$t\bar{t}$ Production via Vector Boson Fusion at High Energy e^+e^- Colliders *

Mikuláš Gintner and Stephen Godfrey
Ottawa-Carleton Institute for Physics
Department of Physics, Carleton University, Ottawa CANADA, K1S 5B6

ABSTRACT

We examine $t\bar{t}$ production via vector boson fusion at high energy e^+e^- colliders using the effective vector-boson approximation. We show cross sections as functions of CM energy for various Higgs masses ranging from 100 GeV up to 1 TeV, and also for $M_H = \infty$ which corresponds to the LET. We give expressions for $\sigma(V_iV_j \to t\bar{t})$ in the $2M_{W,Z}/\sqrt{s} = 0$ approximation and show how this approximation effects the results.

I. INTRODUCTION

Understanding the mechanism responsible for electroweak symmetry breaking is one of the most pressing problems in particle physics. Quite generally, there are two scenarios. In the first, the Higgs boson is light and at high energies the weak sector remains weakly coupled. In this scenario the Higgs boson should be observed at one of the high energy colliders under consideration. In the second scenario the Higgs boson is heavy and at high energies the weak sector becomes strongly interacting. If this were the case a new spectroscopy is likely to manifest itself at energies beyond a TeV. In our view this is a very intriguing possibility. Unfortunately, it will be very difficult to detect and even more difficult to understand. Considerable work has been devoted to understanding this problem using vector boson scattering at high energy colliders with questionable success. However, it has been shown that $t\bar{t}$ production via vector boson fusion at high energy colliders also provides a potentially powerful tool for understanding a strongly interacting weak sector [2] and this subject has attracted growing interest [3, 4]. This is simply a manifestation of the equivalence theorem where longitudinal bosons take on the couplings of the scalars from which they acquire mass and a consequence of the fact that the Higgs boson couples most strongly to the most massive particle available.

Due to the tremendous complexity of the algebra and expressions involved in exact calculations of $t\bar{t}$ production via vector boson fusion, the full analysis of these processes depends heavily on computer techniques. As a consequence, analytic expressions are not usually presented. Nevertheless, it is often useful to have concise analytic expressions available for the purpose of comparison and to gain insights into the process under consideration. An efficient way to simplify the complexity of the calculations is to use the effective vector boson approximation (EVA). In this approximation, vector bosons are treated as constituents of colliding particles and the calculational requirements are reduced to finding the cross section for the subprocess $V_iV_j \to t\bar{t}$, with V_i being a real particle, and convoluting this cross section with the appropriate W/Z distribution functions. The EVA was used for the investigation of several processes in the past, including heavy Higgs boson production, vector boson pair scattering, and the production of heavy leptons at both hadron and e^+e^- colliders. In all of these studies the EVA has been shown to be quite accurate in the region of its expected validity when compared to the full calculations. Although some calculations have also been performed for vector boson fusion $t\bar{t}$ production the picture is far from complete. To the best of our knowledge no full calculations of these processes exist in the literature. The total cross section for $e^+e^- \to \ell\bar{\ell}t\bar{t}$, using the EVA, has been calculated by Kauffman at fixed CM energy of 2 TeV [2]. Closely related to this problem are calculations of heavy fermion production at pp colliders by Dawson and Willenbrock [5], and heavy lepton production by Yuan (pp and e^+e^- colliders) [7], and Éboli *et al* (pp collider) [8]. Approximate expressions for $V_iV_j \to t\bar{t}$ cross sections can be found in Kauffman's paper [2] and can be adapted from expressions for $V_iV_j \to L\bar{L}$ in Éboli *et al*'s paper [8]. Unfortunately, some of the published expressions contain errors. The exact helicity amplitudes for $V_iV_j \to L_1\bar{L}_2$ have been published in the Appendix of Yuan's paper [7].

