Hybrid Rings of Fixed 8T Superconducting Magnets and Iron Magnets Rapidly Cycling between -2T and +2T for a Muon Collider *

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ABSTRACT

Two 2200 m radius hybrid rings of fixed superconducting magnets and iron magnets ramping at 200 Hz and 330 Hz are used to accelerate muons. Muons are given 25 GeV of RF energy per orbit. Acceleration is from 250 GeV/c to 2400 GeV/c and requires a total of 86 orbits in both rings; 82% of the muons survive. The total power consumption of the iron dipoles is 4 megawatts. Stranded copper conductors and thin Metglas laminations are used to reduce power losses.

I. INTRODUCTION

For a $\mu^+\mu^-$ collider, muons must be rapidly accelerated to high energies while minimizing the kilometers of radio frequency (RF) cavities and magnet bores. Cost must be moderate. Some muons may be lost to decay but not too many.

Consider a ring of fixed superconducting magnets alternating with iron magnets rapidly cycling between full negative and full positive field [1]. Table I shows the range of average dipole magnetic field for various mixes of the two types of magnets. One might use more than one ring in succession. Now proceed with a few *back–of–the–envelope* calculations.

Table I: Hybrid ring parameters.

8T Magnets	±2T Magnets	Initial B Field	Final B Field
22%	78%	0.2T	3.3T
25%	75%	0.5T	3.5T
35%	65%	1.5T	4.1T
40%	60%	2.0T	4.4T
50%	50%	3.0T	5.0T
52%	48%	3.2T	5.1T
55%	45%	3.5T	5.3T
60%	40%	4.0T	5.6T
70%	30%	5.0T	6.0T
80%	20%	6.0T	6.8T

II. MAGNET SAGITTAS

The sagitta of a muon in a magnet increases linearly with increasing magnetic field, B. It decreases linearly with increasing momentum, p. And it increases as the square of the length of a magnet, ℓ. The size of the sagitta directly affects the size of magnet bores because the sagitta *changes* throughout a cycle. Table II shows sagitta for various magnets and momenta. As momentum increases, the sagitta in the 8 Tesla magnets decreases towards zero and the sagitta in the 2 Tesla magnets goes somewhat past zero. Note that for a given bore size the magnets can be longer given a higher injection momentum.

$$\text{Sagitta} = R - \sqrt{R^2 - (\ell/2)^2}; \quad R = \frac{p}{.3B} \tag{1}$$

Table II: Sagitta as a function of momentum, magnetic field, and magnet length.

Momentum (GeV)	B Field (Tesla)	Length (meters)	Sagitta (mm)
250	8	1.5	3
250	2	4.5	6
250	8	2	5
250	2	6	11
250	8	3	11
250	2	9	24

III. POWER CONSUMPTION

Consider the feasibility of an iron dominated design for a magnet which cycles from a full -2 Tesla to a full +2 Tesla [2]. First calculate the energy, W, stored in a 2 Tesla field in a volume 6 m long, .03 m high, and .08 m wide. The permeability constant, μ_0, is $4\pi \times 10^{-7}$.

$$W = \frac{B^2}{2\mu_0}[\text{Volume}] = 23\,000 \text{ Joules} \tag{2}$$

Next given 6 turns, an LC circuit capacitor, and a 250 Hz frequency; estimate current, voltage, inductance, and capacitance. The height, h, of the aperature is .03 m. The top and bottom coils may be connected as two separate circuits to halve the switching voltage.

$$B = \frac{\mu_0 N I}{h} \quad \rightarrow \quad I = \frac{Bh}{\mu_0 N} = 8000 \text{ Amps} \tag{3}$$

$$W = .5\, L\, I^2 \quad \rightarrow \quad L = \frac{2\,W}{I^2} = 720\ \mu\text{H} \tag{4}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \quad \rightarrow \quad C = \frac{1}{L\,(2\pi f)^2} = 560\ \mu\text{F} \tag{5}$$

$$W = .5\, C\, V^2 \quad \rightarrow \quad V = \sqrt{\frac{2W}{C}} = 9000 \text{ Volts} \tag{6}$$

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Now calculate the resistive energy loss, which over time is equal to one-half the loss at the maximum current of 8000 Amps. The one-half comes from the integral of cosine squared. Table III gives the resistivities of copper and other metals. A six-turn copper conductor 3 cm thick, 10 cm high, and 7800 cm long has a power dissipation of 15 kilowatts.

$$R = \frac{7800\ (1.8\ \mu\Omega\text{-cm})}{(3)\ (10)} = 470\ \mu\Omega \tag{7}$$

$$P = I^2 R \int_0^{2\pi} \cos^2(\theta)\ d\theta = 15\,000 \text{ watts/magnet} \tag{8}$$

Table III: Conductor, cooling tube, and soft magnetic material properties of resistivity, magnetic saturation in Tesla, and coercive force in Oersteds [3].

