# A Diffractive Model for Short-Range Wakefields in Rectangular Accelerating Structures<sup>\*</sup>

S. Ben-Menahem and D. Yu

DULY Research Inc., 1912 MacArthur St., Rancho Palos Verdes CA 90275-1111

and

Stanford Linear Accelerator Center, Stanford CA 94309

Standard RF accelerating structures are cylindrically symmetrical, and consist of a drift tube periodically loaded by pillbox shaped cavities.<sup>1</sup> For such structures, Bane *et al.*, [1] have developed a simple diffractive model which estimates the short range wakefields due to a single bunch (see also a thorough discussion by Wilson [2]). The Bane *et al.*, calculation is a singlecell estimate, although it is not difficult to extend the diffractive model to the multiple cells. Since inter-cell interference tends to decrease wakefields, it is justifiable to regard the single cell wakefield estimates as upper bounds.

The requirement of high efficiency dictates that future linear colliders for particle physics (with energies of at least 2 TeV) operate at high RF frequencies (at least 30 GHz). This, in turn, necessitates smaller dimensions and tight manufacturing tolerances for accelerating structures. The combination of such manufacturing constraints and challenges, and the emergence of microfabrication techniques based on deep etch X-ray lithography, encourages a shift to accelerating structures based on *rectangular* shapes instead of the traditional cylindrical structures [3]. These rectangular structures typically have high aspect ratios. Specifically, the dimension in one of the two transverse directions (which we denote as Y here) is much smaller than in the X and Z (beam) directions. We refer to this new class of structures as *Planar Accelerating Structures*, or PAS for short.

In the present paper, we describe an extension of the Bane *et al.*, diffractive model to the PAS case. Specifically, we assume that the various sections of the accelerating structure — both drift tube and cavity — are of rectangular rather than circular shape. By `rectangular shape' is meant any polygonal shape in which all interior vertex angles are either  $90^0$  or  $270^0$ , and in which each side is along either the X or Y transverse directions.

X is commonly referred to as the *vertical* direction, while Y is *horizontal*. The drift tube axis, as well as structure periodicity, are along the Z direction.

The drift tube itself is assumed to have a uniform (z independent) cross section which is, furthermore, a simple rectangle. The PAS geometries we have investigated are symmetric under the two reflections,  $x \to -x$  and  $y \to -y$ .

We have developed simple, analytical diffractive model estimates for short range wakes. A FORTRAN code was then written to evaluate these wakefield formulae. In the remainder of the paper, we first render a brief description of our model; then display wakefield curves (for one particular nontrivial, high frequency PAS design) generated using the FORTRAN code; and finally, we compare these results with a fully numerical integration of Maxwell's equations.

# I. THE BANE-SANDS DIFFRACTIVE MODEL FOR SHORT TIME WAKES

Our single-cell diffractive model for the PAS is a straightforward adaptation of the corresponding theory for a cylindrically symmetric structure, to be found in [1] [2]. As explained in detail in [1], the single-cell diffractive model consists in the following conceptual and calculational steps:

- The Poynting flux of RF energy, incident upon a cavity through the beam tube, is evaluated (in frequency domain) using 2D electrostatic methods.
- The step between tube and cavity is treated as infinite in height, and (following Lawson) the RF energy deposited in the cavity is equated with that portion of the incident Poynting flux which is diffracted by the step into the geometric shadow (except for a factor of two that arises due to the Babinet Principle).
- The energy deposited in a cavity at a given frequency, normalized to the power spectrum of the incident bunch current, is simply related to the longitudinal wakefield which is then Fourier-transformed back to the time domain.
- The above-mentioned procedure is also used to calculate the *transverse* wakefields (per unit beam displacement in either the *x* or *y* directions), by making appropriate use of the Panofsky-Wenzel theorem.

## II. ADAPTATION OF DIFFRACTIVE MODEL TO GENERALIZED RECTANGULAR CELL

The modifications we have had to make in adapting this analytical procedure to the PAS geometry, are as follows:

(M1) The 2D electrostatics of a line charge (point charge in 2D language) inside a cross section of the beam tube, is more involved—and is analyzed via a combination of conformal transformation methods and the method of images.

