Wake-fields in Planar Dielectric-loaded Structures*

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ABSTRACT

Several promising schemes for high-gradient acceleration of charged particles in planar electromagnetic structures have been recently proposed. In this paper we investigate, by both computer simulation and theoretical analysis, the longitudinal and transverse wake-fields experienced a relativistic charged particle in a planar structure. We show that, in the limit of a large aspect ratio beam, the net deflecting wake-fields vanishes. Practical implications of this result for short wavelength advanced accelerators are discussed.

I. INTRODUCTION

The use of planar structures for advanced accelerator applications has been discussed recently in the context of both metallic, disk-loaded millimeter wave structures for linear collider applications[1] and high gradient dielectric loaded-structures excited by lasers[2]. It has been noted that this type of electromagnetic structure may have advantages over the usual axisymmetry, in ease of external power coupling, and lowered space-charge forces[2]. More importantly, it has also been speculated[1-2] that the transverse wake-fields associated with this class of structure are mitigated, thus diminishing the beam break-up (BBU) instability which typically limits the beam current in short wavelength accelerators. This instability arises from off-axis beam current excitation of dipole mode wake-fields which in turn steer trailing particles; a measure of the strength of this problem is the amplitude of the transverse wake-fields.

In this paper we show, by both theoretical and computational analysis, that the transverse wake-field amplitude is in fact diminished for asymmetric ($\sigma_x >> \lambda_0 / 2\pi > \sigma_y$) relativistic bunched beams indeed mitigate transverse wake-fields in planar structures. Indeed, in the limit of a structure and beam which is much larger in x than in y, the transverse wake-fields vanish, in analogy to the monopole modes of axisymmetric structures. We begin our analysis in Section II by considering the wake-fields of a very wide beam and structure in this limit. In Section III, we generalize the treatment of the wake-fields to analyze the effects of finite horizontal beam size through a Fourier decomposition of the beam and wakes. Finally, these results are used to illustrate the mitigating effects on multi-bunch BBU, and beam-loading phenomena in general, that a slab-symmetric, highly asymmetric beam linear accelerator design allows.

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II. THE LIMIT OF THE INFINITELY WIDE BEAM IN A PLANAR STRUCTURE

The structure considered here is a plane-symmetric dielectric-loaded geometry, with a dielectric material of permittivity $\varepsilon > 1$ ($\mu = 1$) in the regions $a \le |y| \le b$, a vacuum gap ($\varepsilon = 1$) and conducting boundaries at $|y| = b$. This structure is chosen both for ease of calculation and because of the recent interest displayed in dielectric-loaded devices[1,3-5]. In the usual fashion, the Cerenkov radiation induced wake-fields are assumed to travel in the beam propagation direction z, with phase velocity equal to the beam velocity, with both ultrarelativistic, $v_\phi = v_b \cong c$. Instead of the common Green function approach to calculation of the wake-field response, we analytically calculate the wake-fields by use of energy balance arguments, requiring only that we determine the mode characteristics of the structure, then summing the wake-field coupling of each mode with a further convolution integral over the beam current profile.

We begin by examining the limiting case of no x dependence of the structure, beam, or resultant mode. By assumption, the longitudinal and temporal dependence of the n-th mode excited by the beam is of the form $\exp[ik_n\zeta]$, where $\zeta \equiv z - ct$, and $k_n = \omega_n / c$. The excited modes in this limit are purely transverse magnetic (TM), because the existence of nonvanishing B_z implies through the condition $\vec{\nabla} \cdot \vec{B} = 0$ a nonvanishing B_y, giving rise to a horizontal force, which is forbidden by symmetry. In fact, it is straightforward to show that these conditions also imply that the net vertical force associated with such a wake-field vanishes, as $B_x = -E_y$. We shall verify below that this behavior is obtained in the limit of a very wide, yet finite sized, beam.

