CRAB CROSSING IN A LARGE HADRON COLLIDER

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ABSTRACT

Since its invention by Palmer[1] in 1988, crab crossing has been explored by many people for both linear and storage ring colliders to allow for an angle crossing without a loss of luminosity. Various crab crossing scenarios have been incorporated in the design of newly proposed linear colliders and B-factory projects. For a hadron collider, this scheme can also be employed to lower β^* at the interaction point for a higher luminosity.

In this paper, we first review the principle and operational requirements of various crab crossing schemes for storage ring colliders. A Hamiltonian formalism is developed to study the dynamics of crab crossing and the related synchro-betatron coupling. Requirements are obtained for the operational voltage and frequency of the crab cavities, and for the accuracy of voltage matching and phase matching of the cavities.

For the recently proposed high-field hadron collider,[2, 3] a deflection crabbing scheme can be used to reduce β^* from 0.1 m to 0.05 m and below, without a loss of luminosity due to angle crossing. The required voltage of the storage rf system is reduced from 100 MV to below 10 MV. With the same frequency of 379 MHz operating in a transverse mode, the required voltage of the crab cavities is about 3.2$\sim$4.4 MV. The required accuracy of voltage and betatron-phase matching is about 1%.

I. INTRODUCTION

The subject of angle crossing at the interaction point (IP) of a storage ring collider has been studied for many years with the realization that the synchro-betatron resonance induced by the crossing angle severely limits the luminosity. In 1988, a beam-beam collision scheme was invented by Palmer[1]to allow a large crossing angle for a linear collider without a loss of luminosity. The Palmer scheme (or deflection crabbing scheme) employs transverse rf deflectors placed at locations where the betatron phase advance is -90° from the IP. Both colliding bunches are tilted by the cavities by half the crossing angle at the IP so that they collide head-on. Subsequently, several alternative schemes[4, 5, 6, 7] were also introduced to apply crab crossing to storage ring colliders. Recently, various crab crossing scenarios have been incorporated in the design of newly proposed linear colliders[8] and B-factory projects.[9]

The design goal of a high-field hadron collider[2, 3, 10] is a 50 TeV storage ring that can achieve a peak luminosity of 10^{34} cm$^{-2}\cdot$s^{-1} with a small number of interactions per bunch-bunch collision. With an experimental drift space of $\pm$25 m and a focusing strength of 360 T/m at the triplet quadrupoles, a β^* of 0.1 m can be achieved at the energy of 50 TeV. To enjoy the benefits of a relatively small bunch spacing, the beams must cross at an angle α of about 70μr to avoid more than one bunch-bunch collision in each experimental straight section. Such a non-zero crossing angle causes a degradation in luminosity. Consequently, it demands a short bunch length ($\sigma \approx 2$ cm) which can only be achieved with a large rf voltage (100 MV) when operating at a frequency of 379 MHz. The problem can be solved by crab crossing the two counter-circulating proton beams to make them collide head-on.

In Section II of this paper, we first summarize the principle and operational requirements of three crabbing schemes for storage ring colliders. A Hamiltonian formalism is developed in Section III to calculate the emittance growth and crabbing quality degradation produced by the errors in voltage and betatron-phase matching of the crab cavities. The results are applied to a conceptual design of angle crossing in the proposed high-field hadron collider. Conclusions and a discussion are given in Section IV.

II. CRAB CROSSING SCHEMES

The goal of crab crossing in a storage ring collider is to make the two counter-circulating angle-crossing beams collide head-on at the IP without sacrificing beam quality and luminosity lifetime. In this section, we present three schemes: deflection crabbing,[1] dispersive crabbing,[4] and η^* crabbing.[5] The angle crossing is assumed to occur in the horizontal (x–z) plane.

