Coherent Synchrotron Radiation in the Isochronous Muon Collider Ring*

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ABSTRACT

We show that the coherent (shielded) synchrotron radiation is negligible in a 4 TeV c-of-m muon collider.

I INTRODUCTION

To achieve the luminosity of $\mathcal{L} = 10^{35}\,\mathrm{cm}^{-2}s^{-1}$ in a $\mu^+\mu^-$ collider [1], two bunches per sign of $N = 2\times10^{12}$ particles each and a betatron function of $\beta^* = 3\,\mathrm{mm}$ at the interaction point (IP) are required. This small β^* at the IP constrains the size of the bunch to be $\sigma_z \sim \beta^*$. To maintain this rather short bunch without excessive rf power consumption, an isochronous lattice has been chosen for the final collider ring. [2] One of the important advantages of muons as opposed to electrons is that at up to at least 2 TeV energy it is possible to accelerate muons in circular machines as their synchrotron radiation is reduced by a factor of $(\frac{m_e}{m_\mu})^2 \sim 23\times10^{-6}$ with respect to electrons. Nevertheless, the large number of muons in a short bunch suggests the possibility of strong shielded coherent synchrotron radiation. First, we use the well known formulae[3][4] to evaluate the power of shielded coherent synchrotron radiation in the isochronous muon collider ring. Finally, following the results obtained by Kheifets and Zotter [5] for a bunch with a Gaussian longitudinal charge distribution we show that the coherent synchrotron radiation in the isochronous $\mu^+\mu^-$ collider ring is negligible if the rms bunch length is larger than ≈ 0.3 mm.

II INCOHERENT SYNCHROTRON RADIATION

The main parameters of the present design of a muon collider ring are shown in Tb. I. To calculate the incoherent radiation power we start by recalling the Larmor formula for a circular machine [5];

$$P_\gamma\left[\frac{GeV}{s}\right] = 8.8575\times10^{-5}\frac{c}{2\pi}\frac{E^4\,[GeV]}{\rho\,[m]}\left(\frac{m_e}{m_\mu}\right)^4 \quad (1)$$

is the instantaneous radiation power per particle. For a beam of N_e particles the total synchrotron (incoherent) radiation power per turn is

$$P_\gamma\,[MW] = 8.8575\times10^{-2}\frac{E^4\,[GeV]}{\rho\,[m]}\left(\frac{m_e}{m_\mu}\right)^4 I\,[A] \quad (2)$$

where the current $I[A] = eN_\mu\frac{c\beta}{2\pi\rho} \approx 13.75\,\mathrm{mA}$.

For the $4\,\mathrm{T}$ in center-of-mass muon collider ring, $P_\gamma \simeq 10kW$ which it corresponds to $\sim 8.5\mathrm{W/m}$. This has to be compared with 1 KW/m produced by muon decay [6] (electrons and electron synchrotron radiation); therefore the shielding of the super-conducting magnets is determined by the products of muon decay ($\mu^\pm \to e^\pm + \nu_e + \nu_\mu$). Although the energy is deposited mainly at 90 and 270 azimuthal angle, the simplest solution to bring the energy deposition on the super-conducting coils to safe levels is to include a 6 cm radius of Tungsten liner in the beam pipe [7]. Any increase in the synchrotron radiation would thus be rather expensive and/or unmanageable from a cooling power point of view or realistic pole-tip fields. Therefore, it is natural to study the possibility of coherent synchrotron radiation.

Table I: High energy-high luminosity $\mu^+\,\mu^-$ collider

Maximum c-m Energy [TeV]	4
Luminosity $\mathcal{L}[10^{35}\mathrm{cm}^{-2}\mathrm{s}^{-1}]$	1.0
Circumference [km]	8.08
Time Between Collisions [μs]	12
Energy Spread σ_E[units 10^{-3}]	2
Pulse length σ_z[mm]	3
Free space at the IP [m]	$\pm$6.25
Luminosity lifetime [No.turns]	900
Horizontal betatron tune, ν_x	55.79
Vertical betatron tune, ν_y	38.82
rms emittance, $\epsilon_{x,y}$ [$10^{-6}\pi$ m-rad]	0.0026
rms normalized emittance, $\gamma\epsilon_{x,y}$ [$10^{-6}\pi$ m-rad]	50.0
Beta-function values at IP, $\beta^*_{x,y}$ [mm]	3
rms Beam size at IP [μm]	2.8
Quadrupole pole fields near IP [T]	6.0
Peak beta-function, $\beta_{x\max}$ [km]	284
Peak beta-function, $\beta_{y\max}$ [km]	373
Magnet Aperture closest to IP [cm]	12
Beam-Beam tune shift per crossing	0.05
Repetition Rate [Hz]	15
rf frequency [GHz]	1.3
rf voltage [MeV]	130
Particles per Bunch [units 10^{12}]	2
No. of Bunches of each sign	2
Peak current $\mathcal{I} = eNc/\sqrt{2\pi}\sigma_z$ [kA]	12.8
Average current $\mathcal{I} = eNc/\mathrm{Circum}$ [A]	0.032
Bending Field [T]	8.5

