An IR and Chromatic Correction Design for a 2-TeV Muon Collider*

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ABSTRACT

To obtain the design luminosity of $10^{35}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$ for a 2-TeV muon collider[1], considering the projected muon production rate[1] and lifetime, demands very low β-function values at the IP; a β^* of $3\,\mathrm{mm}$. This very small β^* is particularly difficult to achieve because the superconducting magnets must be heavily shielded from the decay products of the muon beam and because space must be reserved for the detector about the interaction point, thereby reducing further quadrupole gradients which are already weak for 2-TeV muons. The Interaction Region and its chromatic correction remain one of the major challenges of the muon collider lattice design.

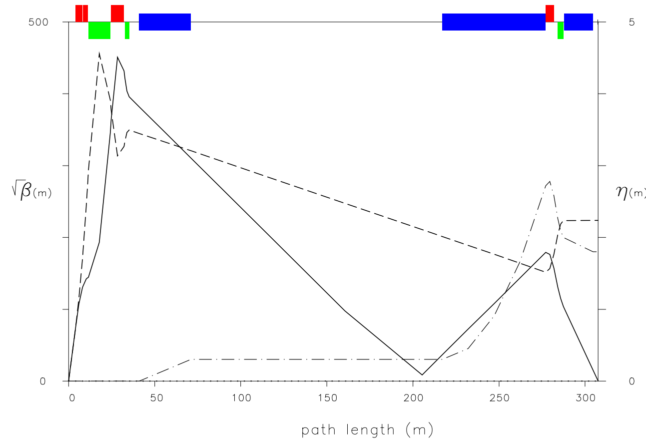

Figure 1: The IR region and match into the Chromatic Correction Module (β_x: solid line, β_y: dash-line, dispersion: dot-dash line)

I. GENERAL DESCRIPTION

Fig. 1 gives the layout and lattice functions of an Interaction Region (IR) design with a β^* of $3\,\mathrm{mm}$ for a 2-TeV muon collider. The basic design of the final focus consists of a quartet of quadrupoles, mostly superconducting, followed by a long drift and match into a chromatic correction module. At the IP, finite dispersion and, for the most part, its first derivative are undesirable for they dilute the luminosity and complicate the optics of the IR. Consequently, only linear optics have been considered in designing the final-focus system and dispersion has been specifically suppressed from the IR quadrupoles through the IP. The full IR is symmetric with reflection about the IP. Antisymmetric versions did not exhibit as much momentum aperture due to a fine cancellation of residual chromatic terms across the IP for the symmetric case.

The poletip fields of the superconducting IR quadrupoles are assumed to be 9.5 T; a value which is high, but achievable with present superconductor technology. The quadrupole nearest the IP, which is difficult to resolve in the figure, is actually a Bitter quadrupole[2] with a poletip of $4\,\mathrm{T}$. This $2\,\mathrm{m}$-long, focussing Bitter quadrupole is stationed $4\,\mathrm{m}$ from the IP and is followed by a string of superconducting quadrupoles. Because of its compact coil design it can be placed nearer the IP than superconducting magnets, and without the tungsten shielding required by superconductors[3]. By comparison, there must be at least $6\,\mathrm{m}$ between the IP and a superconducting quadrupole to maintain the detector acceptance. Permanent magnets were also considered and could, in principle, be placed even closer to the IP. However, the high radiation environment raised concern over their magnetic field lifetime and they are not used in the present IR design.

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II. THE FINAL FOCUS SYSTEM

To achieve the design luminosity, the β^* at the collision point must be only $3\,\mathrm{mm}$; the smallest β^* of any IR design to date. Because of the dynamics of the cooling process, μ^+ and μ^- emerge from the cooling stage with roughly equal emittances. The strengths of aberrations arising from nonlinearities in the final focus system increase as β^* decreases. A round beam at the IP implies, therefore, that the $\beta^*\mathrm{s}$ in both planes should be equal in order to optimally contain aberrations. Optimization of flat-beam optics, on the other hand, requires two different $\beta^*\mathrm{s}$ at the IP. Because of the smaller β^*, most aberrations and nonlinearities are more pronounced in the muon collider than in the even the Next Linear Collider[4], where the smallest β^* (for an equivalent emittance) is 1 cm[5].