A. Deflection Crabbing

With the deflection crabbing scheme, as shown in Fig. 1, two transverse deflectors (rf cavities operating at their transverse

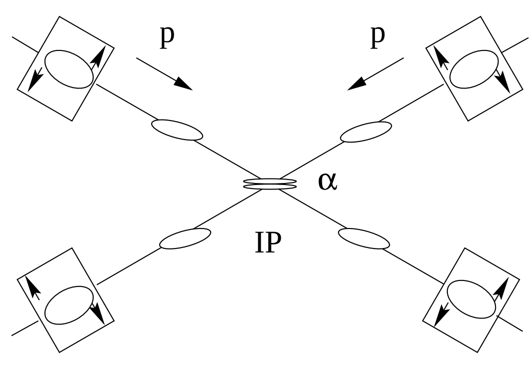

Figure 1: Schematic view of deflection crab crossing of two counter-circulating beams. The crossing angle between the two beam trajectories is α.

modes) are positioned on each side of the IP, preferably at high-β locations with betatron phase advances of ±90° from the IP. At an azimuthal location s_0 with a betatron phase of $-90°$ from the IP, the particle receives a kick in the horizontal direction x, along with a change in momentum ($\delta \equiv \Delta p/p$),

$$\begin{aligned} \Delta x' &= \frac{K_0 R}{h_c} \sin\left(\frac{h_c}{R} z\right), \\ \Delta \delta &= K_0 x \cos\left(\frac{h_c}{R} z\right), \end{aligned} \quad (1)$$

where $x' \equiv dx/ds$, z is the longitudinal displacement from the rf bucket center, $p = \beta E/c$ is the momentum, $C = 2\pi R$ is the circumference, and the strength K_0 of crabbing is related to the peak voltage V_{c0} of the cavity by

$$K_0 = \frac{qeV_{c0}h_c}{RE}, \quad (2)$$

with qe the electric charge of the particle, and h_c, an integer, the harmonic number of the cavity. If the crabbing wavelength is much larger than the bunch length, i.e.,

$$\frac{2\pi R}{h_c} \gg \sigma_z, \quad (3)$$

the kick in x' is approximately linearly proportional to the displacement z. At the IP, this kick results in a z-dependence of the horizontal displacement x. Thus, the bunch can be tilted by an angle $\alpha/2$ in the x–z plane with $\tan(\alpha/2) \approx \sqrt{\beta_0 \beta^*} K_0$, where β^* and β_0 are the β functions at the IP and the cavity (s_0), respectively. The voltage required is thus

$$qeV_{c0} = \frac{RE}{h_c\sqrt{\beta_0\beta^*}} \tan\frac{\alpha}{2}. \quad (4)$$

Obviously, for given α and β^*, a high-β location and a high operational frequency (or harmonic number h_c) is preferred for the cavity, provided that the condition Eq. 3 is satisfied. The second cavity located at s_1 with a betatron phase of $+90°$ from the IP needs to operate at a voltage

$$V_{c1} = \sqrt{\frac{\beta_0}{\beta_1}} V_{c0} \quad (5)$$

to restore the particle motion to its unperturbed state.

For the high-field hadron collider, the crab cavities can be positioned at places with $\beta_0 \approx 50$ km. With a frequency of 379 MHz, the required voltage for the transverse cavities is between 3.2 and 4.4 MV for a crossing angle α between 70 and 97 μr, as shown in Table I.

B. Dispersive Crabbing

An alternative scheme for crab crossing is to employ two *regular* (instead of *transverse deflecting*) cavities located at dispersive regions, where the betatron phase advances are ±180° from the IP. At the first cavity where the dispersion is η_0, the particle receives a kick $\Delta\delta$ in momentum,

$$\Delta\delta = \frac{qeV_0}{\beta^2 E} \sin\left(\frac{h_0}{R} z\right), \quad (6)$$

Table I: Comparison between nominal and crabbing operations.