Before discussing coherent radiation we recall some useful expressions from Jackson [8]. First, the critical frequency is

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given by:

$$\omega_c = \frac{3}{2}\gamma^3\frac{c}{\rho} \quad , \quad \epsilon_c[keV] = 0.665E^2[GeV]B[T]\left(\frac{m_e}{m_\mu}\right)^2 \tag{3}$$

and harmonic number[9] $n_c = 3\gamma^3$.

Second, assuming a Gaussian angular distribution of the radiation (θ is the angle between the observation direction and the tangent to the trajectory of the particle at the moment of emission) then the rms divergence is:

$$\sigma_\theta \approx \begin{cases} \frac{1.07}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3} & \omega \ll \omega_c \\ \frac{0.64}{\gamma} & \omega = \omega_c \\ \frac{0.58}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/2} & \omega \gg \omega_c \end{cases} \tag{4}$$

Notice that at low frequencies ($\omega \ll \omega_c$) the radiation is much wider than the average $\sigma_\theta \sim \frac{1}{\gamma}$ and that at high frequency ($\omega \gg \omega_c$), the angular range is much narrower than the average. This concludes this brief summary of very well known results of incoherent synchrotron radiation.

III COHERENT SYNCHROTRON RADIATION

A Free space

Coherent radiation at wavelength λ is obtained if the relative distance between particles is approximately of order λ. In this case the radiation fields emitted by these particles interfere constructively and the resulting field is the radiation of a representative particle (charge e) multiplied by the total change of the ensemble ($\sum_{j=1}^{N_\mu} e = eN_\mu$). This leads to the characteristic N_μ^2 behavior of coherence.

If the longitudinal length of the bunch of muons is σ_μ, then the condition for coherence in free space is

$$\sigma_\mu \lesssim \frac{\lambda}{2\pi} \equiv \frac{c}{\omega} \tag{5}$$

To understand this fact we refer the reader to Jackson[10]. Assume a set of N_μ muons moving in a circle with fixed relative positions around the circle, they produce a power

$$P = P_n\left|\sum_{i=1}^{N_\mu} e^{-in\phi_i}\right|^2 = P_nN_\mu + N_\mu(N_\mu - 1)P_nf_n \tag{6}$$

where P_n is the incoherent power emitted by one particle in the frequency $\omega = n\omega_0$ and f_n is a form factor

$$f_n = \left(\int d\phi\cos(n\phi)S(\phi)\right)^2 \tag{7}$$

where $S(\phi)$ is the longitudinal distribution of the bunch and $\omega_0 = c/\rho$ is the revolution frequency. For a Gaussian distribution with rms bunch length σ_s

$$S(\phi)d\phi = \frac{1}{\sqrt{2\pi}}\left(\frac{\rho}{\sigma_s}\right)e^{-\frac{\phi^2\rho^2}{2\sigma_s^2}} \tag{8}$$

and after an elementary integration

$$f_n = e^{-\frac{n^2\sigma_s^2}{\rho^2}} \tag{9}$$

From this expression we can derive the coherent condition in Eq.3. The second term in Eq.5 gives the power of coherent radiation $N_\mu(N_\mu - 1) \simeq N_\mu^2$, i.e. proportional to the square of the number of particles or equivalently to the square of the bunch current I_n at the n-th harmonic. This dependence suggests to assign an effective impedance to the coherent radiation following Ohm's law $P_n = Z_nI_n^2$.

