1. Introduction

The gated integrator has been used very often in electronics instrumentation to measure signal charge. Its basic principle is to use a signal current source, charging a capacitor during a certain period of gate time, so the signal charge is converted to a voltage on the capacitor and then measured by an ADC.

Because any real current source has a finite output impedance, a fraction of the signal charge will be lost when it charges the capacitor. This problem will seriously influence the charge measurement accuracy, especially when a rather long gate time is used.

For instance, in the Beijing Spectrometer the output signal of the main drift chamber has a duration of ~ 100–200 ns, and the maximum drift time is about 650 ns, so a gate time as long as ~ 800–1000 ns has to be used to cover all the signals from a collision. In this situation the integrator droop is an important parameter to be carefully considered.
This paper will analyze the circuit behavior of a gated integrator and give different ways to improve the parameter of the droop.

2. Charge Loss After Integration

A simplified equivalent circuit of a gated integrator is shown in Figure 1(a) where \( i(t) \) is the signal current, \( R_i \) is the output impedance of the current source, and \( C \) is the holding capacitor.

Assume \( R_i \) is a constant and suppose \( i(t) = 0 \) when \( 0 \leq t < t_0 \), or \( t_0 + P_W < t \leq T_g \) where \( t_0 \geq 0 \) and \( T_g \geq t_0 + P_W \) as shown in Fig. 1(b). The impulse response of the \( R_i C \) network is

\[
    h(t) = \frac{1}{C} \exp \left\{ \frac{-t}{\tau_0} \right\} , \quad (\tau_0 - R_i C)
\]

\[
    \approx \frac{1}{C} \left( 1 - \frac{t}{\tau_0} \right)
\]

with an approximateion error less than 0.5% because \( \tau_0 > 10T_g \geq 10t \) is always the case in this kind of application.

The signal charge is

\[
    Q = \int_0^{T_g} i(t) \, dt = \int_{t_0}^{t_0+P_W} i(t) \, dt .
\]
But the charge collected on capacitor $C$ at the moment of $T_g$ is

\[
Q_c = C U_c(T_g)
\]

\[
= C \int_0^{T_g} i(\tau) h(T_g - \tau) \, d\tau
\]

\[
= C \int_0^{T_g} i(\tau) \frac{1}{C} \left( 1 - \frac{T_g - \tau}{\tau_0} \right) \, d\tau
\]

\[
= \int_{t_0}^{t_0 + P_w} i(\tau) \left( 1 - \frac{T_g - \tau}{\tau_0} \right) \, d\tau
\]

\[
= \left( 1 - \frac{T_g}{\tau_0} \right) Q + \int_{t_0}^{t_0 + P_w} \frac{\tau}{\tau_0} i(\tau) \, d\tau
\]

\[
= \left( 1 - \frac{T_g}{\tau_0} \right) Q + A = Q - \left( \frac{T_g}{\tau_0} Q - A \right)
\]

where $A = \int_{t_0}^{t_0 + P_w} (\tau/\tau_0) i(\tau) \, d\tau$ is a waveform-dependent constant. This result indicates that the signal charge will be reduced after integration by an amount $[\left(\frac{T_g}{\tau_0}\right) Q - A]$ which is a function of the gate time $T_g$, internal resistance $R_i$, holding capacitor $C$ and the waveform of the signal.

Suppose input signal is an impulse at the moment of $t_0$

\[
i(t) = I_0 \delta(t_0)
\]

where $I_0$ is the intensity of the impulse, then

\[
A = \int_{t_0}^{t_0 + P_w} \frac{\tau}{\tau_0} i(\tau) \, d\tau = \int_{t_0}^{t_0 + P_w} \frac{\tau}{\tau_0} I_0 \delta(t_0) \, d\tau = \frac{t_0}{\tau_0} I_0 = \frac{t_0}{\tau_0} Q
\]

and
\[ Q_c = \left( 1 - \frac{T_g}{\tau_0} \right) Q + \frac{t_0}{\tau_0} Q = Q \left( 1 - \frac{T_g - t_0}{\tau_0} \right). \]

