Inclusive Production of $t\bar{t}$ Pairs in Hadronic Collisions*

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ABSTRACT

We summarize our calculation of the total cross section for top quark production at hadron colliders within the context of perturbative quantum chromodynamics, including resummation of the effects of initial-state soft gluon radiation to all orders in the strong coupling strength.

I. INTRODUCTION AND MOTIVATION

In hadron interactions at collider energies, $t\bar{t}$ pair production proceeds through partonic hard-scattering processes involving initial-state light quarks q and gluons g. In lowest-order perturbative quantum chromodynamics (QCD), $\mathcal{O}(\alpha_s^2)$, the two partonic subprocesses are $q+\bar{q} \to t+\bar{t}$ and $g+g \to t+\bar{t}$. Calculations of the cross section through next-to-leading order, $\mathcal{O}(\alpha_s^3)$, involve gluonic radiative corrections to these lowest-order subprocesses as well as contributions from the $q+g$ initial state [1]. A complete fixed-order calculation at order $\mathcal{O}(\alpha_s^n), n \geq 4$ does not exist.

The physical cross section for each production channel is obtained through the convolution

$$\sigma_{ij}(S,m) = \frac{4m^2}{S}\int_0^{\frac{S}{4m^2}-1} d\eta \Phi_{ij}(\eta,\mu)\hat{\sigma}_{ij}(\eta,m,\mu). \quad (1)$$

The square of the total hadronic center-of-mass energy is S, the square of the partonic center-of-mass energy is s, m denotes the top mass, μ is the usual factorization and renormalization scale, and $\Phi_{ij}(\eta,\mu)$ is the parton flux. The variable $\eta = \frac{s}{4m^2} - 1$ measures the distance from the partonic threshold. The indices $ij \in \{q\bar{q}, gg\}$ denote the initial parton channel. The partonic cross section $\hat{\sigma}_{ij}(\eta,m,\mu)$ is obtained commonly from fixed-order QCD calculations [1], or, as described here, from calculations that go beyond fixed-order perturbation theory through the inclusion of gluon resummation [2, 3, 4] to all orders in the strong coupling strength α_s. We use the notation $\alpha \equiv \alpha(\mu = m) \equiv \alpha_s(m)/\pi$. The total physical cross section is obtained after incoherent addition of the contributions from the the $q\bar{q}$ and gg production channels.

Comparison of the partonic cross section at next-to-leading order with its lowest-order value reveals that the ratio becomes very large in the near-threshold region. Indeed, as $\eta \to 0$, the "K-factor" at the partonic level $\hat{K}(\eta)$ grows in proportion to $\alpha \ln^2(\eta)$. The very large mass of the top quark notwithstanding, the large ratio $\hat{K}(\eta)$ makes it evident that the next-to-leading order result does not necessarily provide a reliable quantitative prediction of the top quark production cross section at the energy of the Tevatron collider. The large ratio casts doubt on the reliability of simple fixed-order perturbation theory for physical processes for which the near-threshold region in the subenergy variable contributes significantly to the physical cross section. Top quark production at the Fermilab Tevatron is one such process, because the top mass is relatively large compared to the energy available. Other examples include the production of hadronic jets that carry large values of transverse momentum and the production of pairs of supersymmetric particles with large mass. To obtain more reliable theoretical estimates of the cross section in perturbative QCD, it is important first to identify and isolate the terms that provide the large next-to-leading order enhancement and then to resum these effects to all orders in the strong coupling strength.

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II. GLUON RADIATION AND RESUMMATION

The origin of the large threshold enhancement may be traced to initial-state gluonic radiative corrections to the lowest-order channels. We remark that we are calculating the inclusive total cross section for the production of a top quark-antiquark pair, i.e., the total cross section for $t + \bar{t} + \mathtt{anything}$. The partonic subenergy threshold in question is the threshold for $t + \bar{t} +$ any number of gluons. This coincides with the threshold in the invariant mass of the $t+\bar{t}$ system for the lowest order subprocesses only.

