

Precision Measurement of $\sin^2 \theta_w$ and the Beam Polarizations at the NLC

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ABSTRACT

The large event rates expected from Bhabha and Møller scattering at NLC may be used to determine simultaneously $\sin^2 \theta_w$ and the polarization of both beams with very high accuracy. A high degree of polarization and a good polar angle coverage of the detectors turn out to be very important.

I. INTRODUCTION

A welcome feature of the NLC is the high degree of polarization which can be obtained for the electron beams. Beam polarizations exceeding 80% are by now routinely obtained at SLAC and are steadily improving. A final 90% electron polarization seems a quite sensible assumption. Concerning the positron beam, although at present no scheme for polarizing positrons has been proven to be implementable, there are reasonable hopes that some practicable technology may be available by the time the NLC is operating. This ingredient is an important additional lever arm to increase the sensitivity of the precision measurements and searches for new phenomena. It is therefore of great importance to be able to measure the degree of polarization with high accuracy.

We propose here a simple method to determine the polarization of both beams in e^+e^- and e^-e^- collisions [1]. This procedure takes advantage of the large cross sections of Bhabha and Møller scattering to obtain a good analyzing power, competitive with Compton polarimetry [2]. Moreover, as the polarizations are measured from the distributions of the final state electrons and positrons, we are guaranteed to take into account all depolarizing effects which can spoil the initial beam polarization at the interaction point. A similar procedure has been illustrated for the Z^0 peak in Ref. [3].

An interesting characteristic of this measurement is that it simultaneously provides a very accurate determination of $\sin^2 \theta_w$. At present, parity violating asymmetry measurements in Z^0 decays have allowed its most precise determination: combining the SLD measurement of the left-right asymmetries with the various asymmetries from LEP, the effective leptonic $\sin^2 \theta_w$ is now constrained to 0.2316 ± 0.0003 [4]. An early discussion of the determination of the weak mixing angle from Bhabha scattering at LEP1 can be found in [5]. After the end of operation of the e^+e^- colliders on the Z^0 peak, the situation is unlikely to improve significantly, although interesting proposals have been put forward, for both low [6] and at high energy [7] experiments. It is therefore particularly interesting to study the potential of the NLC in this respect.

A similar and more detailed analysis of the material presented in these proceedings can be found in Ref. [8].

II. CROSS SECTIONS

Neglecting the Z^0 width, the polarized differential Bhabha and Møller scattering cross sections are

$$\begin{aligned} \frac{d\sigma^{e^+e^-}}{dt} = \frac{4\pi\alpha^2}{s^2} \times & \quad (1) \\ & \left\{ \frac{1 + P_1 + P_2 + P_1P_2}{4} \left[\left(\sum_i R_i^2 \left(\frac{u}{s - m_i^2} + \frac{u}{t - m_i^2} \right) \right)^2 \right. \right. \\ & \quad \left. \left. + \left(\sum_i L_i R_i \frac{t}{s - m_i^2} \right)^2 \right] \right. \\ & + \frac{1 - P_1 - P_2 + P_1P_2}{4} \left[\left(\sum_i L_i^2 \left(\frac{u}{s - m_i^2} + \frac{u}{t - m_i^2} \right) \right)^2 \right. \\ & \quad \left. \left. + \left(\sum_i L_i R_i \frac{t}{s - m_i^2} \right)^2 \right] \right. \\ & \left. + \frac{1 - P_1P_2}{2} \left(\sum_i L_i R_i \frac{s}{t - m_i^2} \right)^2 \right\}, \end{aligned}$$

$$\begin{aligned} \frac{d\sigma^{e^-e^-}}{dt} = \frac{2\pi\alpha^2}{s^2} \times & \quad (2) \\ & \left\{ \frac{1 + P_1 + P_2 + P_1P_2}{4} \left(\sum_i R_i^2 \left(\frac{s}{t - m_i^2} + \frac{s}{u - m_i^2} \right) \right)^2 \right. \\ & + \frac{1 - P_1 - P_2 + P_1P_2}{4} \left(\sum_i L_i^2 \left(\frac{s}{t - m_i^2} + \frac{s}{u - m_i^2} \right) \right)^2 \\ & + \frac{1 - P_1P_2}{2} \left[\left(\sum_i L_i R_i \frac{t}{u - m_i^2} \right)^2 \right. \\ & \quad \left. + \left(\sum_i L_i R_i \frac{u}{t - m_i^2} \right)^2 \right] \right\}, \end{aligned}$$