Comparing two impulses arriving at \( t_0 = 0^+ \) and \( t_0 = T_g^- \), the collected charges on the capacitor \( C \) at the moment of \( T_g \) are, respectively,

\[ Q_{C1} = Q \left( 1 - \frac{T_g}{\tau_0} \right) \]
\[ Q_{C2} = Q \]

The integrator droop in this case could be defined as

\[ \alpha = \frac{Q_{C2} - Q_{C1}}{Q_{C2}} = \frac{T_g}{\tau_0}. \]

In this simplest case, \( \tau_0 \) greater than 20 \( \mu s \) is required to keep \( \alpha \) less than 5% if \( T_g = 1 \) \( \mu s \).

Suppose the signal has an exponential waveform

\[ i(t) = I_0 \exp \left\{ \frac{t - t_0}{\tau_1} \right\} G_{t_0} (P_W + t_0) , \]

where \( G_{t_0} (t_0 + P_W) \) is a unit gate function. This is reasonable when \( P_W \) is greater than \( \sim (4-5) \tau_1 \). We can have

\[ A = \int_{t_0}^{t_0 + P_W} \frac{\tau}{\tau_0} i(\tau) \, d\tau \]
\[ = \int_{t_0}^{t_0 + P_W} \frac{\tau}{\tau_0} I_0 \exp \left\{ -\frac{\tau - t_0}{\tau_1} \right\} \, d\tau \]
\[ = Q \frac{t_0 + \tau_1}{\tau_0} \]

4
therefore

\[ Q_c = \left(1 - \frac{T_g}{\tau_0}\right) Q + \frac{t_0 + \tau_1}{\tau_0} Q \]

\[ = Q \left(1 - \frac{T_g - t_0 - \tau_1}{\tau_0}\right) . \]

Comparing the earliest pulse \((t_0 = 0)\) and the latest pulse \((t_0 = T_g - P_W)\), the respective charges on the capacitor at the moment of \(T_g\) are

\[ Q_{C1} = Q \left(1 - \frac{T_g - \tau_1}{\tau_0}\right) \]

\[ Q_{C2} = Q \left(1 - \frac{P_W - \tau_1}{\tau_0}\right) . \]

The integrator droop also could be defined as

\[ \alpha = \frac{Q_{C2} - Q_{C1}}{Q_{C2}} = \frac{T_g - P_W}{\tau_0 - P_W + \tau_1} , \quad (5) \]

which is a little less than the result from formula (4), in the practical case.

3. Circuit Analysis

Figure 2 shows a gated integrator circuit in the SHAM4 module,\(^1\) designed by SLAC and originally used for MWPC at LASS experiment. The specification of the integrator droop is 8%/1\(\mu\)s which is adequate for a multiwire proportional chamber because a gate as narrow as 100 ns can be used. However, if we want to use this module in the BES main drift chamber, we have to do some modifications to reduce the droop substantially.

The equivalent circuit for droop calculation is shown in Fig. 3, where \(K\) represents the gain of the Darlington circuit. There are three ways to discharge the holding capacitor \(C_4\) when the gate is closed.
<1> THE INPUT IMPEDANCE OF K—$\frac{\Delta U_C}{\Delta i_1}$

Referring to Fig. 2, the input impedance of $Q_4$ is approximately

$$r_4 \approx \beta \left( R_{13} \parallel R_{14} \right) \approx 140 \text{ K} \ ,$$

The input impedance of $Q_5$, that is $\frac{\Delta U_C}{\Delta i_1}$, is approximately

$$r_5 \approx \beta \left( R_{15} \parallel r_4 \right) \approx 1.8 \text{ M} \Omega \ ,$$

which is large enough so we can neglect its influence on the integrator droop.

