	3.3	2.1
RF peak power/cavity (MW)	1.8	1.1	4.8
RF avg power/cavity at 15 Hz rep rate, kW	17	26	43
Stored energy (J)	165	261	423
Accel section length	6	18	18
Total avg. RF power (kW)	85	390	640

For the induction linac a single cell driving voltage of 50 kV was chosen which is about the upper limit that can be achieved with commercially available 50 kV thyratrons discharging a Blumlein pulse line connected to the cells. Phase rotation of the muon pulse then requires ~ 3600 of these acceleration gaps, each approximately 2 cm in length. The first cell was located 24 m from the production target. The pulse length at the entrance to the induction linac is 50 nsec, increasing to 110 nsec at 55 m from the entrance and to 130 nsec at 171 m, the end of the linac. Over most of the accelerator length the pulse length is ~ 125 nsec according to the core loss constraint discussed in the previous section. Since the muon collider ring is filled with two bunches of each sign of muon fifteen times per second, the induction linac must be pulsed at 60 Hz. It is conceivable that this could be done with a single accelerating structure with a bipolar pulsed power supply although this would need further study. For the RF linac, since the ~ 100 μsec cavity fill time is long compared to the bunch separation time in the collider ring, the rep rate is 15 Hz. The four bunches in the collider ring would be phase rotated with a single fill of the RF linac.

Table II: Parameters of the induction phase rotation linac.

Parameter	Value
Cell voltage (kV)	50
Number of cells	3642
Cell length (cm)	1.9
Core inside radius (cm)	30
Core outside radius (cm)	46.2
Core flux swing (T)	2.5
Core weight per cell (kgm)	28.5
Pulse length (nsec)	130
Core loss per cycle (J)	14.7
Cell capacitance (nF)	8.5
Total energy per cell per cycle (J)	38.1
Accelerator length (m)	171
Rep. rate (Hz)	60
Total avg. pulsed power (MW)	8.3

IV. COMPARISON

We now briefly compare the parameters of the RF and induction linacs given in Tables I and II. The total length of RF linac is 42 m compared to 171 m for the induction linac. About half of this factor of four is due to the higher accelerating gradient possible with the RF linac. The remaining factor of two occurs because the RF linac decelerates the head of the pulse and accelerates the tail so the phase rotated muons leave the accelerator with energy ~ (150 + 350)/2 = 250 MeV, whereas the induction linac was monopolar and accelerated all muons to ~ 350 MeV. The possibility of driving the induction linac with a bipolar pulse has not been analyzed in detail and could possibly gain back some of this last factor of two. The average total RF power delivered to the RF linac is 1.1 MW whereas the average total pulsed power delivered to the induction linac is 7.5 times higher, 8.3 MW. About 40% of the pulsed power goes to hysteresis loss in the induction cores, ~ 25% to charging the capacitance of the cells and the remainder to impedance mismatch losses. Attempting to reduce the core loss by changing the core material to ferrite would result in an unworkable increase in core volume due to the factor of ~ 4 reduction in saturation flux. Increasing the pulse length and decreasing the induction rate of change would reduce the core loss ($\sim \dot{B}^2$) but result in a longer accelerator and the ~ 100 nsec pulse length already causes difficulty matching into the downstream phase space cooling section. The full width output pulse length of the RF phase rotation linac, ~ 5 nsec, is about right for injection into the phase space cooling section.

When the problem of phase rotation was initially being contemplated it was thought that the phase rotation linac would be placed some tens of meters from the production target where the decay of pions to muons would be essentially complete. The frequencies for the RF linac approach were then rather low (~ 6 MHz), the cavities were very large and consideration of an induction device seemed a viable option. Subsequently, a solution based on placing the first RF cavity close to the target was examined. This seems to work, the bunch length is reduced so the allowable frequencies for phase rotation are much higher and the size of cavities seems reasonable. The practical question that must be addressed is whether an RF cavity can work within 2 - 3 meters of the target and the accompanying high radiation background. A test of this question at an existing accelerator is very important for establishing the practical validity of the present concept for RF linac phase rotation. If it turns out that there are difficulties, then the induction linac approach could be revisited. A problem with the induction linac solution that would need to be addressed is bunching the beam so that the pulse width is acceptable for injection into the downstream phase space cooling section. One possibility is to continue the acceleration of the tail of the pulse until it slightly exceeds the head and then allow the pulse to drift compress before entering the phase space cooling section. In order that the drift section not be too long the pulse compression must be done at low total energy ~ 150 MeV, less than the 300 MeV for the study reported here, and the remaining instantaneous energy spread, even after phase rotation, may cause difficulty compressing the beam by drift. The output energy of the induction linac could be reduced to ~ 150 MeV by phase rotating a lower energy portion of the muon spectrum, or decelerating the head of the pulse instead of accelerating the tail and not changing the selection of input muons.

Some work has been done on the conceptual design of superconducting solenoids for transporting the muons through the phase rotation linac [1]. Additional work is needed integrating these concepts with the acceleration cavities while achieving satisfactory field ripple.

V. REFERENCES

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