where α is the fine structure constant, P_1 stands for the positron polarization in the case of Bhabha scattering, s , t , u are the Mandelstam variables, the summations are over $i = \gamma, Z^0$ and the couplings are defined by

$$R_\gamma = L_\gamma = 1, \quad R_Z = -\frac{\sin \theta_w}{\cos \theta_w}, \quad L_Z = \frac{1 - 2 \sin^2 \theta_w}{2 \sin \theta_w \cos \theta_w}. \quad (3)$$

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III. METHOD

Apart from the overall coupling, which will eventually drop out in the asymmetries, the cross sections (1,2) solely depend on the weak mixing angle and the polarization of each beam:

$$\sin^2 \theta_w \quad P_1 \quad P_2 . \quad (4)$$

It is our purpose to determine these three parameters as precisely as possible.

The experimental determination of the absolute cross sections is hindered by the systematic error on luminosities, acceptances and efficiencies, which dominate the statistical errors when the event rates are as large as in Bhabha and Møller scattering. It is therefore of great advantage to use three independent differential polarization asymmetries, for example

$$A_1 = \frac{\delta n_{LL} - \delta n_{RR}}{\delta n_{LL} + \delta n_{RR}} \quad (5)$$

$$A_2 = \frac{\delta n_{RR} - \delta n_{LR}}{\delta n_{RR} + \delta n_{LR}} \quad (6)$$

$$A_3 = \frac{\delta n_{LR} - \delta n_{RL}}{\delta n_{LR} + \delta n_{RL}} , \quad (7)$$

where R and L refer to positive and negative polarizations $P_{1,2}$ and for each angular bin

$$\delta n = \mathcal{L} \int_{\text{bin}} d \cos \theta \frac{d\sigma}{d \cos \theta} . \quad (8)$$

For these observables the systematic errors cancel out to a very large extent. As long as the correlations between the three asymmetries are correctly taken into account and the statistical errors dominate, it does not matter which triplet of independent asymmetries is chosen. Any choice other than (5–7) yields the same results.

From the physics point of view there is no difference between the two combinations LR and RL in the asymmetry A_3 (7) for the e^-e^- mode. However, since the electron guns may have different efficiencies, it is important to consider them both in order to measure this hardware asymmetry. It is essential that the polarization of the beams be flipped randomly at short time intervals, a technique in use at SLC [9]. In this case, if the absolute value of the polarization is on average constant, random and systematic fluctuations cancel out.

The accuracy with which the parameters (4) can be measured is such that we can safely assume a linear dependence of the cross sections in the region of interest, *i.e.*, within the error bands around the central values. The error bands corresponding to one standard deviation are therefore given by the quadratic form

$$(\Delta \sin^2 \theta_w, \Delta P_1, \Delta P_2) W^{-1} \begin{pmatrix} \Delta \sin^2 \theta_w \\ \Delta P_1 \\ \Delta P_2 \end{pmatrix} = 1 , \quad (9)$$

where the inverse covariance matrix W^{-1} is given by

$$W_{ij}^{-1} = \sum_{k,l=1}^3 \sum_{\text{bins}} V_{kl}^{-1} \left(\frac{\partial A_k}{\partial \epsilon_i} \right) \left(\frac{\partial A_l}{\partial \epsilon_j} \right) \quad (10)$$

$$\epsilon_i = \sin^2 \theta_w, P_1, P_2 . \quad (11)$$

Since the asymmetries are correlated, their covariance matrix V contains off-diagonal terms. This covariance matrix is given by

$$V_{kl} = \langle (A_k - \bar{A}_k)(A_l - \bar{A}_l) \rangle \quad (12)$$

$$= \sum_{i=1}^4 (\Delta n_i)^2 \left(\frac{\partial A_k}{\partial n_i} \right) \left(\frac{\partial A_l}{\partial n_i} \right) + (\Delta \theta)^2 \left(\frac{\partial A_k}{\partial \theta} \right) \left(\frac{\partial A_l}{\partial \theta} \right)$$