In this report we concentrate on e^+e^- colliders and calculate $e^+e^- \to \ell\bar{\ell}t\bar{t}$ cross sections as a function of CM energy for various Higgs boson masses. In our calculations we use the EVA with exact expressions for $V_iV_j \to t\bar{t}$ cross sections. For the $e^+e^- \to \ell\bar{\ell}t\bar{t}$ via $W_{L/T}W_{L/T}$ and $Z_{L/T}Z_{L/T}$ we evaluate errors introduced by neglecting terms proportional to $2M_{W,Z}/\sqrt{s}$ in $\sigma(WW/ZZ \to t\bar{t})$. In the appendix we give expressions for $\sigma(V_iV_j \to t\bar{t})$ where we made the approximation $2M_{W,Z}/\sqrt{s} = 0$ for $\sigma(WW \to t\bar{t})$ and $\sigma(ZZ \to t\bar{t})$.

II. RESULTS

A. Cross Sections

We calculated the full standard model expressions for the total cross sections for:

- $W^+_{L/T}W^-_{L/T} \to t\bar{t}$
- $Z_{L/T}Z_{L/T} \to t\bar{t}$
- $\gamma Z_{L/T} \to t\bar{t}$
- $\gamma\gamma \to t\bar{t}$

The expressions for these cross sections are given in the appendix where $\sigma(WW \to t\bar{t})$ and $\sigma(ZZ \to t\bar{t})$ are given in the approximation $2M_{W,Z}/\sqrt{s} = 0$. (The complete expressions

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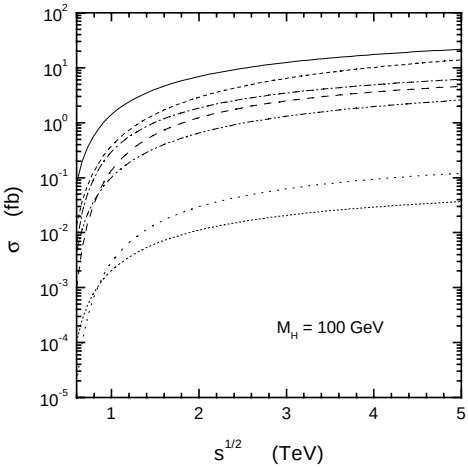

Figure 1: Cross sections for $t\bar{t}$ production via vector boson fusion using the effective $W/Z/\gamma$ approximation with $M_H = 100$ GeV. The solid line is for $\gamma\gamma \to t\bar{t}$, the dot-dashed line for $\gamma Z_L \to t\bar{t}$, the dot-dot-dashed line for $\gamma Z_T \to t\bar{t}$, the dashed line for $W_L W_L \to t\bar{t}$, the short-dashed line for $W_T W_T \to t\bar{t}$, the dotted line for $Z_L Z_L \to t\bar{t}$, and the densely-dotted line for $Z_T Z_T \to t\bar{t}$

are prohibitively long to present in the limited space allocated.) In fig. 1 we show the cross sections for $e^+e^- \to \ell\bar{\ell}t\bar{t}$ for the various subprocesses where the lepton ℓ is either an e or ν as appropriate. To obtain these cross sections we use the effective vector boson approximation (EVA), take $m_t = 175$ GeV, and take $M_{t\bar{t}} > 500$ GeV. The results are given for the full expressions including all orders in $2M_W/\sqrt{s}$ and $2M_Z/\sqrt{s}$. We will discuss the effect of taking $M_{W,Z}/\sqrt{s} \to 0$ below. Of these processes, it is the subprocesses involving longitudinal vector bosons which are of interest since the V_L are, in some sense, the Higgs bosons. The subprocesses with transverse gauge bosons are backgrounds. We also note that the effective vector boson approximation is far from reliable for transverse vector bosons and those results should be viewed with caution.