For $i + j \to t + \bar{t} + g$, we define the variable z through the invariant $(1-z) = \frac{2k\cdot p_t}{m^2}$, where k and p_t are the four-vector momenta of the gluon and top quark. In the limit that $z \to 1$, the radiated gluon carries zero momentum. After cancellation of soft singularities and factorization of collinear singularities in $\mathcal{O}(\alpha_s^3)$, there is a left-over integrable large logarithmic contribution to the partonic cross section associated with initial-state gluon radiation. This contribution is often expressed in terms of "plus" distributions. In $\mathcal{O}(\alpha_s^3)$, it is proportional to $\alpha^3 \ln^2(1-z)$. When integrated over the near-threshold region $1 \geq z \geq 0$, it provides an excellent approximation to the full next-to-leading order physical cross section as a function of the top mass. At $m = 175$ GeV, the ratio of the next-to-leading order to the leading order physical cross sections in the leading logarithmic approximation is $\sigma_{q\bar{q}}^{(0+1)}/\sigma_{q\bar{q}}^{(0)} =1.22$. This ratio shows that the near-threshold logarithm builds up cross section in a worrisome fashion. It suggests that perturbation theory is not converging to a stable prediction of the cross section. The goal of gluon resummation is to sum the series in $\alpha^{n+2}\ln^{2n}(1-z)$ to all orders in α in order to obtain a more defensible prediction.

Different methods of resummation differ in theoretically and phenomenologically important respects. Formally, if not explicitly in some approaches, an integral over the radiated gluon momentum z must be done over regions in which $z \to 1$. Therefore, one significant distinction among methods has to do with how the inevitable "non-perturbative" region is handled. In the approach of Laenen, Smith, and van Neerven (LSvN) [2], an undetermined infrared cutoff (IRC) μ_o is introduced, with $\Lambda_{QCD} \le \mu_o \le m$. The presence of an extra scale spoils the renormalization group properties of the overall expression. The unfortunate dependence of the resummed cross section on this undetermined cutoff is important numerically since it appears in an exponent [2]. It is difficult to evaluate theoretical uncertainties in a method that requires an undetermined infrared cutoff.

III. PERTURBATIVE RESUMMATION

The method of resummation we employ [3] is based on a perturbative truncation of principal-value resummation [5]. This approach has an important technical advantage in that it does not depend on arbitrary infrared cutoffs. Because extra scales are absent, the method permits an evaluation of its perturbative regime of applicability, i.e., the region of the gluon radiation phase space where perturbation theory should be valid. We work in the $\overline{\rm MS}$ factorization scheme.

Factorization and evolution lead directly to exponentiation of the set of large threshold logarithms in moment (n) space in terms of an exponent E^{PV}:

$$E^{PV}(n, m^2) \equiv -\int_P d\zeta \frac{\zeta^{n-1}-1}{1-\zeta} \int_{(1-\zeta)^2}^{1} \frac{d\lambda}{\lambda} g\left[\alpha\left(\lambda m^2\right)\right]. \tag{2}$$

The function $g(\alpha)$ is calculable perturbatively. All large soft-gluon threshold contributions are included through the two-loop running of α. The integral in the complex plane runs along a contour P with endpoints 0 and 1 that is symmetric under reflections across the real axis.

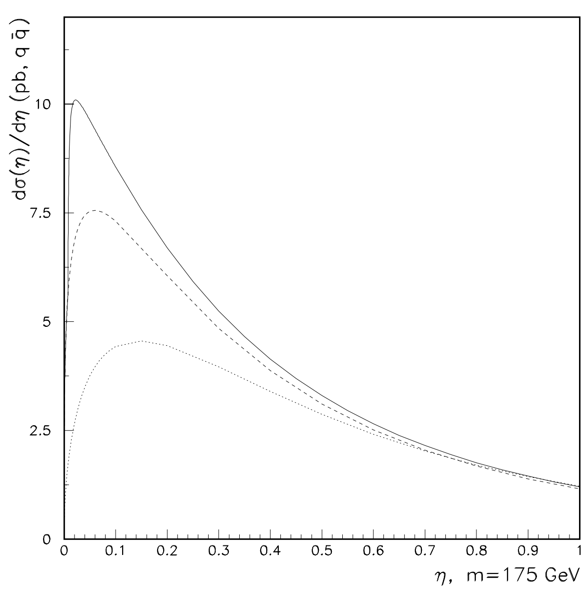

Figure 1: Differential cross section $d\sigma/d\eta$ in the $\overline{\rm MS}$ scheme for the $q\bar{q}$ channel: Born (dotted), next-to-leading order (dashed) and resummed (solid).