This very small β^* requirement is particularly difficult to achieve because of quadrupole gradients which are inherently weak for 2-TeV muons. Increased beam sizes (as compared with linear colliders), and the shielding needed to protect the superconducting magnets reduce further the quadrupole gradients. In initial design efforts, the superconducting quadrupoles nearest the IP had to accomodate a $6\,\mathrm{cm}$ thick tungsten liner [3] in order to dissipate heat generated by the decay products of the muon beam. Recent work[6] has been successful in reducing the required shielding to $2\,\mathrm{cm}$ using combinations of sweep dipoles and collimators. With the thicker liner, quadrupole gradients were reduced by about a factor of 4 near the IP. With the thinner liner, the strength reduction is a factor of 2, since the beam size and liner thickness are comparable. Additionally, and equally important, is the large amount of magnet-free space reserved for the detector on either side of the IP. The drift from the IP to the first quadrupole completely determines the beam size at the first and, also, to a large extent, at subsequent quadrupoles. Since the beam size scales proportionally as the drift distance, s, the aperture, and, therefore, the gradient of the quadrupole nearest the IP scales as s and $\sim 1/s$, respectively (minus the tungsten liner). The apertures of subsequent quadrupoles are similarly affected.

The nonlinear performance of the IR is a function primarily of the peak β functions in the final focus quadrupoles and the gradients and lengths of these quadrupoles. The β_{max} values are determined by the value of β^*, the drift from the IP to the first quadrupole, the strength of the first quadrupole doublet, and the proximity of the IR quadrupoles to each other. For small excursions, β_{max} scales roughly as $1/\beta^*$. In addition, as implied above, β_{max} increases significantly with the length of both the IP drift and that of the first quadrupole. A compact Bitter quadrupole was used near the IP rather than a superconducting one to allow the length of the IP drift to be decreased by 30%, from $6\,\mathrm{m}$ to $4\,\mathrm{m}$, without compromising detector acceptance. The Bitter quadrupole is followed by a superconducting one, also focussing and $6\,\mathrm{m}$ from the IP. Increasing the gradient strength in this quadrupole proved to be crucial in lowering the peak β functions. The maximum gradient achievable in the superconducting quadrupoles, in particular the one nearest the IP, is greatly dependent on the amount of shielding required to protect the coils. Shadowing all IR quadrupoles with $15\,\mathrm{cm}$-long, tungsten collimators, permitted the thickness of the poletip shielding to be reduced by a factor of three[6], which resulted in a factor of two increase in gradient strength. (The tungsten collimators were set to a 4-sigma aperture and the IR quadrupoles to a 5-sigma one.) Finally, all four of the final-focus quadrupoles were spaced as close as possible ($.5\,\mathrm{m}$), leaving room for only the tungsten collimators between the elements.

Optimum conditions, insofar as nonlinearities are concerned, were obtained by both minimizing and (like β^*) equalizing the peak β functions in each plane. The peak β functions were equalized primarily by adjusting the first quadrupole doublet. Strictly speaking, the first pair of quadrupoles is not a doublet because the focussing quadrupoles nearest the IP are not strong enough to focus the beam in the horizontal plane; they merely split the beta function values sufficiently to allow the high-beta quadrupoles which follow to function as a focussing doublet. The lowest peak β function in the vertical plane is produced by finding the minimum workable strength for the first focussing quadrupole string. Practically, this point is the smallest strength that still permits the beam to be focussed in both planes by the high-beta quadrupoles. It should be mentioned, however, that quadrupole gradients were set at their maximum, and quadrupole strengths were varied by changing lengths more so than gradients. The maximum gradient in the final focus is determined not only by the collider energy and the maximum poletip field, but also by unusually large apertures due to β functions which are hundreds of kilometers.