B. Sensitivity to M_H

Given that it is the processes $V_L V_L \to t\bar{t}$ and their sensitivity to the Higgs boson mass that are of primary interest, in fig. 2 we show the cross sections for $e^+e^- \to \nu\bar{\nu}W_L W_L \to \nu\bar{\nu}t\bar{t}$ and $e^+e^- \to e^+e^- Z_L Z_L \to e^+e^- t\bar{t}$ for $M_H = 100$ GeV, $M_H = 500$ GeV, $M_H = 1$ TeV, and $M_H = \infty$ (which corresponds to the low energy theorem (LET)). In fig. 3 we show the same cross sections as a function of the Higgs mass at fixed CM energy $\sqrt{s} = 1$ TeV. For the $W_L W_L$ case the cross sections at $\sqrt{s} = 1$ TeV are ~ 0.1 fb, ~ 0.4 fb, and ~ 0.3 fb for $M_H = 100$ GeV, 1 TeV, and ∞ respectively. For the expected yearly integrated luminosity of 200 fb^{-1} these should be distinguisheable. However, once $t-$quark detection efficiencies and kinematic cuts to reduce backgrounds are included the situation is not so clear. We remind the reader that we already included a cut of $M_{t\bar{t}} > 500$ GeV and reducing this may increase the cross

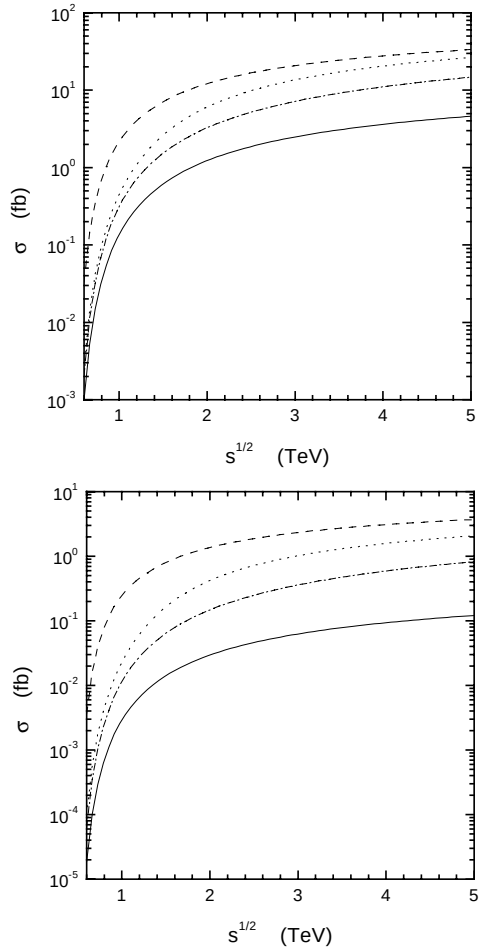

Figure 2: $\sigma(e^+e^- \to \nu\bar{\nu}W_L W_L \to \nu\bar{\nu}t\bar{t})$ (upper figure) and $\sigma(e^+e^- \to e^+e^- Z_L Z_L \to e^+e^- t\bar{t})$ (lower figure) vs $\sqrt{s}$ for several values of M_H. The solid lines are for$M_H = 100$ GeV, the dashed lines for $M_H = 500$ GeV, the dotted lines for $M_H = 1$ TeV, and the dot-dashed lines for $M_H = \infty$ (LET).

section enough to distinguish the cases. Although in fig. 3 the cross section is the same for certain values less than and greater than $M_H \simeq 500$ GeV this does not concern us here since if $M_H < 500$ GeV the Higgs boson should be observed directly at the LHC. As the centre of mass increases, the cross section increases to 6 fb and 3 fb for $M_H = 1$ TeV and $M_H = \infty$ respectively at $\sqrt{s} = 2$ TeV and ∼30 fb and ~ 15 fb respectively at $\sqrt{s} = 5$ TeV. Thus, the different scenarios should be distinguisheable at these energies.

