A Preliminary Lattice for a 2-TeV Muon Collider Ring*

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ABSTRACT

A ring design for a high-luminosity, 2-TeV muon collider is particularly challenging with its low beta, isochronicity, and heavy shielding requirements [1]. This paper presents a preliminary design of an entire collider ring which is intended to meet the constraints, including the technical ones, of such a collider.

I. INTRODUCTION

A 2 TeV on 2 TeV muon collider has been proposed [1] which has a design luminosity of $10^{-35}\,\mathrm{cm}^2\,s^{-1}$. For a ring with a circumference near $8\,\mathrm{km}$, the muon lifetime at 2 TeV allows for approximately 1000 turns, or 1000 collisions, before the beam luminosity degrades substantially. Conceivably, this ring could be cycled at $15\,\mathrm{Hz}$. The dynamics of the cooling process produces a round beam with a normalized emittance of approximately $50x10^{-5}\,\mathrm{mrad}$ and containing $2x10^{12}$ muons per bunch. Because of a strong hourglass effect, a short bunch length of $3\,\mathrm{mm}$ is also required. (Shorter bunches are inconsistent with the cooling dynamics.) The beam's large emittance and beam size, as compared with linear colliders, for example, means that the β^* at the IP must be exceptionally small; i.e. $3x10^{-3}\,\mathrm{m}$, in order to reach the design luminosity given the specifications. The ring design is further complicated by one additional requirement, that of isochronicity. To prevent the short $3\,\mathrm{mm}$ bunch from spreading in time, without applying substantial rf, implies that the momentum compaction factor must be 10^{-6}, or less. A highly nonlinear Interaction Region (IR) combined with the isochronicity condition make designing a lattice for a muon collider exceptionally challenging.

II. LATTICE

The lattice for a 2-TeV on 2-TeV muon collider must satisfy three major design constraints. The first and most difficult of these is provision of an Interaction Region (IR) with an extremely low β^* (~ 3 mm) consistant with an acceptable dynamic aperture. This requirement is complicated by the necessity to include considerable shielding in the superconducting magnets to protect them from the high muon-decay backgrounds [2]. This reduces their gradients and leads to higher peak β-function values. Second, the ring must exhibit a high degree of isochronicity in order to preserve short $3\,\mathrm{mm}$ long bunches with a modest rf system. Lastly, there must be small corrected chromaticity, so that the momentum-dependent tune spread of the beam does not severely restrict the momentum aperture. The following sections describe a preliminary lattice, which is intended to meet the above requirements.

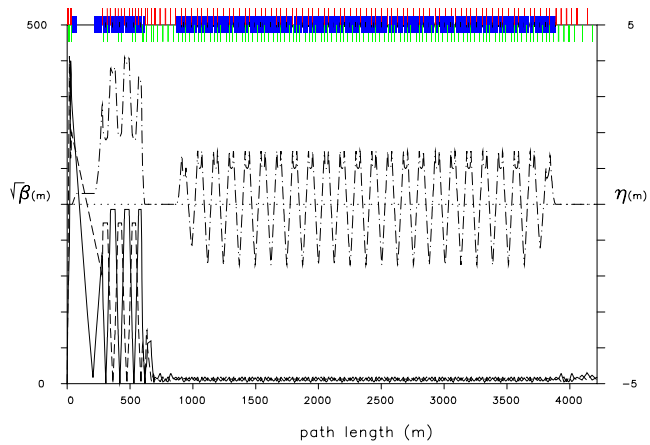

Figure 1: The lattice of half the collider ring (β_x: solid, β_y: dashed, dispersion: dot-dash)

A. Overview

The ring has an oval shape, with reflection symmetry about two perpendicular axes. The lattice has two nearly circular arcs joined by the experimental and utility insertions.

The two arcs are identical; each contains 22 periods and two dispersion suppressors. A phase trombrone is placed between the dispersion suppressors and each end of the insertions for global tune adjustments. The insertions are geometrically identical; each is symmetric about its center. Each half insertion has three parts: two straight sections separated by a bending section. The bending sections are identical in the experimental and utility insertions, except for sextupole strengths, while the straight parts have different quadrupole lengths and gradients. The focusing structure of the ring has one superperiod with reflection symmetry about the line joining the centers of the two insertions. A plot of half of the collider ring design is given in Fig. 1.