<sup>\*</sup> Work supported by DOE STTR grant no. DE-FG03-94ER86007.

<sup>&</sup>lt;sup>1</sup>One may deviate from strict periodicity to achieve detuning, thus minimizing long range wakefields. However, our focus in this paper is on estimating *short* range wakes, so in the interest of simplicity we assume a periodic structure.

- (M2) For PAS cell geometries, such as the `TU' example considered below, in which the drift tube intersects different portions (x-intervals) of a cavity at different axial locations (z values), the simplifying approximation is made that the RF energy deposition in the various x-interval portions is additive.
- (M3) Care must be exercised in applying the Panofsky-Wenzel theorem to evaluate transverse wakes (see 4th bullet above), due to lack of orthogonality between the monopole and quadrupole responses to a transversely displaced beam. Specifically, we must extract the  $x^2$  ( $y^2$ ) term in the energy deposition by a beam displaced in the vertical (horizontal) direction, rather than evaluate the energy depositions of imaginary `dipole beams' in the respective transverse directions. (The two procedures are equivalent for the cylindrically symmetric structures considered in Refs. [1] [2], but are inequivalent for a PAS.)

Wakefields calculated using a single-cell model are larger than the correct multi-cell wakes, because of inter-cavity interference. Future modeling of PAS wakes should utilize a multicell diffractive model. [1] [2]

## A. 2D electrostatics for general rectangular shapes

Thanks to the `modularity' afforded by our approximation (M2), it suffices to consider a rectangular beam tube (pipe) incident upon an infinite (i.e., y-direction) step. The incident RF power (Poynting flux) is obtained by integrating the square of the modulus of the electric field (due to a unit line charge) along the edge of that step. The unit charge is either at x = y = 0 or displaced, depending on whether it is the longitudinal or transverse wakefield that is being calculated. The integration of  $|E|^2$ is along the appropriate x-segment of the beam tube.

We first consider a beam tube infinite in the  $\pm x$  directions; its boundary then consists of the two infinite lines  $y = \pm a$ , where a is the tube half-height. Let the unit charge (beam drive) be situated at x + iy = ic; c is real for a horizontally offset drive, imaginary for a vertical offset, and vanishes for a centered (non-offset) beam. Let z = x + iy. The complex z plane can be conformally mapped into a second complex plane, w, in such a way that the two-line tube boundary becomes a single infinite line (the Im(w) = 0 line) in the new plane; the requisite transformation is

$$w = \coth\left(\frac{z - ia}{4a}\pi\right) \tag{1}$$

with the transformed unit-charge source situated at the point  $w_s$ , corresponding to  $z_s = ic$ . The electrostatic potential then follows from a trivial application of the images method in the w plane:

$$\phi = \ln |w - w_s| - \ln |w - w_s^*|$$
(2)

Next, consider a rectangular beam tube. It is bounded not only by the lines  $y = \pm a$ , but also by the additional lines  $x = \pm b$ , with 2b the tube's (vertical) width. The  $E_y$  electric field component along the y = a or y = -a edge is obtained by applying  $\partial/\partial y$  to the potential of Eq. (2), and then adding to it

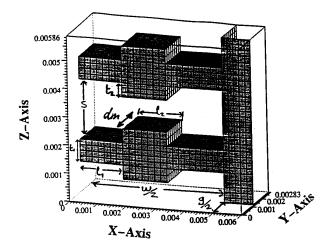


Figure 1: A quarter of single cell for the canonical TU structure. numerical values for the dimensions are shown in Table I.

an infinite series of alternating-sign image terms due to the new  $x = \pm b$  metallic boundaries. (The tangential  $E_x$  component vanishes at  $y = \pm a$ ). The resulting  $E_y$  is then squared and integrated along the entire edge (i.e. that portion of the two lines  $y = \pm a$  which constitutes the tube's intersection with a cavity). This integral, having been evaluated for unit line charge, is directly proportional to the longitudinal wakefield for the drive position  $z_s$ . In the case of a TU cell geometry — the concrete PAS geometry considered below — several different edge integrals are needed. Each is over a finite x-interval corresponding to the intersection of the corresponding cavity portion with the beam tube.

Table I: Dimensions of the canonical TU cell shown in Fig. 1.