C. Errors Introduced with Approximations

Although our figures are obtained using the complete expressions for the subprocess cross sections, for brevity, we have given in the appendix expressions in the limit that $2M_{W,Z}/\sqrt{s} = 0$ for $WW \to t\bar{t}$ and $ZZ \to t\bar{t}$. The complete expressions are not only lengthy but their calculation is sufficiently complex to diminish the simplifying motivation for using the EVA. To help decide whether it is necessary to use the

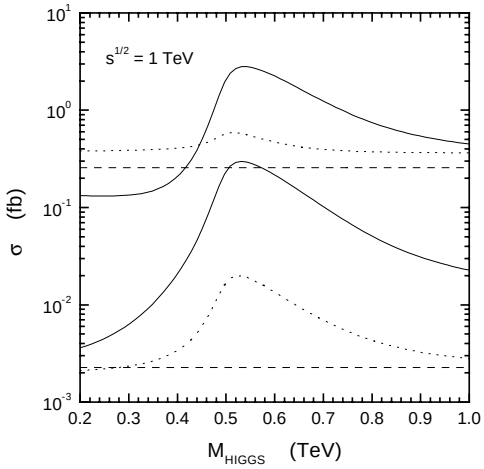

Figure 3: $\sigma(e^+e^- \to \nu\bar{\nu}W_LW_L \to \nu\bar{\nu}t\bar{t})$ and $\sigma(e^+e^- \to e^+e^-Z_LZ_L \to e^+e^-t\bar{t})$ for $\sqrt{s} = 1$ TeV as a function of M_H. The upper set of curves is for WW fusion and lower set of curves for ZZ fusion. The solid curves are for W_LW_L (Z_LZ_L) fusion, the dashed curves for W_LW_T (Z_LZ_T) fusion, and the dotted curves for W_TW_T (Z_TZ_T) fusion.

full expressions rather than the approximate ones shown in the appendix and to know how large an error is introduced by neglecting terms proportional to $2M_{W,Z}/\sqrt{s}$, the differences between the exact and approximate expressions for $e^+e^- \to \ell\bar{\ell}t\bar{t}$ via WW and ZZ fusion are shown in fig. 4 as functions of the CM energy and the Higgs mass. For CM energies between 0.6 TeV and 5 TeV and Higgs masses between 100 GeV and ∞, deviations for W_LW_L and Z_LZ_L range up to about 10%. This is comparable to the precision of the EVA which is claimed to be better than 10%. (Of course, the precision of EVA for $e^+e^- \to \ell\bar{\ell}t\bar{t}$ in particular has not been established yet.) Errors for W_TW_T, W_LW_T, Z_TZ_T, and Z_LZ_T range up to 24%, 13%, 15%, and 3% respectively. However, as has been mentioned earlier, the EVA is expected to be less reliable for the LT and TT modes than the LL modes. The errors tend to diminish with energy but not dramaticaly and not always uniformly. The convergence of the approximation is slowed down by the convolution of $\sigma(WW/ZZ \to t\bar{t})$ with the distribution functions which have large contributions from low energy V_iV_j collisions where the approximate expressions for $\sigma(WW/ZZ \to t\bar{t})$ are not as accurate.

III. ACKNOWLEDGEMENTS

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A. CROSS SECTION FORMULAS

In this appendix we give the formulas for the tree-level cross sections for $V_iV_j \to t\bar{t}$.