Arc module In order to have very short $3\,\mathrm{mm}$ bunches in the 2-TeV muon collider, the storage ring must be quasi-isochronous, which requires that the momentum compaction α be very close to zero[3]. Furthermore, the lattice must be designed so that over the required momentum range, the momentum compaction remains small.

In a FODO lattice α is positive. Since this ring design has bending regions in the insertions with a FODO structure whose contributions to α are positive, the contributions of the arcs must correspondingly be negative with nearly the same magnitude as those of the insertions.

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A negative value of $\alpha_{\rm arc}$ can be obtained by building an arc whose periods are FMC modules. An FMC module[4] is a structure composed of two FODO cells separated by a matching insertion which transforms $(\beta_x, \alpha_x, \beta_y, \alpha_y, D, D')$ to $(\beta_x, -\alpha_x, \beta_y, -\alpha_y, D, -D'.)$

The contribution to α of the module can be adjusted by choosing the appropriate value of D with $D' = 0$ at the end of the module. For the module design used here (see Fig. 2), the matching insertion contains two quadrupole doublets and two dipoles. The two quadrupole gradients and drift lengths are adjusted to bring α_x, α_y and D' to zero at the center of the module. The number of modules and the bending angles of the dipoles are chosen to give the entire arc the bending angle needed to close the ring.

The collider ring lattice has been adjusted to be approximately isochronous for the reference particle. That is, the lattice has been designed so that the momentum-compaction factor, $\alpha(p)$, defined by

$$\alpha(p) = \frac{p}{C}\frac{dC}{dp} \tag{1}$$

is approximately zero. In practice, in order to maintain a $3\,\mathrm{mm}$ bunch and a modest rf, $\alpha(p_0)$ must be about 10^{-6}.

However, over the desired momentum range of $\pm .004$, $\alpha(p)$ varies such that it exceeds 10^{-5}. It is therefore necessary to study higher-order terms in the momentum-compaction equation. The total $\alpha(p)$ can be expanded in powers of $\delta = p/p_0 - 1$:

$$\alpha(p) = \alpha_1 + \alpha_2\delta + \alpha_3\delta^2 + \mathcal{O}(\delta^3). \tag{2}$$

Furthermore, it is possible to correct the second-order term, α_2, in the expansion using sextupoles.

Initially, horizontal and vertical chromaticities (but not the α_2) of the arcs, experimental insertion and utility insertion were cancelled using three independent pairs of sextupole families. Alternatively, the chromaticities of the ring, most of which arise from the experimental insertion, can be cancelled by using only the insertion sextupoles. This frees the arc sextupoles to control α_2. Specifically, by inserting a horizontal sextupole next to each of the central F quadrupoles in the arc modules, the second-order, or α_2 term, can be eliminated. It is also possible to control both the linear α_2 and the quadratic α_3 coefficients by using either an additional sextupole family, or by placing the α_2-correction sextupoles in pairs separated by phase intervals of π. (Pairing sextupoles in the arcs cancels the significant sextupole contribution to the α_3 term.) Thus we conclude that control over both α_2 and α_3 can be achieved to a precision of 10^{-7}; however, this degree of correction may not be necessary (especially for α_3).

In summary, the isochronicity of the ring can be controlled precisely. The final momentum-compaction coefficients chosen for the ring will be based on rf bucket and collective instability calculations.

Dispersion suppressor A dispersion suppressor module is located at each end of the arc. The purpose of these modules is to bring the dispersion and its slope to zero values in the adjacent insertions.

The suppressor on the downstream end just before an insertion is shown in Fig.3; the upstream suppressor is obtained by reflection. This suppressor module is identical to a regular module except that the first four dipoles have been replaced by two dipoles with normal length and different field values. The missing dipoles have been replaced by drift spaces so that the quadrupoles and sextupoles are not changed.

Phase Trombone The dynamic aperture of the muon collider was found to be sensitive to the global tune of the ring. Therefore, a phase trombone was introduced at each end of the experimental and utility insertion to adjust the tune independently in the horizontal and vertical plane without disturbing the rest of the lattice. By simply adjusting the tune, the dynamic aperture of the ring could be varied between approximately one and five sigma.