For the TM case we need only to consider the vertical dependence of the longitudinal electric field to determine the mode fields completely. Inside of the gap ($|y| < a$), we have

$$E_{z,n} = E_0 \exp[ik_n\zeta], \tag{1}$$

where E_0 is an arbitrary amplitude, while in the dielectric ($a < y < b$) we must have

$$E_{z,n} = A_n E_0 \exp\left[ik_n \zeta\right] \sin\left[s_{y,n}(y-b)\right] \qquad (2)$$

with $s_{y,n} = k_n \sqrt{\varepsilon - 1}$. Application of the boundary conditions at $y = a$ (continuity of E_z and D_y, which is trivially derived from E_z from the relation $\vec{\nabla} \cdot \vec{D} = 0$) allows determination of the eigenvalue of each mode through the transcendental relation

$$\cot\left(k_n \sqrt{\varepsilon - 1}(b-a)\right) = k_n a \frac{\sqrt{\varepsilon - 1}}{\varepsilon}, \qquad (3)$$

and the amplitude of the longitudinal electric field within the dielectric,

$$A_n = \csc\left[s_n(b-a)\right]. \qquad (4)$$

Once the fields have been determined, the response of the structure to the passage of an ultrarelativistic, horizontally oriented line charge of constant density ($\rho_e = \lambda\delta(y - y_0)\delta(\zeta)$) within the vacuum gap ($y_0 < a$) can be calculated by energy balance. It can be shown that for a linear wake-field[6] the net decelerating field on a line charge associated with a wake amplitude of E_0 is $E_{\text{dec}} = E_0/2$. We can equate this energy loss per unit length with the field energy per unit length left behind as $\lambda E_{\text{dec}} = \int \left(\langle u_{em} \rangle - \langle S_z \rangle\right) dy$, where the longitudinal Poynting flux $S_z = (4\pi)^{-1} E_y H_x$, and the EM energy density $u_{em} = \frac{1}{2}\left[\varepsilon\left(E_z^2 + E_y^2\right) + \mu H_x^2\right]$, with $H_x = -\varepsilon E_y$. The longitudinal wake-field behind the charge obtained from this expression is simply

$$W_{z,n} = E_{z,n} = \frac{4\pi\lambda}{a + \varepsilon A_n^2 (b-a)} \cos\left[k_n \zeta\right] \Theta(-\zeta) \ , \qquad (5)$$

where Θ is the Heaviside function which explicitly shows the causal nature of the wake-fields. It should be noted from this expression that the longitudinal wake-field is largest for the lowest frequency mode in general. For the slab-symmetric, laser-pumped accelerator proposed in Ref. 2, however, the examples given have the device operating on a higher frequency mode of the structure. This could pose a beam-loading problem, as the beam gains energy from a mode which it is poorly coupled to, in comparison to the modes which it loses energy to in the form of wake-fields.

To obtain the full wake-field response from a beam with an arbitrary current profile ($\rho_e = \lambda\delta(y - y_0) f(\zeta)$), we must perform a longitudinal convolution over the point response,

$$W_z = \sum_n \int_{\zeta}^{\infty} f(\zeta') W_{z,n}(\zeta' - \zeta) d\zeta' . \qquad (6)$$

The predictions of Eq. 6 have been verified by use of numerical simulation of the wake-fields in a planar structure, performed using a custom 2-D finite difference time domain code. The beam is assumed to be a rigid current distribution, infinitesimally thin in the vertical (y) direction and with a fixed offset from the symmetry plane $y = 0$. The beam is also taken to be ultra-relativistic and travelling in the $+z$ direction, with a gaussian longitudinal of standard deviation σ_z. The line charge density of the beam in x is normalized to $\sqrt{2/\pi}\sigma_z$ statcoul/μm, with σ_z in μm. The fields are advanced using the standard leapfrog time integration algorithm. Figure 1 shows a false-color contour map of W_z for a case similar to the infrared wavelength examples given in Ref. 2. One can clearly see the Cerenkov nature of the wake-field in the dielectric, which displays the expected propagation angle, as well as the feature that, even though the beam current is asymmetric with respect to the $y = 0$ plane, the excited longitudinal wake is nearly symmetric (after propagation away from the simulation boundary).