The function E^{PV} is finite, and $\lim_{n\to\infty} E^{PV}(n, m^2) = -\infty$. Therefore, the corresponding partonic cross section is finite as $z \to 1$ ($n \to +\infty$). The function E^{PV} includes both perturbative and non-perturbative content. The non-perturbative content is not a prediction of perturbative QCD. We choose to use the exponent only in the interval in moment space in which the perturbative content dominates. We use the attractive finiteness of Eq. (2) to derive a perturbative asymptotic representation of $E(x, \alpha(m))$ that is valid in the moment-space interval

$$1 < x \equiv \ln n < t \equiv \frac{1}{2\alpha b_2}. \tag{3}$$

The coefficient $b_2 = (11C_A - 2n_f)/12$; the number of flavors $n_f = 5$; $C_{q\bar{q}} = C_F = 4/3$; and $C_{gg} = C_A = 3$.

The perturbative asymptotic representation is

$$E_{ij}(x,\alpha) \simeq E_{ij}(x,\alpha,N(t)) = 2C_{ij} \sum_{\rho=1}^{N(t)+1} \alpha^\rho \sum_{j=0}^{\rho+1} s_{j,\rho} x^j . \tag{4}$$

Here

$$s_{j,\rho} = -b_2^{\rho-1}(-1)^{\rho+j} 2^\rho c_{\rho+1-j}(\rho-1)!/j!\,; \tag{5}$$

and $\Gamma(1+z) = \sum_{k=0}^{\infty} c_k z^k$, where Γ is the Euler gamma function. The number of perturbative terms $N(t)$ in Eq. (4) is obtained [3] by optimizing the asymptotic approximation $\Big|E(x,\alpha) - E(x,\alpha,N(t))\Big| = \text{minimum}$. Optimization works perfectly, with $N(t) = 6$ at $m = 175$ GeV. As long as n is in the interval of Eq. (3), all the members of the family in n are optimized at the same $N(t)$, showing that the optimum number of perturbative terms is a function of t, i.e., of m only.

Because of the range of validity in Eq. (3), terms in the exponent of the form $\alpha^k \ln^k n$ are of order unity, and terms with fewer powers of logarithms, $\alpha^k \ln^{k-m} n$, are negligible. Resummation is completed in a finite number of steps. Upon using the running of the coupling strength α up to two loops only, all monomials of the form $\alpha^k \ln^{k+1} n$, $\alpha^k \ln^k n$ are produced in the exponent of Eq. (4). We discard monomials $\alpha^k \ln^k n$ in the exponent because of the restricted leading-logarithm universality between $t\bar{t}$ production and massive lepton-pair production, the Drell-Yan process.

The exponent we use is the truncation

$$E_{ij}(x,\alpha,N) = 2C_{ij} \sum_{\rho=1}^{N(t)+1} \alpha^\rho s_\rho x^{\rho+1}, \tag{6}$$

with the coefficients $s_\rho \equiv s_{\rho+1,\rho} = b_2^{\rho-1} 2^\rho/\rho(\rho+1)$. The number of perturbative terms $N(t)$ is a function of only the top quark mass m. This expression contains no factorially-growing (renormalon) terms. It is valuable to stress that we can derive the perturbative expressions, Eqs. (3), (4), and (5), without the principal-value prescription, although with less certitude [3].