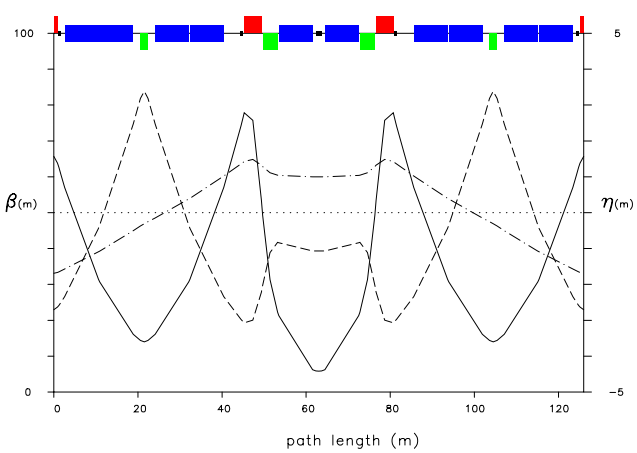

Figure 2: Lattice functions of an arc module (β_x: solid, β_y: dashed, dispersion: dot-dash).

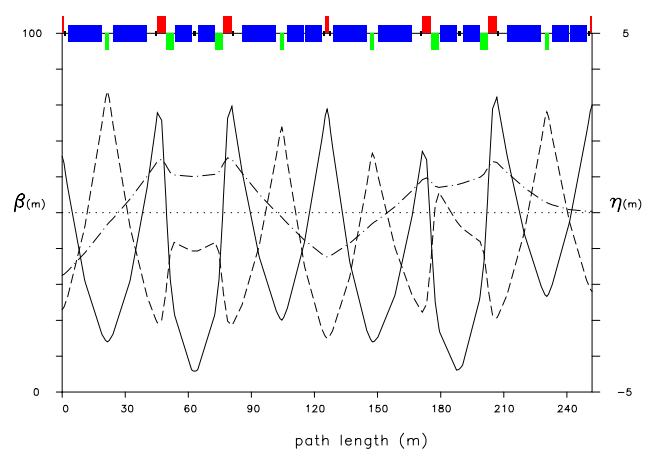

Figure 3: Dispersion Suppressor Module (β_x: solid, β_y: dashed, dispersion: dot-dash)

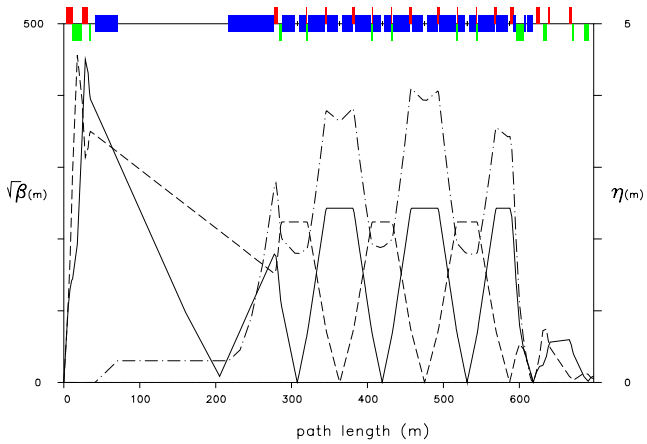

Figure 4: The experimental insert (half) with 3 mm beta functions at the IP (β_x: solid, β_y: dashed, dispersion: dot-dash)

Experimental insertion The design of an insertion with an extremely low-beta interaction region for a muon collider presents a challenge similar to that encountered for the Next Linear Collider (NLC)[5]. The design used here for each half of the symmetric low-beta insertion follows the prescription proposed by Brown[6]; it consists of two telescopes with a chromatic correction section between. Therefore, the experimental insertion consists of three parts: the IR, or Final Focus Telescope (FFT), a Chromatic Correction Section (CCS), and a Matching Telescope (MT). Fig.4 shows the right half of the insertion, starting at the end of the phase trombrone and ending at the IP.

The low beta-function values at the IP are obtained with four strong quadrupoles in the FFT, most of which are superconducting. The quadrupole nearest the IP, is actually a Bitter quadrupole[7] with a poletip of $4\,\mathrm{T}$ which can be placed closer to the IP than superconducting magnets. The $2\,\mathrm{m}$-long, focussing Bitter quadrupole is stationed $4\,\mathrm{m}$ from the IP and is followed by a string of superconducting quadrupoles. The IR quadrupoles are then followed by a long drift and match into a chromatic correction module. Substantial bend was incorporated into this drift in order to make an efficient transition into the CCS. Only linear optics have been considered in designing the final-focus system and dispersion has been specifically suppressed from the IR quadrupoles through the IP. In the present IR design, β_{max} in both planes is $200\,\mathrm{km}$. A more detailed description of the experimental insertion, and, in particular, the FFT can be found in these proceedings[8].