$$n_i = \delta n_{LL}, \delta n_{RR}, \delta n_{LR}, \delta n_{RL} , \quad (13)$$

where the statistical errors originating from the uncorrelated polarized event rates in each bin are given by

$$\Delta n_i = \sqrt{n_i} , \quad (14)$$

whereas the systematic error (second term in Eq.(12)) stems from the inaccurate measurement of the scattering angle. A realistic value that we employ in our analysis is

$$\Delta \theta = 0.5 \text{ mrad} . \quad (15)$$

Since the small angle singularities of the differential cross sections cancel out in the asymmetries, the latter have a rather smooth angular dependence. As a result, the contribution of the second term in Eq. (12) is almost negligible.

The quadratic form (9) defines a 3-dimensional ellipsoid in the $(\sin^2 \theta_w, P_1, P_2)$ parameter space. The inverse square root of the diagonal elements of the inverse covariance matrix W^{-1} are the values of the intersections of the error ellipsoid with the corresponding parameter axes. These correspond to the one-standard-deviation errors on this parameter, assuming the other two parameters are known exactly. In contrast, the square roots of the diagonal elements of the covariance matrix W are the values of the projections of this ellipsoid onto the corresponding parameter axes. These correspond to the one-standard-deviation errors on this parameter, whatever values the other two parameters assume. In presenting our results we choose the latter for our estimates of the errors on $\sin^2 \theta_w$ and the beam polarizations.

IV. RESULTS

For the integrated yearly e^+e^- luminosity of the NLC, we use the following scaling relation

$$\mathcal{L}_{e^+e^-} [\text{fb}^{-1}] \approx 200 s [\text{TeV}^2] , \quad (16)$$

or $\mathcal{L}_{e^+e^-} \approx 7.5 \times 10^7 s$ in $c = \hbar = 1$ units. For the luminosity of the e^-e^- mode we take

$$\mathcal{L}_{e^-e^-} \approx \frac{1}{2} \mathcal{L}_{e^+e^-} , \quad (17)$$

because this mode will suffer to some extent from the anti-pinch effect [10]. Since statistical errors are largely dominating, it is straightforward to modify our results for different luminosities.

We assume the integrated luminosities to be equally distributed over the four possible combinations of beam polarizations LL, RR, LR and RL .