After inversion of the Mellin transform from moment space to the physically relevant momentum space, the resummed partonic cross sections, including all large threshold corrections, can be written

$$\hat{\sigma}_{ij}^{R;pert}(\eta, m) = \int_{z_{min}}^{z_{max}} dz \mathrm{e}^{E_{ij}(\ln(\frac{1}{1-z}),\alpha)} \hat{\sigma}'_{ij}(\eta, m, z). \quad (7)$$

The leading large threshold corrections are contained in the exponent $E_{ij}(x, \alpha)$, a calculable polynomial in x. The derivative $\hat{\sigma}'_{ij}(\eta, m, z) = d(\hat{\sigma}_{ij}^{(0)}(\eta, m, z))/dz$, and $\hat{\sigma}_{ij}^{(0)}$ is the lowest-order $\mathcal{O}(\alpha_s^2)$ partonic cross section expressed in terms of inelastic kinematic variables. The upper limit of integration, $z_{max} < 1$, is set by the boundary between the perturbative and non-perturbative regimes, well specified within the context of the calculation, and z_{min} is fixed by kinematics.

Perturbative resummation probes the threshold down to $\eta \geq \eta_0 = (1 - z_{max})/2$. Below this value, perturbation theory, resummed or otherwise, is not to be trusted. For m = 175 GeV, we determine that the perturbative regime is restricted to values of the subenergy greater than 1.22 GeV above the threshold ($2m$) in the $q\bar{q}$ channel and 8.64 GeV above threshold in the gg channel. The difference reflects the larger color factor in the gg case. The value 1.22 GeV is comparable to the decay width of the top quark.

IV. PHYSICAL CROSS SECTION

Other than the top mass, the only undetermined scales are the QCD factorization and renormalization scales. We adopt a common value μ for both, and we vary this scale over the interval $\mu/m \in \{0.5, 2\}$ in order to evaluate the theoretical uncertainty of the numerical predictions. We use the CTEQ3M parton densities [6]. A quantity of phenomenological interest is the differential cross section $\frac{d\sigma_{ij}(S,m^2,\eta)}{d\eta}$. Its integral over η is the total cross section. In Fig. 1 we plot this distribution for the $q\bar{q}$ channel at $m = 175$ GeV, $\sqrt{S} = 1.8$ TeV, and $\mu = m$. The full range of η extends to 25, but we display the behavior only in the near-threshold region where resummation is important. We observe that, at the energy of the Tevatron, resummation is quite significant for the $q\bar{q}$ channel. A similar figure for the gg channel may be found in our publications [3].

In Fig. 2, we show our total cross section for $t\bar{t}$-production as a function of top mass in $p\bar{p}$ collisions at $\sqrt{S} = 1.8$ TeV. The central value is obtained with the choice $\mu/m = 1$, and the lower and upper limits are the maximum and minimum of the cross section in the range $\mu/m \in \{0.5, 2\}$. At $m = 175$ GeV, the full width of the uncertainty band is about 10% . We consider that the variation of the cross section over the range $\mu/m \in \{0.5, 2\}$ provides a good overall estimate of uncertainty. For comparison, we note that over the same range of μ, the strong coupling strength α varies by $\pm 10\%$ at m = 175 GeV. In estimating uncertainties, we do not consider explicit variations of the non-perturbative cutoff, expressed through z_{max}. This is justified because, for a fixed m and μ, z_{max} is obtained by enforcing dominance of the universal leading logarithmic terms over the subleading ones. Therefore, z_{max} is *derived* and is not a source of uncertainty. At fixed m, the cutoff necessarily varies as μ and thus α vary. Using a different choice of parton densities [7], we find a 4% difference in the central value of our prediction [3] at $m = 175$ GeV. A comparison of the predictions [3] in the $\overline{\mathrm{MS}}$ and DIS factorization schemes also shows a modest difference at the level of 4%.

Our calculation is in agreement with the data [8]. We find $\sigma^{t\bar{t}}(m = 175 \text{ GeV}, \sqrt{S} = 1.8 \text{ TeV}) = 5.52^{+0.07}_{-0.42}$ pb. This cross section is larger than the next-to-leading order value by about 9%.