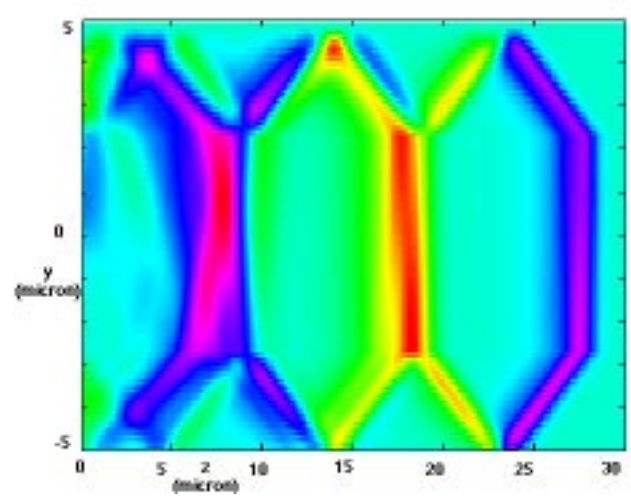

Figure 1. False color contour map of W_z for a slab-symmetric structure with vacuum gap half-height $a = 2.5$ μm, dielectric (of permittivity $\varepsilon = 4$) in the regions $a < |y| < b$ between the gap and the conducting boundaries at $|y| = b = 5$ μm, from time-domain electromagnetic field solver. The ultra-relativistic beam distribution is infinite in x, with line charge density $\lambda = dq/dx = \sqrt{2/\pi}\sigma_z$, infinitesimal in y (at $y = 1$ μm) and gaussian in z with standard deviation $\sigma_z = 0.5$ μm, centered about $z = 27.5$ μm. The color map in linear and in spectral order.

Figure 2 shows the net vertical wake-field $W_y = E_y + B_x$ excited by the beam in this case; one can see that W_y essentially vanishes inside of the vacuum

gap. Figure 3 displays a comparison between the simulation and analytical results for W_z in this case. The results are in good agreement, with some discrepancies due to the transient fields found in the time-domain simulation which are not present in the analytical treatment.

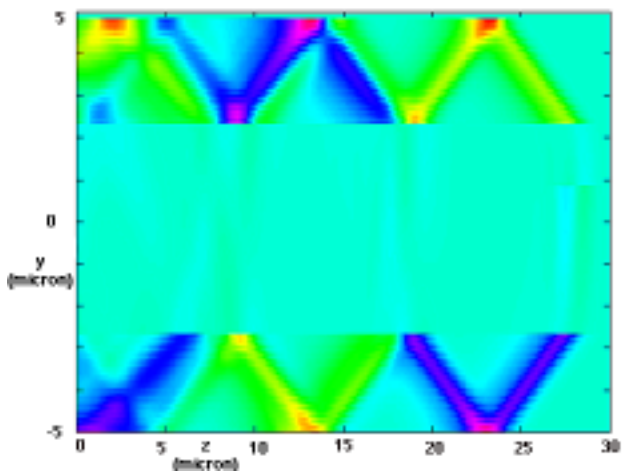

Figure 2. False color contour map of W_y for structure and beam described in Figure 1. The transverse wake-field essentially vanishes in the vacuum gap for an infinitely wide (in x) beam. The lack of full cancellation of $E_y + B_x$ is due mainly to the electric and magnetic field centers in the calculation being one-half of a spatial and time step apart.

The wake-fields due to a beam of finite horizontal extent can be found by generalizing the above analysis to beams of horizontally harmonic charge profile, *e.g.* $\rho_{e,k_x} = \lambda(k_x)\cos(k_x x)\delta(y-y_0)\delta(\zeta)$, where λ is now the peak line charge density. This profile can be viewed as a Fourier component of a finite beam, *i.e.* $\rho_e(x) = \sum \rho_{e,k_x} \cos(k_x x)$, where we implicitly assume that the waveguide now has conducting side-walls with separation in L_x (allowed wave-numbers $k_x = m\pi / L_x$, $m = 1,3,5\ldots$) and the beam distribution in x is centered and symmetric within these walls. Partial wake-fields obtained from this harmonic analysis can therefore be summed to find the complete wake-fields.

The longitudinal electric field associated with the *n*-th mode of the wake-fields that the harmonic beam can couple to has the following form in the vacuum region;

$$E_{z,n} = E_0 \exp[ik_n\zeta]\begin{Bmatrix}\cosh(k_x y)\\ \sinh(k_x y)\end{Bmatrix}\cos(k_x x). \qquad (7)$$

The $\cosh(k_x y)$ dependence indicates the monopole-like, or accelerating, component (independent of y in first order for small vertical offsets) and the $\sinh(k_x y)$ is the dipole-like, or deflecting component, which produces deflecting forces nearly independent of y for small vertical offsets.