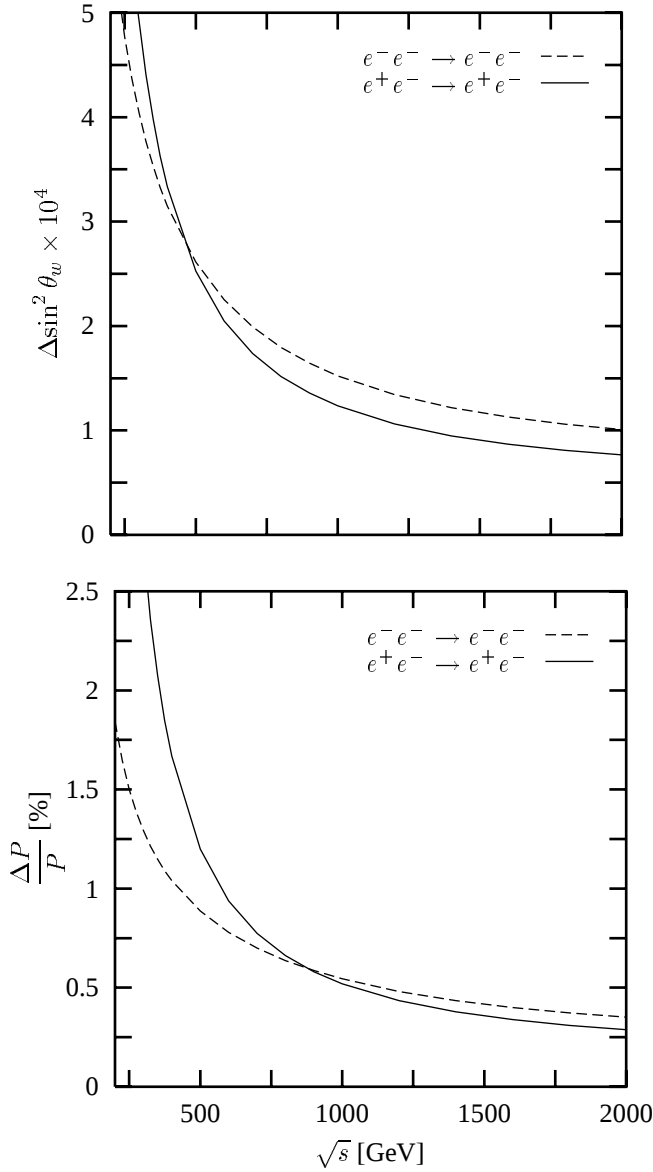


Figure 1: Energy dependence of the errors on $\sin^2 \theta_w$ and the beam polarizations (assumed equal) in Møller and Bhabha scattering.

Unless stated otherwise, we choose from now on the following values for the expectation values of the parameters and the angular acceptance of the detector:

$$\begin{cases} \sin^2 \theta_w = .2315 \\ |P_1| = |P_2| = 90\% \\ |\cos \theta| < .995 \quad \text{or} \quad 5.7^\circ < \theta < 174.3^\circ. \end{cases} \quad (18)$$

To take into account the angular dependence of the asymmetries, we have chosen to work with 200 equal size bins in $\cos \theta$ over the angular range (18). This is easy to implement experimentally, as the scattering angles can be measured with very high accuracy (15). Since the asymmetries have a relatively smooth angular behaviour, increasing the number of bins be-

yond 50 does not significantly improve the accuracy of the measurement. We have checked that, as expected, the results approach very closely the Cramér-Rao minimum variance bound [11].

The two polarization measurements turn out to be highly correlated, in the sense that the average polarization can be determined much more precisely than the polarization difference of the two beams. In contrast, $\sin^2 \theta_w$ is only weakly correlated to the beam polarizations, as long as both polarizations do not differ too much.

Since we assume the luminosities to scale proportionally to the square of the collider energy (16,17), the resolution of the measurement improves at higher energies. This is displayed in Fig. 1, where we plot the center of mass energy dependence of the one standard deviation errors on the measurements of $\sin^2 \theta_w$ and the beam polarization. We observe a clear saturation beyond 1 TeV for both Bhabha and Møller scattering.

At $\sqrt{s} = 500$ GeV, $\sin^2 \theta_w$ can be measured with an error of about 2.5×10^{-4} . Although this may not improve the combined LEP-SLC accuracy, it will provide an independent check. On the other hand, at 2 TeV the resolution on $\sin^2 \theta_w$ can reach up to 8×10^{-5} . Similarly, the polarization can be determined at 500 GeV down to 1.2% in Bhabha and less than 1% in Møller scattering. Compton polarimetry currently yields a similar accuracy of 1.7% [2] and is constantly improving. However, at 2 TeV both Bhabha and Møller scattering can measure the polarization down to 0.3%, a very promising result.

As we mainly rely on the $\gamma - Z^0$ interferences to measure $\sin^2 \theta_w$, it is essential to probe small scattering angles. This is depicted in Fig. 2, where we display the errors as a function of the polar angle coverage. Improving the angular coverage beyond 5° does not appear to be useful. The slight decrease in sensitivity observed for very small polar angle is due to the finite bin size. The error on the polarization is not very sensitive to the detector acceptance, especially for Møller scattering.

High degrees of polarization turn out to be an important asset, especially at lower energies. This should not present any problem for the electron beams and the Møller scattering experiment. In Bhabha scattering, however, it appears that at 500 GeV the resolution degrades significantly for positron polarizations less than 50%. For 2 TeV collisions positron polarizations as small as 30% still yield interesting results.

In the event the positrons cannot be polarized at all, a strong correlation develops between $\sin^2 \theta_w$ and the electron polarization so that these two parameters remain effectively unconstrained. Still, $\sin^2 \theta_w$ can be determined accurately if the electron polarization is also known precisely from the onset (from Compton polarimetry for instance) and its resolution is treated as a systematic error. In this case we observe in Fig. 3 that the resolution on $\sin^2 \theta_w$ is approximately degraded by 50%. At lower energies the systematic error stemming from the measurement of the electron polarization is not important.