Material	Composition	ρ ($\mu\Omega$-cm)	B Max (Tesla)	H_c (Oe)
Copper	Cu	1.8	—	—
Stainless 316L	Fe 70, Cr 18, Ni 10, Mo 2, C .03	74	—	—
Stainless 330	Fe 43, Ni 35, Cr 19	103	—	—
Hastelloy B	Ni 66, Mo 28, Fe 5	135	—	—
Thermostat [4]	Mn 72, Cu 18, Ni 10	175	—	—
Thermenol	Fe 80, Al 16, Mo 4	162	0.61	.02
Pure Iron [5]	Fe 99.95, C .005	10	2.16	.05
1008 Steel	Fe 99, C .08	12	2.09	0.8
Grain–Oriented	Si 3, Fe 97	47	1.95	.1
Supermendur [6]	V 2, Fe 49, Co 49	26	2.4	.2
Hiperco 27 [7]	Co 27, Fe 71, C .01	19	2.36	1.7
Metglas 2605SC [8, 9]	Fe 81, B 14, Si 3, C 2	135	1.6	.03

Calculate the dissipation due to eddy currents in this conductor, which will consist of transposed strands to reduce this loss [10–12]. To get an idea, take the maximum B-field during a cycle to be that generated by a 0.05m radius conductor carrying 24000 amps. This ignores fringe fields from the gap which will make the real answer higher. The eddy current loss in a rectangular conductor made of square wires 1/2 mm wide with a perpendicular magnetic field is as follows. The width of the wire is w.

$$B = \frac{\mu_0 I}{2\pi r} = 0.096 \text{ Tesla} \tag{9}$$

$$P = [\text{Volume}]\frac{(2\pi f B w)^2}{24\rho} \tag{10}$$

$$= [.03\ .10\ 78]\ \frac{(2\pi\ 250\ .096\ .0005)^2}{(24)\ 1.8 \times 10^{-8}} = 3000 \text{ watts}$$

Cooling water will be needed, so calculate the eddy current losses for cooling tubes made from type 316L stainless steel. More exotic metals with higher resistivities are also available as shown in Table III. Choose 2 tubes per 3 cm × 10 cm stranded copper conductor for a total length of 78 × 2 = 156 m. Take a 12 mm OD and a 10 mm ID. Subtract the losses in the inner *missing* round conductor. The combined eddy current loss in the copper plus the stainless steel is 4200 watts (3000 + 2400 - 1200).

$$P\,(12 \text{ mm}) = [\text{Volume}]\ \frac{(2\pi f B d)^2}{32\rho} \tag{11}$$

$$= [\pi\ .006^2\ 156]\ \frac{(2\pi\ 250\ .096\ .012)^2}{(32)\ 74 \times 10^{-8}}$$

$$= 2400 \text{ watts}$$

$$P\,(10 \text{ mm}) = [\text{Volume}]\ \frac{(2\pi f B d)^2}{32\rho} \tag{12}$$

$$= [\pi\ .005^2\ 156]\ \frac{(2\pi\ 250\ .096\ .010)^2}{(32)\ 74 \times 10^{-8}}$$

$$= 1200 \text{ watts}$$

Eddy currents must be reduced in the iron not only because of the increase in power consumption and cooling, but also because they introduce multipole moments which destabilize beams. If the laminations are longitudinal, it is hard to force the magnetic field to be parallel to the laminations near the gap. This leads to additional eddy current gap losses [13]. So consider a magnet with transverse laminations as sketched in Fig. 1 and calculate the eddy current losses. The yoke is either 0.28 mm thick 3% grain oriented silicon steel [14–17] or 0.025 mm thick Metglas 2605SC [8, 9]. The pole tips are 0.1 mm thick Supermendur [6] to increase the field in the gap [18].

