TRANSVERSE SINGLE PARTICLE INSTABILITY
IN THE NAL 500 GeV ACCELERATOR

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Abstract

Experiments reveal that single particle resonances, driven by field imperfections limit the 8 GeV acceptance of the NAL main accelerator to protons with betatron oscillation amplitudes less than 8 mm. It is observed that resonances driven by random azimuthal field fluctuations are much weaker than those driven by average field errors acting in concert with the super-periodicity of the accelerator. This consideration dictates the choice of operating tune. Correction magnets are installed to compensate zero harmonic skew quadrupole, sextupole and octupole errors. Separate control of dv_x/dp , du_y/dp , d^2v_x/dp , and d^2v_y/dp is possible in an energy-dependent manner. High azimuthal harmonic patterns correction magnets are used to eliminate, dipole, quadrupole, sextupole and skew sextupole resonances adjacent to the operating point. A description of these systems and the improvements obtained are given.

1. Introduction

The NAL main ring is fed with twelve batches of 8 GeV protons delivered at 15 Hz repetition rate by the fast cycling booster. These are loaded head to tail to fill the circumference of the main ring and during the loading sequence which lasts 0.8 seconds the main ring must act as a d.c. storage ring. Once the loading is complete the guide field rises parabolically until, at about 20 GeV, the maximum acceleration rate is reached and sustained to full energy.

Soon after this injection scheme was first implemented it became clear that a considerable fraction of the injected beam was lost during the loading sequence. Each batch decayed exponentially with a time constant of less than 1 second and this loss continued during the parabola, tailing off to zero near 20 GeV (Fig. 1). The fraction of the injected beam which survived to full energy was rarely higher than 35% and the peak intensity was limited to 10^{12} protons per pulse,

Improvements in the intensity delivered by the booster, which inevitably resulted in a larger transverse emittance, did little to raise the high energy intensity but merely augmented the losses.

Preliminary experiments showed that switching off the r.f. did not affect the loss rate nor was the loss diminished on those days when the brightness of the injected beam was low. This suggested that the loss

FIG.1 Loss during main ring loading and parabola

was due to a transverse single particle acceptance limitation rather than a longitudinal or collective phenomenon.

The acceptance limitation seemed not to be a purely mechanical obstruction. Measurements of beam profile and the closed orbit distortion after correction revealed that the beam occupied only a 20 mm diameter central core of the 100 mm x 50 mm section vacuum chamber. Moreover, one could bump the beam locally at any point in the machine almost to the vacuum chamber wall before any mechanical obstacle killed the beam.

On the other hand we found that the loss rate was very sensitive to the current in the main quadrupoles of the FODO lattice which affect the tune (v_x , v_y) and to the strength of the sextupole magnets which had been installed to reduce the large chromatic time spread in the injected beam.

We suspected that there were strong resonance lines in the working diamond driven by non linear imperfections in the guide field and that these were responsible for the losses. We therefore commenced a program of studies, which included improvements in techniques for measuring betatron motion and the installation of multipole correction magnets, with the intention of compensating the non linearities and reducing the tune spread in the beam. The remainder of this report describes this work which, at the time of writing, has improved the main ring to the point that it will accept and accelerate over 10^{13} protons per pulse rejecting under the best conditions only 10 to 20% of the beam presented by the booster.

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2. Exploration of the NAL Working Diamond

Figure 2 shows the position of the nominal injection tune of the main ring in relation to the more important resonance lines. The lines are excited by random azimuthal variations in the non linear imperfections in the guide field.

FIG 2 Working diagram

The F and D sets of main ring quadrupoles are connected in series but are powered independently of the dipoles. By changing the ratio of quadrupole to bending magnet current we were able to scan the working point along a line $v_x - v_y = -0.08$, parallel to the diagonal coupling resonance $v_x - v_y = 0$. The small difference in the tunes is due to field imperfections. The configuration of the F and D quadrupole bus bars allowed us to shunt a small fraction of the current from the F to the D system through a pulsed vacuum tube power supply: the tune splitter. In this way we were able to control the tune split, $v_x - v_y$, and scan along a variety of lines mapping the working diamond at injection.

We calibrated these manipulations of the F and D quadrupole currents against measurements made by kicking the beam with a "pinger" dipole. This consisted of two turns of copper pulsed within a machine revolution from a small capacitor bank. By injecting a single booster batch and photographing an oscilloscope trace of the signal from one of the standard capacitive beam position detectors one could examine the beat frequency of the coherent betatron oscillations and calculate the fractional part of v_x or v_y , dividing the number of beats by the number of machine revolutions. We also deduced the tune spread in the beam from the rate of decay of the coherent signal and from the vertical oscillations induced by a radial kick could measure coupling between the two planes. We did not feel we could entrust disentangling these three signatures to a more convenient automated process at this stage.

Since we were interested in the influence of the resonances on beam loss we measured the fraction of the beam surviving by sampling and digitising a beam intensity monitor just after the injection of a single booster batch and again 400 milliseconds later. This quantity, the transmission, we measured with steady 8 GeV magnet excitation and with the r.f. switched on. It proved to be a powerful probe in exposing the resonance lines and reliable index of the overall transmission of the main ring to high energy.

We were fortunate in that we were able to make these injection studies parasitically with a special booster batch injected each machine cycle before the main batch sequence destined for acceleration to high energy.

FIG 3 Contours of transmission for 0.4 sec at 8 GeV with original air cored sextupole pattern

Figure 3 shows a contour plot obtained at the outset of these studies by measuring 8 GeV transmission as a function of v_x and v_y . Clearly the influence of the resonance line: extends over the whole working space and there is no one place where more than 70% of the beam survives.

Beam survival is best close to the diagonal coupling resonance $v_x - v_y = 0$. But in this region there is a danger that this coupling will transfer a large horizontal emittance into the vertical plane and our method of v measurement became confused. One of the first corrections applied was to compensate the skew quadrupole imperfections in the guide field which drive this coupling.

3. Correction of Skew Quadrupole Coupling

Coupling between vertical and horizontal phase planes can be driven by systematic skew quadrupole imperfections in the guide field such as might result if all the quadrupoles of one type were rotated by a fraction of a milliradian about the beam axis. An

average roll of this magnitude could quite easily creep in during the surveying of the ring but could be compensated by rolling a few quadrupoles in the opposite sense. Significant coupling is to be expected in a region centred on the line $v_x - v_y = 0$, of width:

$$v_x - v_y = \frac{1}{2(B\rho)} \int \sqrt{\beta_x \beta_y} k ds$$

where: k is the skew quadrupole $\partial B_x / \partial x$, $\partial B_y / \partial y$
 $B\rho$, the magnetic rigidity
 $\beta_x \beta_y$, the betatron functions

Within this band a vertical ping would generate a coupled horizontal coherent oscillation of frequency $v_x - v_y$ and of amplitude:

$$\frac{\Delta b}{a} = \frac{1}{2(B\rho)(v_x - v_y)} \int \sqrt{\beta_x \beta_y} k ds$$

where Δb is the coupled amplitude horizontally
 a is the pinged amplitude vertically.

and the oscillograms clearly showed the beat frequency $v_x - v_y$ (ten turns per wavelength for $v_x - v_y = 0.1$). These beats were of opposite phase in the two planes. Using above expression we calculated the angle of roll, about 8 milliradians, to be applied to 12 equally spaced quadrupoles to neutralise the coupling. This adjustment reduced the coupling effect by almost an

4. Correction of Chromaticity

Magnetic measurements show that the NAL main ring dipole magnets have a remanent field whose predominant imperfection is a sextupole term. The field falls parabolically towards the edges of the poles. The average derivative is:-

$$d^2 B_y / dx^2 \approx 0.05 \text{ T m}^{-2}$$

This field distortion can be understood qualitatively if one remembers that flux lines in the iron rooted at the centre of the poles are longer than those from the edge. The remanent magnetomotive force, $\int H_c dl$, is larger near the pole centre.

The chromaticity in each phase plane produced by this sextupole shape is

$$(dv/v)/(dp/p) = 9$$

seven times larger than that due to the momentum dependence of the focusing strength of the lattice. Uncorrected it would introduce a tune spread of $\Delta v = \pm 0.09$ for a momentum spread within the beam of $\pm 0.5 \text{ o/o}$, measured by observing the time taken for the coasting injected beam to lose its 53 MHz r.f.

structure. The half widths quoted are those at the half height of the distributions.

During the early commissioning of the machine air cored sextupoles had been installed to correct this large chromaticity. These had been placed in all but a few of the short straight section spaces which follow each main ring quadrupole. Those near F quadrupoles and those near D quadrupoles should ideally have been separately powered to neutralise $\partial v / \partial p$ and $\partial v_y / \partial p$ independently but for practical reasons all sextupoles were in series and powered with a simple square pulse covering only the injection period and the parabola. They were therefore not ideal tools for accurate chromaticity control.

We measured the v spread in each plane at a number of points in the acceleration cycle by counting the number of machine revolutions that a pinged coherent betatron oscillation took to smear to half its original signal strength. For a Gaussian tune distribution the reciprocal of this number is a measure of the extreme (2σ) spread in tune. Figure 4 shows the way in which

Fig.4 Tune Spread During Acceleration (early d.c. sextupoles)

from almost perfect compensation in the vertical plane the chromaticity changes in each plane during the parabola as eddy current sextupole fields develop in the vacuum chamber. The momentum spread also swells, peaking at almost double its injection value in passing through transition. This further adds to Δv . The edges of the proton population cross the third and fourth order stopbands.

We therefore installed a new set of iron cored sextupoles with independent control of the set next to F quadrupoles and the set next to D quadrupoles. The currents in F and D sets are related to the dv/dp and dv_y/dp via a simple 2×2 matrix transformation which was built into the control software to give independent orthogonal control in the two planes.

The time development of sextupole imperfections contained three components: a steady one due to remanent fields, a component proportional to magnet excitation (B) and due to the geometry of the main ring dipoles and finally a component growing as dB/dt , generated by eddy currents in the vacuum chamber. We therefore powered the correctors with a current waveform:

$$I = a + \frac{b dB}{dt} + cB$$

deriving the dB/dt and B signals from the main ring reference magnet.

Each of the coefficients, a, b, c multiplied by the 2×2 matrix could be controlled directly from a page on the main ring console display.

We set up each coefficient in turn at energies at which its influence predominates (8, 17 and 60 GeV), maximising the decay time of pinged betatron oscillations and, repeating the tune spread experiments, we found $\Delta v < \pm 0.005$ throughout the acceleration cycle,

As expected beam survival improved as a result of this correction and the machine became much less sensitive to adjustments in the ratio of quadrupole to dipole current during the parabola.

5. Correction of Third Order Stopbands

Our early transmission experiments had revealed the strong influence of the four third order stopbands which intersect at $v = 20.33$ close to the nominal working point:

$$\begin{aligned} 3v_x &= 61, & \int \beta_x^{3/2} (\partial^2 B_y / \partial x^2) e^{i61\theta} d\theta \\ v_x + 2v_y &= 61, & \int \beta_x^{1/2} \beta_y (\partial^2 B_y / \partial x^2) e^{i61\theta} d\theta \\ 2v_x + v_y &= 61, & \int \beta_x \beta_y^{1/2} (\partial^2 B_x / \partial y^2) e^{i61\theta} d\theta \\ 3v_y &= 61, & \int \beta_y^{3/2} (\partial^2 B_x / \partial y^2) e^{i61\theta} d\theta \end{aligned}$$

The table lists the resonances and their driving terms which are all 61st Fourier harmonics of the azimuthal pattern of sextupole imperfections weighted according to the local betatron functions. The first two resonances are driven and may be corrected by regular sextupoles with symmetry about a vertical plane; the same symmetry as one expects from the main ring dipoles. The last pair are driven by skew sextupoles of symmetry about the median plane. To correct all four stop bands one must use sextupole magnets of both regular and skew types.

The weighting factors are such that a further subdivision of correctors into those near F quadrupole locations where β_x is large and β_y small and those near D locations where the opposite is true gives one almost orthogonal control of each resonance.

To generate the correct phase and amplitude of the 61st harmonic and neutralise the driving term the current settings of the individually powered sextupoles were ganged up through the control system to increment the sine or cosine amplitude in the strength pattern.

Tuning the machine to sit exactly on one of the stopbands we could adjust the amplitudes of the sine and cosine components to achieve 70% transmission. A contour plot versus the two variables revealed a simple maximum.

We calculated the width of the stopband from the strength of the correctors. For a proton at the periphery of the beam (1 cm amplitude at $\beta \approx 100$ m) Δv is greater than ± 0.01 . We had expected these stopbands to be due to variations in the azimuthal pattern of the remanent sextupole fields responsible for the chromaticity. We knew that the remanent fields in the dipole magnets constructed from unshuffled laminations varied by as much as $\pm 40\%$. Yet this could only account for a stopband $\Delta v \approx \pm 0.002$.

Clearly other, stronger sextupoles were present and we soon discovered that the uneven pattern of chromaticity correction sextupoles determined by the need to leave room in some short straight sections for other equipment contained driving terms of the appropriate magnitude and phase to explain the observed widths.

We redeployed these chromaticity sextupoles in a pattern considerably less rich in 61st harmonic taking into account the differing weighting factors in the Table and repeated our transmission scan. The line widths had clearly become very much smaller and this was reflected in an improvement from 1.8 to 3.4×10^{12} in the intensity which could be accepted from the booster and accelerated to full energy.

We went on to compensate these much thinner resonances and from the corrector strength deduced a width commensurate with the theoretical estimate based on magnet measurements.

Our transmission scan showed all four lines to be comparable in width. We presume that sextupole fields of the skew configuration were present coming from assembly errors in the quadrupoles upon which we had little measurement data. We therefore installed F and D sets of skew sextupoles and set about compensating each of the four resonance lines scanning through each in turn along a line $v_x - v_y = -0.1$ where they are clearly resolved in the transmission scan. Figure 5 shows the effect of this compensation which raised the intensity which could be accepted and accelerated intensity a further 50%.

FIG 5.(a). 80 GeV transmission with harmonic sextupole correction off

FIG 5(f). 8 GeV transmission after harmonic sextupole correction.

6. Compensation with Octupoles

Seeking to improve the linearity of the machine still further we measured the amplitude dependence of the tune by pinging coherent oscillations of a range of amplitudes and adjusting a set of d.c. series powered octupoles to make $\partial^2 \nu / \partial a^2$ zero. Zero harmonic octupole imperfections in the lattice also generate a curvature to the chromaticity $\partial^2 \nu / \partial p^2$. We measured the shape of $\nu(p)$ by pinging and steering to various mean radii with an r.f. bump. We optimised the octupoles to give a flat curve. Both techniques arrived at the same octupole strength. They proved effective in improving the tune spread. One could maintain coherent oscillations undiminished for several hundred turns indicating a $\Delta \nu$ of a few $\times 10^{-3}$. Not surprisingly the machine became so linear at this point that a resistive wall instability emerged. The diagnosis and cure of this instability is the subject of another paper in this conference.

7. Alternative Working Point

The definition of our resonant scans had improved sufficiently by this time that we were able to construct three dimensional models of the transmission over the diamond: $20 < \nu_x, \nu_y < 20.5$ and could identify the valleys of the third integer resonances as well as less well defined depressions at the location of fourth and higher order resonances.

The integer stopbands $\nu_x, \nu_y = 20$ were very wide and we suspected that one of their main components was the systematic resonance

$$3\nu = 60 = 10S$$

where S is the superperiodicity of the long straight sections and thence of the systematic sextupole imperfection.

Such systematic relations reoccur at all even ν values for $S = 6$ and give rise to very wide stopbands. We therefore decided to explore the diamond just above 19 to see if the integer was less strong.

Fig. 6 - Contour model of transmission.

Figure 6 shows a model of this diamonds contours with no harmonic correction applied. The integer has a width of $\Delta \nu = \pm 0.05$ compared with ± 0.1 in the original diamond. The region around 19.2 gives somewhat higher transmission and much more elbow room to accommodate changes in the working point due to magnet ripple and quadrupole power supply regulation. Accordingly, it was decided to change the operational working point permanently from 20.2 to 19.2 and improvement in accelerated beam has resulted.

Conclusions

These experiments demonstrated the deleterious effect of non linear field imperfections in the guide field. It is apparent from transmission scans like Fig. 5 that beam survival is affected over a much wider band than the classical stopband width even when the measures have been taken to reduce the momentum and amplitude dependence of the tune. The reason for this line broadening is still being investigated. It may be due to ripple in the excitation of the dipoles and quadrupoles.

As a result of these experiments and improvements the four dimensional acceptance of the main ring increased by almost an order of magnitude to match improvements in Booster intensity and reduce the fraction of the beam lost after injection.

Yet even when the third order resonances had been corrected the acceptance was barely larger than the diameter of the booster beam. To enlarge this still further to be commensurate with the geometrical size of the vacuum chamber will require correction of fourth and higher order stopbands which are not resolved in transmission scans at injection but which we know are there from measurements at higher energy.

DISCUSSION

Andrei Kolomensky (Lebedev Institute): Do you know the mechanism of the pressure instability?

R. Stiening (NAL): No, we do not. We haven't explored it in great detail because the instability occurs under conditions which are very far from our normal operating conditions.

Karl Reich (CERN): Is the narrowing of stop-bands done just during the low energy period or further on in the cycle?

Stiening: Some are and some aren't. The average sextupole corrections are made throughout the cycle. It's necessary to carry them up to about 50 GeV. It doesn't matter, at present, whether the octupole corrections are made or not.

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Summary

The fast damping phenomenon observed in SPEAR is discussed in this paper. Some of the effects of various beam parameters such as chromaticity, rf voltage, octupole lens strength and beam current upon the fast damping are presented.

Introduction

Fast damping of coherent transverse oscillations of the center of charge of a single bunch beam has been observed in VEPP-2¹ and SPEAR. This damping for horizontal oscillations has been studied in SPEAR and the results seem to be consistent with those predicted by the "Head-Tail" theory for wake fields that vary rapidly over the length of the bunch.^{2,3,4,5} This phenomenon was investigated experimentally by shock-exciting the beam horizontally and then observing the decay of the amplitude of the coherent signal. In order to establish that the decay of the coherent signal corresponds to a decay in particle amplitude and not to a randomization of phases of the coherent motion, the transverse particle density was also observed as a function of time. This damping rate has been measured for various parameters of the beam. It has proven possible to eliminate this fast damping of particle amplitude by means of octupole fields which produce a spread in the particle betatron frequencies, leading to Landau damping of the coherent motion.⁶

Experimental Observations

The two kickers which produce the required beam bump during injection were used to produce an initial amplitude of the horizontal oscillation for the center of charge of a stored beam.⁷ By varying the relative strength of the kicker pulses, it was possible to excite horizontal amplitudes that were large enough to be detected and studied.

The response of the beam after the kick is observed by two types of detectors.⁸ The first of these is a strip-line monitor that gives a signal proportional to the position of the center of charge of the beam. The second is a profile monitor that scans the synchrotron light emitted by the beam; the maximum output signal from this monitor is proportional to the maximum density of the beam averaged over times longer than the transverse betatron oscillation period. Thus the maximum signal from the profile monitor is inversely proportional to the beam width. The beam width derived from the profile monitor includes the contribution from the incoherent particle motion and coherent motion at the betatron frequency. An example of the fast damping as observed by the two detectors is shown in Figs. 1 and 2. The two figures are for identical beam parameters but different sweep speeds. The top traces are the envelope of the coherent beam motion while the bottom traces are the evolution of the peak particle density.

If the peak particle density increases while the coherent signal decays, then this would be evidence that the amplitude of the oscillation decays, while if the peak particle density does not increase, this would

Fig. 1. Fast Damping

Fig. 2. Fast Damping

be evidence that the decay of the coherent signal is due to a randomization of the phases of the coherent motion. Figures 1 and 2 are thus evidence that the amplitude of the oscillation decays. Since the active feedback system was off during these experiments and the radiation damping time is long (~ 60 msec for beam parameters in Figs. 1 and 2) compared to the fast damping times measured in SPEAR, there must be a feedback field produced by the beam in its passive surroundings.

Low Current Phenomena

In this section, the fast damping phenomenon discussed will be for currents in a single bunch < 40 mA. The fast damping rate of horizontal coherent oscillations as a function of horizontal chromaticity ξ_x is shown in Fig. 3 for three values of the rf voltage.

In this figure, the energy was 1.5 GeV, the average current 20 mA and the chromaticity ξ_x is defined by $\Delta\nu_x = \xi_x(\Delta p/p)$. One can see from Fig. 3 that the fast damping rate is a linear function of the chromaticity.

The bunch length in SPEAR shortens with increasing rf voltage; hence the results in Fig. 3 show that the variation of the damping rate with chromaticity, $d\alpha/d\xi_x$, decreases with increasing bunch length. This last fact is consistent with the "Head-Tail" instability with fast varying wake fields.

* Work supported by the U. S. Atomic Energy Commission.

** J. M. Paterson, B. Richter, A. P. Sabersky, H. Wiedemann, P. B. Wilson, M. A. Allen, J. E. Augustin, G. E. Fischer, R. H. Helm, M. J. Lee, M. Matera, P. L. Morton

Fig. 3. Fast Damping rate versus chromaticity.
E = 1.5 GeV; I = 20 mA

Below currents of 40 mA, the decay of the coherent oscillation signal observed is similar to the decay presented in Figs. 1 and 2. The damping rate versus the average current is shown in Fig. 4 for various values of the rf voltage with an energy of 1.5 GeV and chromaticity $\xi_x = 2.4$. Due to the bunch lengthening phenomenon in SPEAR, the length of the bunch increases with current and is probably the explanation of why the fast damping rate varies more slowly than linearly with current.

Fig. 4. Fast damping rate versus average current.
E = 1.5 GeV; $\xi_x = 2.4$

The fast damping rate, α , is normally expected to be inversely proportional to the energy so that the decay rate is plotted as a function of $(1/E)$ in Fig. 5. Because the bunch length decreases as $(1/E)$ increases (i.e., as the energy decreases), we see from Fig. 5 that α increases faster than $(1/E)$.

Fig. 5. Fast damping rate versus $(\text{Energy})^{-1}$
I = 20 mA; V = 200 kV; $\xi_x = 2.4$

High Current Phenomena

When the current in a single bunch circulating in SPEAR exceeds 40 mA, the fast damping phenomenon is not as simple to interpret as it was for the lower current. The response of a 100-mA beam is shown in Figs. 6, 7 and 8. Again, the top traces are the envelope of the coherent horizontal motion for the center of charge in the beam while the bottom traces are the peak particle density averaged over the transverse betatron period. In all three figures, the beam parameters were identical except for the chromaticity and the rf voltage. The horizontal chromaticity $\xi_x = 2.1$ in both Figs. 6 and 7, but the rf voltage was 100 kV in Fig. 6 and 225 kV in Fig. 7. The rf voltage was 100 kV in Figs. 6 and 8, but ξ_x was 2.1 in Fig. 6 and 4.2 in Fig. 8. The motion displayed in Figs. 6, 7 and 8 seems to indicate that the center of charge is oscillating horizontally at more than one coherent frequency, and that the difference between these frequencies produces a beating that depends upon the bunch length and the chromaticity.

Fig. 6. I = 100 mA; $\xi_x = 2.4$; V = 100 kV

Fig. 7. $I = 100$ mA; $\xi_x = 2.4$; $V = 225$ kV

Fig. 9. $I_{oct} > -3A$

Fig. 8. $I = 100$ mA; $\xi_x = 4.2$; $V = 100$ kV

For the case where the "Head-Tail" instability theory assumes a hollow bunch in longitudinal phase space, there should be only one frequency for the transverse coherent oscillation of the center of charge. However, if the bunch is not hollow in longitudinal phase space, it is possible to have several transverse coherent dipole modes with slightly different frequencies^{2,4} and it is possible that these are what we observe in SPEAR for high current.

Landau Damping

In the SPEAR lattice, there are octupole lenses that can be powered to produce a variation of the transverse oscillation frequency with particle amplitude, a .

The octupole may be powered with up to ± 20 amps, which yields up to $(\Delta\nu/a^2) = \pm 0.004/cm^2$ for an amplitude variation of betatron wave numbers. The fast damping has been observed as a function of the octupole lens strength. It has been found that for a 10-mA beam, the fast damping is independent of the octupole lens strength in the range $-3A < I_{oct} < 12A$. On the other hand, if the octupole lens strength is sufficiently negative; i.e., $I_{oct} < -5.6A$, the fast damping disappears. This is shown in Figs. 9 and 10.

In Fig. 9, the $I_{oct} > -3A$, and we see that we have the usual fast damping; however, in Fig. 10, the $I_{oct} = -5.6A$, and we see that the coherent signal decays rapidly and the peak particle density is lower than was obtained for the fast damping case. We have interpreted these results as a Landau damping of the coherent motion. When the Landau damping is sufficient to

damp the coherent motion, the resulting beam size is larger (peak particle density smaller) and the oscillation amplitudes are damped by the slower radiation process.

Acknowledgements

It is a pleasure to thank the SPEAR Operations Group, who not only aided in the operation of the storage ring but also actively participated in the experimental measurements.

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PART I on the PS, by J. Gareyte

PART II on the Booster, by J. Gareyte and F. Sacherer

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Summary

A single-bunch instability of the head-tail type has been observed for several years in the PS. Only mode zero is seen even though the phase shift between head and tail is changed from zero to over 100 radians by varying the machine chromaticity. According to theory, the bunch spectrum is centered on the "chromatic" frequency $f_g = \text{phase shift/bunch length}$, and has a width of about 100 MHz (inverse bunch length). Near transition, f_g can be varied over a range of 2 GHz, thus providing an excellent probe of the transverse coupling impedance seen by the beam. One finds a broad-band spectrum with a flat maximum near 1 GHz. This is consistent with the absence of higher modes. Thresholds versus octupole strength and beam emittances have been measured and compared with theory. The present cure is octupoles, but reduction of the chromaticity to zero is foreseen for higher intensities ($\sim 10^{13}$ ppp).

In the Booster, head-tail modes zero, one, two and three occur, the higher modes occurring for large chromaticities and mode zero for zero or small chromaticities. The growth rate increases rapidly as the Q-value approaches 5 from below. This is a coupled-bunch multi-turn phenomenon that is well explained by the resistive wall effect. The e-folding times of a few ms require a total wall resistance of about one Ω m at low frequencies, in agreement with the resistance of the thin walled vacuum chamber.

PART I : THE CPSHistory

Already in 1967 when the CPS accelerated intensity exceeded 0.8×10^{12} p/p, the beam was from time to time deteriorated by a vertical instability. The phenomenon manifested itself as partial or total disappearance of some of the 20 bunches. The cure was to lengthen the bunches by means of a dilution in the longitudinal phase space, obtained by mistiming transition phase jump.

Soon after, the accelerated intensity increased to 2×10^{12} p/p and the ISR asked for a CPS longitudinal density as high as possible. The Q-jump method was devised to cure transition problems and maintain nominal phase-space area up to the available peak intensity. The result of this was a catastrophic vertical explosion of the beam soon after transition (~ 7 GeV/c), leading to complete loss of the particles. Fortunately, some of the octupolar lenses foreseen during construction of the CPS were still in the ring, and proved to be sufficient to Landau damp the instability. A systematic study programme was then launched with the aim of controlling the instability even at the increased intensity of 10^{13} p/p.

ObservationsSingle-Bunch Instability

Usually several, if not all, of the twenty bunches become unstable. Nevertheless, a regular pattern,

which is the signature of coupled motion, was never observed. Also, working near the threshold, one can observe situations where only one bunch oscillates (Fig. 1).

Fig. 1 : A single bunch instability

Dependence on bunch length

Increasing bunch length, either by increasing longitudinal emittance or by lowering longitudinal focusing, reduces growth rate and increases the threshold of the instability.

Dependence on octupoles and emittances

Octupoles are currently used in operation to suppress the instability. A rule of thumb which can be drawn from dispersion relation analysis^{1,2} is that the spread in betatron number Q due to octupoles must exceed $|\Delta Q_{inc} - \Delta Q_{coh}|$, the difference between incoherent and coherent Laslett Q-shifts. In fact, half this value is sufficient to avoid any beam blow-up under operational conditions. During the experiment described below, the required spread was found to be even much smaller.

The experiment : Threshold was determined in terms of octupole current needed to suppress the instability for given energy and intensity, and varying emittances. Vertical instability was investigated, horizontal and vertical emittances being varied by means of multiple pulsing of a fast kicker followed by filamentation in the presence of strong non-linearities. The clear and reproducible threshold values obtained are plotted on Figs 2 and 3. Minimum spread is about 20 times smaller than $|\Delta Q_{inc} - \Delta Q_{coh}|$.

Dependence on chromaticity

At medium energy ($P < 15$ GeV/c), the CPS is a linear machine with both horizontal and vertical chromaticities

$$\xi = \frac{\partial Q}{\partial Q} / \frac{\partial P}{\partial P} = -1$$

In this case, instability grows in the vertical plane. At high energies, ξ_H and ξ_V become respectively -2 and zero due to saturation, and instability grows in the horizontal plane. Above transition, it was found by varying the chromaticities with sextupoles that the instability grows in the plane in which $|\xi|$ is the largest ($\xi < 0$). For zero or positive chromaticities, no instability has ever been seen up to intensities of 5×10^{12} p/p. Below transition, the same behaviour is

found for the reverse sign of chromaticity (unstable for $\xi > 0$).

Figure 2

Octupole current and estimated Q_V spread at threshold versus vertical emittance (10 GeV/c, 1.7×10^{12} p/p) :

————— positive current
 - - - - - negative current

- ΔQ_V is the quadratic sum of the Q_V spreads induced by the octupoles via both vertical and radial betatron amplitudes. Each of these contributions corresponds to full-width at half-height of the considered distribution.
- $|\Delta Q_{inc} - \Delta Q_{coh}| = U$ is plotted, divided by 20 for reasons of scale.
- Emittances correspond to twice the r.m.s. radius of the beam.

Fin. 3 same as 2, but for varying horizontal emittance.

Fig. 4 shows the signal of a single bunch coming from a position observation station (APU) on five consecutive passages. According to theory¹, such a signal should be

$$p(t) = P_m(t) e^{j\omega_\xi t + j2\pi k Q} \quad (1)$$

on the k^{th} revolution, where $\omega_\xi \approx \frac{\xi}{\eta} Q \omega_0$,

$$\eta = \gamma_T^{-2} - \gamma^{-2}$$

ω : revolution angular frequency,

and P_m is approximately sinusoidal with m nodes for head-tail mode m (Fig. 7). The phase shift from head to tail of the bunch due to non-zero chromaticity is

$$\chi = \omega_\xi \tau_L \text{ radians} \quad (2)$$

(τ_L = bunch length in seconds).

Fig. 4 shows a bunch oscillating in mode $m = 0$, with $\chi \approx 12$ radians. For smaller values of χ , oscillations approaching rigid-bunch motion are observed; the instability disappears for $\chi = 0$. One can check that the oscillation mode is $m = 0$ up to $\chi \approx 15$ radians or chromatic frequency $f_\xi \approx 200$ MHz, which is the bandwidth limit of the APU. For higher values of χ , the instability can be monitored (Fig. 5) by the Ionization Beam Scanner (IBS³), but a signal is no longer visible on the ΔP_U . This is expected if mode 0 is still the dominant mode at these high chromatic frequencies; low frequency signals would be observed if higher order modes grew (see Fig. 9).

Figure 4 :
 Δ signal for
 a single bunch
 on 5 consecutive
 revolutions :

a) mode 0 observed for $\chi = 12$ radians (2 ns/div.)

b) computed from equation 1.

- 5 traces

- 1 trace

Figure 5 :
 Mountain range
 display (5 ms/cm)
 of the Ionic Beam
 Scanner during
 the instability.

Very large phase shifts ($\chi > 100$ radians, chromatic frequencies f_ξ in the GHz range) can be investigated, on a magnetic flat-top near to transition energy (η small). Growth-rates have been measured as a function of f_ξ . Different energies (hence different η) have been used to look more closely at one region or the other of the frequency spectrum. For $f_\xi < 200$ MHz, the growth rate was monitored using the signal from our P.U. station. Above 200 MHz, the IBS was used. It was set to collect electrons from a small region centered on the beam - the rate of decrease of this signal is related to the growth-rate. This is a rather crude measurement, but it has the advantage of being independent of frequency. At low frequency, it agreed quite well with the P.U. measurements. Fig. 6 shows the results of these investigations : the striking feature is that the growth rate increases markedly with frequency.

Figure 6 :

Growth rate as a function of chromatic frequency f_ξ was measured for three energies (error bars are drawn only for last point of each curve). The impedance deduced from eq. (4) is indicated as well as the resistive-wall impedance ($\eta \approx \omega/\omega_0$).

Discussion and Interpretation

The single-bunch feature and the disappearance of the instability at zero chromaticity is well explained by the head-tail mechanism described by Pellegrini⁴ and Sands⁵. For short-range wake fields, the growth rate for mode m is given by¹ :

$$\tau_m^{-1} = \frac{-1}{m+1} \frac{1}{2Q\omega_0} \frac{e\beta}{\gamma m_0} \frac{I_0}{L} \frac{\int_{-\infty}^{+\infty} \text{Re } Z_\perp(\omega) h_m(\omega - \omega_\xi) d\omega}{\int_{-\infty}^{+\infty} h_m(\omega) d\omega} \quad (3)$$

where $Z_\perp$ is the transverse coupling impedance which can often be related to the usual longitudinal impedance $Z_\parallel$ (see eq. 6); $h_m(\omega)$ is the frequency spectrum of mode m (Fig. 9); m_0 is the rest mass of the proton; I_0 is the current in one bunch of length L meters. This formula can be applied with the hollow-bunch modes of Pellegrini and Sands, or with the more realistic sinusoidal modes defined by Sacherer¹.

The experimental observations are consistent with

the assumption that $Z_\perp$ is a broad-band impedance, slowly varying with frequency. Then (3) can be simplified for mode 0 to

$$\tau_0^{-1} = \frac{-1}{2Q\omega_0} \frac{e\beta}{\gamma m_0} \frac{I_0}{L} \text{Re } Z_\perp(\omega_\xi) \quad (4)$$

which is the coasting beam result for ω_5 , but increased by the bunching factor.

Measuring τ gives then $\text{Re } Z_\perp(\omega)$ with a resolution of order $\delta\omega = 2\pi/\tau_L$, which is the width of the frequency spectrum for mode 0. On Fig. 5, a double vertical scale shows the correspondence between growth rate and impedance, according to this interpretation. The resistive-wall impedance is plotted for comparison : it cannot explain the observed phenomena.

An impedance that increases with frequency is also consistent with the absence of higher-order modes. A

search for equipment that could explain Fig. 5 has been made. It was concluded⁶ that ceramic vacuum chamber sections, introduced to allow fast field penetration from pulsed magnets, and ferrite structures used in fast kickers, are good candidates. Unfortunately, it is hard to see how to modify them to reduce their impedance without affecting the CPS capabilities.

Damping with octupoles : It seems likely that non-linear space-charge fields of the beam contribute to the useful Q -spread², and can help or counteract the action of octupoles : this can perhaps explain why the necessary spread is sometimes much smaller than what is expected and the rather complicated behaviour observed. The subject needs more study, both theoretical and experimental, to be understood in detail.

Acknowledgements

Thanks are due to C.D. Johnson, builder of the IBS, for his participation in growth rate measurements.

Observations

Dependence on Q-value

The Q-value can be varied over the range 4 to 5.3. One finds that both horizontal and vertical instabilities depend strongly on Q-value, with the growth-rate increasing rapidly as Q_V or Q_H approaches 5 from below. Other conditions being equal, the horizontal instability grows faster.

Dependence on chromaticity

The chromaticities can be varied over the range $-2 > \xi_H > +.5$ and $-4 > \xi_V > +1$ with the zero-harmonic sextupoles. For zero chromaticity, rigid-bunch motion occurs (Fig. 7a). As $|\xi|$ increases, a phase-shift develops between head and tail of a bunch (Fig. 7b). For larger $|\xi|$, the higher-modes start to grow (Figs 7c-f). The demarcation between modes is not sharp, with often two or three modes growing for the same value of ξ . The higher modes appear to grow less fast than the lower modes.

a) mode $m = 0$, $\chi = 0$

b) $m = 0$, $\chi = 2.3$ radians

b) $m = 1$, $\chi = 6.9$ radians

d) $m = 2$, $\chi = 6.9$ radians

e) $m = 2$, $\chi = 8.9$ radians

f) $m = 3$,
 χ not recorded.

Fig. 7 : A single bunch seen on about 20 consecutive revolutions. Vertical axis : APU signal. Horizontal axis : time (50 ns/div.).

a-b-c-d : with wide-band PU (bandwidth ≈ 150 MHz)
e-f : with normal PU (bandwidth ≈ 40 MHz, and baseline restitution circuit acting).

Coupled-motion : Fig. 8a shows coupled motion for rigid-bunches ($\xi_V = 0$) with $Q_V = 4.96$. The frequency observed is

$$f_n = |n - Q_V| f_{rev} = 25.6 \pm 1 \text{ KHz}$$

for a revolution frequency $f_{rev} = 678$ kHz, and therefore the coupled-bunch mode number is $n = 5$. The evidence for coupled motion of the higher modes is less direct, but it appears from photos such as Fig. 7e that the motion is coupled. In any case, when the growth rate is reduced by lowering the Q-value, the motion is less strongly coupled with sometimes only two of the five bunches growing.

a) 10 μ s/div.

b) 5 ms/div.

Fig. 8 : Coupled bunch mode pattern a) and growth rate b).

Dependence on octupoles : Small currents in the hero-harmonic octupoles cure the instability except for Q-values near to but below 5 where larger currents are required. In all cases, the required spread in Q is less than the expected value $|\Delta Q_{coh} - \Delta Q_{incoh}|$.

Discussion

The growth-rates fit a resistive-wall impedance, or in general a narrow bandwidth impedance centered at zero frequency. The fact that the higher modes grow as the chromaticity is increased is explained by the frequency spectrum of the various modes (Fig. 9). One sees that increasing $|\chi|$ displaces the spectrum so that progressively higher-order modes overlap the assumed high impedance region at the origin. The numerical value of the phase-shift χ necessary for a given mode to grow agrees reasonably well with the theory (Eq. 5).

The frequency spectrum shown in Fig. 9 is actually discrete with lines at $f_n = (n+Q)f_{rev}$. The strong dependence on Q-value and the long-range nature of the wake fields imply that the impedance is sufficiently narrow that only the line f_n nearest the origin contributes (Fig. 10). In this case, the growth-rate for mode m is ¹ (MKS units)

$$\tau^{-1} = - \frac{1}{1+m} \frac{e \beta I}{2Q\omega_0 \gamma m_0} \frac{\text{Re } Z_{\perp}(f_n)}{2\pi R} F'_m(\chi) \quad (5)$$

which is just the coasting beam result but reduced by the factors $(1+m)^{-1}$ and $F'_m(\chi)$. Here, $F'_m(\chi)$ is proportional to the amplitude of the Fourier component at f_n (Fig. 9 and Fig. 7 of Ref. 1). The other quantities are: $\beta = v/c$, $\omega_0 = v/R$, I = total current for coupled motion or the current in one bunch for decoupled motion, m_0 = rest mass of particles, $2\pi R$ = machine circumference, and $Z_{\perp}$ is the transverse coupling impedance in Ohm/meter. For a circular pipe of radius b and surface resistivity R_{surf} , it is related to the more familiar longitudinal coupling impedance $Z_{||}$ by

$$Z_{\perp}(\omega) = \frac{2c}{b^2} \frac{Z_{||}(\omega)}{\omega} \quad (6)$$

where
$$Z_{||}(\omega) = \frac{2\pi R}{2\pi b} R_{surf} \quad (7)$$

and $R_{surf} = (1+j)\rho/\delta$ provided the wall thickness l exceeds the skin depth δ , or $R_{surf} = \rho/l$ if $\delta > l$. Here ρ is the resistivity in Ohm-meters. It follows that the growth-rate varies as $|n-Q|^{-1/2}$ for thick walls, or as $|n-Q|^{-1}$ for thin walls and that the modes with $|n|$ larger than Q are unstable. In the limit $\chi = 0$ and for the thick-wall impedance, (5) reduces to the familiar Courant and Sessler⁷ result for mode $m = 0$.

Measurements of growth rate for different Q-value are shown in Fig. 11. The chromaticity was set to zero so that only mode $m = 0$ grows. The frequency $|n-Q|f_{rev}$ was determined to an accuracy of 1 or 2 kHz (Fig. 8a), and the growth rate to about $\pm 5\%$ (Fig. 8b). The results have the thin-wall form with $\bar{Z}_{\perp} \approx 10.2 \times 10^5$ Ohm/m at 30 kHz, or $Z_{||} = 321 b^2$ Ohms independent of frequency. On the other hand, the major contribution to the vacuum chamber impedance in the Booster comes from the 52 m of thin-walled ($l = 0.4$ mm, $\rho = 1.3 \cdot 10^{-6}$ Ohm-meters) chamber in the bending magnets. The skin depth is much larger than the wall thickness ($\delta \geq 1.7$ mm for $Q \geq 4.8$) so one expects $Z_{||} = 0.027/b$ Ohms independent of frequency. This agrees with the measured value for $b = 4.4$ cm, which is about what one expects for the actual rectangular chamber of half-height 3 cm and half-width 6.6 cm.

Fig.9 Frequency spectrum for modes 0,1 and 2 drawn for $\chi = 2\pi$

Fig. 10 : Spectrum lines for mode $m = 0$ and Q just below an integer.

Fig.11 e- folding time vs Q on 70 MeV flat top normalized to $I = 100$ mA

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DISCUSSION

Gus Voss (DESY): I see that there are no questions anymore about the existence of higher modes of the head-tail instability.

Lee Teng (NAL): In what way does the bunched beam resistive wall effect differ from that on an extended beam? Assume the beam is continuous and just take a section and add the appropriate boundary conditions. Wouldn't that be exactly the same thing?

Frank Sacherer (CERN): If you put all the boundary conditions on it's probably the same. It's the same electromagnetic fields, and the particles move in the same ways. You can get quite confused if you use coasting beam theory in the case of bunched beams. The growth rates are bigger by the bunching factor, for example. But the electro-dynamics and all that are roughly the same, although that's certainly not the way it was arrived at.

Alessandro Ruggiero (NAL): Now let me comment on that. The difference depends on the synchrotron motion. If you take a coasting beam, chopped, but with no synchrotron motion, there is one kind of instability. If you take a bunched beam with synchrotron motion, there is another kind of instability.

Introduction and Summary

A glance at Figs. 1 and 2 shows some of the contortions that a single bunch can undergo as it progressively destroys itself. The instability is driven by the surrounding environment, which extracts energy from the longitudinal directed motion of the bunch and converts it into growing transverse oscillations. As pointed out by Pellegrini¹ and Sands², simple rigid-bunch motion is possible only in the exceptional case of zero machine chromaticity; otherwise there is always a phase-shift between head and tail of a bunch. While the original and still widely used theory of Courant and Sessler³ is restricted to rigid-bunch motion driven by long-range wake forces, the more recent head-tail theory of Pellegrini and Sands is restricted to the opposite limit of short-range wakes that act only from head to tail of a bunch, and not from bunch to bunch or over many revolutions. In addition, it was developed mainly for the unrealistic hollow-bunch distribution (see Fig. 3).

This paper presents a unified approach for a parabolic bunch that includes both single-turn and multi-turn effects. The main ingredients are

- i) oscillation modes
- ii) transverse coupling impedance $Z_{\perp}(\omega)$
- iii) stability criterion.

The derivations are given in other papers^{4,5} and only the results are presented here. A companion paper⁶ presents the recent experimental observations in the CERN PS and Booster.

Classification of modes

If all particles have the same betatron frequency $\omega_B = Q\omega_0$ and synchrotron frequency ω_s , and we ignore the transit time of the bunch past a fixed observer, the first few head-tail modes appear as in Fig. 1a. The difference signal from a position monitor has the form

$$A\text{-signal} \propto p_m(t) e^{j2\pi k Q}, \quad (1)$$

for the k^{th} revolution.

Usually both Q and the revolution frequency ω_0 depend on momentum, so ω_B varies as a particle moves

Fig. 1 Contortions of a single bunch on separate revolutions, and with six revolutions superimposed. Vertical axis is difference signal from position monitor, horizontal axis is time, and $Q = 4.833$.

Fig. 2 Head-tail modes observed in the CERN Booster⁶.

around a synchrotron orbit. The important quantity is the betatron phase of a particle at each position along the bunch as compared with the phase of the synchronous particle. The total phase-shift χ between head and tail has contributions from the Q -variation, the ω_0 variation, plus the finite transit time. In fact, the last two contributions cancel, and we are left with

$$\chi = \frac{\xi}{\eta} Q \omega_0 \tau_L \quad (\text{radians}), \quad (2)$$

where $\xi = (dQ/Q)/(dp/p)$ is the chromaticity, $\eta = \gamma_T^{-2} - \gamma^{-2}$, and τ_L is the bunch length in seconds. The difference signal has the form (Figs. 1b and 1c)

$$\Delta\text{-signal} \propto p_m(t) e^{j\omega_\xi t + j2\pi k Q}, \quad (3)$$

where

$$\omega_\xi = \frac{\chi}{\tau_L} = \frac{\xi}{\eta} Q \omega_0. \quad (4)$$

Note that for the example shown in Fig. 1c, mode 2 has appreciable contre-of-mass motion and would leave a long-range resistive-wall wake whereas mode 0 would not.

For a parabolic bunch, the modes $p_m(t)$ are approximately sines and cosines as shown in Fig. 1, while for a hollow bunch they appear as in Fig. 3 with

$$p_m(t) = \frac{T_m(t/\tau_L)}{\pi \sqrt{1 - (t/\tau_L)^2}}, \quad (5)$$

where T_m is a Chebychev polynomial.

Fig. 3 Hollow-bunch modes for the same parameters as in Fig. 1c

Coupling impedance

Longitudinal

$$Z_{\parallel} = \frac{2\pi R}{2\pi b} \mathcal{R}_{\text{surf}} \quad (6)$$

where $2\pi R$ = machine circumference, b = vacuum chamber radius, and $\mathcal{R}_{\text{surf}}$ is the surface impedance in ohms per square. In this case the beam sees a uniform longitudinal electric field set up by the return currents flowing in the vacuum chamber walls.

Transverse

The wall currents flow in opposite directions on either side of the vacuum chamber (or whatever is enclosing the beam) and set up a transverse magnetic field and a longitudinal electric field that varies in strength across the aperture (Fig. 4). Energy

Fig. 4

Currents:

$$\begin{aligned} I_w &= \text{wall current} \\ J_0(x, y, z) e^{j\omega t} &= \text{beam current density} \\ &\approx J_0(x, y, z) - \frac{\partial J_0}{\partial y} \Delta e^{j\omega t} \end{aligned}$$

$$\text{with } I = \text{beam current} = \int J_0(x, y, z) dx dy$$

Fields:

$$\begin{aligned} E &= E_0 \frac{y}{b} e^{j\omega t} \quad (\text{median plane}) \\ B_x &= \frac{j}{\omega} \frac{E_0}{b} e^{j\omega t} \quad (\text{from } \nabla \times E + \dot{B} = 0) \end{aligned}$$

extracted from the directed motion of the beam by the longitudinal electric field drives the wall currents, which set up the dipole magnetic field, which deflects the beam. Expressed in equations, the power lost per unit length by the beam is

$$\begin{aligned} \int E \cdot J dx dy &= -\frac{E_0}{b} A \int y \frac{\partial J_0}{\partial y} dx dy \\ &= \frac{E_0 \Delta}{b} I \end{aligned} \quad (7)$$

while the power flow into the walls is $-2I_w E_0$, and therefore

$$I_w = -\frac{1}{2} \frac{\Delta}{b} I \quad (8)$$

This current is related to the electric field at the wall by the wall impedance,

$$E_0 = 4 \frac{I_w Z_{\parallel}}{2\pi R} \quad (9)$$

where in place of the actual current distribution in the wall, we assume that I_w is confined within a strip of width $\frac{1}{4}$ the pipe circumference. This gives the correct result for a circular pipe, which has a $\cos \theta$ current distribution. The deflecting magnetic field is (from Fig. 4)

$$B_x = -j \frac{2A}{\omega b^2} \frac{Z_{\parallel} I}{2\pi R} e^{j\omega t} \quad (10)$$

and when this is inserted into the definition

$$\frac{\int [E + v \times B] dz}{\quad} \quad (\text{ohms/metre})$$

one finds

$$Z_{\perp} = \frac{2c}{b^2} \frac{Z_{\parallel}}{\omega} \quad (12)$$

where c is the speed of light (3×10^8 m/sec) and $\beta = v/c$. The definition (11) was introduced by Hereward, and is used extensively by the ISR group.

The convenient relation (12) between $Z_{\perp}(\omega)$ and $Z_{\parallel}(\omega)$ is strictly valid for a round pipe with surface impedance $\mathcal{R}_{\text{surf}}$ and for frequencies sufficiently below cut-off that the fields have the simple form shown in Fig. 4. On the other hand, for perfectly conducting walls ($\mathcal{R}_{\text{surf}} = 0$)

$$Z_{\perp} = -j \frac{R Z_0}{\beta^2 \gamma^2} \left(\frac{1}{a^2} - \frac{1}{b^2} \right) \quad (13)$$

where $Z_0 = 377$ ohms, a = beam radius, and b = pipe radius. The additional contribution due to wall resistivity can be found from (6) and (12) with

$$\mathcal{R}_{\text{surf}} = (1 + j) \frac{\rho}{\delta} \quad (\text{thick-wall}) \quad (14)$$

where ρ is the resistivity (ohm-m) and δ is the skin depth, which is assumed to be smaller than the wall thickness. At low frequencies where $\delta >$ wall thickness ℓ ,

$$\mathcal{R}_{\text{surf}} = \frac{\rho}{\ell} \quad (\text{thin-wall}) \quad (15)$$

provided the impedance of the outside material (air, magnets, etc.) is sufficiently high that all currents

flow through the walls. At low frequencies, $Z_{||}$ is just the d.c. resistance of the vacuum chamber, typically about one ohm for stainless steel, and increases with frequency as $\sqrt{\omega}$ owing to the skin effect. Interruptions in the conducting vacuum chamber for ceramic or ferrite sections leads to much larger impedances. More elaborate and accurate calculations of $Z_{||}$ and $Z_{\perp}$ or equivalently U and V can be found elsewhere⁷⁻¹⁰, but the above approach is often sufficient.

Growth-rates in the absence of frequency spreads

The growth-rate is

$$\frac{1}{\Delta\omega} = -\text{Im } \Delta\omega \quad (16)$$

and the motion is unstable if $\text{Im } \Delta\omega$ is negative. For purpose of comparison, the coasting-beam growth-rate is found from

$$\Delta\omega = \frac{j}{2\omega_{\beta}} \frac{e\beta}{\gamma m_0} \frac{Z_{\perp}(\omega) I}{2\pi R} \quad (\text{coasting beam}) \quad (17)$$

$$= U + (1 - j)V$$

with $\omega = (n + Q)\omega_0 + \Delta\omega$ for the mode with $|n|$ wavelengths around the machine circumference $2\pi R$, beam current I , particle rest mass m_0 , $\beta = v/c$, and $\omega_{\beta} = Q\omega_0$. MKS units are used throughout with an assumed time dependence $\exp(j\omega t)$. To compare with papers using $\exp(-i\omega t)$, replace j with $-i$ in all formula.

For a bunched beam, the growth-rate involves a sum over the bunch spectrum. We need

$$h_m(\omega) = |\tilde{p}_m(\omega)|^2$$

where $\tilde{p}_m(\omega)$ is the Fourier transform of $p_m(t)$ (see Fig. 5).^m The spectrum is discrete with lines at $\omega_p = (p + Q)\omega_0$, $-\infty < p, \infty$ for a single bunch or several bunches oscillating independently. For coupled motion of M bunches, only every M^{th} line occurs with $p = n + kM$, $-\infty < k < \infty$, where n is the coupled-bunch mode number. It specifies the phase difference $2\pi|n|/M$ between adjacent bunches.

The growth-rate for mode m is found from

$$\Delta\omega_m = \frac{1}{1+m} \frac{j}{2\omega_{\beta}} \frac{e\beta}{\gamma m_0} \frac{I_0}{L} \frac{\sum_p Z_{\perp}(\omega_p) h_m(\omega_p - \omega_{\xi})}{\sum_p h_m(\omega_p - \omega_{\xi})}, \quad (18)$$

Fig. 5 Frequency spectrum for modes 0 to 3 with $\chi = 0$.

where I_0 is the current in one bunch of length L metres. The factor $(1 + m)^{-1}$ arises because the motion for the higher-order modes is constrained more and more to the few particles with large synchrotron amplitudes; this factor is absent for the hollow-bunch. Equation (18) is the general result. In the limit of short-range fields or smoothly varying $Z_{\perp}(\omega)$, it reduces to the classic head-tail effect, while in the opposite limit of long-range fields or rapidly varying $Z_{\perp}(\omega)$, it gives the multiturn contribution.

Fig. 6

As an example, consider Fig. 6, which is drawn for a positive phase shift χ so one is above transition with $\xi > 0$ or below transition with $\xi < 0$. The phase shift $\chi = 3 \times 2\pi$ corresponds to three oscillations along the bunch. Only the resistive or real part of $Z_{\perp}(\omega)$ causes instabilities, and this is drawn for a resistive-wall type impedance. Regardless of the type of impedance, the resistive part of $Z_{\perp}$ is positive for positive frequencies and negative for negative frequencies [from Eq. (12)]. From the figure, one sees that mode $m = 0$ is stable for χ positive and unstable for χ negative, and that this is true for any type of impedance.

If $Z_{\perp}(\omega)$ is sufficiently smooth that the sum in (18) can be replaced by an integration (discrete spectrum replaced by continuous spectrum), the growth-rate is independent of betatron frequency. In fact, the impedance shown is sufficiently smooth that it can be removed from the sum, and we find

$$\Delta\omega_m = \frac{j}{2\omega_{\beta}} \frac{e\beta}{\gamma m_0} \frac{Z_{\perp}(\omega_{\xi}) I}{2\pi R B} \quad (19)$$

which is just the coasting beam result (17) for the frequency ω_{ξ} with a bunching factor included ($B = ML/2\pi R$; $I = MI_0$).

It is convenient to rewrite (18) as

$$\Delta\omega_m = \frac{1}{1+m} \frac{e\beta I}{2\omega_{\beta} \gamma m_0 2\pi R} \times \quad (20)$$

$$\left[\frac{\frac{1}{B} \int_{-\infty}^{\infty} Z_{\perp}(\omega) h_m(\omega - \omega_{\xi}) d\omega}{\int_{-\infty}^{\infty} h_m(\omega) d\omega} + Z_{\perp}(\omega_n) F'_m(\chi - \omega_n \tau_L) \right]$$

near field,
independent of Q ,
 $\rightarrow 0$ or $\xi \rightarrow 0$

multiturn fields, de-
pend on Q , multiply by
 $1/M$ for independent
bunch motion

with the slowly varying part of $Z_{\perp}(\omega)$ separated from the rapidly varying part, where only one or a few lines $\omega_n = (n + Q)\omega_0$ contribute. The form factor $F'_m(\chi)$ is plotted in Fig. 7.

Fig. 7

If only a single line is important as for the narrow spectrum shown in Fig. 8, Eq. (20) again leads to the coasting beam result (17), but reduced by the factors F'_m and $(1+m)^{-1}$.

Fig. 8

A less obvious case is the resistive-wall impedance shown in Fig. 9, but here also only a single line contributes strongly to the long-range wake, namely the line $\omega_n = (n+Q)\omega_0$ nearest the origin. One easily recovers the usual rule that the mode number $|n|$ just above Q grows fastest. In the limit $\xi \rightarrow 0$ and for the thick-wall impedance (14), the multiterm part of (20) reduces to the familiar Courant and Sessler formula³ (as corrected by Morton^{11,12}). However, ξ is rarely zero, and often the frequency $|n-Q|\omega_0$ is sufficiently small that the thin-wall impedance (15) applies with consequently larger growth-rates. Although Fig. 9 is drawn for mode 0, for large enough χ the spectrum of

Fig. 9 The frequencies $\omega_n = (n+Q)\omega_0$ drawn for mode $m=0$ and Q just below an integer

mode m overlaps the high impedance region near the origin: for example for $\chi = 9$ radians, mode $m=2$ grows fastest (see Figs. 1c and 7).