Quantity	Unit	Nominal	Crab I	Crab II
β^*	m	0.1	0.1	0.05
α	μr	70	70	97
V_{rf}	MV	100	10	10
V_{crab}	MV	0	3.2	4.4
f_{crab}	MHz	–	379	379
σ_z	mm	22	41	41
$\mathcal{L}_{ini}$	10^{34} cm^{-2}s^{-1}	1.1	1.1	2.2

which results in an offset in the betatron closed orbit,

$$\Delta x_{\beta 0} = -\eta_0 \Delta\delta, \quad (7)$$

where V_0 and h_0 are the peak voltage and harmonic number of the rf cavity. At the IP where the dispersion is zero, the bunch is tilted by an angle $\alpha/2$, with

$$\tan\frac{\alpha}{2} \approx -\sqrt{\frac{\beta^*}{\beta_0}} \frac{\Delta x_{\beta 0}}{z}. \quad (8)$$

The voltage V_0 required is thus

$$qeV_0 = \frac{RE\beta^2}{h_0\eta_0}\sqrt{\frac{\beta_0}{\beta^*}} \tan\frac{\alpha}{2}. \quad (9)$$

A second cavity located at a place with a betatron phase of $+180°$ from the IP operates at the same voltage as the first one to restore the particle motion. The dispersion and β function at the second cavity needs to be the same as at the first one.

The dispersive scheme usually requires a large dispersion at the cavity locations along with a large operating rf voltage. For the high-field collider, the rf cavities are assumed to be positioned at places with $\beta_0 \approx 200$ m. Even with a high dispersion of $\eta_0 = 10$ m, an impractically high voltage of more than 1 GV is required for a $\alpha = 70\mu$r crossing. Furthermore, since the required dispersions at the two cavities are the same, additional dipoles are needed to make the dispersion at the IP zero. The longitudinal slippage between the two cavities produced by these dipoles will inevitably degrade the crabbing accuracy.

C. η^* Crabbing

Another scheme for crab crossing is *not* to employ any dedicated cavities. Instead, the storage rf cavity is placed near the IP, and the dispersion η^* at the IP is made non-zero. With a peak voltage V_{rf} and harmonic number h, the rf cavity changes the momentum of the particle,

$$\Delta\delta = \frac{qeV_{rf}}{\beta^2 E} \sin\left(\frac{h}{R} z\right). \quad (10)$$

Due to the dispersion η^*, the horizontal displacement at the IP resulting from this momentum change produces a tilt in the bunch with $\tan(\alpha/2) \approx \eta^* \Delta\delta/z$. The voltage required is thus

$$qeV_{rf} = \frac{RE\beta^2}{h\eta^*} \tan\frac{\alpha}{2}. \quad (11)$$

For a moderate voltage V_{rf}, this scheme often requires a large dispersion η^* at the IP. Such a dispersion effectively enlarges the horizontal beam size σ^* at the IP, which inevitably causes a degradation in luminosity. Furthermore, the dispersion modulates the beam-beam interaction to give nonlinear synchro-betatron coupling.

For the high-field collider, we assume that an rf cavity of 50 MV is located near each one of the two IPs. To achieve an angle of $\alpha = 70\mu$r, the η^* needed is more than 4 m. With a momentum spread of $\sigma_p \approx 2 \times 10^{-5}$ in the beam, such a large dispersion makes the spot size at the IP intolerably large.

III. THEORETICAL ANALYSIS

In this Section, we develop a Hamiltonian formalism to study the dynamics of deflection crab crossing, which so far is the only practical scheme for the high-field hadron collider. Using this formalism, we evaluate the sensitivity of beam quality and machine performance to errors in voltage and phase matching of the crab cavities. A matrix formalism is also introduced for the linearized system to describe the coupling.

A. Hamiltonian Formalism

The single-particle motion can be described by a Hamiltonian expressed in terms of the variables (x, p_x, y, p_y, ϕ, W; s) as

$$H = H_0 + H_1, \tag{12}$$

where $\phi = -hz/R$ is the rf phase, $W \equiv -\Delta E/h\omega_s$, ω_s is the angular revolution frequency, and $\Delta E = \beta c \Delta p$. Here, H_0 governs the unperturbed particle motion,[11, 12]