$$\text{P(3\% Si–Fe)} = [\text{Volume}]\frac{(2\pi f B t)^2}{24\rho} \tag{13}$$

$$= [6\ ((.42\ .35) - (.20\ .23))]\ \frac{(2\pi\ 250\ 1.6\ .00028)^2}{(24)\ 47 \times 10^{-8}}$$

$$= 27\,000 \text{ watts}$$

$$\text{P(Metglas)} = [\text{Volume}]\frac{(2\pi f B t)^2}{24\rho} \tag{14}$$

$$= [6\ ((.42\ .35) - (.20\ .23))]\ \frac{(2\pi\ 250\ 1.6\ .000025)^2}{(24)\ 135 \times 10^{-8}}$$

$$= 75 \text{ watts}$$

$$\text{P(Supermendur)} = [\text{Volume}]\frac{(2\pi f B t)^2}{24\rho} \tag{15}$$

$$= [6\ .09\ .02]\ \frac{(2\pi\ 250\ 2.2\ .0001)^2}{(24)\ 26 \times 10^{-8}}$$

$$= 210 \text{ watts}$$

Eddy currents are not the only losses in the iron. Hysteresis losses, $\int \mathbf{H} \cdot d\,\mathbf{B}$, scale with the coercive force, H_c, and increase linearly with frequency. Anomalous loss [5] which is difficult to calculate theoretically must be included. Thus I now

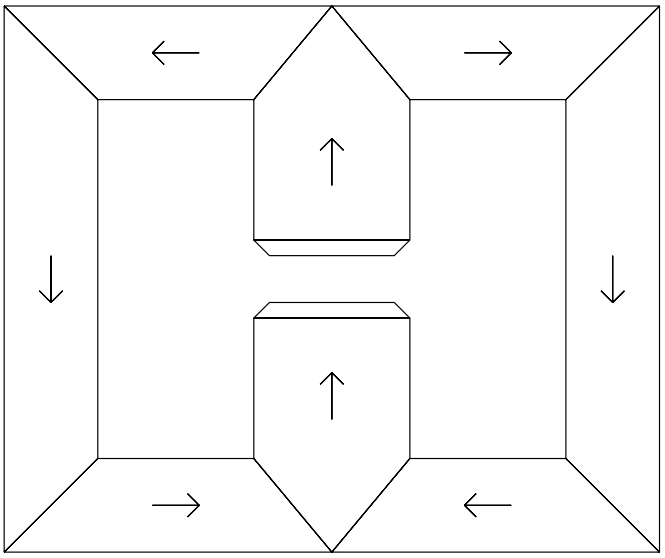

Figure 1: A two dimensional picture of an H frame magnet lamination with grain oriented 3% Si–Fe steel. The arrows show both the magnetic field direction and the grain direction of the steel. Multiple pieces are used to exploit the high permeability and low hysteresis in the grain direction [19]. If Metglas 2605SC is used for the yoke, multiple pieces are not needed, except for the poles. The pole tips are an iron–cobalt alloy for flux concentration exceeding 2 Tesla.

use functions fitted to experimental measurements of 0.28 mm thick 3% grain oriented silicon steel [20], 0.025 mm thick Metglas 2605SC [8], and 0.1 mm thick Supermendur [20].

Table IV: Magnet core materials.

Material	Thickness (mm)	Density (kg/m^3)	Volume (m^3)	Mass (kg)
3% Si–Fe	0.28	7650	0.6	4600
Metglas	0.025	7320	0.6	4400
Supermendur	0.1	8150	0.01	90

$$\begin{aligned} \text{P(3\% Si–Fe)} &= 4.38 \times 10^{-4}\, f^{1.67}\, B^{1.87} \\ &= 4.38 \times 10^{-4}\, 250^{1.67}\, 1.6^{1.87} \\ &= 10.7 \text{ watts/kg} \\ &= 49\,000 \text{ watts/magnet} \end{aligned} \tag{16}$$

$$\begin{aligned} \text{P(Metglas)} &= 1.9 \times 10^{-4}\, f^{1.51}\, B^{1.74} \\ &= 1.9 \times 10^{-4}\, 250^{1.51}\, 1.6^{1.74} \\ &= 1.8 \text{ watts/kg} \\ &= 7900 \text{ watts/magnet} \end{aligned} \tag{17}$$

$$\begin{aligned} \text{P(Supermendur)} &= 5.64 \times 10^{-3}\, f^{1.27}\, B^{1.36} \\ &= 5.64 \times 10^{-3}\, 250^{1.27}\, 2.2^{1.36} \\ &= 18 \text{ watts/kg} \\ &= 1600 \text{ watts/magnet} \end{aligned} \tag{18}$$

Table V: Power consumption for a 250 Hz dipole magnet.

