Electroweak Radiative Corrections to $W$ Boson Production at the Tevatron

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ABSTRACT

We present some results of a new calculation of the $O(\alpha)$ electroweak radiative corrections to $W$ boson production at hadron colliders with special emphasis on the transverse mass distribution.

I. INTRODUCTION

Despite the remarkable success of the Minimal Standard Model (MSM) in describing elementary particle interactions at presently accessible energies, there is little direct experimental information on the mechanism which generates mass for the $W$ and $Z$ bosons. In the MSM, spontaneous electroweak symmetry breaking is responsible for mass generation. The existence of the Higgs boson in the MSM is a direct consequence of this mechanism.

Complementary to the direct Higgs boson search at colliders ($M_H > 58.4$ GeV [1] from LEP1, where $M_H$ is the Higgs boson mass), indirect information on $M_H$ can be extracted by confronting theoretical predictions for radiative corrections to electroweak observables with high precision measurements. Assuming that the MSM is valid, a global fit to the currently available data from LEP and SLC with $\alpha_s$, the top quark mass ($m_t$), and $M_H$ as free parameters yields $M_H = 146^{+112}_{-68}$ GeV [2]. Similar results have been obtained in Ref. [3]. The indirect constraints on $M_H$ are expected to improve considerably in the future with more precise measurements of the top quark mass and the $W$ boson mass ($M_W$). Presently, their world averages are $m_t = 175 \pm 6$ GeV [4] and $M_W = 80.356 \pm 0.125$ GeV [5].

In this phase space region, the soft photon approximation ($\sqrt{s} \gg m_T$) is usually valid. However, the soft photon region is defined by requiring that the photon energy in the parton center of mass frame, $E_{\gamma}$, is smaller than a cutoff $\Delta E = \delta_s \sqrt{s}/2$, where $\sqrt{s}$ is the parton center of mass energy. In this phase space region, the soft photon approximation can be used to calculate the cross section as long as $\delta_s$ is sufficiently small. Throughout the calculation the soft singularities have been regularized by giving the photon a fictitious mass. In the sum of the virtual and soft photon terms the unphysical photon region is de®ned by requiring that the photon energy in the parton center of mass frame, $E_{\gamma}$, is smaller than a cutoff $\Delta E = \delta_s \sqrt{s}/2$, where $\sqrt{s}$ is the parton center of mass energy. In this phase space region, the soft photon approximation can be used to calculate the cross section as long as $\delta_s$ is sufficiently small. Throughout the calculation the soft singularities have been regularized by giving the photon a fictitious mass. In the sum of the virtual and soft photon terms the unphysical photon term is IR finite.

Obviously, to measure $M_W$ with high precision, it is crucial to fully control higher order electroweak (EW) and QCD radiative corrections.

In this contribution we present some results of a new calculation of the $O(\alpha)$ corrections to $W$ boson production in hadronic collisions. In particular, we study the effect of these corrections on the $W$ transverse mass ($M_T(\ell\nu)$) distribution from which $M_W$ is extracted at the Tevatron. In a previous calculation [10], only the final state photonic corrections had been included, using an approximation in which the sum of the virtual and soft part is indirectly estimated from the inclusive $O(\alpha^2)$ $W \to \ell\nu(\gamma)$ width and the final state hard bremsstrahlung contribution. Our calculation includes both initial and (complete) final state corrections, as well as their interference. As a result of our calculation, the current systematic uncertainty of $\Delta M_W = 20$ MeV [11, 12] originating from EW radiative corrections will be reduced. We shall only discuss the $W \to \ell\nu$ decay channel here. More details and a discussion of the $W \to \mu\nu$ decay channel will be presented elsewhere [13].

II. THE $O(\alpha)$ CONTRIBUTION TO RESONANT $W$ PRODUCTION

When calculating the EW radiative corrections to resonant $W$ boson production the problem arises how to treat an unstable charged gauge boson consistently in the framework of perturbation theory. This problem has been studied in Ref. [14] with particular emphasis on finding a gauge invariant decomposition of the EW $O(\alpha)$ corrections into a QED-like and a modified weak part. Unlike the $Z$ boson case, the Feynman diagrams which involve a virtual photon do not represent a gauge invariant subset here. In Ref. [14], it was demonstrated that gauge invariant contributions can be extracted from the infrared (IR) singular virtual photon corrections. These contributions can be combined with the real photon corrections in the soft photon region to form gauge invariant QED-like contributions corresponding to initial state, final state and interference corrections. The soft photon region is defined by requiring that the photon energy in the parton center of mass frame, $E_{\gamma}$, is smaller than a cutoff $\Delta E = \delta_s \sqrt{s}/2$, where $\sqrt{s}$ is the parton center of mass energy. In this phase space region, the soft photon approximation can be used to calculate the cross section as long as $\delta_s$ is sufficiently small. Throughout the calculation the soft singularities have been regularized by giving the photon a fictitious mass. In the sum of the virtual and soft photon terms the unphysical photon mass dependence cancels, and the QED-like contributions are IR finite.