$$\begin{aligned} H_0 \;=\; & -p + \frac{1}{2p}\left(p_x^2 + p_y^2\right) + \frac{p}{2}\left(K_x x^2 + K_y y^2\right) - \\ & - \frac{h^2\omega_s^2}{2\beta^3 Ec}\left(\frac{\eta_x}{\rho} - \frac{1}{\gamma^2}\right) W^2 - \\ & - \frac{qeV}{h\omega_s}\sum_{n=-\infty}^{\infty}\delta(s - nC)\cdot \\ & \cdot\left\{\cos\left[\phi_s + \phi + \frac{hs}{R} - \frac{h}{Rp}\left(\eta_x p_x - p\eta_x' x\right)\right] + \right. \\ & \left. + \sin\phi_s \cdot \left[\phi - \frac{h}{Rp}\left(\eta_x p_x - p\eta_x' x\right)\right]\right\}, \end{aligned} \tag{13}$$

with η_x the dispersion, $\eta_x' = d\eta_x/ds$, and H_1 represents the contribution from the two crab cavities of strength K_0 and K_1 located at s_0 and s_1, respectively,

$$\begin{aligned} H_1 \;=\; & \frac{pR}{h_c} x \sin\left(\frac{h_c}{h}\phi\right)\cdot \\ & \cdot\sum_{n=-\infty}^{\infty}\left[K_0\delta(s - nC - s_0) + K_1\delta(s - nC - s_1)\right]. \end{aligned} \tag{14}$$

For simplicity without losing generality, we assume that the crab cavities operate at the same frequency as the storage rf system ($h_c = h$), and that the machine operates above transition in storage mode with the synchronous phase $\phi_s = \pi$. We average the contribution of the storage rf cavities over the circumference, and define

$$C_\phi \equiv \frac{qeV}{\pi h}, \quad \text{and} \quad C_W \equiv \frac{h^2\omega_s^2\eta}{2E\beta^2}, \tag{15}$$

where $\eta = \langle \eta_x/\rho\rangle - 1/\gamma^2$ is the slippage factor. The Hamiltonian (Eq. 13) can be expressed in terms of the action-angle variables (ψ_x, J_x, ψ_y, J_y, ψ_z, J_z; s) using a canonical transformation. The new variables are related to the old ones as

$$\begin{aligned} x,\, y \;&=\; \sqrt{\frac{2\beta_{x,y}J_{x,y}}{p}}\cos(\nu_{x,y}\phi_{x,y} + \psi_{x,y}), \\ p_{x,y} \;&=\; -\sqrt{\frac{2pJ_{x,y}}{\beta_{x,y}}}\left[\alpha_{x,y}\cos(\nu_{x,y}\phi_{x,y} + \psi_{x,y}) + \right. \\ & \qquad \left. + \sin(\nu_{x,y}\phi_{x,y} + \psi_{x,y})\right], \\ \phi \;&=\; 8\sum_{m=1,odd}^{\infty}\frac{c_m}{m}\cos\left[m\left(-\nu_z\theta + \psi_z\right)\right], \\ W \;&=\; \frac{2\pi}{\mathrm{K}(k)}\sqrt{\frac{C_\phi}{C_W}}\sum_{m=1,odd}^{\infty} c_m \sin\left[m\left(-\nu_z\theta + \psi_z\right)\right], \end{aligned} \tag{16}$$

where the betatron phase $\phi_{x,y}$ and the azimuthal angle θ are defined as

$$\phi_{x,y} \equiv \int \frac{ds}{\nu_{x,y}\beta_{x,y}}, \quad \theta \equiv \frac{s}{R}. \tag{17}$$

The nonlinear synchrotron motion of ϕ and W is reflected by the series expansion with the coefficients

$$c_m \equiv \frac{\xi^{m-\frac{1}{2}}}{1 + \xi^{m-\frac{1}{2}}}, \quad \xi \equiv \exp\left[-\frac{\pi \mathrm{K}(\sqrt{1-k^2})}{\mathrm{K}(k)}\right], \tag{18}$$