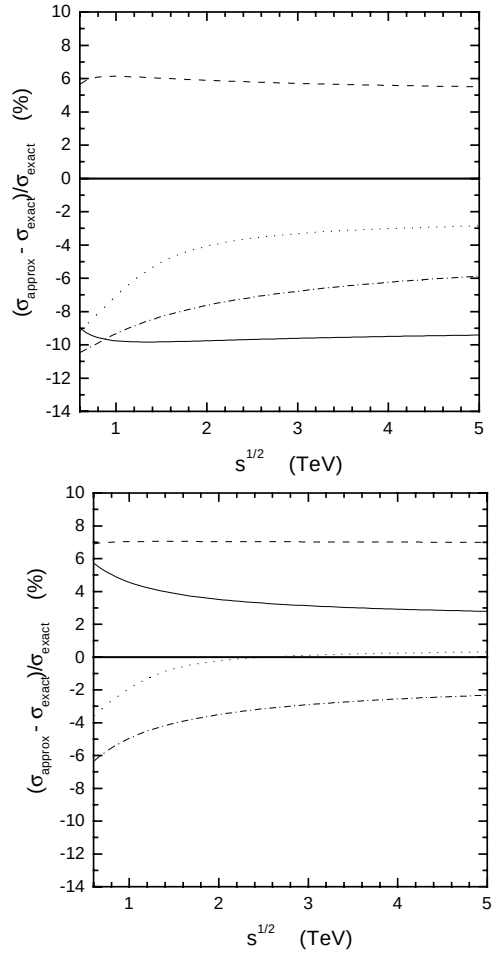

Figure 4: Errors introduced by taking $2M_{W,Z}/\sqrt{s} \to 0$ for the W_LW_L mode (upper set of curves) and the Z_LZ_L mode (lower set of curves) for $e^+e^- \to \ell\bar{\ell}t\bar{t}$. In both cases the solid curves are for $M_H = 100$ GeV, the dashed curves for $M_H = 500$ GeV, the dotted curves for $M_H = 1$ TeV, and the dot-dashed curves for $M_H = \infty$.

<u>$\gamma\gamma \to t\bar{t}$:</u>

$$\sigma(\gamma\gamma \to t\bar{t}) = \frac{4\pi\alpha^2 Q_t^4 N_c}{s}[(1+x_t^2-\frac{x_t^4}{2})\mathcal{L}-\beta_t(1+x_t^2)] \quad (1)$$

<u>$\gamma Z_L \to t\bar{t}$:</u>

$$\sigma(\gamma Z_L \to t\bar{t}) = \frac{4\pi\alpha^2 Q_t^2 N_c}{s_W^2 c_W^2 s(1-M_Z^2/s)^3}\frac{m_t^2}{M_Z^2} \times\{[a_t^2(1-x_Z^2+\frac{x_t^2x_Z^2}{4})-(a_t^2+2v_t^2)\frac{x_t^2x_Z^2}{16}\frac{M_Z^2}{m_t^2}]\mathcal{L} +\frac{1}{4}\beta_t x_Z^2[2a_t^2+(a_t^2+v_t^2)\frac{M_Z^2}{m_t^2}]\} \quad (2)$$

<u>$\gamma Z_T \to t\bar{t}$:</u>

$$\sigma(\gamma Z_T \to t\bar{t}) = \frac{\pi\alpha^2 Q_t^2 N_c}{s_W^2 c_W^2 s(1-M_Z^2/s)^3}$$

$$\times\{(a_t^2+v_t^2)[(\mathcal{L}-\beta_t)(1+\frac{1}{16}x_Z^4)-\frac{1}{2}\beta_t x_Z^2]$$
$$+(a_t^2-v_t^2)[\beta_t-(1-\frac{1}{2}x_t^2)\mathcal{L}]x_t^2\} \quad (3)$$

<u>$W_LW_L\to t\bar{t}$:</u> ($x_W=0$ approximation)

$$\sigma(W_LW_L\to t\bar{t})=\sum_i\sigma_i^{LL} \quad \text{where} \quad (4)$$

$$\sigma_{\gamma\gamma}^{LL}=\frac{\pi\alpha^2Q_t^2N_c}{3}\frac{s}{M_W^4}\beta_t(1+\frac{1}{2}x_t^2) \quad (5)$$

$$\sigma_{ZZ}^{LL}=\frac{\pi\alpha^2N_c}{12s_W^4(1-M_Z^2/s)^2}\frac{s}{M_W^4}\beta_t \times[v_t^2(1+\frac{1}{2}x_t^2)+a_t^2(1-x_t^2)] \quad (6)$$

$$\sigma_{HH}^{LL}=\frac{\pi\alpha^2N_c}{8s_W^4}\frac{m_t^2}{M_W^4}\beta_t^3s^2\chi_H \quad (7)$$

$$\sigma_{bb}^{LL}=\frac{\pi\alpha^2N_c}{24s_W^4}\frac{s}{M_W^4}\beta_t^2[\beta_t(1+\frac{3}{2}x_t^2)+\frac{3}{4}x_t^4\mathcal{L}] \quad (8)$$

$$\sigma_{\gamma Z}^{LL}=\frac{\pi\alpha^2Q_tN_cv_t}{3s_W^2(1-M_Z^2/s)}\frac{s}{M_W^4}\beta_t(1+\frac{1}{2}x_t^2) \quad (9)$$