<2> THE EQUIVALENT RESISTANCE OF THE BOOTSTRAP CIRCUIT—
$\frac{\Delta U_C}{\Delta i_2}$

Suppose $C_3 \gg C_4$, $C_2R_{13} \gg$ time of interest, then

$$\Delta i_2 = \frac{\Delta U_C - \Delta U_C \ K}{R_{12}} = \frac{\Delta U_C \left( 1 - k \right)}{R_{12}}$$

$$r_{eq} = \frac{\Delta U_C}{\Delta i_2} = \frac{R_{12}}{1 - K} \ (K \approx 1, \ \text{but} \ K < 1) .$$

In the circuit the gains of $Q_4$ and $Q_5$ are respectively

$$K_4 = \frac{R_{13} \parallel R_{14}}{r_{e4} + R_{13} \parallel R_{14}}$$

$$K_5 = \frac{R_{15} \parallel r_4}{r_{e5} + R_{15} \parallel r_4}$$

$$K = K_4 K_5 = \frac{(R_{13} \parallel R_{14})(R_{15} \parallel r_4)}{(r_{e4} + R_{13} \parallel R_{14})(r_{e5} + R_{15} \parallel r_4)} .$$

Because $V_C$ has a rather large dynamic range (say $\sim 0-4$ V), $r_{e4}$, $r_{e5}$ are not constant, which makes the problem more complicated. We will discuss it in more detail later.
For simplicity we give two different values of $K$ and $r_{eq}$, corresponding to $V_{CO} = 0$ (quiescent point) and $V_{CP} = 4$ V (maximum output):

$$
K_{4O} = 0.957 \quad K_{5O} = 0.994 \quad K_0 = 0.951 \quad r_{eqO} = 104 \text{ K}
$$

$$
K_{4P} = 0.978 \quad K_{5P} = 0.997 \quad K_P = 0.974 \quad r_{eqP} = 196 \text{ K}
$$

which in any case is not big enough for a small droop.

<3> **THE OUTPUT RESISTANCE OF $Q_3$ — $\Delta U_C/\Delta i_3$**

Suppose $C_1$ is big enough, then we can use the modified $h$ parameters to determine the output resistance of the transistor $Q_3$ in this circuit. As shown in Fig. 4, the modified $h$ parameters are

$$
h_{11} = r_{be} + \frac{R_{11}(1 + \beta)}{1 + R_{11} h_{oe}}
$$

$$
h_{12} = \frac{h_{re} + R_{11} h_{oe}}{1 + R_{11} h_{oe}}
$$

$$
h_{21} = \frac{\beta}{1 + R_{11} h_{oe}}
$$

$$
h_{22} = \frac{h_{oe}}{1 + R_{11} h_{oe}}
$$

in contrast to the $h$ parameters of $Q_3$ in Fig. 3.

because

$$
\Delta i_b = -\frac{h_{12} \Delta U_C}{R_5 + h_{11}}
$$

$$
\Delta i_c = \Delta U_C h_{22} + h_{21} \Delta i_b = \Delta U_C \left( h_{22} - \frac{h_{21} h_{12}}{R_5 + h_{11}} \right)
$$
therefore

\[ r_c = \frac{\Delta U_C}{\Delta i_c} = \frac{1}{h_{22} - \frac{h_{21} h_{12}}{R_o + h_{11}}} \approx \frac{1}{\frac{h_{re}}{h_{oe}}} \frac{1}{\frac{h_{re} + R_1}{\frac{h_{oe}}{\beta} + R_1}}. \] (8)

We know that \( 1/h_{oe} \) has a value of \( \sim 30-100 \) K for the transistor 2N3906 at the working point of this circuit, but \( r_c \) could be a few hundred kilohms determined by the parameters of \( h_{re}, h_{oe}, r_{be} \) and \( \beta \) of the transistor and the external components of \( R_1, R_5 \).