$k = \sqrt{H_0/C_\phi} \le 1$ is related to the action variable J_z by the relation

$$J_z = \oint W d\phi = 8\sqrt{\frac{C_\phi}{C_W}}\left[(k^2 - 1)\mathrm{K}(k) + \mathrm{E}(k)\right], \tag{19}$$

where $\mathrm{K}(k)$ and $\mathrm{E}(k)$ are the complete elliptical integrals[13] of first and second kind. The unperturbed synchrotron tune ν_z can be expressed in terms of the linear tune ν_{s0} as

$$\nu_z = \frac{\pi}{2\mathrm{K}(k)}\nu_{s0}, \quad \nu_{s0}\omega_s = \sqrt{C_\phi C_W}. \tag{20}$$

Using the relation[14, 15] $\sin\phi = \nu_{s0}^{-2} d^2\phi/d\theta^2$, the new Hamiltonian can be obtained as $\tilde{H}_0 = 0$, and

$$\begin{aligned} \tilde{H}_1 \;=\; & -\sum_{n=-\infty}^{\infty}\left[K_0\delta(s - nC - s_0) + K_1\delta(s - nC - s_1)\right]\cdot \\ & \cdot\frac{2\pi^2 R}{h\mathrm{K}^2(k)}\left(2p\beta_x J_x\right)^{1/2}\cos(\nu_x\phi_x + \psi_x)\cdot \\ & \cdot\sum_{m=1,odd}^{\infty} m c_m \cos\left[m\left(-\nu_z\theta + \psi_z\right)\right]. \end{aligned} \tag{21}$$

Note that the dependence on the longitudinal action J_z is contained in c_m and k.

B. Error-Induced Coupling Resonance

Eq. 21 indicates that if the crabbing is not completely compensated, synchro-betatron couplings can be excited. Suppose that $\Delta\phi$ is the deviation of the betatron phase advance from 180° between the two crab cavities, and that ΔK is the deviation of the crabbing strength from the matching value,

$$\phi_1 - \phi_0 = \pi + \Delta\phi, \quad \text{and} \quad K_1 = \sqrt{\frac{\beta_0}{\beta_1}}(K_0 + \Delta K). \quad (22)$$

Keeping only slowly varying resonance terms satisfying

$$\nu_x \pm m\nu_z = l + \epsilon, \quad \epsilon \ll 1, \quad m = \text{odd}, \quad (23)$$

and performing another canonical transformation with

$$\begin{aligned} \tilde{J}_x &= J_x, \qquad \tilde{\Psi}_x = \Psi_x, \\ \tilde{J}_z &= J_z, \qquad \tilde{\Psi}_z = \Psi_z \pm \frac{\epsilon}{m}\frac{s}{R}, \end{aligned} \quad (24)$$

the final Hamiltonian becomes

$$H_0 = \pm\frac{\epsilon}{mR}J_z, \quad (25)$$

$$\begin{aligned} H_1 &= -\sum_{n=-\infty}^{\infty}\left[K_0\delta(s - nC - s_0) + K_1\delta(s - nC - s_1)\right]\cdot \\ &\quad \cdot\frac{m\pi^2 Rc_m}{h\mathrm{K}^2(k)}\left(2p\beta_x J_x\right)^{1/2}\cos\left(\psi_x \mp m\psi_z\right), \end{aligned} \quad (26)$$

where the tildes in J and Ψ are omitted for brevity. From Eq. 26 and the canonical equations of motion, the relation

$$mJ_x \pm J_z = \text{constant} \quad (27)$$

holds near a resonance $\nu_x \pm m\nu_z = l$. Hence, the growth in action is limited for a sum (or difference) resonance above (or below) transition. However, since J_x is usually much smaller than J_z, the growth in the betatron amplitude is important even for a difference resonance above transition.[12] The change of action J_x and J_z in one revolution can be evaluated as

$$\begin{aligned} \Delta J_x &= -\frac{m\pi^2 Rc_m}{h\mathrm{K}^2(k)}\left(2p\beta_x J_x\right)^{1/2}\left[K_0\sin\left(\psi_x \mp m\psi_z\right) + \right. \\ &\quad \left. +(K_0 + \Delta K)\sin\left(\psi_x \mp m\psi_z + \Delta\phi\right)\right], \\ \Delta J_z &= \mp m\Delta J_x. \end{aligned} \quad (28)$$

