Measuring Chiral Parameters in the Strongly Interacting W System at a Linear Collider*

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ABSTRACT

In the absence of a Higgs particle vector boson scattering amplitudes are generally described by an electroweak chiral Lagrangian below the resonance region. For a Linear Collider with CMS energy \(\sqrt{s} = 1.6 \text{TeV}\) and an integrated luminosity of 200 fb\(^{-1}\) we estimate the sensitivity on the chiral parameters \(\alpha_4\) and \(\alpha_5\). We consider the processes \(e^+e^- \rightarrow W^+W^-\bar{\nu}\nu\) and \(e^+e^- \rightarrow ZZ\nu\bar{\nu}\), performing a complete calculation which includes all relevant Feynman diagrams at tree level, without relying on the Equivalence Theorem or the Effective W Approximation. The dominant backgrounds and W/Z misidentification probabilities are accounted for.

I. INTRODUCTION

Without the cancellations induced by a Higgs resonance, the scattering amplitudes of massive vector bosons grow with rising energy, saturating the unitarity bounds in the TeV region [1]. Thus there is a strongly interacting domain which lies within the reach of the next generation of collider experiments. One usually expects new resonances which manifest themselves as peaks in the invariant mass distribution of massive vector boson pairs \(VV\) in reactions which contain the nearly on-shell scattering \(VV' \rightarrow VV'\) as a subprocess.

As Barger et al. have shown [2], with a suitable set of kinematical cuts different resonance models (in particular, a 1 TeV scalar and a 1 TeV vector) can clearly be distinguished by analyzing the two modes \(e^+e^- \rightarrow W^+W^-\bar{\nu}\nu\) and \(e^+e^- \rightarrow ZZ\nu\bar{\nu}\) at a Linear Collider with 1.5 TeV CMS energy. A number of similar analyses for hadron, e\(^-\)e\(^-\), and muon collisions have also been performed [3, 4, 5].

This result encourages one to consider the same processes in the more difficult case when resonances do not exist or are out of reach of the particular experiment. In the following we present results for the sensitivity on details of the strong interactions as a two-parameter analysis, carried out in the framework of a complete tree-level calculation.

II. CHIRAL LAGRANGIAN

Below a suspected resonance region the electroweak theory is properly parameterized in terms of a gauged chiral Lagrangian which incorporates the spontaneous breaking of the electroweak symmetry. This Lagrangian induces a low-energy approximation of scattering amplitudes organized in powers of the energy [6].

Models for strong vector boson scattering are usually embedded in Standard Model calculations via the Equivalence Theorem [7] and/or the Effective W Approximation [8]. However, they are not needed for our purpose, since the very nature of chiral Lagrangians as effective low-energy theories allows a complete calculation without approximations. For an accurate estimate of the sensitivity the correct treatment of transversely polarized vector bosons and interference effects is essential, and the full kinematics of the process must be known in order to sensibly apply cuts necessary to isolate the signal.

In our study we made use of the automated calculation package CompHEP [9]. For technical reasons, the chiral Lagrangian has been implemented in \(^\dagger\)Hooft-Feynman gauge:

\[
\mathcal{L} = \mathcal{L}_G + \mathcal{L}_{GF} + \mathcal{L}_{FP} + \mathcal{L}_e + \mathcal{L}_0 + \mathcal{L}_4 + \mathcal{L}_5
\]

where

\[
\mathcal{L}_G = -\frac{1}{8} \text{tr}[W^2_{\mu\nu}] - \frac{i}{4} B_{\mu\nu}^2 \\
\mathcal{L}_{GF} = -\frac{1}{4} \left( \partial^\rho W^\rho_{\mu\nu} + i \frac{\text{Im}}{\text{Re}} \text{tr}[U\tau^]\right)^2 \\
- \frac{1}{4} \left( \partial^\rho B_{\mu\nu} - i \frac{\text{Im}}{\text{Re}} \text{tr}[U\tau^]\right)^2 \\
\mathcal{L}_e = \frac{1}{2} \text{tr}[D_{\mu}\bar{\chi}] + \frac{1}{2} \text{tr}[\bar{\chi}D_{\mu}] + \text{h.c.} \\
\mathcal{L}_0 = \frac{1}{4} \text{tr}[D_{\mu}\bar{\chi}] + \frac{1}{4} \text{tr}[\bar{\chi}D_{\mu}] \\
\mathcal{L}_4 = \alpha_4 \text{tr}[V_{\mu}V_{\rho}]^2 \\
\mathcal{L}_5 = \alpha_5 \text{tr}[V_{\mu}V_{\rho}]^2
\]

with the definitions

\[
U = \exp(-i\omega^a\tau^a/v) \\
V_{\mu} = U^\dagger D_{\mu} U
\]

III. PARAMETERS

To leading order the chiral expansion contains two independent parameters which give rise to W and Z masses. The fact that they are related, i.e., the \(\Delta\rho\) (or \(\Delta T\)) parameter is close to zero, suggests that the new strong interactions respect a custodial \(SU^L_2 \times SU^R_2\) symmetry [10], spontaneously broken to the diagonal \(SU_2\).