The IR finite remainder of the virtual photon corrections and the weak one-loop corrections can be combined to separately gauge invariant modified weak contributions to the $W$ boson production and decay process. Both the QED-like and the modified weak contributions can be expressed in terms of form factors, $F_{\text{QED}}^s$ and $F_{\text{weak}}^s$, which multiply the Born cross section [14]. The superscript $s$ in the form factors denotes the initial
state, final state or interference contributions.

The complete $O(\alpha^3)$ parton level cross section of resonant $W$ production via the Drell-Yan mechanism $q_i\bar{q}_j \to f'f(\gamma)$ can then be written as follows [14]:

$$
\frac{d\sigma^{(0+1)}}{d\hat{s}} = d\sigma^{(0)} \left[ 1 + 2\pi\beta F_{\gamma}(\hat{s}, \hat{t}) + F_{\gamma}(\hat{s}, \hat{t}) \right] + \sum_{\text{ren., final}} \left[ d\sigma^{(0)} F_{\gamma}^{\text{collinear}}(\hat{s}, \hat{t}) + d\sigma^{\text{collinear}}_{2\to3} \right],
$$

where the Born cross section, $d\sigma^{(0)}$, is of Breit-Wigner form and $\hat{s}$ and $\hat{t}$ are the usual Mandelstam variables in the parton center of mass frame. The modified weak contributions have to be evaluated at $\hat{s} = M_W^2$ [14]. The IR finite contribution $d\sigma^{\text{collinear}}_{2\to3}$ describes real photon radiation with $E_\gamma > \Delta E$.

Additional singularities occur when the photon is emitted collinear with one of the charged fermions. These collinear singularities have been regularized by retaining finite fermion masses. Thus, both $d\sigma^{\text{collinear}}_{2\to3}$ and $F_{\gamma}^{\text{collinear}} (\alpha = \text{initial, final})$ contain large mass singular logarithms which have to be treated with special care. In the case of final state photon radiation, the mass singular logarithms cancel when inclusive observables are considered (KLN theorem). For exclusive quantities, however, these logarithms can result in large corrections, and it may be necessary to perform a resummation of the soft and/or collinear photon emission terms. For initial state photonic corrections, the mass singular logarithms survive. These terms are universal to all orders in perturbation theory, and can therefore be cancelled by universal collinear counterterms generated by ‘renormalizing’ the parton distribution functions (PDF), in complete analogy to gluon emission in QCD.

To increase the numerical stability, it is advantageous to extract the collinear part from $d\sigma^{\text{collinear}}_{2\to3}$ for both the initial and final state, and perform the cancellation of the mass singular logarithms analytically. The collinear region is defined by requiring that the angle between the fermion and the emitted photon is smaller than a cutoff parameter $\delta_s$. The reduced $2 \to 3$ contribution, which comprises the real photon contribution away from the soft and collinear region, is evaluated numerically using standard Monte Carlo techniques. Our method is very similar to the phase space slicing method described in Ref. [15].

It should be noted that the analytic cancellation of the final state mass singular logarithms is only possible when realistic experimental electron identification requirements are taken into account. This will be discussed in more detail in Sec. II.B.

In the remainder of this section, we extract the collinear behavior of $d\sigma^{\text{collinear}}_{2\to3}$ for both the initial and final state, and perform the ‘renormalization’ of the PDF.

### A. Initial state photon radiation

In the collinear region, for sufficiently small values of $\delta_s$, the leading pole approximation can be used, and the initial state real photon contribution can be written as a convolution of the Born cross section with a collinear factor (see also Ref. [16]):

$$
d\sigma^{\text{init}}_{\gamma} = \int_0^{1-\delta_s} dz \, d\sigma^{(0)}
$$

i.e. an incoming parton $q_i$ with momentum $p_i$, mass $m_i$ and charge quantum number $Q_i$ splits into a parton $q_j$ with momentum $p_j$ and a photon of momentum $(1-z)p_i$. The value for the upper limit of the $z$ integral avoids overlapping with the soft photon region.