After the first quadrupole strength was optimized, the length of the second (defocussing) quadrupole was adjusted until an equal peak β value was achieved in the opposite, or horizontal plane. The strength of the third quadrupole was adjusted

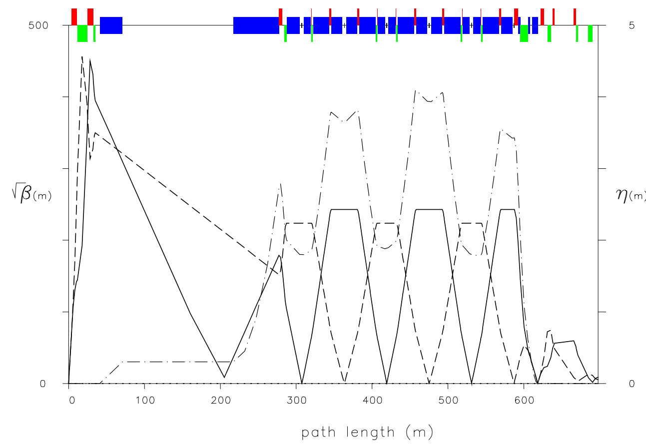

Figure 2: Experimental Insert (half) with a 3 mm beta function at the IP. (β_x: solid line, β_y: dash-line, dispersion: dot-dash line)

to fix the characteristic demagnification and focal length of the IR without changing peak β values. A fourth, defocussing quadrupole, was added because it was found to be essential to maintaining optimized quadrupole strengths while satisfying matching conditions. That is, the optical properties of the IR remain more or less fixed despite the matching conditions imposed at the ends of the IR.

This optimization procedure produced the minimum β_{max} achievable in both planes, which is approximately $200\,\mathrm{km}$ for the present IR design. (Given the parameters, peak beta-function values of several hundred kilometers are unavoidable in the collider IR.) With round-beam optics, equal $\beta_{max}s$ and minimal focussing by the final focus system were found to produce the smallest aberrations overall. Even so, peak β functions of several hundred kilometers in long, weak quadrupoles produce extremely high chromaticities; several thousand before correction. (The larger the quadrupole gradient the more compact the final focus system becomes and this is accompanied by a decrease in both the peak β-function values and chromaticities. Reaching higher poletip fields and reducing apertures are very important to the design of an IR with very low β^*s.) Once the maximum gradients of the IR quadrupoles are determined, it is then important to minimize as much as possible the overall focal length of the final-focus quadrupoles. Reducing the length, or integrated strength, of these quadrupoles significantly reduces their chromatic contributions. Ideally, the IR quadrupoles should exhibit a long, soft focus about a kilometer long, because of the extremely high peak β functions. Since the muon lifetime is a major consideration in keeping the circumference of the collider ring down, the focal length has to be much shorter; less than a couple of hundred meters in at least one plane. Higher chromatic terms, both second and third order, remain significant in the muon collider and must be corrected.

The depth of focussing by the high-beta quadrupoles determines most of the length of the IR region. The approach used is to present the softest focus in the plane with the largest chromatic behavior, and this corresponds to the plane which has its first focussing quadrupole farthest away from the IP; i.e. the vertical plane in this design. From Fig. 1 it can be seen that after the high-beta quadrupoles it is approximately $150\,\mathrm{m}$ to the horizontal waist and much further to the vertical one. A gradual decline in beta functions reduces not only first-order chromaticity but second and third orders even more – as much as two orders of magnitude from previous designs. Sharp, short waists increase the higher-order chromaticities. Fig. 2 depicts a complete half experimental insert which includes half of the IR and a Chromatic Correction Section (CCS). Although the total length of the IR and the CCS is $1.2\,\mathrm{km}$, substantial bend was incorporated to make efficient use of the long drift following the high-beta quadrupoles. Consequently, despite the longer distance to the waists, the overall circumference is still about $8\,\mathrm{km}$ [1].