Now consider only the near-field part of (20). As an illustration of the graphical approach, the growth-rates of modes 0 and 2 for a resistive-wall impedance are sketched in Fig. 10b. As pointed out above, mode 0 is stable for positive χ , but one sees that mode 2 is unstable for small positive χ , and the reason is evident

Fig. 10

from Fig. 10a. For χ sufficiently large, both modes have the same stability character as the coasting beam. The exact result for the thick-wall impedance (14) and sinusoidal modes is shown in Fig. 11, and the growth-rate is found from

$$\Delta\omega_m = \frac{j}{1+m} \frac{e\beta I}{2\omega_\beta \gamma m_0} \frac{1}{2\pi R} \times \left[\frac{\sqrt{2\pi}}{\sqrt{MB}} Z_\perp(\omega_0) F'_m(\chi) + Z_\perp(\omega_n) F'_m(\chi - \omega_n \tau_L) \right]. \quad (21)$$

At this point some history is in order. The formula of Pellegrini⁷ and Sands⁸ is just the near-field part of (20), but expressed in the time domain and written explicitly for the hollow-bunch modes (5) and the resistive-wall impedance (14). They simplified the integration by considering only small χ , and found that the modes with $m > \bar{n}$ are unstable when mode $\bar{n}$ is stable, and vice versa. Later Zotter¹³ carried out the integration numerically and found results similar to Fig. 11. However, the resistive-wall impedance is not sufficient to explain the fast growth-rates or absence of higher-order modes in electron storage rings. This is also true for the PS: Gareyte⁶ has made detailed measurements of growth-rate as a function of phase-shift χ and has deduced from (19) that above 100 MHz the impedance rises slowly with frequency to a broad maximum around 1 GHz. Such an impedance can result from the several metres of ceramic and ferrite elements required for extraction magnets. A slowly rising impedance also explains the absence of higher-order modes: from

Fig. 11

Fig. 12 one sees that in this case the broad positive part of the spectrum contributes more than the narrow region of negative frequencies near the origin. This was also pointed out by Ruggiero¹⁴. One should have no trouble imagining other types of impedance and estimating the corresponding growth-rates.

Fig. 12

Stability criterion

A sufficient spread in bunch frequencies prevents the coupled-bunch type instability. Such a spread may arise from a difference ΔN in bunch populations, via the coherent Laslett frequency shift $\Delta\omega_c = \Delta Q_c \omega_0$. For decoupled motion, the r.m.s. spread in bunch frequencies should exceed the growth-rate*

$$|\Delta\omega_c| \left(\left| \frac{\Delta N}{N} \right| \right)_{\text{rms}} > |\tau^{-1}|, \quad (22)$$

and this may occur for large space-charge forces.

Octupoles cure either single-bunch or coupled-bunch instability if they produce enough frequency spread within a bunch to prevent its coherent motion, that is pro-

vided the spread in betatron frequencies exceeds the frequency shift $\Delta\omega_m$,

$$|\text{full-spread at half-height of } Q\omega_0| > |\Delta\omega_m|. \quad (23)$$

Sextupoles or changes in machine chromaticity change the phase-shift χ , but do not contribute to Landau damping. For the long-range resistive-wall instability observed in the PS Booster, increasing χ shifts the instability to higher-order modes which have slower growth-rates. The opposite occurs for the PS or electron storage rings, namely the growth-rate increases as χ is made more negative.

Acknowledgements

I have profited very much from discussions with J. Gareyte, H.G. Hereward, K. Hübner, D. Möhl, and B. Zotter.

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I. Introduction

The phenomenon of current dependent bunch lengthening discovered at ACO and ADONE about seven years ago can be used as an engineering diagnostic tool to examine how the longitudinal electromagnetic field of a beam couples to its surroundings.¹ The importance of isolating such coupling factors is that they may give rise to damaging instabilities. Another practical reason for studying the effect is that if bunch lengths grow anomalously beyond those calculated on the basis of classical theory,² the effective luminosity of storage rings, particularly those employing low beta insertions, may be reduced.³ As in most other electron storage rings, the effect at SPEAR is observed to be large and may play a role in proton rings if the beam is highly bunched.

Two sets of theories explaining the effect exist. In one set it is suggested that the effect results from modification of the effective azimuthal potential well by beam loading. These theories can also lead to bunch shortening but not to an increase in the energy spread of the beam. A very general treatment, together with many examples of coupling impedance is put forth, for example, by Pellegrini and Sessler.⁴ The theoretical calculation of specific coupling impedances has received a great deal of attention in the last several years.⁵ On the other hand, an alternative explanation of bunch lengthening was advanced by Lebedev⁶ in which the effect arises due to instabilities of internal coherent synchrotron oscillations of the bunch. This concept has recently been extended to include the turbulent motion of the collective higher modes.⁷ The latter theories predict that the anomalous bunch lengthening is directly related to an increased energy spread of the beam. We believe that some ingredients from both sets of theories contribute.

In the following section we compare the functional dependence of the SPEAR bunch length with those of the various theories. In Section III the potential distortion model is used to extract a coupling impedance. The existence of coherent bunch shape oscillations is demonstrated in Section IV. Frequency shifts with current, and line widths are discussed. The excitation of theoretically predicted vacuum chamber modes is shown in the next section after which we estimate beam power absorption to those elements of the structure believed to cause bunch lengthening. In the final section, the prediction that the ferrite kicker magnet cores are presently the dominant elements is confirmed.

II. Bunch-Length Measurements and Data Reduction

The rms length σ_z of a single circulating bunch was measured as a function of wide-ranging values of average current I , total rf cavity voltage V_0 , and machine energy E_0 , by observing on a fast sampling oscilloscope the output from a specially mounted light-sensitive diode exposed to the optical part of the synchrotron-radiation spectrum. Details of this technique are reported elsewhere.⁸ A typical output scan is shown in Fig. 1. It is noted that at high currents the distribution tends to deviate from a purely Gaussian shape, and at times displays an asymmetric tail so that the values of σ_z calculated from the measured full width at half-maximum tend to lose their traditional meaning. (A discussion of bunch shape was recently presented by Haissinski.⁹) The data were corrected for instrumental rise time. The correction ranged from 30 percent at very low currents and very high voltages to an average of a few percent over most of the points taken.

*Work supported by the U. S. Atomic Energy Commission.

FIG. 1--A typical bunch length recording and calibration trace.

The so-called "natural bunch length" σ_{z0} , which results from the balance of quantum fluctuation and radiation-damping terms is calculated following the analysis of Sands² and is proportional to $E_0^{3/2} \alpha^{1/2} (V_0 \cos \phi)^{-1/2}$ and is of course independent of I . (Symbols used throughout this paper are identified in Table I.) The momentum compaction factor α and V_0 are singled out here because, in contrast to other machine parameters, they are not so well known and must be measured. Fortunately, the quantum lifetime near zero current, and the synchrotron frequency have a functional dependence on both α and V_0 and by a series of measurements of these quantities the cavity voltage readout was calibrated and α determined. The value of α found in this way differed from that calculated by the magnet-lattice program by only a few percent. Measurements taken over a span of one year with various different kinds of equipment reproduced to better than 10 percent.

Values of $R = (\sigma_z / \sigma_{z0})$ are plotted in Figs. 2, 3, and 4 as functions of I , E_0 , and V_0 , respectively. Some of the points shown in Fig. 2 were interpolated by less than 10% in

FIG. 2--The ratio $R = \sigma_z / \sigma_{z0}$ at various rf voltages and energies vs average single bunch current.

either current or voltage to facilitate intercomparison. For large R , the data demonstrate an $I^{1/3}$ dependence, which is also found at ACO¹⁰ and ADONE,¹¹ and which is common to

both sets of theories. For large I , the $E^{-5/3}$ dependence shown in Fig. 3 is also consistent with data from ACO and

FIG. 3-- R vs E_0 for various currents.

and ADONE. The potential distortion model⁴ predicts $E^{-3/2}$, the coherent synchrotron oscillation model⁵ $E^{-3/3}$ and the simplified turbulent higher-mode model⁷ $E^{-3/2}$. The dependence of R on cavity voltage is not so easily deciphered. Figure 4 shows that it is independent of voltage at very low

FIG. 4-- R vs rf voltage at 1.5 GeV and various currents. The fit to the data indicates a power-law dependence on voltage, the exponent of which is proportional to current.

currents, but that it rises to as much as $V^{1/4}$ for higher currents. The ACO and ADONE results are fit approximately by $V^{1/6}$ while the existing theories predict $V^{1/6}$, $V^{1/2}$, $V^{1/6}$, respectively. From these considerations only, therefore, it is hard to distinguish between the various models.

To fit the data over a wide range of the ratio R , which must tend to 1 as the current approaches zero, the function

$$R = \frac{2}{\sqrt{3}} \cos(1/3 \arccos KI) \quad (1)$$

derived in Ref. 6 as formula (24) was chosen.

For large R , $R \approx (KI)^{1/3}$. A reasonable phenomenological fit to all SPEAR data is

$$KI = 0.92 I_{mA} \left(\frac{1.5}{E_{GeV}} \right)^5 \left(\frac{V_{kV}}{100} \right)^{\frac{3I_{mA}}{427}} \quad (2)$$

in which only the constant and the power of V were adjusted.

The solid lines drawn in Fig. 2 obtain from the formula for R shown above. Relevant machine parameters are listed in Table I.

TABLE I. SPEAR I Parameters

r	= average ring radius = 37.3 meters
$\omega_0/2\pi$	= revolution frequency = 1.28 MHz
ρ	= magnetic radius = 12.7 meters
α	= momentum compaction factor = 0.04124 (configuration INJEQ)
h	= harmonic number = 40, rf frequency = 51.22 MHz
i_e	= energy partition constant = 2
τ_{rad}	= synchrotron damping time at zero current = 33 msec at 1.5 GeV
U	= radiation loss per turn in kV
Ω_0	= synchrotron frequency in kHz
ϕ	= synchronous phase angle in degrees
σ_{e0}	= rms energy spread at zero current in GeV
σ_0	= rms bunch length at zero current in cm
R	= the ratio σ_z/σ_{z0}
V_{rf}	= total cavity voltage
$\mathcal{R}$	= resistance

GeV	kV		kV	degrees	kHz	cm
1.5	35	0.539	100	159	5.05	13.6
			300	173	9.03	7.6
			500	176	11.68	5.9
2.0	111	0.958	300	157	7.56	12.1
			500	167	10.00	9.2
2.5	270	1.49	500	147	8.2	13.8

III. Application of Bunch-Length Data

Two calculations have recently been performed using the previously published SPEAR data.¹² Sessler⁷ has found a surprisingly good fit using the turbulent mode model and finds a value of coupling impedance $Z_n = i46n$, where the integer n indicates the coupling frequency in multiples of the revolution frequency. However, the derived values of energy spread are about a factor of 3 larger than those actually found at SPEAR from measurements of the quantum lifetime of large current beams. Further attempts at measuring the energy spread are under way. Sacherer,¹³ on the other hand, applied the potential distortion model⁴ (no increased energy spread is contained in this theory) and found either $Z_n = i30n$, or $i360n^{1/2}$ to fit the data. He pointed out that this is much too large to be caused by the conductivity of the vacuum-chamber walls. More will be said on this point further on. It may be instructive to follow his analysis in a very simplified form.

Formula (2.8) of Ref. 4 relates the current dependent synchrotron frequency shift with bunch length as follows:

$$(\Omega^2 - \Omega_0^2)/\Omega_0^2 = 1 - \left(\frac{\sigma_z}{R}\right)^2 = 1 - \left(\frac{1}{R}\right)^2 \equiv x \quad (3)$$

In terms of an effective impedance Z_{eff} , x is also the ratio of the beam-induced voltage to the impressed voltage, i. e., $x = I Z_{\text{eff}} / h V_0 \cos \phi$.

In the case when Z_{eff} is dominated by the high frequency components of the beam one can write

$$Z_{\text{eff}} = i \sum_n n Z_n e^{-1/2 n^2 \sigma_z^2 \omega_0^2}$$

in which ω_0 is the revolution frequency for a single bunch machine and σ_z is in time units. If, for example, one assumes that Z_n is of a nonresonant form and can be fitted by a power law $Z_n = -i R |n|^s$, then the sum over n may be simply carried out and, for $s=1$, yields

$$Z_{\text{eff}} = \frac{2.5 R}{(\sigma_z \omega_0)^3} \quad (4)$$

Substituting this in the above, we have

$$x = \frac{2.5 R}{h V_0 \cos \phi \omega_0^3} \left(\frac{1}{\sigma_z}\right)^3 \quad \text{or} \quad R I = \frac{x \sigma_z^3 V \cos \phi h \omega_0^3}{2.5} \quad (5)$$

For Eq. (3) one notes for large $R = \sigma_z / \sigma_{z0}$, x approaches 1, and from Eq. (5) the bunch length should then vary as $I^{1/3}$, consistent with observation. $R I$ is plotted in Fig. 5, which

FIG. 5--The quantity $R I = \frac{x \sigma_z^3 V \cos \phi h \omega_0^3}{2.5}$ vs average current I .

demonstrates that a single value of $R=30 \Omega$ fits the data quite well at low rf voltage so that simple assumption made about the form of Z_n is borne out. Data in the very short bunch regime, i. e., high voltage, low current regime, however, does not fit this simple model,

Several comments may be made at this point. First: Since σ_z is of the order of nanoseconds, very high harmonics of the revolution frequency are involved in the above sum and Z_{eff} is quite large, in fact, around 10^8 ohms, and one would expect the beam to lose a good deal of power somewhere around the ring. Second Since x approaches 1, one would expect large shifts of the synchrotron frequency of particles in the bunch with current. These shifts will not be observed for the coherent center-of-mass motion or dipole frequency because the induced defocusing force of the beam on itself can be thought to be traveling around the ring with the bunch, but might be seen in the frequencies of high-order shape oscillations.^{6, 14}

IV. Observation of Coherent Bunch-Shape Oscillations

In summer of 1973, a high-gain low-frequency spectrum analyzer* was installed to permit more accurate measurement of betatron frequencies. This instrument can also be used to measure the frequencies of synchrotron oscillations, either those occurring naturally or those excited by frequency-modulating one main rf cavity driver. Again, the details of technique are reported elsewhere.⁸ Under certain operating conditions, especially for low cavity voltage and high beam current, a rich spectrum of naturally occurring lines was observed. Typical sweep traces are shown in Fig. 6. The

FIG. 6--Higher mode frequency spectrum tracings taken at 1.5 GeV, $V_{\text{rf}} = 100$ kV, and various currents.

frequencies of the lines seen are proportional to $(V \cos \phi)^{1/2}$ and arise from high-mode synchrotron oscillations. The current dependence of the frequencies is shown in Fig. 7. One may note that the shifts, down from integral multiples of the dipole synchrotron frequency, vanish at zero current. Also, at higher currents the frequency shifts become constant. We do not believe that the coherent dipole frequency shifts significantly with current at current levels below 60 mA.

During these measurements the cavity gap voltage was stabilized to compensate for beam loading.¹⁵

*Hewlett Packard Model 3950A.

FIG. 7--Higher mode frequencies vs single bunch average current. The shifts from multiples of the synchrotron frequency are indicated on the right in kHz.

Further demonstration that the lines arise from shape oscillation stems from the following experiment. One main rf cavity was externally phase-modulated at the shifted frequency $\Omega_Q - 2\Omega_s = \Delta\Omega$ and the bunch shape was photographed using the sampling oscilloscope with its trigger locked to the modulating frequency. A series of photographs taken at various fixed times in the modulating cycle therefore represent a slow motion sequence of the oscillation and clearly showed the "breathing" nature of the quadrupole mode. Under certain conditions of cavity voltage and beam current, specific naturally occurring modes become dominant. A multiple trace photograph of a bunch executing sextupole oscillations is shown in Fig. 8. This shape is precisely that

FIG. 8--Multiple sweep photograph of sampling oscilloscope showing self-excited

coherent sextupole oscillations of the bunch shape. Sweep speed=1 nsec/cm. Single bunch e^- current 134 mA at 1.5 GeV. Cavity voltage 235 kV.

outlined by Sacherer (Fig. 1, $m=3$ in Ref. 14). The octupole mode could also be seen in this way.

We turn now to the magnitude of the frequency shifts as a function of mode number m . The convention used is: For the dipole, $m=1$, quadrupole, $m=2$, etc. The importance of examining this dependence is that various impedance structures would yield differing results.¹⁴ Examining Fig. 7, one sees that the frequency shifts do not bear a fixed relationship to each other until high currents are reached. Something more complicated must be going on. In spite of this, the higher mode frequencies at large current are well fitted by expression (45) of Ref. 18, $\omega_{mn} = nQ_z + 2nC_{mn}\Delta Q$ in which $Q_z = Q_{z0} - \Delta Q$ and we have set $m=n$ for the fastest growing modes. The coefficients C_{mn} used were those calculated by Sacherer for the so-called Legendre modes that arise from a parabolic charge distribution. From the fit, one finds $\Delta Q = 0.83$ kHz at $Q_z = 5.0$ kHz. Since $C_{11} = 0.5$ the dipole frequency does not shift with current in this theory.

As stated before, specific modes are preferentially excited at certain currents. In fact, this is observed to occur for those currents at which the dashed line in Fig. 7 intersects the frequency of that mode. Even harder to understand is the appearance in the spectra of a relatively broad line, whose frequency is given by this dashed line. The first observation might be explained as follows. Suppose there exists in the ring a beam-excited resonant gap of spacing $D = \lambda/2$, so that when the total bunch length $4\sigma_z = m\lambda/2$, the m th mode is preferentially coupled. This picture corresponds to the form-factor treatment of Ref. 14. Unfortunately the predicted even spacing in bunch length with m is not borne out. The second observation suggests that there exists inside the bunch a separate population of particles, which oscillate coherently at frequencies linearly proportional to the total current and when frequencies coincide, preferential mode excitation occurs. Although the observed modes are clearly coherent, it is not yet possible to tell precisely what fraction of the bunch population is oscillating coherently.

The above discussion concerns the real part of the frequency shift. A measure of the imaginary part was obtained in two ways: swept line widths and decoherence times following shock excitation. The techniques are reported elsewhere⁸ and were first tried out on the dipole line. Near zero current, this line displays a proper Lorentzian shape and yields the proper classical damping time. Both cavity detuning, producing Robinson¹⁵ damping and phase feedback (when used),¹⁵ broaden the line correctly. Both instrumental methods give the same results. Although rf-system damping should be different for the quadrupole mode, the observed line shape, taken with high resolution, is not understood. Moreover, the full width at the base is too narrow to fit the generally accepted stability criterion¹⁴ that the spread be greater than the shift. This last fact may, however, be consistent if mode mixing, suggested in the turbulent equilibrium theory,¹⁷ contributes. At the highest operating energies (2.5 GeV) quantum fluctuations appear to wash out the coherent modes.

V. Observation of Vacuum-Chamber Resonances

In SPEAR, the aluminum-extruded, curved vacuum chambers that thread through the bending magnets are connected to each other by round, 2-meter long 20-cm-diameter stainless-steel tubes. The tubes house a variety of machine elements. Common to each straight section is a subsection containing a set of beam position pickup electrode buttons. Signals from these electrodes were observed to contain very

high frequency components which were found not to be characteristic of the electrodes and their associated circuitry.¹⁹ A detailed spectrum-analyzer examination of all twenty stations showed lines at 1.1, 1.4, and 1.8 GHz (Fig. 9). These

FIG. 9--Spectrum analyzer traces showing resonances in cylindrical vacuum-chamber straight sections at 1.1, 1.4, and 1.8 GHz. (a) 100 MHz/cm, (b) and (c) expanded scales showing lines which are high harmonics of the 1.28 MHz revolution frequency. The ordinates are proportional to power. Taken at 20 mA, 1.5 GeV, 500 kV.

frequencies are just those of the so-called "waveguide modes," which are independent of cavity length and are mentioned in Ref. 4. Their usual designation is TM_{010} , TE_{210} (TM_{110} - TE_{010}), respectively. Transverse magnetic modes certainly could be excited by the axial electric field of the beam, and the cavity impedance for these modes can be estimated. In fact, one of the first calculations explaining bunch lengthening correctly was due to Robinson²⁰ who considered the fields due to vacuum chamber discontinuities. For the TM_{010} mode for example, one has the relation,

$$\frac{R}{Q} = 308 \sin(\pi \ell / \lambda) \quad (5)$$

where ℓ is the length of the cavity and λ the resonant wavelength. The Q of the 1.1GHz mode is estimated from Fig. 9 to be about 400. For twenty elements having a length $\ell \sim \lambda/2$, the preceding relation then gives a total impedance of about 2.5×10^6 ohms, a significant value.

Since the purpose of this investigation is to attempt to determine which machine elements couple strongly to the beam, further miscellaneous tests were performed. The electric separation plates (normally terminated) and the electric quadrupole conductors were shorted to ground or opened. The 3rd harmonic cavity¹⁵ shorting bar was removed. The tuning paddle in an idling rf cavity was moved to resonance. None of these impedance changes affected the bunch length measurably.

VI. Beam-Power-Loss Estimates

A. Kicker Magnets

Following Sacherer,¹³ one may write for the power dissipation

$$P = \frac{1}{2} I^2 \sum_n \operatorname{Re} Z_n e^{-n^2 \sigma_z^2 w_0^2 / \omega_0^2} \quad (6)$$

Assuming the resistive part of Z_n to be of the same order as the reactive part, i.e., $\operatorname{Re} Z_n \approx \operatorname{Im} Z_n$, substituting, and performing the sum gives

$$P = \frac{1}{2} \frac{I^2 \mathcal{R}}{(\sigma_z \omega_0)^2} \quad (7)$$

For a beam current of 50 mA and using the value of $\mathcal{R}$ obtained from the fits in Fig. 5, one finds a loss of between 400 and 800 watts, the higher figure obtained from shorter bunches obtained with higher cavity voltages. The elements most likely to be responsible for this loss are the ferrite-kicker magnet cores which have been observed to have a large thermal outgassing rate, not associated with synchrotron radiation, in the presence of high current beams. The power dissipated per unit volume in the ferrite core at frequency ω is

$$P(\omega)/v = \frac{1}{2} \omega \mu'' \mu_0 H^2(\omega) \quad (8)$$

in which μ'' is related to the loss tangent and relative permeability by $(\mu''/\mu_r) = \tan \delta$. Using values of μ'' and μ_r for the 4C4 Ferroxcube material found in the literature,^{21,22} one can perform the sum over the high-frequency components of the beams numerically and find a power loss of about 100 watts for each of the four kickers, the loss peaking very broadly at frequencies of about 100 MHz. Unfortunately, it is not known exactly what material the manufacturer actually provided, so this rather good agreement is only qualitative but is consistent with core heating times.

B. Cavity Modes

An estimate of power loss to the cylindrical cavity modes can also be made. In this case, one is dealing with low- Q resonators at specific high frequencies. The excitation of, for example, the 1.1GHz TM_{010} mode depends strongly on bunch lengths, which in turn is a function of current. The total beam loading voltage due to all twenty resonators can be written

$$V_T = I R_T f_1(\sigma_z) f_2(T_0/T_F) \quad (9)$$

where $f_1 = e^{-\omega^2 \sigma_z^2 / 2c^2}$ gives the component of the current at frequency ω resulting from the finite bunch length. The function f_2 gives the enhancement in the energy loss which results when the bunch revolution time, T_0 becomes comparable to the decay time, T_F , for the mode in question. For SPEAR, T_0 is 0.78 μ sec, for a cavity with a Q of 400 at 1.1GHz, $T_F = 0.12$ psec giving $T_0/T_F = 6.6$. In Ref. 23 it is shown that for $T_0/T_F \gg 1$, $f_2 \sim 1/2 (T_0/T_F) = 3.3$. The function f_1 decreases rapidly with increasing current because of the increase in bunch length. The product $I f_1$, in fact, reaches a maximum value of 0.9 mA at a current $I = 5$ mA. Using $R_T = 2 \times 10^6$ ohms as previously calculated, V_T is about 6 kV. The total power dissipation is $I f_1 V_T = 5$ W. The power dissipated in all of the cavity modes might be several times this, or about 15 W. In contrast to the situation at DORIS,²⁴ resistive losses due to the aluminum vacuum chamber wall with these bunch lengths amount to no more than a few watts.

VII. Effect of Removing Half the Kicker-Magnet Core

In January, 1974, two of the four magnets were replaced with ferriteless magnets of a new design. Bunch lengths and frequency shifts were remeasured. The results are shown best in Fig. 5. If the ferrite cores were the dominant bunch-lengthening component and half of them were removed from the ring, one should expect the slope of $\mathcal{R}I$ to be halved. This is indeed nearly the case. For higher voltages, i.e., short bunches, the effect is less dramatic. The resultant ~ 20 percent reduction in bunch length (recall Eq. (5)) also shows up in the mode spectra. The rich spectrum up to 6th order previously seen at 70 mA now contains prominent lines only up to order 2, but can be reproduced at 170 mA. The frequency shifts of the higher modes are also uniformly reduced by 25 percent. Removal of the remaining two kicker cores is scheduled for May, 1974, at which time large further reduction of bunch length is expected. How close to the theoretical value it becomes will depend then on the influence of the aluminum-chamber impedance, cavity-mode excitation, and as yet unidentified elements whose effects have been masked by those of the kickers. Further, an entirely new regime will obtain after conversion of SPEAR to higher energies. The present RF system will be replaced with one of seven times the frequency and capable of providing accelerating voltages up to 7 MeV/turn. The resulting extremely high peak currents of a single bunch will no doubt raise interesting new problems.

VIII. Conclusions

On the basis of the data obtained at SPEAR, ACO, and ADONE, we conclude that no single theory presently explains the bunch lengthening effect, however, use of existing theories has lead us to identify the dominant element responsible in SPEAR. The search for other elements will continue after all the ferrite kicker magnet cores have been removed. We find that the existence of higher order bunch-shape oscillations, and measurements of their real and imaginary frequency shifts provide an additional, perhaps simpler tool for the study of the electromagnetic beam-environment interaction. It remains to compare such observations with theory in a quantitative way.

Acknowledgments

We wish to thank the other members of the SPEAR Group, R. Helm, M. Lee, P. Morton, J. M. Paterson, B. Richter, and the operating staff for their assistance.

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DISCUSSION

Karl Reich (CERN): Could you give some recipe on what should be done about the ferrites?

Gerry Fischer (SLAC): At SPEAR we have decided to remove them, and at PEP we have also decided not to use them.

Reich (CERN): Then replace them by what?

Fischer: Air core magnets.

Andrei Kolomensky (Lebedev Institute): What do you expect with your new machine which will go up to 4 GeV? There may be new resonant instabilities as well as the present ones.

Fischer: The bunch will be considerably shorter because the harmonic of the cavities has been raised a factor of 7, and the voltage will go up.

Kolomensky: So, it will improve?

Fischer: No. The bunch will be shorter, and therefore the bunch spectrum will now overlap the higher order resonances in the vacuum boxes, and I imagine that one should design mode suppressors for these boxes to prevent the impedance from having a bad effect. I'd just like to comment on the impedance that Sacherer mentioned. The longitudinal and transverse impedances are very much tied together, and if you have a bad impedance longitudinally you can probably get into trouble transverse.

Gus Voss (DESY): I think the data on the energy widening may perhaps be consistent with the observed bunch lengthening, if one keeps in mind that the bunch lengthening is observed with a photodiode where you observe plus or minus one or two standard deviations, whereas the energy spread is inferred from the quantum lifetime where one observes what happens at five standard deviations. If we no longer have a Gaussian distribution one would underestimate the energy widening in the center. If I understand it correctly, in all other storage rings the bunch lengthening data is consistent with an energy widening, and I believe it still might be here too.

Fischer: Our present thought is that the bunch energy widening is only a factor of two down from what is required to explain bunch lengthening. I really believe in potential distortion because at very high energies where the coherence seems to wash out, we have no bunch widening but we have bunch lengthening.

Andrew Sabersky (SLAC): There is strong evidence for potential well distortion which was not explicitly mentioned. That is, the bunch under certain conditions becomes highly asymmetrical. The tail is much longer than the head. This has been carefully checked to be sure that it's not an instrumental effect. We think that the only way to explain such an asymmetry is very strong potential well distortion.

Fischer: From the SPEAR magnetic detector data, looking at the time of arrival of Bhabha events, we feel that the leading edge is sharper than the tail.

INVESTIGATION AND CURES OF LONGITUDINAL INSTABILITIES OF BUNCHED BEAMS IN THE ISR

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(Presented by W. Schnell)

Summary

The present status of longitudinal bunched beam instabilities in the ISR is described. Experiments to test the theory have been conducted with a tunable, passive cavity. In order to cope with the instability at higher intensities we expect in the future, possible cures were investigated. A bunch-by-bunch feedback system has been built. It damps dipole and quadrupole oscillations of individual bunches. Stabilization by means of a passive cavity was also successfully demonstrated.

1. Introduction

The RF system in the ISR is mainly used to accelerate the injected beam for stacking in longitudinal phase space¹. It works at the 30th harmonic of the revolution frequency. The injector, the CERN proton synchrotron, has the same RF frequency but its circumference is only 2/3 of the ISR. Hence, the injected beam consists normally of 20 consecutive bunches. They are trapped on the injection orbit in 20 large, stationary buckets with the help of a phase lock system; the remaining 10 buckets are left empty. Each bunch covers a longitudinal phase space area of 0.16 eVs. The phase oscillation frequency is around 60 Hz and the injected beam current equals ~ 75 mA. The bunches stay in these large buckets for approximately 10 phase oscillation periods until the shutter protecting the stack from the stray field of the inflector magnet has opened. Subsequent acceleration with the same voltage displaces the beam towards the stack. Before the beam approaches the stack the voltage is reduced to fit the buckets tightly to the bunches. This avoids bringing empty phase space into the stack, which would dilute its density. The stable phase angle, whose typical value is 350, is kept constant during the whole acceleration. After a final acceleration with the reduced voltage to the stacking orbit the voltage is cut off abruptly. Fig. 1 shows the voltage and frequency programmes. The total length of such a stacking cycle is determined by the repetition rate of the injector. It is around 2 s.

During the time the bunch spends in the large bucket, phase oscillations develop and subsequent filamentation leads to dilution of the longitudinal phase space density. Since the maximum current which can be stacked is proportional to the longitudinal phase space density, performance will be reduced. Therefore, a study of these oscillations as well as of possible cures was in order.

2. Observation of Phase Oscillations

In order to observe these phase oscillations we kept the beam usually on injection orbit with a constant RF voltage, rather than going through the normal RF programme. A wide-band, fast intensity monitor and a "mountain range" display were our main tools for beam observation. With this set-up the longitudinal position

Fig. 1. Voltage and frequency programmes during an acceleration cycle.

of a bunch as well as its shape could be observed during the development of the instability.

The first objective of our investigation was to find the mode of the oscillation of a particular bunch. The dipole mode (rigid bunch mode) oscillation, which is described here with the mode number $m = 1$, was usually dominant, but also some quadrupole mode, $m = 2$, could be observed. Fig. 2 shows these oscillations for two different RF voltages ~ 3.8 s after injection on a "mountain range" display.

Fig. 2. "Mountain range" display of bunch oscillations for two different RF voltages

For the lower RF voltage, the bunch is longer and the quadrupole mode is more pronounced, as expected.

As a next step we studied the coupling between the bunches which is characterized by the phase relation between the oscillations of different bunches. An obvious such relation can be seen in Fig. 3 which shows the dipole oscillations of four bunches ~3.8 s after injection.

Fig. 3. Coupled bunch oscillations. Bunches with the same phase are connected by the oblique, dashed line.

On this picture the phase difference from one bunch to the next is

$$\Delta\phi \sim \frac{25}{30} 2\pi.$$

This phase difference can be described by a mode number n so that

$$\Delta\phi = 2\pi \frac{n}{h}$$

where h is the harmonic number of the RF.

Finally we measured the growth rate $\Delta\omega_m$ of the instability. By taking "mountain range" pictures at different times after injection we can observe the growth of the amplitudes of the different modes m . In the case where the dipole mode is dominant, we took "mountain range" pictures with many sweeps, triggered every revolution, on the same trace. Each trace then shows in superposition several phase oscillations and the growth of their amplitude can be measured directly from a single picture (Fig. 4).

These methods were used to investigate the longitudinal instability we presently observe for bunched beams in the ISR. We found that for standard conditions ($V_{RF} = 16$ kV, $I \sim 80$ mA) our instability is a coupled bunch phenomenon consisting mainly of dipole mode oscillations with a growth rate of

$$\Delta\omega_1 \sim 1.6 \text{ s}^{-1}$$

and some weaker quadrupole oscillations.

Fig. 4. Growth of dipole instability

3. Comparison of Theory and Experiment

Review of the Theory

Basically the longitudinal stability of the bunches in the ISR is determined by two effects: (a) the spread S of phase oscillation frequency ω_s due to the non-linearity of the RF waveform; (b) the longitudinal space charge force which shifts the coherent phase oscillation frequency with respect to the incoherent distribution and hence can make ineffective the Landau damping due to the spread S .

For typical ISR bunches in stationary buckets this space charge effect is quite large and Landau damping is ineffective. The situation is therefore potentially unstable which means that a resonator of arbitrarily small shunt impedance can cause an instability.

A theory describing the effect of such resonators on bunched beams has been worked out by F. Sacherer². Below we summarize it briefly and apply it to the ISR. This theory applies to beams with equidistant bunches and gives the modes m and n as well as the growth rates of the oscillations which are excited by a resonator having a resonant frequency f_{res} and a shunt impedance R_s . Since we have in the ISR 20 bunches followed by 10 empty buckets, rather than equidistant bunches, Sacherer's theory is not directly applicable and a correct treatment is complicated. However, for a resonator which has a quality factor Q large enough for its signal not to decay too much during the time of $\sim 1 \mu s$ while the empty buckets pass by, we can use this theory as a good approximation. For the ISR beam in stationary buckets the growth rate of the instability is then:

$$\Delta\omega_m \approx \frac{\omega_s R_s I D F_m(\Delta\phi)}{2\pi V_{RF} B}$$

where

I = beam current,

V_{RF} = RF voltage and

B = bunching factor = full bunch length/distance between bunches.

The form factor $F_m(\Delta\phi)$ specifies how efficiently the resonator can drive a certain mode m . It depends on

the phase change $\Delta\phi = 2\pi \times \text{bunch length in seconds} \times f_{\text{res}}$, which occurs during the passage of a bunch (cf. Fig. 5). The term D depends on the attenuation of the resonator signal between two bunches and on the ratio

$$k = \frac{f_{\text{res}}}{f_0}$$

between the resonant frequency f_{res} and the revolution frequency f_0 . This factor D has in general an imaginary part which is, for our assumption of large Q, of about value 1 if

$$k \approx \text{integer} \times h \pm n,$$

but of value zero for the coupled modes $n = 0$ and $n = 15$.

The above equation determines which coupled bunch mode n will be excited. How well it has to be satisfied depends on the bandwidth of the resonator.

Fig. 5. Form factors $F_m(\Delta\phi)$ (from F. Sacherer²)

Experimental Test of the Theory

We tested the theory and its validity for the ISR case using a cavity with variable shunt impedance and resonant frequency. The excited modes and their growth rates were measured using the methods described in the last chapter. The experimental results are listed in Table 1 and compared with the calculations. The agreement is quite good considering the rather large measurement errors. For each case in Table 1 only the mode with the strongest oscillation is shown, because its presence may change the condition for the weaker modes and also makes their measurement difficult.

Application of the Theory

Having now tested the theory (at least for some range of resonant frequencies) we can use it to derive a criterion for the maximum shunt impedance R_{max} a resonator in the ISR is allowed to have, if a certain instability growth rate $\Delta\omega_{\text{max}}$ is tolerated. We assume a beam current of $I = 0.1\text{A}$, stationary buckets with $V_{\text{RF}} = 16\text{ kV}$ and we approximate the curves $F(\Delta\phi)$ by their envelope and get for $k > 75$:

$$R_{\text{max}} \sim 40\sqrt{k}\Delta\omega_{\text{max}} \quad \Omega$$

Since we assumed $|D| = 1$, this criterion might be pessimistic.

Table 1

Comparison of Theory and Experiment

beam energy	GeV	26	26	26	22	22
beam current	mA	90	30	90	90	90
RF voltage	kV	16	16	5.8	16	16
bunch length	ns	16.5	15.5	22	17.5	17.5
shunt impedance	k Ω	5.2	5.2	5.2	13	13
$k = f_{\text{res}}/f_0$		175	175	175	153	177
strongest observed mode m		2	2	2	2	2
growth rate						
observed	s ⁻¹	12.5	2.7	2.4	11.8	16.7
calculated	s ⁻¹	6.9	2.6	8.5	16.8	11.4
mode n						
observed		-	5	-	27	
expected		5	5	5	27	3

As a further application of the theory we can use instability observations to estimate the parameters of the possible resonator. By measuring the growth rate of the dominant modes m for different RF voltages and hence bunch lengths, we get some idea of where on the curves $F_m(\Delta\phi)$ we are operating. This determines $\Delta\phi$ which is directly related to the resonator's frequency. Observation of the coupled bunch mode n may give additional information about f_{res} . A comparison between the oscillation amplitudes of the first bunch (after the gap of the 10 empty buckets) and later bunches can be used to estimate the quality factor Q. The measured growth rate and Q determine the shunt impedance of the resonator. This method has been successfully used in the past to identify an instability-causing resonator in the ISR which then could be shorted out. The present instability observations, as described in the last chapter, indicate a resonator with $R_s \sim 1.5\text{ k}\Omega$, $f_{\text{res}} \sim 20\text{ MHz}$ and $Q \sim 15$, if the instability is caused by only one resonator.

4. Increase of the Landau Damping with a Cavity Operating at a Harmonic of the RF Frequency

A cavity operating at a harmonic p of the RF frequency can increase the Landau damping and provide longitudinal stability for bunched beams⁴. With a passive cavity we made some experiments to study this method. While dipole and quadrupole mode oscillations can be damped with the feedback system described in the next chapter, a cavity could be used to damp possible higher modes. Ideally such a cavity should be driven by an oscillator and have a low shunt impedance. Operating with the correct phase and with a voltage V_p it would produce a spread S_p in phase oscillation frequencies of approximately⁴

$$S_p \sim S_0 \left[1 + \frac{V_p}{V_{\text{RF}}} (p^3 - p) \right],$$

where S_0 is the spread obtained with the normal RF system alone. This is a good approximation as long as the bunch length is smaller than the wavelength of the higher harmonic oscillation and $V_p/V_{RF} < 1/p$.

It is also possible to work with a passive cavity which is driven by the beam. For optimum operation it has to be tuned to a frequency which is about half a bandwidth below $p \cdot f_{RF}$ so that the phase ψ between this cavity oscillation and the bunch is $\sim 45^\circ/p$ as shown in Fig. 6,5. The spread obtained is then

$$S_p \sim S_0 \left[1 + \frac{I_p R_s}{2V_{RF}} (p^3 - p) \right]$$

where I_p is the $p \cdot f_{RF}$ - Fourier component of the beam current. Due to the shunt impedance R_s and the finite bandwidth such a cavity can itself excite instabilities. The damping provided by the cavity has to cope with their growth rates and the space charge frequency shift.

Fig. 6. Phase relation between the oscillation of the higher harmonic cavity, the bunch and the RF voltage.

In our experiments we used a cavity with variable shunt impedance operating at the 6th harmonic of the RF

frequency. With $p = 6$ this cavity is not quite ideal because the wavelength of its oscillation is shorter than the bunch length at the end of the acceleration cycle. This cavity can therefore not be used for stacking and all the experiments were done with bunches in stationary buckets.

With this method all bunches could be kept stable for a long time as shown in Fig. 7. Usually some oscillations showed up after some time. We do not know if they are caused by a real instability or by noise. Such oscillations either got damped out (as in Fig. 7) or they upset the driving of the cavity, in which case a violent instability occurred. The lowest shunt impedance of the cavity necessary to provide stability was measured and found to be in good agreement with theory. To appreciate the damping effect of the cavity Fig. 7 should be compared with Fig. 4 which refers to normal conditions.

Fig. 7. A bunch stabilized with the higher harmonic cavity.

Fig. 8. Block diagram of the feedback system

One method of damping bunch oscillations of the dipole and quadrupole mode is to act directly on the phase and amplitude of the RF voltage. The ISR RF system works with beam control¹ (missing bunch phase-lock), where the beam phase-reference is taken from one single bunch. The dipole mode of bunch oscillations thus appears as a phase-modulation of individual bunches with respect to the reference bunch, with a frequency ω_s equal to the synchrotron frequency. A pure quadrupole oscillation appears as an amplitude modulation of the individual bunches. The frequency of the amplitude modulation is $2\omega_s$.

By detecting these phase and amplitude modulations of individual bunches and using the detected signals to modulate the RF wave seen by the same bunches, these oscillations can be damped. Such a system has been built for the ISR.

Fig. 8 shows a simplified block diagram of the feedback system. The signal induced on a pick-up station is fed into an analogue gating system, where the 20 bunches are separated. Twenty phase-detectors measure the instantaneous phase difference between each individual bunch and the RF voltage on the cavities. In the multiplexer, the error signal from each detector is sampled once per revolution for approximately 105 ns (one RF period). The output signal from the multiplexer is a signal representing the phase-deviation between the individual bunches and the RF voltage. This signal is applied to a fast phase-modulator that varies the phase of the RF cavity voltage.

Due to a relatively low Q-value of the RF cavities⁶, fast changes of phase and amplitude of the RF voltages are possible, when these changes are small.

Twenty peak-detectors measure the amplitude modulation of individual bunches. An identical multiplexing system as described above is used to extract a signal related to the amplitude of the bunch oscillations for each individual bunch. This signal modulates the amplitude of the RF voltage.

All the switching processes are guided from the beam-synchronized timing system.

Experimental Results

The feedback system is built so that any number of bunches from 1 to 20 can be damped individually. Dipole oscillations of 1 and 2 of the 20 bunches have been successfully damped, in the presence of violent oscillations of both the leading and lagging neighbouring bunches. Fig. 9a shows two neighbouring bunches when no feedback is applied. Fig. 9b shows the same bunches when feedback is applied only to the last bunch.

Fig. 9a. Dipole oscillations without feedback

Fig. 9b. Second bunch stabilized by feedback

An experiment measuring the overall effect on the final phase plane density in a stack when feedback is applied to all 20 bunches has not yet been performed.

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DISCUSSION

Matt Sands (UCSC): Can you explain how the feedback system works for the quadrupole mode?

Wolfgang Schnell (CERN): First, there is a separate detector and separate channel for each bunch with multiplexers. I think that's not what you wonder about. The other thing is rather well known, the basic principle, the quadrupole oscillation means that the bunch gets long and short periodically. When it's short, it's dense and so the peak induced voltage in the pickup is high. When it's long, it's low. The next stage is that you modulate the rf voltage with twice the phase oscillation frequency and you find that it spirals in phase space. This is well known and is used in several machines. I know it's used in the PS and the AGS for many years. The new thing for us is that it's individual from bunch to bunch.

Gus Voss (DESY): At the PS it is called Aragon damping, is it not?

Schnell: Yes. I believe it's called Racah damping at Brookhaven.

Arie Van Steenbergen (BNL): Would you give us some more detail on your injection kicker which has no shutter.

Schnell: We are working on this—the beam height is smaller than we expected and we usually make it even smaller by shaving away a lot of beam. Then instead of having a moving shutter you can perhaps get away with having the beam in a narrow slot where the field attenuates very rapidly when it penetrates into the slot. The trouble is that we can probably not use it at the lowest energies that we have to run, so we will try to work this into the moving shutter and then have both systems.

Gus Voss (DESY): You lose a little bit of radial aperture, I guess, with such an arrangement.

Schnell: A few millimeters, yes. This doesn't work yet, this is under development.

Alberto Renieri (Frascati): What is the order of magnitude of modulation of the rf for damping the quadrupole mode?

Schnell: A few percent. I think it's no more than two or three percent.

CONTROL OF THE BUNCH LENGTHENING PHENOMENON IN ELECTRON STORAGE RINGS*

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Introduction

A dependence of the bunch length on the bunch total charge in electron storage rings was first reported by the Orsay group¹. Since that time, the phenomenon has been observed at Frascati², Kharkov³, and SLAC⁴. The phenomenon was not observed at the Stanford electron-electron storage rings⁵, VEPP I and VEPP II⁶ at Novosibirsk and, until recently, on Tantalus I⁷, the 240 MeV electron storage ring at the Physical Sciences Laboratory of the University of Wisconsin.

Because of the importance of bunch lengthening in determining the luminosities achievable in electron-positron storage rings, a considerable amount of theoretical work aimed at explaining the phenomenon has been carried out⁸⁻¹¹. In 1971, Pellegrini and Sessler¹² gave a general theory of the equilibrium length of high current bunches in electron-positron storage rings. This theory requires inductive coupling between the beam and its surroundings for bunch lengthening to occur above transition energy. Here, the term inductive coupling means that the impedance seen by the beam is inductive, i.e. the structure with which the beam interacts is driven at a frequency below resonance. The theory also allows for the possibility of bunch shortening to occur through the same coupling mechanism, though, to our knowledge, before this present work no such observations have been reported.

Subsequently, Sessler extended this work and developed a dynamic theory which describes the turbulent steady state associated with stable non-zero amplitude collective modes¹³.

Recently, bunch lengthening has been observed at Tantalus I. A series of measurements, to be described below, have been made of the bunch length dependence on both current and energy which are in good agreement with both the original Pellegrini-Sessler Theory and the more recent development by Sessler. Further, bunch shortening has also been observed and, while the measurements made so far on this manifestation of the phenomenon are not as complete as those made of bunch lengthening, they have been included because of their possible importance.

I. Theory

In this section we give a brief summary of the theory developed in Ref. 12. The theory makes use of the concept of self fields arising from the interaction of the bunch itself with components of the accelerator or storage ring near the particle path such as RF cavities, clearing electrodes, pick up loops, etc. These self fields then modify the accelerating potential seen by the electrons in the bunch and there then results a change in the synchrotron oscillation frequencies of the electrons in the neighborhood of the synchronous particle. The oscillation amplitudes of the electrons, now oscillating at some new synchrotron oscillation frequency Ω , change, and this results in the bunch taking on a new length.

The dependence of the mean square of the half bunch length, Δ^2 , on the synchrotron oscillation frequency, Ω , as given by Bruck¹⁴, is

$$\Delta^2 = A/R^2 \quad (1)$$

where A is a quantity dependent on machine parameters and energy. Thus, the perturbed bunch length Δ can be related to the unperturbed bunch length Δ_0 through

$$\Delta^2 = \Delta_0^2 [1 + (\Omega^2 - \Omega_s^2)/\Omega_s^2]^{-1} \quad (2)$$

where Ω_s , the synchrotron oscillation frequency in the presence of the self fields, is calculated by including a term of the form $e dU/d\phi$ to account for the fluctuation in energy gain per turn arising from the self fields in the usual equation for Ω ¹⁴.

The frequencies of the self fields may be either lower or higher than a critical frequency defined as $\omega_{crit} = \beta c/A = R_s \omega_s/\Delta$. We will confine the discussion to the case $\omega > \omega_{crit}$. In this case, Ref. 12 gives the dependence of Ω on the impedance $Z(\omega)$ which gives rise to the self fields as

$$(\Omega^2 - \Omega_s^2)/\Omega_s^2 = F IC \Delta^{-3} \quad (3)$$

where

$$Z(\omega) d\omega + \int_{\omega_{crit}}^{\omega} Z(\omega) d\omega$$

and C is a constant that depends only on machine parameters and energy. Pellegrini and Sessler show that with the use of this expression, Eq. 2 can be written in the form

$$\Delta^2 = \Delta_0^2 (1 + KI/E_s^3 \Delta) \quad (5)$$

where the constant K is proportional only to F .

The impedance $Z(\omega)$ may arise from several structures in the machine or it may be presented to the beam by one high impedance object. In the work being reported here the latter is the case. If $\Delta \gg \Delta_0$, the following development of the theory is pertinent. Equation 5 becomes under these conditions

$$\Delta^3 = \Delta_0^2 KI/E_s^3 \quad (6)$$

Thus, in this limit, the parametric dependence of Δ itself becomes $\Delta \propto (I/V_{rf} \cos \phi_s)^{1/3}$ and is independent of energy.

In Ref. 13, Sessler derives a similar expression for the standard deviation, σ_Δ , as

$$\sigma_\Delta^3 \propto I |Z_n/nZ_0| V_{rf}^{-1} \quad (7)$$

where Z_n is the coupling impedance of the n th mode of the structure interacting with the beam and Z_0 is the impedance of free space.

*Work performed under the auspices of the AFOSR-Contract No. F44620-70-C-0029.

II. Observation

The theoretical root mean square bunch length in Tantalus I, calculated from Eq. 1 is .22 nsec. The observed RMS bunch length is .54 nsec and normally is constant for a range of beam currents of over two orders of magnitude (100 pA- 20 mA). The reasons for the discrepancy between theory and observation are not known at the present time.

Recently (1972), the injection system of the machine was modified and the original fixed, septumless inflector was replaced with an inflector with septum, which is remotely controllable in position. A plan view of the new inflector and its associated section of vacuum chamber is shown in Fig. 1. The inflector and its stem are made of OFHC copper and it is grounded to the vacuum chamber by spring fingers at the point of stem penetration. The fundamental resonant mode of the inflector and stem occurs at a frequency near 240 MHz, which is approximately three times ω_{crit} for the storage ring. However, it is clear from the geometry of the inflector that it can support higher modes; certainly up to several kHz. Finally, because of the shape of the inflector, it is reasonable to believe that the beam may couple to the inflector structure either magnetically or electrostatically, depending on the standing wave distribution along the inflector in the region of closest approach of the electron beam. The standing wave distribution will be determined by which mode the beam happens to excite. Movement of the inflector tunes the structure through two effects: first, by changing the length of the inflector stem protruding beyond the ground point and, second, by changing the capacitive loading to ground at the other end of the inflector body.

During the adjustment of this new inflector it was noted that the beam bunch length became either longer or shorter than normal, depending on radial position of the inflector.

These observations of the bunch length were carried out by processing the signal from an electrostatic induction pickup mounted in the storage ring vacuum chamber through a Tektronix 1S1 sampling system and displaying the composite signal on an oscilloscope. The signal observed, then, was not a "real time" signal and questions concerning the reality of bunch length measurements based on these observations may legitimately be raised. However, beam lifetimes on Tantalus I, at beam currents of more than one or two mA, are strictly a function of bunch density. Thus, the fact that beam lifetimes reflected the observed bunch length variations make a reasonably convincing argument for the accuracy of the bunch length observations. Fig. 2 shows the normal bunch and Fig. 3 shows a typical lengthened bunch. A beam decay curve showing the beam lifetime for normal and long beam bunch lengths appears in Fig. 4-A.

A series of measurements of the beam bunch length as a function of circulating current have been made under both normal and lengthened bunch conditions. The results of these measurements appear in Fig. 5. The curve shown for the long bunch is a least squares fit of Eq. 5 to the observational data. The fit is sufficiently good so as to argue strongly for the correctness of the functional form of Eq. 8.

From these data, the derived value of the quantity F is $9.9 \times 10^3 \text{ MHz}^{-1}$. Taking $\omega = 1.4 \times 10^9 \text{ sec}^{-1}$, which corresponds to the fundamental mode of the inflector, gives a value for $Z(\omega) = 2.4 \times 10^3 \Omega$.

The value of $|Z_n|/n$ calculated on the basis of Eq. 7 for large beam currents and taking $n = 1$, is $2.2 \times 10^3 \Omega$. This is not unreasonable, considering the expected

electrical properties of the perturbing structure. Further, for constant circulating current, $|Z_n|/n$ is reasonably constant as is shown by the small variation in bunch length with energy displayed in Fig. 4-B. We note the agreement of this result with that obtained on the basis of the theory developed in Ref. 12.

Finally, the variation of bunch length as a function of accelerating voltage is shown in Fig. 4-C. Once again, the theory tends to predict the observational data. The discrepancy at low accelerating voltage may well be the result of the fact that the total bunch length has become almost equal to $\lambda_{RF}/4$.

As one expects, the beam is a strong source of radio frequency signals up to at least the 100th harmonic of its revolution frequency. When operating with normal bunch length the spectrum of the beam signal in the region of the fundamental and any of its harmonics, appears as shown in Fig. 6. When the inflector is positioned so as to excite bunch lengthening, the spectra exhibit very little change at the fundamental and the lower harmonics. However, beginning at the eighth harmonic, ($\omega = 1.4 \times 10^9 \text{ sec}^{-1}$) the spectra take the form shown in Fig. 7. This spectrum is typical of a frequency modulated signal. The modulating frequency is 60 kHz which is the small amplitude synchrotron oscillation frequency of the beam at 240 MeV. As can be seen, the level of the sidebands fluctuate violently. At higher harmonics the spectra become richer in structure as is shown in Fig. 8. Again, the prominent sidebands occur at 60 kHz intervals but other sideband spacings can be seen. These spectra suggest that the electron motion within the bunch is to some extent collective and is, indeed, strongly turbulent.

Since the bunch length is longer, it is reasonable to expect, and, indeed, the theory developed in Ref. 13 predicts that the beam should be wider. Observation once again confirms this prediction. The data are not as extensive as in the previous measurements, however, Fig. 9 shows that the standard deviation of the beam width increases by a factor of approximately 2 between the normal and perturbed conditions.

As was mentioned above, the bunch lengthening phenomenon was not seen in Tantalus I during the first several years of its operation. However, bunch shortening was observed several times. Unfortunately, the observations were misinterpreted at the time (1969) and we have only recently begun to study this effect. For the sake of historical completeness, we show Fig. 10. Here, the total beam bunch length is less than one nsec. The structure that the beam was interacting with is not known, but it is suspected that it was the original inflector which was not very different in form from the one presently installed.

Recently, the oscillogram shown in Fig. 11 was obtained. To produce this reduction in total bunch length (from 2.5 nsec to approximately 1 nsec) adjustment of the inflector position both vertically and radially was necessary.

III. Conclusions:

Controlling the bunch length in order to control beam lifetime is now standard practice at the Synchrotron Radiation Center of The University of Wisconsin. In developing this technique we studied the effects of the interaction of a specific resonant structure with the electron beam circulating on Tantalus I. The effect of this interaction is usually to cause the bunch length to increase but under some circumstances the interaction can cause the bunch length to decrease. The observational data are in good agreement

with the predictions of the theory developed by Sessler and Pellegrini. As a result, there appears to be reason to believe that passive resonant structures could be developed that would reduce the bunch length in large electron and positron storage rings and thus increase the luminosities attainable in these machines. The capability of reducing bunch length would also be of some use at electron storage rings used as synchrotron radiation sources for measurement of short fluorescence lifetimes.

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FIG. 1--Plan view of inflector. 1. Injection beam path; 2. Electron orbit; 3. Scintillator; 4. Inflector body; 5. Inflector electrode; 6. High voltage terminal; 7. Photon beam line from upstream magnet.

FIG. 2--Normal bunch. Sweep speed: 10^{-9} sec cm^{-1} .

FIG. 3--Wide bunch. Sweep speed: 10^{-9} sec cm^{-1} .

FIG. 4--A. Beam decay, long and short bunch. B. Bunch length dependence on energy. C. Bunch length dependence on accelerating voltage.

FIG. 6--Bunch signal spectrum at sixth harmonic of particle revolution frequency, normal bunch length. Dispersion: 10^5 hertz cm^{-1} .

FIG. 8--Bunch signal spectrum at thirtieth harmonic of particle revolution frequency, long bunch length. Dispersion: 10^5 hertz cm^{-1} .

FIG. 10--"Historical" short bunch signal, Ca 1969. Sweep speed: 10^{-9} cm sec^{-1} .

FIG. 5--Bunch length dependence on circulating current for normal and long bunches.

FIG. 7--Bunch signal spectrum at eighth harmonic of particle revolution frequency, long bunch length. Dispersion: 10^5 hertz cm^{-1} .

FIG. 9--Beam width, normal and long bunch length,

FIG. 11--Short bunch. Sweep speed: 10^{-9} cm sec^{-1} .

DISCUSSION

Phil Morton (SLAC): You showed a curve of the bunch decay for shorter and longer bunches. Why did a short bunch decay faster?

E. Rowe (University of Wisconsin): The density of the charge in the bunch is very much higher for the short bunch, and apparently our lifetime is determined by the Touschek effect.

Andrew Sessler (LBL): Do you have observations of the radial bunch width at the same time that you see shortening in length?

Rowe: That was shown on about the third slide from the end.

Sessler: That was while the beam was getting long.

Rowe: There were two curves there. One showed the width in the normal, short bunch mode, and the other curve showed the width in the long bunch mode. We don't have data on the super-short mode. We're trying to get it. These observations are difficult. We are fortunate that our machine is rather small and operates at a rather low energy so that some of these effects tend to be gross, like the bunch length going up by a factor of 7 or 8.

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Summary. In the paper is described the feedback system, actually working on Adone, that acts on the longitudinal relative modes of oscillation of the two e^+, e^- beams, of three bunches each. The stability condition for small oscillations of rigid bunches is analitically found. The system, implemented with two resonators tuned at the 8th harmonic of the revolution frequency, is described. The results are discussed.

1. - Introduction

The instabilities of the synchrotron oscillations of particle bunches in a circular accelerator are known and their mechanism has been discussed by many authors^{1,2,3} together with the means to counteract them. However the damping systems designed or implemented for storage rings up to the present time were made for beams containing only one bunch. In this work we shall describe the damping system for the e^+, e^- beams in Adone, containing three bunches each.