Since the modes under consideration are not pure TM, but are hybrid modes, we must find the longitudinal magnetic field to specify all the fields. In the gap region, this field has the form

$$B_{z,n} = E_0 \exp[ik_n\zeta]\begin{Bmatrix}\sinh(k_x y)\\ \cosh(k_x y)\end{Bmatrix}\sin(k_x x). \qquad (8)$$

We again have obviously taken the ultra-relativistic limit, and in this case one must be very careful in finding the transverse components of the fields in the vacuum region. They are:

$$\begin{aligned}
E_{x,n} &= -i\frac{k_n E_0}{2k_x}\exp[ik_n\zeta]\begin{Bmatrix}\cosh(k_x y)\\ \sinh(k_x y)\end{Bmatrix}\sin(k_x x)\\
E_{y,n} &= -i\frac{k_n E_0}{2k_x}\exp[ik_n\zeta]\begin{Bmatrix}\sinh(k_x y)\\ \cosh(k_x y)\end{Bmatrix}\cos(k_x x) \qquad (9)\\
B_{x,n} &= iE_0\left(\frac{k_n}{2k_x} - \frac{k_x}{k_n}\right)\exp[ik_n\zeta]\begin{Bmatrix}\sinh(k_x y)\\ \cosh(k_x y)\end{Bmatrix}\cos(k_x x)\\
B_{y,n} &= -iE_0\left(\frac{k_n}{2k_x} + \frac{k_x}{k_n}\right)\exp[ik_n\zeta]\begin{Bmatrix}\cosh(k_x y)\\ \sinh(k_x y)\end{Bmatrix}\sin(k_x x)
\end{aligned}$$

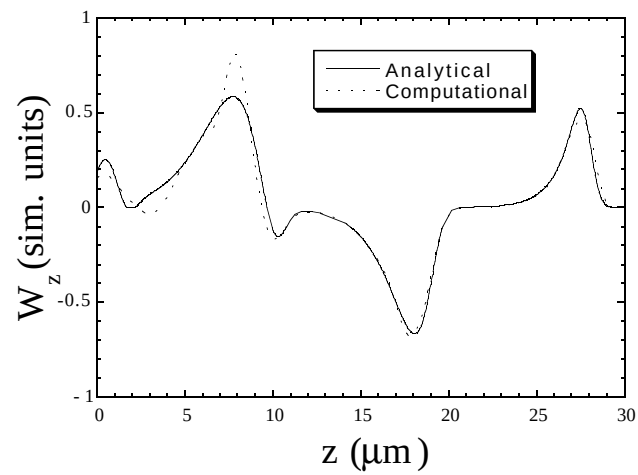

Figure 3. Comparison of the values of W_z (as a function of z at $y = 1$ μm) given by time-domain electromagnetic field solver and the predictions of Eqs. 5 and 6, for case of Figs. 1 and 2.

It should be noted that one cannot obtain the case of the uniform beam by taking the limit $k_x \Rightarrow 0$ in Eqs. 9, because these expressions were obtained by assuming that $k_x >> k_n / \gamma$, where $\gamma = (1-\beta^2)^{-1/2}$ is the Lorentz factor of the beam. One also obtains immediately from Eqs. 9 the gratifying result that the transverse forces on a relativistic particle of charge q, due to the modes described by Eqs. 8,

$$F_{x,n} = i\frac{k_x}{k_n}E_0 \exp\left[ik_n\zeta\right]\begin{Bmatrix}\cosh(k_x y)\\ \sinh(k_x y)\end{Bmatrix}\sin(k_x x)$$
$$F_{y,n} = -i\frac{k_x}{k_n}E_0 \exp\left[ik_n\zeta\right]\begin{Bmatrix}\sinh(k_x y)\\ \cosh(k_x y)\end{Bmatrix}\cos(k_x x) \quad , \quad (10)$$

vanish in the limit that $k_x \Rightarrow 0$, as we had found in the uniform line charge ($k_x = 0$) beam case. This is perhaps the primary result in this paper, simply stated; for highly asymmetric ($\sigma_x >> \sigma_y$, with associated $k_x \sim \sigma_x^{-1}$ the Gaussian beams typical of a linear collider) beams in slab-symmetric structures, transverse wake-fields are strongly suppressed. In fact, since all wake-fields are proportional to λ, the transverse wake-fields scale as σ_x^{-2}. This result mitigates one of the major objections to use of high frequency accelerating structures, that the transverse wake-fields scale prohibitively with frequency. This objection holds for cylindrically symmetric structures, but can be greatly eased by use of slab structures.