The top quark cross section increases quickly with the energy of the $p\bar{p}$ collider. We provide predictions in Fig. 3 for an upgraded Tevatron operating at $\sqrt{S} = 2$ TeV. We determine $\sigma^{t\bar{t}}(m = 175 \text{ GeV}, \sqrt{S} = 2 \text{ TeV}) = 7.56^{+0.10}_{-0.55}$ pb. The central value rises to 22.4 pb at $\sqrt{S} = 3$ TeV and 46 pb at $\sqrt{S} = 4$ TeV.

Extending our calculation to much larger values of m at $\sqrt{S} = 1.8$ TeV, we find that resummation in the principal $q\bar{q}$ channel produces enhancements over the next-to-leading order cross section of 21%, 26%, and 34%, respectively, for m = 500, 600, and 700 GeV. The reason for the increase of the enhancements with mass at fixed energy is that the threshold region becomes increasingly dominant. Since the $q\bar{q}$ channel also dominates in the production of hadronic jets at very large values of transverse momenta, we suggest that on the order of 25% of the excess cross section reported by the CDF collaboration [9] may well be accounted for by resummation.

Turning to pp scattering at the energies of the Large Hadron Collider (LHC) at CERN, we note a few significant differences from $p\bar{p}$ scattering at the energy of the Tevatron. The dominance of the $q\bar{q}$ production channel is replaced by gg dominance at the LHC. Owing to the much larger value of $\sqrt{S}$, the near-threshold region in the subenergy variable is relatively less important, reducing the significance of initial-state soft gluon

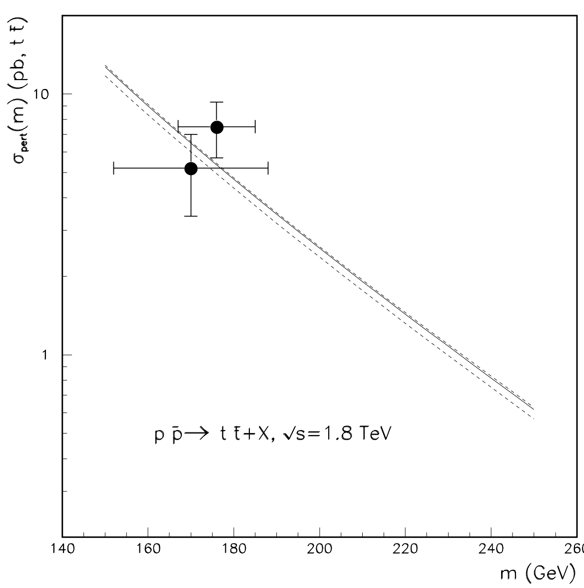

Figure 2: Inclusive total cross section for top quark production in $p\bar{p}$ collisions at $\sqrt{S}$ = 1.8 TeV. The dashed curves show the upper and lower limits while the solid curve is our central prediction. CDF and D0 data are shown.

radiation. Lastly, physics in the region of large subenergy, where straightforward next-to-leading order QCD is also inadequate, becomes significant for $t\bar{t}$ production at LHC energies. Using the approach described in this paper, we estimate $\sigma^{t\bar{t}}(m = 175 \text{ GeV}, \sqrt{S} = 14 \text{ TeV}) = 760$ pb.

V. OTHER METHODS OF RESUMMATION

Two other groups have published calculations of the total cross section at $m = 175$ GeV and $\sqrt{s} = 1.8$ TeV: $\sigma^{t\bar{t}}$(LSvN [2]) = $4.95^{+0.70}_{-0.40}$ pb; and $\sigma^{t\bar{t}}$(CMNT [4]) = $4.75^{+0.63}_{-0.68}$ pb. From the purely numerical point of view, all agree within their estimates of theoretical uncertainty. However, the resummation methods differ as do the methods for estimating uncertainties. Both the central value and the band of uncertainty of the LSvN predictions are sensitive to their arbitrary infrared cutoffs. To estimate theoretical uncertainty, we use the standard μ-variation, whereas LSvN obtain theirs primarily from variations of their cutoffs.