The extremely high beta values in the FFT quadrupoles produce large chromaticities which must be corrected locally with sextupoles. The natural chromaticity of the FFT is -2000 in the horizontal and -3000 in the vertical. The purpose of the CCS, then, is to correct these large first-order chromaticities locally about the IR by using noninterleaved sextupoles pairs. The sextupoles are located at positions with large values both in the dispersion and in the beta functions. In this design, β at the sextupoles is $50\,\mathrm{km}$ in the plane being corrected. The dispersion is $3.7\,\mathrm{m}$ at the horizontal sextupoles and $1.8\,\mathrm{m}$ at the vertical ones. The sextupoles which comprise each pair are separated by betatron-phase intervals of $\phi = \pi$. Additionally, they are located at positions where the phase interval from the IP is an odd multiple of $\pi/2$. This sextupole arrangement cancels the second-order geometric aberrations of the sextupoles, which reduces the second order tune shift by several orders of magnitude. The horizontal-correction sextupole pair is farthest from the IP, and the vertical-correction pair is closest since the chromaticity in the vertical is much larger.

The Matching Telescope (MT), on the far right of the figure, brings the beta functions from the phase trombone to a focus of a few centimeters, matching to the CCS.

Utility insertion Changes will be made to the utility insertion, Fig. 5, to accommodate systems for injection, RF, and scraping as needed.

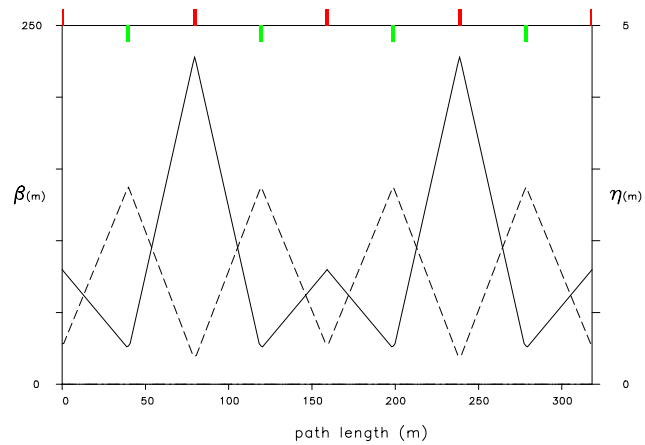

Figure 5: The utility insertion (β_x: solid, β_y: dashed, dispersion: dot-dash)

B. Summary

Studies have been underway to improve the experimental insertion. Improvements to the IR concentrated on reducing its chromaticity which in turn determines its nonlinear characteristics. A more detailed discussion of the improvements can be found in these proceedings[8].

Initially, the dynamic aperture using the improved experimental insertion did not increase as a consequence of the IR improvements; it remained less than 1 sigma for on-momentum particles. The reason for this proved to be the CCS. Optimization work on the CCS proved to be as important as the improvements made to the IR. When peak beta functions in the CCS were lowered from 100 to 50 km, dispersion was raised at the insertion sextupoles, and the entire lattice was globally tuned using a phase trombone, tracking showed that the dynamic aperture increased to 5 sigma. Presently a $10-\mathrm{km}$ version of the CCS with same final focus structure is being tested.

The momentum bandwidth of the system is limited by third-order aberrations and residual second-order amplitude-dependent tune shifts. These aberrations arise from: a) small

phase errors between the sextupoles and the final quadruplet; b) finite length of the sextupoles. After the FFT and CCS optimization is considered complete using only sextupoles, the addition of octupoles and perhaps decapoles will be studied to further reduce the amplitude-dependent terms. The third-order aberrations may require even higher multipole correctors. Also, in future, it is hoped that the Bitter quadrupole, which has a high power consumption, can be removed if high T_c superconductor research indicates that we can employ stronger quadrupole gradients in the final focus.

III. REFERENCES

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