As it is known, the longitudinal modes of oscillation of the bunches of a beam may be classified in center of mass mode and relative modes. The latter are the most difficult to deal with and may be damped in two ways. One way is to split the synchrotron frequencies of the various bunches by means of a sinusoidal voltage whose frequency is a multiple of the revolution frequency but not of the main radio-frequency. In Adone we have introduced 2KV peak by means of a small resonator tuned at 71.4 MHz which is the 25th harmonic of the revolution frequency. This voltage alone has not been sufficient to damp the instabilities of the relative modes, especially at low energy, and it has been necessary to introduce the feedback damping system illustrated hereafter.

The centre of mass mode of oscillation of one beam has been damped by a feedback system that uses the main accelerating cavities to interact with the beam. This system has already been mentioned elsewhere⁴ and will not be described in this paper. The mode of oscillation of the centers of mass of the two beams in phase opposition cannot be acted upon by any of the systems here described. However it can be influenced by splitting the synchrotron frequencies of the two beams. This can be achieved in Adone by varying suitably the phases of the R.F. voltages on two adjacent main accelerating gaps.

2.1. - Description of the feedback system for relative modes.

The block scheme of the feedback system is

shown in Fig. 1. For the symbols used see Appendix A.

FIG. 1

As shown in the scheme, the system is made of two amplifying chains (A and B) and two cavities. The two amplifier chains correspond to two different equivalent positions of the pick-ups on the circumference of the accelerator and two different gains $(\Delta V / \Delta S)_A$ and $(\Delta V / \Delta S)_B$. From pick-up A the same signal is sent to the two cavities, from B the signal to cavity 2 is inverted w.r. to that sent to cavity 1. The signals from the two beams (e^+, e^-) are separated by a gate system, so that it is possible to shift the equivalent position of the pick-up for each beam merely by inserting delays. The pick-up gives a signal proportional to the radial displacement of the beams. There are various reasons for employing a transverse position pick-up instead of a longitudinal one. The most immediate one is that the radial displacement signal has a 90° phase shift w.r. to the longitudinal displacement, so that its phase is already the correct one to be fed back on the beam. There is however a fundamental reason that excludes the use of a longitudinal position pick-up in a feedback system on many bunches, because with such a signal it is impossible to damp simultaneously all the modes of oscillation.

2.2. - Equations of motion.

The linearized motion equations, in the absence of radiation damping and noise, written as iterative equations turn by turn, are³:

$$\phi_{1,m}^{\pm} - \phi_{1,m-1}^{\pm} = e \varepsilon_{1,m}^{\pm} \quad (1)$$

$$(1=1, 2, 3 \dots, -\infty < m < +\infty)$$

$$\begin{aligned}
& \epsilon_{1,m}^{\pm} - \epsilon_{1,m-1}^{\pm} = -\frac{\Omega^{2\pm}}{\varrho} \phi_{1,m}^{\pm} + \\
& + N \sum_{k=1}^3 \sum_{n=-\infty}^{+\infty} (C_{k-1,n-m}^{\mp} \phi_{k,n}^{\mp} + C_{k-1,n-m}^0 \phi_{k,n}^{\pm}) + \\
& + N \sum_{k=1}^3 \sum_{n=-\infty}^{+\infty} (S_{k-1,n-m}^{\mp} \epsilon_{k,n}^{\mp} + D_{k-1,n-m}^{\pm} \epsilon_{k,n}^{\pm})
\end{aligned} \quad (1)$$

The meaning of the symbols is the following:

$\phi_{k,m}^{\pm}, \epsilon_{k,m}^{\pm}$ Displacement from synchronous phase and energy of the k^{th} bunch $e^{\pm}$ at the m^{th} turn.

$$\varrho = \frac{2na}{E}$$

a = momentum compaction

E = energy of synchronous particle

$\Omega^{\pm}$ = synchrotron frequency of $e^{\pm}$ beam in unities of the revolution frequency ω_0 .

$C_{r,s}^{\pm}, C_{r,s}^0$ coefficients that give a weight to the signals left in the cavities in the preceding turns.

$D_{r,s}^{\pm}, S_{r,s}^{\pm}$ coefficients that take into account the action of feedback.

In Appendix A are reported the expression of:

$$\Omega^{2\pm}, C_{r,s}^{\pm}, C_{r,s}^0, D_{r,s}^{\pm}, S_{r,s}^{\pm}$$

Equations (1) are obtained using the approximation of rigid bunches. We also suppose that in the accelerator are circulating 3 bunches per beam with an identical number of particles, and that the synchronous phase is equal for the various bunches.

2.3. - Damping coefficients.

Neglecting the difference ($\Omega^{2+} - \Omega^{2-}$) and supposing the two feedback cavities identical, we obtain from equations (1) the damping coefficients (see again ref. 3) for the four relative modes:

$$\begin{aligned}
\delta_r^{\pm} = 3N \{ & g_A [\cos(m_0(\theta_A - \theta_1)) + \cos(m_0(\theta_A - \theta_2))] + \\
& + g_B [\cos(m_0(\theta_B - \theta_1)) - \cos(m_0(\theta_B - \theta_2))] + \\
& + \gamma \text{Re}(\alpha_{r\pm} \sqrt{\Delta_r}) \} \quad r = 1, 2
\end{aligned} \quad (2)$$

with: $\text{Re}(\) = \text{real part of}(\)$.

$$\begin{aligned}
\Delta_r = \{ & g_A [\cos(m_0(\theta_A + \theta_1)) + \cos(m_0(\theta_A + \theta_2))] + \\
& + g_B [\cos(m_0(\theta_B + \theta_1)) - \cos(m_0(\theta_B + \theta_2))] + \gamma \alpha_r^c \}^2 + \\
& + \{ g_A [\sin(m_0(\theta_A + \theta_1)) + \sin(m_0(\theta_A + \theta_2))] +
\end{aligned} \quad (3)$$

$$\begin{aligned}
& + g_B [\sin(m_0(\theta_B + \theta_1)) - \sin(m_0(\theta_B + \theta_2))] + \gamma \alpha_r^s \}^2 - \\
& - \{ g_A [\sin(m_0(\theta_A - \theta_1)) + \sin(m_0(\theta_A - \theta_2))] + \\
& + g_B [\sin(m_0(\theta_B - \theta_1)) - \sin(m_0(\theta_B - \theta_2))] \}^2
\end{aligned} \quad (3)$$

where:

$$g_{A,B} = \frac{e\psi}{4E} \left(\frac{\Delta V}{\Delta S} \right)_{A,B}, \quad \gamma = \frac{e^2 \omega_0 \varrho}{2 \Omega_0}$$

ψ = closed orbit function at the pick-up position;

$\theta_{1,2}$ = longitudinal abscissa of the feedback cavities 1, 2;

$\theta_{A,B}$ = equivalent longitudinal abscissa of pick-up for chains A, B;

$\Omega_0 \approx \Omega^+ \approx \Omega^-$

m_0 = harmonic of the revolution frequency to which are tuned the feedback cavities;

$\alpha_r, \alpha_r^c, \alpha_r^s (r=1,2)$ = coefficients that depend on the impedances of the resonant modes of the cavities interacting with the beam along its path; for their expressions see Appendix A.

In order that the four relative oscillation modes be stable, it is necessary that the four coefficients $\delta_1^+, \delta_1^-, \delta_2^+, \delta_2^-$ be positive.

As the sign before the term $\sqrt{\Delta_r}$ is ambiguous, it is convenient to choose the parameters of the feedback system so that the dependance of Δ_r on g_A and g_B disappears.

If we choose:

$$\begin{aligned}
a &= \frac{n}{m_0} + \frac{\theta_1 + \theta_2}{2} \\
\theta_B &= \frac{\pi s}{m_0} + \frac{n}{2m_0} + \frac{\theta_1 + \theta_2}{2} \\
g_A &= (-1)^n g \sin(m_0 \frac{\theta_1 - \theta_2}{2}) \quad n = 0, \pm 1, \pm 2, \dots \\
g_B &= (-1)^s g \cos(m_0 \frac{\theta_1 - \theta_2}{2}) \quad s = 0, \pm 1, \pm 2, \dots
\end{aligned} \quad (4)$$

we obtain for $\sqrt{\Delta_r}$ an expression that is independent from gain parameters:

$$\sqrt{\Delta_r} = \gamma \sqrt{(\alpha_r^c)^2 + (\alpha_r^s)^2}$$

and we have:

$$\delta_r^{\pm} = 3N [2g \sin(m_0(\theta_2 - \theta_1)) + \gamma \text{Re}(\alpha_{r\pm} \sqrt{(\alpha_r^c)^2 + (\alpha_r^s)^2})] \quad (5)$$

The physical meaning of the choice made above is that the equivalent pick-up for chain A is placed, with respect to the intermediate point between the two feedback cavities $(\theta_1 + \theta_2)/2$, in a crossing point for the displacement of the bunches at the harmonic m_0 of the revolution frequency, while pick-up B is placed in a non-crossing point. Thus chain A sees only the in-phase modes of the two beams, while B sees only the ones in phase opposition. The conditions on g_A and g_B exclude effects arising from interference between in-phase and counter-phase modes.

3.1. - Implementation of the system.

With reference to the scheme of Fig. 1, and to equations (4) the feedback system can be implemented with two cavities tuned at the harmonic m_0 of the revolution frequency ω_0 , where m_0 is not a multiple of the main radio frequency and with two pick-ups placed one at a crossing point and the other one at a non crossing point at frequency $m_0 \omega_0$. From the same equations we see that the gains of the two chains, g_A and g_B , are equal if the distance between two feedback cavities along the circumference is a whole number of half wavelengths plus one quarter wavelength, still at frequency $m_0 \omega_0$. Imposing the latter condition, as it simplifies the system, and given the physical distance L between the two gaps, the harmonic m_0 is determined:

$$2\pi m_0 \frac{L}{C} = \frac{\pi}{2} + k\pi \quad k = 0, 1, \dots$$

Where C is the perimeter of the accelerator. In Adone $C = 105$ m, $L = 16.5$ m, so, for $k = 2$ it results $m_0 \approx 8$ with an error of about 3 degrees on the angular distance. As $\omega_0 = 2.856$ MHz the resonant frequency of the cavities results $m_0 \omega_0 = 22.8$ MHz. Another condition on the cavities is that their bandwidth be large with respect to the synchrotron oscillation frequency.

Furthermore the pick-ups must behave as magnetic ones, i.e. the signal must depend only on transverse displacement and not on the sign of the particle. The two pick-ups can be reduced to only one by means of suitable delays added in the chains, as it was already mentioned. To see this let us choose the origin of coordinates in the mid point between feedback cavities as in Fig. 2, where θ_1, θ_2 are the coordinates of the cavities, θ_A, θ_B those of the pick-ups.

FIG. 2

We may write, for the signals from two e^+, e^- bunches on a pick-up situated at an abscissa θ^x , at frequency $m_0 \omega_0$, using phasor notation:

$$w^+(\theta^x) = w_e^+ e^{i\varphi^+ + im_0\theta^+} = W^+ e^{+im_0\theta^x}$$

$$w^-(\theta^x) = w_e^- e^{-i\varphi^- - im_0\theta^-} = W^- e^{-im_0\theta^x}$$

where φ^+, φ^- are the phases of each bunch through the origin; w^+, w^- are proportional to the bunch intensities. Remembering that $\theta_A = n\pi/m_0$, $\theta_B = (s\pi/m_0) + (\pi/2m_0)$ we may write:

$$w^+(\theta_A) = W^+ (-1)^n; \quad w^-(\theta_A) = W^- (-1)^n$$

$$w^+(\theta_B) = +iW^+ (-1)^s; \quad w^-(\theta_B) = -iW^- (-1)^s$$

Calling v_1, v_2 the voltages on cavities C_1 and C_2 and choosing n, s both even or odd, we obtain:

$$v_1 \propto W^+(1+i) + W^-(1-i) \propto W^+ e^{+i\pi/4} + W^- e^{-i\pi/4}$$

$$v_2 \propto W^+(1-i) + W^-(1+i) \propto W^+ e^{-i\pi/4} + W^- e^{+i\pi/4}$$

We may rewrite these equations as follows:

$$v_1 \propto \left[W^+ e^{+im_0\theta^x} e^{-i(\frac{\pi}{4} + m_0\theta^x)} + W^- e^{-im_0\theta^x} e^{-i(\frac{\pi}{4} - m_0\theta^x)} \right] \quad (5)$$

$$v_2 \propto -i \left[W^+ e^{+im_0\theta^x} e^{-i(\frac{\pi}{4} + m_0\theta^x)} - W^- e^{-im_0\theta^x} e^{-i(\frac{\pi}{4} - m_0\theta^x)} \right]$$

If the signals due to e^+, e^- are separated by means of gates, they can be delayed independently and equations (5) can be translated into the block scheme of Fig. 3.

FIG. 3

In Fig. 3 the signals are taken from one only pick-up placed in θ^x , the e^+ and e^- channels are separated by gating and the signals are combined according to equations (5). Taking into account the effective position of the pick-up as it results from Fig. 4, the shown values of the delays turn out.

FIG. 4

3.2 - Pick-up sensitivity and loop gain.

The signals proportional to transverse displacement are taken from a strip-line pick-up similar to that already used in Stanford'. The difference between the signals from the two electrodes is made by means of hybrid junctions, as illustrated in Fig. 5.

FIG. 5

The pick-up system therefore behaves as a magnetic pick-up, i.e. the polarity of the signal depends only on the transverse displacement and not on the sign of the particle. The wave form, after integration, is a train of approximately rectangular pulses, whose width is about 5 nsec (of the order of twice the transit time of a wavefront along the electrodes and whose period is 350 nsec. The peak voltage is $S = 4.2 \text{ mV} / \langle \text{mA} \rangle \text{ bunch}$. The 8th harmonic of such a waveshape is $\approx 2.7 \times 10^{-2} \text{ S}$, the gain of the chains has resulted about 100 db, therefore $(1/e\omega_0)(\Delta V/\Delta S) = \approx 11.5 \text{ V/mm} \langle \text{mA} \rangle \text{ bunch}$. Supposing the oscillations to be caused by one only resonant line at $\approx 100 \text{ MHz}$, and remembering that

$$g = \frac{-e}{4} \left(\frac{\Delta V}{\Delta S} \right) \frac{\omega}{E} \approx \gamma \alpha_s$$

with

$$\gamma = \frac{\pi \alpha e^2 \omega_0}{\Omega E}, \quad (\hbar B - s - \frac{\Omega}{2\pi}) \approx 35$$

$\alpha_s \approx 35 Z(35 \omega_0)$, $\alpha = 10^{-2}$, $\Omega = 3.3 \times 10^{-3}$ at 1 GeV, $\omega_0^s = 2 \pi \times 3 \text{ MHz}$, $\psi = 2 \text{ m}$, we obtain $Z(35 \omega_0) = 17 \text{ K}\Omega$, i.e. the instability could be caused by a spurious mode of the cavities of shunt impedance $17 \text{ K}\Omega$ at 100 MHz.

4. -Conclusions.

The adjustment of the system is simple enough. It is made with only one beam and with only one feedback cavity working at a time, by acting separately on the variable delays shown on each chain.

The feedback system described is working on Adone. In Fig. 6 we show a photo of straight section 10 with one of the feedback cavities and the synchrotron frequency splitting cavity.

FIG. 6

Straight section with 71.4 MHz cavity and one of the feedback cavities.

The system has resulted essential for a good injection and to avoid sudden beam losses, especially at low energies. Care must be taken that the signal level, depending on the stored current level, does not saturate the amplifiers.

Acknowledgments.

Thanks are due to all the Adone staff and particularly to D. Fabiani for collaboration.

Appendix A.

Definitions :

B = n° of bunches per beam (in Adone B=3)
N = n° of particles per bunch
N_C = number of spurious cavities
N_{RF} = n° of accelerating cavities
v_j, ψ_j = peak voltage and phase of the jth accelerating cavity.
Q = 2π a/E, a = momentum compaction, E = energy of synchronous particle.
e = charge of the electron
φ₀ = synchronous phase (supposed equal for all bunches)
θ_{A,B} = azimuth of pick-up of chain A, B.

Z_j(ω), θ_j = impedance and azimuth of jth cavity (indexes 1,2 refer to the feedback cavities).
ω₀ = 2π/T = revolution frequency.
i = √-1
μ_{A,B} = $\frac{e\psi}{2E\omega_0 Z^x} \left(\frac{\Delta V}{\Delta S}\right)_{A,B}$
 $\left(\frac{\Delta V}{\Delta S}\right)$ = voltage in cavity (chain A,B) for a radial displacement ΔS at the pick-up.
oy = closed orbit function at the pick-up.
Z^x = real part of feedback-cavities impedance at their harmonic frequency.

From reference 3 we have:

$$\Omega^{2\pm} = -e Q B \sum_{j=1}^{N_{RF}} V_j \cos[B(\theta_0 + \theta_j) - \psi_j] + i e Q B N \sum_{j=1}^{N_C} \sum_{h=-\infty}^{+\infty} h B \omega_0 Z_j(h B \omega_0) (1 + e^{\mp 2 i h B \theta_j})$$

$$C_{r,s}^0 = \frac{i e^2}{\omega_0} \sum_{j=1}^{N_C} \int_{-\infty}^{+\infty} \omega d\omega Z_j(\omega) e^{i\omega(s + \frac{r}{B})T}$$

$$C_{r,s}^{\pm} = \frac{i e^2}{\omega_0} \sum_{j=1}^{N_C} \int_{-\infty}^{+\infty} \omega d\omega Z_j(\omega) e^{i\omega(s + \frac{r}{B} \pm \frac{\theta_j}{B})T}$$

$$D_{r,s}^{\pm} = \sum_{j=1}^2 \int_{-\infty}^{+\infty} d\omega Z_j(\omega) e^{i\omega(s + \frac{r}{B})T} \left(\mu_A e^{\pm i\omega \frac{\theta_A - \theta_j}{2\pi} T} - (-1)^j \mu_B e^{\pm i\omega \frac{\theta_B - \theta_j}{2\pi} T} \right)$$

$$S_{r,s}^{\pm} = \sum_{j=1}^2 \int_{-\infty}^{+\infty} d\omega Z_j(\omega) e^{i\omega(s + \frac{r}{B})T} \left(\mu_A e^{\pm i\omega \frac{\theta_A + \theta_j}{2\pi} T} - (-1)^j \mu_B e^{\pm i\omega \frac{\theta_B + \theta_j}{2\pi} T} \right)$$

$$a_r = \sum_{j=1}^{N_C} \sum_{h=-\infty}^{+\infty} (hB - r - \frac{\Omega_0}{2\pi}) Z_j \left[(hB - r - \frac{\Omega_0}{2\pi}) \omega_0 \right]$$

$$a_r^c = \sum_{j=1}^{N_C} \sum_{h=-\infty}^{+\infty} (hB - r - \frac{\Omega_0}{2\pi}) Z_j \left[(hB - r - \frac{\Omega_0}{2\pi}) \omega_0 \right] \cos \left[2(hB - r - \frac{\Omega_0}{2\pi}) \theta_j \right]$$

$$a_r^s = \sum_{j=1}^{N_C} \sum_{h=-\infty}^{+\infty} (hB - r - \frac{\Omega_0}{2\pi}) Z_j \left[(hB - r - \frac{\Omega_0}{2\pi}) \omega_0 \right] \sin \left[2(hB - r - \frac{\Omega_0}{2\pi}) \theta_j \right]$$

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Summary

When a bunch of stored particles passes a gap, it induces fields which affect the energy of the particles, and hence the bunch shape. An analytical formula giving the transient voltage is derived for a short bunch of any shape. The equilibrium bunch shape is calculated using Haissinski's self consistent theory. The bunch lengthens, but comparison with experiment shows that beam-cavity fast interaction is not the main reason for bunch lengthening in rings having only one cavity.

Introduction

The longitudinal equilibrium shape of a bunch circulating in a storage ring is related to the effective accelerating voltage by¹

$$I(\tau) = K \exp \left[- \frac{eV(\tau) - U_0}{H_0} \right]$$

where τ is the time displacement between a particle of the bunch and an origin which will be defined below,

$I(\tau)$ is the instantaneous intensity in the bunch,

$V(\tau)$ is the effective accelerating voltage,

U_0 is the average energy loss per turn,

T_0 is the revolution period in the ring,

$K = I(0)$ is a constant which, as we shall see, is related to the total charge of the bunch,

H_0 is a constant which depends on ring characteristics and energy.

The effective accelerating voltage $V(\tau)$ is the sum of two terms : the sinusoidal voltage $V_{RF}(\tau)$, and a transient voltage $U(\tau)$ due to the self fields induced by the bunch

$$V(\tau) = V_{RF}(\tau) + U(\tau) \quad (2)$$

$V_{RF}(\tau) = \hat{V} \sin(\omega\tau + \phi_s)$ is the accelerating voltage, taking into account sinusoidal beam loading.

The time origin $\tau = 0$ is defined such that $eV_{RF}(0)$ compensates exactly the average energy loss U_0 .

Since the bunches are much shorter than the wavelength, V_{RF} may be linearized :

$$eV_{RF}(\tau) = U_0 + e \dot{V}_{RF} \times \tau \quad (3)$$

where $\dot{V}_{RF} = \omega \hat{V} \cos \phi_s = \omega \sqrt{\hat{V}^2 - (U_0/e)^2}$ is the slope of the sinusoidal voltage at origin.

At low current, the transient self fields are weak, and $U(\tau)$ may be neglected.

Under such conditions, the instantaneous equilibrium current has a gaussian shape :

$$I(\tau) = K \exp \left(- \frac{\tau^2}{2\sigma_0^2} \right)$$

where $\sigma_0 = \sqrt{T_0 H_0 / e \dot{V}_{RF}}$ is the standard deviation of the gaussian distribution.

At higher currents, the bunch shape depends on the transient induced voltage $U(\tau)$:

$$I(\tau) = K \exp \left[- \frac{\tau^2}{2\sigma_0^2} - \frac{1}{\sigma_0^2 \dot{V}_{RF}} \int_0^\tau U(y) dy \right] \quad (4)$$

The transient voltage $U(\tau)$ results from interaction between the bunch and the elements of the ring (accelerating cavity, vacuum chamber, ...). Then it depends on $I(\tau)$ and geometrical environment.

We give here below an analytical formula of $U(\tau)$ expressing only bunch-cavity interaction. Then we obtain the shape, length and position of the bunch for DCI and ACO. Finally, we compare the results with measurements on ACO.

Evaluation of the transient voltage $U(\tau)$

The detailed calculations have been reported in the reference². We restrict ourselves here to the main points.

Geometry of the cavity gap

Figure 1 shows the DCI cavity. The apertures for the beam are neglected. The field front generated by the bunch propagates radially at light velocity ; the influence of the radius R_0 of the gap plate appears on the axis only after a time $2R_0/c$. Thus, if the bunch is short enough, the cavity may be replaced by any cylinder of height g and radius $R > R_0$ (fig. 2).

Fig. 1 : Actual geometry of the DCI cavity.

Fig. 2 : Equivalent simplified geometry.

Self field and voltage generated by the bunch are calculated by summing the response to a unit step of current

We denote by $U_H(x)$ the transient voltage acting on the particle entering the cavity at a time x later than the head of the unit step of current to which it belongs. If the actual instantaneous intensity $I(\tau)$ in the bunch is zero outside the interval $[-b, +b]$ and continuous inside, the transient voltage $U(\tau)$ is :

$$U(\tau) = - \int_0^{b-\tau} U_H(x) \dot{I}(\tau+x) dx \quad (5)$$

$U_H(x)$ is evaluated by eigenmode method^{3,4,5}

We obtain :

$$U_H(x) = \frac{-1}{\pi \epsilon_0 c} \sum_{p=0}^{\infty} \sum_{n=1}^{\infty} \frac{\alpha_p}{v_{on}^2 J_1^2(v_{on}) \omega_{onp}} \left\{ 2 \sin \omega_{onp} x - (-1)^p \sin \left[\omega_{onp} \left(x - \frac{g}{c} \right) \right] - (-1)^p \sin \left[\omega_{onp} \left(x + \frac{g}{c} \right) \right] \right\} \quad (6)$$

$\alpha_p = 1$ if $p = 0$ and $\alpha_p = 2$ if $p \neq 0$.

v_{on} is the n^{th} root of Bessel function J_0 and $\omega_{onp} = c \sqrt{(v_{on}/R)^2 + (p\pi/g)^2}$.

Using properties of Fourier series and Fourier-Bessel series, we find :

$$\sum_{p=0}^{\infty} \sum_{n=1}^{\infty} \frac{\alpha_p}{v_{on}^2 J_1^2(v_{on})} \frac{\sin \theta \sqrt{\left(\frac{v_{on}}{R}\right)^2 + \left(\frac{p\pi}{g}\right)^2}}{\sqrt{(v_{on}/R)^2 + (p\pi/g)^2}} = \begin{cases} -\frac{g}{2} \ln \frac{\theta}{R} & \text{when } 0 < \theta < 2g < R \\ -\frac{g}{2} \left(\ln \frac{\theta}{R} + \ln \frac{\theta^2 - 4g^2}{R^2} \right) & \text{when } 2g < \theta < 4g < R \end{cases} \quad (7)$$

$$\sum_{p=0}^{\infty} \sum_{n=1}^{\infty} \frac{(-1)^p \alpha_p}{v_{on}^2 J_1^2(v_{on})} \frac{\sin \theta \sqrt{\left(\frac{v_{on}}{R}\right)^2 + \left(\frac{p\pi}{g}\right)^2}}{\sqrt{(v_{on}/R)^2 + (p\pi/g)^2}} = \begin{cases} 0 & \text{when } 0 < \theta < g < R \\ -\frac{g}{2} \ln \frac{\theta^2 - g^2}{R^2} & \text{when } g < \theta < 3g < R \\ -\frac{g}{2} \left| \ln \frac{\theta^2 - g^2}{R^2} + \ln \frac{\theta^2 - 9g^2}{R^2} \right| & \text{when } 3g < \theta < 5g < R \end{cases}$$

Using (7), the double sum (6) simplifies into :

$$U_H(x) = \begin{cases} -\frac{1}{2\pi\epsilon_0 c} \ln \left(1 + \frac{2g}{cx} \right) & \text{when } 0 < x < \frac{2g}{c} \\ -\frac{1}{2\pi\epsilon_0 c} \ln \left(1 + \frac{2g}{cx+2g} \right) & \text{when } \frac{2g}{c} < x < \frac{4g}{c} \end{cases}$$

We can take for $U_H(x)$ practically without appreciable error :

$$U_H(x) = -\frac{1}{2\pi\epsilon_0 c} \ln \left(1 + \frac{2g}{cx} \right). \quad (8)$$

Expression for $U(\tau)$

From (5) and (8) we get :

$$U(\tau) = \frac{1}{2\pi\epsilon_0 c} \int_0^{b-\tau} \ln \left(1 + \frac{2g}{cx} \right) \dot{I}(x+\tau) dx. \quad (9)$$

Determination of the bunch shape equilibrium

From the formulas (4) and (9), we obtain the equation satisfied by $I(\tau)$:

$$I(\tau) = K \exp \left[-\frac{\tau^2}{2\sigma_0^2} - \frac{1}{2n\epsilon_0 c \sigma_0^2 V_{RF}} \right]$$

$$\left| \ln \left(1 + \frac{2g}{cx} \right) \dot{I}(x+y) dx dy \right|$$

This equation may be written in the form $I(\tau) = F[I(\tau)]$ where F is the operator defined on the rhs of (10). $I(\tau)$ is determined by iteration as the limit of the set of functions

$$I_n(\tau) = F[I_{n-1}(\tau)]$$

where $I_0(\tau)$ is an arbitrary function; for example $I_0(\tau) = 0$, then $I_1(\tau) = K \exp(-\tau^2/2\sigma_0^2)$.

Equation (10) is solved numerically using an IBM 1130 computer by noticing that :

(i) given σ_0 and V_{RF} , the solutions $I(\tau)$ of (10) depend on K . For each value of K , we can solve $I(\tau)$ and find the bunch charge

$$Q = \int_{-b}^b I(\tau) d\tau$$

It can be noticed that K determines finally the value of Q .

(ii) according to the latter remark and to the form of (10), it follows clearly that the ratio $I(\tau)/Q$ depends only on the two parameters σ_0 and $Q/\dot{V}_{RF}$.

σ_0 is the standard deviation of the gaussian distribution when the self fields are disregarded. $Q/\dot{V}_{RF}$ depends on the parameters of the storage ring

$$\frac{Q}{\dot{V}_{RF}} = \frac{i T_0^2}{2\pi h [\hat{V}^2 - (U_0/e)^2]^{1/2}}$$

where $i = Q/T_0$ is the mean current in the ring, and h is the harmonic number.

	T_0 (ns)	h	i (mA)	$\hat{V} \cos \phi_s$ (kV)	$Q/\dot{V}$ (10^{-20} C.s.V $^{-1}$)
ACO	73	2	30	17.5	.0726
ADONE	350	3	60	120	.325
SPEAR	780	40	50	400	.0302
DCI	316	8	350	350	.198

Table I gives the value of the parameter Q/V for some storage rings.

Application to DCI

With the above assumption we have calculated the bunch shape in DCI using the following characteristics :

- number of accelerating cavities : 1,
- operating frequency of the cavity : 25.352 MHz ($h=8$),
- transit time through the gap $g/c = .555$ ns,
- order of magnitude of $\dot{V}_{RF}(0) \approx 10^{13}$ V/s,
- order of magnitude of charge $Q : 10^{10}$ to 10^{12} particles.

The exact bunch shape and the transient voltage are shown in figures 3 and 4 for the particular case $\sigma_0 = 0.2 \times 10^{-9}$ s and $Q/\dot{V}_{RF} = .115 \times 10^{-20}$ C.s.V $^{-1}$.

In figure 3, it can be noticed that the gaussian distribution which describes the bunch shape when the self fields of the cavity are disregarded is modified : the bunch is longer and the center of charge is shifted forward. The dotted straight line on figure 4, whose slope is $\dot{V}_{RF}/Q$, allows comparison between the transient voltage and the RF voltage. We systematically study the deformation effect of the bunch versus σ_0 and $Q/\dot{V}_{RF}$ by calculating the following parameters :

- half height bunch length ℓ (for the gaussian bunch, this length is $\ell_0 = 2.35 \sigma_0$),
- position τ_0 of the center of charge defined by

$$\tau_0 = \frac{1}{Q} \int_{-b}^{+b} \tau I(\tau) d\tau$$

- average energy loss $\langle U(\tau) \rangle$ defined by

$$\langle U(\tau) \rangle = \frac{1}{Q} \int_{-b}^{+b} U(\tau) I(\tau) d\tau$$

Fig. 3 : Bunch shape for $\sigma = .2$ ns.

$$Q/\dot{V}_{RF} = .115 \times 10^{-20} \text{ C.s.V}^{-1}.$$

The dotted curve represents the gaussian obtained by neglecting the self-fields.

Fig. 4 : Transient voltage for $\sigma = .2$ ns,

$$Q/\dot{V}_{RF} = .115 \times 10^{-20} \text{ C.s.V}^{-1}.$$

The dotted line represents the sinusoidal voltage of the cavity.

The results are summed up in the curves of figures 5, 6 and 7 obtained by solving (10) for different values of σ_0 and $Q/\bar{v}_{RF}$. These curves stop where the iteration method becomes divergent and does not permit solutions. Notice in these conditions that the relative lengthening ($l - l_0$)/ l_0 is at most 30 % and the relative center of charge shift τ_0/l_0 can reach 30 %.

Fig. 5 : Evolution of bunch half-height length.

Fig. 6 : Relative shift of the center of charge.

Fig. 7 : Average energy loss.

Application to ACO

The gap shape of ACO is quite complicated and can hardly be regarded as a cylindrical cavity.

However, since the expression of $U(\tau)$ only depends on the transit time g/c , we determined the lengthening on ACO using $g/c = 0.35$ ns.

Comparing with experimental results⁶ (fig. 8), we notice that the bunch-cavity interaction does not explain more than 25 to 40 % of the total lengthening.

Fig. 8 : Evolution of the ACO bunch length versus beam intensity.

— experimental length
----- calculated length

Conclusion

The bunch-cavity interaction is not the main cause of bunch lengthening observed at high intensity in storage rings using only one cavity. Actually, at each discontinuity of the vacuum chamber, the bunch loses energy and a transient voltage similar to those we have studied appears. The geometry of the vacuum chamber being very complicated, even a simplified calculation is difficult. However, when storage rings are equipped with several accelerating cavities, the bunch-cavity interactions can generate important bunch lengthening.

Acknowledgments

We wish to express our thanks to Professor Haïssinski, Dr P. Brunet, Dr D. Potaux and Dr H. Zyngier for their encouragement and helpful suggestions.

† Work supported by the Laboratoire de l'Accélérateur Linéaire d'Orsay.

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Summary and Conclusions

Increasing the PSB and CPS intensities to 10^{13} ppp will require more powerful octupoles in order to overcome the transverse instabilities that are to be expected. Unfortunately, theory fails to give quantitative predictions at low energies, where direct space-charge forces dominate over image forces. To be able to specify the future needs more precisely, an extension of the theory would be desirable, particularly since experimental evidence is such that it is hard to extrapolate to higher intensities.

The theory presented here solves the dispersion relation, including the influence of external and space-charge induced non-linearities in both transverse dimensions as well as a spread in the LNS coefficient U . Results are preliminary in the sense that modulation of the space-charge forces due to synchrotron motion is not included. They show that the combined effect of external and space-charge non-linearities in the two transverse planes can considerably enhance Landau damping in low-energy machines (up to 10 GeV, say). The sign of the octupole current is important especially for a flat beam, and should be chosen in such a way that ν increases with amplitude in the direction where the beam is wide.

Stability diagrams for several typical conditions are presented and applied to the PSB.

1. Introduction

Octupole lenses have been installed both in the CPS and its Booster years ago, and they have successfully cured transverse instabilities. It is planned to use more powerful lenses in the CPS, and possibly in the Booster too, in order to tame future intensities. It is puzzling, however, to observe that the currents needed to avoid instabilities are considerably lower than predicted by theory^{1,2}. In addition, lower thresholds for radial rather than for vertical resistive wall instability were observed frequently in the PSB. These observations suggest that the influence of space-charge non-linearities and the spreads due to motion in the second transverse direction might be important.

An attempt to include these effects³ already showed the importance of the variations due to the betatron amplitudes in the second transverse plane. This approach, however, still predicted too high octupole strengths and failed to give the correct behaviour in the limit of vanishing external octupole force. In this limit incoherent space-charge forces should have no influence on dipole motion, a fact which can be deduced from the single particle-equation⁴ and is fully confirmed by computer simulation⁵.

To make more reliable extrapolations, it was felt necessary to extend the theory in order to remove these deficiencies. The main difference of the present work in comparison with Ref. 3 is that we include space-charge non-linearity in both the driving term (the U -term of Ref. 1) and in the single-particle frequency.

A general model would have to include:

- i) frequency spreads due to external non-linearities in both transverse (x and y) directions;
- ii) spreads due to x - and y -non-linearities of space-charge forces;
- iii) spreads due to the energy distribution of the beam;

iv) spreads due to longitudinal variation of transverse space-charge forces;

and probably other effects. In the present paper we content ourselves with considering the effects (i) and (ii). This is a valid approximation at least for the coasting beam experiments which were performed in the Booster and which exhibited the "anomalies" discussed above. We shall find that the combination of external and space-charge non-linearities can considerably enhance the stability conditions, and that the spreads from both transverse directions are important.

2. Dispersion Relation

Let us discuss dipole oscillations in one transverse (x) direction. Neglecting energy spread, the dispersion relation of Laslett, Neil and Sessler (LNS) can be written as

$$(U + V + iV) \int_0^\infty \frac{-\frac{1}{2} h'(a) a^2 da}{\omega - (n - \nu_x) \Omega} = 1. \quad (1)$$

Here the tune $\nu_x = \nu_x(a)$ depends on a , the amplitude of the incoherent betatron oscillation; $h(a)$ is the corresponding distribution function, $\int_0^\infty h(a) a da = 1$. The coefficient $U + V + iV$ can be expressed in terms of the revolution frequency (Ω) and the incoherent ($\Delta\nu_{ic}$) and coherent ($\Delta\nu_c$) Laslett tune shifts (generalized to include wall resistivity and equipment interaction):

$$U + V + iV = \Omega(\Delta\nu_c - \Delta\nu_{ic}). \quad (2)$$

Finally $\omega = (n - \nu) \Omega$ is the mode frequency (coasting beam mode n) to be obtained by solving Eq. (1); $\text{Im}(\omega)$ is the growth rate, ν being the collective betatron frequency.

To include space-charge non-linearities as well as y -variation of ν_x we can use the derivation of Eq. (1) given by Hereward⁶. We start from the single-particle equation

$$\ddot{x}_i + \Omega^2 \left[\nu_i^2(x_i, y_i) - 2\nu_0 \Delta\nu_{ic}(x_i, y_i) \right] x_i = 2\Omega^2 \nu_0 \left[\Delta\nu_c - \Delta\nu_{ic}(x_i, y_i) \right] \bar{x}, \quad (3)$$

where $\bar{x}$ is the dipole motion of the beam and x_i the motion of the test particle (coherent and incoherent).

The corresponding dispersion relation is

$$\int_0^\infty \frac{(U + V + iV)(a, b) (-\frac{1}{2} h'(a) a^2) b g(b) da db}{\omega - [n - \nu(a, b)] \Omega} = 1. \quad (1a)$$

Here b is the incoherent y -amplitude, $g(b)$ the corresponding distribution, $\int_0^\infty g(b) b db = 1$. The tune $\nu(a, b) = \nu_0(a, b) - \Delta\nu_{ic}(a, b)$ includes the incoherent tune shift, and so does $U + V + iV$ through Eq. (2). The terms $\nu(a, b)$ and $(U + V + iV)(a, b)$ are obtained by averaging over the incoherent betatron motion.

To evaluate Eq. (1a) we make some further approximations: We assume that only the incoherent tune shift is non-linear, whereas the image term $\Delta\nu_c$ is the same for all particles. This is a valid approximation at low energy, where $\Delta\nu_c \ll \Delta\nu_{ic}$ and/or for thin beams. The case where both $\Delta\nu_c = \Delta\nu_c(a, b)$ and $\Delta\nu_{ic} = \Delta\nu_{ic}(a, b)$ but where external non-linearities are negligible ($\nu_i = \text{const}$) was discussed in Ref. 4. We shall further expand $\nu(a, b)$ and only retain terms up to the octupole moment:

$$\Delta v_{ic}(a,b) = \Delta v_{ic}(0) + \frac{\partial \Delta v_{ic}}{\partial a^2} a^2 + \frac{\partial \Delta v_{ic}}{\partial b^2} b^2$$

$$= A - A a' - \Delta_b b^2, \quad (4)$$

and

$$v_0(a,b) = v_0(0) + \frac{\partial v_0}{\partial a^2} a^2 + \frac{\partial v_0}{\partial b^2} b^2$$

$$= v_0 + v_a a^2 + v_b b^2.$$

The symbols A , Δ_a , Δ_b (all positive) and v_a , v_b are hereby defined and will be used throughout the rest of the paper. Quantities such as $v_a \hat{a}^2$, $\Delta_a \hat{a}^2$, etc., will be denoted as "external spread" and "space-charge spread", respectively; $\hat{a}$ and $\hat{b}$ are typical amplitudes to be defined below; $\hat{a}$ corresponds to the plane of the instability.

3. Results

Here we shall present the solution of the dispersion relation (1a) for two different distributions and for several typical conditions. We assume $v_a = 0$, $v_b = 0$ in Section 3.1; $\hat{a} \gg \hat{b}$ in Section 3.2 and Figs. 1 and 2; $2\hat{a} = \hat{b}$ in Section 3.3 and Figs. 3 and 4; and $\hat{a} \ll \hat{b}$ in Section 3.3 and Fig. 5.

3.1 No external non-linearities (i.e. $v_a = 0$, $v_b = 0$)

In this case incoherent space-charge has no effect, as was found already in Ref. 4. In fact it is easily verified from the single-particle equation (3) that $x_i = \bar{x}$ is a solution provided that $v_i^2 = v_0^2 - 2v_0 \Delta v_{ic}$ is the same for all particles. Under this condition Δv_{ic} simply drops out from Eq. (3).

3.2 v-spread due to betatron amplitudes in the plane of the instability only (i.e. $\hat{a} \gg \hat{b}$)

3.2.1 "Semicircular" distribution

It is instructive to start with the distribution which -- without space charge forces -- leads to a circular range of stability and gives the rule-of-thumb criterion

$$4 \frac{|U + V + iV|}{\Omega} \leq \delta v_{FWHH} \quad (5)$$

for the stabilizing v-spread. The derivative of this function which enters into Eq. (1a) is a half circle:

$$-\frac{1}{2} h'(a^2) da^2 = \frac{2}{\pi \hat{a}^2} \sqrt{1 - \left(\frac{a^2}{\hat{a}^2} - 1\right)^2} da^2, \quad (6)$$

$$0 \leq a^2 \leq 2\hat{a}^2$$

Including now space-charge, the solution of the dispersion relation gives the stability boundary

$$v^2 + \frac{(U + V)^2}{\left(1 + \frac{2}{q_a}\right)^2} = \frac{\Omega^2}{4} (v_a \hat{a}^2)^2 \simeq \frac{\Omega^2}{16} (\delta v_{FWHH})^2. \quad (7)$$

Here we have introduced a parameter $q_a = v_a/\Delta_a$, the ratio of external spread to space-charge spread. U involves an average of Δv_{ic}

$$\frac{U}{\Omega} = \Delta v_c - (A - \Delta_a \hat{a}^2). \quad (8)$$

For $1/q_a = 0$ we recover the rule of thumb (5). With space-charge spread, the stability circle is distorted into an ellipse with real half-axis:

$$\hat{U} = (\Omega/4) \delta v_{FWHH} (1 + 2/q_a).$$

The stable range is thus increased or decreased depending on the sign of q_a , i.e. depending on the polarity of the octupoles. Typically the space-charge non-linearity is

such that $2/q_a = + |U|/(\Omega v_a \hat{a}^2)$ (see Appendix). Hence for $V \ll |U|$ -- the case of interest in the PSB and in the CPS below 10 GeV -- the stability condition reduces to

$$4 \frac{|U|}{\Omega} \leq \frac{\delta v_{FWHH}}{1 \pm \frac{1}{2}} \simeq \delta v_{FWHH} \begin{cases} 2 \\ 0.67 \end{cases}, \quad (9)$$

and the stabilizing octupole v-spread is about 0.67 or twice the value obtained from conventional theory ($1/q_a = 0$).

Normalization: In the following we shall use a suitable normalization to reduce the number of parameters. We normalize frequencies (U, V) and spreads to the space-charge spread $S = \Omega \Delta_a \hat{a}^2$ in the plane of the instability. We denote

$$u = \frac{U + V}{S} \Big|_{\text{beam centre}}, \quad v = \frac{V}{S}. \quad (10)$$

This normalization is convenient because the range over which u can reasonably vary is small (values between -3 and -5 depending on the emittance ratio and the plane considered, are typical for the PSB), see Appendix.

Figure 1 displays stability boundaries' for the distribution (6) in this normalization. Note that the normalization (10) shifts the ellipse (7) towards more negative u -values because u refers to the beam centre.

The reduction of the stable area for the "wrong" octupole polarity ($q_a < 0$) can be seen from a comparison between Fig. 1a and Fig. 1b for given $|q_a|$, and is obviously due to cancellation of external and space-charge spread.

3.2.2 Parabolic distribution

Figure 2 gives similar results for a parabolic distribution:

$$h(a) = \frac{1}{\pi \hat{a}^2} \left[1 - \frac{a^2}{\hat{a}^2} \right], \quad 0 \leq a^2 \leq 2\hat{a}^2. \quad (11)$$

Again for negative q_a the stable area is largely reduced. One particularity of Fig. 2 is that all curves intersect the point $v = 0$, $u = -2$. It appears that this effect is due to the sharp cut-off of this distribution.

3.3 v-spread due to betatron amplitudes in both transverse directions (parabolic distribution in each plane)

Now we have to introduce another two parameters:

$$p = \frac{v_b \hat{b}^2}{v_a \hat{a}^2}, \quad r = \frac{\Delta_b \hat{b}^2}{\Delta_a \hat{a}^2}; \quad (12)$$

p denotes the ratio of the two external spreads and r the ratio of the space-charge spreads; $\hat{b}$ is the equivalent of $\hat{a}$ for the second transverse direction. Note that r is determined by the emittance ratio and varies from 0 for $\hat{a} \gg \hat{b}$ to about 2 for $\hat{a} \ll \hat{b}$; $r = 1$ corresponds to $\hat{b} = 2\hat{a}$.

Figures 4 and 5 refer to parabolic distributions $h(a)$ $g(b)$ in Eq. (1a) and demonstrate the influence of the second transverse plane. In Figs. 3 and 4, $r = 1$, i.e. the same spreads $\Delta_a \hat{a}^2$ and $\Delta_b \hat{b}^2$ have been assumed.

The case $p = 1$ as taken in Fig. 3 assumes that two sets of octupoles (in F- and D-sections say) are used such that v_a and v_b have the same sign. In this case we recover the reduction of the stable area for negative q as for the one-dimensional case (Figs. 1 and 2).

In Fig. 4, v_a and v_b have been taken of opposite sign, which is typical for simple lens arrangements.

In this case, the reduction of stable areas is less noticeable, because the tendency of cancellation for the one octupole polarity is to some extent smoothed out : cancellation in one plane goes together with addition in the other plane.

Finally Fig. 5 corresponds to a beam which is wide in the direction perpendicular to the plane of instability ($\hat{b} \gg \hat{a}$), as is usually the case for vertical instability of a multiturn beam. Hence we take $r = 2$ and assume in addition "simple octupoles" with large negative p .

One finds again that one octupole polarity is favourable although the "right" sign is now the opposite to the preferred one in the case of a beam which is wide in the other direction ($p = 0, r = 0$). We conclude that the octupole moment in the direction where the beam is wide should be chosen with care.

Application to the PSB

The coasting beam and bunched beam instabilities observed in the PSB^{7,8} occur sometimes horizontally, sometimes vertically, in an apparently irregular way. In order to explain this feature, we apply the results of Section 3.3. to the PSB. We compute the thresholds for both planes as a function of the emittance ratio E_H/E_V , assuming the product $E_H E_V$ and hence the area of the beam cross-sections to be a constant. We include image forces and averaging over the strongly varying beam dimensions within a machine period. The arrangement of the octupoles is as described in Ref. 9. Thresholds are expressed by the octupole currents required to stabilize the beam.

Figure 6 shows the result for $N = 2.5 \cdot 10^{12}$ p/p and $E_H E_V = 5200$ (π mrad mm)² at 50 MeV. One observes that positive octupole current ($\partial v / \partial r^2 > 0$) is more favourable and that for this polarity vertical stability requires stronger octupole currents for $E_H/E_V \leq 1.7$ whereas horizontal stability is more critical for $E_H/E_V \gg 1.7$. The intersection point $E_H/E_V = 1.7$ falls into the region of emittance ratios actually observed in many machine experiments. This might explain why slight differences in beam parameters can favour the one or the other direction.

The predicted octupole currents are still higher than measured values. This is probably owing to the neglect of synchrotron motion in our model for the case

Fig. 1 : Stability diagram for a beam with amplitude distribution (6). The LNS-coefficients u and v are normalized to the space-charge v -spread (for this distribution and vanishing image forces, u is around -3). The beam is stable if the point (u,v) is on the left-hand side of the curve.

Fig. 2 : Same physical conditions as in Fig. 1, but parabolic amplitude distribution (11). For this distribution u is typically between -3.5 and -5 .

q_a is a measure of the octupole v -spread due to betatron oscillations in the plane of the instability. Parts (a) and (b) of Figs 1-5 refer to the two polarities of the octupoles, respectively; p is the ratio of the octupole u -spreads due to incoherent motion in the two transverse directions; r is the corresponding ratio for space-charge v -spreads. In Figs 1 and 2, p and r are 0 because the beam is wide in the direction of the instability ($\hat{a} \gg \hat{b}$).

Fig. 3 : Parabolic amplitude distributions (11) in both transverse directions : $r=1$ means $b \approx 2a$; $p \approx -1$ is typical for one set of octupoles and not too flat a beam

Fig. 4 : As for Fig. 3, but $p \approx 1$ requires two sets of octupoles.

Fig. 5 : As for Fig. 3, but for a beam wide in the direction perpendicular to that of the instability ($b \gg a$).

Fig. 6 : Octupole currents required to stabilize the PSB beam versus emittance ratio, calculated for parabolic amplitude distributions (11) in both transverse planes. Nominal parameters at 50 MeV : $E_H E_V = 130 \times 40$ (π mrad mm) 2 , $N = 2.5 \cdot 10^{12}$ p/p.

of bunched beams and because of neutralization in the coasting beam case. To conclude, let us make a comparison with conventional theory (external spread in one transverse direction, no spread in Δv_{ic}): the latter requires stabilizing octupole currents of 400 A (horizontal) and 520 A (vertical instability), for the nominal $E_H/E_V = 130/40$, all other parameters as for Fig. 6. This is to be compared with the values of 180 A and 70 A taken from Fig. 6.

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Appendix

Calculation of incoherent v-shift and v-spreads

In order to assure self-consistency with the parabolic distribution that we mainly use, we should solve the potential problem for the corresponding charge distribution. This seems to be difficult for the kind of factorized amplitude distribution $h(a)g(b)$ assumed above, which represents a beam of rectangular cross-section. Hence we restrict ourselves to the computation of v-shifts and v-spreads for a beam of elliptical cross-section and parabolic density

$$\rho(x,y) = \frac{\lambda}{\hat{a}\hat{b}\pi} \left[1 - \frac{(x - \bar{x})^2}{2\hat{a}^2} - \frac{y^2}{2\hat{b}^2} \right], \quad (A1)$$

where λ is the linear density and $\hat{a}\sqrt{2}$, $\hat{b}\sqrt{2}$ are the beam radii, corresponding to the ellipse $\rho(x,y) = 0$, which contains all particles. This choice provides at least approximate self-consistency: projected densities obviously do agree with those obtained from the factorized amplitude distribution.

Neglecting image forces, the force in the x-direction is given by

$$eF_x = \frac{e^2\lambda}{\pi\epsilon_0} \frac{1}{\hat{a}(\hat{a} + \hat{b})} \left\{ (x - \bar{x}) - \frac{1}{6} \frac{2\hat{a} + \hat{b}}{\hat{a}^2(\hat{a} + \hat{b})} (x - \bar{x})^3 - \frac{1}{2\hat{b}(\hat{a} + \hat{b})} y^2 (x - \bar{x}) \right\}. \quad (A2)$$

Performing an averaging process over incoherent betatron motion (e.g. by the method of harmonic balance), $x - \bar{x} = a \cos(v_x \Omega t)$, $y = b \cos(v_y \Omega t)$, we take

$$\langle (x - \bar{x})^3 \rangle = \frac{3}{4} a^3 \cos v_x \Omega t = \frac{3}{4} a^2 (x - \bar{x}) \quad (A3)$$

$$\langle y^2 (x - \bar{x}) \rangle = \frac{1}{2} b^2 a \cos v_x \Omega t = \frac{1}{2} b^2 (x - \bar{x}),$$

and obtain for $\Delta v_{ic}(a,b)$:

$$\Delta v_{ic}(a,b) = \frac{Nr_p R}{\epsilon_0 \pi \beta^2 \gamma^3} \frac{1}{\hat{a}(\hat{a} + \hat{b})} \left[1 - \frac{2\hat{a} + \hat{b}}{8(\hat{a} + \hat{b})} \frac{a^2}{\hat{a}^2} - \frac{\hat{b}}{4(\hat{a} + \hat{b})} \frac{b^2}{\hat{b}^2} \right], \quad (A4)$$

where the r.h.s. should be averaged over the circumference. We identify the quantities introduced in Eq.(A4):

$$\begin{aligned} \Delta &= \frac{Nr_p R}{\epsilon_0 \pi \beta^2 \gamma^3} \frac{1}{\hat{a}(\hat{a} + \hat{b})} \\ A &\approx \frac{A}{\hat{a}^2} \frac{1 + \frac{a}{\hat{a} + \hat{b}}}{8} \\ \Delta_b &\approx \frac{\Delta}{\hat{b}^2} \frac{1}{4} \frac{\hat{b}}{\hat{a} + \hat{b}}. \end{aligned} \quad (A5)$$

This gives a rough estimate of the order of u [of Eq.(10)] to be expected. For our parabolic distribution, u represents the normalized v-shift in the beam centre:

$$u = \frac{\Delta - \Delta_{vc}}{\Delta_a \hat{a}^2} \approx - \frac{\Delta}{\Delta_a \hat{a}^2} = \frac{8}{1 + \frac{\hat{a}}{\hat{a} + \hat{b}}}.$$

This would imply values of u between -4 and -8, which is valid only for a machine of small wiggle in the β -functions.

Computed values for the PSB, assuming nominal emittances ($E_H = 130\pi$, $E_V = 40\pi$ mrad at 50 MeV) and including image contributions, give values of

$$u = -3.8 \text{ (horizontal)} \text{ and } u = -4.8 \text{ (vertical)}.$$

Note that r is approximately given by $r = 2\hat{b}/(2\hat{a} + \hat{b})$ and can only take values of $0 < r < 2$ (Eqs.(12), (A5)).

For the semicircular distribution (6), a numerical estimate gives $u = -\Delta/\Delta_a \hat{a}^2 \approx -3$.

This value of u is used to derive the stability criterion (9). From (8) and (10) we obtain for $V \ll |U|$

$$\begin{aligned} \frac{1}{q_a} \frac{A_a \hat{a}^2}{v \hat{a}^2} \frac{U/\Omega + A}{v \hat{a}^2} \frac{u}{\Omega v_a \hat{a}^2} \frac{u}{q_a} &= \dots \\ \text{and} \\ \frac{2}{q_a} &= \pm \frac{U}{|u + 1|} = \pm f \left| \frac{U}{\Omega v_a \hat{a}^2} \right|, \quad (A6) \\ f &= \frac{2}{|u + 1|} \approx 1. \end{aligned}$$

Hence

$$|U| < \hat{U} = \frac{\Omega}{2} v_a \hat{a}^2 \left| 1 - \frac{2}{a} \right|,$$

$$2 \frac{|U|}{\Omega} \left| 1 \pm \frac{f}{2} \right| \leq v_a \hat{a}^2 \sim \frac{1}{2} \delta v_{FWHH},$$

or

$$4 \frac{|U|}{\Omega} \leq \delta v_{FWHH} \frac{1}{|1 \pm \frac{f}{2}|} \approx \delta v_{FWHH} \left\{ \begin{array}{l} 2 \\ 0.67 \end{array} \right\}.$$

Acknowledgements

We are grateful to K.H. Reich for the encouragement to carry out the work presented here, and to Mrs. M. Lelaizant for carrying out numerical computations.

THE EFVCT OF THE BEAM SELF-FIELD
ON THE TRANSVERSE BETATRON OSCILLATION FREQUENCY

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Summary

The electric and magnetic fields generated by the accelerator beam are computed using a mesh-iteration program. This allows the self-fields to be computed for beams having any desired charge or current distribution, and for vacuum chambers and magnet pole faces of any desired shape. The shift of the transverse betatron oscillation frequencies due to the self-fields of the beam can then be computed. The results are applied to various beam geometries including ISABELLE, the AGS, and a small aperture conventional storage accelerator. Comparisons are made with the results found using the analytical results for simple geometries.

I. Introduction

The electric and magnetic fields generated by an accelerated beam, the self-field of the beam, give rise to forces on individual particles in the beam which change their betatron oscillation frequencies, ν_x and ν_y . Depending on the distribution of charge and current within the beam, the shift in ν_x and ν_y caused by the self-field of the beam will vary across the cross section of the beam.

The fields produced by the beam also depend on the surroundings such as the vacuum tank and the iron pole face of the magnet. Calculations have been done of the change of the ν -value by Kerst,¹ Laslett,^{2,3,13} Resegotti,³ Lapostolle,⁴ Gluckstern,^{5,6} Month,⁶ Claus,⁷ Zotter⁸ and Hardt.⁹ These were calculations usually involving some simplification of the charge and current distribution or of the vacuum tank or pole face geometries.

In this paper, the change in the ν -value caused by the beam self-field and the beam images are computed using a mesh-iteration program, GRACY,¹⁰ which allows the calculation to be done for any distribution of current and charge and for any geometry of the vacuum tank and the pole face. Generally, one finds that results found using the simple analytical formulas are fairly good. However, in some instances, appreciably different results are found using the more exact mesh-iteration calculation.