The counterterms generated by the ‘renormalization’ of the PDF can simply be derived from Ref. [17] by performing the replacement

$$
\frac{\alpha}{\pi} \rightarrow \frac{\alpha}{\pi} Q_i^2.
$$

One then finds:

$$
q_i(x_i, Q^2) = q_i(x) \left[ 1 + \frac{\alpha}{\pi} Q_i^2 \left( 1 - \log \delta_s - \log^2 \delta_s \right) + \left( \log \delta_s + \frac{3}{4} \right) \log \left( \frac{Q^2}{m_i^2} - \frac{1}{4} \lambda_{\gamma F} \right) \right] + \int_{x_i}^{1-\delta_s} \frac{dz}{z} \frac{\alpha}{\pi} Q_i^2 \left( \frac{1 + z^2}{1 - z} \log \left( \frac{Q^2}{m_i^2} \right) - \frac{1 + z^2}{1 - z} + \lambda_{\gamma F} \right)
$$

with

$$
f_{e} = 9 + \frac{2\pi^2}{3} + 3 \log \delta_s - 2 \log^2 \delta_s
$$

and

$$
f_{\gamma} = \frac{1 + z^2}{1 - z} \log \left( \frac{1 - z}{z} \right) - \frac{1}{2} \frac{1 - z}{1 - z} + 2 \log z + 3.
$$

The $q_i(x_i)$ and $q_i(x_i, Q^2)$ are the unrenormalized and renormalized parton distribution functions, respectively. The parameter $\lambda_{\gamma F}$ distinguishes between the $\overline{\text{MS}}$ ($\lambda_{\gamma F} = 0$) and the DIS scheme ($\lambda_{\gamma F} = 1$). The scheme dependent contributions $f_e$ and $f_{\gamma +}$ can be derived from Ref. [18]. $Q$ is the factorization scale.

The cross section for $p\gamma \to W(\gamma) \to lv(\gamma)$ is then obtained in two steps: first, the parton level cross section of Eq. (1) is convoluted with the unrenormalized PDF $q_i(x)$, and second, $q_i(x)$ is replaced by the renormalized PDF $q_i(x, Q^2)$ by using Eq. (3).

The initial state QED-like contribution $F_{\gamma}^{\text{initial}}$ and the collinear part $d\sigma^{\text{init}}_{\gamma}$, including the effect of mass factorization, can be grouped into a single $2 \to 2$ contribution:

$$
d\sigma^{\text{init}}_{\gamma} = \sum_{i,j} \int dz_1 dz_2 \left[ q_i(x_1, Q^2) q_j(x_2, Q^2) + (1 \leftrightarrow 2) \right] F_{\gamma}^{\text{initial}}
$$

$$
\frac{\alpha}{\pi} \left( Q_i^2 + Q_j^2 \right) \left[ \left( \log \delta_s + \frac{3}{4} \right) \log \left( \frac{Q^2}{Q_i^2} \right) + \frac{\pi^2}{6} \right] - 2 \log^2 \delta_s + \frac{1}{4} \lambda_{\gamma F} f_{\gamma +} - \log \delta_s + \frac{3}{2} + \frac{\pi^2}{24}
$$
As expected, the mass singular logarithms cancel completely.

In our calculation, we have not taken into account the QED radiative corrections to the Gripov-Lipatov-Altarelli-Parisi evolution of the PDF. This introduces an uncertainty that needs to be quantified. We will address this question in Ref. [13]. A study of the effect of QED on the evolution indicates that the change in the scale dependence of the PDF is small [19]. To treat the QED radiative corrections in a consistent way, they should be incorporated in the global fitting of the PDF.

B. Final state photon radiation

Similar to initial state radiation, the final state real photon corrections in the collinear region can be described as a convolution of the Born cross section with a collinear factor. Using the leading pole approximation one finds (see also Ref. [16]):

\[
\begin{aligned}
\sigma^{\text{final}}_{\text{coll}} &= \int_{0}^{1-\delta_{s}} dz \; d\sigma^{(0)} \\
\frac{\alpha}{2\pi} &\left\{ Q_{f}^{2} \left[ 1 + \frac{z^{2}}{1 - z} \log \left( \frac{\hat{\sigma} (1 - z)}{\hat{\pi}^{2} Q_{f}^{2} z_{f}} \right) - \frac{2z}{1 - z} \right] \\
+ \left( f \rightarrow f' \right) \right\}.
\end{aligned}
\]

(7)