Defined the rms horizontal emittance ϵ_x and longitudinal bunch area S,

$$\epsilon_x = \frac{2}{p}\langle J_x\rangle \quad \text{and} \quad S = 2\pi\langle J_z\rangle, \quad (29)$$

where $\langle\ \rangle$ denotes the average over all the particles. The growth rate of the emittance ϵ_x can be obtained as

$$\begin{aligned} \frac{1}{\epsilon_x}\frac{d\epsilon_x}{dt} &= \frac{2m\langle c_m\rangle\beta cK_0}{\pi h}\left(\frac{2\beta_0}{\epsilon_x}\right)^{1/2}\cdot \\ &\quad \cdot\left[\left(\frac{\Delta K}{K}\right)^2 + (\Delta\phi)^2\right]^{1/2}. \end{aligned} \quad (30)$$

For a difference (or sum) resonance above (or below) transition, the growth rate of the bunch area S is

$$\begin{aligned} \frac{1}{S}\frac{dS}{dt} &= \frac{2m^2\langle c_m\rangle K_0\beta^2 E}{hS}\left(2\beta_0\epsilon_x\right)^{1/2}\cdot \\ &\quad \cdot\left[\left(\frac{\Delta K}{K}\right)^2 + (\Delta\phi)^2\right]^{1/2}, \end{aligned} \quad (31)$$

where the quantity $\langle c_m\rangle$ can be approximated by[14]

$$\langle c_m\rangle \approx \left(\frac{\langle\phi_{max}\rangle}{8}\right)^m, \quad \langle\phi_{max}\rangle^2 \approx \frac{h^2\eta\omega_s S}{\pi\beta^2 E\nu_z}, \quad (32)$$

with ϕ_{max} the maximum phase of the particle synchrotron oscillation.

For a hadron storage ring, the synchrotron tune is usually small. The number m satisfying the resonance condition Eq. 23 is very large. In the case of the high-field collider, $\nu_z \approx 0.003$, $\phi_{max} = 0.18$, and $\langle c_m\rangle < 10^{-18}$ for $m > 10$. The emittance growth caused by the synchro-betatron coupling, which is excited by the crabbing errors, is negligible.

On the other hand, for an electron-electron or electron-positron storage ring collider, the synchrotron tune is often large. The mismatch in crabbing voltage and phase can excite strong synchro-betatron couplings of low order m, which results in emittance growth in both horizontal and longitudinal directions.

C. Off-Resonance Crabbing Degradation

Even though the beam is off resonance, the perturbation in action can still make the crabbing process less accurate. If $\Delta\nu_x$ is the average horizontal tune spread in the bunch, the characteristic decoherence time for the betatron motion is $\Delta\nu_x^{-1}$ turns. The off-resonance condition for an accurate crabbing is thus

$$\frac{1}{\Delta\nu_x}\frac{\Delta\epsilon_x}{\epsilon_x} \ll 1. \quad (33)$$

Using Eqs. 28, 29, and 32 with $m = 1$, this condition can be sufficiently met if the accuracy of voltage and betatron-phase matching between the two cavities satisfies

$$\left[\left(\frac{\Delta K}{K}\right)^2 + (\Delta\phi)^2\right]^{1/2} \ll \frac{2h\omega_s\Delta\nu_x}{\langle\phi_{max}\rangle\beta cK_0}\left(\frac{\epsilon_x}{2\beta_0}\right)^{1/2}, \quad (34)$$

where $\langle\phi_{max}\rangle$ is given by Eq. 32.

For the high-field hadron collider, the tune spread produced by beam-beam interactions, magnetic multipoles, etc. is of the order 10^{-3}. If the tolerable deviation in ϵ_x is 1%, the matching crabbing voltage needs to be accurate to about 1%, and the betatron phase advance between the two cavities needs to be within about 2° of 180° (Eq. 34).