In next-to-leading order there are eleven CP-even chiral parameters. Two of them correspond to the \(S\) and \(U\) parameters [11]. Four additional parameters describe the couplings of three gauge bosons. They can be determined, e.g., at \(e^+e^-\) colliders by analyzing \(W\) boson pair production [12]. In our study we assume that these parameters are known with sufficient accuracy. For simplicity, we set them to zero.

The remaining five parameters are visible only in vector boson scattering. If we assume manifest custodial symmetry, only two independent parameters \(\alpha_4\) and \(\alpha_5\) remain. They can be determined by measuring the total cross section of vector boson scattering in two different channels.

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In the present study we consider the two channels $W^+W^- \rightarrow W^+W^- \rightarrow W^+W^-$ and $W^+W^- \rightarrow ZZ$ which are realized at a Linear Collider in the processes $e^+e^- \rightarrow W^+W^-\nu\bar{\nu}$ and $e^+e^- \rightarrow ZZ\nu\bar{\nu}$. In the limit of vanishing gauge couplings the amplitudes for the two subprocesses are related:

$$a(W^+_L W^-_L \rightarrow Z_L Z_L) = A(s, t, u)$$

(10)

$$a(W^+_L W^-_L \rightarrow W^+_L W^-_L) = A(s, t, u) + A(t, s, u)$$

(11)

where

$$A(s, t, u) = \frac{s}{v^2} + \alpha_4 \frac{4(t^2 + u^2)}{v^4} + \alpha_5 \frac{8s^2}{v^4}$$

(12)

with $v = 246$ GeV. These relations hold only for the longitudinal polarization modes. Although in the present study all modes are included, they lead us to expect an increase in the rate for both processes with positive $\alpha_4$ and $\alpha_5$. Negative values tend to reduce the rate as long as the leading term is not compensated.

**IV. CALCULATION**

Using the above Lagrangian, the full squared matrix elements for the processes $e^+e^- \rightarrow W^+W^-\nu\bar{\nu}$ and $e^+e^- \rightarrow ZZ\nu\bar{\nu}$ have been analytically calculated and numerically integrated at $\sqrt{s} = 1600$ GeV (omitting $Z$ decay diagrams, see below). The backgrounds $e^+e^- \rightarrow W^+W^-e^+e^-$ and $e^+e^- \rightarrow W^{\pm}Z\pi\nu$ are relevant if the electrons escape undetected through the beampipe. In that region they receive their dominant contribution through $\gamma\gamma$, $\gamma Z$ and $\gamma W$ fusion which has been calculated within the Weizsäcker-Williams approximation [13].

A set of optimized cuts to isolate various strongly interacting $W$ signals has been derived in [2]. It turns out that similar cuts are appropriate in our case:

- $|\cos \theta(W)| < 0.8$
- $150$ GeV $< p_T(W)$
- $50$ GeV $< p_T(W^+W^-) < 300$ GeV
- $200$ GeV $< M_{inv}(\nu\bar{\nu})$
- $700$ GeV $< M_{inv}(W^+W^-) < 1200$ GeV

The lower bound on $p_T(W^+W^-)$ is necessary because of the large $W^+W^-e^+e^-$ background which is concentrated at low $p_T$ if both electrons disappear into the beampipe. We have assumed an effective opening angle of 10 degrees. The cut on the $\nu\bar{\nu}$ invariant mass removes events where the neutrinos originate from $Z$ decay, together with other backgrounds [2]. For the $ZZ$ final state the same cuts are applied, except for $p_T^{\text{min}}(ZZ)$ which can be reduced to 30 GeV.

The restriction to a window in $M_{inv}(W^+W^-)$ between 700 and 1200 GeV keeps us below the region where (apparent) unitarity violation becomes an issue. Furthermore, it fixes the scale of the measured $\alpha$ values, which in reality are running parameters, at about 1 TeV. In any case, including lower or higher invariant mass values does not significantly improve the results.

For the analysis we use hadronic decays of the $W^+W^-$ pair and hadronic as well as $e^+e^-$ and $\mu^+\mu^-$ decays of the $ZZ$ pair. In addition, we have considered $WW \rightarrow j j \ell\nu$ decay modes which are more difficult because of the additional neutrino in the final state. We find that with appropriately modified cuts the backgrounds can be dealt with also in that case, although the resulting sensitivity is lower than for hadronic decays. In the following results the leptonic $W$ decay modes are not included.