Material	3% Si–Fe	Metglas
Coil Resistive Loss	15 000 watts	15 000 watts
Coil Eddy Current Loss	4200 watts	4200 watts
Core Eddy Current Loss	27 210 watts	285 watts
Total Core Loss	50 600 watts	9500 watts
Total Loss	69 800 watts	28 700 watts

In summary, a 250 Hz dipole magnet close to 2 Tesla looks possible as long as the field volume is limited and one is willing to deal with stranded copper and thin, low hysteresis laminations. Total losses can be held to twice the I^{2}R loss in the copper alone, using Metglas.

IV. MUON ACCELERATION AND SURVIVAL

Now with a rough design for a fast ramping magnet in hand, work out the details of ring radii, RF requirements, and the fraction of muons that survive decay. The fraction of the circumference packed with dipoles is set at P_F = 70%. As an example, consider two rings in a 2200 m radius tunnel with an injection momentum of 250 GeV/c. The first has 25% 8T magnets and 75% $\pm$2T magnets and ramps from 0.5T to 3.5T. The second has 55% 8T magnets and 45% $\pm$2T magnets and ramps from 3.5T to 5.3T.

$$B = \frac{250 \text{ GeV/c}}{.3\, P_F\, R} = \frac{250}{(.3)\,(.7)\,(2200)} = 0.54 \text{ Tesla} \tag{19}$$

$$\begin{aligned} p &= (3.5 \text{ Tesla})\,(.3)\,(P_F)\,(R) \\ &= (3.5)\,(.3)\,(.7)\,(2200) = 1600 \text{ GeV/c} \end{aligned} \tag{20}$$

$$\begin{aligned} p &= (5.3 \text{ Tesla})\,(.3)\,(P_F)\,(R) \\ &= (5.3)\,(.3)\,(.7)\,(2200) = 2400 \text{ GeV/c} \end{aligned} \tag{21}$$

Provide 25 GeV of RF. The first ring accelerates muons from 250 GeV/c to 1600 GeV/c in 54 orbits. The second ring accelerates muons from 1600 GeV/c to 2400 GeV/c in 32 orbits. At what frequency do the two rings have to ramp?

$$\begin{aligned} \text{Time}\,(0.5T \rightarrow 3.5T) &= \frac{(54)\,(2\pi)\,(2.2)}{300\,000} \\ &= 2.5 \text{ ms} \\ &\rightarrow 200 \text{ Hz} \end{aligned} \tag{22}$$

$$\begin{aligned} \text{Time}\,(3.5T \rightarrow 5.3T) &= \frac{(32)\,(2\pi)\,(2.2)}{300\,000} \\ &= 1.5\,\text{ms} \\ &\rightarrow 330\,\text{Hz} \end{aligned} \tag{23}$$

How many muons survive during the 86 orbits from 250 GeV/c to 2400 GeV/c? N is the orbit number, $\tau = 2.2 \times 10^{-6}$ is the muon lifetime, and $m = .106$ GeV/c^2 is the muon mass.

$$\text{SURVIVAL} = \prod_{N=1}^{86} \exp\left[\frac{-2\pi R\, m}{[250 + (25\,N)]\, c\tau}\right] = 82\% \tag{24}$$

Only 1/6 of the 18% loss occurs in the second ring, so it is not crucial to run it as fast as 330 Hz; but the RF does allow this speed.

V. SUMMARY

The 250 → 1600 GeV/c ring has 1200 6 m long dipole magnets ramping at 200 Hz. The 1600 → 2400 GeV/c ring has 725 6 m long dipole magnets ramping at 330 Hz. The weighted average rate is 250 Hz. If running continuously, the 1925 magnets would consume a weighted average of 29 kilowatts each for a total of 56 megawatts. But given a 15 Hz refresh rate for the final muon storage ring [21], the average duty cycle for the 250 → 2400 GeV/c acceleration rings is 6%. So the power falls to 4 megawatts, which is small.

Finally note that one can do a bit better than 82% on the muon survival during final acceleration if the first ring is smaller, say 1000 meters, rather than 2200 meters. Given that RF is expensive, a single line of cavities could still be used for all rings.

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