As described above, the internal resistance of the current source is

\[ R_i = r_5 \parallel r_{eq} \parallel r_c \]

Because \( r_{eq} \) is a nonlinear resistance, it is not easy to calculate the droop by simply using the time constant \( \tau_0 = R_i C_4 \). The solution can be found based on the equivalent circuit shown in Fig. 5, where \( r_o = r_5 \parallel r_6, r_{o4} = R_{15} \parallel r_{e4} \) is the output impedance of \( Q_4, K_1 \) is the gain of the Darlington circuit without consideration of \( R_{13} \).

Assuming a voltage \( V_p \) on capacitor \( C_4 \) at the moment of \( t = 0 \), and a droop \( \Delta V_c \) in a time element \( \Delta t \) at the moment of \( t \), we have the following simultaneous equations:

\[
\begin{align*}
-C_4 \Delta V_c &= (i_1 + i_2) \Delta t = \left( \frac{V_c(t)}{r_0} + i_2 \right) \Delta t \\
i_2 &= \frac{V_c(t) - V(t)}{R_{12}} \\
V(t) &= K_1 V_c(t) - r_{o4} i_3 - \frac{1}{C_2} \int_0^t i_3(t) \, dt = (i_2 + i_3) R_{13}
\end{align*}
\]

From this we deduce the following:

\[
\begin{align*}
-C_4 \frac{dV_c(t)}{dt} &= \frac{1}{r_0} V_c(t) + i_2(t) \\
(1 - K) V_c(t) - R_{12} i_2(t) &= -\frac{1}{i_1} \int_0^t [V_c(t) - i_2(t) (R_{12} + R_{13})] \, dt
\end{align*}
\]
where $K$ is the gain of the Darlington circuit with consideration of $R_{13}$ as defined in formula (7'), and $r_1 = C_2 R_{13}$. Let's discuss the solution of Eqs. (9).

Suppose $C_2$ is large enough or $t$ is not too long, then

$$\frac{1}{C_2} \int_0^t i_3(t) \, dt \approx 0$$

and Eqs. (9) become

$$\begin{cases} 
-C_4 \frac{dV_c(t)}{dt} = \frac{1}{r_0} V_c(t) + i_2(t) \\
(1 - K) V_c(t) = R_{12} i_2(t)
\end{cases}$$

or

$$r_{\parallel} \frac{dV_c}{V_c} = -\frac{1}{C_4} \, dt,$$

where

$$\frac{1}{r_{\parallel}} = \frac{1}{r_0} + \frac{1}{\frac{R_{12}}{1-K}}.$$

Remember $K$ is a function of $V_c(t)$, referring to formula (7')

$$K = f[V_c(t)] = \frac{R_{13} R_{14}}{r_4 + R_{13} R_{14}} r_{\parallel}$$

$$= \frac{R_{13} R_{14}}{V_c(t) + V_0 - 2V_{be}} + \frac{R_{15} r_4}{r_5 + R_{15} r_4}$$

$$= \frac{0.026 R_{14}}{V_c(t) + V_0 - 2V_{be}} + \frac{R_{15} r_4}{V_c(t) + V_0 - V_{be} + R_{15} r_4}.$$

where $V_0$ is the quiescent bias voltage of the $Q_3$ collector and $V_{be} \approx 0.65$ V.

The solution of this equation is

$$\begin{cases} 
P \ln[V_c(t) - x] + Q \ln[V_c(t) - y] + Z \ln V_c(t) + \text{const} = -\frac{1}{r_0 C_4} t \\
\text{const} = -[P \ln (V_p - x) + Q \ln (V_p - y) + Z \ln V_p]
\end{cases}$$