II. Theory

The beam will generate an electric field E_x , E_y and a magnetic field H_x , H_y which will give rise to a force F_x , F_y acting on the individual particles of the beam where

$$F_y = e(E_y + \beta H_x) ,$$

$$F_x = e(E_x - \beta H_y) ,$$

At any instant of time, the charge distribution longitudinally is given by

$$\lambda = \lambda_0 + \lambda_1 \cos kz$$

where λ is the charge per cm in the beam, λ_0 is the average charge/cm around the accelerator and the bunching factor is given by

$$B = \lambda_0 / (\lambda_0 + \lambda_1) ,$$

B is the ratio of the average charge per cm to the peak charge per cm.

The beam has both a time varying component and a steady component which have different boundary conditions. Following the usual treatment,² we write

$$E_y = E_{y,ac} + E_{y,dc}$$

$$H_x = H_{x,ac} + H_{x,dc} .$$

Then

$$E_{y,ac} = \frac{\lambda_1}{\lambda_0} E_{y,dc} = (1/B - 1) E_{y,dc} ,$$

$$H_{x,ac} = \beta E_{y,ac} = -\beta(1/B - 1) E_{y,dc} ,$$

since E_{ac} , H_{ac} , and E_{dc} satisfy the same boundary conditions that the tangential components of E and H vanish on the surface of the vacuum tank.

We thus find

$$E_y = (1/B) E_{y,dc}$$

$$H_x = -\beta(1/B - 1) E_{y,dc} + H_{x,dc} .$$

H_{dc} satisfies a more complicated boundary condition on the iron surface. We find for the force F_y

$$F_y/e = E_{y,dc} \frac{1}{\sqrt{2}B} + \beta^2 [E_{y,dc} + H_{x,dc}/\beta] .$$

In the absence of the vacuum tank, and the iron surface, $E_{y,dc}$ and $H_{x,dc}/\beta$ would exactly cancel each other. The presence of the image fields due to the surroundings gives the result

$$F_y/e = E_{y,dc} \frac{1}{\sqrt{2}B} + \beta^2 [E_{y,image} + H_{x,image}/\beta] .$$

This force leads to a change in the ν_y by the amount

$$2\nu_y \Delta \nu_y = \frac{NRr}{2\pi\gamma\beta^2} \left\{ \frac{1}{2} \frac{EG_{y,dc}}{\sqrt{2}B} + (EG_{y,image} + MG_{x,image}/\beta)\beta^2 \right\} ,$$

where EG_y and MG_x stand for the components of the electric and magnetic gradients for a beam having a unit charge per cm.

There are various simplifications in the above expression for $\Delta\nu_y$ which could be removed if desired. The magnetic image term $MG_{x,image}$ needs to be reduced by the factor ρ/R because the magnet iron only covers ρ/R of the circumference, where ρ is the magnet radius of curvature and R is the average radius of the accelerator. The various gradients are varying around the accelerator ring as the beam dimensions change. As a first approximation one should use the average dimensions of the beam around the ring, or the beam dimensions where the β -factor has its average value.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

A similar expression can also be found for the horizontal force F_x . Since, as in the above

$$E_x = (1/B)E_{x,dc} ,$$

$$H_y = \beta(1/B - 1)E_{x,dc} + H_{y,dc} ,$$

$$F_x/e = \frac{1}{\gamma^2 B} E_{x,dc} + \beta^2 [E_{x,dc} - H_{y,dc}/\beta] ,$$

$$F_x/e = \frac{1}{\gamma^2 B} E_{x,dc} + \beta^2 [E_{x,image} - H_{y,image}/\beta] ,$$

and

$$2v_x \Delta v_x = - \frac{NRr_p}{2\pi\gamma\beta^2} \left\{ \frac{1}{\gamma B} EG_{x,dc} + (EG_{x,image} - MG_{y,image}/\beta)\beta^2 \right\}.$$

Finding the electric field generated by the beam for the geometry shown in Fig. 1 is fairly simple. One simply solves the equation

$$\nabla^2 V = - 4\pi\rho ,$$

with the boundary condition that $V = \text{constant}$ on the surface of the vacuum tank. To find the image field, the GRACY program also computes the free space electric field when no vacuum tank is present. It does this by subdividing the beam into small rectangles and adding up the electric fields due to each rectangle which it computes using an analytical result for the electric field due to a rectangular beam with a uniform charge density.

Finding the magnetic field generated by the beam shown in Fig. 1 is somewhat more complicated. The vector potential satisfies the equation.

$$\nabla^2 A = - 4\pi S ,$$

where S is the current density. However, the usual boundary condition that the tangential component of H vanish at the surface of the iron is not valid,^{2,6} for this would imply that $\int H d\ell = 0$ along the surface of the iron, which cannot be true as there is a non-zero net current enclosed by the iron.

One approach to solving this problem is to solve the problem with a finite permeability of about $\mu = 10$ or $\mu = 100$ in the iron, and then find the limit as $\mu \rightarrow \infty$. The boundary condition at the iron is then

$$(\partial A / \partial n)_{\text{air}} = 1/\mu (\partial A / \partial n)_{\text{iron}} .$$

This approach usually involves some convergence problems when one tries to run the mesh-iteration program with large value of μ in the iron.

III. Results

In this case the magnet is circular with an inner iron surface with a radius of about 8 cm, and the vacuum tank is circular with an inner radius of 4 cm. The beam is assumed to 2 cm wide and 0.8 cm high. The charge distribution in either direction is taken as

$$f(x) = 1 , \quad x \leq w_x ,$$

$$f(x) = 0.5(1 - \sin Z) , \quad w_x \leq x \leq w_x + H_x ,$$

$$Z = n(x - w_x - H_x/2)/H_x ,$$

which represents a beam which has a uniform distribution over the distance w_x and falls gradually to zero over the distance H_x . All the results given here

assume that the fall-off region H_x is 20% of the beam extent.

The circular vacuum tank has the property that the image fields vanish for a circular charge distribution. However, during the stacking process, the ISABELLE beam will be moved off center.

Figure 1 shows the variation of the image electron gradient, $EG_{x,image}$ across the horizontal extent of the beam. Results are shown when beam is moved $Ax = 1$ cm off center, and $Ax = 2$ cm off center. Also shown are the simple analytical results found using the image field for a point charge concentrated at the center of the beam. For the proposed geometry and injection procedure of ISABELLE, $R = 427$ m, $I = 10$ A, $v = 20$, $\gamma = 30$, $r_p = 1.535 \times 10^{-18}$ m, the Δv caused by $EG_{x,image}$ is

$$\Delta v_x = - 0.48 EG_{x,image} .$$

The analytical result for a circular vacuum tank is

$$EG_{x,image} = 2/(x - a/x_0)^2 ,$$

where x_0 is the center of the beam, a is the radius of the vacuum tank and the beam has unit charge per unit length.

In storage rings, the v -variation across the beam becomes important especially in connection with the "brick wall" effect. One sees that in particular one is interested in the quadratic variation of v with x , since this may require correction with octupole coils. One sees from Fig. 1, that the simple analytical result is fairly good, except when the beam gets close to the vacuum tank and the v -variation with x becomes increasingly nonlinear.

Table I lists the magnetic and electric gradients across the beam in the median plane. The magnetic gradient is given for a case when the inner radius of the iron is 4 cm. The proposed ISABELLE magnet will have an inner iron radius of 8 cm, for which the image magnetic field is about 50 times smaller than for the 4 cm radius case, and produces a negligible effect.

Warm ISABELLE Magnets

The case considered here is that of the storage accelerator using warm conventional magnets. We assume that the magnet is a window frame magnet with an iron aperture of 4×20 cm. The vacuum tank is assumed to be elliptical and 3×14 cm. The beam is assumed to be 2 cm wide and 1 cm high, with the same kind of charge distribution assumed for the cold superconducting ISABELLE magnets.

Table II lists the results for the electric and magnetic gradients along the horizontal x -axis and along the vertical y -axis of the beam. The electric results are for a beam having a unit charge per cm of beam, and the magnetic results are divided by β .

Figure 2 shows the variation of the image electric gradient $EG_{x,image}$ and the image magnetic gradient $MG_{y,image}$ across the horizontal extent of the beam. The analytical result for $MG_{y,image}$ is also shown which result is obtained assuming a point current between 2 parallel iron surfaces. This analytical result is¹²

$$MG_{y,image} = \frac{2n^2}{g} \sinh^2(\pi x/g) - \frac{2}{x^2} ,$$

where g is the gap between the iron surfaces, and a unit

current is assumed.

For a possible 100 GeV storage accelerator using warm H-magnets, with $R = 428.2$ m, $I = 3$ A, $v = 15$, $r_p = 1.535 \times 10^{-18}$ m, $p = 218$ m, $\gamma = 30$, Δv_x generated is^p

$$\Delta v_x = -0.19 (EG_{x,image} - 0.45 \frac{MG_{y,image}}{\beta})$$

ACS Magnets

The ACS beam at injection is assumed to be 6×10 cm. The vacuum tank is elliptical whose dimensions are 3×6 in. or 7.5×15 cm. The pole is roughly hyperbolic with a gap at the center of 3 in. or 7.64 cm. The same charge and current distributions are used as in the previous 2 cases.

Table III lists the results for the electric and magnetic gradients along the horizontal x-axis and along the vertical y-axis for a beam having unit charge per cm. The magnetic results are divided by β . In this case since the injection occurs at 200 MeV, and $\gamma \approx 1$, and the field term $EG_{x,dc}$ plays an important role.

Figure 3 shows the variation of the electric gradient across the horizontal extent of the beam. Also shown is the magnetic gradient $MG_{y,image}$ computed using the ACS pole face, and computed by replacing the complicated ACS pole with a window frame pole which is 3.75×30 cm in aperture.

For the ACS, $R = 120$ m, $I = 0.7$ A, $v \approx 8.75$, $p = 87$ m, $\gamma \approx 1$ and the Δv_x generated is

$$\Delta v_x = -0.32 \left\{ \frac{1}{B} EG_{x,dc} + \beta^2 (EG_{x,image} - 0.72 \times \frac{MG_{y,image}}{\beta}) \right\}$$

Acknowledgments

We wish to thank J. Claus for drawing our attention to this problem.

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Table I. Field gradients due to the beam self-field for a cosine magnet. The vacuum tank radius is 4 cm; the iron inner radius is 4 cm. The beam is 0.8×2 cm and off-center by 2 cm. Gradients are in cm^{-2} .

X	EG_x	$EG_{x,image}$
0.75	-1.98	0.043
1	-1.02	0.046
1.25	3.55	0.051
1.5	3.16	0.056
1.75	2.59	0.062
2	2.44	0.068
2.25	2.61	0.077
2.5	3.19	0.087
2.75	3.59	0.099
3	-0.95	0.112
3.25	-1.9	0.122

X	MG_y	$MG_{y,image}$
1.0	-1.09	-0.030
1.2	2.25	-0.035
1.4	3.46	-0.043
1.6	2.76	-0.047
1.8	2.42	-0.052
2.0	2.32	-0.060
2.2	2.41	-0.065
2.4	2.74	-0.073
2.6	3.43	-0.079
2.8	2.21	-0.090
3.0	1.15	-0.100

X	EGY	$EG_{y,image}$
0	7.28	-0.0685
0.1	7.30	-0.683
0.2	7.98	-0.0680
0.3	4.90	-0.0675
0.4	1.20	-0.0665
0.5	-1.31	-0.0655
0.6	-1.78	-0.0647

Y	MG_x	$MG_{x,image}$
0	-2.32	-0.059
0.126	-2.30	-0.058
0.251	-2.27	-0.055
0.377	-2.07	-0.050
0.503	-1.66	-0.041
0.628	-1.43	-0.028
0.754	-1.21	-0.027

Table II. Field gradients due to the beam self-field for a window frame magnet with a 4×20 cm aperture. The beam is 1×2 cm and the elliptical vacuum tank is 3×14 cm. Gradients are in cm^{-2} .

X	EG_x	$EG_{x,image}$
0	1.982	-0.328
0.25	2.119	-0.322
0.5	2.586	-0.304
0.75	2.801	-0.276
1	-1.089	-0.241
1.25	-1.974	-0.202

X	MG _y	MG _{y,image}
0	2.57	0.253
0.2	2.65	0.251
0.4	2.92	0.247
0.6	3.44	0.238
0.8	2.29	0.227
1.0	-0.64	0.214
1.2	-1.75	0.198

Y	EG _y	EG _{y,image}
0	5.78	0.320
0.1	5.80	0.324
0.2	5.86	0.328
0.3	5.96	0.336
0.4	0.415	0.348
0.5	0.420	0.352
0.6	-1.35	0.364

Y	MG _x	MG _{x,image}
0	-5.31	0.253
0.1	-5.33	0.255
0.2	-5.39	0.258
0.3	-5.48	0.259
0.4	-3.66	0.262
0.5	+0.084	0.267
0.6	+1.88	0.272

Table 111. Field gradients due to the beam self-field for the AOS magnet. The beam is 6 x 10 cm. The elliptical vacuum tank is 7.5 x 15 cm. Gradients are in cm⁻².

X	EG _x	EG _{x,image}
0	0.0594	-0.0301
1	0.0620	-0.0302
2	0.0703	-0.0306
3	0.0863	-0.0305
4	0.0464	-0.029
5	-0.0516	-0.0245

X	MG _y	MG _{y,image}
-5.08	0.032	0.064
-3.81	0.159	0.071
-2.54	0.180	0.071
-1.27	0.159	0.065
0	0.144	0.054
1.27	0.134	0.040
2.54	0.134	0.025
3.81	0.099	0.011
5.08	-0.033	-0.002

Y	EG _y	EG _{y,image}
0	0.199	0.0301
1	0.203	0.0315
2	0.208	0.0293
3	0.024	0.0280

Y	MG _x	MG _{x,image}
0	-0.115	0.054
1.27	-0.118	0.056
2.54	-0.028	0.062
3.18	0.100	0.067

FIG. 1

FIG. 2

FIG. 3

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Summary

Acceleration by phase displacement is used in the ISR to accelerate stacked protons from 26 GeV/c to 31.4 GeV/c. In the phase displacement process empty RF buckets are moved through the stack from higher to lower momentum which results in a shift of the stacked beam towards higher momentum. The total momentum shift is determined by the available RF voltage, the stable phase angle ϕ_s and the number of passages of the empty buckets through the stack. The stable phase angle ϕ_s should be as small as the time and RF noise permit, in order to reduce the dilution of longitudinal phase space density inherent in the process. One of the main difficulties encountered was the synchronisation of the magnetic field adjustments and the RF system. Twenty-six power supplies are adjusted simultaneously by the control computer in order to keep the betatron frequencies constant during the acceleration. To reduce losses resulting from beam-beam effects, the betatron frequency band used must be free of resonances and the beams have to be separated in the intersections during acceleration.

Introduction

The first attempts to accelerate stacked protons in the ISR to 31.4 GeV/c were made by means of re-bunching. The available RF voltage, however, limits the total current that can be accelerated to 31.4 GeV/c to about 1 A¹. In order to overcome this limitation the method of phase displacement acceleration has been developed. This method is at present operational and currently used in the ISR for preparation of colliding beams for physics experiments at 31 GeV/c.

Beam currents of up to 6 A have been accelerated by this method. The highest luminosity so far achieved has been $0.36 \cdot 10^{30} \text{ cm}^{-2} \text{ sec}^{-1}$, but a luminosity of $10^{30} \text{ cm}^{-2} \text{ sec}^{-1}$ seems well within reach.

Phase displacement acceleration

Large stacks can be accelerated or decelerated with a medium power radio frequency system by means of "phase displacement". In this process the frequency of the radio frequency system is modulated in such a way that empty RF buckets are generated which are carried all the way through the stack repetitively. The change in momentum which the stack experiences when an empty bucket is carried through is however accompanied by a reduction in the longitudinal phase-space density, which is the main disadvantage of the process.

The choice of the RF parameters - stable phase angle, $\sin \phi_s$, and RF voltage V - for the phase displacement process is a compromise between an acceptable density reduction and the total time spent during the process.

The density reduction, which can be described by the root mean square momentum spread - δp_{rms} - introduced in the stack by the passage of n empty buckets, is given by

$$\delta p_{\text{rms}} = \sin \phi_s \cdot \delta p_0(p, V) \sqrt{n} \quad (1)$$

where $2\pi\delta p_0(p, V)$ is the area of the stationary bucket that would be generated by the actual RF voltage V at $\sin \phi_s = 0$. This relationship, which was derived theoretically² and studied with a computer simulation³, has been verified experimentally and good agreement has been found, as can be seen from Fig. 1.

Fig. 1. Normalised r.m.s. momentum spread as function of $\sin \phi_s$.

The time t spent by one passage of an empty bucket through the stack is given by

$$t = \frac{s \Delta p}{\dot{p}} = \frac{s \Delta p \cdot 2\pi R \cdot m_0 c}{V \sin \phi_s} \quad (2)$$

where Δp is the width of the stack in momentum at the beginning of the process and $\dot{p}$ is the rate of change of momentum of the empty bucket, which is independent of p when $\sin \phi_s$ and V are kept constant during the process.

The safety factor s has to be introduced since the total momentum bite traversed by the empty buckets should always be considerably bigger than the width of the stack in order to allow for the growth in Δp inherent in the process and to avoid additional density reduction due to switching transients, which occur when the RF voltage is switched on and off at the beginning and the end of the empty bucket sweeps.

The total number of passages n required to change the average momentum of a stack from p_1 to p_2 ($p_2 > p_1$) is given by

$$n = \frac{p_2 - p_1}{\delta p_0(p, V) \cdot \alpha(\sin \phi_s)} \quad (3)$$

with $\alpha(\sin \phi_s)$ the bucket area parameter.

Combining (1) and (3) one finds

$$\delta p_{rms} = \frac{\sin \phi_s}{\sqrt{\alpha(\sin \phi_s)}} \cdot \sqrt{p_2 - p_1} \cdot \sqrt{\delta p_0(p, V)} \quad (4)$$

which shows a strong dependence of δp_{rms} on the choice of $\sin \phi_s$ and a weak influence of the voltage V since δp_0 is proportional to $\sqrt{V}$. The dependence of δp_{rms} on momentum p is also very weak when p is well above transition energy.

To minimize time and density reduction the highest possible RF voltage should be used together with the lowest value of $\sin \phi_s$ which seems practical from the point of view of the total time spent.

The working line

The band covered by the tune in the stack is shown in Fig. 2. This working line is similar to the line frequently used for the preparation of medium intensity stacks for physics.

Fig. 2. The working line used for the phase displacement acceleration.

The chosen region provides 35 mm of radial space, free of resonances of an order lower than 8. Space-charge effects limit the region slightly but no compensation has been necessary hitherto. The series of 8th order resonances, $nQ_V + (8-n)Q_H = 69$, and the series of 11th order resonances, $mQ_V + (11-m)Q_H = 95$, are those of the lowest order present.

Beam loss due to the excitation of these resonances is not observed during the normal operation of the ISR. However, during the acceleration process these resonances are excited much more strongly, especially

by beam-beam effects⁴ when a beam is present in the other ring.

Furthermore the resonances will move slightly relative to the stack because of the imperfections when tuning the magnet. This movement can cause serious beam losses depending directly on the precision in controlling the magnetic field distribution.

Tuning the magnet

The configuration of the magnetic field is kept proportional to the beam momentum at central orbit over the full energy range by the excitation of the pole-face windings, the focussing and defocussing magnet units, as well as the excitation of the localized sextupole lenses⁵. The corrections are highly non-linear due to the saturation in the magnet core. An example of the distribution of the pole-face winding currents as function of radial position and momentum is shown in Fig. 3.

Fig. 3. Normalized currents in the pole-face windings of the focussing magnet units as function of radial position and momentum.

The resulting compensation of the magnetic field gradient is shown in Fig. 4.

Fig. 4. Normalized field-gradient compensation as a function of radial position and momentum.

The working lines were measured and adjusted for six different values of beam momentum. To establish working lines at intermediate beam momenta linear interpolation is used.

Application of the phase displacement acceleration in the ISR

In the ISR the phase displacement method of acceleration is used to accelerate stacked proton beams from 26.5 GeV/c to 31.4 GeV/c.

The maximum voltage that can be generated by the ISR RF system is 20 kV which, however, cannot be used due to severe beam loading of the RF cavities. This beam loading is the result of the modulation of the stack by the empty buckets carried through, which results in induced voltages, equivalent to what would be induced by about 1A beam current, on the cavity gap. At present 12 kV is used which has proved to be a safe value.

The safety factor s has been chosen so that the total momentum bite travelled by the empty bucket is approximately three times the initial stack width.

The value of $\sin \phi_s$ has been set to 0.1 which yields a total time for the acceleration process of about 20 minutes which is acceptable for routine operations. The bucket area for 12 kV RF voltage and $\sin \phi_s = 0.1$ is 0.168 ($\Delta p/m_0 c$, ϕ_{RF}) at 26 GeV/c and increases to 0.177 ($\Delta p/m_0 c$, ϕ_{RF}) at 31 GeV/c. A total of 184 empty bucket traversals is required to change the average momentum from 26.5 to 31.4 GeV/c. In Fig. 5 the RF empty bucket scans are shown at the beginning and at the end of a typical acceleration process, whereby an initial stack of 11.9A gave 6.2A at 31.4 GeV/c.

Fig. 5. Empty bucket signal induced on p.u. station
(a) initial 11.9A stack at 26.5 GeV/c
(b) 6A stack at 31 GeV/c after phase displacement

The acceleration process is at present performed under complete computer control. Main field and pole-face winding currents are changed in small steps, derived by linear interpolation from the currents measured at six different momenta. The magnitude of the steps is such that the increase in central orbit momentum due to main field change and one empty-bucket traversal are identical. The current changes in all power supplies and the empty-bucket sweep are initialized at the same time by the computer.

Synchronization of main field adjustments and RF system is guaranteed since field changes can only be applied when a signal, indicating the end of the empty bucket sweep, has been received by the computer.

The matching between empty bucket sweeps and field changes is, however, still not perfect since the variation of the bucket area as function of momentum has up till now not yet been taken into account.

Current losses during the acceleration process

During the acceleration process a considerable amount - typically between 40 and 50% - of the current stacked at 26 GeV/c is lost, which makes the process less efficient than would be expected from the density reduction described above.

The current losses seem to depend on both density and/or intensity, since the losses are always much bigger at the beginning of the acceleration process than towards the end (see Fig. 6).

Fig. 6. Current as function of momentum
(1) single ring acceleration
(2) acceleration in presence of a stack

Several sources can be evoked for these current losses.

(a) Current loss due to RF noise

Due to noise in the RF system, which modulates both the phase and the amplitude of the RF voltage, particles are collected in the buckets when passing through the stack. If a significant number of particles is collected in the buckets, a bunched-beam-like signal should be observed on the pick-up stations as the bucket emerges from the stack. A signal of this nature can indeed be observed on a sufficiently sensitive pick-up. The particles collected in the buckets are most likely to be found near the separatrix and since $\sin \phi_s \approx 0.1$ the bunching factor of the particles in the bucket will be small. The peak value of the signal induced on the pick-up stations, together with the low bunching factor, indicates that a total number of particles is trapped in the buckets corresponding to approximately 10 mA, which shows that RF noise is only partially responsible for the heavy current losses during the acceleration.

The number of particles collected in the bucket is a function of the density in the stack and the time the bucket spent in the stack. The RF noise is thus setting a lower limit to the value of $\sin \phi_s$. When $\sin \phi_s$ is lowered to minimize δp_{rms} more time will be spent per empty bucket passage and the current loss due to particles trapped in the bucket will thus increase.

(b) Current losses due to non-linear resonance excitations

When an empty bucket moves through a stack heavy current losses occur even when the magnetic field is kept constant. These losses can be as high as 200 mA for a single empty bucket sweep through a stack which only contains relatively high order - 8th and 11th - resonances.

The reasons for these types of losses are the following: The presence of an empty bucket in the stack influences the particle trajectories and particles on orbits near the non-linear resonances are moved into the resonances and lost. Furthermore, the moving bucket does modulate the space charge detuning and as a consequence the resonances are not kept at the same position during the empty bucket sweep.

During the actual phase displacement the movements of the stack due to imperfect synchronisation between the RF system and magnetic field changes add to the losses.

The increase in momentum spread inherent in the process also contributes to the current losses since particles are moved into the 3rd and 5th order resonances which are the boundaries of our present "resonance free" region.

The relative strength of the different loss mechanisms is at present unknown and will be studied in the future.

The beam losses due to non-linear resonances are

very much enhanced by beam-beam effects. Only about 1A of beam current is left after acceleration to 31 GeV/c if a beam of 5A is circulating in the other ring (see Fig. 5, curve 2). These losses are biggest in the region of the 8th order resonances. When separating the beam vertically by 10 mm in all the inter-sections, these losses can be avoided.

That the empty bucket in the stack is at the origin of the heavy current losses has been very clearly demonstrated by the beam-beam effect mentioned above, which appears during acceleration much more strongly than during normal stacking.

Acknowledgements

We are very indebted to many colleagues, in particular to R. Keyser who wrote all the necessary computer programs both for the incremental field changes and the synchronisation with the RF system.

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Detailed examination of computed particle trajectories has revealed a complexity and disorder that is of increasing interest to accelerator specialists. To introduce this topic, I would like you to consider for a moment the analysis of synchrotron oscillations for a particle in a coasting beam, regarded as a problem in one degree of freedom. A simple analysis replaces the electric field of the RF-cavity system by a traveling wave, having the speed of a synchronous reference particle, and leads to a pair of differential equations of the form

$$dy/dn = -K \sin \pi x, \quad (1a)$$

where y measures the fractional departure of energy from the reference value, πx measures the electrical phase angle at which the particle traverses the cavity, and K is proportional to the cavity voltage; and

$$dx/dn = \lambda' y, \quad (1b)$$

in which λ' is proportional to the change of revolution period with respect to particle energy. It will be recognized that these equations can be derived from a Hamiltonian function

$$H = (1/2)\lambda' y^2 - (K/\pi) \cos \pi x. \quad (2)$$

Because this Hamiltonian function does not contain the independent variable explicitly, it will constitute a constant of the motion and possible trajectories in the x, y phase space will be just the curves defined by $H = \text{Constant}$, namely the familiar simple curves in phase space that are characteristic of a physical (non-linear) pendulum.

If we note, however, that a localized cavity can affect the energy of a particle only when the particle encounters the cavity, it is natural to replace the differential equations by difference equations. Thus, measuring energy y_n at the n^{th} entry to the cavity, we write the transformation

$$\left. \begin{aligned} y_{n+1} &= y_n - K \sin \pi x_n \\ x_{n+1} &= x_n + \lambda' y_{n+1} \end{aligned} \right\} \quad (3a,b)$$

(which can readily be shown to be area-preserving). Although alternatively the motion in this case could again be expressed by differential equations derivable from a Hamiltonian function, the Hamiltonian now would contain a periodic δ -function of the independent variable as a factor multiplying the term $-(K/\pi) \cos \pi x$ and hence could not be taken as a constant of the motion. (The differential equations, moreover, would be non-linear, so that Floquet theory could not be applied.) The use of such a Hamiltonian formulation nonetheless can be helpful in analytic work, but difference equations of course are attractive for computational investigations.

It is of interest to take a quick look at some computational results obtained through use of a transformation equivalent to (3a,b) but written in terms of work-

ing variables $Y = y - (K/2) \sin \pi x$, $X = x$, so that the transformation assumes the form

$$\left. \begin{aligned} X_{n+1} &= X_n + \lambda' [Y_n - (K/2) \sin \pi X_n] \\ Y_{n+1} &= Y_n - (K/2) [\sin \pi X_n + \sin \pi X_{n+1}] \end{aligned} \right\} \quad (3a',b')$$

with the result that the resulting phase diagrams will necessarily have a desirable symmetry about both the X - and Y -axes. With $K/\pi = 0.1$ and $\lambda' = 0.1$ we find what appear to be conventional bucket diagrams with buckets separated in Y by $2/\lambda'$ for successive harmonic modes, although we may wish to return to the question of whether the bucket boundaries are as simple and definite as appears on Fig. 1.

Fig. 1. - X, Y phase plot for a coasting beam under the influence of an R.F. cavity with $K/\pi = 0.1$, $\lambda' = 0.1$ - as computed by Eqns. (3a',b'). X is plotted mod. 2.

We also find evidence of some "sub-harmonic" structure (with higher order fixed points) that, if enlarged some 60X, has the appearance shown in Fig. 2.

R F Phase Plots $K/\pi = 0.1$
 $\lambda' = 0.1$

10.2

9.8
-1.0

1.0

Fig. 2. - Circa 60-fold vertical enlargement of central portion of Fig. 1, near $Y = 10.0$, showing sub-harmonic structure.

If the cavity voltage is increased eight-fold (so $K/\pi = 0.8$), the bucket areas are expected to become larger, and we indeed find this to be the case (Fig. 3), with an accompanying very marked increase of complexity

R F Phase Plots $K/\pi = 0.8$
 $\lambda' = 0.1$

26.0

-6.0
-10

1.0

Fig. 3. - Phase plot similar to Fig. 1, but for operation with $K/\pi = 0.8$, showing the obvious development of complex structure.

that is immediately apparent in the phase plot. Of particular interest is the evident diffuse character of phase trajectories generated by points launched close to the first-order unstable fixed points situated at $X = \pm 1$, since the bucket boundary in consequence no longer appears clearly defined.

In the first example ($K/\pi = 0.1$), on the other hand, where the bucket width is some two and one-half times smaller *in* relation to the bucket separation, the presence of structure in the separatrix can be revealed computationally only with considerable care.⁴ To do this, one can extend from the unstable fixed points the eigenvector directions of the transformation linearized about these fixed points, and examine whether such curves intersect smoothly. One finds in fact that they do not quite do so, but generate loops (of a nature to be illustrated later) that *in* this instance ($K/\pi = 0.1$) have a very small area that amounts to only about $1/(5 \times 10^3)$ of the area of the bucket itself.

Similar questions concerning the character of phase trajectories and the possible erratic or stochastic behavior of canonical mappings can arise in problems with more than one degree of freedom. As an example, Hénon and Hiles⁵ and subsequently Walker and Ford⁶ studied a model of an astronomical system, for which the Hamiltonian function was taken to be

$$H = \frac{1}{2}(p_1^2 + p_2^2 + q_1^2 + q_2^2) + q_1^2 q_2 - \frac{1}{3} q_2^3. \quad (4)$$

The cubic terms appearing here as coupling terms become increasingly significant for increasingly large values of H -- which is itself a constant of the motion. With the coupling terms present, however, and in the absence of any simple constant of the motion other than H , a given phase trajectory might be expected to wander (ergodically) over virtually all of a three-dimensional surface specified by $H = \text{Constant}$ (and that will be a closed surface for values of H below the dissociation energy). If, on the other hand, some additional integral of the motion were in fact also acting, the phase points of a given trajectory then would be constrained to lie on a two-dimensional surface, and graphs of the intersection of such surfaces with some selected plane or other surface (a "surface-of-section") would lead to simple curves in this plane rather than to a scattering of points. Computations of this nature indicated that for sufficiently small values of energy (e.g., $H \leq 1/12$) only curves that to computer accuracy were smooth (and relatively simple) were formed by intersection with the plane $q_1 = 0$ (and $p_1 \geq 0$). Examples in which the energy of the particles was successively raised, however, resulted in the development of ragged island structures or of apparent stochastic behavior over increasingly large portions of this surface-of-section (Fig. 4).

Fig. 4. - Phase plots, in the surface of section $q_1 = 0$, resulting from the equations implied by the Hamiltonian function (4) -- for increasing values of the energy. [After Walker and Ford.⁶]

Such behavior appears concordant with the "KAM" (Kolmogorov-Arnol'd-Moser) theory (see Refs. 58, 59, & 60 of our Ref. 1c), which suggests that many of the invariant curves or surfaces present in the absence of the perturbation will persist, with only minor distortion, in the presence of a sufficiently small perturbation (see, however, Note 7). It is of interest, of course, to determine or to estimate the circumstances (e.g., perturbation strength) at which the KAM theory becomes inapplicable and extended regions of erratic (or stochastic) behavior develop. As was suggested by our first examples, and has been extended more extensively by Zaslavskij and Chirikov, one means for obtaining such estimates may be by determining the ratio of resonance width $[\delta\omega = (d\omega/dI)_r \delta I]$ to the distance $(\Delta\omega)$ to the nearest neighboring resonance.

Additional tests (to be mentioned below) may be required to determine the degree of disorder associated with the movement of phase points in such stochastic regions. We may first note, however, that the existence of nested closed invariant curves in a plane -- as suggested by the KAM theorem for a problem in one degree of freedom -- prevents phase points from moving outward or inward to regions of substantially different "amplitude" (in the absence of noise). With more than one degree of freedom, however, stochastic layers may intersect, to form an intricate system of channels along which a phase point can slowly diffuse and result in instability. The possibility of such "Arnol'd diffusion" has been demonstrated by Arnol'd [Ref. 35 of our Ref. 1c; stated simply the example considered by Arnol'd is comprised of a physical pendulum and a simple-harmonic oscillator, with a time-dependent coupling (that also depends on the phases, or angle variables, of these oscillations)].

It should be pointed out that some non-linear transformations -- say for a system with one degree of freedom -- will not lead to the disappearance of some or all of the invariant phase curves at substantial amplitudes. Thus for transformations of the form

$$x_{n+1} = y_n, \quad y_{n+1} = -x_n + f(y_n), \quad (5a,b)$$

McMillan⁹ has shown that if $f(y)$ can be written as $\phi(y) + \phi^{-1}(y)$ (where ϕ^{-1} denotes the function inverse to ϕ), then the curves $y = \phi(x)$ and $x = \phi(y)$ will constitute invariant curves. Such curves will pass through the first-order fixed point(s) situated at the intersection(s) of $y = (1/2)f(x)$ with the principal diagonal. An enclosed area can thereby be formed from which phase points cannot escape even if the behavior in portions of the interior becomes highly stochastic. This is illustrated by an example (Fig. 5) in which

$$f(y) = \frac{1}{2}(3y-1) - \frac{1}{2} \frac{k^2}{y+1} + \sqrt{y^2+k^2} \quad (6a)$$

and

$$\phi(x) = x - 1 + \sqrt{x^2+k^2}. \quad (6b)$$

Such a situation also can develop when $f(y)$ is a step-wise linear function of y with discontinuities of slope, as has been noted by Dr. Judd [see, for example, Figs. 13 and 14 (pp. 27-28) of Ref. 10]. If $f(y)$ is of the form

$$f(y) = -(By^2 + Dy)/(Ay^2 + By + C), \quad (7)$$

moreover, the entire phase plane will be covered by a family of simple invariant curves -- see, for example, the cases⁹ $f(y) = 2ky/(1+y^2)$, with the invariants $x^2y^2 + x^2 + y^2 - 2kxy = \text{Constant}$, and $f(y) = 2ky/(1-y^2)$, with the invariants $x^2y^2 - x^2 - y^2 + 2kxy = \text{Constant}$, illustrated by Figs. 6-8.

Fig. 5. - Phase diagram for the transformation (5a,b), with $f(y)$ given by Eqn. (6a). The scattered points result from computations initiated with $x_0 = y_0 = 0.25$, but must remain within the separatrix defined by the function ϕ [Eqn. (6b)]. $k=0.1$

Fig. 6. - Invariant curves for the transformation (5a,b) with $f(y) = 2ky/(1+y^2)$ and $k = 2/3$. [Figs. 6-10 after McMillan.⁹]

Fig. 7. - Invariant curves for the same transformation as in Fig. 6, but with $k = 1.36$.

Fig. 8. - Invariant curves for the transformation (Sa,b) with $f(y) = 2ky/(1-y^2)$ and $k = 0.64$.

It is of interest to examine the mechanism whereby irregular behavior can develop in the neighborhood of unstable fixed points, taking as an illustration an example suggested by Professor deVogelaere that [when generalized and rewritten in variables leading to the form (Sa,b) advocated by McMillan] employs

$$f(y) = 2[Ty + (1 - T)y^2]. \quad (8)$$

First-order fixed points appear at $(0,0)$ and at $(1,1)$. For $T = 0$, this transformation, when linearized about the unstable fixed point at $(1,1)$, can be represented by the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 4 \end{pmatrix}$, with eigenvalues and eigenvector slopes

$$\lambda = 2 \pm \sqrt{3}, \quad dy/dx = \lambda. \quad (9)$$

A line segment extending downward from the fixed point $(1,1)$ with the slope $2 + \sqrt{3}$, if subjected to repeated applications of the transformation, generates the loops shown in Fig. 9; similarly a line segment of slope $2 - \sqrt{3}$, if extended by the inverse transformation, generates the mirror-image curve (mirrored about the principal diagonal). Points such as A, B, C ... progress toward the fixed point in smaller and smaller steps and, since the transformation is area-preserving, the associated loops clearly must become increasingly elongated as they become increasingly narrow from repeated applications of the forward transformation. The evolution of such loops clearly will become quite intricate (Fig. 10),

Fig. 9. - Plot of the extensions of the eigenvector directions from the unstable fixed point at $(1,1)$, for the deVogelaere transformation expressed in McMillan's variables [Eqns. (Sa,b) and (8), with $T = 0$]. The areas of the loops marked L are all equal, by virtue of the area-preserving character of the transformation and the inherent symmetry about the principal diagonal.

Fig. 10. - A partial extension of the curves shown on Fig. 9.

but the loops apparently need not permeate the entire "interior" -- portions of an inward loop can, in fact, enter, on a later iteration, into the interior of an outward-lying loop (as indicated on Fig. 10).¹¹ It is clear, however, that the development of such a loop system can readily give rise to an apparent stochastic motion of phase points in portions of the phase diagram -- most particularly near an unstable fixed point such as that mentioned here.

The existence of a firm separatrix, or of an extensive family of invariant curves generally, can be extremely sensitive to the exact form of the transformation.¹² A case of some physical interest arises in computational studies relating to the Toda Lattice.¹³ "his one-dimensional lattice consists of particles interacting through exponential pair potentials and can propagate certain non-linear wave forms ("solitons") without change of shape. One computational investigation¹⁴ of stability for a three-particle lattice (with periodic boundary conditions) has commenced with a Hamiltonian function

$$H = \frac{1}{2}(P_1^2 + P_2^2 + P_3^2) + e^{-(Q_1 - Q_3)} + e^{-(Q_2 - Q_1)} + e^{-(Q_3 - Q_2)}. \quad (10)$$

By a canonical transformation of variables, in recognition of the invariance of this system to translation -- so that $I_1 = P_1 + P_2 + P_3$, constitutes a constant of the motion -- the Hamiltonian (10) becomes expressible as a function of two pair of conjugate variables in the form

$$H = \frac{1}{2}(p_1^2 + p_2^2) + \frac{1}{24}[e^{(2q_2 + 2\sqrt{3}q_1)} + e^{(2q_2 - 2\sqrt{3}q_1)} + e^{-4q_2}], \quad (11)$$

which is identical to the Hénon-Heiles Hamiltonian function (4) through terms of third order. It is of interest to examine whether in the present case constants of the motion other than H act to restrict the motion. Computationally it was found -- again using the surface-of-section $q_1 = 0 (p_1 > 0)$ -- that in this case simple invariant curves apparently continued to exist in the q_2, p_2 plane, even for very large values of H . Stimulated by this result, Hénon¹⁵ has directed attention to an additional integral of the motion that is valid in this case; the constants of the motion for the three-particle lattice then can be written in a form that we may express as¹⁶

$$H = \text{Constant} \quad (12a)$$

$$P_1 + P_2 + P_3 = \text{Constant, and} \quad (12b)$$

$$P_1 P_2 P_3 - P_1 e^{-(Q_3 - Q_2)} - P_2 e^{-(Q_1 - Q_3)} - P_3 e^{-(Q_2 - Q_1)} = \text{Constant.} \quad (12c)$$

[Exactly] further analytic work in fact has now established that the n -particle Toda lattice with periodic boundary conditions (or with fixed ends) is a "completely integrable" system.]

It is of some interest to seek means for anticipating whether stochastic behavior will occur in various portions of a phase diagram and to examine the character of such stochastic behavior as does occur. What we here have loosely termed stochastic behavior can be catalogued with respect to a hierarchy of properties (ergodicity, mixing, ...), indicative of increasing disorder, that are fundamentally significant for statistical mechanics.^{1a,e} Of particular interest to the accelerator designer, of course, is the determination of a threshold beyond which stochastic behavior will set in and may act to carry a phase point to unacceptably large amplitudes. As noted earlier, stochastic behavior appears to be associated with overlapping resonances,^{1c} and this concept has served as the basis for some analytic esti-

mates of stochasticity limits.^{1c,17} It has been noted by René deVogelaere and confirmed in subsequent computations¹⁸ that for a particular class of fixed-point families -- say those with rotation of the form $m/(4m+1)$ -- there is a closely linear relationship between the order of the resonance $(4m+1)$ and $\ln|1 - \frac{1}{2} \text{Trace}|$ through many decades ("Trace" denoting the trace of the tangential-mapping or differential matrix associated with the $4m+1$ iterations required to map a given fixed point onto itself). Such regularities, and others relating to the apparent size of the stable areas about high-order fixed points (e.g., as estimated from the intersection angle of eigenvectors), have been considered useful indicators of the change in character of a mapping at certain amplitudes.^{19,8,20}

A computational procedure of considerable interest for recognizing stochasticity is that in which one follows the evolution of the distance between two initially very close points in phase space. In practice it can prove desirable to reduce the separation from time to time by a recorded factor whenever the separation becomes excessive during the computations, or, perhaps preferably, to evaluate the growth of an infinitesimal vector through use of the cumulative tangential-mapping matrix. A high degree of stochasticity can be ascribed to the behavior of the transformation if there are such vectors whose length generally grows beyond the first iteration by a factor greater than unity (while others may similarly contract). (Ref. 1a, p. 55; for examples, see Ref. 21.) An analogous procedure -- that can be more attractive, although possibly of a less direct basic significance -- is an investigation of the growth of the eigenvalue(s) of the cumulative tangential mapping. Such eigenvalues can change sign repeatedly during the course of many iterations, and hence will be seen to decrease from time to time, but an exponentially increasing trend in eigenvalue magnitude is likely to be associated with a similar type of increase for the lengths of the vectors mentioned previously. The nature of eigenvalue growth has been illustrated by Froeschlé²² for the transformation²³

$$\left. \begin{aligned} x_{n+1} &= x_n \cos \alpha - (y_n - x_n^2) \sin \alpha \\ y_{n+1} &= x_n \sin \alpha + (y_n - x_n^2) \cos \alpha \end{aligned} \right\}. \quad (13a, b)$$

The general characteristics of this transformation, expressed in variables such that the transformation has the symmetry of McMillan's form, is seen on Fig. 11. On an expanded scale (X10), we see (Fig. 12) the sudden onset of erratic behavior as the starting values for the transformation are successively increased (in steps $\Delta x_0 = 0.0025$, for $y_0 = 0$), and on a scale expanded by a further factor 100/6 we see (Fig. 13) the presence of a great deal of additional structure within a portion of this "stochastic" region. Associated with the transition to the stochastic region there appears to be a marked change in the manner of growth of $\psi_n = \log|\lambda_n|$ (linear, vs. n , in the stochastic case -- indicative of an exponential trend for $|\lambda_n|$) or of the "Cesaro

mean" $\mu_n = \frac{1}{n} \sum_{m=1}^n \frac{1}{m} \psi_m$ (constancy in the stochastic case,

monotonically decreasing otherwise -- Fig. 14).²⁴ Such methods indeed may prove useful in investigating computationally the possible development of stochastic motion in storage-ring devices. Extended computations of this nature can present challenging problems with respect to computer accuracy.²⁵

Fig. 11. - Apparently-smooth phase curves and a scattering of points resulting from iteration of the transformation (11a,b), with $\cos \alpha = 0.22$ and coordinates X, Y appropriate to expressing the transformation in the form (5a,b).²³ Five islands of stability (containing stable fixed points of order 5) are seen surrounding the area associated with the order-1 fixed point at the origin. The outermost smooth curve, shown as bounding this inner area, resulted from the starting values $x_0 = 0.5350, y_0 = 0$ (Froschlié notation), and the scattered points result from $x_0 = 0.5375, y_0 = 0$. Scale (as indicated by the coordinate axes): -1.0 to 1.0

Fig. 12. - Enlarged portion (10X) of Fig. 11, showing seven smooth phase trajectories resulting from starting values $x_0 = 0.5200, 0.5225, \dots, 0.5350$ (and $y_0 = 0$) and a scattering of points resulting from $x_0 = 0.5375, y_0 = 0$. Note the occurrence of open areas within the region covered by the scattered points -- for example the area surrounding an (unplotted) stable fixed point of order 65 at $X \approx 0.476, Y \approx 0.521$. Scale: 0.38 to 0.58

Fig. 13. - Detailed multiple-island structure in the immediate neighborhood of an order-65 stable fixed point (shown here just below the center of the diagram) of which mention has been made in the caption to Fig. 12. Scales: 0.470 to 0.482 for X , 0.516 to 0.528 for Y .

Fig. 14. - Plots of the "sliding mean", v_n (Note 24), $\overline{v}_n$, obtained from computations begun (i) with initial conditions leading to the last smooth curve of Fig. 12 ($x_0 = 0.5350$) and (ii) with initial conditions leading to the scattered points on that Figure ($x_0 = 0.5375$), of which only the results for the latter case indicate a general exponential upward trend of $|\lambda_n|$.

References and Notes

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24. The curves of Fig. 14 are plots of
$$v_n \equiv \sum_{m=1}^n \frac{1}{m} \psi_m \exp(-\frac{n-m}{\tau}) / \sum_{m=1}^n \exp(-\frac{n-m}{\tau})$$
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26. I am deeply indebted to Paul J. Channell (LBL) for many stimulating and helpful conversations concerning topics discussed here. Responsibility for the views expressed in this paper, however, remains exclusively my own.

DISCUSSION

Lee Teng (NAL): Are you proposing the parameter ν as the parameter which specifies whether you are close to this stochastic paradox?

Jackson Laslett (LBL): I would like to propose that for the consideration of anyone doing this. I have noticed recently that people in this country have been concerned with whether interacting beams, a weak beam on a strong beam, give stable or unstable motion. Some of these people have taken two points a finite distance apart, followed them with the equations, and then tried to see how the distance between two points grows. We see the structure in some cases can be very detailed, so if one took two points, not an infinitesimal distance apart, but a considerable distance apart to get some computer accuracy on the difference, one might worry whether one should continue too long before they would get on opposite sides of the diagram. I think that Chirikov has suggested that one occasionally push them back together and keep a record of which one is pushed back. It might seem more reasonable to use infinitesimal vectors, and do *this* by means of keeping track of this tangential mapping transformation. One does get into computational difficulties in that connection and the same would apply for the eigenvalues. I don't know that it makes too much difference. Froeschlé has discussed the use of eigenvalues, and he has done some very nice work in transformations in two spatial degrees of freedom where he had four phase variables, and in that case he insisted that one should look not only at the growth of the largest eigenvalue but also at the next fellow, and that put very difficult computational requirements on his program. So, I am not sure whether that's a practical thing to do or not. In fact, in a plane I think if you have a scope presentation you could see the erratic behavior developing about as quickly as you could see the evolution. On the other hand, people interested in statistical mechanics are sometimes interested in the qualitative manner in which the erratic behavior develops. So I'm not sure. It's an open question to me. I'd be interested in your opinion.

Teng: Have you checked that this indicator behaves the same way as some others usually used to assess whether the distance between infinitesimally separated particles grows.

Laslett: I think that's what Froeschlé is suggesting and I've been trying to see whether I believe it, and I'm beginning to believe it from some of my computer experiments. That is to say, it's correlated with the scatter points, or with the evolution at least of the infinitesimal distances. The matrix elements sort of control both these things.

Francis Cole (NAL): One of the transformations that flashed by, I think it was the one right after de Vogelaere's, when it was put into MacMillan's form, seemed to be linear. It had $\sin \alpha'$ and $\cos \alpha$. Was it linear?

Laslett: No, it was not linear. There was a $(y-x_n)^2$, quadratic term in it.

Henry Blosser (Michigan State University): Occasionally, in tracking orbits in a computer, one's near the limit. I have seen them be well behaved for a long period of time and then sort of go into the stochastic sort of behavior. Do you think that's a really physical possibility, that it would change behavior? Or that it just might be round-off?

Laslett: I would think it could be either. I mean one certainly has to be on guard against computer artifacts, and there are ways of trying to check on those things, as I'm sure you very well know. However, one can imagine situations where, for example, if one is extending one of these eigenvector directions from an unstable fixed point, in the manner indicated, the extensions of such a segment will generate a curve which only after some time may develop noticeable loops of the sort that we saw so readily with Professor de Vogelaere's transformation. And any single point of course ultimately would show a greater scatter as a result of following along on those loops. I don't know whether you feel differently. I think it's an open question in any particular instance. One just has to check his computations carefully and see whether it's right.

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Abstract

It is well known that nonlinear detuning stabilizes high order isolated nonlinear resonances. On the other hand, for a system with changing tune, small amplitude particles can "lock-into" a resonance and be carried to large amplitudes. In coasting beams, such as the ISR beams, there is no obvious dynamic process to provide tune changes. However, it has been pointed out that scattering processes, such as intrabeam p-p scattering, induce momentum diffusion and, through the chromaticity, tune diffusion. In this way, particles can "pass through resonances". Using a random-walk model of the diffusion process, a simple expression for the fractional current loss rate is derived. Estimates for the loss rate are compared with ISR observations and are found to be in agreement. It might be emphasized that the model of beam loss presented here uses only the classical theory of nonlinear resonances.

1. Introduction

The stacked coasting beams in the ISR storage rings exhibit a loss rate which cannot be explained by a single, direct mechanism. A typical fractional beam loss rate might be $10^{-5}/\text{min}$. This is much faster than can be described by the existing diffusion processes in the ISR, e.g. scattering of protons off the residual gas in the chamber or the electromagnetic scattering of protons off each other (intrabeam scattering). On the other hand, it is apparent from ISR observations that the beam loss is associated with resonances of high order. In fact 5th and 8th order resonances have been identified and correlated with the loss of beam.¹ The difficulty here is that single, isolated nonlinear resonances, by themselves, cannot cause beam loss. Actually, it is well known that nonlinear detuning stabilizes these high order nonlinear resonances.² Here, then, is the dilemma: In the first place, there are diffusion processes which can cause beam growth and loss to the aperture. But, these are too slow to explain the observed beam loss rate. Secondly, the beam loss is correlated with high order resonances; but, in a coasting beam, the particles should be stable against these.

To resolve the situation, two basic approaches have been tried. One is to discard the traditional resonance picture and replace it with a multi-resonance model.^{3,4} Under the influence of many nonlinear resonances, particles are (so it is suggested) driven into complex pathways in the four-dimensional betatron phase space, pathways which can transport particles to large betatron amplitudes and thereby to the aperture limit. The transport process, known as Arnold diffusion, is thought to be related to the stochastic nature of strong nonlinearities. Thus, it is suggested that the combination of many nonlinear resonances can manifest itself as a diffusive beam growth, with the time scale determined by the quasi-random nature of the multi-resonance system. The ISR observation that beam loss is associated with single high order resonances presents somewhat of a problem in this model. It is conceived, however, that certain strong single resonances may act as feeding mechanisms for the Arnold diffusion pathways.

The final word on this rather elegant and radical approach is not yet in and we will not pursue it further here. The second approach to an explanation of storage ring beam loss is more conventional. It follows simply from the fact that, although isolated high order resonances are stable in a static system, for a system with changing tune, particle amplitudes may grow. Thus, it is suggested that diffusion processes, such as intrabeam scattering, can feed resonances which then transport particles to the aperture limit.⁵ We will here take this approach and present a model for beam loss in the ISR, utilizing a diffusion process which feeds a resonance which in turn transports particles to the aperture limit.

The ingredients of our model are as follows: Intrabeam scattering induces a diffusion of momentum and, through the machine chromaticity, a diffusion in particle tune. In this way, particles "pass through resonances". Under certain conditions, particles may "lock-into" a resonance during their passage through it and thereby be carried to large amplitudes. Thus, we have, in our model, the two essential features, feeding and transport.⁶ The feeding process is described by a random-walk model of the tune diffusion due to intrabeam scattering, while particle transport during resonance passage will be described by a trapping mechanism⁷, obtained using only the classical theory of nonlinear resonances. Combining the feeding and transport processes, we derive an expression for the fractional beam loss rate.⁸

Our analysis applies to one-dimensional resonances. The extension to two dimensions has not been done. However, in the coupling resonance case, the excitation width and nonlinear detuning are still the dominant characteristics; and, we believe that, by an appropriate redefinition of tune and a proper interpretation of resonance width and detuning strength, our one-dimensional results are also applicable in the coupling resonance case.

In Section 2, we discuss the trapping mechanism, while in Section 3, we present our beam loss model, and compare our predicted loss rates for the ISR with experimental observations. We find that they are in agreement. We conclude that a combination of intrabeam scattering and classical resonance theory provides an adequate explanation of beam loss rates at the ISR.

2. Particle Trapping in Resonance Passage

A particle passes through resonance when its linear tune passes through some rational fraction. As this occurs, the particle's phase-space topology is transformed. If the passage through resonance is in the proper direction, stable islands appear at the origin and begin to move outward to regions of larger betatron amplitude as the tune changes. As these islands move outward, they also grow in phase-space area. The moving, growing islands can trap particles in their migration. This therefore represents a particle transport mechanism. Note that under quasi-static (adiabatic) conditions, particles can stream into the stable islands through the regions around the unstable fixed points, since these are intrinsically nonadiabatic phase-space regions. If a particle is trapped in one of these islands, and if the island reaches a physical aperture, then the trapped particle will be deposited

*Work performed under the auspices of the U.S. Atomic Energy Commission.

on the boundary walls and be lost. This process can be formally described by a trapping efficiency, which is simply the fraction of particles trapped (i.e. capable of being transported to "large" amplitudes) during a single resonance passage.

The detailed trapping mechanism is clearly a complex one; thus, the trapping efficiency is a complicated function of the resonance crossing parameters. First, there is the speed of resonance crossing. In the limit of fast crossing, we expect no trapping. Now, this is not to say that in the fast crossing limit, there is no influence on the beam. However, the time scale for beam growth, in this case, is far different from what exists under conditions when trapping occurs. In any case, we ignore the effect on the "remaining" beam. In other words, we presume that the only significant effect of resonance crossing is trapping. The question we confine ourselves to is, then, what is the trapping efficiency as a function of crossing speed? A second characteristic which influences trapping is the beam distribution. In this regard one might ask, is the trapping probability dependent on the particle phase-space position at the time the islands pass? Are particles near the outside of the beam more easily trapped than those near the center? For our purposes here, the influence of beam distribution can be neglected. Finally, we must determine the dependence of the trapping efficiency on the two nonlinear parameters: the resonance width and the nonlinear detuning strength.

The evaluation of the trapping efficiency has been considered in detail elsewhere.⁷ The results of that analysis allows us to make substantial simplifications in order to obtain an expression for the trapping efficiency. The main point is that the trapping efficiency can be approximated by the area of the moving islands (relative to the beam area) near the exit of these islands from the beam. The crossing speed has been taken into account and enters through a multiplicative factor with an exponential fall-off. If we take for the island area the area at the amplitude corresponding to one rms beam emittance, ϵ_{rms} , we find for the trapping efficiency,

$$P_T = \frac{\sqrt{2}}{\sqrt{p}} \left| \frac{\Delta_e}{\Delta_{NL}} \right|^{\frac{1}{2}} e^{-\gamma} \quad (2.1)$$

where p is the order of the resonance, Δ_e is the resonance excitation width at $E = \epsilon_{rms}$, Δ_{NL} is the nonlinear detuning at $E = \epsilon_{rms}$, and γ is a function of the crossing speed given by

$$\gamma = \left[\frac{\delta v_{rev}}{4\pi \left| \frac{\Delta_e}{\Delta_{NL}} \right|} \right]^{2/p}, \quad (2.2)$$

with δv_{rev} the average tune change per particle revolution. It is demonstrated in Ref. (7), that this expression for the trapping efficiency is in remarkable agreement with a computer simulation of a 5th order resonance crossing.