When realistic experimental conditions are taken into account (see Sec. III), the electron and photon four-momentum vectors are recombined to an effective electron four-momentum vector if their separation \( \Delta R_{\gamma} \) in the azimuthal angle – pseudorapidity plane is smaller than a critical value \( R_{c}. \) If the cutoff parameter \( \delta_{s} \) is chosen to be smaller than \( R_{c} \), the integration over the momentum fraction \( z \) in Eq. (7) can then be performed analytically. In this case, the mass singular logarithms in the sum of \( d\sigma^{\text{final}}_{\text{coll}} \) and the QED-like contribution \( F^{\text{final}}_{\text{QED}} \) explicitly cancel, and one obtains:

\[
\begin{aligned}
d\sigma^{\text{final}}_{2 \rightarrow 2} &= \sum z_{1}, z_{2} \left[ \sigma_{i}(z_{1}, Q^{2}) \sigma_{i}(z_{2}, Q^{2}) + (1 \leftrightarrow 2) \right] \\
\frac{\alpha}{\pi} &\left\{ -\log \delta_{s} + \frac{3}{2} + \frac{\pi^{2}}{24} + (Q_{f}^{2} + Q_{f'}^{2}) \left[ \frac{5}{4} - \frac{\pi^{2}}{6} \right. \\
&\left. - \left( \log \delta_{s} + \frac{3}{4} \right) \log \left( \delta_{s}^{2/3} \right) \right] \right\}.
\end{aligned}
\]

(8)

The approximation [10] used so far in modeling the EW radiative corrections to \( W \) boson production at the Tevatron differs from our calculation only in the \( 2 \rightarrow 2 \) contribution. At the parton level, the difference between Eq. (8) and the approximation is given by

\[
\Delta \sigma = \frac{\alpha}{\pi} \left\{ \log \left( \frac{\hat{\sigma}}{M_{W}^{2}} \right) + 1 + \frac{\pi^{2}}{12} \\
+ (Q_{f}^{2} + Q_{f'}^{2}) \left[ \left( \log \left( \frac{\hat{\sigma}}{M_{W}^{2}} \delta_{s} \right) + \frac{3}{2} \right) \log \left( \frac{\hat{\sigma}}{M_{W}^{2}} \delta_{s} \right) \right] \\
+ \frac{\pi^{2}}{6} - \frac{\pi^{2}}{6} \right\}
\]

(9)

As we shall see, this difference has a non-negligible effect on the transverse mass distribution, and thus on the \( W \) boson mass extracted from experiment.

III. NUMERICAL IMPACT ON THE TRANSVERSE MASS DISTRIBUTION

Since detectors at the Tevatron cannot directly detect the neutrino produced in the leptonic \( W \) boson decay, \( W \rightarrow e\nu_{e}, \) and cannot measure the longitudinal component of the recoil momentum, there is insufficient information to reconstruct the invariant mass of the \( W \) boson. Instead, the transverse mass distribution of the final state lepton pair is used to extract \( M_{W}. \) In the following, we therefore focus on the \( M_{T}(e\nu_{e}) \) distribution. The following acceptance cuts and electron identification criteria [12] are used to simulate detector response:

- the uncertainty in the energy measurement is simulated by Gaussian smearing of the lepton momenta corresponding to the \( D\bar{O} \) electromagnetic energy and missing transverse energy resolution,
- the photon and electron are treated as separate particles only if \( \Delta R_{\gamma} > 0.3. \) For smaller values of \( \Delta R_{\gamma}, \) the four momentum vectors of the two particles are combined to an effective electron momentum four-vector (recombination cut). In the region \( 0.2 < \Delta R_{\gamma} < 0.4 \) the event is rejected if \( E_{\gamma} > 0.15 E_{e} \) (isolation cut),
- we impose a cut on the electron transverse energy of 25 GeV, a missing transverse energy cut of 25 GeV, and require the electron pseudorapidity to be \( |\eta| < 1.2. \)

We use the MRSA set of parton distributions [20] with the factorization and renormalization scales set equal to \( M_{W}. \) For the numerical evaluation of \( d\sigma^{\text{initial}}_{2 \rightarrow 2} \) (see Eq. (6)) we use the \( \overline{\text{MS}} \) scheme. The leading order (LO) and next-to-leading order (NLO) \( M_{T}(e\nu_{e}) \) differential cross sections are shown in Fig. 1. As can be seen, the overall effect of the \( O(\alpha) \) corrections is to reduce the cross section.