(10)
where
\[
\begin{align*}
  P &= \frac{1}{x-y} \left( x - \frac{c}{ay} + \frac{b}{a} + \frac{c}{a} \frac{x+y}{xy} \right) \\
  Q &= 1 - \frac{c}{axy} - \frac{1}{x-y} \left( x - \frac{c}{ay} + \frac{b}{a} + \frac{c}{a} \frac{x+y}{xy} \right) \\
  Z &= \frac{c}{axy}
\end{align*}
\]

\[
\begin{align*}
  x &= -\frac{1}{2a} \left( b + d \frac{r_0}{R_{12}} \right) + \sqrt{\frac{1}{4a^2} \left( b + d \frac{r_0}{R_{12}} \right)^2 - \left( e \frac{r_0}{R_{12}} + c \right) \frac{1}{a}} \\
  y &= -\frac{1}{2a} \left( b + d \frac{r_0}{R_{12}} \right) - \sqrt{\frac{1}{4a^2} \left( b + d \frac{r_0}{R_{12}} \right)^2 - \frac{1}{a} \left( e \frac{r_0}{R_{12}} + c \right)}
\end{align*}
\]

and
\[
\begin{align*}
  a &= 39 \text{ V}^{-1} \\
  b &= d + a(2 V_0 - 3 V_{be}) \\
  c &= a(V_0 - 2V_{be})(V_0 - V_{be}) + \left( 1 + \frac{R_{14}}{R_{13}} \right) (V_0 - V_{be}) \\
  &\quad + \left( 1 + \frac{R_{15}}{r_4} \right) (V_0 - 2V_{be}) \\
  d &= 2 + \frac{R_{14}}{R_{13}} + \frac{R_{15}}{r_4} \\
  e &= \left( 1 + \frac{R_{14}}{R_{13}} \right) (V_0 - V_{be}) + \left( 1 + \frac{R_{15}}{r_4} \right) (V_0 - 2V_{be})
\end{align*}
\]

It is not very difficult to calculate $V_c(t)$ by computer for different time $t$ from Eq. (10). For $V_p = 4$ V and $r_0 = 500$ K, the calculated droop is about 7%/1 \mu s.
If $C_2$ is not large enough, or we want to look at the circuit behavior for a longer time, we can get the following from Eq. (9):

\[
\begin{align*}
V_c''(t) &+ \left[ \frac{1}{r_2} \left( 1 + \frac{R_{12}}{r_0} - K \right) + \frac{\tau_3}{r_1 r_2} \right] V_c'(t) \\
- \frac{1}{r_2} \frac{dK}{dt} V_c(t) + \frac{1}{r_1 r_2} \left( 1 + \frac{R_{12} + R_{13}}{r_0} \right) V_c(t) & = 0 , \quad (11)
\end{align*}
\]

\[
V_c(0) = V_p
\]

\[
V_c'(0) = - \frac{1}{C_4} V_p \left( \frac{1}{r_0} + \frac{1 - K_p}{R_{12}} \right)
\]

where $\tau_1 = C_2 R_{13}$, $\tau_2 = C_4 R_{12}$, and $\tau_3 = C_4 (R_{12} + R_{13})$.

Suppose $K = \text{constant}$, then Eq. (11) has an analytical solution:

\[
\begin{align*}
V_c(t) & = \exp \{\lambda_1 t\} \left( A_1 \cos \lambda_2 t + A_2 \sin \lambda_2 t \right) \\
\lambda_1 & = - \frac{1}{2} \left[ \frac{1}{r_2} \left( 1 + \frac{R_{12}}{r_0} - K \right) + \frac{\tau_3}{r_1 r_2} \right] \\
\lambda_2 & = \sqrt{\frac{1}{r_1 r_2} \left( 1 + \frac{R_{12} + R_{13}}{r_0} \right) - \lambda_1^2} \quad . \quad (12)
\end{align*}
\]

$A_1$, $A_2$ can be determined from the initial values as

\[
\begin{align*}
A_1 & = V_p \\
A_2 & = \frac{V_c'(0) - A_1 \lambda_1}{\lambda_2}
\end{align*}
\]

For $K = K_0 = 0.951$ or $K = K_p = 0.974$, we get:

\[
V_c(t) = \exp \{-0.062 t\} \left( 4 \cos 0.093 t - 2.32 \sin 0.093 t \right) \quad , \quad (12')
\]

or
\[ V_c(t) = \exp\{-0.039 \, t\} \, (4 \cos 0.104 \, t - 1.19 \sin 0.104 \, t) \quad , \quad (12'') \]

and the corresponding curves are shown in Fig. 6 as (1) and (2), which agree very well with experiment.