3. Beam Loss Rate

To produce beam loss from the trapping mechanism described in the previous section, there must be a process which causes a particle's tune to vary with time. In a coasting beam, of course, there is no synchrotron motion. However, in a high intensity beam, scattering processes may be strong enough to produce significant momentum diffusion. Through the machine chromaticity, this momentum diffusion translates into tune diffusion. In applying our model to the ISR, we will use p-p intra-beam scattering, which is a substantial effect in the

ISR, as the diffusion source.^{1,5}

Let us assume a momentum diffusion process, with a diffusion coefficient

$$D_p = \frac{d}{dt} \left(\frac{\Delta p}{p} \right)_{rms}^2 \quad (3.1)$$

The tune diffusion arising from this can be described by

$$D = (\nu')^2 D_p, \quad (3.2)$$

where ν' , the change in ν per unit fractional momentum change, is proportional to the chromaticity. We consider the diffusion process as a simple random walk in tune. However, there is a boundary condition: If the particle is trapped by the stable outward moving islands, then at a certain tune distance from the resonance, say 6, the particle will be deposited at the aperture and be lost. If we consider a set of n particles at resonance at time $t = 0$, then some fraction of these will be lost. In fact, if the trapping efficiency is P_T , then the beam loss rate at time $t = T$ can be written

$$\frac{\dot{n}}{n} = - \frac{P_T \delta}{2T(\pi D T)^{\frac{1}{2}}} e^{-\delta^2/4D T} \quad (3.3)$$

For a beam with uniform density in tune, we can find the fractional loss rate at time T by integration (some care must be exercised in performing this integration since only particles which cross the resonance in the proper direction are to be included):

$$\frac{2P_T}{\Delta} \sqrt{\frac{D}{\pi T}} e^{-\delta^2/4D T} \quad (3.4)$$

where Δ is the total tune spread in the beam, and N is the total number of particles in the beam. Since this function has a maximum at

$$T = \frac{\delta^2}{2D}, \quad (3.5)$$

the maximum percentage loss rate is

$$\left(\frac{\dot{N}}{N} \right)_{\max} = - 0.97 \frac{P_T D}{\Delta \delta} \approx - \frac{P_T D}{\Delta \delta} \quad (3.6)$$

Note that the loss rate as a function of T falls off slowly, like $T^{-\frac{1}{2}}$ and so we can use this last expression as a measure of the average loss rate.

Thus, the beam loss rate is a simple function of 4 parameters: 1. The trapping efficiency, P_T ; 2. the tune diffusion coefficient, D ; 3. the total beam tune spread, Δ ; and 4. the change in tune as trapped particles move from the resonance to the aperture, 6. The quantity 6 is just the change in linear tune which causes the stable fixed points at the island centers to move from the origin of the betatron phase space to the aperture limit. It can be approximated by the nonlinear detuning at the aperture--that is,

$$\delta = \Delta_{NL} \frac{E}{\epsilon_{rms}} \quad (3.7)$$

where ϵ_{ap} is the emittance value corresponding to the aperture amplitude. The average tune change per revolution is determined by the diffusion process. The relation between the diffusion coefficient D and δv_{rev} is easily found to be

$$\delta v_{rev} = (2D T_{rev})^{\frac{1}{2}}, \quad (3.8)$$

where T_{rev} is the revolution period of the machine.

Let us now apply this theory to the ISR. We try to estimate the required parameters for a 5th order resonance. Using a head-on collision model and noting that the 5th order resonances are excited only if the various interactions are not identical, we estimate the resonance excitation and nonlinear detuning (at one rms beam width) to be⁴

$$\Delta_e = 0.1 \eta \Delta_{bb} ,$$

and

$$\Delta_{NL} = 3/8 \Delta_{bb} ,$$

where η is a factor (< 1) describing the departure from symmetry of the interaction points, and Δ_{bb} is the linear beam-beam tune shift. Typically, for the ISR, we can take $\Delta_{bb} = 3.5 \times 10^{-4}$, which leads to $\Delta_e = 3.5 \times 10^{-5} \eta$ and $\Delta_{NL} = 1.3 \times 10^{-4}$. Taking $D_v = 4 \times 10^{-11} \text{ sec}^{-1}$, we have $\delta v_{rev} = 1.5 \times 10^{-8}$. For 5% symmetry breaking, $\eta = .05$, $\Delta_e = 1.75 \times 10^{-5}$, which leads to $\gamma = 1.94$ and $P_T = 0.033$. If we assume the aperture limit to be 3 times the beam size, $\delta = 1.2 \times 10^{-3}$. Thus, if $\Delta = 0.01$, we have the loss rate $\dot{N}/N = \dot{I}/I = 0.7 \times 10^{-5} \text{ min}^{-1}$. If η is as high as 10%, the loss rate increases to $1.5 \times 10^{-5} \text{ min}^{-1}$. These loss rates are consistent with observations made at the ISR.¹

4. conclusions

We have proposed a model which can account for the beam loss rate observed at the ISR. The model is composed of a mechanism for beam loss due to a high-order isolated nonlinear resonance fed by a diffusion process, which was taken to be intrabeam scattering. It is perhaps important to emphasize that only the classical theory of nonlinear resonances was used.

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DISCUSSION

Rae Stiening (NAL): Have you studied more extreme cases? Where more of the beam goes away faster? In particular with regard to what the situation is with the remaining core of these dips.

Melvin Month (BNL): No, in fact, this calculation is only an onset calculation. It doesn't attempt to look at the beam distribution dynamically as the process is proceeding. In one of the slides I showed, there was the beginning of an attempt to see the effect on the distribution.

Andrei Kolomensky (Lebedev Institute): Besides this process of scattering of proton on proton, there can be other processes. Is there an effect of other random processes to shift the tune?

Month: I'm afraid I'm not familiar with detailed calculations. I've just used the results from the ISR reports, where it is claimed that the dominant process of this nature is the intra-beam scattering, and I think that's correct.

Kolomensky: About 10 years ago we tried to use stochastic modulation in the ν_r, ν_z plane to detune this resonance. But we got diffusion separation, and the harm was greater than the benefit. So we cancelled this work. But maybe, in principle, with a controlled stochastic mechanism one could counteract resonances. But in our machine in the USSR, this stochasticity did more harm than good.

Month: I think it's true here, too.

Andrew Sessler (LBL): In a bunched beam there will be a very systematic and large variation of the synchrotron motion. And I wonder if you have tried to calculate that, and put it into your theory because then you can make comparisons with experiments which have already been tried on the ISR. It would be a stringent test of your theory, and interesting as a predictive thing for other machines.

Month: No, I haven't tried that.

Mervyn Hine (CERN): There is another feeding mechanism, which I believe can contribute in some conditions. As you lose current, the space-charge v -shift changes, so the whole beam slips fairly sideways across resonances.

Month: That's certainly a possibility. I haven't looked at that.

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Summary

The excitation and damping of betatron oscillations and the energy spread due to intra-beam scattering is investigated. It is shown that below transition energy an equilibrium for the particle distribution exists which does not depend on the number of the particles. The rise times and damping times for betatron oscillations and energy spread are calculated. The investigation shows that this effect sets a limit to the intensity of stored proton beams at energies below a few GeV.

Introduction

The intra-beam scattering or multiple scattering, i.e. the scattering of the particles within the beam, was investigated in several reports¹⁻⁸). The result of these investigations showed an increase of all beam or bunch dimensions and an increase of the energy spread. But in these investigations two facts have been disregarded, the energy spread within the beam and the influence of the coordinates of the scattered particle on the scattering angle.

The energy spread in the beam, i.e. the energy difference of two colliding particles before the collision, leads to an excitation of betatron oscillations which is, for small energies, the main contribution to the increase of the beam dimensions.

The second point, the linear dependence of the change of the coordinates on these same coordinates, leads to a damping of the betatron oscillations and to a reduction of the energy spread. Without this damping one obtains an infinite increase of the beam dimensions and the energy spread. This seems to be impossible for particles below transition energy.

Far below transition energy the influence of the dispersion can be neglected as will be shown later, and the particles in the beam behave like the particles of a gas in a closed box. Here the focusing forces play the same role as the walls of the box. Since the collisions within a gas cannot lead to an increase of the temperature the collisions within the beam cannot, below transition, lead to an increase of the total oscillation energy and the energy spread. One can only expect a transfer of oscillation energy from one direction into another. Thus, there must exist an equilibrium distribution where the intra-beam scattering does not change the beam dimensions. It is shown that the equilibrium distribution is a Gaussian distribution.

Above transition energy the situation is changed by the property of the particles that is often characterized by the so called "negative mass" behaviour. Here the comparison with a gas in a closed box is not valid, and the calculation shows that the total oscillation energy can increase. The behaviour of the beam can be described with help of an invariant which is given by

$$\frac{1}{n} \left(\frac{1}{\gamma^2} - \alpha_M \right) \left\langle \frac{\Delta p}{p^2} \right\rangle + \langle x'^2 \rangle + \langle z'^2 \rangle = \text{const} \quad (1)$$

with

$p, \Delta p$ = momentum and momentum deviation, respectively
 x', z' = betatron angles for horizontal and vertical direction, respectively
 γ = particle energy divided by its rest energy
 α_M = momentum compaction factor
 $n = 1$ for bunched beams
 $n = 2$ for unbunched beams

The momentum compaction factor α_M is, in Eq.(1), precise only for a weak focusing machine. For a strong focusing machine α_M is a good approximation for the mean value of D^2/β^2 , where D denotes the dispersion and β_x the amplitude function.

If $1/\gamma^2 - \alpha_M$ is positive, i.e. below transition, the three mean values in Eq.(1) are limited. But for negative $1/\gamma^2 - \alpha_M$ the three mean values can increase so far as they do not exceed other limitations, and an equilibrium distribution does not exist.

In case of an equilibrium the three terms of the sum in Eq.(1) are equal and one can calculate the equilibrium dimensions starting from the initial dimensions. Eq.(1) further shows that the equilibrium dimensions do not depend on the number of particles, but only on the initial mean values. The number of particles determines the relaxation time for the equilibrium and the rise time for the increase above transition energy.

Kinematics

If s, x and z denote the longitudinal, horizontal and vertical coordinates, the momenta of two particles before a collision are given by (see fig.1)

$$\vec{p}_{1,2} = p_{1,2} \{ 1, x'_{1,2}, z'_{1,2} \} \quad (2)$$

$x'_{1,2}$ and $z'_{1,2}$ are the betatron angles, and we consider in this investigation only linear and quadratic terms of $x'_{1,2}$ and $z'_{1,2}$. We define a coordinate system $\{u, v, w\}$ with help of the unit vectors

$$\begin{aligned} \vec{e} &= (\vec{p}_2 + \vec{p}_1) / |\vec{p}_2 + \vec{p}_1|, \\ \vec{e}_v &= \vec{p}_1 \times \vec{p}_2 / |\vec{p}_1 \times \vec{p}_2| \\ \vec{e}_w &= \vec{e}_u \times \vec{e}_v \end{aligned} \quad (3)$$

The momenta can then be written in the form

$$\vec{p}_{1,2} = p_{1,2} \{ \cos \alpha_{1,2}, 0, \pm \sin \alpha_{1,2} \}_{u,v,w} \quad (4)$$

The angles α_1 and α_2 are defined by

$$p_1 \sin \alpha_1 = p_2 \sin \alpha_2 \quad (5)$$

and

$$\alpha_1 + \alpha_2 = 2\alpha = \sqrt{(x'_1 - x'_2)^2 + (z'_1 - z'_2)^2} \quad (6)$$

A Lorentz transformation parallel to the u -axis gives the representation of the momenta in the center of

mass system

$$\vec{p}_{1,2} = \pm \{ \vec{p}_u, 0, \vec{p}_w \} \vec{u}, \vec{v}, \vec{w}$$

$$= p_{1,2} \{ \gamma_{tr} (\cos \alpha_{1,2} - \beta_{tr} / \beta_{1,2}), 0, \pm \sin \alpha_{1,2} \} \vec{u}, \vec{v}, \vec{w} \quad (7)$$

where β_{tr} and γ_{tr} are defined by

$$\beta_{tr} = \frac{\beta_1 \gamma_1 \cos \alpha_1 + \beta_2 \gamma_2 \cos \alpha_2}{\gamma_1 + \gamma_2} \quad (8)$$

$$\gamma_{tr} = \frac{1}{\sqrt{1 - \beta_{tr}^2}}$$

The bar denotes all quantities in the center of mass system. After the collision the momenta are changed by the polar angle $\bar{\psi}$ and the azimuthal angle $\bar{\phi}$, and the momenta are

$$\vec{p}'_{1,2} = \pm \{ \vec{p}_w \sin \bar{\psi} \cos \bar{\phi} + \vec{p}_- \cos \bar{\psi}, \vec{p}_- \sin \bar{\psi} \sin \bar{\phi}, \vec{p}_w \cos \bar{\psi} - \vec{p}_- \sin \bar{\psi} \cos \bar{\phi} \} \vec{u}, \vec{v}, \vec{w} \quad (9)$$

where the prime denotes all quantities after the collision. The inverse Lorentz transformation gives the momenta in the $\{u, v, w\}$ system, and the change of the momenta has the form

$$\vec{p}'_{1,2} - \vec{p}_{1,2} = p_{1,2} \{ \gamma_{tr} (\pm \sin \bar{\psi} \cos \bar{\phi} \sin \alpha_{1,2} + (\cos \bar{\psi} - 1) \gamma_{tr} (\cos \alpha_{1,2} - \beta_{tr} / \beta_{1,2})) \pm \sin \bar{\psi} \sin \bar{\phi} / p_{1,2}, -\sin \bar{\psi} \cos \bar{\phi} \gamma_{tr} (\cos \alpha_{1,2} - \beta_{tr} / \beta_{1,2}) \pm (\cos \bar{\psi} - 1) \sin \alpha_{1,2} \} \vec{u}, \vec{v}, \vec{w}$$

We now assume that the velocity of the particles in the center of mass system is non relativistic. Then the relation

$$\beta^2 = \frac{1}{4} \beta^2 \gamma^2 ((\eta_1 - \eta_2)^2 / \gamma^2 + (x'_1 - x'_2)^2 + (z'_1 - z'_2)^2) \ll 1$$

with

$$\eta_{1,2} = \frac{\Delta p_{1,2}}{p}$$

is satisfied, and one obtains

$$\gamma_{tr} = \gamma, \quad \cos \alpha_{1,2} - \beta_{tr} / \beta_{1,2} = \pm \frac{\eta_1 - \eta_2}{2\gamma^2}$$

The change of the momenta can now be written in the form

$$\vec{p}'_{1,2} - \vec{p}_{1,2} = \pm \frac{p}{2} \{ 2\alpha \gamma \sin \bar{\psi} \cos \bar{\phi} + \gamma \xi (\cos \bar{\psi} - 1), (\xi \sqrt{1 + \xi^2 / \alpha^2 / 4} \sin \bar{\phi} + \xi \theta / \alpha / 2 \cos \bar{\phi}) \sin \bar{\psi} + \theta (\cos \bar{\psi} - 1), (-\theta \sqrt{1 + \xi^2 / \alpha^2 / 4} \sin \bar{\phi} + \xi \zeta / \alpha / 2 \cos \bar{\phi}) \sin \bar{\psi} + \zeta (\cos \bar{\psi} - 1) \} \vec{s}, \vec{x}, \vec{z}$$

with

$$(\eta_1 - \eta_2) / \gamma = \xi, \quad x'_1 - x'_2 = \theta, \quad z'_1 - z'_2 = \zeta$$

Change of the emittances and the momentum spread

The emittance ϵ is defined by

$$\beta_x \epsilon_x = x_x^2 + \beta_x^2 x_x'^2$$

where we have neglected β_x' . The change of the emittance due to a scattering event is

$$\beta_x \delta \epsilon_x = 2x_x \beta_x \delta x_x + \delta^2 x_x + 2\beta_x^2 x_x' \delta x_x' + \beta_x^2 \delta^2 x_x'$$

$$= -2x_x \beta_x D \frac{\delta p}{p} + D^2 \frac{\delta^2 p}{p^2} + 2\beta_x^2 x_x' \frac{\delta p_x}{p} + \beta_x^2 \frac{\delta^2 p_x}{p^2} \quad (11)$$

For the vertical oscillations one obtains

$$\beta_z \delta \epsilon_z = 2\beta_z^2 z_z' \frac{\delta p_z}{p} + \beta_z^2 \frac{\delta^2 p_z}{p^2} \quad (12)$$

For the momentum spread one can define the following invariant

$$H = \begin{matrix} \eta^2 + \hbar^2 / \Omega^2 & \text{for a bunched beam} \\ \eta^2 & \text{for an unbunched beam} \end{matrix}$$

where Ω is the synchrotron frequency and η the relative momentum deviation. The change of H due to a scattering event is in both cases

$$\delta H = 2\eta \frac{\delta p}{p} + \frac{\delta^2 p}{p^2} \quad (13)$$

Determination of an invariant

Since the particle velocity should be non relativistic in the center of mass system we may employ the Rutherford cross section which has the form

$$d\sigma = \left(\frac{r_0}{4\beta^2 \sin^2 \bar{\psi} / 2} \right)^2 \sin \bar{\psi} d\bar{\psi} d\bar{\phi} \quad (14)$$

with r_0 = classical particle radius.

The impact parameter for the smallest scattering angle $\bar{\psi}_m$ is given by

$$\tan \frac{\bar{\psi}_m}{2} = \frac{r_0}{2\beta^2 b} \quad (15)$$

We assume $\bar{b}$ to be equal one half of the average distance between the particles in the center of mass system

$$\bar{b} = \frac{1}{2} (\gamma / \rho)^{1/3} \quad (16)$$

To calculate the mean value of the change of the emittance or the momentum deviation for one particle we have to average with respect to all betatron angles and momentum deviations of the second particle. For the total mean value of the change of the emittance and momentum deviations of all particles we have to average additionally with respect to all betatron angles, momentum deviations and positions of the first particle. Thus, we have to integrate with the following density function

$$P = \rho_s^2(s) \rho_{x\beta}(x_{\beta 1}) \rho_{x\beta}(x_{\beta 2}) \rho_z^2(z) \rho_\eta(\eta_1) \rho_\eta(\eta_2) \cdot \rho_x(x'_1) \rho_x(x'_2) \rho_z(z'_1) \rho_z(z'_2)$$

since

$$s_1 = s_2 = s, \quad z_1 = z_2 = z$$

but

$$x_{1\beta} + \eta_1 D = x_{2\beta} - \eta_2 D$$

The functions ρ_i are normalized to one. We replace the variables $x_{\beta 1,2}, \eta_{1,2}, x'_{1,2}, z'_{1,2}$ by the variables $x_{\beta}, \eta, x', \xi, 0, \zeta$ with help of the relations

$$x_{\beta 1,2} = x_{\beta} \mp D\gamma\xi/2, \quad \eta_{1,2} = \eta \pm \gamma\xi/2$$

$$x'_{1,2} = x' \pm \theta/2, \quad z'_{1,2} = z' \pm \zeta/2$$

The function P is now symmetric with respect to ξ, θ and ζ . The relative velocity between two colliding particles in the center of mass system is $2c\beta$. The mean value of the change of the emittances and the momentum deviation per unit time is then given by

$$\int_{-\psi_m}^{\psi_m} 2c\beta P \int_0^{\pi} \int_0^{2\pi} \frac{d\sigma}{d\omega} \begin{pmatrix} \delta H_1 / \gamma^2 \\ \delta \epsilon_{x1} / \beta_x \\ \delta \epsilon_{z1} / \beta_z \end{pmatrix} \sin \bar{\psi} d\bar{\phi} d\bar{\psi} d\tau =$$

$$= \frac{\pi}{2} c r_o^2 \gamma \int P \begin{pmatrix} \theta^2 + \zeta^2 - 2\xi^2 \\ \xi^2 + \zeta^2 - 2\theta^2 + \frac{D^2\gamma^2}{\beta_x^2} (\theta^2 + \zeta^2 - 2\xi^2) \\ \xi^2 + \theta^2 - 2\zeta^2 \end{pmatrix}$$

$$\ln\left(\frac{2\beta c b}{\dots}\right) \frac{d\tau}{\dots}$$

$$d\tau = ds dx dz d\eta dx' dz' d\xi d\theta d\zeta$$

$$4\beta^4 b^2 / r^2 \gg 1$$

From Eq. (17) we get

$$\int_{-\psi_m}^{\psi_m} \int_0^{\pi} \int_0^{2\pi} P \left(\frac{1}{\gamma^2} - \frac{D^2}{\beta_x^2} \right) \delta H_1 + \frac{\delta \epsilon_{x1}}{\beta_x} + \frac{\delta \epsilon_{z1}}{\beta_z} \frac{d\sigma}{d\omega} \bar{\psi} \sin \bar{\psi} d\bar{\phi} d\bar{\psi} d\tau = 0$$

and

$$\left(\frac{1}{\gamma^2} - \frac{D^2}{\beta_x^2} \right) \langle H \rangle + \langle \epsilon_x \rangle / \beta_x + \langle \epsilon_z \rangle / \beta_z = \text{const.}$$

where the brackets denote the average with respect to all particles of the beam. With the relations

$$\langle H \rangle = \begin{cases} 2 \frac{\Delta^2 p}{p^2} & \text{for bunched beams} \\ \frac{\Delta^2 p}{p^2} & \text{for unbunched beams} \end{cases}$$

$$\langle \epsilon_x / \beta_x \rangle = 2 \langle x'^2 \rangle, \quad \langle \epsilon_z / \beta_z \rangle = 2 \langle z'^2 \rangle \quad (19)$$

we obtain finally for the invariant the expression

$$\frac{1}{n} \left(\frac{1}{\gamma^2} - \frac{D^2}{\beta_x^2} \right) \frac{\Delta^2 p}{p^2} \langle x'^2 \rangle + \langle z'^2 \rangle = \text{const.}$$

Rise times and damping times

For the calculation of the rise time and damping time we assume a Gaussian distribution for x_{β}, z, η, x' and z' . The distribution in longitudinal direction may be uniform or Gaussian-like. With Eq. (17) one gets for the change of the mean value of the emittances and the momentum spread per unit time

$$\begin{pmatrix} \frac{d}{dt} \langle H \rangle / \gamma^2 \\ \frac{d}{dt} \langle \epsilon_x \rangle / \beta_x \\ \frac{d}{dt} \langle \epsilon_z \rangle / \beta_z \end{pmatrix} = 2c \int_{-\psi_m}^{\psi_m} P \int_0^{\pi} \int_0^{2\pi} \bar{\beta} \frac{d\sigma}{d\omega} \begin{pmatrix} \delta H_1 / \gamma^2 \\ \delta \epsilon_{x1} / \beta_x \\ \delta \epsilon_{z1} / \beta_z \end{pmatrix} \sin \bar{\psi} d\bar{\phi} d\bar{\psi} d\tau$$

$$= 2A \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left\{ -\frac{\xi^2 \gamma^2}{4} \left(\frac{D^2}{\sigma_{x\beta}^2} + \frac{1}{\sigma_{\eta}^2} \right) - \frac{\theta^2}{4\sigma_{x'}^2} - \frac{\zeta^2}{4\sigma_{z'}^2} \right\} \cdot \begin{pmatrix} \theta^2 + \zeta^2 - 2\xi^2 \\ \xi^2 + \zeta^2 - 2\theta^2 + \frac{D^2\gamma^2}{\beta_x^2} (\theta^2 + \zeta^2 - 2\xi^2) \\ \xi^2 + \theta^2 - 2\zeta^2 \end{pmatrix} \cdot \ln \left(\frac{q^2}{4} (\xi^2 + \theta^2 + \zeta^2) \right) \frac{d\xi d\theta d\zeta}{(\xi^2 + \theta^2 + \zeta^2)^{3/2}}$$

with

$$A = \frac{c r_o^2 \Lambda}{16\pi\sqrt{\pi} \sigma_{x\beta} \sigma_z \sigma_{\eta} \sigma_{x'} \sigma_{z'} \beta^3 \gamma^4}$$

$$\Lambda = \begin{cases} N_b & \text{for bunched beams} \\ 2\sqrt{\pi} \sigma_s & \text{for unbunched beams} \end{cases}$$

$\sigma_{s,x,z}$ = standard deviations for longitudinal, horizontal and vertical bunch dimensions

N_b, N = number of particles in a bunch and in the beam, respectively

C = circumference, $c\beta$ = particle velocity

With Eq. (19) one obtains

$$\frac{1}{\tau_{\eta}} = \frac{1}{2\sigma_{\eta}^2} \frac{d\sigma_{\eta}^2}{dt} = nA \left(1 - \frac{\eta}{\sigma_{\eta}} \right) f\left(\frac{y}{\sigma_{x'}}, \frac{y}{\sigma_{z'}}, q\sigma_y\right)$$

$$\frac{1}{\tau_{x'}} = \frac{1}{2\sigma_{x'}^2} \frac{d\sigma_{x'}^2}{dt} = A \left(f\left(\frac{\sigma_{x'}}{\sigma_y}, \frac{\sigma_{x'}}{\sigma_{z'}}, q\sigma_{x'}\right) + \frac{D^2\sigma_{\eta}^2}{\sigma_{x'}^2} f\left(\frac{\sigma_y}{\sigma_{x'}}, \frac{\sigma_y}{\sigma_{z'}}, q\sigma_y\right) \right)$$

$$\frac{1}{\tau_{z'}} = \frac{1}{2\sigma_{z'}^2} \frac{d\sigma_{z'}^2}{dt} = A f\left(\frac{\sigma_{z'}}{\sigma_y}, \frac{\sigma_{z'}}{\sigma_{x'}}, q\sigma_{z'}\right)$$

with

$$\sigma_y = \frac{\sigma_{\eta} \sigma_{x\beta}}{\gamma \sigma_x}, \quad q = \beta \gamma \sqrt{\frac{b}{r_o}}$$

The function $f(a, b, c)$ is defined by

$$f(a, b, c) = 2 \int_0^{\pi} \int_0^{\pi} \int_0^{\pi} \exp \left\{ -\rho (\cos^2 \mu + (a^2 \cos^2 \nu + b^2 \sin^2 \nu) \cdot \sin^2 \mu) \right\} \cdot \ln(c^2 \rho) (1 - 3 \cos^2 \mu) \sin \mu d\nu d\mu d\rho \quad (21)$$

f cannot be evaluated analytically. It is calculated by a computer program and plotted in fig. 2 and 3.

To reduce the range of numerical values for f one can use the relations

$$f(a, b, c) = f(b, a, c)$$

$$f(a, b, c) + \frac{1}{a^2} f\left(\frac{1}{a}, \frac{b}{a}, \frac{c}{a}\right) + \frac{1}{b^2} f\left(\frac{1}{b}, \frac{a}{b}, \frac{c}{b}\right) = 0$$

One can proof that

$$f(1, 1, c) = 0$$

Thus, with

$$\sigma_{\eta} \sqrt{\frac{1}{\gamma^2} - \frac{D^2}{\beta_x^2}} = \sigma_{x'} = \sigma_{z'}$$

an equilibrium is reached and the Gaussian distribution

remains stable, This equilibrium can only be reached for

$$\gamma < \left(\frac{D^2}{\beta_x^2} \right)^{-1/2} \approx \frac{1}{\sqrt{\alpha_M}}$$

i.e. below transition energy.

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Fig. 1

DISCUSSION

Andrew Sessler (LBL): Could you remind me how the rise time goes with γ , as one goes to higher energy.

Anton Piwinski (DESY): The risetimes are proportional to $1/A$, and A is proportional to $1/\gamma^4$, so the risetimes are proportional to γ^4 . But, there is also a $\nu\gamma$ weaker γ dependence in the function f .

Andrei Kolomensky (Lebedev Institute): Are the risetimes independent or are they connected by some rule?

Piwinski: There is a linear relation between $1/\tau_n$ and $1/\tau_z$, but multiplied with certain factors.

Wolfgang K. H. Panofsky (SLAC): How is the numerical agreement with the 2 GeV tests made in the ISR?

Piwinski: Buchner has made an estimate and calculation, and he found that the values that come from measurement have the right order of magnitude, but I don't remember the exact measurement.

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1. Abstract and Introduction

In the normal operating conditions of ACO the two beam behaviour is dominated by the beam-beam interaction. Single beam behaviour in the range of energy and current described below ($250 \leq E \leq 510$ MeV, $I \leq 35$ mA) exhibits no effect such as resonances, beam losses, etc..., in comparison with the ones observed with two beams. In this clear situation a detailed study of the beam-beam interaction has been performed on ACO in 1973.

The normal operating conditions are : one bunch per beam, two crossing points and head-on collisions. Radial and vertical wave numbers per turn are set just below an integer, near the coupling resonance $\nu_x - \nu_z = 2$ (usually $\nu_z \sim .835$, $\nu_x \sim 2.845$). Table 1 shows some relevant ring parameters at the crossing points.

The main result obtained is the important part played by high order non-linear resonances in the beam loss induced by the beam-beam interaction. This appears in the study of the beam-beam limit as a function of the operating point in the wave number diagram.

Other results are reported on :

- The tune shift parameter ξ at the beam-beam limit.
- The influence of the number of bunches.
- The absence of any difference between the interaction of two strong beams versus the strong beam-weak beam interaction.
- Coherent motion induced by small vertical separations at the crossing points.

More details can be found in recent ACO reports'.

Betatron amplitude function β^*	radial	~ 2 m
	vertical	~ 4 m
Off-momentum function η^*		~ 2 mm
Transverse beam dimensions	radial σ_x^*	$\sim .5$ mm
	vertical σ_z^*	$\sim .5$ mm
Longitudinal beam dimension	σ_L	~ 15 cm
Single beam lifetime at 30 mA	τ	~ 35 h

Table 1 : Ring parameters at $E = 510$ MeV and

$$V_{RF} = 17.5 \text{ kV.}$$

2. Operating points and non-linear resonances

It has been studied how the operating point can be moved in the ν_x, ν_z plane as a function of the intensity for two equal beams at 510 MeV.

The ν_x, ν_z diagram has been explored in the region^t

$$.67 < \nu_z < .90 \quad 2.67 < \nu_x < 2.90$$

According to the intensity, three cases can be distinguished :

2.1 LOW current : $I^+ \sim I^- < 5$ mA

Whatever the tune, the beam-beam interaction does not affect the beam lifetime. Away from the coupling resonance $\nu_x - \nu_z = 2$, many beam transverse enlargements are observed, indicating the excitation of non-linear resonances. No enlargement is observed with only one single beam stored in the ring. Hence we may infer that these resonances are excited by the beam-beam interaction. Between two well separated non-linear resonances, the beams are flat, but the beam-beam limit (very short lifetime) is already reached at $I \sim 5$ mA.

2.2 High current : $I^+ \sim I^- > 10$ mA

Using a lifetime criterion ($\tau > 1$ h), the possible tunes are found to be restricted to small areas along the coupling resonance $\nu_x - \nu_z = 2$. Figure 1 shows 4 different areas which were specially investigated. They are separated by stopbands near rational numbers p/q . Other areas between rationals may exist, but were not searched for.

Fig. 1 : ν_x, ν_z diagram

Studied operating points are indicated by small circles.

Rational values of ν_x and ν_z are indicated by vertical and horizontal lines.

The number between parenthesis gives the order $2n$ of the non-linear resonances associated to the rational number m/n .

^tThe values ν_x, ν_z are determined by the quadrupole settings, calibrated with a single beam stored in the ring. This calibration does not take into account the small perturbation of wave numbers on the coupling resonance.

The existence of such stopbands can be attributed to the influence of non-linear resonances in the beam loss mechanism induced by the beam-beam interaction.

High order resonances seem to be involved. For instance the stopband near 5/6 is related to 12th order resonances in the case of two crossing points per turn*. Many 12th order resonances cross at $\nu_x = 2+5/6$, $\nu_z = 5/6$. The stopband may be due to a single or a cluster of such resonances. There is no experimental answer to this question.

2.3 Medium current : $5 \text{ mA} < I^+ \sim I^- < 10 \text{ mA}$

The size of the area is larger than for higher currents, but still restricted to the neighbourhood of the coupling resonance (Figure 2). Beam enlargements appear when ν_z is approaching a rational value. Sometimes these resonances can be crossed, by varying ν_z , without beam loss.

Fig. 2 : Range of ν versus the radial wave number ν .

Enlargement of one beam near a resonance is indicated by R or R.

Low order rational values $\nu = m/n$ are indicated by horizontal dotted lines.

(x : limit of the ν_z range when the lifetime drops)

*Here the order of a resonance is defined as the smallest degree of the terms which excite this resonance in the Taylor expansion of the space charge potential. The wave number $\nu = k \times p/q$ (k : the number of crossing points, p and q : relatively prime integers) corresponds to a resonance of the order either q (q even) or $2q$ (q odd), since the potential parity is even. For $k = 2$, and writing $\nu = m/n$, the resonance order is $2n$.

For instance, at $I^+ \sim I^- \sim 7 \text{ mA}$, three groups of resonances have been observed (Figure 2), corresponding to $\nu_z = 6/7, 5/6$ and $4/5$. One may guess that the strength of these resonances is too low to reduce the lifetime. For higher current ($I > 10 \text{ mA}$), their strength increases and the lifetime is reduced before a beam enlargement appears.

At high current, the boundary of the area available for an operating point is characterized by a sharp decrease of the beam lifetime (from a few hours to much less than one hour). For a fixed value of ν_x , the beam lifetime remains almost constant as ν_z is varied, and a sharp decrease is observed when the boundary is reached.

The size of the ν_x, ν_z area decreases when the current is increased (Figure 3). At the same time, its center shifts below the coupling resonance.

Fig. 3 : Range of ν versus the radial wave number ν for different intensities.

The coupling resonance $\nu_x - \nu_z = 2$ is indicated by a dotted line. x

Enlargement of one beam at an end point of the ν_z range is indicated by R.

Figure 4 shows how the limits of the ν_z range vary with the intensity I for three different operating points. The lower bound $\nu_{z \text{ min}}$ is almost independent of I , but the upper bound $\nu_{z \text{ max}}$ decreases as I increases.

Fig. 4 : Limits of ν_z range versus the intensity of each beam and for 3 values of ν_x .

Low order rational values $\nu = m/n$ are indicated by horizontal dotted lines.

x : one bunch per beam.
o : two bunches per beam.

The $\nu_{z \text{ min}}$, $\nu_{z \text{ max}}$ bounds, extrapolated to zero current, are quite close to rationals, as shown in table 2. The same behaviour has been found for the ν_x bounds.

ν_x	$\nu_{z \text{ min}}$	$\nu_{z \text{ max}}$
2.162	.752 $\sim 3/4$	.774 $\sim 7/9$
2.817	.798 $\sim 4/5$	.830 $\sim 5/6$
2.845	.829 $\sim 5/6$	.848 $\sim 6/7$

max,

All these experimental results can be summarized as follows :

- At high intensity, operating points are separated by stopbands near rational numbers.
- Beam enlargements appear at low and medium current near such values.
- The $I = 0$ extrapolated values of the ν_x, ν_z bounds are also close to rational numbers.

The excitation of high order non-linear resonances by the beam-beam interaction may explain such a behaviour.

The beam-beam limit is said to be reached at an intensity for which the beam lifetime is less than one hour, inside the available area of the operating point. At this limit, the ν_x or ν_z range is about $3 \cdot 10^{-3}$.

Since the operating point is set below an odd integer, and the beam-beam interaction shifts the tune upwards, it is currently believed that the half integer resonance is responsible of the beam-beam limit observed on ACO^{2,3}. The experimental result reported here is not inconsistent with this idea, but it shows that the limit is also due to the excitation of non-linear resonances.

At 510 MeV the intensity limit, and the maximum luminosity, are about the same for the three operating points studied : $\nu_x = 2.845, 2.817$ and 2.762 .

$$I_{\text{max}} \sim 35 - 40 \text{ mA}$$

$$L_{\text{max}} \sim 10^{29} \text{ cm}^{-2} \text{ s}^{-1}.$$

These maximum values scale with the energy like :

$$I_{\text{max}} \sim E^{3.5 \pm .2} \quad L_{\text{max}} \sim E^{5.4 \pm .6}$$

in the ACO energy range : 250-540 MeV. The L_{max} scaling law appears to be inconsistent in the cubic law ($L_{\text{max}} \sim E^3$) predicted by a simple model⁴. It is worth point out that the measured beam cross section varies as $E^{2. \pm .2}$.

3. Tune shift parameter

The tune shift parameter :

$$\xi_{x,z} = \frac{Nr_e}{2\pi\gamma (\sigma_x^* + \sigma_z^*)} \times \left(\frac{\beta}{\sigma_{x,z}} \right)^*$$

(N is the number of particles per bunch, and γ the ratio of the particle energy to their rest mass energy) has been derived from the luminosity measurements with a double Bremsstrahlung monitor.

At 510 MeV, and at the two operating points $\nu_x = 2.817$ and 2.845 , the maximum ξ values are :

$$\xi_{x \text{ max}} = .021 \quad \xi_{z \text{ max}} = .030$$

Systematic errors on these values, due to biases on the intensity and luminosity measurements, are less than 30 %.

These values disagree with former values derived from transverse beam dimensions measured inside a bending magnet : $\xi_{x \text{ max}} \approx .03$, $\xi_z \approx .055$.

It is believed that the new measurements, using a luminosity monitor, are more reliable. A bias on the former values could arise from the correcting factor

relating the measured dimensions to the actual beam dimensions at the crossing point. The factor used does not account for optical perturbations due to the beam-beam interaction.

From the energy law of the maximum intensity and luminosity, a slight dependance of $\xi_{\max}$ is deduced :

$$\xi_{\max} \sim E^{1/2}.$$

The exponent is smaller than the one observed at ADONE.

4. Beam-beam interaction with two bunches per beam

Assuming a constant $\xi_{\max}$, the maximum luminosity should be proportional to the number of bunches. Already at the first stages of ACO⁵, it has been observed that the maximum luminosity is the same with two bunches per beam, at the usual operating point $\nu_x = 2.845$.

This result has been confirmed at the other operating point $\nu_x = 2.817$. More precisely the ν_z range as a function of the intensity is about the same with one or two bunches per beam (Figure 4).

5. Strong beam-weak beam interaction

Simple models for the beam-beam interaction study the motion of one particle crossing an opposite beam. Such models are only relevant for strong beam-weak beam interaction.

Since storage rings operate with two equal beams, it is important to check if there is any qualitative difference.

The strong beam-weak beam interaction has been studied at the usual operating point $\nu_x = 2.845$, and at 510 MeV. The weak electron beam intensity was about 1/20 of the strong positron beam intensity. No qualitative difference has been observed.

The ν_x, ν_z range, and its variation with the strong beam intensity, are about the same than those obtained with two equal beams. In particular the conclusion about the excitation of non-linear resonances is the same.

6. Coherent motion induced by the beam-beam interaction

In the head-on collision mode, only a very small coherent motion of the beams can be observed (about 3 % of $\sigma_{x,z}$) whatever the operating conditions⁶. The coherent frequency is approaching the revolution frequency at the beam-beam limit, but with no amplitude increase. Therefore it is difficult to imagine how such a small amplitude motion can participate to the beam loss mechanism.

When the two beams are slightly separated in the vertical direction at the crossing points, the situation is quite different.

A large amplitude (about $\sigma_{x,z}$) coherent motion appears in a small range of ν_z , when the beam separation reaches a threshold around $\frac{\sigma}{20}$. At high intensity (above 20 mA) this coherent motion leads to a poor lifetime*.

It is worth to try to relate the observation of the difficulties encountered in various rings with beams crossing at angle'. Obviously a longitudinal displacement of the crossing point leads to a small separation, which in itself has already a bad effect, as observed on ACO.

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*Preliminary results, published at the San Francisco Conference⁷ must be corrected on this point.

DISCUSSION

Sergio Tazzari (Frascati): What parameters did you measure with the strong beam-weak beam?

J. Buon (Orsay): We have measured the ν_x, ν_z areas.

Alberto Renieri (Frascati): What is the threshold for radial coherent motion of separated beams?

Buon: There is a sharp threshold, true, but the result at the moment is not very reliable. But it's quite small, something like the transverse dimension divided by 10.

Gerhard Fischer (SLAC): Is the value of ξ that you quote per interaction region or total?

Buon: It's per interaction region.

A. Renieri

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Abstract.

The motion of a single particle colliding with a strong beam in a storage ring is studied by computer simulation, under the assumptions that there is no radiation noise and no damping and that the strong beam has a gaussian transverse charge distribution. The phase space regions of stochastic motion and the corresponding minimum ΔQ values are determined. There is evidence that the motion stochasticity is due to overlapping of neighbouring resonances (Chirikov criterion).

1. - Introduction.

The stochastic phase space behaviour of many one-dimensional non-linear conservative dynamic systems has been extensively studied by I. Gumowski and C. Mira^{1,2,3}, E. Keil^{4,5}, using the resonance overlapping Zaslavskii-Chirikov⁶ criterion, evaluated the transverse amplitude and ΔQ limit for beam-beam non linear space charge effect.

In this paper we investigate the one-dimensional transverse phase space motion of a single particle colliding with a strong beam in a storage ring. The ΔQ threshold for the stochastic motion and the stochastic phase space regions are found using the properties of the phase space periodic points (cycles). The investigation is performed under the following conditions,

- head-on collision,
- strong beam shorter than β value at crossing point,
- the space charge forces from the strong beam are simulated by δ -function-like kicks given to the weak beam particle (point-like strong beam),
- no radiation noise and no damping,
- off energy closed orbit $\eta = 0$ at interaction point,
- no chromaticity,
- gaussian charge distribution function of the strong beam

$$f(x, z) = \frac{1}{2\pi a^2} e^{-\frac{(x^2+z^2)}{2a^2}} \quad (1)$$

- only one transverse mode excited.

We will use the following notations,

- β = betatron δ -value at crossing point,
- μ = betatron phase difference between two neighbouring crossing points,
- x = transverse displacement,
- $x' = dx/dS$, where S is the machine longitudinal coordinate,
- $\theta(x)$ = transverse kick experienced by the particle with x transverse displacement.

For the strong beam distribution function (1), $\theta(x)$ is given by the eq.

$$\theta(x) = -\frac{r_0 N}{\gamma \sigma^2} \left\{ x \left(\frac{1-e^{-\frac{x^2}{2\sigma^2}}}{\frac{x^2}{2\sigma^2}} \right) \right\} \quad (2)$$

Where r_0 is the particle classical radius, γ is the ratio between the total and the rest energy, N is the number of particles of the strong beam. Note that all computations have been carried out with at least 14 digits.

2. - Motion equations.

We write the motion equations as recurrences between the middle of one crossing and the middle of the next one. We assume that the machine is perfect, and that all crossing points are identical. We have,

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{n+1} = \begin{pmatrix} \cos \mu & \beta \sin \mu \\ -\frac{1}{\beta} \sin \mu & \cos \mu \end{pmatrix} \begin{pmatrix} x \\ x' + \frac{1}{2} \theta(x) \end{pmatrix}_n + \frac{1}{2} \begin{pmatrix} 0 \\ \theta(x) \end{pmatrix}_{n+1} \quad (3)$$

where n labels the crossing.

If we put,

$$F(y) = \frac{1-e^{-\frac{y^2}{2a^2}}}{\frac{y^2}{2a^2}}$$

equations (3) become (see eq. (2)),

$$\begin{aligned} x_{n+1} &= x_n \cos \mu + \beta x'_n \sin \mu - 2\pi \xi F\left(\frac{x_n^2}{2a^2}\right) \sin \mu x_n \\ x'_{n+1} &= -\frac{x_n}{\beta} \sin \mu + x'_n \cos \mu - \\ &\quad - 2\pi \xi F\left(\frac{x_n^2}{2a^2}\right) \cos \mu x_n - 2\pi \xi x_{n+1} F\left(\frac{x_{n+1}^2}{2a^2}\right) \end{aligned} \quad (4)$$

ξ is related to the linear ΔQ shift by the eq.,

$$\cos(\mu + 2\pi \Delta Q) = \cos \mu - 2\pi \xi \sin \mu$$

3. -Phase space variables.

The recurrence equations (4) can be derived

from the Hamiltonian,

$$H(q, p) = \frac{1}{2} \left(p^2 + \left(\frac{\mu}{L} \right)^2 q^2 \right) + \frac{4\pi\xi\mu\sigma^2}{L} G\left(\frac{q^2}{2\sigma^2}\right) + \sum_{n=-\infty}^{\infty} \delta(s-nL) \quad (5)$$

Where we have put,

$$q = x, \quad p = \frac{\mu}{L} x'$$

L = longitudinal distance between two neighbouring crossing points.

$\delta(s)$ = Dirac function

$$G(x) = \int_0^x F(y) dy$$

In order to obtain variables more manageable than (q, p) , let us define,

$$H_0(q, p) = \frac{1}{L} \int_0^L H(q, p|s) ds = \frac{1}{2} p^2 + \frac{\mu^2 \sigma^2}{L^2} V\left(\frac{q^2}{2\sigma^2}\right) \quad (6)$$

$$(V(x) = x + \frac{2\xi}{\mu} G(x)).$$

From this Hamiltonian we derive two new variables (T, W) that will be used in describing the motion derived from the complete time-dependent Hamiltonian (5). We define,

$$T(q, p) = \frac{1}{\mu} \int_{\lambda_M(q, p)}^{\lambda_M(q, p)} \frac{d\lambda}{\sqrt{V(\lambda_M^2(q, p)) - V(\lambda^2)}} \quad (7)$$

$$W(q, p) = \frac{|p|}{p} \frac{2\pi}{T(q, p)} \int_{q/\sqrt{2}\sigma}^{\lambda_M(q, p)} \frac{d\lambda}{\sqrt{V(\lambda_M^2(q, p)) - V(\lambda^2)}} \quad (8)$$

where $\lambda_M(q, p)$ is the solution of the equation,

$$V(\lambda_M^2(q, p)) = \left(\frac{L}{\mu\sigma} \right) H_0(q, p) \quad (9)$$

The physical meaning of T , λ_M and W is obvious. T is the motion period, in units of L , $\lambda_M = A/\sqrt{2}\sigma$ where A is the maximum q value, W is the phase conjugate variable. The solutions of the dynamic equations derived from the time-independent Hamiltonian H_0 are,

$$T(S) = T(0) \quad (10)$$

$$W(S) = W(0) - \frac{2n}{T(0)} - \left(\frac{S}{L} \right) \pmod{2\pi}$$

In the following we restrict our investigation to the phase space strip enclosed between $W=0$ and $W=\pi/2$.

This is allowed by the symmetry of recurrence equations (4) with respect to the x and x' axis, that is, in terms of (T, W) , with respect to $W=0, \pi/2, \pi, 2\pi$.

4. - Cycles and separatrices.

With the new (T, W) variables, the recurrence equations (4) become,

$$T_{n+1} = A(T_n, W_n)$$

$$W_{n+1} = B(T_n, W_n).$$

Let us define the functions,

$$\begin{aligned} A_k(T_n, W_n) &= T_{n+k} \\ B_k(T_n, W_n) &= W_{n+k} \end{aligned} \quad (12)$$

We call cycles of order N , the points (T_n, W_n) satisfying the equations,

$$\begin{aligned} T_n &= A_N(T_n, W_n) \\ W_n &= B_N(T_n, W_n) \end{aligned} \quad n = 1, 2, \dots, N \quad (13)$$

A further property of a cycle, beside N , is the number of turns (rotational number) made around the origin ($x=x'=0$) by the phase space representative point. In the following we use the notation "cycle N/r " for a cycle of order N and rotational number r . The characteristic matrix of a cycle N/r is,

$$\hat{C} = \begin{pmatrix} \frac{\partial A_N(T, W)}{\partial T} & \frac{\partial A_N(T, W)}{\partial W} \\ \frac{\partial B_N(T, W)}{\partial T} & \frac{\partial B_N(T, W)}{\partial W} \end{pmatrix} \quad (14)$$

The determinant of $\hat{C}$ is always 1, because our recurrence is conservative. If the absolute value of the trace of $\hat{C}$ is lower (greater) than 2 the cycle is stable (unstable) and is called centre (saddle). Separatrices start from saddles. They divide the phase space into stable and unstable phase regions. As an example we have, in Fig. 1, cycles 6/1 and 5/1 with their separatrices. Inside the 6/1 separatrices we have stable motion (particle E), while outside the phase motion is unstable (particle A and H). In the computer graphs we use the notations, $\times$ = saddle; $\circ$ = centre; N = separatrix of cycle $N/1$; $\times$ = overlap between two or more separatrices. The vertical scale on the right (A/σ) is related to T by eq. (7) ($A = \sqrt{2} \lambda_M \sigma$). Roughly speaking A is the "instantaneous motion amplitude". From the physical meaning of T and W , we have, for a cycle N/r ,

$$T_1 \approx T_2 \approx \dots \approx T_N \approx \frac{N}{r}$$

$$W_n - W_{n-1} \approx -\frac{2nr}{N}$$

FIG. 1

5. - Stochasticity threshold.

The phase space pattern of $N/1$ cycles, for $\mu/2\pi=0.15$, is given in Figs. 2, 3 and 4,

- Fig. 2, $\xi=0.10$ ($\Delta Q=0.087$); we have cycles $6/1$ and $5/1$,
 Fig. 3, $\xi=0.16$ ($\Delta Q=0.136$); there is an overlap region between cycles $6/1$ and $5/1$; a new cycle ($4/1$) appears,
 Fig. 4, $\xi=0.20$ ($\Delta Q=0.171$); the whole phase space region between cycles $6/1$ and $4/1$ is stochastic, with the exception of the stable phase islands around the centres.

FIG. 2

FIG. 3

FIG. 4

In Fig. 5 ($\xi=0.20$) we have the phase space trajectories of particles A, B, H outside and W inside the stochastic region. From the vertical right scale we see that A/σ ranges, for particle W, from 1 to 10, so that this stochastic region may be very dangerous.

FIG. 5

If we add, in somehow artificial way, radiation noise and damping to our model, we obtain that the r.m.s. transverse dimension of the weak beam does not change until stochastic regions appear (See Fig. 6). When the phase space between 6/1 and 4/1

FIG. 6

is fully stochastic, the weak beam enlarges by a factor of five. This behaviour does not depend on the damping time value (τ), in the range

$$10^2 \lesssim \frac{\tau}{\delta} \lesssim 10^4$$

where δ is the time distance between two neighbouring crossing points. From this observation we may derive the energy independence of ξ in the weak beam-strong beam incoherent limit. However, we must remember that our model is very rough. The minimum requirement, for taking into account experimental data, is to have a bi-dimensional model (four-dimensional phase space).

6. - Conclusions.

According to Zaslavskii and Chirikov⁶, we may say that the stochastic region in Fig. 3 and 4 is due to the overlapping of the stochastic layers of neighbouring resonances. On the other hand, as showed by Gumoski^{2,3}, when a centre bifurcates into a saddle, the region in the neighbourhood of such a cycle becomes stochastic. We have indeed that the phase space between cycles 5/1 and 4/1 becomes stochastic just when centres 19/4 and 17/4 bifurcate into saddles. Equally we have that the bifurcation of centre 23/4 (21/4) makes the region between 6/1 and 11/2 (11/2 and 5/1) stochastic. Fig. 7 shows the plot of ξ bifurcation values versus μ of the centres 19/4 and 17/4. We have tested this behaviour for

$$0.05 \leq \mu/2\pi \leq 0.15;$$

for this range of μ we may write the following empirical rule.

The phase space region between cycles $N/1$ and $(2N+1)/2$ becomes stochastic when the centre $(4N+1)/4$ bifurcates into a saddle.

FIG. 7

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TWO-DIMENSIONAL RESONANCE EFFECTS DUE TO A LOCALIZED BI-GAUSSIAN CHARGE DISTRIBUTION*

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The effect of a nonlinear kick supplied by one beam to the other in a colliding device had already been investigated by E. Keil¹ and J. LeDuff². The nonlinear kick was multiple analyzed and a single resonance was considered, but only the average term and the lowest Fourier mode driving the resonance were taken into account. More recently, A.G. Ruggiero and L. Smith³ approached the problem again, but with a different technique. They found that it is possible to describe a single resonance taking the exact analytical expression of the nonlinear kick and the contribution of all the higher Fourier modes driving the same resonance. Nevertheless, their calculation was limited to the one-dimensional case. The purpose of this paper is to extend this kind of calculation to the bi-dimensional case. We shall still assume a round beam with bi-gaussian distribution of standard deviation σ . The nonlinear kick is taken to occur lumped over a zero length interval. The main application is the calculation of the motion in proximity of a single, isolated and weak resonance; the calculation of the resonances width; 2nd of the stochasticity limit. Our result for the stochastic limit is higher than the one obtained by Keil. Because of the amount of the topics we shall report here the essential results. The details of the calculations can be found in another paper⁴.

Equations of Motion

The equations of motion are

$$\ddot{x}(s) = \xi_x \frac{1 - e^{-u^2}}{u^2}$$

and similarly for y,

$$u^2 = \frac{x^2 + y^2}{2\sigma^2}$$

Both beams are considered ultrarelativistic i.e. $v \approx c$. k_x and k_y are the two unperturbed linear focusing functions. δ_{int} is the periodic delta function which represents a kick every revolution of circumference $2\pi R$. Also, it is

$$\xi_x/\beta_x^* = \xi_y/\beta_y^* = \xi/\beta^* = Nr_0/4\pi\sigma^2\gamma$$

N is the number of particles in the strong bunch, r_0 the classical radius of the test particle, to be taken as positive for charges of equal sign and negative for charges of opposite sign, γ the ratio of the total energy to the rest energy. β_x^* and β_y^* are constant and denote the values of the beta-functions at the crossing point. ξ_x and ξ_y are the usual linear tune shifts per interaction.

*Work Supported by the U.S.A.E.C.

We perform the transformation to the two pairs of angle-action variables ψ_x, I_x and ψ_y, I_y . The angle $\theta = s/R$ is the independent variable and prime denotes derivative with respect to θ . By performing a triple Fourier expansion we obtain

$$\psi_x' = \nu_x + 2\xi_x \sum_{n,m} f_{nm}(I_x, I_y)$$

$$\cdot e^{i(n\psi_x + m\psi_y - l\theta)}$$

$$I_x' = -2\xi_x I_x \sum_{n,m,l} g_{nm}(I_x, I_y)$$

$$\cdot e^{i(n\psi_x + m\psi_y - l\theta)}$$

and similarly for y, where

$$f_{nm} = \frac{1}{4\pi} \int_{-\infty}^{+\infty} e^{i\omega(\tau_x + \tau_y)} (\log \frac{\omega^2 + 1}{\omega^2}) (-i)^{\frac{m+n}{2}} \cdot$$

$$\cdot J_{\frac{n}{2}}(\omega\tau_y) \left\{ J_{\frac{n}{2}}(\omega\tau_x) - i J_{\frac{n}{2}}'(\omega\tau_x) \right\} d\omega$$

$$g_{nm} = \frac{1}{4\pi} \int_{-\infty}^{+\infty} e^{i\omega(\tau_x + \tau_y)} (\log \frac{\omega^2 + 1}{\omega^2}) (-i)^{\frac{m+n}{2}} \cdot$$

$$\cdot J_{\frac{n}{2}}(\omega\tau_x) J_{\frac{m}{2}}(\omega\tau_y) \frac{n d\omega}{\omega\tau_x}$$

for n and m both even numbers, otherwise $f_{nm} =$

Single Isolated Resonance

We now define a resonance by choosing three integer numbers, with no common divisor, such that the quantity, $X = p\psi_x + q\psi_y - r\theta$, can be considered as slowly varying, and retain in the triple sum only terms of the form, $\exp[i(p\psi_x + q\psi_y - r\theta)]$. It can be proven that one invariant of the motion is

$$W_1 = qI_x - pI_y = \text{constant.} \quad (2)$$

The other invariant is the Hamiltonian W_2 which relates the action variables I_x and I_y to the new angle variables $\alpha_x = \psi_x - \frac{r}{p}\theta$, $\alpha_y = \psi_y$

$$W_2 = (\nu_x - \frac{r}{p})I_x + \nu_y I_y + 2\sigma^2 \left(\frac{\xi_x}{\beta^*} \right) \int_0^1 \frac{dt}{t} \left\{ 1 + \right.$$

$$- e^{-t(\tau_x + \tau_y)} \sum_{l=-\infty}^{+\infty} I_{lp_0}(\tau_x) I_{lq_0}(\tau_y) \cdot \cos(lX_0) \}^* \quad (3)$$

where p_0 , q_0 and X_0 equal, respectively, p , q and $2X$ in the case at least one of p and q is odd, and equal, respectively, $p/2$, $q/2$ and X in the case both p and q are even. Also it is

$$\tau = \frac{X}{X_0}, \quad \tau_y = \frac{Y}{Y_0},$$

In the case of a single resonance the motion is bounded, because for large τ_x and τ_y the dependence of W_2 on X_0 vanishes.

The equations of motion are easily derived from (3)

$$I_x' = -4p_0 \sigma^2 \left(\frac{\xi_x}{\pi} \right) \int \frac{dt}{t} e^{-t(\tau_x + \tau_y)} \sum_{l=-\infty}^{+\infty} l I_{lp_0}(\tau_x) I_{lq_0}(\tau_y) \sin(lX_0) \quad (4)$$

and similarly for y .

In particular we derive

$$\begin{aligned} X' &= p\alpha_x' + q\alpha_y' = \varepsilon_{pq} + \\ &+ p\xi_x \int_0^1 dt e^{-t(\tau_x + \tau_y)} \sum_{l=-\infty}^{+\infty} \left[I_{lp_0}(\tau_x) + \right. \\ &- I_{lp_0}'(\tau_x) \left. \right] I_{lq_0}(\tau_y) \cos(lX_0) + \\ &+ q\xi_y \int_0^1 dt e^{-t(\tau_x + \tau_y)} \sum_{l=-\infty}^{+\infty} \left[I_{lq_0}(\tau_y) + \right. \\ &- I_{lq_0}'(\tau_y) \left. \right] I_{lp_0}(\tau_x) \cos(lX_0) \end{aligned} \quad (5)$$

where $\varepsilon_{pq} = p\nu_x + q\nu_y - r$.