Parameter	Loaded M.tin
w (mm)	9.8
dm (mm)	1.6
s (mm)	2.13
<i>t</i> <sub>1</sub> (mm)	0.8
<i>t</i> <sub>2</sub> (mm)	0.5
<i>l</i> <sub>1</sub> (mm)	1.8
$l_2$ (mm)	1.5
g (mm)	2.46
Frequency (GHz)	34.29
Q	2090.
$R/Q$ ( $\Omega$ /m)	11.4
$R (M \Omega/m)$	23.8
% Falloff $(E)$	0.31

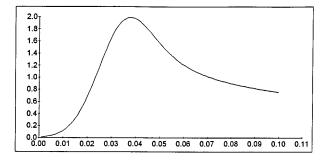


Figure 2: Theoretical (Single-cell Diffractive Model) longitudinal (z) wake in units of V/pC/mm, versus s (distance behind bunch head) in units of cm. Cell design is TU.

This exercise in 2D electrostatics, in conjunction with the theoretical framework presented above, results in expressions for the  $W_x, W_y$  and  $W_z$  short-time wakefields, for any given PAS cell geometry. These expressions involve x integrals and infinite sums. Since the integrations and summations cannot, in general, be performed in closed form, a simple FORTRAN code was written to implement them. The code also produces plottable wakefield datasets. In general, the code is applied to each portion of the tube-cavity intersection, and the results added together to yield the total wakefields (approximation (M2) above).

#### III. AN EXAMPLE

The concrete example we consider is a PAS design from [4], which we call `TU' or `tugboat' cavity. A quarter (X, Y positive) of a single cell is depicted in Fig. 1, with the dimensions given in Table I; the full cell is then obtained using the two reflection symmetries. The TU cell in Fig. 1 is for an operating frequency of 34.3 GHz, yet our theoretical wakes were computed for 11.4 GHz (all linear dimensions scaled up by the appropriate factor). The bunch length was assigned a realistic value of 100 microns. For this example, the three diffraction theoretical, short range, single-cell wakes are shown in Figures 2 through 4. Note that the X-wake is orders of magnitude below the Y-wake, as it should be due to the high aspect ratio: for the simple rectangle (a,b) discussed below eq. (2), the  $b \to \infty$ limit corresponds to vanishing  $W_x$ .

A fully *numerical* simulation of the same physics, using the general electromagnetic simulation program MAFIA3, yielded results which are quite similar to Figs. 2—4, both quantitatively and qualitatively. However, the very fine mesh needed for a reliable MAFIA3 simulation, resulted in some systematic errors which precluded a precise comparison. We feel that at this stage of the computations, the diffraction model wakefield estimates are more reliable than the corresponding MAFIA results.

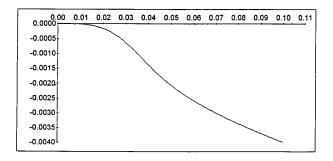


Figure 3: Theoretical (Single-cell Diffractive Model) vertical (z) wake in units of  $V/pC/(mm)^2$ , versus s (distance behind bunch head) in units of cm. Cell design is TU.

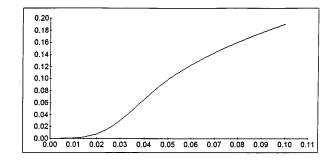


Figure 4: Theoretical (Single-cell Diffractive Model) horizontal (y) wake in units of  $V/pC/(mm)^2$ , versus s (distance behind bunch head) in units of cm. Cell design is TU.

### IV. REFERENCES

- K. Bane and M. Sands, "Wakefields of Very Short Bunches in an Accelerating Cavity', SLAC-PUB-4441, 1987.
- [2] P.B. Wilson, "Introduction to Wakefields and Wake Potentials", SLAC-PUB-4547, January 1989.
- [3] D. Yu *et al.*, "Design and Microfabrication of Planar Accelerating Structures", Proc. of the 6th Advanced Accelerator Concepts Workshop, Lake Geneva, Wisconsin, June 12-18, 1994.
- [4] H. Henke, "Planar Structures for Electron Acceleration", TET preprint; presented in PAC 1995. U.S. Patent Pending.