The group of Catani, Mangano, Nason, and Trentadue (CMNT) [4] calculate a central value of the resummed cross section (also with $\mu/m = 1$) that is less than 1% above the exact next-to-leading order value. There are similarities and differences between our approach and the method of CMNT. We use the same universal leading-logarithm expression in moment space, but differences occur after the transformation to momentum space. The differences can be stated more explicitly if we examine the perturbative expansion of the resummed hard kernel $\mathcal{H}^R_{ij}(z,\alpha)$. If, instead of restricting the resummation to the universal leading logarithms only, we were to use the full content of $\mathcal{H}^R_{ij}(z,\alpha)$, we would arrive at an analytic expression that is equivalent to the numerical inversion of CMNT,

$$\mathcal{H}^R_{ij} \simeq 1 + 2\alpha C_{ij}\left[\ln^2(1-z) + 2\gamma_E \ln(1-z)\right] + \mathcal{O}(\alpha^2). \quad (8)$$

In terms of this expansion, in our work we retain only the leading term $\ln^2(1-z)$ at order α, but CMNT retain both this term and the subleading term $2\gamma_E \ln(1-z)$. Indeed, if the subleading term $2\gamma_E \ln(1-z)$ is discarded in Eq. (8), the residuals $\delta_{ij}/\sigma^{NLO}_{ij}$ defined by CMNT [4] increase from 0.18% to 1.3% in the $q\bar{q}$ production channel and from 5.4% to 20.2% in the gg channel. After addition of the two channels, the total residual δ/σ^{NLO} grows from the negligible value of about 0.8% cited by CMNT to the value 3.5%. While still smaller than the increase of about 9% that we obtain, the increase of 3.5% vs. 0.8% shows the substantial influence of the subleading logarithmic terms retained by CMNT.

We judge that it is not appropriate to keep the subleading term for several reasons: it is not universal; it is not the same as the subleading term in the exact $\mathcal{O}(\alpha^3)$ calculation; and it can be changed arbitrarily if one elects to keep non-leading terms in moment space. The subleading term is negative, and it is numerically very significant when integrated throughout the phase space. In the $q\bar{q}$ channel at $m = 175$ GeV and $\sqrt{S} = 1.8$ TeV, its inclusion eliminates more than half of the contribution from the leading term. In our view, the presence of numerically significant subleading contributions begs the question of consistency. The influence of subleading terms is amplified at higher orders where additional subleading structures occur in the CMNT approach with significant numerical coefficients proportional to π^2, $\zeta(3)$, and so forth. We will present a more detailed discussion of these points elsewhere.

Our theoretical analysis and the stability of our cross section under variation of the hard scale μ provide confidence that our perturbative resummation yields an accurate calculation of the inclusive top quark cross section at Tevatron energies and exhausts present understanding of the perturbative content of the theory.

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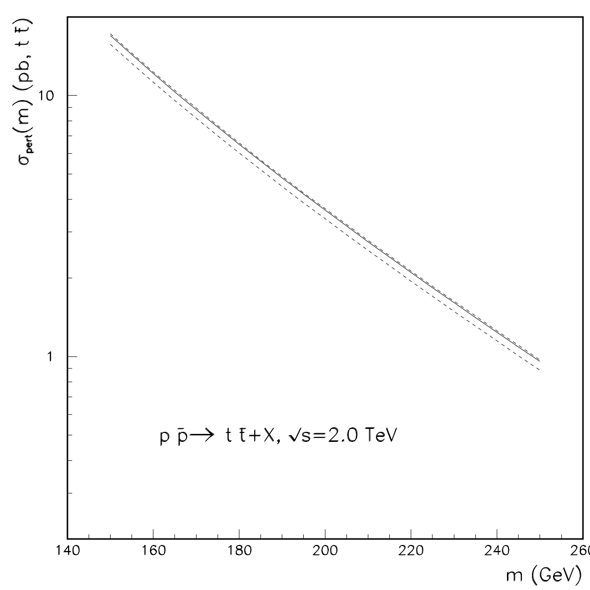

Figure 3: Inclusive total cross section for top quark production in $p\bar{p}$ collisions at $\sqrt{S}$ = 2.0 TeV.