$$\sigma_{Hb}^{LL}=-\frac{\pi\alpha^2N_c}{4s_W^4}\frac{m_t^2}{M_W^4}[\beta_t(1-\frac{x_t^2}{2})-\frac{x_t^4}{4}\mathcal{L}]s(s-M_H^2)\chi_H \quad (10)$$

$$\sigma_{\gamma b}^{LL}=-\frac{\pi\alpha^2Q_tN_c}{6s_W^2}\frac{s}{M_W^4}[\beta_t(1-\frac{x_t^2}{4}-\frac{3}{8}x_t^4)-\frac{3}{16}x_t^6\mathcal{L}] \quad (11)$$

$$\sigma_{Zb}^{LL}=-\frac{\pi\alpha^2N_c(a_t+v_t)}{12s_W^4(1-M_Z^2/s)}\frac{s}{M_W^4} \times[\beta_t(1-\frac{1}{4}x_t^2-\frac{3}{8}x_t^4)-\frac{3}{16}x_t^6\mathcal{L}] \quad (12)$$

$$\sigma_{\gamma H}^{LL}=\sigma_{ZH}^{LL}=0 \quad (13)$$

<u>$W_TW_T\to t\bar{t}$:</u> ($x_W=0$ approximation)

$$\sigma(W_TW_T\to t\bar{t})=\sum_i\sigma_i^{TT} \quad \text{where} \quad (14)$$

$$\sigma_{\gamma\gamma}^{TT}=\frac{2\pi\alpha^2Q_t^2N_c}{3s}\beta_t(1+\frac{1}{2}x_t^2) \quad (15)$$

$$\sigma_{ZZ}^{TT}=\frac{\pi\alpha^2N_c}{6s_W^4s(1-M_Z^2/s)^2}\beta_t \times[v_t^2(1+\frac{1}{2}x_t^2)+a_t^2(1-x_t^2)] \quad (16)$$

$$\sigma_{HH}^{TT}=\frac{\pi\alpha^2N_c}{4s_W^4}m_t^2\beta_t^3\chi_H \quad (17)$$

$$\sigma_{bb}^{TT}=\frac{\pi\alpha^2N_c}{2s_W^4s}[\beta_t(-\frac{7}{6}+\frac{17}{12}x_t^2-\frac{1}{2}x_t^4) +(1-x_t^2+\frac{7}{8}x_t^4-\frac{1}{4}x_t^6)\mathcal{L}] \quad (18)$$

$$\sigma_{\gamma Z}^{TT}=\frac{2\pi\alpha^2Q_tN_cv_t}{3s_W^2s(1-M_Z^2/s)}\beta_t(1+\frac{1}{2}x_t^2) \quad (19)$$

$$\sigma_{Hb}^{TT}=\frac{\pi\alpha^2N_c}{s_W^4}\frac{m_t^4}{s^2}[(1-\frac{x_t^2}{2})\mathcal{L}-\beta_t](s-M_H^2)\chi_H \quad (20)$$

$$\sigma_{\gamma b}^{TT}=-\frac{\pi\alpha^2Q_tN_c}{3s_W^2s}[\beta_t(1+\frac{5}{4}x_t^2-\frac{3}{8}x_t^4) +(\frac{3}{4}-\frac{3}{16}x_t^2)x_t^4\mathcal{L}] \quad (21)$$

$$\sigma_{Zb}^{TT}=-\frac{\pi\alpha^2N_c}{6s_W^4s(1-M_Z^2/s)} \times\{\beta_t[a_t(1-\frac{1}{4}x_t^2-\frac{3}{8}x_t^4)+ v_t(1+\frac{5}{4}x_t^2-\frac{3}{8}x_t^4)] +\frac{3}{4}x_t^4[v_t(1-\frac{1}{4}x_t^2)-\frac{1}{4}a_tx_t^2]\mathcal{L}\} \quad (22)$$

$$\sigma_{\gamma H}^{TT}=\sigma_{ZH}^{TT}=0 \quad (23)$$

<u>$W_LW_T\to t\bar{t}$:</u> ($x_W=0$ approximation)