Fixed Lines

The equation of a fixed line is obtained by setting

$$I_x' = I_y' = 0 \quad \text{and} \quad X' = 0.$$

From eq. (4), we notice that there are two fixed lines, namely for $X_0 = 0$ and $X_0 = \pi$. From eq. (5), then, the equations of the two fixed lines are, respectively,

$$p\xi_x \int_0^1 dt e^{-t(\tau_x + \tau_y)} \sum_{l=-\infty}^{+\infty} \left[I_{lp_0}(\tau_x) + \right.$$

$$\begin{aligned} &- I_{lp_0}'(\tau_x) \left. \right] I_{lq_0}(\tau_y) + \\ &+ q\xi_y \int_0^1 dt e^{-t(\tau_x + \tau_y)} \sum_{l=-\infty}^{+\infty} \left[I_{lq_0}(\tau_y) + \right. \\ &- I_{lq_0}'(\tau_y) \left. \right] I_{lp_0}(\tau_x) + \varepsilon_{pq} = 0 \end{aligned}$$

and

$$\begin{aligned} &p\xi_x \int_0^1 dt e^{-t(\tau_x + \tau_y)} \sum_{l=-\infty}^{+\infty} \left[I_{lp_0}(\tau_x) + \right. \\ &- I_{lp_0}'(\tau_x) \left. \right] I_{lq_0}(\tau_y) (-1)^l + \\ &+ q\xi_y \int_0^1 dt e^{-t(\tau_x + \tau_y)} \sum_{l=-\infty}^{+\infty} \left[I_{lq_0}(\tau_y) + \right. \\ &- I_{lq_0}'(\tau_y) \left. \right] I_{lp_0}(\tau_x) (-1)^l + \varepsilon_{pq} = 0. \end{aligned} \quad (7)$$

The motion around the fixed lines can be investigated by expanding the right hand side of eqs. (4) and (5), respectively, around $X_0 = 0$ and $X_0 = \pi$. It is found that the $X_0 = \pi$ fixed line is a stable line and the $X_0 = 0$ fixed line is an unstable line. It has been proven that the condition for existence of the fixed lines is, for $p > 2$

$$-1 < \frac{\varepsilon_{pq}}{p\xi_x + q\xi_y} < 0. \quad (8)$$

Let us consider the three-dimensional space with ε_{pq} on the abscissa and I_x, I_y on the other two axis. Eqs. (6) and (7) represent two surfaces in this space. For fixed I and I_y we can go from one surface to another moving parallel to the ε_{pq} -axis. The distance $\Delta\varepsilon_{pq}$ which separates the two surfaces defines the "width" of the resonance at amplitude I_x and I_y . This is obtained by simply subtracting (7) from (6).

The general feature of the motion in the presence of a single resonance ($p > 2$) is that the phase space is clearly divided in two regions, one close to the origin and the other reaching the infinite. In both regions, a particle executes stable betatron oscillations with little amplitude and frequency modulation. This modulation vanishes as the unperturbed amplitude gets either smaller or very large. These two regions confine with an intermediate region. In this, the perturbation of the resonance is the largest and several islands appear. The motion inside an island is also stable, but on top of the betatron frequency another frequency of precession movement appears; this is the resonance frequency. The precession movement disappears when (8) is not satisfied.

This is pretty much all that we can say, from a general point of view, about a single

* I_n is the modified Bessel function.

resonance. To have a better insight of the motion one should have a more compact expression for the summation

$$R = \sum_{\ell=-\infty}^{+\infty} I_{\ell p_0}(\tau_{\ell y}) I_{\ell q_0}(\tau_{\ell y}) \cos(\ell X_0)$$

which we have been able to derive only for some special cases such as (a) $q = 0$ and (b) $p = |q|$, or $\tau_y = 0$ (see ref. [3]).

First Mode Approximation

Taking into account all the higher Fourier modes, we have found that even as well as odd order resonances are possible. This is in contrast to what we get by using the "first mode approximation" where only the average term $R = 0$ and the next $|\ell| = 1$ are retained. In this case, only even order resonances are possible. On the other hand, in the more exact approach, a resonance is defined only when the three integer numbers p , q and r do not have a common divisor. In contrast, in the "first mode approximation", the (p, q, r) -resonance is considered independent of the $(\ell p, \ell q, \ell r)$ -resonance.

In the "first mode approximation" the width of a resonance is

$$\begin{aligned} & \xi_x \int dt e^{p(\tau_x)} \\ & - \frac{I_p'(\tau_x)}{2} \left[\frac{I_q(\tau_y)}{2} + \right. \\ & + 4q\xi \left. \left| \frac{I_q(\tau_y)}{2} \right| \right. \\ & \left. - \frac{I_q'(\tau_y)}{2} \right] \frac{I_p(\tau_x)}{2} \end{aligned} \quad (9)$$

Numerical calculations to compare the width calculated according to the above formula and the one calculated more exactly from (6) and (7) show appreciable differences only for large τ_x and τ_y and low order p .

One-Dimensional Resonances

These are defined by setting $q = 0$. The Hamiltonian is

$$\begin{aligned} W_2 = & (v_x - \frac{E}{p}) I_x + v_y I_y + 2 \frac{\sigma^2}{p_0} \left(\frac{\xi}{\beta} \right) \sum_{s=1}^p \left[\frac{1}{\beta} \int_0^1 dt \left[1 + \right. \right. \\ & \left. \left. - I_0(\tau_y) e^{-t(\tau_x + \tau_y - I_x)} \cos \frac{X_0 - 2\pi s}{p_0} \right] \right] \end{aligned}$$

The analysis proceeds in the same way as outlined in another paper.³ Let us consider a few cases.

$P = 1$

This resonance does not exist in the "first mode approximation", but it is there when all the Fourier modes are taken into account. In the (x, x') -plane there is one unstable fixed point, the origin, and two stable points, diametrically opposite, with coordinate given by the equation

$$r - v_x = 2\xi_x e^{-(\tau_y + 2\tau_x)} \left\{ I_0(\tau_y) + \frac{I_1(\tau_y)\tau_y}{\tau_y + 2\tau_x} \right\}. \quad (10)$$

The fixed points exist only when $2\xi_x/(r - v_x) > 1$. This resonance can certainly be responsible for beam loss. Particles injected in proximity of the origin can be spilled out along the separatrix as shown in Fig. 1. Also, if the collision between two beams occurs in "adiabatic" way, the two beams would be split and locked each inside their own stable areas. In this case the separation of the two beams would be of the order of $2\tau_x$, where τ_x is the solution of (10).

$P = 2$

The flow diagram is still the one shown in Fig. 1. The origin is again the unstable fixed point, and the other two points are stable and symmetric, their coordinate being still obtained by solving eq. (10) with r replaced by $r/2$. This also, of course, can cause a beam growth and then a limitation on the luminosity achievable.

$P = 4$

The flow diagram is the one shown in Fig. 2. There are four stable and four unstable fixed points which exist only when $4\xi_x/(r - 4v_x) > 1$.

The location τ_x of the stable and unstable fixed points are obtained by solving the following respective equation

$$\begin{aligned} (r - 4v_x) &= 4\xi_x e^{-(\tau_x + \tau_y)} \left[I_0(\tau_y) + \frac{\tau_y I_1(\tau_y)}{\tau_x + \tau_y} \right] \\ (r - 4v_x) &= 4\xi_x e^{-(\tau_y + \frac{r}{2})} \left[I_0(\tau_y) + \frac{\tau_y I_1(\tau_y)}{2\tau_y + \tau_x} \right] \end{aligned}$$

Coupling Resonances

For $q \neq 0$, (3) is a two-dimensional Hamiltonian. Nevertheless, because of the existence of the first invariant (2), it is possible to get a one-dimensional Hamiltonian with a proper nonlinear rotation of the (x, x', y, y') four-dimensional phase-space around the origin. The rotation is accomplished by means of the following generating function

$$S = (p\alpha_x + q\alpha_y)W + \alpha_x W_1$$

which transforms the old variables ψ_x, I_x and ψ_y, I_y in the new variables X_1, W_1 and X, W through the relations

$$I_x = pW + W_1, \quad X = p\alpha + q\alpha_y$$

$$I_y = qW, \quad X_1 = \alpha_x.$$

We shall investigate here only the case $P = |q|$.

We have

$$W_2 = (v_x - \frac{r}{p})I_x + v_y I_y + \frac{2\sigma}{p_0} \left(\frac{\xi}{\beta} \right) \sum_{s=1}^{\infty} \int_0^{\frac{X-2\pi s}{p_0}} \frac{dt}{t}$$

$$- e^{-t(\tau_x + \tau_y)} I_0 \left(\sqrt{\tau_x^2 + \tau_y^2 + 2\tau_x \tau_y \cos \frac{X-2\pi s}{p_0}} \right).$$

Let us consider the lowest orders $p_0 = 1$ and the case $\xi_x = \xi_y$, i.e. $\beta_x^* = \beta_y^*$.

Difference Resonance, $q < 0$

We have the invariant $\tilde{W}_1 = \tau_x + \tau_y$ and the new variable $\tilde{W} = \frac{1}{q} \tau_y$.

On the $(X, \tilde{W})$ -plane there are two fixed points. The origin $\tilde{W} = 0$ is the unstable fixed point, and the stable fixed point is obtained by solving the following equation

$$1 + \frac{E_{pq}}{2\beta\xi} (2p\tilde{W} + \tilde{W}_1) - e^{-\tilde{W}_1} (1 + \tilde{W}_1) I_0(2p\tilde{W} + \tilde{W}_1) + \\ - e^{-\tilde{W}_1} (2p\tilde{W} + \tilde{W}_1) I_1(2p\tilde{W} + \tilde{W}_1) = 0.$$

Observe that the location of the unstable fixed point depends on the invariant $\tilde{W}_1$. The picture of the motion on the $(X, \tilde{W})$ -plane is thus similar to the one shown in Fig. 1.

Sum Resonance, $q > 0$

We have the invariant $W_1 = \tau_x - \tau_y$ and the new variable $W = \tau_y/q$. On the (X, W) -plane there are two fixed points. The stable one has for coordinate the solution of

$$E_{pq} + 2p\xi e^{-(2p\tilde{W} + \tilde{W}_1)} \left[I_0(\tilde{W}_1) + \frac{\tilde{W}_1 I_1(\tilde{W}_1)}{2p\tilde{W} + \tilde{W}_1} \right] = 0$$

and the unstable one has for coordinate the solution of

$$E_{pq}(2p\tilde{W} + \tilde{W}_1) + 2p\xi \left[1 - e^{-(2p\tilde{W} + \tilde{W}_1)} I_0(2p\tilde{W} + \tilde{W}_1) \right] = 0.$$

Also here, the location of the fixed points depends on the invariant $\tilde{W}_1$. The picture of the motion of the $(X, \tilde{W})$ -plane is shown in Fig. 3.

Several Crossings Per Turn

The generalization to n_c crossings per turn or to n_r revolutions between crossings is easily done. The only changes are: (a) ξ_x and ξ_y are now replaced by $\xi_x n_c$ and $\xi_y n_c$ or ξ_x/n_r and ξ_y/n_r , (b) the only possible resonances are those with the third integer number r that is an algebraic multiple of n_c or $1/n_r$. Thus the strength of a resonance is either increased by the factor n_c or decreased by a factor n_r . But the density of the resonances is also, respectively, decreased or increased by the same amount.

The Stochasticity Limit

According to Chirikov⁵ the stochasticity limit is reached when many nonlinear resonances overlap. As done by Keill¹, we take as criterion for resonance overlapping that the area covered by resonances in a square region in the (v_x, v_y) -plane of unit area becomes unity. The extension of a resonance in the (v_x, v_y) -plane is given by the quantity $\Delta\epsilon_{pq}$ we have calculated above. This quantity gives the range of $p v_x + q v_y - r$ which is locked to the resonance. The extension of the same resonance along the v_x -axis is obviously given by $\Delta\epsilon_{pq}/p$, and the extension along the v_y -axis by $\Delta\epsilon_{pq}/|q|$. The sum of the areas occupied by resonances is obtained by summing all $\Delta\epsilon_{pq}/p$ for resonances $p > |q|$ and all $\Delta\epsilon_{pq}/|q|$ for resonances $p < |q|$. The width of a resonance, $\Delta\epsilon_{pq}$, does not depend on the number r , also in the "first mode approximation". For assigned p and q there are exactly p resonances all with the same width, in a square region of the (v_x, v_y) -plane of unit area, if $p > |q|$ and $|q|$ resonances if $p < |q|$. Also, to calculate the sum of the area we should use eqs. (6) and (7) for the width $\Delta\epsilon_{pq}$. Nevertheless, one should take into account in the sum only those triplets (p, q, r) that have no common divisor. Since the "first mode approximation" gives an accurate estimate of the resonance width, we shall use eq. (9). But in this way, odd order resonances do not give any contribution. To balance this, we shall sum over all possible triplets (p, q, r) including those that do have common divisors.

Denoting the sum by S , we have

$$S = 4\xi_x T(\tau_x) + 4\xi_y T(\tau_y) \quad (11)$$

where the function

$$T(x) = e^{-x} \left[I_0(x) + I_1(x) \right]$$

is plotted in Fig. 4. The details of the summation can be found in [4]. All the contributions to S come only from the one-dimensional resonances, namely the $q = 0$ resonances with $I_y = 0$ contribute to the first term, in ξ_x , and the $p = 0$ resonances with $I_x = 0$ to the second term, in ξ_y .

In the case one of β_x^* and β_y^* is much smaller than the other, the corresponding term at the right hand side of (11) can be neglected. The stochasticity limit ($S=1$) is then reached for $\xi \sim 0.25$. Conversely, if $\beta_x^* = \beta_y^*$, the stochasticity limit is reached

for $\xi \sim 0.125$. In both cases, the limit occurs at $x = y = 0$.

Observe that most of the contribution to the function $T(x)$ comes mostly from the lowest order resonance $p = 2$ or $q = 2$. Indeed if the contribution of this resonance is ignored, $T(x)$ is smaller and given by the lower curve in Fig. 4. In this case the stochastic limit is reached at, $\tau_x = \tau_y = 2.5$, for $\xi \sim 0.8$ if $\beta_x^* \gg \beta_y^*$, and for $\xi \sim 0.4$ if $\beta_x^* \sim \beta_y^*$.

This result is in disagreement with Keil's results. We found that the contribution of the higher order resonance is smaller than the contribution of the few lowest order resonances. Keil found just the opposite. The discrepancy can be due to (a) the different definition of the resonance width, and/or (b) to the fact that Keil performs multiple expansion of the nonlinear kick and stops the summation to the order 30.

If we are to believe our result, (which is in much better agreement with the experimental observations) we infer that the experimental beam-beam limit is mainly caused by few low-order resonances, and that it is rather below the stochastic limit.

In the case of one kick every n_r revolutions or n_c kicks every revolution, we would still obtain the same result if the summation of the resonance widths is taken over a square of area, respectively, $1/n_r^2$ and n_c^2 .

Clearly, what is more important, especially in the second case, is a local summation of the widths. Likely the stochastic limit is a function of a tune. To prove this, we limited ourselves to the one dimensional case ($\beta_y^* = 0$ and $\tau_y = 0$), then we summed the widths of those resonances that fall in a smaller interval of tune, let us say, between 0.1 and 0.2, or 0.2 and 0.3, and so on. The results are shown in the next table where the maximum tune shift ξ_{max} allowable is reported versus the tune. The amplitude τ_{max} is also shown in the table.

Range of the Tune (incl.) - (excl.)	ξ_{max}	$2\tau_{max}$
0.0 - 0.1	0.050	0.0
0.1 - 0.2	0.984	4.9
0.2 - 0.3	0.473	1.4
0.3 - 0.4	0.984	4.9
0.4 - 0.5	2.884	19.0
0.5 - 0.6	0.050	0.0
0.6 - 0.7	0.984	4.9
0.7 - 0.8	0.473	1.4
0.8 - 0.9	0.984	4.9
0.9 - 1.0	2.884	19.0

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INSTABILITIES DETECTED IN SATURNE

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I. Introduction

After the injection energy had been increased from 3.6 MeV to 20 MeV, some problems arised at Saturne.

The theoretical number of captured particles is about $6 \cdot 10^{11}$. This figure corresponds to a v_z shift from 0.88 to 0.5. But we did not succeed to accelerate more than $1.6 - 2 \cdot 10^{11}$. Moreover it was very difficult to keep particles up to the end of the accelerating process. Strong losses appeared in the first 100 ms together with, occasional coherent oscillations phenomena in both directions. In order to overcome these difficulties, we empirically created horizontal magnet field index variation for coherent oscillations damping and we had to work out some cumbersome radial exploration during the acceleration process.

This refined tuning depended upon the beam intensity and was unstable from pulse to pulse. These difficulties were especially inconvenient for resonant extraction because of the necessary good optical properties of the beam needed for this extraction type and of the imposed magnetic field index variation.

For these reasons we have undertaken the experimental and theoretical studies described in this paper.

2. Experimental study

In order to get the machine characteristics the measurements were made by means of the beam itself, in the following way. [1]

At a given time of the cycle, the beam is accelerated on a given radius $R_0 + x$. Then, by means of the current i of the pole face windings, we change the n -value continuously. If we measure the intensity of the beam before and after the field index n changes we know the variation **loss** versus i . Maxima losses appear for a few i values. Then this measurement is repeated for different x positions. If we plot the i values giving maximum loss versus x , we see that we obtain parallel straight lines. Moreover, the v_z is measured and is used to obtain n -value from the formula $v_z = \sqrt{n(1+k)}$. From these measurements we deduce machine characteristics as shown on figure n° 1.

The magnetic field index (integrated on one turn) varies with x ($n' \sim 0.5$ meter). Losses occur when the field index n reaches the indicated values. Later, we shall see what

kind of resonances are concerned.

The same investigations have been made at various times during the cycle. On figure n° 2 one can see how the magnetic field index varies. We have not yet found a good explanation of this phenomena : remanent magnetic field, eddy currents, saturation effect, etc..

3. Resonances

According to the instabilities theory we have to deal with relations such as $av_x + bv_z = c$, where $(|a| + |b|)$ is the resonance order. On figure n° 4 we can see the stability diagram (v_x, v_z) with the resonance lines up to the fourth order and the circle $v_x^2 + v_z^2 = 1 + k$, wich represents operating points of the machine. [2] [3]

Instabilities occur at the intersection of the circle with the lines. If one refers to figure n° 3, one can see that there is a precise correspondance with experimental results.

Sextupolar or octupolar defects responsible for instabilities proceed from non linear index variation versus x inside the quadrant and from end effects of the quadrants. We tried to correct this last defect. But this correction cannot be perfect and the first defect remains and hence we did not notice a very important improvement.

4. Coherent oscillations

Experimental results show that coherent oscillation can occur together with incoherent oscillation. We have not yet found any theoretical explanation but it seems that the induced signal on the vacuum chamber walls due to the increase of the beam dimension helps the coherent oscillation to develop.

Moreover if the $\frac{dn}{dx}$ value is decreased in order to improve acceleration or ejection process, the beam becomes immediatly unstable whatever is the n -value. This is due to the lack of n -value dispersion inside the beam.

5. Consequences on the beam behaviour

We are, now, able to understand losses occurring at injection and beam instabilities during acceleration. We can then consider the

possibility of improving the performances.

Let us consider figures n° 2 and 3.

The beam closed orbit dispersion due to energy spread from injection up to 150 ms is larger than 10 cm. Hence each particle crosses resonance lines many times during synchrotron motion.

The $3\nu_x + \nu_z = 3$ resonance is the most dangerous since the betatron amplitudes can increase infinitely. We understand then why we can avoid losses by changing beam position during the acceleration process. But if the injection itself is not exactly reproducible from pulse to pulse, the capture efficiency changes, the energy spread too and the beam becomes unstable.

Referring to figure n° 2, we can see that the beam would be more stable if we could inject particles between $n = 0.561$ and $n = 0.5$ and attempt to keep the beam inside this range during the whole acceleration process. The losses would occur in the few first ten milliseconds and the beam would remain stable. If there is any emittance increase the betatronic damping would begin earlier so that the beam diameter would be smaller at the time we intend to eject. We could check experimentally this assumption and we now inject, at n -value = 0.55.

Unfortunately we have to achieve a n -value = 0.658 for the resonant ejection process and we have again to cross resonances. But another experiment shows that the emittance increase is small if we stay for less than a few millisecond in the resonance vicinity. Therefore we are studying a power supply able to change n -value from $n = 0.55$ to $n = 0.658$ within 10 ms.

A more accurate analysis of phenomena occurring at injection, taking into account space charge, has been made and shows another particle intensity limitation. The operating point moves in the stability diagram inside the circle : $\Delta\nu_x = -4.10^{-5} \times N$,

$$\Delta\nu_z = -1.3.10^{-4} N,$$

if N is the number of particles. Whatever the n -value between $n = 0.5$ and $n = 0.7$, the operating point reaches the $2\nu_x + 2\nu_z = 3$ resonance line and the N value we calculate from $\Delta\nu_x$ or $\Delta\nu_z$ corresponds exactly to the maximum amount of particles captured i.e. $1.6.10^{12} - 2.10^{12}$.

In figure n° 5 we have plotted the beam extension in the (ν_x, ν_z) diagram taking into account the energy spread, as the n -value depends on radial position or energy. Since this extension is a parallel line to $2\nu_x + 2\nu_z = 3$ we understand why we cannot cross this line because all the particles are unstable at the same time.

6. Conclusion

All these theoretical and experimental studies show that resonances are the limiting factors for improving Saturne performances. Further developments have been recently stopped due to the decision to strongly increase the performances of Saturne : the magnetic components of the main ring will be rebuilt, leading to a version number 2 of the Saturne accelerator.

The new magnetic structure will be a strong focusing separated function scheme. [4]

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FIG 1 MAGNETIC FIELD INDEX AND RESONANCES

FIG 2 INDEX VARIATION VERSUS TIME

FIG 3 RESONANCES IDENTIFICATED IN SATURNE

FIG 5 OPERATING POINT VARIATION DUE TO SPACE CHARGE

Some estimates of the stochasticity domain of a particular Hamiltonian dynamic system with a continuously increasing non-linearity have been given elsewhere's'. These estimates are not directly applicable to longitudinal motion in a circular accelerator because in the latter case the non-linearity is periodic and bounded. An extension of the method used in Refs. 1 and 2 shows that in accelerators such as the CERN PS and PSB the accelerating voltage non-linearity produces stochastic behaviour only in the close vicinity of the separatrix. The usual description of a microtron is shown to imply the existence of a large stochastic domain.

Introduction and Statement of the Problem

The longitudinal motion in an accelerator can be described in a straightforward manner if the assumption is made that the ring is composed of discrete elements only. One formulation without space charge, taking into account one RF-cavity gap, is³

$$\begin{aligned} \Delta p_{n+1} &= \Delta p_n + \frac{eV}{\beta_n c} (\sin \phi_n - \sin \phi_s) \\ \phi_{n+1} &= \phi_n + \frac{2\pi h}{\beta_n} (\alpha - \gamma_n^2) \Delta p_{n+1} \end{aligned} \quad (1)$$

where p is the particle momentum, Δp the momentum deviation, α the momentum compaction factor, ϕ the phase, n the number of cavity gap traversals, V the amplitude of a sinusoidal accelerating voltage, h the harmonic number, e the particle charge, and β , γ , c the usual relativistic quantities. The subscript s refers to synchronous values. When radiation damping is taken into account, the recurrence (or point-mapping) (1) is equivalent to the known alternate forms^{4,5}. It is also equivalent to the differential equation⁶

$$\frac{d\phi}{d\tau} = \psi, \quad \frac{d\psi}{d\tau} = \Omega^2 \left(-\sin \phi + 2\gamma \phi \cdot \sin^2 \frac{\phi}{2} \right) \left(1 + 2 \sum_{m=1}^{\infty} \omega_m \tau \right) \quad (2)$$

where τ and Ω are the normalized time and small-amplitude synchrotron frequency, respectively.

The object of this paper is the determination of the solution structure of the recurrence (1) in the discrete phase plane $\Delta p_n, \phi_n$, and the application of the results to the CERN PS and PSB. When the differential equation (2) is solved by a method of series truncation⁶ (perturbations with averaging, or a finite number of canonical transformations which reject the explicit time-dependence toward higher-order terms), the solution structure so obtained is valid only in the region where the recurrence (1) admits closed trajectories (invariant curves without "stochastic" intersections). The region where the invariant curves of (1) intersect is called stochastic, because the intersection pattern is so complex that superficially it appears to be random. Since this pattern is unambiguously defined by (1), it is of course completely deterministic.

Determination of the solution structure of the recurrence (1)

The approach used in this paper is based on the previously tested conjecture that the qualitative features of the solution structure of (1) can be deduced from the distribution in the phase plane of zero- and one-dimensional singular solutions (point- and line-singularities)^{1,2,7-11}. Consider, in fact, the particular conservative recurrence

$$\begin{aligned} x_{n+1} &= y_n - F(x_n) = g(x_n, y_n) \\ y_{n+1} &= -x_n + F(x_{n+1}) = f(x_n, y_n) \end{aligned} \quad (3)$$

where $F(x)$ is an arbitrary differentiable function. From (3) it is possible to construct unambiguously the iterated recurrence

$$\begin{aligned} x_{n+k} &= g_k(x_n, y_n), \quad y_{n+k} = f_k(x_n, y_n) \\ g_1 &= g, \quad f_1 = f, \quad k = \text{integer} \end{aligned} \quad (4)$$

The simplest point-singularity of (3) is a cycle of order k (periodic point of order k), defined by a real root x, y of the two algebraic equations

$$x_{n+k} = x_n = x, \quad y_{n+k} = y_n = y \quad (5)$$

deduced from (4), provided x, y is not simultaneously a root of (5) when k is replaced by one of its divisors. A cycle of order $k=1$ is called a fixed point.

An invariant curve of (3) is described by $G(x, y) = \text{const.}$ if the function $G(x, y)$ satisfies the functional equation

$$G(g_k(x, y), f_k(x, y)) = G(x, y) \quad (6)$$

If the curve $G(x, y) = \text{const.}$ has at least one single-valued branch $y = \theta(x)$ passing through a point (x_0, y_0) , then an equivalent formulation of (6) for this branch is

$$f_k(x, \theta(x)) = \theta[g_k(x, \theta(x))] \quad , \quad y_0 = \theta(x_0). \quad (7)$$

The function $\theta(x)$ and a sufficient number of its derivatives are assumed to possess at least an asymptotically convergent Taylor series at and near x_0 . The simplest line-singularity of (3) is an invariant curve passing through a point of a cycle.

Cycles are classified according to the eigenvalues $\lambda_{1,2}$ of their characteristic equation. The special form of (3) implies $\lambda_1 \cdot \lambda_2 = 1$. The indices are so chosen that either $\lambda_1 > +1$ or $\lambda_2 < -1$. Cycles are called saddles (or hyperbolic points) if the λ are real (of type 1 if $\lambda > 0$ and of type 3 if $\lambda < 0$) and centres (or elliptic points) if $\lambda = \exp(\pm i\phi)$. The constant ϕ is called the rotation angle of the centre. An additional property of a cycle is its rotation number r (integer), describing the least number of turns made around an interior point when following the k successive points of the cycle in the phase plane. For conciseness, a cycle of order k and rotation number r is designated by $k_{(r)}$.

If the point (x_0, y_0) is a saddle, a singular invariant curve segment passing through (x_0, y_0) can be sought in the form

$$y = \theta(x) = y_0 + \sum_{i=1}^{\infty} \beta_i (x - x_0)^i, \quad (8)$$

where the β_i , $i = 1, 2, \dots$ are real constants. Because of the postulated smoothness of $\theta(x)$, β_1 is equal to one of the eigenslopes p_1, p_2 at (x_0, y_0) . A known segment $y = \theta(x)$ can be continued by means of (4). The slope β_1 can be continued by means of the recurrence

$$dx_{n+k} \left(a + b \frac{dy_n}{dx_n} \right)^{k-1} \cdot \left(c + d \frac{dy_n}{dx_n} \right), \quad \frac{dy_0}{dx_0} = \beta_1, \quad (9)$$

where a, b, c, d are elements of the Jacobian matrix of (4), whose determinant is unity for any k . Similar recurrences exist for the continuation of the β_i , $i > 1$. If $b > 0$, the eigenslopes of a saddle are given by $p_{1,2} = (A_{11} \pm a)/b$.

It is straightforward to show that the recurrence (1) can be transformed into the form (3). In fact, when $\theta_n = b(\phi_n - \phi_s)$, $\psi_n = b(\phi_{n-1} - \phi_s)$, $b \approx \pi - 2\phi_s$, (1) becomes

$$\psi_{n+1} = \theta_n, \quad k_0 = \frac{h e V}{E_0} \cdot \frac{x_n^2 - 1}{x_n(x_n^2 - 1)} \quad (10)$$

$$\theta_{n+1} = -\psi_n + 2\theta_n + \frac{2\pi k_0}{b} [\sin(b\theta_n + \phi_s) - \sin \phi_s]$$

where E_0 is the particle rest energy. The parameter k_0 is negative below transition. Comparing (10) to the diagonal form of (3):

$$x_{n+1} = u_n, \quad u_{n+1} = -x_n + 2F(u_n) \quad (11)$$

obtained by means of the transformation $u_n = y_n + F(x_n)$, yields the required equivalence relation

$$F(x) = x - \frac{1-\mu}{b \cos \phi_s} [\sin(bx + \phi_s) - \sin \phi_s] \quad (12)$$

$$\mu = 1 + \pi k_0 \cos \phi_s, \quad 0 \leq \phi_s \leq \frac{\pi}{2}, \quad -1 < \mu < +1$$

The function F in (12) involves two independent parameters μ and ϕ_s , rendering a graphical display of the singular solutions of (3), (12) very inconvenient. A preliminary study has shown, however, that the stochastic features of (3), (12) do not change qualitatively as long as $\phi_s \neq 0$. For the purposes of this paper it is therefore possible to choose ϕ_s arbitrarily. For convenience ϕ was fixed as follows:

$$\phi_s = \frac{\pi}{2} - \arctan \frac{1-\mu}{\pi} \quad (13)$$

The recurrence defined by (3), (12), (13) is periodic in x . In order to assure uniqueness of the graphical representation for a fixed μ , a cylindrical phase space may be used, i.e. the phase plane is thought to be wrapped around a cylinder of radius $1/\pi$ centred on $x = 0$.

Some of the singularities of (3), (12), (13) are easily found, such as the two fixed points at (0,0) and (1,0). The former is a centre of rotation angle $\phi = \arccos \mu$ and the latter a saddle of type 1. The locations of other singularities depend on the value of μ and must be sought numerically. A partial list of cycles is given in Tables 1-3. The invariant curves passing through the "main" saddle (1,0) are shown in Figs. 1-3.

From an inspection of Figs. 1-3 it is clear that the recurrence (2), (12), (13) admits homoclinic points. Hence by a theorem of Birkhoff¹² it admits an infinity of cycles. An inspection of Tables 1-3 suggests that the cycles found so far form well-ordered sets. Some properties of the ordering can be found by rearranging the tables so that the ratio k/r is roughly constant, or by tracing the locations of the points in the phase plane, together with the "main" invariant curves traversing the saddle (1,0), as illustrated in Figs. 1-3. The first deduction is that the points of the cycles are located on curves which approach a discrete set of homoclinic points'. In addition to the x -axis, one such curve is $y = x - F(x)$. A more fundamental way consists, however, in ordering the cycles according to conditions of their appearance, disappearance, or other change of properties, i.e. in ordering them from the point of view of Poincaré's theory of bifurcation.

By present knowledge, bifurcations take place only when the eigenvalue of a cycle satisfies

$$\lambda^q = +1 \quad \text{or} \quad \lambda^q = -1, \quad q = \text{integer} \geq 1 \quad (14)$$

The case $q = 1$ is called a critical case¹³ and the case $q > 1$ an exceptional one^{14,15}. The latter arises in (3) whenever $\phi = 2\pi p/q$, $p = \text{integer}$. In the case (3), (12), (13) an exceptional case occurs at (0,0) whenever

$$\mu \rightarrow \bar{\mu} = \cos(2\pi r/k), \quad k > r \quad \text{and relatively prime} \quad (15)$$

It has been found¹ that the traversal of $\bar{\mu} \neq -\frac{1}{2}$ in the direction of decreasing μ (increasing strength of non-linearity) releases from (0,0) a cycle saddle $k(r)$ of type 1 and a cycle centre $k(r)$. The "bifurcated" cycles

$k(r)$ exist only for $\mu < \bar{\mu}$ and they merge with (0,0) when $\mu \rightarrow \bar{\mu}$ from below. Furthermore

$$\begin{aligned} k/r &< \text{finite constant}, \quad \lambda_1(k(r)) \rightarrow +1 \\ p_1 - p_2 &\rightarrow 0, \quad \phi(k(r)) \rightarrow 0 \quad \text{as} \quad \mu \rightarrow \bar{\mu}, \end{aligned} \quad (16)$$

where $\lambda(k(r))$, $\phi(k(r))$ designate the eigenvalue and rotation angle of the cycles $k(r)$, respectively. The first enumerable set of cycles of (3), (12), (13) consists therefore of cycles bifurcated from (0,0) at the exceptional values (14), provided $r/k \neq 1/3$, according to the schematic rule:

$$\begin{aligned} \text{centre (0,0)} &\rightarrow \text{centre (0,0)} + \text{centre } k(r) + \\ &\quad + \text{saddle } k(r) \text{ of type 1} \end{aligned} \quad (17)$$

The geometric distance $s(x,y)$ between points of a cycle $k(r)$ so bifurcated and (0,0) is found to increase simultaneously with the parametric distance $s(\mu) = \bar{\mu} - \mu$. The parameters $\lambda_1(k(r))$ and the angle between p_1 , p_2 also increase with $s(\mu)$.

For any given $\mu \neq -\frac{1}{2}$ the centre (0,0) is thus surrounded by an infinite but enumerable set of concentric cycle pairs $k(r)$, bifurcated according to the scheme (17), with k/r verifying the inequality $\mu < \cos(2\pi r/k)$. When the difference $\lambda_1(k(r)) - 1$ is sufficiently small, the computed invariant curves passing through the saddles $k(r)$ are for all practical purposes indistinguishable from regular invariant curves, i.e. from invariant curves not traversing any singular points. The assumption that these invariant curves are not "analytically" closed for arbitrarily small values of $s(x,y)$ implies that homoclinic points can exist arbitrarily close to (0,0). It is, however, known¹ that homoclinic points $(\bar{x}, \bar{y})$ are accumulation points of a set of cycles $k(r)$ of coordinates $(x_m, y_m)_k$, and that the "effective" order k/r , as well as the eigenvalues $\lambda_1(k(r))$ of the saddles of type 1 of this set increase indefinitely as the geometric distance between $(\bar{x}, \bar{y})$ and $(x_m, y_m)_k$ decreases, i.e.:

$$\begin{aligned} k/r \rightarrow \infty, \quad \lambda_1(k(r)) \rightarrow \infty \quad \text{and} \quad p_1 - p_2 \rightarrow \text{finite} \\ \text{constant as } (x_m, y_m)_k \rightarrow (\bar{x}, \bar{y}). \end{aligned} \quad (18)$$

The assumption that sufficiently near (0,0) the invariant curves are not (truly) closed leads to a contradiction between the properties (16) and (18). A small neighbourhood of (0,0) is therefore filled with an infinite but enumerable set of island structures [formed by the invariant curves which traverse the saddles $k(r)$ of type 1, surround the centres $k(r)$ and join without other intersections]. Since two close island structures have been bifurcated from (0,0) at two distinct values of $\phi = 2\pi r/k$, and thus of μ (because k and r are relatively prime) the area between them contains either regular closed invariant curves or closed invariant curves passing through an infinity of singular points [points of cycles $k(r)$ for which $k \rightarrow \infty$, $r \rightarrow \infty$, $\lim k/r < \infty$, $\lambda_1(k(r)) \rightarrow 1$, $p_1 - p_2 \rightarrow 0$, $\phi(k(r)) \rightarrow 0$]. The distribution of closed invariant curves near (0,0) is thus extremely irregular in theory but quite smooth in practice.

Because the construction of an iterated recurrence like (4) does not produce any intrinsically new data, the solution structure near a centre $k(r)$, $k > 1$, is the same as near the centre (0,0), i.e. one has a "box within a box" behaviour. The growth of the rotation angle as a function of $s(\mu)$ of the centre $k(r)$, bifurcated from the centre $k(r)$ and characterized by the property that k/r contains the factor k/r , is much faster than the growth of $\phi(k(r))$. In other words, the bifurcation speed is higher inside the inner boxes.

Since the rotation angle of a centre increases with the parametric distance from its generating bifurcation, it may eventually reach the value $\phi = \pi$. If the

traversal (in the direction of decreasing μ) of the corresponding critical case $\lambda = -1$ is examined from the bifurcation point of view, it is found¹¹ that the cycle centre $k(r)$ turns into a cycle saddle $k(r)$ of type 3 with a simultaneous release of a cycle centre $(2k)_{(2r)}$, or schematically

$$\begin{aligned} \text{centre } k(r) \rightarrow \text{saddle } k(r) \text{ of type 3} + \\ \text{centre } (2k)_{(2r)} \end{aligned} \quad (19)$$

Since the companion saddles $k(r)$ of type 1 remain unchanged, the bifurcation (19) converts the centre-saddle of type 1-pair into a saddle of type 3-saddle of type 1-pair. Several examples of such saddle-saddle pairs appear in Tables 2 and 3. The presence of cycles generated by means of the bifurcation (19) appears to be a sufficient condition of the presence of local stochasticity, i.e. of the presence of interesting invariant curves in the neighbourhood of the saddle-saddle pair. Small islands of closed trajectories may persist near the centres $(2k)_{(2r)}$.

The bifurcations (17) and (19) are insufficient to explain the origin of all cycles of the recurrence (3), (12), (13). There are, for example, two cycles saddle $13(2)$ of type 1 in Table 3, and it is found that the cycle pair $3(1)$ exists also when $\mu > -\frac{1}{2}$, whereas according to (17) it should appear at $\mu = -\frac{1}{2}$. The study of this difficulty has disclosed the existence of two additional bifurcations, related to the critical case $\lambda = +1$ ($\phi = 0$)¹¹. The first is of the type:

$$\begin{aligned} \text{composite cycle } k(r) \rightarrow \text{centre } k(r) + \\ \text{saddle } k(r) \text{ of type 1} \end{aligned} \quad (20)$$

The composite cycle $k(r)$ appears as an isolated double root of (5), and is unrelated to any close simple root. After traversal of the bifurcation the double root separates into two simple ones, giving rise to the usual pair centre $k(r)$ -saddle $k(r)$ of type 1. The second cycle saddle $13(2)$ of Table 3 and the cycle pair $3(1)$ originate in this way (at $\mu \approx 0.23$ and $\mu \approx 0.41$, respectively).

When $\mu \rightarrow \bar{\mu} = \cos(2\pi/3) = -\frac{1}{2}$, only the $3(1)$ saddles merge with the centre $(0,0)$, the $3(1)$ centres remaining some distance away [one point at $(-0.33, 0)$]. Furthermore, $\lambda_1(3(1)) \rightarrow 1$ as $\mu \rightarrow \bar{\mu}$. The bifurcation is of the type

$$\begin{aligned} \text{saddles } k(r) \text{ of type 1} \rightarrow \left\{ \begin{array}{l} \text{composite cycle} \\ \lambda(k(r)) = 1, \varphi(k(r)) = \frac{2\pi p}{q} \end{array} \right\} \rightarrow \dots \\ \text{centres } \bar{k}(r) \rightarrow \left\{ \begin{array}{l} \text{saddles } k(r) \text{ of type 1} \\ \text{centres } \bar{k}(r), k \neq \bar{k} \end{array} \right\} \end{aligned} \quad (21)$$

with $k = 3$ and $\bar{k} = 1$. Six invariant curve segments traverse the composite singular point $(0,0)$ when $\mu = \bar{\mu}$, resulting from the coalescence of the three saddles $3(1)$ of type 1. The bifurcation (21) produces an unstable point singularity.

Consider now the structure of invariant curves of (3), (12), (13) when $\mu \neq -\frac{1}{2}$. According to the theory of Birkhoff¹⁶ the island structures around $(0,0)$ should turn into instability rings (degenerate island structures formed when the invariant curves traversing the saddles of type 1 have other intersections but remain nevertheless inside a domain bounded by a finite closed curve) as their geometric distance from $(0,0)$ increases. Stochasticity thus exists inside an instability ring, but it does not lead to an "orbital" instability. Since instability rings require the existence of a centre-saddle pair, they must give way to a different configuration when the bifurcation (19) is reached.

The corresponding geometric distance from $(0,0)$ can be estimated from Tables 1-3. It has been found, however, that instability rings cease to exist sooner¹⁷. From Tables 1-3 it can be seen that $\lambda_1(k(r))$ and the angle between p_1 and p_2 increase simultaneously with the effective order k/r . Beyond a critical value the geometric distance between $(0,0)$ and a point on a singular invariant curve through a saddle $k(r)$ is found to increase first at an algebraic rate and then at an exponential one. The slow increase has been called diffusion and the fast one stochastic instability¹⁷.

Application to longitudinal motion

When the solution structure of the recurrence (3) (12), (13) is expressed in the terminology of accelerator theory [recurrence (1)], RF acceleration is found to produce bunches with a dense structured core (region of closed trajectories, including island structures), surrounded first by a dense but unstructured shell (instability rings) and then by a halo (diffusion region). Since the separatrix of the RF bucket (main invariant curve) is not closed, random particle losses occur from the halo (stochastic instability). The size of each region depends on the specific accelerator parameters. The RF acceleration process is unstable at the 1/3 parametric resonance between the synchrotron and beam revolution frequencies [exceptional case $\phi = 2\pi/3$ at $(0,0)$].

As far as the PSB is concerned, all longitudinal stochastic effects are negligible ($\mu \geq 0.99$). Analogous circumstances prevail in the CPS ($\mu \geq 0.90$). Stochastic effects may, however, occur inside self-buckets produced by beam-induced high-frequency voltages.

An accelerator likely to be subject to considerable stochastic effects is the microtron. Assume that the recurrence^{4,17,18}

$$x_{n+1} = x_n + y_n, \quad y_{n+1} = y_n + 2\pi \left(\frac{\omega x_{n+1}}{\omega \phi_n} - 1 \right) \quad (22)$$

is a valid description of the longitudinal motion (some doubts have been expressed in a paper by Turrin¹⁹). By a linear change of variables (22) transforms into

$$z_{n+1} = v_n, \quad v_{n+1} = -z_n + 2v_n + \frac{2\pi}{\omega \phi_n} [\omega(v_n - \phi_n) - \omega \phi_n] \quad (23)$$

Since an RF voltage $V \sin \phi$ below transition is equivalent to $-V \sin(\phi - 2\phi_s)$ above transition²⁰, transforming (22) into an equivalent form "below transition" and replacing $\cos \phi$ by $\sin \phi$ yields the recurrence (3), (12), (13) with $0 < \phi_s < \arctg(2/\pi) \approx 32.5^\circ$. The solutions of (22) therefore exhibit considerable stochasticity even for relatively small ϕ_s . Figure 3 corresponds to $\phi_s = 15.6^\circ$; on the x-axis, stochastic instability starts at $x \approx 0.51$.

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Table 1

Cycles of the recurrence
(3), (12), (13), $\mu = 0.8$

k	r	x	Centres or Saddles of type 3				Saddles of type 1				
			Y	ϕ or λ_2 (> 0) (< 0)	p ₁	p ₂	x	y	λ_1	p ₁	p ₂
10	1	0.250529									
11	1	0.841459	0	0.0001			0.241025	0.036592	1.0001	-1.900	-1.893
12	1	0.701253	0.049673	0.0037			0.553773	0	1.0037	38.1	
13	1	0.704250	0	0.0103			0.690883	0.042725	1.0103	-2.135	-2.021
14	1	0.852660	0.033848	0.0188			0.792358	0	1.0190	48.78	
15	1	0.869912	0	0.0296			0.846697	0.025965	1.0300	-2.269	-2.023
16	1	0.923064	0.019596	0.0434			0.894539	0	1.0444	25.71	
17	1	0.942253	0	0.0616			0.920418	0.014651	1.0635	-2.425	-1.964
18	1	0.949829	0.010888	0.0859			0.944776	0	1.0897	14.19	
19	1	0.969920	0	0.1187			0.957934	0.008060	1.1258	-2.706	-1.840
19	1	0.969314	0.005949	0.1631			0.970704	0	1.1767	7.818	

k	r	Centres or Saddles of type 3					Saddles of type 1				
		x	y	ϕ or λ_2 (> 0) (< 0)	p ₁	p ₂	x	y	λ_1	p ₁	p ₂
7	1	-0.453247	0	0.4374			0.611988	0	1.531	2.349	
8	1	-0.519721	0	1.1355			0.763198	0.090499	2.766	1.084	-4.284
9	1	-0.541178	0	2.9517			0.872783	0	5.685	1.277	
10	1	-0.548847	0	-9.366	30.47		0.915006	0.038910	13.10	1.057	-1.375
11	1	-0.551694	0	-29.21	25.90		0.931111	0.054332	32.41	1.040	-1.419
12	1	0.971773	0	-81.08	-1.043		0.968120	0.015436	82.94	1.120	-1.082
13	1	0.961612	0.037471	-216.9	-1.072	0.881	0.973774	0.021412	215.3	1.127	-1.086
13	2	-0.364999	0	0.0319			0.455575	0		36.39	
15	2	-0.495642	0	0.5326			0.707595	0		5.355	
17	2	-0.532975	0	-2.357	128.0		0.848055	0		1.665	
19	2	0.899388	0.024261	-43.33	-0.732	1.318	0.885209	0		1.345	
19	3	-0.313851	0	0.0013			0.375926	0		386.2	

Table 2

Cycles of the recurrence
(3), (12), (13), $\mu = 0.5$

Table 3

Cycles of the recurrence
(3), (12), (13), $\mu = 0.125$

k	r	Centres or Saddles of type 3					Saddles of type 1				
		x	Y	ϕ or λ_2 (> 0) (< 0)	p ₁	p ₂	x	Y	λ_1	p ₁	p ₂
5	1	-0.502858	0	2.525			0.551511	0	3.849	1.011	
6	1	-0.577941	0	-25.97	2.731		0.754058	0.163018	20.18	1.297	-1.084
7	1	-0.594668	0	-114.5	2.511		0.892309	0	80.35	1.347	
8	1	0.953152	0	-423.3	-1.458		0.932867	0.054883	291.3	1.423	-1.446
9	1	-0.600442	0	1494	2.452		0.970033	0	1024	1.514	
9	2	-0.288774	0	0.1467			0.318515	0	1.157	4.245	
11	2	-0.545848	0	-85.46	3.311		0.621978	0	70.28	1.259	
12	2	0.861806	0.029977	-395.1	-1.282	1.141	0.641711	0	251.8	1.377	
13	2						0.647291	0	874.8	1.418	
13	3						0.802214	0.226028	1611	1.202	
14	3	0.455590	0	0.4915			0.440250	0.052857	1.617	-4.440	-1.292
15	3	-0.460141	0	-8.748	1.772		-0.511499	0	5.072	-4.768	
16	3	0.608929	0	-286.8	-1.183		0.597869	0.029811	253.1	1.089	-1.300
17	3						0.613179	0.040647	802.0	1.117	-1.527
19	4	-0.387920	0	1.048			0.505232	0	2.615	1.917	
20	4						-0.523779	0	40.96	-0.413	
25	5	-0.504186	0	0.0378

Fig. 1 $\mu = 0.8$. Main invariant curves and positions of some cycle

Fig. 2 $\mu = 0.5$. Main invariant curves and positions of some cycles

Fig. 3 $\mu = 0.125$. Main invariant curves and positions of some cycles

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Summary

In 1973, the former Q-jump system has been replaced by 8 fast and 6 slowly pulsed quadrupoles forming 4 doublets and 2 triplets respectively. They allow presently a change of γ_t by ≈ 2.3 (instead of ≈ 0.23), limited only by the power supply within 0.8 ms (< 0.4 ms with a new supply under construction). Transition is crossed 50 times (instead of 5 times) faster than without jump. More than $5 \cdot 10^{12}$ p/p have been accelerated operationally and brought through transition within a bunch area of ≈ 10 mrad. In the future, it should be possible to handle 10^{13} p/p within the same bunch area. The scaling laws for achievable longitudinal density Ψ (apart from logarithmic terms $\Psi \propto N^{1/3} |\Delta\gamma_t|^{1/2}$ for bunch matching and $\Psi \propto N^{1/5} |\Delta\gamma_t|^{-1/10}$ for negative mass instability) are in good agreement with experiments.

1. Introduction

High longitudinal phase space density is of particular importance in the CPS as it serves as injector for the ISR and, in the future for the SPS. The most severe limitation occurs at transition where a blow-up due to negative mass instability must be avoided. As shown in chapter "The negative mass instability", the only efficient method to achieve this at high intensity is to cross transition much faster than it would be crossed without special precautions. The method used to manipulate the transition energy is the subject of chapters 2 to 5. Chapter 6 deals with bunch matching, chapter 7 with the negative mass instability leading to a simple criterion for avoiding blow-up trouble. Chapter 8 summarizes the practical experience with the new scheme.

2. The Lens Configuration

A convenient way to influence γ_t without affecting the betatron tune is to form doublets: two lenses of opposite polarity are placed half a betatron wavelength apart. The phase advance per cell of the CPS being $\pi/4$ and the structure being FOFDOD, the doublet lenses can all be placed in mid-F straight sections. A doublet leaves the amplitude function β_H unperturbed outside but not the dispersion function x_p . Two consecutive doublets of opposite polarity form a triplet whose center lens is twice as strong as the outer lenses.

The lens configuration consists of two superperiods, each containing: 1 triplet, 2 cells, 1 doublet, 1 cell, 1 doublet, 6 cells. In the following three chapters, expressions for β , x_p and γ_t are derived for a machine assuming:

- i) all unperturbed β and x_p values at the lenses are equal to β_F and x_{pF} respectively;
- ii) the phase advances are precisely as required.

3. The Excursion of the Amplitude Function

The knowledge of β is not really needed for obtaining γ_t but we want an expression for $\hat{\beta}$ and we shall prove that the Q-value remains constant. As a measure for the lens strength we introduce the dimensionless parameter:

$$q = \beta_F / f \quad (f : \text{focusing length})$$

With $\psi = \int \frac{ds}{\beta}$ the phase advance of the unperturbed machine counted from the first quadrupole, we have in short lens approximation with initial conditions

$$\Delta\beta = 0 \quad \text{and} \quad \frac{d}{d\psi} \left(\frac{\Delta\beta}{\beta} \right) = 0$$

just before the lens:

$$\Delta\dot{\beta} = -\beta \Delta q \sin 2\psi \quad \text{for small } \Delta q.$$

For large q , we have:

$$\frac{\partial \tilde{\beta}}{\partial q} = -\tilde{\beta} \sin 2\tilde{\psi} \quad (3.1)$$

where $\tilde{\beta}$ and $\tilde{\psi}$ are the new values such that

$$ds = \tilde{\beta} d\tilde{\psi} = \beta d\psi$$

It follows together with (3.1)

$$\frac{d^2 \tilde{\psi}}{dq^2} = \frac{d\tilde{\psi}}{dq} \sin 2\tilde{\psi} \quad (3.2)$$

The system (3.1, 3.2) has a solution which can be written:

$$q = \text{ctg } \psi - \text{ctg } \tilde{\psi} \quad \text{from which}$$

$$\tilde{\psi} = \pi \quad \text{for } \psi = \pi \quad \text{q.e.d.}$$

$$\frac{\tilde{\beta}}{\beta} = \cosh \chi - \sinh \chi \sin(2\psi + \arctg \frac{q}{2})$$

$$\frac{\beta}{\tilde{\beta}} = \cosh \chi + \sinh \chi \sin(2\tilde{\psi} - \arctg \frac{q}{2})$$

$$\frac{q}{2} = \sinh \left(\frac{\chi}{2} \right)$$

It follows:

$$\left(\frac{\tilde{\beta}}{\beta} \right)_{\max}^{\min} = e^{\pm\chi} = 1 + \frac{q^2}{2} \pm q \sqrt{1 + \frac{q^2}{4}}$$

4. The Excursion of the Dispersion Function

A simple way of treating x_p is to consider $\Delta x_p = x_p - x_{p0}$ as a betatron oscillation excited by kicks at the lenses, the magnitude of the kick being proportional to x_p . A convenient measure for Δx_p is

$$\xi = \frac{\Delta x_p}{x_p}$$

ξ must obey the differential equation

$$\frac{d^2 \xi}{d\psi^2} + \xi = -q_k (1 + \xi_k) \delta(\psi_k)$$

With complex notation $z = \xi + i \frac{d\xi}{d\psi}$ and the initial conditions $z(\psi_1) = z_1$ at $\psi_1 = 0$, one finds after a lens of strength q_1 at $\psi_1 = 0$

$$z = \left(z_1 - iq_1(1+\xi_1) \right) \exp(-i\psi) ; 0 < \psi < \pi \quad (4.1)$$

and after the second doublet lens of strength $-q_1$ at $\psi = \pi$:

$$z = (z_1 - 2iq_1) \exp(-i\psi) ; \pi < \psi \quad (4.2)$$

The term $-q_1 \xi_1 \sin\psi$ which occurs inside the doublet deserves particular attention. As we shall see in chapter 5, this term causes the γ_t change. With $\varphi_k = \psi_{k+1} - \psi_k$, the tracing law is

$$z_{k+1} = (z_k - 2iq_k) \exp(-i\varphi_k) \quad (4.3)$$

For n doublets within a superperiod, the periodicity condition leads to

$$z_1 = \left(-\operatorname{ctg} \frac{\phi}{2} + i \right) \exp(i\phi)$$

where $\phi = \sum_{k=1}^n \varphi_k$ is the phase advance over the superperiod.

In the CPS, there are two sets of lenses powered differently. Let $q_1 = -q_2 = T$; $-q_3 = q_4 = D$. The choice $\varphi_3 = \phi$ eliminates the term containing D^2 in the characteristic polynomial P_2 (see 5.2) and simultaneously maximizes the term containing DT if φ_2 is put $3\pi/2 \pmod{2\pi}$ and $\varphi_4 = 5\pi/2 \pmod{2\pi}$. Modifications arise from shifting doublets by π and reversing the polarity. The scheme

$$(T) \pi (-T) \frac{3\pi}{2} (-D) \phi (D) \frac{5\pi}{2}$$

yields with (4.3) and (4.4)

$$\begin{cases} \frac{z_1}{2} = \frac{z_T}{2} = \left(-\operatorname{ctg} \frac{\phi}{2} + i \right) T + D ; \frac{z_2}{2} = \frac{z_{-T}}{2} = \operatorname{ctg} \frac{\phi}{2} T - D \\ \frac{z_3}{2} = \frac{z_D}{2} = \left(-1 + i \operatorname{ctg} \frac{\phi}{2} \right) T - iD ; \frac{z_4}{2} = \frac{z_{-D}}{2} = \left(1 + i \operatorname{ctg} \frac{\phi}{2} \right) T \end{cases} \quad (4.5)$$

from which results

$$\sum_{k=1}^4 q_k \xi_k = 4T \left(2D - \operatorname{ctg} \left(\frac{\phi}{2} \right) T \right)$$

In the CPS, $|\operatorname{ctg} \phi/2|$ could be kept small by going to the antisymmetric disposition which makes

$$\phi = \frac{6.25 - 1}{2} 2\pi$$

and

$$-\operatorname{ctg} \frac{\phi}{2} = \operatorname{tg} \frac{\pi}{8} = \sqrt{2} - 1$$

(a corresponding scheme for the AGS is feasible with $\phi = 8.75/2 \cdot 2\pi$ or $-\operatorname{ctg} \phi/2 = -(\sqrt{2} - 1)$ with lenses placed in FD or DF straights for which the desired phase distances can be approximated rather closely i.e. $\pi \approx 3\frac{1}{2}$ cells; $3\pi/2 \approx 5$ cells; $7\pi/4 \approx 6$ cells).