Vertical and horizontal chromaticities in the IR are not equal. The chromaticity in the plane which is focussed nearest the IP is substantially smaller than in the opposite plane. Even though peak β functions are the same, the length of the high-beta quadrupole is shorter for the plane which is focussed initially. (In this design this is the horizontal plane.) With focussing quadrupoles located nearest the IP, the length ratio between the horizontal and vertical high-beta quadrupoles becomes roughly 2/3. Therefore, the first-order chromatic integral in the horizontal plane is less by approximately this factor when compared with the vertical.

The natural first-order chromaticities of the IR were minimized by lowering peak β functions and by employing quadrupole doublets. Even after optimization the chromaticity in the horizontal plane is -2000 and in the vertical it is -3000. Higher orders are minimized by extending the focal length of the high-beta quadrupoles, although they still remained large (second order chromaticity is $\sim\ 10^5$ and third order $\sim\ 10^6$). Such high chromaticities arising in the IR cannot be compensated for in the arcs and must be corrected locally about the region. (Dynamic aperture is nonexistent when chromatic corrections are performed nonlocally in the arc.) A chromatic module which corrects the large first-order chromaticities is discussed in the following section.

III. CHROMATIC CORRECTION OF THE FINAL FOCUS

Because of the strong focussing and large β functions in the final-focus quadrupoles, the chromatic effects are large and must be compensated locally with sextupole pairs. Although the β functions are high in the IR, there is no advantage to propagating dispersion through the IR and using sextupoles there. The β functions are not sufficiently different in the two planes to chromatically correct in the IR itself. A separate section, shown in Fig. 3, has been specifically designed with greatly disparate β functions which allow distinct, and independent chromatic corrections to be performed. Chromatic correction starts about $300\,\mathrm{m}$ away from the IP (Fig. 2) and requires about $300\,\mathrm{m}$ for noninterleaved sextupoles.

The chromatic correction scheme is the conventional approach of placing sextupole pairs in high-dispersion regions which are separated by a phase advance of π [7]. Originally, the

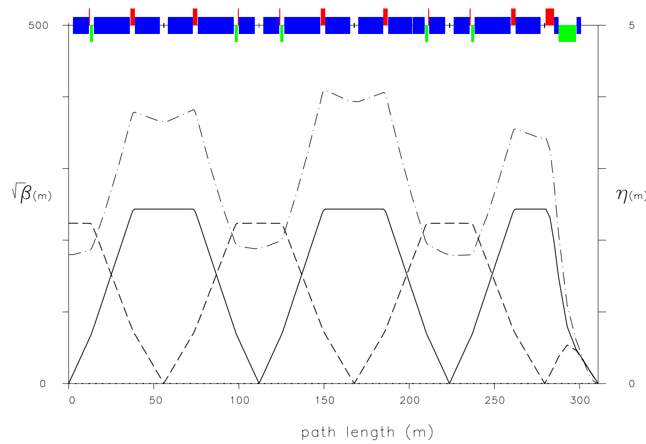

Figure 3: Experimental Insert (half) with a 3 mm beta function at the IP. (β_x: solid line, β_y: dash-line, dispersion: dot-dash line)

β function in the sextupoles were comparable to the peak β functions in the final focus quadrupoles. This reduces the sextupole strength required to correct the first-order chromaticity; however it produces a tuneshift with amplitude which is too large to sustain circulating beam. Because of the large tuneshifts, the previous design did not have acceptable dynamical aperture. Reducing the beta function in the chromatic correction sextupoles from 100 to $50\,\mathrm{km}$ opened the dynamic aperture by more than a factor of 5. (It is presently about 5σ, although there is a strong tune dependence.)

Chromatic correction is always initiated in the plane with the largest chromaticity. The plane with the largest chromaticity is the one which is defocussed in the quadrupole nearest the IP. The opposite plane is corrected immediately after. Sextupoles cannot be interleaved as it leads to large correlation terms which is indicative of insufficient separation for effective correction.

As was mentioned in the previous section, the match into the chromatic correction should be as gentle as possible avoiding deep waists. Higher order chromatic terms due to the CCS itself are reduced several orders of magnitude by lowering as much as possible peak β functions and by avoiding sharp minima (centimeter-sized waists following βs which are hundreds of kilometers).