We adopt the dijet reconstruction efficiencies and misidentification probabilities that have been estimated in [2]. Thus we assume that a true $W$ ($Z$) dijet will be identified as follows:

$$W \rightarrow 85\% \ W, \ 10\% \ Z, \ 5\% \ reject$$

(13)

$$Z \rightarrow 22\% \ W, \ 74\% \ Z, \ 4\% \ reject$$

(14)

With $b$ tagging the $Z \rightarrow W$ misidentification probability could be further reduced, improving the efficiency in the $ZZ$ channel. Including the branching ratios and a factor 2 for the $WZ$ background, we have the overall efficiencies

$$\epsilon(WW) = \ 34\%$$

$$\epsilon(ZZ) = \ 34\%$$

(15)

$$\epsilon(WZ) = \ 18\% \ \text{id. as WW, 8\% as ZZ}$$

![Figure 1](image-url)

**Figure 1:** Differential distributions in $p_T$ and $M_{inv}$ of the $W$ pair (after cuts). The dark area shows the background from $WWee$ and $WZee$ final states; the light area is the rate after the signal process $e^+e^- \rightarrow W^+W^-\nu\bar{\nu}$ with $\alpha_4 = \alpha_5 = 0$ has been added; the upper curve denotes the corresponding distribution for $\alpha_4 = 0, \alpha_5 = 0.005$. The $WW$ reconstruction efficiency has not been included.

**V. RESULTS**

The simulations have been carried out for a number of different values of the two parameters $\alpha_4$ and $\alpha_5$, such that a two-parameter analysis was possible for all observables. Fig. 1
shows the differential distributions in the transverse momentum and invariant mass of the $WW$ pair for $e^+e^- \rightarrow W^+W^-\bar{\nu}\nu$ including backgrounds after all cuts have been applied. The shown signal distribution is similar in shape to a broad scalar (Higgs) resonance; however, the total signal rate is smaller.

Both channels are enhanced by positive values of the two parameters, the $ZZ$ channel being less sensitive to $\alpha_4$ than the $WW$ channel. With actual data at hand one would perform a maximum-likelihood fit to the various differential distributions. In our analysis, however, we only use the total cross sections after cuts. For $\alpha_4 = \alpha_5 = 0$ we find 80 $WW$ and 67 $ZZ$ events if 200 fb$^{-1}$ of integrated luminosity with unpolarized beams and the efficiencies (15) are assumed.

In Fig. 2 we show the $\pm 1\sigma$ bands resulting from the individual channels as well as the two-parameter confidence region centered at $(0,0)$ in the $\alpha_4\alpha_5$ plane. The total event rate allows for a second solution centered roughly at $(-0.017,0.005)$ which corresponds to the case where the next-to-leading contributions in the chiral expansions are of opposite sign and cancel the leading-order term. This might be considered as unphysical; in any case, this part of parameter space could be ruled out by performing a fit to the differential distributions or by considering other channels such as $WZ$ [possibly including results from the LHC].

![Figure 2: Exclusion limits for unpolarized beams. The shaded bands display the $\pm 1\sigma$ limits resulting from either one of the two channels; the lines show the combined limits at the $\chi^2 = 1, 3, 5$ level. [For Gaussian distributions, this corresponds to a 39%, 78%, 92% confidence level, respectively.]

Since in both channels the signal part is generated only by the combination of left-handed electrons and right-handed positrons, polarizing the incident beams enhances the sensitivity of the experiments. Assuming 90% electron and 60% positron polarization, the signal rate increases by a factor 3. For the $WZ$ background the enhancement is 1.75, whereas the $W^+W^-e^+e^-$ background remains unchanged. We now find 182 $WW$ and 193 $ZZ$ events.

![Figure 3: Exclusion limits for polarized beams.]

Here we have not taken into account that part of the intrinsic background to $e^+e^- \rightarrow W^+W^- (Z\bar{Z})\bar{\nu}\nu$ is not due to $WW$ fusion diagrams and will therefore not be enhanced, and that the cuts could be further relaxed in the polarized case. Thus the actual sensitivity will be improved even more.

VI. SUMMARY

As our analysis shows, a Linear Collider is able to probe the chiral parameters $\alpha_4$ and $\alpha_5$ down to a level of $10^{-3}$ which is well in the region where the actual values are expected by dimensional analysis. Full energy ($\sqrt{s} = 1.6$ TeV) and full luminosity (200 fb$^{-1}$) is needed to achieve that goal. Electron and positron beam polarization both improve the sensitivity. With several years of running time a precision measurement of chiral parameters seems to be a realistic perspective, rendering a meaningful test of strongly interacting models even in the pessimistic case where no resonances can be observed directly.

VII. REFERENCES