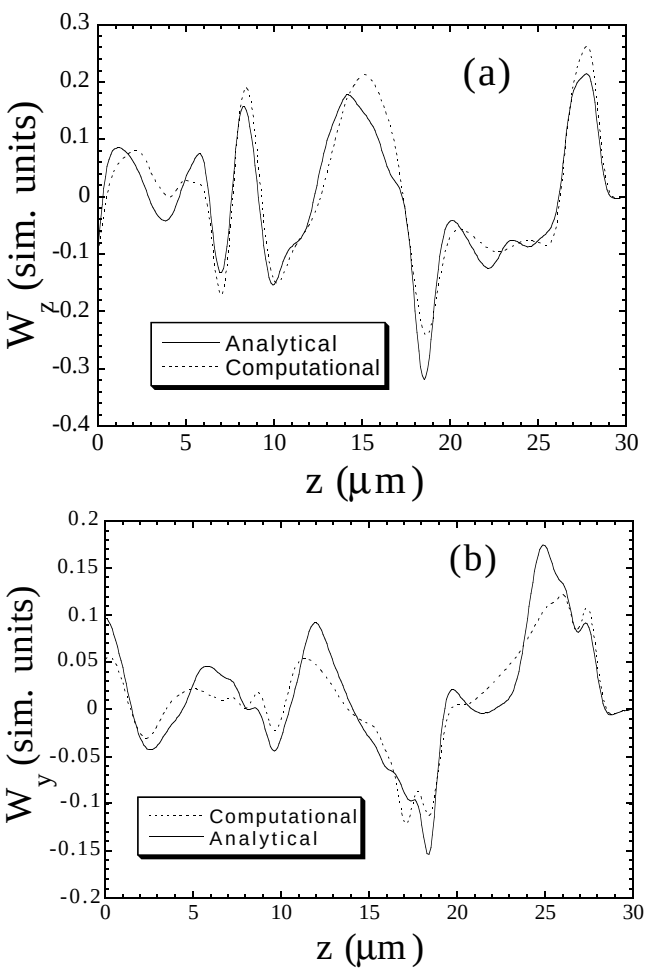

Figure 4. Comparison of the values of (a) W_z and (b) $W_y \equiv F_y / q$ (as a function of z at $y = 1$ μm) given by time-domain electromagnetic field solver and the predictions of Eqs. 6 and 12, for the identical structure and beam of Figs. 1-3, but with beam charge distribution modulated with $k_x = 0.4$ μm^{-1}.

The fields in the dielectric, unlike those in the gap, can be found by standard wave-guide analysis; for brevity their derivation is omitted. Following the same prescription used to obtain Eqs. 3-5, we obtain a transcendental expression for the eigenvalues of the symmetric modes,

$$\cot\left[\sqrt{(\varepsilon-1)k_n^2 - k_x^2}\,(b-a)\right]\coth(k_x a)\sqrt{(\varepsilon-1)\left(\frac{k_n}{k_x}\right)^2 - 1} = \frac{(\varepsilon-1)}{2\varepsilon}\left(\frac{k_n}{k_x}\right)^2 + 1. \quad (11)$$

The eigenvalues of the antisymmetric modes are simply obtained by substitution of $\tanh(k_x a)$ for $\coth(k_x a)$ in Eq. 11. The longitudinal wake-fields associated with the symmetric modes are

$$W_{z,n}(k_x) = \quad (12)$$
$$\frac{4\pi\lambda\cosh(k_x y_0)\cosh(k_x y)\cos(k_x x)\cos(k_n\zeta)\Theta(-\zeta)}{\frac{\sinh[2k_x a]}{2k_x}\left[\left(\frac{k_x}{k_n}\right)^2+1\right]+\left[\frac{\varepsilon\cosh^2(k_x a)}{\sin^2[s_n(b-a)]}+\frac{\sinh^2(k_x a)}{\cos^2[s_n(b-a)]}\right]\left[\left(\frac{k_x}{s_n}\right)^2+2\right]\left(\frac{b-a}{2}\right)+\ldots}$$
$$\frac{\sin[2s_n(b-a)]}{4s_n}\left[\left[\frac{\sinh^2(k_x a)}{\cos^2[s_n(b-a)]}-\frac{\varepsilon\cosh^2(k_x a)}{\sin^2[s_n(b-a)]}\right]\left(\frac{k_x}{s_n}\right)^2+\frac{4\varepsilon k_x s_n}{k_n^2(\varepsilon-1)}\frac{\cosh(k_x a)\sinh(k_x a)}{\sin[s_n(b-a)]\cos[s_n(b-a)]}\right]$$

those of the antisymmetric modes are obtained by substitution of $\sinh(k_x\tilde{y})$ for $\cosh(k_x\tilde{y})$ and vice versa, where $\tilde{y}$ takes on the values of $a, y,$ and y_0 in Eq. 12.