5. The new Transition Energy

As γ_t is defined by $1/\gamma_t^2 = \Delta C/C$ where ΔC is the additional circumference of a particle with unit momentum displacement $\Delta p/p = 1$ we have to sum over the contributions of all elements $\Delta C = \sum \delta C_k$. The k th doublet contributes to this sum

$$\begin{aligned} \delta C_k = \int \frac{\Delta x}{\rho} ds \approx \frac{x_{pF}}{R \sqrt{\beta_F}} \int_0^\pi \beta^{3/2} \left[\xi_k \cos \psi + \left(\left(\frac{d\xi}{d\psi} \right)_k - q_k(1+\xi_k) \right) \sin \psi \right] d\psi \\ - \frac{x}{pR \sqrt{\beta_F}} 2 \left(\left(\frac{d\xi}{d\psi} \right)_k - q_k(1+\xi_k) \right) \end{aligned} \quad (5.1)$$

When summing over the whole circumference the only remaining terms involve the combinations $q_k \xi_k$ leading to

$$\left. \begin{aligned} \left(\frac{\gamma_{to}}{\gamma_t} \right)^2 &= 1 - c_P P_2 \\ c_P &= 8 \frac{pF}{\pi R^2 \sqrt{\beta}} \quad \text{for the CPS} \\ P_2 &= T \{ 2D + (\sqrt{2} - 1) T \} \end{aligned} \right\}$$

With this P_2, γ_t can be changed to either side as with a first order perturbation by keeping one set (triplets T) fixed and pulsing the other set (doublets D). Applying reasonable criteria, this scheme is superior to those studied previously²⁾.

An additional application has arisen which helps to ease debunching the beam prior to the continuous transfer into the SPS³⁾. The combination $|Z| I \eta^{-1} (\Delta p/p)^{-2}$ should be small (Z : coupling impedance; I : beam current). As long as $|Z|$ is rather high as at present, it is helpful to increase η via a decrease of γ_t . The phase advance over the superperiod ϕ can be made 6.25π by reversing the polarity of one triplet and of one doublet within each of the two superperiods so that P_2 becomes

$$P_2 = -T \left(2D + (\sqrt{2}+1) T \right)$$

The quadratic term is so efficient now that with the (water-cooled) triplet lenses alone, a gain of 2.5 in η is obtained with $T \approx 1$; $\gamma_t \approx 4.32$; $\beta \approx 60$ m; $\hat{x}_p = 23$ m. The peak excursion of x_p is high, but as for this application $\Delta p/p$ is small, the synchrotron width of the beam is not excessive. Also betatron width and synchrotron width add in quadrature to the total beam width; this is rigorous for the rms values of symmetric and uncorrelated distributions, and 4σ are a good measure for the total beam width.

The scheme is fairly flexible and allows for other modes of operation if the criteria (e.g. permissible β , $\hat{x}_p$) are altered.

6. Bunch Matching

Since we shall describe the features of the negative mass instability also in terms of the matching theory, it is justified to recall briefly the most important facts of the matching theory⁴. They are

i) RF forces are linearized, so trajectories are elliptic ;

ii) Space-charge forces are also linearized by assuming a parabolic shape for the linear density which results from phase space density of semi-circle shape across trajectories ;

iii) Time is measured in units of the non-adiabatic time τ for ordinary transition crossing

$$\tau = \left(\frac{\gamma_{cr}^4 |tg \phi_s|}{4 \pi v_{RF} \dot{\gamma}_s^2} \right)^{1/3}$$

and the new time given by $x = t/\tau$; $d/dx = \dot{\gamma}_s$; γ_{cr} is the γ at which transition crossing occurs (i.e. γ_t equals γ_s the value of the particles), ϕ_s the synchronous phase angle and v_{RF} the RF frequency.

iv) Manipulations on γ_t are described by a function $f(x) = \eta/\eta_s'$ where

$$\eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma_s^2} \quad \text{and} \quad \eta_s' = - \frac{d}{dx} (\gamma_s^{-2}) \simeq \frac{2Y}{\gamma_{cr}}$$

Note that $f(x) \simeq x$ for $\gamma_t = \text{const.}$

v) A longitudinal space-charge parameter n_0 is introduced by

$$n_0 = \frac{3\pi^2}{2} \frac{r_p N}{R A^{3/2}} \left(\frac{v_{RF}}{\dot{\gamma}_s} \right)^{1/2}$$

$$n = 0.772334 n_0(0) ; n_0(0) \text{ of ref. 4.}$$

r_p = classical proton radius; $2\pi R$ = machine circumference; N = number of particles; A = bunch area in units of $\Delta(\beta\gamma) \cdot \text{RF-angle}$ (cut-off value); $g_0 \simeq 1 + 2 \ln b/a$ (see 7.i).

vi) A variable θ is introduced denoting the half length of a normalized bunch of area π . θ is related to the physical bunch half length $\hat{\theta}$ by

$$\hat{\theta} = \frac{2\sqrt{A} v_{RF} \gamma_s' \tau}{\gamma_s^2} e$$

θ obeys a second order differential equation. Far away from transition, the equilibrium values of θ are below (-) and above (+) transition given by

$$|f_{\pm}| = \theta_{\pm}^4 \pm n_0 \theta_{\pm} \quad (6.1)$$

A large and fast jump is aimed at such that $\theta_{\pm}$ is re-established $\theta_+ = \theta_- = \theta$. For this case, summation and

subtraction of (6.1) yields

$$\theta = \left(\frac{\Sigma f}{2} \right)^{1/4} \quad \text{and} \quad n_0 = \left(\frac{\Delta f}{2\theta} \right) \quad (6.2)$$

with $\Sigma f = f_+ + |f_-|$ and $\Delta f = f_+ - |f_-|$. For $f_- = 0$, the minimum jump size⁵ is $\Sigma f = f_+ = 2(n_0)^{4/3}$.

The system installed now in the CPS allows at least twice this jump.

The advantages of such a "superjump" are :

i) The mismatch is little sensitive against space-charge parameter variations. This is important at the CPS since each of the four Booster rings has some individuality. Roughly one has :

$$\frac{\partial}{\partial n_0} \left(\frac{\Delta \theta}{\theta} \right) \propto \left(\frac{\Sigma f}{\theta} \right)^{-3/4}$$

ii) For systematic intensity modulations from cycle to cycle, as planned, it is sufficient to change only the timing. This is much simpler than to change current amplitudes too.

iii) The negative mass unstable region is crossed near the peak of $|\dot{\gamma}_t|$, a very important aspect which leads us to the next chapter.

7. The Negative Mass Instability

The linear region of negative mass (n.m.) instability⁶ is characterized by the fact that (small) modulations of the linear density grow exponentially with a growth rate which is a function of both the mode number and, in our case, of time, since we change η rapidly with the aim of reaching the threshold before the modulations have grown too much. In order to treat the phenomenon conveniently, we shall make a few assumptions and simplifications none of which are unrealistic or too rough.

i) The dependence of the geometrical factor g on the mode number is given for a beam of radius a , within a perfectly conducting pipe of radius b , at the axis by

$$g = \frac{4}{w} \left(\frac{1}{w} - K_1(w) - I_1(w) \frac{K_0\left(\frac{wb}{a}\right)}{I_0\left(\frac{wb}{a}\right)} \right)$$

$$\text{with} \quad w = \frac{a k_c}{\gamma_{cr} R} = \frac{\pi a h k_b}{\gamma_{cr} R \hat{\theta}}$$

k_c, k_b : mode number per circumference or per bunch length respectively.
 I_0, I_1, K_0, K_1 : modified Bessel functions.

The space-charge parameter n_0 contains :

$$g_0 = \lim_{w \rightarrow 0} g = 1 + 2 \ln \frac{b}{a}$$

For larger arguments w and b/a not too big, g is approximated by

$$g = \frac{g_0}{1 + \frac{w}{(1.6 \frac{a}{b} + 0.52)}} \quad (7.1)$$

$$g = \frac{g_0}{2} \quad \text{at} \quad k_{c\frac{1}{2}} = \gamma_{cr} R \left(\frac{1.6}{b} + \frac{0.52}{a} \right)$$

using the simplified kernel' for the space-charge potential, together with a numerical fit.

ii) From inspecting the Vlasov equation, it can be concluded that for $k_b \gg 1$, the coasting beam model is a good approximation as proposed earlier by Pease⁸.

iii) For the unperturbed density, the same distribution is assumed as in chapter 6.

iv) The initial modulation is assumed to be given entirely by statistical fluctuations due to the finite number of particles within the bunch $N_b = N/h$. The small amplitude approximation being based on Fourier decomposition, we can calculate the expectation value of the modes when referred to the average linear density $A = N/2\phi$ and find them independent of the mode number and the distribution function to be (see the Appendix)

$$|c_k(0)| = \frac{2}{\sqrt{N_b}}$$

A detailed study of the initial value problem shows that the behaviour after transition crossing as function of the growth rate $G(x, k)$ is

$$|c_k(x)| \approx |c_k(0)| \cosh \int G(x, k) dx$$

so that after a short time

$$|c_k(x)| \approx \frac{\exp \int G(x, k) dx}{\sqrt{N_b}} \quad (7.2)$$

With these assumptions, the growth rate and the threshold can be found via the Vlasov equation and the dispersion relation. The threshold can be expressed conveniently by the parameters of chapter 6 except that

$$n = \frac{g}{g_0} n_0 \text{ is to be used (7.1) : } \frac{n}{n_{th}} = \frac{f}{n\theta} \quad (7.3)$$

This relation is valid along the whole bunch and allows to define a threshold value $f_{th} = n\theta$ which is always smaller than f_+ the value to be reached for matching. The growth rate varies along the bunch as the square root of the linear density and involves additional parameters. At the bunch center, the growth rate is

$$G(x, k_c) = \frac{k_c n}{h} \sqrt{\frac{A |tg \phi_s|}{\pi \gamma_s'}} \frac{1 - \left(\frac{f}{n\theta}\right)}{\sqrt{2 \frac{n\theta}{f} - 1}} \quad (7.4)$$

For $\theta = \text{const.}$ and $f = f'x$, the integral throughout the unstable region which measures the accumulated e-folding times of the n.m. instability is

$$E_{acc}(k_c) = \int G(x, k_c) dx = \frac{k_c n^2 \theta}{h f'} \left(1 - \frac{\pi}{4}\right) \sqrt{\frac{A |tg \phi_s|}{\pi \gamma_s'}} \quad (7.5)$$

E_{acc} has a maximum $\hat{E}_{acc}$ for $k_c = k_{c2}/\sqrt{3}$, yielding

$$k n^2 = \frac{3 \sqrt{3}}{16} k_{c2} n_0^2$$

$\hat{E}_{acc}$ must remain smaller than some critical value E_{crit}

in order to avoid a blow-up which happens when

$$(\Sigma |c_k|^2)^{\frac{1}{2}}$$

of equation (7.2) approaches unity.

The implied summation

$$\sum_k \exp(2E_{acc}(k)) = N_b$$

can be carried out by second order expansion of $E_{acc}(k)$ around E_{crit}

$$E_{acc}(k_b) \approx E_{crit} \left(1 - \left(\frac{3}{4} \frac{\Delta k_b}{k_{b2}}\right)^2\right)$$

k_{b2} has to be taken in order to achieve the proper mode spacing within a bunch. The sum may then be approximated by the error integral yielding the transcendental equation for E_{crit}

$$\frac{k_{b2}}{3} \sqrt{\frac{8\pi}{E_{crit}}} \exp(2E_{crit}) \underbrace{\text{erf} \left[3 \sqrt{\frac{E_{crit}}{8}} \right]}_{\approx 1} = N_b$$

An approximate solution is

$$E_{crit} = \frac{1}{2} \left[\ln N_b - \ln \left(\frac{k_{b2} \sqrt{8\pi}}{3 \sqrt{\frac{1}{2} \ln N_b}} \right) \right] \approx 10$$

($\frac{1}{2} \ln N_b$ is E_{crit} for only one mode; the second term arises from the presence of many modes).

The criterion for no blow-up due to n.m. instability is then

$$\frac{k_{eff} n_0^2 \theta}{h f'} \left(1 - \frac{\pi}{4}\right) \sqrt{\frac{A |tg \phi_s|}{\pi \gamma_s'}} < E_{crit} \quad (7.6)$$

with $k_{eff} \approx \frac{\gamma_{cr} R}{3} \left(\frac{1.6}{b} + \frac{0.52}{a} \right)$; $k_{eff}(\text{CPS}) \approx 2 \cdot 10^4$

This should be considered as a condition for f' indicating how much faster transition energy is to be crossed by the jump. If f' proves to be insufficient, the magnitude of the blow-up might be estimated by applying Dory's law⁹ together with (7.4).

The inequality (7.6) allows also to study other methods, for example, playing with ϕ_s . Since they only provide factors of the order unity, at least in the good direction, we conclude that a fast jump is indispensable*.

If we put $\theta = \left(\frac{\Sigma f}{2}\right)^{\frac{1}{2}}$ and note that n_0^2 contains A^{-3} , we see that the longitudinal phase-space density scales as $N/A \propto N^{1/5} (f')^{2/5} (\Sigma f)^{-1/10}$. The scaling law for bunch matching taken from (6.2) goes as $N/A \propto N^{1/3} (\Delta f)^{2/3} (\Sigma f)^{-1/6}$ and simplifies to $N/A \propto N^{1/3} (\Sigma f)^{\frac{1}{2}}$ if $\Sigma f / \Delta f$ remains constant.

* A promising method at a first glance seems to be passive compensation¹⁰, i.e. reducing n_0 via g_0 . As we need really to reduce $n(k)$ via $g(k)$, a critical frequency characteristic of the wall impedance over a wide range is required to achieve this without over-compensation¹¹. Since that is also sensitive to the density distribution, an awkward problem would have to be mastered.

8. Experimental observations

All quadrupoles are compact in length (< 0.25 m), the doublet quadrupoles (except one) are also compact in cross-section. The present power supplies allow for $|D| \approx 0.45$ and $T \approx 0.82$ corresponding to $\hat{\gamma}_t \approx 7.9$ and $\hat{\gamma}_t \approx 5.6$, the jump is performed within 0.8 ms corresponding to $f' \approx 50$ ($\dot{\gamma}_s = 50 \text{ s}^{-1}$; $\gamma_s' \approx 0.09$).

The scheme came into operation smoothly. Some initial beam loss due to closed orbit perturbation could be cured by centering carefully the mean radial position of the beam. The pulse form of the triplet current is rather uncritical, the peak should approximately coincide with the jump. The doublet current should not rise too quickly after the jump (see Fig. 1). The triple switch on the RF phase gives perfect bunch matching but a single switch is sufficient (see Fig. 2). The bunch size of the booster seems to decrease with increasing intensity. Above $6 \cdot 10^{12}$ p/p in the CPS, a negative-mass instability blow-up was sometimes observed at $A \approx 8$ mrad (in quantitative agreement with (7.6)). In order to improve the situation, more powerful supplies are under design for a larger jump and more than doubled speed aiming at $|D| \geq 0.55$, $T \geq 1.0$, $f' \approx 200-400 \cdot 10^{13}$ p/p could then be handled within a bunch area of 10 mrad or less. The betatron tunes Q_H and Q_V change only by $\Delta Q = \text{a few } 10^{-3}$ in agreement with computations by a lattice programme which also show that the real CPS yields the analytical results of chapters 3-5 very closely, in particular for γ_t .

Appendix

Let N_b = number of particles per bunch, $2\hat{\phi}$ = bunch length, $F(\phi)$ = bunch distribution function normalized by

$$\int_{-\hat{\phi}}^{\hat{\phi}} F(\phi) d\phi = 2\hat{\phi}$$

Subdivide the bunch into M bins indexed by m (or n). Each bin contains $F(\phi)$ AN particles where $AN = N_b/M$ is the average number of particles per bin. Random fluctuations lead to particle numbers per bin different by δN from that determined by the distribution function. The expectation value of that difference is assumed to be given by

$$E \left\{ \frac{\delta N}{\Delta N}(m) \frac{\delta N}{\Delta N}(n) \right\} = \frac{F}{\Delta N} \delta_{mn}$$

The implied step function $f(\phi) \approx (\delta N/\Delta N)(m)$ for $(m-1)/M < (\phi+\hat{\phi})/2\hat{\phi} < m/M$ represented by Fourier expansion is to be

$$\begin{aligned} f(\phi) &= \sum_{k=-\infty}^{\infty} c_k \exp\left(\frac{i2\pi k}{2\hat{\phi}} \phi\right) \rightarrow c_k = \frac{1}{2\hat{\phi}} \int_{-\hat{\phi}}^{\hat{\phi}} f(\phi) d\phi \\ \exp\left(-\frac{i2\pi k}{2\hat{\phi}} \phi\right) d\phi &= \frac{1}{2\hat{\phi}} \sum_{m=1}^M \int_{\frac{m-1}{M}2\hat{\phi}}^{\frac{m}{M}2\hat{\phi}} \frac{\delta N}{\Delta N}(m) \\ \exp\left(-\frac{i2\pi k}{2\hat{\phi}} (\phi + \hat{\phi})\right) d\phi \\ &= \frac{1}{M} \sum_{m=1}^M \frac{\delta N}{\Delta N}(m) \exp\left(-\frac{i2\pi km}{M}\right) \rightarrow E \left\{ c_k \bar{c}_k \right\} \\ &= \frac{1}{M^2} \sum_{m=1}^M \sum_{n=1}^M E \left\{ \frac{\delta N}{\Delta N}(m) \frac{\delta N}{\Delta N}(n) \right\} \exp\left(-\frac{i2\pi k}{M}(m-n)\right) \\ &= \frac{1}{M^2} \sum_{m=1}^M \frac{F}{\Delta N} = \frac{1}{2\hat{\phi}} \int_{-\hat{\phi}}^{\hat{\phi}} \frac{F}{N_b} d\phi = N_b \end{aligned}$$

The modulus of the Fourier coefficient c_k is composed of c_k and c_{-k} so that $|c_k| = 2/\sqrt{N_b}$ q.e.d.

Acknowledgements

The hardware for the present γ_t -jump system was provided under the responsibility of F. Rohner, the power supply built by H. Dijkhuizen. The scheme was brought into operation by M. Bôle-Feyso, E. Brouzet, J. Gareyte, J. Guillet, R. Ley, F. Rohner, G. Roux and further people. With D. Möhl and F. Sacherer, I had fruitful discussions on the theory. B. Schorr contributed to the subject in the appendix.

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Figure 1
Triplet current
(upper trace).
Doublet current
(lower trace).
Sweep : 5 ms/div.

Figure 2
Wide-band pick-up
station signal,
inversely propor-
tional to bunch
length. First
peak : doublet
current starts
rising. Second
peak : Jump.
 $N = 5.5 \cdot 10^{12}$ p/p.
Sweep : 10 ms/div.

DIFOLE TWO-BEAM INSTABILITY IN CIRCULAR BEAMS
WITH AZIMUTHAL NON-UNIFORMITY

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Summary. The modulation of the beam transversal dimensions in the strong focusing accelerators causes the spread of the oscillation frequency of the electrons or ions which are created by ionization of residual gas. This paper will discuss the influence of such frequency spread on the stability of the two-beam dipole transversal oscillations.

After publication of paper ¹ on two-beam instability in electron-ion rings some theoretical ²⁻⁵ and experimental investigations on two-beam instabilities in circular accelerators and storage rings have been made (such instabilities appear due to the interaction of the rotating beam with ionization particles-electrons or ions created by ionization of residual gas). It seems that the growth of the beam transversal dimensions observed at the Bevatron ⁶ and at the CERN storage ring ⁷ is due to two-beam instabilities.

It is evident that in high energy accelerators and storage rings such instabilities have some specific features which are beyond the theory developed in paper ¹. In this work we will mainly deal with the effects which are connected with azimuthal non-uniformity of interacting beams. Besides, some other effects have been considered qualitatively.

1. Influence of Transversal Dimension Modulation of Rotating Beam

In paper ⁴ it is noted that the transversal dimension modulation causes the spread of the oscillation frequency of the ionization particles and, consequently, affects the instability increment and the intensity limits.

Let us consider this problem by means of the linear hydrodynamical theory neglecting

the image forces and forces of the coherent interaction between particles with the same charge sign. Besides, let us restrict ourselves by the vertical motion and suppose that the longitudinal density of the rotating beam is uniform.

Under these assumptions the transversal motion equation of the rotating particles (index 1) and equation of ionization particles (with index 2) will be of the following form

$$\frac{d^2 z_1}{dt^2} + \Omega_1^2(x) z_1 = \Omega_{12}^2(x) \bar{z}_2 \quad (1)$$

$$\frac{d^2 z_2}{dt^2} + \Omega_2^2(x) z_2 = \Omega_2^2(x) \bar{z}_1 \quad (2)$$

In (1) t - the time, z - transversal coordinate, z - the beam centre gravity coordinate, x - the length along the equilibrium trajectory, $\frac{d}{dt}$ - the time derivative.

Supposing the first and the second beam to have the uniform longitudinal densities and the same elliptical cross sections we get the following expression for $\Omega_2^2(x)$ and $\Omega_{12}^2(x)$:

$$\Omega_2^2 = (\Omega_2^*)^2 \alpha^2(x) \quad (3)$$

$$\Omega_{12}^2 = (\Omega_{12}^*)^2 \alpha^2(x)$$

where $\alpha(x)$ is the coefficient describing the azimuthal modulation of transversal dimensions, Ω_2^* and Ω_{12}^* are the values of Ω_2 and Ω_{12} corresponding to the beam with the circular constant radius cross section, Ω_2^* and Ω_{12}^* being determined by

$$\begin{aligned} (\Omega_2^*)^2 &= (1-\eta_1) \Omega^2 \frac{m r_e z_2 z_1 N_1 Q_1}{\pi M_2 V} \\ (\Omega_{12}^*)^2 &= \eta_1 \Omega^2 \frac{m r_e z_2 z_1 N_1 Q_1}{\pi M_1 \gamma_1 V} \end{aligned} \quad (4)$$

where η_1 - the neutralization factor, N_1 - the number of particles in the chamber, the transversal phase volume of the rotating beam, z_1 and z_2 are the particle char

ges, M_1 and M_2 are the particle masses, γ_1 - the relativistic factor, r_e and m are the classic radius and the mass of the electron, Q_1 - the vertical oscillation betatron frequency*

Neglecting the Floquet function modulation due to the random harmonics of the magnetic field gradient we obtain:

$$\mathfrak{z}_1^2 = \frac{2R\Delta}{Q_1 g_z (g_r + g_z)} = \frac{2}{\langle \frac{1}{g_z^2} \rangle g_z (g_r + g_z)} \quad (5)$$

where $2\pi R$ the accelerator length, $\Delta = 1$ m, g - the Floquet function modulus, $\langle \rangle$ means the averaging over the accelerator length. Note that $\mathfrak{z}^2$ is the periodic function of x with the period length $2\pi R/s$ (s is the number of magnetic structure periods along the accelerator length).

In the variables

$$y = \frac{z}{g_z(x)}, \quad d\psi = \frac{\Omega R dt}{Q_1 g_z^2(x)} \quad (6)$$

eq. (1) has the following form

$$\frac{d^2 y_1}{d\psi^2} + Q_1^2 y_1 = \frac{Q_1^2 g_z^4(\psi) \Omega_{12}^2(\psi)}{R^2 \Delta^2 \Omega^2} \bar{y}_2 \quad (7)$$

If the right-hand part of eq. (7) is time-independent, then the solution of this equation cannot be represented as orthogonal Fourier series in ψ . Let us try to solve the system including eqs. (2) and (7) in the following form

$$\bar{y}_{1,2} = \sum_{n=-\infty}^{\infty} \bar{y}_{1,2}^n \exp[i(n\psi - \omega t)] \quad (8)$$

Substituting $\bar{z}_1 = g_z \sum_{n=-\infty}^{\infty} \bar{y}_1^n \exp[i(n\psi - \omega t)]$ into eq. (2) and taking into consideration that all the ionization particles have the same frequency for given ψ we find

$$\bar{y}_2 = \sum_{n=-\infty}^{\infty} \frac{\bar{y}_1^n \Omega_{12}^2(\psi)}{\Omega_{12}^2(\psi) - \omega^2} \exp[i(n\psi - \omega t)] \quad (9)$$

After the substitution (9) into (7) we get the following equation for y_1

$$\frac{d^2 y_1}{d\psi^2} + Q_1^2 y_1 = \sum_{n=-\infty}^{\infty} \frac{\bar{y}_1^n \Omega_{12}^2 \Omega_{12}^2 g_z^4 Q_1^2}{R^2 \Delta^2 \Omega^2 (\Omega_{12}^2 - \omega^2)} \cdot \exp[i(n\psi - \omega t)] \quad (10)$$

The general solution of eq. (10) is determined by the following system of equations

$$\bar{y}_1^{n+ks} = \sum_{K=-\infty}^{\infty} C_{K-K'} \bar{y}_1^{n+KS}.$$

$$\langle [Q_1^2 \Omega_{12}^2 - (n\Omega + K'S\Omega - \omega)^2]^{-1} \rangle \quad (11)$$

where $K' = 0, \pm 1, \pm 2, \dots$, the sign $\langle \rangle$ means the averaging over longitudinal momentum. Thus, the amplitudes of harmonics with the numbers differing by ks are connected by an infinite system of linear equations; supposing that the determinant of this system is equal to zero we get the exact dispersion relation. The resonance condition

$$\omega \approx (n + K'S)\Omega - Q_1\Omega \quad (12)$$

may be, however, realized only for one value of K . Besides, if the focusing is not too strong, $|C_0| \gg |C_{K-K'}|$ (for $|K-K'| \neq 0$). Taking into account of these considerations

we may neglect all the harmonics except the resonant one starting with $K=K'=0$; then after substitution (3) and (4) into (11) we get the following dispersion equation

$$1 = -(\Omega_{12}^* \Omega_{12}^*)^2 \int_{-\infty}^{\infty} \frac{f(p) dp}{Q_1^2 \Omega_{12}^2 - (n\Omega - \omega)^2} \quad (13)$$

where $f(p)$ is normalized p distribution function.

Of course, without dimension modulation the dispersion relation coincides with that for nonmodulated beam obtained in ¹.

Taking into account only resonant terms in the denominator let us write the dispersion equation (13) as

$$1 = -\xi^2 \int_{u_1}^{u_2} \frac{\varphi_1(u) du}{\lambda - u - \eta} \int_{v_1}^{v_2} \frac{\varphi_2(v) dv}{\lambda - v} \quad (14)$$

where

$$\lambda = \frac{\omega}{\Omega_0} - Q_2^*, \quad u = \frac{(n - Q_1)\Omega}{\Omega_0} - (n - Q_1^0),$$

$$\eta = n - Q_1^0 - Q_2^*, \quad v = Q_2 - Q_2^*, \quad (15)$$

$Q_2 = \Omega_2 / \Omega_0$, ψ_1 and ψ_2 are normalized distribution functions, index "0" corresponds to the central particle of the rotating beam. Here we supposed that the shape modulation is not very strong and therefore

$$\frac{2\Delta}{\pi Q_1 (q_z + q_z)^2} \approx (2\pi R)^{-1}$$

If longitudinal ion density does not depend on x we obtain the following formula for the function $\psi_2(v)$

$$\psi_2(v) = \left| \frac{dv}{dx} \right|^{-1} S (2\pi R)^{-1}$$

Parameter η — the frequency shift with respect to the resonance line center, ξ — the monochromatic beam increment divided by the rotation frequency;

$$\xi^2 = \frac{Q_2^* Q_{12}^2}{4Q_1} \approx \frac{Q_2^* \Delta Q_{12}}{2} \quad (17)$$

where ΔQ_{12} is the rotating particle frequency shift due to the ionization particles. It is seen from (14) and (16) that the beam dimension modulation results in Landau damping. However, this damping has some specific features because $\psi_2(v) \rightarrow \infty$ near the Floquet function extremum (when $\frac{dq}{dt} \rightarrow 0$). It is known, that the Landau damping becomes small if the distribution functions have infinite magnitude. To illustrate this effect let us consider a simple example

$$v = \Delta Q_2 \cdot \cos\left(\frac{5x}{R}\right), \quad (18)$$

$$\psi_2(v) = \frac{1}{2\pi} (|\Delta Q_2|^2 - v^2)^{-1/2} \quad (19)$$

Substituting (19) into (14) we obtain

$$1 = -\xi^2 [\mathcal{L}^2 - (\Delta Q_2)^2]^{-1/2} \int_{u_1}^{u_2} \frac{\psi(u) du}{\mathcal{L} - u - \eta} \quad (20)$$

Here $\mathcal{L}$ is the complex variable with $\text{Im } \mathcal{L} > 0$.

For $\psi(u) = \delta(u)$ eq. (20) has the following form

$$\sqrt{\mathcal{L}^2 - (\Delta Q_2)^2} (\mathcal{L} - \eta) = -\xi^2 \quad (21)$$

The solution of eq. (21) depends on the parameters $A = \Delta Q_2 / \xi$ and $B = \eta / \xi$. It may

be shown that the stability limit is determined by

$$|B| \leq B_{\text{lim}}(A) \quad (22)$$

The graph $B_{\text{lim}}(A)$ is given in Fig. 1.

Fig. 1

It is seen that at $A = 0$ $B_{\text{lim}} = 2$. When A increases the instability region is expanded and if $A \gg 1$

$$B_{\text{lim}}(A) \approx A + 1.5A^{-1/3} \quad (23)$$

Within the instability region for one of the roots $\text{Im } \mathcal{L} > 0$.

Fig. 2 shows the dependence $\chi = \text{Im } \mathcal{L} / \xi$ on B and A .

Fig. 2 $A = \frac{n - Q_1^0 - Q_2^0}{\epsilon}$

It is seen from the graph that if $A \gg 1$ X achieves its maximum value at $B \approx A$; this maximal value of X is determined by

$$\varepsilon X = \Gamma m \frac{\omega}{S_0} \approx \frac{\sqrt{3}}{2} \frac{\varepsilon^{5/3}}{(2\Delta Q_2)^{1/3}} \quad (24)$$

So we can see that the frequency spread in one of the beam results in the increase of the resonance width accompanied by some reduction of the increment value. The value of these changes depends on the form of the distribution function.

It seems that this effect causes "Landau antidamping" mentioned by Chirikov⁸.

Analysis of eq.(20) shows that for nonmonochromatic beams the stability condition has the form

$$\langle \Delta Q_1 \rangle \gg \frac{K_1 \varepsilon^2}{\sqrt{(\Delta Q_2)^2 - \eta^2}} \quad (25)$$

where K_1 is the coefficient of order of 1 depending on the form of distribution in u and $\langle \Delta Q_1 \rangle$ is the dispersion of this distribution.

It is seen from (25) that if the rotating particle oscillation frequency coincides with the oscillation frequency of ionization particles in the Floquet function extremum (i.e. at $n = Q_1 + Q_2^* \pm \Delta Q_2$) the dipole oscillations are unstable for any rotating particle frequency spread.

If we formally introduce an additional frequency spread of the ionization particles depending on variable z , the corresponding dispersion integral in eq.(15) will have the modified form

$$\int_{v_1}^{v_2} \frac{\varphi_2(v) dv}{\alpha - v} \rightarrow \int_{v_1}^{v_2} \int_{z_1}^{z_2} \frac{\varphi_2(v) \varphi_2(z) dv dz}{\alpha - z - v} \quad (26)$$

where $\varphi_2(z)$ is the distribution function in z .

The stability condition(25) is also modified and for the most dangerous modes (with $n = Q_1 + Q_2 \pm \Delta Q_2^*$) is determined by

$$K_2 \langle \Delta Q_1 \rangle > \sqrt{\Delta Q_2 \langle \Delta Q_2 \rangle} > \varepsilon^2 \quad (27)$$

where $\langle \Delta Q_2 \rangle$ is the "trial" ionization particles frequency spread connected with variable K_2 is the coefficient depending on $\varphi_2(z)$.

In fact, $\langle \Delta Q_2 \rangle$ may appear only due to non-

linearity of the potential well which stabilizes the ionization particles motion, and consequently $\langle \Delta Q_2 \rangle$ depends on many factors, such as the rotating particles distribution in phase space, the Energy distribution of ionization particles and so on.

If $\eta_1 \ll 1$ and the transversal phase space density of rotating particles is uniform $\langle \Delta Q_2 \rangle / Q_2^* \sim 0.03$. Substituting in to (28) $\Delta Q_2 / Q_2^* = 0.2, \langle \Delta Q_2 \rangle / Q_2^* = 0.03$, and $K_2 = 2$ we get the following stability condition:

$$1.5 \langle \Delta Q_1 \rangle > \Delta Q_{12} \quad (28)$$

where ΔQ_{12} is the betatron frequency shift of rotating particles due to the interaction with the ionization particles. Of course, formula(28) is approximate as it is derived from the hydrodynamical theory. Besides, it should be reminded that this analysis was made neglecting the image force effects resulting in appearance of the shift between the coherent frequency and the incoherent oscillation frequencies. It seems that the Landau damping disappears if this frequency shift exceeds the corresponding incoherent frequency spread.

2. Neutralization Factor and Ionization Particle Life-Time.

The calculation of the neutralization factor is a nontrivial problem. Firstly let us first consider a debunched beam; if the ionization particle life-time is equal to τ_1 , then the neutralization factor is determined by

$$\frac{d\eta_1}{dt} = \mu(\eta_1) - \eta_1 / \tau_1 \quad (29)$$

where μ is the ion birth-rate depending on η_1 due to the variation of the potential well depth (at $\eta_1 \rightarrow 1, \mu(\eta_1) \rightarrow 0$). Thus, the equilibrium value of η_1 is derived from the expression

$$\mu(\eta_1) \tau_1 = \eta_1 \quad (30)$$

Particularly, at $\tau_1 \rightarrow \infty, \mu(\eta_1) \rightarrow 0$ and $\eta_1 \rightarrow 1$.

If the "clearing" is absent the ion life-time is determined by the various coherent and incoherent effects, while in the presence of "clearing" the life-time is determined by the drift time up to the "clearing" electrodes.

If the instability has already appeared the ionization particle life-time begins to depend on its increment. The character of this dependence is sophisticated. For example, if the instability takes place at some definite points of the ring, the longitudinal components of the ionization particles field results in their motion to the loss place. On the other hand, these ionization particles induce the desorption of the molecules in the molecular layer and thereby deteriorate the local vacuum and increase the leakage rate⁷. Note that the influence of finite life time¹⁰ leads to variation of the particles dynamics. With the account of this effect eq.(16) is modified.

$$\int_{v_1}^{v_2} \frac{y_2(v) dv}{2-v} \rightarrow \int_{v_1}^{v_2} \frac{y_2(v) dv}{2+i\delta-} \quad (31)$$

The analysis of the modified dispersion equation (31) shows that outside the region of the coupling resonance this effect results in appearance of weak instability; inside the resonance region this effect, formally analogous to coherent radiation¹⁰ diminishes the increment.

The situation considerably changes with the presence of the r.f. field because the rotating particles bunching creates some "alternating field focusing" for electrons or ions⁵. Of course, the bunching influence may be neglected if

$$Q_2^* \ll q \quad (32)$$

where q is the harmonic number. If inequality (32) is not fulfilled, then to determine q_1 one needs in detail calculation of the stability regions⁵. We believe that the electrons may be accumulated only in the first stability region due to the fluctuation of density from bunch to bunch (such fluctuation may be artificially increased that allows one to check the presence or the absence of effects caused by the electrons oscillating in the high order stability regions. As a rule, the inequality (32) is correct for electrons and therefore in the positive ion circular accelerator a great number of electrons may be accumulated only in the debunch-regime or if the accelerated ions are injected at the constant magnetic field (with synchronous phase $\psi_s = 0$). Consequently, the use of such regimes requires a special attention to "proton-electron" instability.

Conclusion

We see that the ionization particle frequency spread due to the dimension modulation of the rotating beam does not stabilize the dipole two beam instability even in the presence of momentum and frequency spread of the rotating beam. Let us remind that according to the results of paper¹, the stability region may exist only if the frequency spread is present in both beams. Consequently the given necessary condition is insufficient to provide the stability.

The evaluation shows that the two-beam instability may essentially affect the beam parameters in the accelerators and the storage rings if the circulating beam have the uniform density during some nonzero time interval (for example, during the constant field injection or slow injection). Correct calculations of the instability limit is complicated due to the difficulties in the evaluation of the neutralization fac-

tor. However, it is possible to give a simple expression to evaluate the maximal beam disturbance due to the electron-proton dipole interaction in the proton synchrotrons.

Note that in the electron-proton instability the electron oscillation amplitudes exceeds the proton oscillation amplitudes. On the other hand, the electron coherent oscillation amplitude is limited by the chamber dimensions. This allows one to make a simple estimate of a maximal possible effect during the time interval t after the proton injection. Substituting $y_2 = y_{max}$, $\eta_1 = \mu t$ into (2) we get

$$\frac{y_1}{y_{max}} = \frac{\mu \Omega_0 t \omega^2 r_e Z_1 \cdot Z_2 N_1}{2\pi M_1 \gamma_1 V} \quad (33)$$

Substituting the ITEF proton synchrotron data into (33) ($Z_2 = Z_1 = 1$, $\omega = 200 \text{ sec}^{-1}$, $V = 7 \cdot 10^{-5} \text{ m.rad}$, $N_1 = 10^{12}$, $\Omega_0 = 2\pi \cdot 2.5 \cdot 10^5 \text{ sec}^{-1}$) we find

$$\frac{y_1}{y_{max}} = 10^7 \text{ to } 2 \text{ sec}^{-2}$$

i.e. the instability begins to be dangerous at $t = 0.1 \text{ msec}$. Accounting for the difficulty of the correct calculation, the formula (33) may be useful for evaluation of the possible effect in the positive ion circular accelerators.

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THE INTERACTION OF A LONG INTENSIVE ELECTRON BUNCH WITH A PASSIVE CAVITY

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Abstract

The change of energy distribution in an initially monochromatic long electron bunch due to radiation field induced by the bunch in a passive cavity is investigated. The characteristics of "autoacceleration" for a long bunch is obtained, the "head-tail" effect dependence on energy for a short intensive bunch is evaluated and electron ring defocusing averaged over the trajectory is calculated.

The charged bunch passing through a cavity resonator induces the radiation field. Due to this field the bunch as a whole is being retarded. For a long bunch the distribution of the radiation reaction force on a separate particle is not uniform along the bunch. The analysis of this radiation reaction force distribution is of great importance in several applications. It may be noted three of them.

1. The possibility of "autoacceleration" of an electron beam passing through a cavity by the induced field [1]. If the sign of energy which transferred to the separate particle by the longitudinal component of the radiation field changes along the bunch some part of the particles will be accelerated.

2. The influence of radiation also must be taken into account in design of the electron accelerator with intensive bunches [2]. The action of radiation field results in an energy spread of the particles, that may be comparable to energy gain due to the accelerating field.

3. The diffraction radiation may have a significant effect upon longitudinal ring stability in electron ring accelerator. The approximate estimation [3] shows that for the periodical structure diffraction radiation

defocusing force is proportional to the total energy loss of the electron ring.

The points in question will be investigated for a model of cylindrical cavity (radius b , length $2d$) with entrance and exit tubes of radii a [4]. Let the bunch with total charge Q is an uniformly charged filament of length $2h$, that moves along the axis of structure with constant velocity $v = \beta c$. We introduce the coordinate ξ along the bunch with the origin in its centre and normalised to its length ($-1 \leq \xi \leq 1$).

The diffraction radiation reaction force on electron with coordinate $\mathbf{k}$ in the $\mathbf{z}$ point of trajectory is equal

$$T(\xi, z) = e E_z(\vec{r}, t) \Big|_{z=0}^{t=(z-\xi h)/v} \quad (1)$$

where E_z is longitudinal component of electric field induced in the structure by the bunch. We will investigate the radiation reaction force work upon a separate electron during the flight through the structure

$$F(\xi) = \int_{-\infty}^{\infty} T(\xi, z) dz \quad (2)$$

The electromagnetic radiation field may be found following the references [4, 5], where the excitation of cavity with tubes by given current have been investigated.

The total field is presented as a sum $\vec{E}^0 + \vec{E}$ and only the radiation field $\vec{E}$ have been taken into account in calculations of radiation reaction force work: this field for each frequency may be presented by the superposition of waveguide modes. The expression of radiation reaction force work upon the separate charge with coordinate ξ is

$$F(\xi) = -4ea \int_{-\infty}^{\infty} d\omega e^{i\frac{\omega}{v}\xi h} \sum_{n=1}^{\infty} \left\{ \frac{v_n}{J_1^2(v_n) [(ra)^2 + v_n^2]} \right\} \times$$

$$[F_n(ih_n \sin \frac{\omega}{v} d - \frac{\omega}{v} \cos \frac{\omega}{v} d) + G_n(h_n \cos \frac{\omega}{v} d - i \frac{\omega}{v} \sin \frac{\omega}{v} d)] + \\ + i \frac{\beta}{a} \frac{v_n \exp(i g_n d)}{(\Gamma \beta)^2 + v_n^2} [K_n(\frac{\omega}{v} \sin g_n d \cos \frac{\omega}{v} d - g_n \cos g_n d \sin \frac{\omega}{v} d) + \\ + L_n(g_n \sin g_n d \cos \frac{\omega}{v} d - \frac{\omega}{v} \cos g_n d \sin \frac{\omega}{v} d)] \} \quad (3)$$

where $\Gamma = k/\gamma\beta$, $k = \omega/c$, $\gamma = (1 - \beta^2)^{-1/2}$, $g_n = \sqrt{k^2 - (v_n/\beta)^2}$, $h_n = \sqrt{k^2 - (v_n/a)^2}$ and the coefficients F_n, G_n, K_n, L_n can be found from the infinite set of linear algebraic equations, which was outlined in [5]. This coefficients have the simple poles at the resonant frequencies of the structure and the integration path in ω -plane in (3) must come above those poles through a small semi-circles. Thus the integral (3) may be expressed in terms of the principal value of integral and half of residues in those poles. The determination of coefficients and evaluation of principal value of integral and the residues in poles have been found by numerical methods.

We begin to consider the results of calculation with discussion of "autoacceleration" possibility. The plot of reaction force work $F(\xi)$ for different length bunches is presented in the Fig. 1. The function $F(\xi)$ is normalised by factor $f_0 = \frac{N}{N_0} \beta^{-1}$, where N - the total number of particles in the bunch, $N_0 = 10^{12}$, β - is expressed in centimeters; then the value $F(\xi)$ is in MeV. The autoacceleration becomes possible for $h > \beta/2$. For $h \gg \beta$ the beam after passing a passive resonant cavity is turned out to be energy modulated at the frequency close to main resonant mode ones. If the total bunch charge is fixed, then the modulation amplitude varies inversely as its length. If we will determine the acceleration efficiency as the total energy gain of all accelerated particles then this quantity represents by integral $\int F(\xi) d\xi$, where the integration region include only that ξ for which $F(\xi) > 0$. Fig. 2 shows the total energy of accelerated particles vs bunch length. Optimal case is $h \approx \beta$.

As it can be seen from Fig. 1, for length of bunch much less than the wavelength of main resonant mode (this case is typical for using of a resonator as an active accelerating device) the action of radiation reaction force gives the energy spread in the beam. For example, the energy spread for the 20 cm length bunch with $N = 10^{12}$ after passing the resonator with $b = 20$ cm, $d = a = 5$ cm is equal 40 KeV for $\gamma = 1.01$. The energy spread vs relativistic factor γ is shown in Fig. 3; it quickly reaches the limiting value as γ increases.

The bunches in the electron ring accelerator have a small axial extent, which is constant in the ring frame. Our calculation shows that the radiation field gives a defocusing effect and for realistic ring dimension the distribution of the radiation force work $F(\xi)$ is approximately linear. The ratio of average over the trajectory magnitude of defocusing force due to the radiation $T = [F(\xi=1) - F(\xi=-1)] \gamma/2h$ to the total energy loss $W = \frac{1}{2} \int_{-1}^1 F(\xi) d\xi$ in function of γ is shown in Fig. 4. The data are obtained for a thin electron ring with radius $\rho = 0.2a$. For a single cavity the presented ratio increases linearly with γ as distinguished from results in [3]. This discrepancy may be explained by different character of the radiation loss energy dependence for a single cavity and a periodic waveguide structure.

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Fig. 1

Fig. 2

Fig. 3

Fig. 4

BEAM EXTRACTION

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Introduction

Extraction of beams from synchrotrons has progressed over the past few years from an interesting accelerator-physics problem to an important engineering problem, the success of which determines in one sense the ultimate intensity at which machine operation is feasible. The original Piccioni extraction was a natural step away from internal-target physics. Resonant extraction then provided more efficient extraction with less loss and induced radioactivity. The extraction process could be more easily understood and analyzed. Simultaneous extraction to as many as two external lines has been accomplished^{1,2} as has the simultaneous use of an internal target and an extracted beam.^{3,4,5,6} Fast spill necessary for neutrino and bubble-chamber experiments has been obtained by fast kickers⁶ and more recently by shaving^{7,8} and fast resonant extraction.⁹

With the advent of large machines, the philosophy of extraction has changed. Effort has been made to isolate the various experimental areas from the machine proper. No longer are hadron beams made directly in the machine and radiation problems associated with targeting intense high-energy primary beams have been relegated to the various laboratories in the experimental area, where designs specifically directed at the radiation problem can be carried out more easily. The only severe radiation problem associated with the accelerator proper thus becomes the beam-extraction efficiency. The invention of the wire electrostatic septum¹⁰ has been a major step in pushing the extraction efficiency from around 80% using magnetic septa to approximately 97%. Large machines have a slow cycle rate and it has become necessary to provide in each cycle slow extracted beam, bubble-chamber beam, and high-intensity fast extracted beam for weak interaction physics.

Large machines and storage rings have also produced a different type of extraction problem, that is, the problem of extracting from a booster, transporting, and injecting into a larger higher-energy machine or storage ring.

Resonant Extraction

Three types of resonant extraction are used in accelerators at the present time; they are integer, half-integer, and third-integer. Integer and half-integer modes are similar in that they are linear resonances to which a nonlinear perturbation has been added in order to obtain a gradual slow spilling of the beam. The third-integer resonance is nonlinear and does not produce any intrinsic linear instability.

Integer and half-integer extraction rely

on a perturbing quadrupole or set of quadrupoles to both shift the machine tune toward $2\nu = \text{integer}$ and to generate a stopband such that if the perturbing quadrupole is increased sufficiently or if the unperturbed machine tune is changed, it is possible to get inside the stopband. In this case, the solution of the equations of motion predicts exponential growth of all particle orbits (except for an unstable fixed point of zero phase-space population). Tuning to the integer has the disadvantage that equilibrium-orbit distortions may become large. In both the integer and half-integer cases the amplitude function around the machine is modified by the perturbing quad and a superperiodicity of $\Theta(\phi)$ and $\alpha(\phi)$ established. This structure has a repetition rate of 2ϕ , where Θ is the normal betatron phase advance. The β distortions are analytically similar to equilibrium-orbit distortions and the phase of the perturbed amplitude can be rotated by changing the ratio of strengths of two or more perturbing quads. Beta distortions produced by main quadrupole errors are probably better behaved at the half-integer than dipole equilibrium-orbit distortions at the integer produced by main dipole errors. This is especially true for separated function machines where there are a great many more dipoles than quadrupoles. The change in β with the excitation of the perturbing quad produces phase space ellipses that are very long. At the azimuthal location of the extraction septum, the ellipses should lie approximately along the x axis of the x, x' phase plot. When the stopband is entered, the phase ellipse becomes hyperbolic.

For slow extraction, it is necessary to generate a phase-stable region that slowly shrinks so that only a few particles become unstable in any short time interval. To generate this region, a nonlinear element (either intrinsic to the machine or introduced for this purpose) must be used. In the case of integer extraction, this element is generally a sextupole and thus produces one fixed point and one extraction phase-space trajectory. (An octupole nonlinearity could possibly produce the trajectories for simultaneous extraction of two beams.) In the half-integer system, the octupole must be used because the sextupole effect would be cancelled on successive turns.

The third-integer extraction relies on sextupoles themselves to supply both the nonlinearity and to set the phase of the extraction trajectory. The size of the stable region is controlled by how far the tune is from $\nu = 1/3$ or $2/3$. In some cases, a "semi-quad" or similar magnet is used instead of a sextupole, but has the same function.^{4,11}

It is not clear what criteria should be used in selecting one particular resonance over the others. The integer resonances have

possible equilibrium-orbit problems. In half-integer extraction, the separatrix can be considerably curved; consequently the extracted beam phase space will be diluted. In third-integer extraction, octupole components of the guide magnetic field may make extraction difficult. Because there is no stopband, it is also possible to pass through the resonance and either leave some unextracted particles or extract from the other side of the resonance and thus dilute the extracted phase space and increase the extraction losses. Third-integer extraction is presumably least sensitive to magnet ripple, because the growth of the resonant beam near the fixed points is slower than for the other two cases, but the ripple should not sweep the beam tune back and forth through the resonance.

Third integer extraction seems to be better than integer if beam is shared with an internal target. Apparently this is so because of the small momentum aperture of the integer scheme.³ Simultaneous third- and half-integer extraction to two experimental areas has succeeded.^{1,12} Fast spill must also be provided in accelerators; linear resonances make some forms of fast extraction very easy and inexpensive.

In separated-function machines, new considerations enter. There are usually substantially fewer machine quadrupoles than bending magnets. It is therefore likely that the machine octupole moment will be much less important than the sextupole moment. The octupole moment will then be easily controlled for half-integer extraction and the sextupole moment will not matter except for its effect on tune changes caused by bending-magnet power supply ripple. For third-integer extraction, both sextupole and octupole field components may have to be corrected out. Another factor to consider is that quad and bend magnets may be separately powered and the sensitivity to the magnetic-field ripple in both types of magnets should be investigated. CERN has undertaken a study of all three resonant extraction modes for the SPS and probably quantitative comparisons will be forthcoming.^{13,14}

Beam Spill Structure

The reduction of slow-beam spill structure to the point where effective spills are a large fraction of the available spill time is important to the experimental program at all accelerators doing counter experiments. Typically, one or more of the following approaches are used to reduce the spill structure. Power supply ripple is reduced by active or passive filtering and phase adjustment of thyristors or ignitrons. This is done either through feedback from devices which measure the voltage, current, or magnetic field ripple or through manual correction. At the AGS, for instance, the ripple is analyzed in terms of frequency components and manual correction is made by the operators. In some cases an auxiliary device is used to compensate for ripple from the main supply. In most cases, rf structure is not a problem because either the beam has been purposely debunched or the rf has been turned off so that it naturally debunches. At NAL, the rf fre-

quency is high enough that rf structure has not yet become as important as other spill-factor problems. Betatron acceleration is proposed in a paper at this conference as a means of obtaining beams of small instantaneous momentum spread with good spill structure.²⁰ Typical power-supply ripple is approximately 10^{-4} in $\Delta B/B$ at frequencies between 10 and 1000 Hz. At higher frequencies than this where filtering is easier, the ripple is much less. The beam-spill structure itself can be used in feedback loops. This use is discussed in two papers at this conference and is also done at other laboratories.^{2,9} Debunching the beam by the rf to produce a wide momentum spread is used at the AGS and the PS to help control spill.

Rf structure can often be used for time of flight and its quantization of the beam leads to ease in setting up coincidence-circuit gate widths. At the Bevatron, rf knock-out to drive coherent betatron oscillations has been suggested as a means of producing sharp bursts of extracted particles.¹⁶

Most laboratories can obtain what they consider to be a good spill structure with a peak-to-valley ratio of at least 3 to 2. What path is taken and how it is pursued to improve the spill depends to a great extent on personal preference and cost considerations. (Straightforward power-supply filtering is expensive in large systems and at low frequencies.)

In high-energy, high-intensity machines it is likely that only a fraction of the beam will be used in the slow spill. Beam-structure problems will probably become very difficult when say only 10% of the beam is slow extracted over the magnetic flat top of the order of 1 second. Usually the beam phase space in the machine does not grow linearly with intensity. However, extraction of a small fraction of the beam must come from a small fraction of the phase space and the rate of change of phase space will determine the spill quality.

Extraction Efficiency

To first order, the extraction efficiency is given by the septum thickness divided by the resonant step size for n turns of a particle with an oscillation amplitude equal to the distance of the septum from the equilibrium orbit. (n is 1, 2, or 3 depending on whether the resonance is integer, $1/2$ or $1/3$.) For semitransparent septa such as wire electrostatic septa, the extraction efficiency is expected to be better than the purely geometrical prediction.

Extraction efficiencies in machines with magnetic septa are usually 70 to 80%. At Brookhaven, for instance, the calculated efficiency of 85% is also the best observed efficiency.⁶ Internal targets are used in some laboratories to produce secondary-particle beam lines. These targets are relatively thick and have a non-negligible effect on the extracted beam efficiency and emittance. (These targets should not be confused with those of the NAL internal-target area, which uses very light thin targets like 25 microns of polyethylene fibers and does not interfere

with the machine operation and extraction.) Generally, the extraction efficiency drops another 20% if internal targets are used to interact with 10 to 20% of the beam.⁵ CERN (PS) uses an electrostatic septum and obtains 97% extraction efficiency with no internal target. The efficiency with internal targets using up to 40% of the beam stays above 90% and agrees very well with calculations.⁴

Extraction efficiency at NAL, where a wire electrostatic septum is used is also estimated to be about 97%. Calculations¹⁰ had predicted considerably higher efficiencies, but probably there is no cause for alarm, both because the efficiency measurement is very elementary and because residual radioactivity appears to be under control. Probably improvements of the inefficiency of a factor of 2 or 3 are still forthcoming even without the shield suggested in Ref. 10. Possibly of more significance to future machine design is the catastrophic wire failures that have been experienced at NAL at intensities an order of magnitude below the design intensity. These failures are not yet understood.

Fast Extraction

Fast extraction is becoming increasingly more important. It is used at low intensities for conventional bubble-chamber experiments and at very high intensities for neutrino experiments. It is used for the transfer of beam from one machine to another. There are different requirements placed on the time structure, phase space, intensity, and extraction efficiency of the fast beam, depending on its final use. Bubble chambers tend to need 0.5-msec spill with low intensity that should not be disruptive to the slow extraction process because they can be most efficiently mixed with slow spill. Neutrino horns do not have long high-efficiency pulse times and requires spill times of approximately 0.1 msec at very high intensity. Weak-interaction physics often needs high-intensity spills of the order of 1 msec but not as short as 0.1 msec to reduce cosmic-ray background while still not producing too many accidentals. Injection from boosters to larger rings need single turn, very-fast rise and fall-time kickers so that the rings can be efficiently filled.

Fast kickers for extracting beams traditionally have been single-turn or fractional-turn devices that extract a number of rf bunches at one time and can be fired a number of times during one acceleration cycle.¹⁷ They have tended to be rather extravagant and electrically complicated devices so as to ensure flexibility and fast rise time. Actually? for applications simultaneous with slow spill or internal targeting, kickers are not particularly well matched, because they put holes in the circulating-beam structure and consequently in the experimenters beam. The more recent use of beam shaving and enhanced slow spill are more reasonable solutions to the problem of supplying low-intensity fast spikes. The intensity range is much greater and slow spill is not affected.

For high-intensity fast spill at high-energy accelerators, single-turn kickers have

the advantage that the beam should cleanly jump the septum with only low losses resulting. At intensities above 10^{13} , the fraction of the beam required in the fast spill will probably become larger and single-turn extraction may be one of the most effective ways to minimize radioactive buildup. The kicker necessary for fast extraction probably does not have to have a rise time shorter than about 0.5 μ sec because of the long revolution time (20 μ sec) of particles in large rings. Notching of the beam could be done so that losses do not occur during the kicker rise time. Thus the fast kicker is probably a reasonably straightforward device, even though it is acting on particles with momentum in excess of 200 GeV. Of course orbit-bump magnets (either pulsed or slow) and a thin septum are also a necessary part of single-turn extraction or injection.

Fast kickers have become all important in extraction and injection between boosters and large accelerators. Here, ferrite delay-line magnets are used. Rise times of less than 60 nsec have been achieved. Such systems have been very successful and reliable at NAL. A more complicated but similar system is used for transfer of beam from the four ring PS Booster to the PS at CERN.^{18,19}

Beam shaving has been developed at CERN⁷ over the past few years as the means of extracting from the PS for injection into the SPS. Here it is necessary to peel the beam from the PS in eleven revolutions in order to fill the SPS azimuthally.

Beam shaving is conceptually the next step after fast single-turn extraction and consists of a fast-kicker orbit-bump system, the field level of which can be changed quickly on successive turns. The standard slow bumps and septum are of course part of the system. Reasonably large transverse beam size and thin electrostatic septa to peel the beam out are necessary or efficiencies will not be tolerable. An added bonus of the method is that the phase space injected into the SPS will be considerably reduced from that in the PS.

Though the CERN system is complicated in order to produce uniform filling of the SPS, less-complex applications may easily be found.⁸ For instance, beam shaving can be applied to extraction of small fractions of the beam either over a fraction of a turn, a single turn, or a number of turns. Two great advantages of this system over single fast kickers are that the rise time of the shaving magnets does not have to be exceptionally fast, and that, when the beam is not totally extracted, coherent oscillations are not induced from the kicker magnets.

Fast extraction from a resonant machine condition is also possible.⁹ The half-integer resonance with its intrinsic stopband seems exceedingly flexible. Small fast quadrupoles can extract or aid in the extraction of large fractions of the beam. Coherent extraction relies on the resonant build up of a betatron oscillation produced by a fast kicker. One other form of fast resonant extraction that should be mentioned involves the third-integer

resonance and an octupole nonlinearity.²¹ In this scheme, particles are trapped in stable island regions along the arms of the separatrix as the ν value is tuned through the third integer. Three-turn extraction can then be obtained with the use of a fast-kicker orbit bump that lasts over a time interval of three revolutions. It is possible that this type of extraction could be done a number of times in a beam cycle. It would have the advantage of good extraction efficiency because of the isolation of the beams in phase space islands.