In our case, $\lambda \gtrsim 1.8\,\mathrm{cm}$, hence is comparable to the transverse size of the beam pipe. Under this condition the coherent radiation and its characteristics are substantially changed by the presence of metallic walls. The induced currents tend to decrease the radiation fields created by the bunch (destructive interference). This fact is denoted as *shielding*[11],[12]

B Metallic boundaries

In the presence of metallic boundaries the image charges and currents created by the beam itself (self induced fields) have the potential to destructively interfere with the radiation field emitted by the beam; this effect is clearly more effective the closer the beam to the wall, i.e. the smaller is the dimension of the beam pipe.

The condition for coherent synchrotron radiation can be made plausible following the arguments from Kheifets and Zotter[11].

Assume a beam moving between 2 parallel conductive plates; the tangential radiation field must vanish at the conductor, hence $k_\perp = \mid k \mid \sigma_\theta$ with $\mid k \mid^2 = \mid k_\perp \mid^2 + \mid k_\parallel \mid^2$. At the surface $k_\parallel = 0$ then $\mid k \mid^2 = \mid k_\perp \mid_S$. The cutoff wave vector due to the waveguide is $\frac{\pi}{h}$, therefore $k_\perp > \frac{\pi}{h}$ or

$$\sigma_\theta > \frac{\pi\rho}{nh} \tag{10}$$

At the end of the previous section, Eq.5 we recalled that $\sigma_\theta \approx 1.07\left(\frac{3}{2n}\right)^{1/3}$, therefore

$$n > \frac{1}{1.07}\sqrt{\frac{2}{3}}\left(\frac{\pi\rho}{h}\right)^{3/2} \equiv n_{\mathrm{th}} \tag{11}$$

hence the possible coherent radiation harmonic number is limited from below. Now from Eq.8 the condition for coherence limits the harmonic number from above, i.e.

$$n < n_{\mathrm{coh}} = \frac{\rho}{\sigma_s} \tag{12}$$

Combining Eqs.11 and 12 we get approximately (up to small numeric factors)

$$n_{\mathrm{th}} \equiv \sqrt{\frac{2}{3}}\left(\frac{\pi\rho}{h}\right)^{3/2} < n < \frac{\rho}{\sigma_s} \equiv n_{\mathrm{coh}} \tag{13}$$

or in terms of wavelength

$$\sigma_s < \lambda < \frac{h}{\pi}\sqrt{\frac{6h}{\pi\rho}} \equiv \lambda_{\mathrm{th}} \tag{14}$$

Notice that the upper limit λ_{th} is smaller than the waveguide cutoff $\lambda_{c.o.} \approx h(\frac{h}{\rho} \ll 1)$. Consequently, there will be coherent synchrotron radiation only if

$$\sigma_s < 2\pi\rho\sqrt{\frac{3}{2}}\left(\frac{h}{\pi\rho}\right)^{3/2} \tag{15}$$

Before applying Eq.15 to the collider ring to show that there is no coherent radiation possible, we write for completeness the expression for the total power of shielded coherent synchrotron radiation

$$P_{\rm coh} \approx N_\mu^2 \frac{\omega_0}{2\rho}\frac{e^2}{4\pi\epsilon_o}\left(\frac{\rho}{\sigma_s}\right)^{4/3} F(x_{th}, x_{up}) \tag{16}$$

where the form factor $F(x_{th}, x_{up}) = \int_{x_{th}}^{x_{up}} dx e^{-x} x^{-1/3}$, with $x_{th} = \frac{2}{3}\left(\frac{\pi\rho}{h}\right)^3\left(\frac{\sigma_s}{\rho}\right)^2$ and $x_{up} = \frac{9}{4}\gamma^6\left(\frac{\sigma_s}{\gamma}\right)^2$. Expanding the function $F(x_{th}, x_{up})$, assuming that $\frac{x_{up}}{x_{th}} \gg 1$ we get

$$F(x_{th}, \infty) = \begin{cases} \Gamma(2/3) & x_{th} \ll 1 \\ x_{th}^{-1/3} e^{-x_{th}} & x_{th} \gg 1 \end{cases} \tag{17}$$

Next we evaluate n_{th} and n_{coh} obtaining, $n_{coh} = 3.7 \times 10^5$ and $n_{th} = 3.2 \times 10^7$ or $\lambda_{th} \sim 0.2\,\mathrm{mm}$ which is a factor of 10 smaller than σ_s. We conclude that for the isochronous muon collider ring no radiation can be emitted coherently and we have an order of magnitude of safety margin.

IV CONCLUSION

With the present design parameters of a muon collider ring, there is no possible coherent synchrotron radiation

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