IV. SUMMARY

Studies have been underway to improve the experimental insertion. Introducing the high-gradient Bitter quadrupole near the IP and the tungsten collimators and sweep dipoles improved dramatically effective quadrupole gradients in the IR. As a result, peak β functions decreased by almost a factor of two, and chromaticities by a factor of 3 and a factor of 2 in the horizontal and vertical planes, respectively. Higher-order aberrations were reduced by about two orders of magnitude.

Initially, the dynamic aperture of the collider ring[8] did not increase as a consequence of the IR improvements. The reason for this proved to be the CCS. Optimization work on the CCS proved to be as important as the improvements made to the IR. To maximize momentum aperture, the peak β functions in the chromatic correction sextupoles were deliberately large, which had the intended effect of reducing their strength and therefore their contribution to higher-order aberrations. However, the large β functions in the sextupoles increased significantly their contribution to amplitude-dependent tuneshifts. A better way to decrease sextupole strength (and length) is to increase the dispersion function at their locations. By increasing the dispersion and reducing the peak β functions at the sextupoles, the higher-order contributions of sextupoles to amplitude-dependent tuneshifts was minimized without increasing the higher-order aberrations significantly. When peak β functions in the CCS were reduced from $100\,\mathrm{km}$ to $50\,\mathrm{km}$, tracking showed the on-momentum aperture to increase from less than 1 to 5σ with a full momentum acceptance of $.3\%$. Results, however, were found to be strongly tune-dependent and a phase trombone was introduced into the collider ring[8] to adjust tunes independently and without disturbing the lattice. Presently a $10\,\mathrm{km}$ peak β version of the CCS structure is being tested with same final focus.

Further studies of the impact of the β functions in the chromatic correction sextupoles on aperture are in progress. After the FT and CCS optimization is complete using only sextupoles, the addition of octupoles and perhaps decapoles will be studied to further reduce the amplitude-dependent terms. Also, in future, it is hoped that the Bitter quadrupole, which has a high power consumption, can be removed if high T_c superconductor research indicates that stronger quadrupole gradients can be employed in the final focus system.

V. REFERENCES

[1] '$\mu^+\mu^-$ Collider: A Feasibility Study', The $\mu^+\mu^-$ Collider Collaboration, BNL–52503; Fermilab–Conf–96/092; LBNL–38946, July 1996.

[2] R. Weggel, presented at the Workshop on New Directions for High-Energy Physics, Snowmass 96, June-July, 1996.

[3] N. V. Mokhov and S. I. Striganov, 'Simulation of Backgrounds in Detectors and Energy Deposition in Superconducting Magnets at $\mu^+\mu^-$ Colliders', in *AIP Conference Proceedings 372 of the 9th Advanced ICFA Beam Dynamics Workshop: Beam Dynamics and Technology Issues for $\mu^+\mu^-$ Colliders*, Montauk, NY, October 15–20, 1995, pp. 234-256; also Fermilab–Conf–96/011 (1996).

[4] 'Zeroth-Order Design Report for the Next Linear Collider', The NLC Design Group, LBNL-5424; SLAC-474; UCRL-ID-124161; UC-414, May, 1996.

[5] E. Keil, presented at the Workshop on New Directions for High-Energy Physics, Snowmass 96, June-July, 1996.

[6] C. Johnstone and N. Mokhov, 'Optimization of a Muon Collider Interaction Region with Respect to Detector Backgrounds and the Heat Load to the Cryogenic Systems', to appear in the Proceedings of the Workshop on New Directions for High-Energy Physics, Snowmass 96, June-July, 1996.

[7] K. Brown, 'A Conceptual Design of Final Focus Systems for Linear Colliders, SLAC-PUB-4159, (1987).

[8] A. Garren and C. Johnstone, 'A Preliminary Lattice for a 2-TeV Muon Collider Ring', to appear in the Proceedings of the Workshop on New Directions for High-Energy Physics, Snowmass 96, June-July, 1996.