A discrete sum and convolution integral similar to Eq. 6 which sums over both symmetric and antisymmetric modes, as well as the beam charge distribution, must be performed to obtain the full wake-fields for a beam of finite extent in configuration space. The resultant expressions have also been compared to simulations of this periodic (in x) system, with W_y and W_z obtained by both methods shown in Figure 4. The algorithm in the numerical simulations used in this case is based on discretizing the Maxwell equations after Fourier transforming with respect to x. The discrepancies in the two approaches due to transient effects are slightly more pronounced in this case, but again the agreement is quite good. Parametric studies with computer simulations have also verified the suppression of transverse wake-fields for wide beams.

IV. CONCLUSIONS

In conclusion, we have theoretically and computationally analyzed the transverse wake-fields in a slab-symmetric dielectric-loaded structure. We have found a suppression of the transverse wake-fields for wide beams (with small horizontal spatial frequencies) in these structures. This result allows the more serious consideration of short wavelength advanced accelerator schemes, which have potential application to linear colliders as well as radiation producing accelerators (FELs,

Compton scattering sources), as it mitigates the scaling[7] of the transverse wakes which limit the current in cylindrically symmetric devices, from $W'_\perp \equiv \partial_y W_y \propto k_\perp^3 \propto k_0^3$ ($k_\perp$ is the transverse wavenumber of the mode, analogous to our present k_x) in the cylindrical case, to $W'_\perp \propto k_x^3 \propto \sigma_x^{-3}$ in the slab case. In short wavelength accelerators, the limitations on injector pulse length implies that a macroscopic pulse is to be captured and accelerated in the structure as a multiple micro-pulse train. For this reason, the limit on accelerated charge comes from multi-bunch beam breakup. Following the treatment by Ruth and Thompson[8], we have derived a limit on the stable propagation distance before onset of multibunch BBU instability in a channel with only ponderomotive electromagnetic focusing[2] to be $z_{\mathrm{BBU}} \cong \sqrt{\sigma_x^3 k_\perp a / N r_e}$ [9], where N is the number of electrons per microbunch and we consider only the effect of the lowest frequency asymmetric mode. The equivalent expression for a cylindrical structure is $z_{\mathrm{BBU}} \cong \sqrt{a^2 / 4 N r_e k_\perp}$, which displays the disadvantage relative to the slab case in scaling the structure to high frequency.

TABLE I. Comparison beam breakup of cylindrical and slab symmetric resonant standing wave accelerator with an average accelerating gradient of 1 GV/m, fundamental wavelength $\lambda_0 = 2\pi/k_0 = 10.6\ \mu\mathrm{m}$, $a = 2.5\ \mu\mathrm{m}$, quality factor $Q = 500$, and beam loading voltage of ten percent; only lowest frequency dipole-like mode is considered.

	SLAB CASE ($\sigma_x = 100\ \mu\mathrm{m}$)	CYLINDRICAL CASE
Average current eNc / λ_0	630 mA	16 mA
Transverse wake $W'_\perp / eN$	30 V / mm^2 / fC	10^5 V / mm^2 / fC
Stability length z_{BBU}	6.2 cm	0.52 mm

To illustrate this comparison, we give a list of parameters describing two equivalent designs with slab and cylindrical geometry, respectively, in Table 1. In both designs, we consider a resonant standing wave (to provide ponderomotive focusing) accelerator with an average accelerating gradient of 1 GV/m, wavelength $\lambda_0 = 2\pi/k_0 = 10.6\ \mu\mathrm{m}$, $a = 2.5\ \mu\mathrm{m}$, quality factor $Q = 500$, and beam loading voltage of ten percent. It can be seen that in the slab case, with a beam width of $\sigma_x = 100\ \mu\mathrm{m}$, much higher average current can be accelerated at this beam loading level, and it can be propagated stably longer. These advantages, we believe, present significant motivation for further work in this field; plans have been made to test these results using asymmetric, high charge beams, producing cm wavelength wake-fields, in the near future at the Argonne Wake-field Accelerator facility[10].

Extensions of this theoretical work on wake-fields in slab-symmetric structures is presently underway, both by these authors and A. Chao[11]. These efforts include the treatment of Gaussian beams, which produce diffractive wakes which have useful analogies to Gaussian electromagnetic modes in lasers. We have also undertaken an analysis of the flute instability[12] found in this type of system which is driven preferentially by the transverse wake-fields with $k_x \propto a^{-1}$. This instability, which causes the beam to break up into filaments (as opposed to the multibunch BBU discussed above, in the entire bunch train undergoes a dipole instability) can be stabilized by Landau damping. This implies a minimum horizontal temperature (minimum emittance, maximum beta-function) to achieve stability.

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