$$\sigma(W_LW_T\to t\bar{t})=\sum_i\sigma_i^{LT} \quad \text{where} \quad (24)$$

$$\sigma_{\gamma\gamma}^{LT}=\frac{4\pi\alpha^2Q_t^2N_c}{3M_W^2}\beta_t(1+\frac{1}{2}x_t^2) \quad (25)$$

$$\sigma_{ZZ}^{LT}=\frac{\pi\alpha^2N_c}{3s_W^4(1-M_Z^2/s)^2M_W^2}\beta_t \times[v_t^2(1+\frac{1}{2}x_t^2)+a_t^2(1-x_t^2)] \quad (26)$$

$$\sigma_{bb}^{LT}=\frac{\pi\alpha^2N_c}{6s_W^4M_W^2}[\beta_t(1-x_t^2+\frac{3}{4}x_t^4) +\frac{3}{4}x_t^2(1-x_t^2+\frac{1}{2}x_t^4)\mathcal{L}] \quad (27)$$

$$\sigma_{\gamma Z}^{LT}=\frac{4\pi\alpha^2Q_tN_cv_t}{3s_W^2(1-M_Z^2/s)M_W^2}\beta_t(1+\frac{1}{2}x_t^2) \quad (28)$$

$$\sigma_{\gamma b}^{LT}=\frac{2\pi\alpha^2Q_tN_c}{3s_W^2M_W^2}[\beta_t(1+\frac{x_t^2}{8}+\frac{3}{16}x_t^4)+\frac{3}{32}x_t^6\mathcal{L}] \quad (29)$$

$$\sigma_{Zb}^{LT} = -\frac{\pi\alpha^2 N_c}{3s_W^4(1-M_Z^2/s)M_W^2} \times\{\beta_t[a_t(1-\frac{5}{8}x_t^2+\frac{3}{16}x_t^4)+ v_t(1+\frac{1}{8}x_t^2+\frac{3}{16}x_t^4)] -\frac{3}{8}x_t^4[a_t(1-\frac{1}{4}x_t^2)-\frac{1}{4}v_t x_t^2]\mathcal{L}\} \quad (30)$$

$$\sigma_{HH}^{LT} = \sigma_{Hb}^{LT} = \sigma_{\gamma H}^{LT} = \sigma_{ZH}^{LT} = 0 \quad (31)$$

<u>$Z_L Z_L \to t\bar{t}$:</u> ($x_Z = 0$ approximation)

$$\sigma(Z_L Z_L \to t\bar{t}) = \sum_i \sigma_i^{LL} \qquad \text{where} \quad (32)$$

$$\sigma_{ss}^{LL} = \frac{\pi\alpha^2 N_c}{8s_W^4 c_W^4}\frac{m_t^2}{M_Z^4}\beta_t^3 s^2 \chi_H \quad (33)$$

$$\sigma_{tt}^{LL} = \sigma_{uu}^{LL} = \frac{\pi\alpha^2 N_c}{48s_W^4 c_W^4}\frac{s}{M_Z^4}\beta_t[(a_t^4+v_t^4)(1+\frac{1}{2}x_t^2) +6a_t^2 v_t^2(1-\frac{3}{2}x_t^2)] \quad (34)$$

$$\sigma_{st}^{LL} = \sigma_{su}^{LL} = \frac{\pi\alpha^2 N_c a_t^2}{2s_W^4 c_W^4}\frac{m_t^2}{M_Z^4}(\beta_t - \frac{x_t^2}{2}\mathcal{L})s(s-M_H^2)\chi_H \quad (35)$$

$$\sigma_{tu}^{LL} = -\frac{\pi\alpha^2 N_c}{24s_W^4 c_W^4}\frac{s}{M_Z^4} \times\{\beta_t[a_t^4(1-\frac{23}{2}x_t^2)+v_t^4(1+\frac{1}{2}x_t^2) +6a_t^2 v_t^2(1-\frac{3}{2}x_t^2)]+6a_t^4 x_t^4\mathcal{L}\} \quad (36)$$