The bounds to be obtained for a few realistic energies and polarizations are summarized in Table I. They assume of course the validity of the luminosities stated in Eqs (16,17). For different values of the integrated luminosity the results can be easily corrected, since the statistical errors largely dominate the system-

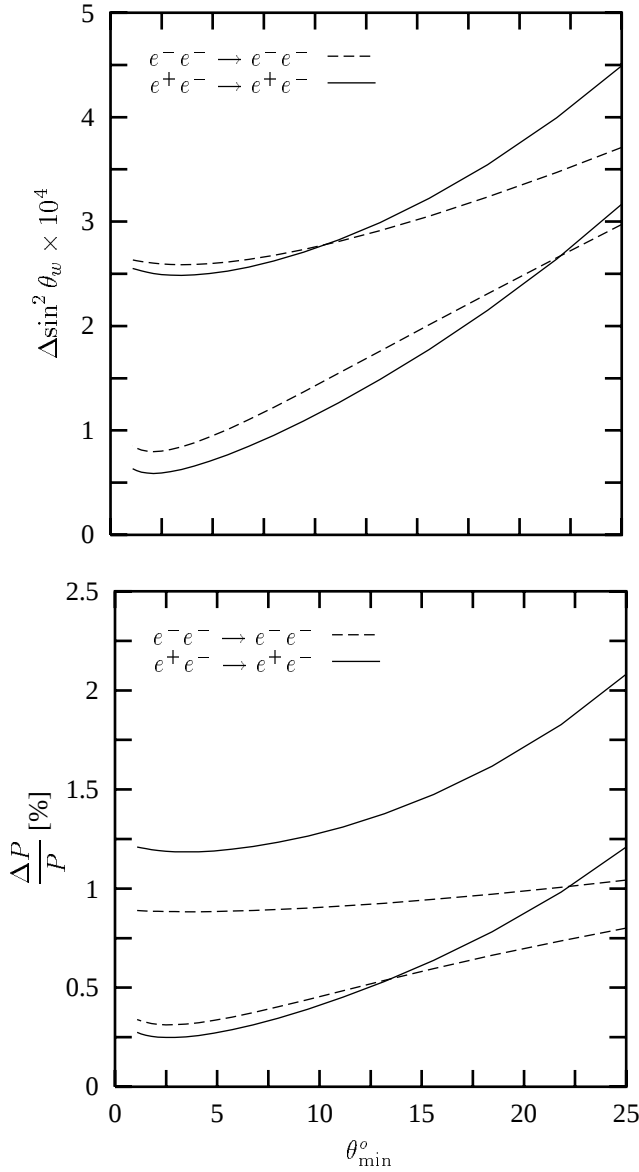


Figure 2: Polar angle acceptance dependence of the errors on $\sin^2 \theta_w$ and the beam polarizations (assumed equal) in Møller and Bhabha scattering. The upper and lower pairs of curves correspond to 500 GeV and 2 TeV center of mass energy collisions.

atic errors included here and scale like $1/\sqrt{\mathcal{L}}$. We also note that, with respect to the processes studied here, the $e^+e^- \rightarrow \mu^+\mu^-$ mode yields much less interesting bounds, about one order of magnitude worse. This is obviously expected because of the absence of the forward Coulomb peak in this case.

The present preliminary analysis has been carried out at the tree level only. Electroweak radiative corrections to Bhabha scattering off the Z^0 peak have been first calculated in [12], and updated to leading two-loop order in [13]. In general, electroweak corrections can be included in the Bhabha amplitudes by means of three complex-valued gauge invariant form factors explicitly depending on θ [13]. We expect a similar factorization