In Fig. 6 we also give curve (3), which is the result of formula (10) \((\tau_0 = 500 \, K\Omega\) is supposed in all the calculations).

In the case where \(K\) cannot be regarded as a constant, we have to solve Eq. (11) by numerical methods. Equation (11) is a stiff problem; it can be solved by GEAR package\(^2\) under SLAC VM. Table 1 gives some results for the circuit shown in Fig. 2.

<table>
<thead>
<tr>
<th>Method</th>
<th>( V_c ) (V) at ( t = ) ((\mu)s)</th>
<th>Droop at ( t = 1 ) ((\mu)s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eq. (11) GEAR</td>
<td>4  3.69  3.34  2.93  2.46  1.91</td>
<td>7.8%</td>
</tr>
<tr>
<td>Eq. (12'')</td>
<td>4  3.71  3.39  3.06  2.71  2.36</td>
<td>7.3%</td>
</tr>
<tr>
<td>Eq. (12')</td>
<td>4  3.54  3.10  2.67  2.25  1.87</td>
<td>11.5%</td>
</tr>
<tr>
<td>Eq. (10)</td>
<td>4  3.72  3.47  3.23  2.99  2.77</td>
<td>7.0%</td>
</tr>
</tbody>
</table>

We can conclude that the formula (12) with the maximum gain of \(K_p\) is a simple and good approximation of the circuit behavior and droop calculation.
4. Improvement

From the analysis above we can see that the integrator droop could be reduced:

1) First, by increasing the equivalent resistance of the bootstrap circuit—\( r_{eq} \) and holding capacitor \( C \). This means increasing the resistance of \( R_{12} \) and the gain \( K \) of the emitter follower.

Considering other characteristics of the circuit, we can use the following component values in the circuit, shown in Fig. 2, without any changing of the printed circuit board of SHAM4:

\[
R_{10} = 3.9 \, \text{K}, \quad R_{12} = R_{13} = 10 \, \text{K}, \quad R_{14} = 5.1 \, \text{K}, \quad C_4 = 200 \, \text{pf}
\]

In this case we can get

\[
V_{CO} = 0 : \quad K_O \approx 0.987, \quad r_{eqO} \approx 770 \, \text{K}\Omega
\]
\[
V_{CP} = 5 \, \text{V} : \quad K_P \approx 0.993, \quad r_{eqP} \approx 1.4 \, \text{M}\Omega
\]

The result from both calculation and experiment shows that the integrator droop is less than 3% for different sizes of signals. If we could pull the resistors of \( R_{14}, R_{15} \) down to \(-24\,\text{V}\) instead of ground, we could reduce the nonlinearity of gain \( K \) dramatically and improve the integrator droop at small signals.

2) The second approach to reducing the integrator droop is to increase the output impedance \( r_e \) of the transistor. From formula (8) we can estimate that the output resistance \( r_e \) of the transistor in the circuit of Fig. 2 is a few hundred Kilohms, which is much larger than \( 1/h_{oe} \) of the transistor, itself. But this is not large enough to cause small integrator droop; it has serious divergences from transistor to transistor, and may even result in unstable circuit behavior.
The simplest way to solve this problem is to use the transistor common base configuration, where in most circumstances the output resistance of the transistor is greater than 1 Megohm. A modified test circuit from Fig. 2 is shown in Fig. 7. Both measurement and calculation from formula (12) demonstrate that the integrator droop is about 1%/1 μs.

We also can reduce the integrator droop more by increasing further the holding capacitor, but maintaining the dynamic range of the output voltage has to be taken into account.

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References

Fig. 1
Fig. 2
Fig. 3
Fig. 4
Fig. 5
Fig. 7