The dependence on the cutoff parameters \( \delta_{e} \) and \( \delta_{s} \) must cancel in the sum of the \( 2 \rightarrow 2 \) and reduced \( 2 \rightarrow 3 \) contributions, provided that the cutoff parameters are chosen sufficiently small such that the soft photon and leading-pole approximation are valid. This is demonstrated in Fig. 2 for the final state contributions. We have also verified the cancellation for the initial state and interference contributions.

In Fig. 3, the effect of the various individual contributions to the \( \overline{\text{EW}} O(\alpha) \) corrections on the \( M_{T}(e\nu_{e}) \) distribution is shown. The following conclusions can be drawn:
The initial state QED-like contribution uniformly increases the cross section by about 1%, but is largely cancelled by the modified weak initial state contribution.

Both the complete $O(\alpha)$ initial state contribution and the interference contribution are small and change the shape of the $M_T(e\nu)$ distribution very little. These contributions therefore, are expected to have a negligible effect on the value of $M_W$ extracted from the data.

The final state QED-like contribution significantly changes the shape of the transverse mass distribution and will, therefore, have a non-negligible effect on the value of $M_W$ extracted from the data. As for the initial state, the modified weak final state contribution reduces the cross section by about 1%, but has only a small effect on the shape of the transverse mass distribution.

The change in the shape of the $M_T(e\nu)$ distribution due to the QED-like final state corrections can be easily understood. Photon radiation reduces the energy of the final state electron and thus the transverse mass when the electron and photon momenta are not recombined.

In Fig. 4, we display the ratio of the $M_T(e\nu)$ distribution obtained with our complete NLO calculation to the one obtained by using the approximation of Ref. [10]. The dependence of the ratio on the transverse mass is described by Eq. (9). For $M_T(e\nu) < M_W$, most events originate from the region $\delta \approx M_W^2$, due to the Breit-Wigner resonance. Consequently, there is very little dependence on $M_T$ in that region. For $M_T(e\nu) > M_W$, the steeply falling cross section in the tail of the Breit-Wigner resonance favors events with $\delta \approx M_W^2$. Therefore, in this region, the terms proportional to $\log(\delta/M_W^2)$ in Eq. (9) cause a change in the shape of the transverse mass distribution. The difference in the line shape occurs in a region of the $M_T(e\nu)$ distribution which is sensitive to $M_W$, and we expect that the $W$ boson mass extracted when using the complete calculation will differ by several MeV from the value obtained using the approximate calculation. Since this difference is much smaller than the present uncertainty for $M_W$, the approximation of Ref. [10] provides an adequate description of $W$ boson production at the Tevatron for the currently available data. However, for future precision experiments, a difference of several MeV in the extracted value of $M_W$ can no longer be ignored, and the complete calculation should be used.

IV. CONCLUSIONS

We have presented the results of a calculation of the EW $O(\alpha)$ corrections to $W$ boson production at hadron colliders. Both initial and (complete) final state corrections, as well as the interference between the initial and final state corrections are included in our calculation. The initial state corrections and the interference contribution are found to be small and uniform in $M_T(e\nu)$, and are expected to have a small effect on the $W$ boson mass extracted from experimental data. As expected, the final state corrections dominate the EW radiative corrections. They significantly change the shape of the transverse mass distribution, and thus the value of $M_W$ extracted from the data.

We also compared the result of our complete calculation with that of Ref. [10] which uses an approximation to estimate the
Figure 3: The ratio of the NLO to LO $M_T(e\nu_e)$ distribution for various individual contributions: the QED-like initial or final state contributions (solid), the complete $\mathcal{O}(\alpha)$ initial and final state contributions (short dashed) and the initial–final state interference contribution (long dashed).

Figure 4: The ratio of the complete to the approximate NLO $M_T(e\nu_e)$ distribution.

The ratio of the NLO to LO $M_T(e\nu_e)$ distribution for various individual contributions: the QED-like initial or final state contributions (solid), the complete $\mathcal{O}(\alpha)$ initial and final state contributions (short dashed) and the initial–final state interference contribution (long dashed).

sum of soft and virtual final state corrections. For $M_T > M_W$, the complete and approximate NLO $M_T(e\nu_e)$ distributions differ in shape, and we expect that they will yield values for the $W$ boson mass which differ by several MeV.

Our calculation substantially improves the treatment of EW radiative corrections to $W$ boson production in hadronic collisions, and will allow to significantly reduce the systematic uncertainties associated with these corrections when the $W$ boson mass is extracted from Tevatron data.

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VI. REFERENCES