D. Matrix Formalism

The coupling between the horizontal and longitudinal motion caused by imperfect crabbing can also be illustrated by a matrix formalism. Consider the one-turn matrix at the location of the deflection crabbing cavity, $-90°$ betatron phase from the IP, for variables (x, x', z, δ). The matrix can be derived by linearizing the kicks by the crabbing cavities,

$$\left[\begin{array}{cc} \mathbf{M} & \mathbf{n} \\ \mathbf{m} & \mathbf{N} \end{array}\right] \tag{35}$$

where, to the first order of error in strength ΔK and phase $\Delta\phi$,

$$\begin{aligned}
\mathbf{M} &= \left[\begin{array}{cc} \cos\mu_x + \alpha_0 \sin\mu_x & \beta_0 \sin\mu_x \\ -\alpha_0 \cos\mu_x \Delta\phi & \\ -\frac{1+\alpha_0^2}{\beta_0}\sin\mu_x & \cos\mu_x - \alpha_0 \sin\mu_x \end{array}\right] \\
\mathbf{N} &= \left[\begin{array}{cc} 1 - C\eta U + C\eta\beta_0 K_0^2 \Delta\phi & -C\eta \\ U - \beta_0 K_0^2 \Delta\phi & 1 \end{array}\right] \\
\mathbf{m} &= \left[\begin{array}{cc} C\eta(\alpha_0 K_0 \Delta\phi + \Delta K) & C\eta\beta_0 K_0 \Delta\phi \\ -\alpha_0 K_0 \Delta\phi - \Delta K & -\beta_0 K_0 \Delta\phi \end{array}\right] \\
\mathbf{n} &= \left[\begin{array}{cc} \beta_0(K_0 \cos\mu_x \Delta\phi - \sin\mu_x \Delta K) & 0 \\ -K_0(\sin\mu_x + \alpha_0 \cos\mu_x)\Delta\phi & 0 \end{array}\right],
\end{aligned} \tag{36}$$

where $U = qeV_{rf}\, h/RE\beta^2$, $\mu_x = 2\pi\nu_x$, and α_0 and β_0 are the lattice functions at s_0. The amount of global x–z coupling is often characterized[7] by the quantity

$$\begin{aligned}
\det(\mathbf{m} + \mathbf{n}^\dagger) &= K_0^2 C\eta\beta_0 \left[-(\Delta\phi)^2 \sin\mu_x + \right. \\
&\quad + \Delta\phi \frac{\Delta K}{K_0}(\cos\mu_x - \alpha_0 \sin\mu_x) - \\
&\quad \left. - \left(\frac{\Delta K}{K_0}\right)^2 \sin\mu_x \right].
\end{aligned} \tag{37}$$

For the high-field collider with a matching error of 10^{-2}, $\det(\mathbf{m} + \mathbf{n}^\dagger)$ is of the order of 10^{-10}, i.e., the global x–z coupling is very small.

IV. CONCLUSIONS AND DISCUSSION

In this paper, we have reviewed the principle and operational requirements of various crab crossing schemes for storage ring colliders. A Hamiltonian formalism has been developed to study the crabbing dynamics. Using this formalism, we evaluate the sensitivity of crabbing performance to errors in voltage and phase matching of the crab cavities. The emittance growth caused by matching error induced synchro-betatron coupling is also estimated. Requirements are obtained for the voltage and frequency of the crab cavities to achieve a head-on collision during angle crossing, and for the accuracy of voltage and phase matching in a deflection crabbing scheme.

For the high-field hadron collider, a deflection crabbing scheme can be used to reduce β^* from 0.1 m to 0.05 m and below. At the same time, luminosity degradation caused by the angle crossing is eliminated, as shown in Table I. Since a longer bunch length can be tolerated when crab crossing is employed, the required voltage of the 379 MHz storage rf system is greatly reduced from 100 MV to about 10 MV. With the same frequency of 379 MHz operating in a transverse mode, the required voltage for the crab cavities is about 3.2 MV for $\beta^* = 0.1$ m, and 4.4 MV for $\beta^* = 0.05$ m. The needed relative accuracy of voltage and betatron-phase matching is about 1%. The beam emittance growth caused by error-induced synchro-betatron coupling is negligible.

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