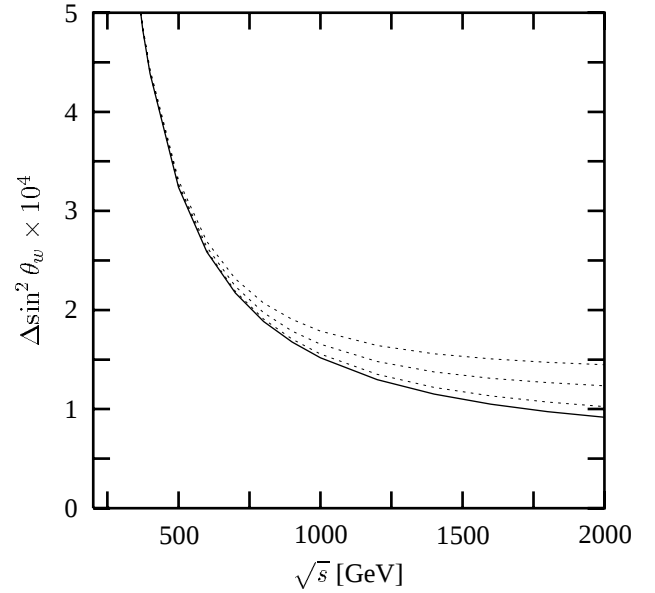


Figure 3: Energy dependence of the errors on $\sin^2 \theta_w$ in Bhabha scattering. The positron beam is unpolarized while the electron beam is polarized to 90%. This polarization is assumed to have been determined by other means. The plain curve is obtained neglecting the systematic error on the polarization measurement. The dotted curves indicate the expectations for 0.5%, 1.0% and 1.5% systematic errors on the measurement of the electron polarization.

of radiative corrections in Møller scattering, for which a full one-loop computation, to the best of our knowledge, is still missing at high energies [6]. The inclusion of these calculable radiative effects should not affect significantly our estimates of the statistical error, particularly because they are dominated by events in the forward peak, where electroweak corrections become less relevant. QED effects are generally quite sizable in large angle Bhabha and Møller scattering [14], and could in principle introduce additional uncertainties. However, soft photons and other QED effects factorize and cancel in the asymmetries, and we do not expect dramatic effects on our error estimates.

V. CONCLUSIONS

We have demonstrated how the large Bhabha and Møller scattering cross sections can be advantageously used at the NLC to measure the polarization of the incoming electron or positron beams down to the percent level or better. The method we propose measures the polarization of the interacting beams through the final states, so that it takes into account all depolarizing effects due to beamstrahlung and disruption.

Simultaneously, the value of $\sin^2 \theta_w$ can be determined in 500 GeV collisions with an absolute error of about 2.5×10^{-4} . This error can be further reduced down to 1×10^{-4} , by increasing the center of mass energy up to 2 TeV. Beyond this energy, however, there is little gain unless the luminosity is increased with respect to Eq. (16,17).

These precision measurements can be easily carried out and do not interfere with the main tasks of the NLC. To reach the above-mentioned accuracies, though, it is essential to have a good polar angle coverage of the detector as well as highly polarized beams.

If electron and positron beams can be polarized with similar efficiency, both Bhabha and Møller scattering yield very similar results. At high energies Bhabha scattering performs marginally better, because of the higher luminosity of the e^+e^- mode with respect to the e^-e^- mode (17). However, if the positron beam cannot be polarized, the resolving power of Bhabha scattering is reduced by less than 50%.

VI. REFERENCES

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reaction	$\sqrt{s}$ [TeV]	P_1	P_2	$\Delta \sin^2 \theta_w \times 10^4$	$\Delta P_1/P_1$ [%]	$\Delta P_2/P_2$ [%]
$e^+e^- \rightarrow e^+e^-$	0.5	0	0.9	3.3	—	1.5*
		0.3	0.9	5.4	2.8	3.0
		0.6	0.9	3.2	1.6	1.6
		0.9	0.9	2.5	1.2	1.2
$e^-e^- \rightarrow e^-e^-$	0.5	0.9	0.9	2.6	0.9	0.9
$e^+e^- \rightarrow e^+e^-$	2	0	0.9	1.4	—	1.5*
		0.3	0.9	1.5	0.75	0.8
		0.6	0.9	1.0	0.4	0.4
		0.9	0.9	0.8	0.3	0.3
$e^-e^- \rightarrow e^-e^-$	2	0.9	0.9	1.0	0.35	0.35

Table I: One standard deviation error bounds on the measurements of $\sin^2 \theta_w$ and the beam polarizations in Bhabha and Møller scattering for various values of energy and polarization. In the case of Bhabha scattering P_1 stands for the positron polarization. If these are not polarized, the polarization of the electrons is assumed to have been determined with a precision of 1.5% by other means (*).