<u>$Z_T Z_T \to t\bar{t}$:</u> ($x_Z = 0$ approximation)

$$\sigma(Z_T Z_T \to t\bar{t}) = \sum_i \sigma_i^{TT} \qquad \text{where} \quad (37)$$

$$\sigma_{ss}^{TT} = \frac{\pi\alpha^2 N_c}{4s_W^4 c_W^4} m_t^2 \beta_t^3 \chi_H \quad (38)$$

$$\sigma_{tt}^{TT} = \sigma_{uu}^{TT} = \frac{\pi\alpha^2 N_c}{8s_W^4 c_W^4 s} \times\{-\frac{4}{3}\beta_t[a_t^4(1-x_t^2)+v_t^4(1+2x_t^2)+12a_t^2 v_t^2] +[a_t^4(1-\frac{1}{2}x_t^2)+v_t^4(1+\frac{3}{2}x_t^2) +6a_t^2 v_t^2(1+\frac{1}{2}x_t^2)]\mathcal{L}\} \quad (39)$$

$$\sigma_{st}^{TT} = \sigma_{su}^{TT} = \frac{\pi\alpha^2 N_c}{4s_W^4 c_W^4}(v_t^2 - a_t^2)\frac{m_t^2}{s}\beta_t^2\mathcal{L}(s-M_H^2)\chi_H \quad (40)$$

$$\sigma_{tu}^{TT} = \frac{\pi\alpha^2 N_c}{12s_W^4 c_W^4 s}\{\beta_t[(a_t^4+v_t^4)(1+5x_t^2) +6a_t^2 v_t^2(1+x_t^2)] -\frac{3}{2}x_t^2[(a_t^4+v_t^4)(1+x_t^2) +6a_t^2 v_t^2(1-\frac{1}{3}x_t^2)]\mathcal{L}\} \quad (41)$$

<u>$Z_L Z_T \to t\bar{t}$:</u> ($x_Z = 0$ approximation)

$$\sigma(Z_L Z_T \to t\bar{t}) = \sum_i \sigma_i^{LT} \qquad \text{where} \quad (42)$$

$$\sigma_{ss}^{LT} = \sigma_{st}^{LT} = \sigma_{su}^{LT} = 0 \quad (43)$$

$$\sigma_{tt}^{LT} = \sigma_{uu}^{LT} = \frac{\pi\alpha^2 N_c}{12s_W^4 c_W^4 M_Z^2} \times\{\beta_t[a_t^4(1-x_t^2)+v_t^4(1+\frac{1}{2}x_t^2) +6a_t^2 v_t^2(1+\frac{1}{4}x_t^2)]-3a_t^2 v_t^2 x_t^2\mathcal{L}\} \quad (44)$$

$$\sigma_{tu}^{LT} = -\frac{\pi\alpha^2 N_c}{6s_W^4 c_W^4 M_Z^2}\{\beta_t[a_t^4(1+2x_t^2) +v_t^4(1+\frac{1}{2}x_t^2)+6a_t^2 v_t^2(1+\frac{1}{4}x_t^2)] -\frac{3}{2}x_t^2(a_t^4+3a_t^2 v_t^2)\mathcal{L}\} \quad (45)$$

In all the above equations we used the substitutions

$$\mathcal{L} = \ln\left(\frac{1+\beta_t}{1-\beta_t}\right), \quad \chi_H = \frac{1}{(s-M_H^2)^2+\Gamma_H^2 M_H^2} \quad (46)$$

$\beta_{t,z,w} = \sqrt{1-x_{t,z,w}^2}$, $x_t = 2m_t/\sqrt{s}$, $x_Z = 2M_Z/\sqrt{s}$, $x_W = 2M_W/\sqrt{s}$, $s_W = \sin\theta_W$, $c_W = \cos\theta_W$, $a_t = \frac{1}{2}$, $v_t = \frac{1}{2} - \frac{4}{3}s_W^2$, Q_t is the electric charge of the top quark in terms of $|e|$, and N_c is the number of colours.

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