Conclusions

Extraction of beams for experimental use will continue to be dominated by the need to reduce beam losses as much as possible. Historically, every particle extracted from a machine first passed a number of times through an internal target before being extracted. Then, with the change to resonant extraction, 20 to 30% of the beam hit the magnetic septum. Now, with the electrostatic septa, 3% of the particles are lost. Possible future advances may involve even further reduction of material in the beam or possibly a way may be found to remove completely the septum material.

Beam-transfer systems from boosters to high-energy machines, from accelerators to storage rings, and beam-abort systems of storage rings probably will continue to use various permutations of fast-kicker and beam-shaving schemes. The more complicated beam-transfer problems will, however, involve rf manipulations necessary for phase locking, or debunching and stacking in longitudinal phase space.

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Abstract

The installation of a digital phase angle control system for the Zero Gradient Synchrotron (ZGS) magnet power supply firing circuits reduced the low frequency ripple to the point where a magnetic beam spill mode with RF off has become feasible. Orbit warping magnets using feedback from the extracted beam are used to compensate for remaining ripple, and main magnet flattop control is used to move the beam to the $\nu_r = 2/3$ extraction point. To date, a combined extraction efficiency for the two beams of 50% has been achieved. There is 0% RF structure and about 20% of low frequency modulation.

Introduction

The primary experimental areas of the ZGS are supplied from two extracted proton beam lines lying 180° apart around the machine circumference as shown in Fig. 1. Under these conditions, it appeared to be very time effective to consider simultaneous extraction to these two areas by using the $\nu_r = 2/3$ resonance extraction scheme. With the $\nu_r = 2/3$ extraction, three separatrices are formed in phase space and beam is extracted into each area every turn and a half around the machine. Since the majority of experiments use counters and spark chambers, their data accumulation rate is sensitive to the time structure of the extracted beam. It was therefore necessary to use magnetic field feedback, rather than RF, to control the beam spill.

Fig. 1. Layout of the ZGS ring showing extraction system components.

*Work performed under the auspices of the U. S. Atomic Energy Commission.

In Section I we describe the resonance extraction scheme and in Section II the method of beam spill control.

I. Description of the ZGS Resonance Extraction

Use of the $\nu_r = 2/3$ resonance to provide simultaneous extraction into the two ZGS external proton beam lines has already been described.¹ At the present time, with the two sextupole magnets located in straight sections L-2 and L-4 (Fig. 1), the locations of the separatrices in the septum magnet straight sections S-2 and S-4 are shown in Fig. 2.

Fig. 2. Phase space plot of extraction in straight sections S-2 and S-4.

Protons move away from the unstable fixed points and are eventually intercepted by the two thin septum magnets located on the inside of the straight sections S-2 and S-4. Those protons which pass close to the septum magnet in either S-2 or S-4 will, after $3/2$ turns, find themselves within the magnet apertures in S-4 or S-2 respectively. There they receive inward kicks to arrive within the apertures of the thick septum magnets located further radially inward in S-1 and S-3 respectively. The S-1 and S-3 magnets in turn kick the particles outward into the outside extraction chains located in L-2 and L-4 respectively.

The beam is extracted slowly by driving the circulating beam radially inward which causes the radial stable phase space to decrease in area. For spills with no RF structure, the RF is turned off and the inward movement of the beam is accomplished by allowing the main magnetic guide field to increase slowly. Feedback from the spilled beam to two

warping magnets located in L-1 and L-3 provides fine control and takes out most of the slow structure.

As described in Ref. 1, the sharing ratio of the amount of beam into each proton area can be controlled either by adjusting the relative positions of the two septum magnets in S-2 or S-4 or by adjusting the strengths of the two dipole fields that are a part of the two resonance extraction magnets (REX) in L-2 and L-4. The warping magnets in L-1 and L-3 can also be used to control the share ratio to a limited extent. Difference signals from the spilled beams in the two proton areas may eventually be fed to these magnets to maintain the desired share ratio during the entire extraction time.

We had anticipated that moving the REX magnets from the positions described in Ref. 1 to their present location would increase the extraction efficiency. We have found, however, that the increase in their septum current necessitated by this move has produced sufficiently strong fringe fields, which have increased the effective septum thickness of the magnet from about 0.1 in. to 0.2 in. In addition, the higher temperature gradient across the septum sheet gives rise to radial field components which affect the vertical size of the beam. Our present plans are to correct these fringe fields. This should improve the extraction efficiency from its present peak of 50%.

II. Magnetic Beam Spill Control

The ZGS has operated its slow spills since 1963 with extraction from energy loss targets. Appropriate choice of the parameters of the target damping lip allows such a spill to come out with little RF structure. Since the RF can be left on during extraction, a closed-loop spill feedback control is utilized which uses the RF to control the rate the beam impinges on the target. This system is used to establish the rate of spill required by the experimenters and to minimize the structure caused by guide magnet field ripple and nonlinear radial beam distribution.

Since a slow, resonantly-extracted beam is most usable when free of RF structure, we have developed methods of magnetic spill control with RF off.

The ZGS guide magnet power supply is a 12-phase, 50-Hz system. The control elements are excitrons. Theoretically, no frequency components less than 600 Hz should be present. Detailed studies of the guide field ripple showed 50 Hz components that sometimes were as large as ± 1 G out of 19,800. These are probably created by supply transformer unbalances. Due to adjustment limitations on our 1963-vintage phase control system, these 50 Hz errors could not be eliminated. Other frequency components were present above their expected levels.

Originally, we had planned an active filter for the guide magnet power supply system to reduce ripple for magnetic spill control of slow resonant extraction. The 50 Hz components drastically increased the power dissipation requirements for the proposed active ripple filter. It was therefore

necessary to improve the guide magnet phase control system. This has been completed with a significant reduction in ripple, as described in companion papers.^{2,3}

Even with this improvement, an active ripple filter would require several hundred water-cooled transistors. It would be expensive, take two man-years of effort, and force a two-month shutdown for installation. An inexpensive alternate method using small warping magnets and feedback control was proposed to eliminate spill structure caused by guide field ripple. This method has the additional advantage of compensating for REX magnet field ripple and radial beam distribution nonlinearities.

A system consisting of two small medium-frequency (dc to 2 kHz) magnets, two 3 kW bipolar transistor power amplifiers, and appropriate beam feedback sensors has been built, installed, and successfully operated in the ZGS. These magnets warp the beam orbits at the REX magnets in response to beam spill signals. A more accurate description might be "the rate at which the stable phase is decreased is modulated by these magnets to control spill structure." As shown below, these magnets are not powerful enough to spill the beam by themselves. The low frequency spill rate is set by ramping the guide field magnets. A tandem spill regulation system is used. That is, the low frequency rate is set by spill feedback into the guide magnet phase control system, while structure is regulated by spill feedback to the warping magnets. Each by itself is inadequate, but together they do a rather nice job, as Figs. 3, 4, and 5 demonstrate.

Fig. 3. Open loop spill.

Trace 1: EPB-I ion chamber readout.
Trace 2: EPB-II ion chamber readout.
Trace 3: Ion Q at 1×10^{12} protons/cm.

Fig. 4. Guide magnet spill feedback.

Trace 1: EPB-I ion chamber readout.
Trace 2: EPB-II ion chamber readout.
Trace 3: Ion Q at 1×10^{12} protons/cm.

Fig. 5. Tandem spill feedback.

Trace 1: EPB-I ion chamber readout.
Trace 2: EPB-II ion chamber readout.
Trace 3: Ion Q at 1×10^{12} protons/cm.

Effect of Guide Field Ripple

It is difficult to appreciate the effect of a small amount of ripple in a weak focusing synchrotron. A few numbers point up the problem.

A 4-inch wide beam in the ZGS can be completely extracted on a $2/3$ resonance with a radial movement of 0.7 in. Extracting that beam in 0.7 s gives a rate of beam movement of 1 in./s.

A disturbance modulation $\left(\frac{dE}{dt} \right)$ of ± 0.3 in./s superimposed on a 1 in./s programmed spill movement creates a spill structure of 30%. This disturbance can be defined as that created by guide field ripple, and the allowable ripple for 30% modulation can be computed. With appropriate ZGS parameters for 12.3 GeV/c operation we have for 30% modulation, an allowed ripple field rate of

$$\frac{dB_r}{dt} = \frac{\nu_r B}{R} \times \frac{dE}{dt} = \frac{(0.7)(19,800)}{1080} \times 0.3 = 3.85 \text{ G/s.}$$

If the ripple has frequency f , the allowed ripple field is

$$B_r = \frac{1}{2\pi f} \frac{dB_r}{dt} = \frac{3.85 \text{ G/s}}{2\pi f}.$$

For 50 Hz, this gives an allowable peak-to-peak ripple of 0.024 G out of 19,800 G. Only ± 0.001 G of the basic 600 Hz ripple of the ZGS would be acceptable. Such small values indicate that a pretty good active filter would be required. The warping magnet fields to compensate for this ripple are not quite so awesome.

Warping Magnets

The radial movement resulting from the 1 G 50 Hz ripple field at 12.3 GeV/c is:

$$\Delta R = \frac{(\Delta B) R}{\nu_r B} = \frac{1(1080)}{(0.7)(19,800)} = 0.078 \text{ in.}$$

Two warping magnets located 180 machine degrees from each other and 90° from the REX magnets can produce in-phase warps at each of the REX magnets, which will remove structure from the spill to each of the extracted proton lines. The required warp magnet strengths to compensate for the 0.078 in. movement are:

$$BL = \frac{(B_p)(2\nu_r)(AX)\sin\frac{\pi}{2}}{R}$$

$$BL = \frac{(1400)(2 \times 0.7)(0.078) \sin 63^\circ}{1080}$$

$$BL = 0.126 \text{ kG ft}$$

This modest requirement is exceeded by our magnet and amplifier design which produces a dc warp of ± 0.297 in. and a 50 Hz warp of ± 0.101 in. at each REX magnet.

Each of the magnets has a tape-wound, 12-mil Selectron "C" core which has a 6 in. x 6 in. cross section. The half cores are symmetrically cut to have a 5.25 in. air gap when assembled. Thirty-six turn coils are mounted at the pole tips. These are wound out of No. 4 AWG square copper. No special cooling is provided since the rms value of the pulsed currents is only about 25 A. When the coils are connected in series, the magnets have an inductance of about 2×10^{-3} H at 60 Hz. The magnets are operated at a peak flux density of 800 G.

Simple "C" magnets such as these have an almost Gaussian flux distribution. Gradients of

Power Amplifier

The magnet and connecting cable form a load impedance of $0.66 \angle 72.4^\circ$ ohms at 50 Hz. The load time constant is about 20 ms. Driving such an inductive load as part of a feedback system presents stability problems. The amplifier is operated with about 30 dB of current feedback so that the phase shift does not build up until the output transistors saturate. This results in a gap flux corner frequency of about 200 Hz.

The Closed Loop

Coupling the beam feedback into the guide magnet power supply is somewhat more complicated. The ZGS magnet flattop is smoothed by a closed loop system. The normal operation of this closed-loop system (without spill feedback) compares B to a reference input (usually zero) and the difference is applied to an analog-to-digital converter (ADC). The output of this ADC is gated into the guide magnet phase control system only on ZGS flattop. The B comparison amplifier is very heavily filtered with an upper corner frequency at about 20 Hz. This frequency was chosen to be below an existing MG set shaft resonance which is at 36 Hz.

Figure 3 shows the open-loop proton spill. The spill structure is mostly caused by phase control commands attempting to maintain the flattop flat.

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Fig. 4 shows the spill down the beam lines with only the guide magnet loop closed. Note the instability caused by the 20 Hz response limit of *the* system. Figure 5 shows the proton spill with both loops closed. The feedback source for spill control was obtained from an ion chamber signal in the EPB-II line. The spike at the beginning is caused by resonant magnet turn-on. The two loops complement each other so well because the frequency response limits are so far apart. The warping magnets can operate more than ten times higher in frequency than the guide magnet systems.

Operating Experience

While we have only operated in a HEP mode for a few days with this system, Figs. 3, 4, and 5 seem to speak for themselves as to its success. Despite the apparently good spills of Fig. 5, the HEP experimenters are complaining of troublesome accidental rates between 800 and 1,000 Hz. Since our spill sensors do not show this rate, the feedback loop cannot work to correct it. Further effort must be spent to determine the sensing, cause and cure of this problem. At the present time, these frequencies produce a 50% spill modulation.

Acknowledgements

Quite a few people have been involved with the development ~~of~~ *the* warping magnet system. Our thanks go to M. Faber, R. Zolecki, K. DeVries, R. Lari, T. Quarnstrom, and L. Donley and last, but certainly not least, F. Brumwell.

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DISCUSSION

Lee Teng (NAL): How thick is the thin septum?

Crosbie: This is somewhat a matter of controversy at the present time. It was supposed to be on the order of 50 - 60 mils. From indirect measurements on the extracted beam, we know that it is thicker than that. It might effectively be as much as 150 - 200 mils.

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Summary

Significant improvements have been made in the efficiency, duty factor, and flexibility of the NAL Main Accelerator extraction system, consistent with the increasing intensity of the synchrotron and the growing diversity of the experimental program. Currently, a typical extraction mode will include a one-second period of slow extraction punctuated by four higher intensity bursts for bubble chamber use, followed by a high intensity fast terminal spike for neutrino experiments. The extraction efficiency - now some 97% - is such that radioactivity levels in extraction system components are not increasing despite operation of the accelerator at record currents. A modified septum design is expected to lead to further gains. A computer-assisted feedback system has raised the spill duty factor to 70%. Recently, the synchrotron has been operated with a front porch, with two periods of slow extraction, each of one-second duration, at two different energies within the same accelerator cycle.

Introduction

When a slow extracted beam was initially obtained from the Main Accelerator at NAL in September of 1972, the synchrotron was operating at an intensity of less than 10^{11} protons per pulse and only one experimental area was in operation. Under those circumstances, the demands placed upon the extraction system were modest, and a low efficiency spill with a ragged time-structure was tolerable for the time being. Today, however, the accelerator intensity has risen by two orders of magnitude and a rich and diversified experimental program is underway at the three experimental areas simultaneously, each receiving a share of the protons from every acceleration cycle, and the demands on the extraction system have risen correspondingly. It is the purpose of this paper to describe the evolution of the extraction system during this period.

The basic features of the extraction system have been reported previously.^{1,2} In brief, half-integral resonant extraction is employed for the slow spill, at a horizontal (extraction-plane) tune of $\nu_x = 39/2$. (Until recently, $\nu_x = 41/2$ was used, but a few months ago, the operating point of the Main Accelerator was lowered by one unit for reasons not associated with extraction.) Prior to extraction, the currents in the Main Accelerator quadrupole busses are adjusted to bring ν_x close to the half-integer. To initiate extraction, a stop-band is created by the excitation of two quadrupoles, so situated that by varying their relative currents a rotation of the phase space area of the extracted beam can be effected. The non-linearity required to differentiate between stable and unstable particles and to give adequate growth rate to betatron oscillation amplitude of the particles to be extracted is provided

by the intrinsic octupole moment of the Main Accelerator, aided when necessary by a single iron-core octupole of adjustable strength. A proton whose unstable betatron oscillation amplitude has grown sufficiently (2 cm typically) will enter the extraction channel, the initial component of which is an electrostatic septum.² Localized orbit bumps are used to adjust the position and angle of the Main Accelerator central trajectory with respect to the extraction channel.

We distinguish three main categories, into which the subsequent discussion is divided, on which our system development efforts have been concentrated. First, the improvement of extraction efficiency has been essential, in order to control the buildup of residual radioactivity in accelerator and extraction system components. Second, a smoothing of the initially ragged time structure of the spill had to be achieved so that experiments utilizing electronic detectors might benefit from increases in accelerator intensity. Third, the various requirements of the three experimental areas had to be satisfied within each beam spill.

Efficiency

Once the various elements of the extraction channel have been carefully aligned with respect to one another and with respect to the beam, the dominant source of losses in the extraction process becomes the first splitting device - the electrostatic septum. The alignment process raised the extraction efficiency above 90% over a year ago; since that time, considerable attention has been given to identifying and correcting septum losses.

The beam splitting surface of the septum consists of a row of vertical tungsten wires having a diameter of 0.002 in. spaced at 0.050 in. intervals. The wires are at ground potential.

Parallel to the plane of the wires, at a distance of 1 cm., a titanium cathode is placed which is charged to a negative high voltage (72 kV at 300 GeV) with respect to the wires. The extraction septum consists of two separate 10 foot units which can be individually positioned or replaced. Within each unit, the wire planes and cathodes are divided into five 2 foot modules. Protons passing between the wire plane and the cathode experience a horizontal deflection of 0.14 mrad, sufficient to enter the gap of a Lambertson magnet 100 feet downstream.

For half-integral resonant extraction, a step size of 1 cm per two turns would imply extraction losses of ~ 1% for ideally fabricated wire septa with our wire thickness. In contrast, a year ago losses were at the 6 - 7%

level. The source of these losses have been studied using two techniques.

At the outset, the origin of extraction inefficiency could not be clearly attributed to any single element of the system. To study the extraction process, a number of remotely operable horizontal and vertical targets were installed in the extraction straight section and external beam line. The first horizontal target is located 0.5 m upstream of the electrostatic septum. The second and third horizontal targets are positioned between the septum and the Lambertson magnets. The fourth horizontal and first vertical targets are located near the downstream end of the long straight section while the fifth horizontal and second vertical targets are situated in the extracted beam line. Ionization chambers are used to detect particles scattered by these targets, and a plot of normalized ionization chamber signal versus target position yields a measure of proton flux as a function of position across the beam. Figure 1 shows a plot of ionization chamber signals as the targets are scanned across the beam. Scattering from targets downstream of the electrostatic septum clearly outline the shadow cast by the septum and from these data effective septum thicknesses were found to be considerably larger, typically a factor of five, than

that suggested by wire size alone. Subsequently, mechanical measurements of the septa verified these results. The target scan plots can be used also to estimate the phase space of the extracted beam. Typical results give areas of 0.14 mm - mr vertically and 0.25 mm - mr horizontally. The phase space measured 1000 meters downstream gives numbers of the order of three times larger and to date the discrepancy is not well understood.

The mechanical technique for measuring septa thickness is illustrated in Figure 2. A wire is stretched from one end of the septum to the other, and electrical contact is maintained with the septum wires as the position of the two ends of the stretched wire are moved in the direction perpendicular to the plane of the septum wires. A graph of the position of one end of the stretched wire versus the other while electrical contact is preserved can then be interpreted to yield the septum thickness. In Figure 3, we show results of measurements by this method before and after straightening a crooked septum (0.030 inch thick) which had exhibited a poor extraction efficiency. By external welding on the septum vacuum skin, to introduce compensating stress, for seam welds, the effective thickness of the septum was reduced to 0.005 ± 0.003 in. A modified wire septum constructed on a single 10 ft. wire frame promises to be straighter, to be constructed more rapidly, and to automatically retract broken wires without the necessity of accelerator shutdown.

Figure 1

EXTRACTED BEAM PROFILE

Figure 2
Diagram of Method Used to Measure Effective Thickness of Septa

Present extraction losses of approximately 3% are sufficiently low that excessive radioactivity buildup in system components is not anticipated at average accelerator intensities close to the 10^{13} protons per pulse.

Absolute calibration of the losses has not been developed to date. In one sense the residual radioactivity is the only meaningful observable. As long as the diagnostic equipment give signals which indicate how to improve the efficiency absolute calibrations have not been necessary, and it is quite possible the extraction efficiency is higher than 97%.

It is clear, however, that as the accelerator intensity rises toward its design goal of 5×10^{13} protons per pulse, corresponding improvements in extraction efficiency must be achieved, and our major effort in this direction is focussed on the septum designs discussed above.

It should be mentioned that, twice in the past three months, the extraction septum has failed catastrophically where all the wires over the first two to four feet of the upstream module have broken. It is believed thermal heating from short duration high losses on the wires is responsible. These failures point out the need to understand high temperature behavior of the septa wires and also show the necessity for very clean fast or coherent extraction in which no major portion of the beam hits the wires.

Duty Factor

The time structure of the slow extracted beam arises from two sources; rather low frequency current ripple, principally in the quadrupole magnets which leads to tune variation, and the 53 MHz radio frequency bunch structure of the beam. To date, the radio

Figure 3
Septum Thickness Measurements. Data before straightening are represented by curve with the points designated x.

frequency structure has not been a serious disadvantage for experimenters, and a number of experiments have found it useful for timing purposes. Current ripple in the main ring power supply system at 360 Hz and below, however, requires compensation. Higher harmonics are reduced to negligible proportion by the active filters on the main quadrupole supplies.³ For the present, we have chosen to define the duty factor, F, by

$$F = \frac{[\int_0^T I(t) dt]^2}{T \int_0^T I^2(t) dt}$$

where T is the duration of the slow spill, I(t) is the instantaneous extracted beam current as a function of time t, and the radio frequency structure is ignored. This definition of F is suggested by the expression for the chance coincidence rate in a two counter telescope.

To counteract the tune variation brought about by ripple, two air-core quadrupoles have been placed in the accelerator. Their excitation currents are derived from a combination of signals which have been found to be effective in improving the duty factor. Each is driven by a current proportional to the ripple component of field in neighboring main ring quadrupoles. The two so called "bucker" quadrupoles are far from each other in the ring so that their signal sources sample potentially different ripple contributions to the overall tune variations. (The accelerator magnet circuit forms a rather complex delay line, so that ripple terms in the current may be non-uniform around the machine.)

In addition, one of these air-core quadrupoles receives an excitation current derived from the actual beam spill. A detector composed of a scintillator and light sensitive diode is placed so as to intercept spray from the extracted beam. The signal from this detector is digitized and sampled by a PDP-8 computer at a rate of 8×720 Hz. A learning algorithm, similar in principle to that used in the Main Accelerator power supply control system⁴, determines the appropriate correction signal which is delivered to the bucker quadrupole advanced in time on subsequent cycles by an amount corresponding to the time delay between a perturbation in the tune and its modulation of the spill (about 1/2 ms). A calculation of the duty factor is made in the computer and displayed to the operators.

The improvement in duty factor effected by the bucker quadrupoles is shown in Figure 4. Figure 4a shows typical ripple signals on the main quadrupole (top) and bend (bottom) busses and Figure 4b the associated spill if no correction signal is sent to the buckers. In Figure 4c, the buckers are receiving their excitation from the local ripple field in the main quadrupole (/Bdt signal), while in Figure 4d the computer processed spill signal alone is used. In Figure 4e both the integrated B signal and computer processed spill signals are used to drive the buckers. In the last case, a duty factor of about 70% has been achieved in contrast with the 15% or so of Figure 4b.

Clearly, many avenues for duty factor improvements remain to be explored. For example, no compensation for ripple on the dipole busses has been introduced as yet, and no real time spill feed back tried. Recognizing that subharmonics of the twelve-phase power supplies must arise from phase unbalance, the 360 Hz and lower components may be removed by power supply improvements, such as trying to induce ripple on the regulating supplies to compensate for the over all ripple and the addition of phase balance compensating circuits in the individual unregulated supplies. Finally, manipulations of the accelerated beam such as spreading the momentum with the r.f. may make extraction less sensitive to power supply ripple.

Extraction Modes

Currently, four extraction modes are in use, combined during each accelerator cycle in a way that reflects the needs of the experimental program.

Slow Spill

Normally, most of the beam is extracted by the slow, resonant, process. The regulation of the spill insofar as its gross time structure is concerned (that is, exclusive of the higher frequency structure discussed in connection with duty factor above) is performed by a feedback system consisting of a MAC-16 mini-computer and an iron-core quadrupole. The intensity of the beam circulating in the accelerator is digitized and sent to the computer. The desired dependence of circulating beam on time throughout the spill is stored in memory; the computer compares the actual

Figure 4
Magnet Ripple, Spill Structure & Bucker Drive
Signals
(10 ms/div)
4a Top-Main Ring Quadrupole Bus 1 div = 0.8×10^{-4}
Bottom-Main Ring Bend Bus
4b Uncompensated Spill
4c through 4e; Top trace is the output of the
diode spill monitor and bottom trace is the
bucker drive input signal.
See text for details.

intensity with that desired and delivers a correction signal to the quadrupole excitation circuit. A learning algorithm similar to that employed for duty factor improvement is also used here. The use of the computer allows for rather complicated time structures like front porch operation and enhanced spill.

Enhanced Spill

The 30 inch bubble chamber is capable of expansion every quarter of a second during a beam pulse. The normal slow spill intensity does not provide sufficient flux for the bubble chamber when exposures with secondary particles, such as negative pions, are being made. In order to provide spill rate enhancement during the chamber's sensitive period, an air core quadrupole is driven with a half-wavelength sinusoidal current pulse. This causes a shrinkage of the stable region in phase space for a short time, thus forcing a number of particles beyond the separatrix and causing them to be extracted. Depending on bubble chamber requirements, typically 10^{10} to 5×10^{11} particles are extracted in each burst, though this method has been used to provide up to 2×10^{12} protons in its 1 msec spike. (See Figure 5.) Pulsed beam line magnets are being constructed to avoid sending this intensified spill to laboratories wishing only slow spill.

The computer maintaining the slow spill rate need only be informed of the times at which spill enhancement occurs. After each such event, it renormalizes its model of the circulating beam intensity to the current remaining after the burst, and continues as before.

Fast Extraction

Fast extraction represents one of two ways to achieve a fast spill of the beam remaining in the synchrotron at the end of the slow extraction period. The fast spill, extracted in one turn, is intended primarily for use with the bubble chamber and neutrino focusing horn (125 μ s pulse width). The fast spill is accomplished by using pulsed orbit bumps to place the beam near but not on the wire septum a few ms before the ferrite kicker is fired. The ferrite kicker gives the beam a large amplitude betatron oscillation and causes it to jump across the septum which is located half way around the ring from the kicker. To achieve the 0.5 μ s rise time and 20.5 μ s duration of the ferrite magnet field a lumped element delay line charged to 50 kV is discharged into a matched 12 ohm load consisting of ferrite magnets and terminating resistors. A picture of the kicker is shown in Figure 6.

Coherent Extraction

If a spill of duration somewhat longer than 20 ps, but still fast, is desired the beam is coherently extracted. As in the enhanced spill described above an air core quadrupole is pulsed to shrink the stable area of phase space while at the same time the ferrite kicker and orbit bumps referred to above are pulsed at a magnetic field reduced from the single turn case. The beam is thus

perturbed coherently in phase space so that it starts to come out after several turns and proceeds to spill on every other turn until all the beam is expelled. Figure 7 shows an example of the integrated spill and extraction SEM intensity for a beam which has been coherently extracted in 200 ps.

Front Porch

The Proton and Neutrino experimental areas can receive proton beams up to 500 GeV. However, the Meson area is limited, by design, to 300 GeV operation. For this reason and for the potential advantages of sending two energies to the proton laboratory a front porch test has been completed in which the front porch and flat top operated at 200 and 300 GeV, respectively. For both levels the beam was spilled with the MAC 16 computer feedback discussed in the slow spill section above. Each of the two spills had a duration of one second with the ripple bucket magnet working over each spill duration (See Figure 8.) Many devices in the external beam lines including the extraction electrostatic septum were ramped. The front porch test showed that this mode of operation was reasonable and that the relative intensities of the two energy extracted beams could be varied over a range of about 5 to 1.

Conclusion

The NAL Main Ring half integer resonant extraction system has operated satisfactorily over the past year with steadily improving performance. The extraction efficiency and duty factor are 97% and 70%, respectively. A front porch test was concluded with no difficulties and suggests it may be a useful way to run as the machine achieves higher energies.

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Figure 5a
Typical Enhanced Spill
(2ms/div)

Figure 5b
Air Core Quadrupole Used
for Enhanced and Coherent Spill

Figure 6
Main Ring One Turn
Extraction Ferrite Pulsed Magnet

Figure 7
Coherent Spill (50 μ s/div)

Figure 8
Front Porch
Top trace is the spill; middle trace is the
main ring intensity; bottom trace is the
magnet ramp.

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Summary

Design and performance of a recently commissioned fast extraction system to serve the requirements of the bubble chamber programs at the AGS are discussed. Several of the major components are described along with the diagnostic instrumentation and system performance. Extraction of various fractions of the internal beam, including total extraction in an essentially single-turn mode, has been accomplished with the AGS beam either bunched or debunched. The expected extraction efficiency, $\sim 95\%$, has been observed and transport losses in the 258 m, 10-cm aperture external beam are $\leq 1\%$.

I. Shaving Extraction Method

In a previous paper,¹ we presented the orbit calculations, configuration and component parameters of the extraction system. A similar system has been developed at CERN² for extraction from the PS. Briefly, the method consists of producing a rapid, local orbit deformation at the azimuth of a thin septum magnet in straight section E10. The fraction of beam displaced across the septum is deflected into the aperture of a septum ejector magnet at straight section H10, approximately $2\frac{1}{2}$ betatron wavelengths (λ_B) from E10. The rapid orbit deformation is produced by two full aperture, single-turn, ferrite core kicker magnets located in straight sections C15 and E15 with a separation of approximately $1\frac{1}{2}$ λ_B . In the fractional extraction mode, the kickers are powered by a half sinusoid current waveform of 2.5 μ s duration. For full extraction, the waveform duration is ~ 6 μ s with amplitude of ~ 8000 A. At 5000 A in the kickers, and at typical extraction momentum 28.7 GeV/c, the measured deflection at E10 is 1.4 cm, which is adequate for complete displacement of a 1.3 cm wide³ AGS bunch across the 0.3 mm E10 septum. A 1.0 mrad deflection at E10, provided by a current pulse of 3000 A peak and 200 μ s duration in the single-turn magnet deflects the shaved fraction across the 2.25 mm H10 ejector septum with a measured separation between circulating and shaved beam of 7.6 mm, in good agreement with calculations. The H10 ejector, also a single turn magnet, provides a 1.5 ms current pulse of 21,000 A peak, and deflects the beam by 22 mrad into the external channel for the 7-ft bubble chamber neutrino beam. The E10 and H10 septa are fixed at mean radial positions relative to the AGS central orbit of 4.6 and 5.6 cm respectively, during the entire accelerating cycle; slower orbit deformations of 9 ms duration in a $1/2$ λ_B configuration centered at the septa are energized prior to extraction and provide measured displacements of 3.95 and 3.7 cm at E10 and H10 respectively, for a 1000 A peak current through backleg windings on selected AGS magnets. We have determined that the septa are not machine apertures at injection energy, even for intensities up to 9×10^{12} protons/pulse.

Another operating mode, presently being commissioned, involves a polarity reversal of the H10 magnet and deflection of the shaved beam by -3 mrad, back into the

AGS aperture, for extraction $3/4$ λ_B downstream at I10 for the 80-in. bubble chamber. The I10 ejector magnet, originally used for single-bunch extraction,⁴ has been reduced in septum thickness to 2.0 cm and a backleg winding $1/2$ λ_B orbit deformation has been deployed in the I superperiod for the present application. In this mode, we will extract only $\sim 2\%$ of the 6×10^{12} p/p circulating beam; the system is capable of four extractions per second at 200 ms intervals. We have achieved the required spill duration of ≤ 1.5 μ s imposed by the present pulse duration of the rf beam separator for the 80-in. chamber. Also, we have extracted the beam both during the acceleration and flat top part of the AGS cycle.

II. Extraction Components

Kicker Power Supply

For the fractional shaving mode, a deuterium thyatron switches a 0.2 μ F capacitor charged to 30 kV, generating a 5 kA peak pulse with 2.5 μ s base. In the full extraction mode, four thyatrons are triggered, each switching a 0.2 μ F capacitor at 30 kV onto the common load. Two thyatrons are triggered initially; 0.6 μ s later, the other two are triggered. The resulting pulse has an 8 kA amplitude and 0.5 μ s rise time with 6 μ s base. This triggering method produces an optimum waveshape for the given constants of cable and magnet.

Ejector Power Supply

The H10 magnet supply consists of an energy discharge system in which magnet inductance and power supply capacitance oscillate with a period of 3 ms. The stored energy is 5500 J and the peak current is 21.5 kA at 800 V. The power supply is completely solid state, featuring water cooled SCR's (thyristors) and silicon rectifiers. The charging supply is a three phase, constant current type utilizing resonant monocyclic networks. This system eliminates the need for a lossy series charging resistor.

Ejector Magnet

The ejector magnet, located at the H10 straight section, has been designed to serve two purposes. The magnet assembly consists of two segments; the length ratio is one-third for the upstream segment and two-thirds for the downstream segment. With a current of 21.5 kA flowing in the same direction in both segments, the beam is deflected 22 milliradians out of the AGS, aiming toward the 7 ft. chamber. With a current of 8 kA flowing in the opposite direction in the downstream segment, the beam is displaced across a thick septum (20 mm) at the I10 straight section. A further kick by the I10 ejector magnet causes the beam to exit from the AGS aiming toward the 80-in. bubble chamber.

*Work performed under the auspices of the U.S. Atomic Energy Commission.

Fig. 1. Cross section of ejector magnet at the H10 straight section.

The cross section of the magnet is shown in Fig. 1. The septum is of copper 2.25 mm thick. The nominal design current is 21.5 kA at induction of 1.05 tesla in the gap. Thermal heating of this septum is rather low due to the short pulse width (1.5 ms). However, the mechanical stress due to magnetic pressure is rather severe. The septum is edge cooled by attaching two monel 400 rectangular tubes at the top and bottom of the septum. The resistance ratio of each tube to septum is only in the order of 0.27%; this results in near-

ly uniform current density in the septum. The septum is very carefully hand fitted into the magnet gap, so that the clearance between the septum and core can be kept at 0.025 mm. The laminations are made of intermediate silicone steel, 0.35 mm thick. The end plate is 31 mm thick, made of welded laminations. The laminations are held together by 4 ceramic insulated tie rods. The tightening torque is kept as low as possible (0.1 kg-m in our case). The stray field 6 mm from the middle of the septum was found to be 0.08% of the central internal field; 6 mm from the end of the septum the value was 1.4%.

III. External Optics

The design of the beam transport was dictated largely by the presence of two superconducting dipoles⁶ at the 8° bend point located ~ 96 m from the ejector magnet and downstream of a 4½° bend with conventional magnets as shown in Fig. 2. The superconducting dipoles are 1.83 m long with a 7 cm ID vacuum pipe; the additional length of the dewar and vacuum chamber increased the length of the vacuum pipe to 2.62 m. Sagitta in the magnet accounted for 3.2 cm of the aperture. Calculations⁷ indicate that as few as $\sim 3 \times 10^6$ hadrons/cm²/AGS cycle impinging on the Nb-Ti superconducting ribbon might cause the critical temperature to be exceeded and the magnet to revert to normal conductivity. It was therefore necessary to provide a beam of small horizontal size at this point with stability against momentum changes or variations in strength of the 4½° bend. We accomplished this by designing an achromatic system giving momentum recombination at the 8°; the five quadrupoles Q3 through Q7, between the 4½° and 8°, powered in series accomplish this objective. In addition, protective collimation upstream of the 8° bend was provided at a betatron phase shift of 180° at positions U5 and U12 in Fig. 2 to remove halo which might quench the superconductor. Antiscattering collimators were also provided at U170 and U259. The expected collimator images have been observed on scintillating screens upstream of the magnet, and the expected beam size there, 9 mm horizontal x 26 mm vertical, based on full extraction emittances of $E_H = 2.42$ mm-mrad and $E_V = 1.86$ mm-mrad, have been qualitatively confirmed.

AG5 NORTH AREA FAST EXTERNAL BEAM

Fig. 2. Plan view of external beam line from ejector magnet to neutrino production target for 7-ft bubble chamber.

The transport downstream of the 8° is intended to provide a minimum spot size at the 5 mm dia. $\times 45.7$ cm long sapphire production target. Qualitative observation on a BeO screen indicate that we have achieved the expected 1.7 mm horizontal $\times$ 1.2 mm vertical spot, which is minimum size for the given target length and emittances.

IV. Diagnostic Instrumentation

Instrumentation was designed to measure beam intensity, position and spatial distribution. All signals are presented as quasistatic analog levels at the computer interface. No electronic components are located within the primary radiation shield. Intensity is measured by current transformers at six locations. These consists of tape wound permalloy cores with $n = 100$ turn secondaries. The signal is fed via coaxial cable to an integrator/peak reader (Fig. 3). The inte-

Fig. 3. Schematic diagram of beam current transformer and integrator.

grators have an output calibration of

$$Q/V = nC_f / 1.6 \times 10^{-19} = 10^{12} \text{ protons/volt.} \quad (1)$$

The integrator incorporates a dynamic reset circuit which prevents runaway drift due to low source impedance but is slow enough (500 μ s) to have a negligible effect over the 3 μ s extraction interval. Bench Calibration using a pulser or capacitor discharge source and a dielectric loaded coaxial fixture to minimize reflections showed less than 1% departure from the theoretical value in Eq. (1). Another calibration compared the integrator output with the beam charge determined from ^{11}C activation of polyethylene foils. This also agreed with the theoretical calibration to within the $\pm 4.6\%$ uncertainty in the ^{11}C production cross section.⁷ A current transformer with a 24 turn secondary has also been used to observe the temporal distribution of the beam (see Fig. 7 below).

Beam position is measured by insulated plates and split transverse ion chambers (STIC). Insulated plates in the AGS ring are situated in the shadows of the extraction magnet septa. The plates are charge sources by virtue of the knock-on electrons ejected by the proton beam. The efficiency is about 10%, i.e. one electron per 10 incident protons. The signal is transported via coaxial cable to integrators similar to those for current transformers; however, the peak detector and dynamic reset circuits are omitted because the source impedance is large. The plates are used to optimize extraction magnet position and skew-angle. Insulated plates are also located upstream of collimator jaws in the external transport, but suffer in this application due to ion collection in the rough vacuum (10^{-3} to 10^{-2} T). The residual gas ionization is exploited in the STIC (Fig. 4). The ions are propelled along field lines transverse to the beam axis to triangular collection electrodes. For uniform ionization density, the relative signal strength is proportional

to beam position. The beam position is given by $D \cdot W/2$, where W is the effective width of the electrodes and D is the normalized difference in integrated signals, $(A-B)/(A+B)$.

Fig. 4. Sketch of split transverse ion chamber (STIC).

Spatial distribution is provided by closed circuit TV viewing scintillating screens of "Radelin" (a fluoroscopy screen from U.S. Radium Corp.) or beryllium oxide, and by multiwire profile monitors. The usefulness of the screens is limited since they are easily damaged by the beam and optical and electronic components of the cameras suffer radiation damage. The profile monitors consist of 16 Al strips, 0.6 mm thick and 5 mm center-to-center spacing, attached to a rotatable frame. A 0.05 mm Al bias electrode is included upstream and downstream of the strips to comprise 16 parallel strip ionization chambers. Screens and profile monitors are mounted on remotely controlled actuators and are removed from the beam when not required. Future designs of the profile monitor will use thinner electrodes and bias foils. The charge from each electrode is stored on the coaxial signal cable which sees $> 10^9 \Omega$ impedance at the FET multiplex switch at the computer interface. In the "on" state, each switch FET has $< 100 \Omega$ resistance and the charge from each cable is sequentially transferred to an integrator and ADC. The read time per channel is 75 μ s. The integrator gain is remotely varied over a 36:1 range by means of a switchable integrating capacitor. Typical beam profile displays are shown in Fig. 5.

Fig. 5. Typical beam profile monitor displays for successive machine cycles. The profiles at right show saturation at low values for the remotely switched integrating capacitor.

We have also deployed 22 ionization chambers along the beam line to measure the longitudinal distribution of beam loss. The associated electronics include discriminators and alarm circuits which activate interlocks to turn the beam off should excessive losses occur. An illustration of the computer-generated loss pattern from these detectors is given in Fig. 6. The readout and display utilizes an amplifier and FET multiplex switch similar to the profile monitor circuit. The data readout and transfer is accomplished by a single-coax digital system, Datacon 11, controlled by a PDP-8E computer interfaced to the AGS control room PDP-10 computer. The system is described elsewhere.'

Fig. 6. Computer generated display of radiation monitor signals along the beam line. The ejector magnet is at the left, and the production target, T, at the right of each abscissa. a - Beam tuned for minimum loss. b - Scintillation screen (-72 mg/cm^2) inserted at U15.

V. Beam Performance

Extraction efficiency for the full extraction mode was measured by comparing the internal AGS "pick-up" electrode beam monitor to an external current transformer, and a monitor foil using carbon-eleven activation. During the calibration run, the measured efficiency was only 85%; however, $\sim 90\%$ is indicated by the current transformer at a circulating beam intensity of $\sim 5 \times 10^{12}$ when the beam is well tuned, and 95% is observed at lower circulating beams of $\sim 2 \times 10^{12}$ p/p. Assuming that three AGS bunches are shaved by the E10 septum due to the finite rise time of the beam kicker pulse, the expected theoretical efficiency is $\sim 98\%$. Indirect evidence from bunch amplitudes from current transformer signals indicate an efficiency of 96% . Thus, the measurements are well within the $\pm 5\%$ uncertainty in calibration of the internal beam monitor. ••

Transport efficiency was measured by comparing current transformers and foil calibrations at the upstream and downstream end of the beam. The result under optimum conditions was $100 \pm 0.1\%$. Note that for this case, the foil activation results do not suffer from the $\pm 4\%$ absolute uncertainty in the ^{11}C activation cross section.

The observed spill duration for total and fractional extraction in a bunched and unbunched mode were observed on an external current transformer and is shown in Fig. 7. For the full extraction case, it is seen that a fraction of the first bunch was shaved by the E10 septum; part of the remainder was extracted on the second turn. For the fractional extraction cases illustrated, the internal beam was only $\sim 2 \times 10^{12}$ p/p and

we extracted about 25% of the beam. The spill duration for smaller fractions will be correspondingly less.

Fig. 7. Top photo - Oscilloscope of external beam current transformer signal-full extraction mode, bunched beam ($0.5 \mu\text{sec/div}$). Middle photo - Fractional extraction mode, bunched beam. Upper trace shows AGS circulating beam intensity during extraction process. Lower trace shows external current transformer signal ($1 \mu\text{sec/div}$). Bottom photo - Current transformer oscillogram for fractional extraction mode, debunched beam ($0.5 \mu\text{sec/div}$).

The beam has been successfully utilized for both bubble chamber and spark chamber neutrino experiments. In these applications, position stability has been of importance for optimum neutrino production as well as to minimize neutron and muon background resulting from transport losses. We have noted that the position variation, ΔX , at an observation station—14 m from the ejection magnet for a change, Δr , in the radial position of the internal beam is $\Delta X/\Delta r \approx 6.4 \text{ mm/mm}$. The corresponding sensitivity at the production target is $(\Delta X)_{\text{target}}/\Delta r = 2.2 \text{ mm/mm}$. Thus, a radial shift $\Delta r = 1 \text{ mm}$ will move the spot at the production target by more than its diameter. We therefore had to optimize the gain of the AGS radial position servo, and have achieved long-term stability of $\Delta r \sim 0.1 \text{ mm}$. Similarly, we have maintained the momentum drift at extraction time to 0.1% by careful adjustment of the motor-generator speed control. The extraction process is also sensitive to internal beam size. The beam size is in turn affected by longitudinal and transverse instabilities in the machine during acceleration. The instabilities are intensity and tune-dependent. We have observed both vertical and horizontal beam blow-up from coherent dipole oscillations and higher order resonances. To avert these difficulties, it is necessary to program the machine radius by a sequence of radius shifts during acceleration, as well as introduce a larger than optimal momentum spread by detuning the rf phase shift at transition energy. This procedure results in a beam loss at transition of $\sim 15\%$ from a circulating beam of $\sim 6 \times 10^{12}$ p/p. We also find that the available aperture at extraction time is only 3 mm. Typical machine tune at the extraction radius is $\nu_H = 8.74$, $\nu_V = 8.66$.

Spot size measurements in the external channel have been in qualitative agreement with calculations based on earlier emittance measurements." However, we plan new measurements to define the emittance better under various extraction conditions. For the new measurements, we plan to measure the beam half widths, W_1 , W_2 and W_3 at three different positions downstream of the 8° bend. It can be shown" the emittance area $\epsilon = E/\pi$ and the ellipse parameter α_1 and β_1 at point one are given by

$$\epsilon = \frac{(\mathcal{D}_1^2 + \mathcal{D}_2^2 + \mathcal{D}_3^2)^2 (\mathcal{D}_1^2 + \mathcal{D}_2^2 + \mathcal{D}_3^2)}{2|a_2 b_3 - a_3 b_2| b_2 b_3} \quad (2)$$

$$\alpha_1 \epsilon = \frac{\left[\frac{W_1^2}{\epsilon} (a_2^2 b_3^2 - a_3^2 b_2^2) - \mathcal{D}_2 + \mathcal{D}_3 \right]}{2(a_2 b_3 - a_3 b_2) b_2 b_3} \quad (3)$$

$$\beta_1 \epsilon = W_1^2 \quad (4)$$

where $\mathcal{D}_1 = \frac{W_1^2}{\epsilon} (a_2 b_3 - a_3 b_2)^2$; $\mathcal{D}_2 = \frac{W_2^2}{\epsilon} b_3^2$; $\mathcal{D}_3 = \frac{W_3^2}{\epsilon} b_2^2$

and a_2 , b_2 and a_3 , b_3 are matrix elements of the transport matrix

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

from point one to points two and three respectively.

Acknowledgements

We thank R. Adams and W. Gefers for the beam security system; R. Glasmann, J. Gabusi and E. Raka for valuable help in machine studies; D. Easler, W. Harrison and J. Schirmer for expert construction of external power supplies and computer interfaces; J. Balsamo, J. Guthy, E. Tomblar and M. Zguris for invaluable contributions to the instrumentation; and A. Pendzick for excellent cooperation in facility installation. We are grateful for much support and consultations from J.G. Cottingham, B. Culwick, G. Danby, H. Foelsche, J. Grisoli, A. van Steenbergen and H. Williams.

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Summary

The slow ejection of the CPS, going either to the West or East Hall, shares the beam regularly with one of the two internal targets feeding the South Hall. A simulation program has been written, in order to understand the process of sharing, especially the losses (about 5% to 7% for typically 30% to 50% on the internal target; the efficiency of the ejection itself is 95% to 97%) and the emittance blow-up (about a factor 2 for the vertical emittance) associated with it. In this program, a number of particles are followed through the resonant extraction process, while the target is treated using a Monte-Carlo method. Calculated values for efficiencies and emittance blow-up are given and compared with measured values.

1. Introduction

The CPS provides protons for counter experiments in four areas : East Hall (slow ejection from s.s. 62), West Hall (slow ejection from s.s. 16), and two internal targets in the South Hall (one in s.s. 1, one in s.s. 8). In order to make efficient use of these facilities, the two slow ejection areas are served turn about (runs of 3 weeks), always sharing the beam with one of the internal targets.

First machine experiments' and calculations' had shown that with the former integer resonant extraction system the efficiency of the sharing process would have been unacceptably low, but with a third integer resonance test scheme, rather favorable results could be obtained³. In parallel with the installation of the final system, which has been running now for almost two years, the sharing process was studied by computer.

2. The Sharing System

The ejection system is schematically shown in Fig. 1. The required sextupolar component is provided by one non-linear lens (semi-quadrupole, SQ 53). A quadrupole (AQ 23) together with the quadrupolar component of the semi-quadrupole shifts the Q-value at the centre of the vacuum chamber from about 6.235 to the resonance $6 \frac{1}{3}$. The accelerated beam is put in the inside half of the aperture, then debunched, and drifts into the resonance due to a slope on the flat-top of the main magnetic field.

Fig.1 : Layout of the system.

The phase of the separatrices is chosen so as to have the maximum jump at the first septum, which is electrostatic (foil ~ 0.15 mm effective thickness). $1/8$ betatron wavelength downstream there follows a thin septum magnet (1.5 mm septum). Depending on the required operation, the beam then enters either the extractor magnet in s.s. 16 (to West Hall), or s.s. 62 (to East Hall). Except for the extractor magnet and the corresponding orbit deforming dipoles, the system is identical for both channels. Further details on some of the extraction channel elements are given in Ref. 4.

The internal target is put at a position as required by the secondary beams, and by means of a servo-loop acting on two orbit deforming dipoles, the sharing ratio is adjusted.

The servo-system for the ejection acts on a separate quadrupole and works independently of the target servo.

3. Description of the Program

The program is an extensively modified version of the program of ref. 5, treating the target similarly to ref. 6.

At the beginning, radial and vertical co-ordinates are chosen corresponding to an approximately Gaussian beam profile. All particles are given the same momentum, as we assume that the sharing process is independent of momentum within the momentum spread of the debunched beam.

After each machine turn, the radial position and accordingly the Q-values are changed to simulate the drift of the beam, and checks are made to see if the target is hit, if the particle enters the field of the first septum or if it is lost on the walls of the vacuum chambers.

While the magnetic septa are taken into account with their actual septum thickness, the thickness of the electrostatic one is neglected. This is justified by the experience that the losses on this septum are proportional to the ejected beam intensity.

3.1 Internal target

As in ref. 6, each particle is assumed to represent a large number of protons, given by its "intensity" μ_i , which is set to 1 at the beginning. If a super-particle hits the target or its support, a certain fraction $\Delta\mu$ is lost by nuclear interactions

$$\Delta\mu = \mu_i \left(1 - e^{-\frac{D\rho}{\lambda}} \right) \quad (1)$$

D is the length of the target (or support), λ the total nuclear mean free path, and ρ the density of the target material. The surviving part loses energy by ionisation

$$\Delta E = -L.D.\rho \quad (2)$$

where L is a material constant characterizing the ionisation energy loss.

Moreover, the angle of the super-particle is changed by multiple Coulomb scattering. The distribution of the projected scattering angles is assumed to be Gaussian with r.m.s. θ (Rossi formula)

$$\theta_s^2 = \left(\frac{E_s}{\beta_{cp}} \right)^2 \frac{Dp}{X_0} \quad (3)$$

with $E_s = 15$ MeV, β_{cp} measured in MeV, and X_0 the radiation length depending on target material.

3.2 Definition of efficiencies

In the program, a superparticle can have the following "fates" :

- after many target traversals, its betatron amplitudes may have grown such that it hits the vacuum chamber either vertically or horizontally; or
- after having passed through the deflecting field of the electrostatic septum, it may hit one of the septum magnets either radially or vertically; or
- after having passed through the deflecting field of the electrostatic septum, it may be extracted.

The sum of the intensities of the latter super-particles divided by the original total intensity, gives the extracted fraction n_e . The sum of all Ap's (eq. 1) divided by the original total intensity, gives the fraction n_t lost by nuclear interactions.

In a separate run, with all ejection lenses switched off, the fraction n_{to} has been computed as reference value. We define then the relative fraction of the beam on the target by

$$f_t = \frac{n_t}{n_{to}} \quad (4)$$

and the computed sharing efficiency by

$$\eta_c = n_e + f_t \quad (5)$$

This corresponds to the efficiency η_m as defined in ³ for measurement purposes

$$\eta_m = \frac{\eta_{es}}{\eta_e} + \frac{\alpha_{st}}{\alpha_t} \quad (6)$$

with η_e the measured ejection efficiency without target, η_{es} the measured ejection efficiency with sharing, α_t and α_{st} being the integrated and normalised signals from a counter near the internal target, the former measured when all ejection lenses are switched off (reference), the latter measured during sharing. All efficiencies and fractions are to be understood with respect to the beam after all preceding operations like fast ejections or short target bursts.

3.3 Emittances

As emittance, two times the mean squared betatron amplitude is taken.

$$\epsilon = 2 \langle a^2 \rangle = 2 \frac{\sum \mu_i a_i^2}{\sum \mu_i} \quad (7)$$

with, for the vertical co-ordinates,

$$a_i^2 = \beta y_i'^2 + 2\alpha y_i y_i' + \gamma y_i^2 \quad (8)$$

For a Gaussian distribution, this emittance contains 86.5 % of all particles; 95% of a measured profile (projected distribution) are between $\pm \sqrt{\beta \epsilon}$.

The radial emittance in the case of a slow ejection is, of course, not at all elliptic, and the distribution is not Gaussian. Nevertheless, a formula similar to (8), with empirical constants, was used to calculate a "betatron amplitude" and the emittance calculated using formula (7).

4. Results

We have concentrated mainly on three systems (for the main parameters see Table 1). While systems I and II correspond essentially to the present situation, system III assumes larger emittances for the internal beam as expected for a future operation at higher intensity⁷. All computations were done for 24 GeV/c. The target dimensions used for calculations are slightly different from the dimension of the targets now used in operation (usually $2 \times 1 \times 10$ mm, 100 to 200 mrad angle), but by a few computer runs we could verify that the influence of target dimension is small. The same argument is valid for a change in target material from Be to B₄C. The material constants used are given in Table 2.

System	Ejection Channel (s.s.)	Emittances of int. beam		Target				
		hor.	vert.	s.s.*	Mat.	width	height	length
		(10 ⁻⁶ π m.rad)				(mm)		
I	16	1.	1.	1	Be	1.	1.	20.
II	16	1.	1.	8	Be	1.	1.	20.
III	62	3.	1.7	8	Be	1.	1.	20.

Table 1 : Main parameters of systems

* Straight section 1 is radially focussing, straight section 8 radially defocussing.

Material : Be		
Ionisation loss L	2.	MeV/(g/cm ²)
Radiation length X ₀	62.5	g/cm ²
Nuclear mean free path λ	56.5	g/cm ²
Density ρ	1.84	g/cm ³

In order to keep the use of computer time at an acceptable level, the calculations were done with relatively small numbers of superparticles (usually 300).

4.1 The process in general

Already for rather modest fractions on the internal target, practically all ejected protons have passed through the target or support at least once (90% for $f_t \sim 0.1$). Some of them have lost so much energy that they become unstable at a radial position outside the normal range (see Fig. 2 for a typical example). They reduce the width of the hole created by the first septum, and a certain part is lost radially on the magnetic septa.

Fig. 2 : Mean radial position of superparticles entering the electrostatic septum. (Typical case).

Some protons are "locked in" to the target, they lose energy so fast that they do not become unstable in spite of the drift and their emittance growth. A small fraction of these protons, however, is scattered across the first septum ($mrp \sim -13$ mm in Fig. 2), creating a diffuse vertical halo around the ejected beam, and is usually lost vertically on one of the magnetic septa (as it has no yoke, the electrostatic septum is no obstacle in the vertical direction).

4.2 Efficiencies and emittances

All results are presented on Figs 3 to 6 as a function of f_t , the relative fraction on the internal target, as defined in section 3.2.

For the sharing efficiency (Fig. 3) we give two curves for each system. The lower curve assumes a rather

poor alignment of the septa, while the upper curve assumes no radial losses on the septa at all. By careful alignment of all septa, efficiencies between the two curves can be expected. A slightly smaller efficiency seems to result when a target in a D-section is used, but this difference has not been found experimentally. The measured points correspond to system I, but with different target dimensions (see above 4.).

Fig. 3 : Calculated and measured sharing efficiencies (Roman numbers are system numbers from Table 1). Single points, which are not measured points, correspond to the pessimistic curve of system I, but with different targets :

- Be, 1x1x10mm
- + Be, 4x1x10mm
- × B₄C, 1x1x20mm

Fig. 4 : Repartition of protons as function of f_t . Unlike the rest of the report, the losses on the electrostatic septum are shown explicitly.

Fig. 4 presents the results for system I in a different way (only the conditions corresponding to the pessimistic curve in Fig. 3 are shown). From the point of view of radiation damage to the whole machine, the additional losses due to the sharing process (region 3 and a small part of 4) are small compared to the damage due to the use of any internal target at all (region 5 and most of 4). Even if the losses on the magnetic septa can partially be avoided by careful alignment, the irradiation of these delicate magnets represents however one of the most important limitations of the sharing scheme.

Figs 5 and 6 show the blow-up of the emittances by the sharing process. Again, it seems that a target in a D-section is slightly worse than a target in an F-section. Measured points for system II³ are probably too optimistic, as they have been measured rather far downstream in the ejected beam line, and there are indications that a small part of the beam might have been shaved off on the way (in our calculation, all particles passing through the extractor magnet have been included).

Fig. 5 : Blow-up of radial emittance. Single points other than measured points have the same meanings as in Fig. 3.

Fig. 6 : Blow-up of vertical emittance. Single points other than measured points have the same meaning as in Fig. 3.

5. Conclusions

On paper, beam sharing between slow ejection and internal targets at the CPS looks rather efficient, and experience supports this result. Under the conditions prevailing at CERN, it allows an optimal use of experimental facilities.

Taking the machine as a whole, the difference in radiation damage between an operation with an internal target alone and a sharing operation is small. The additional losses however occur mainly on the magnetic septa, as these are the aperture limitations in the first part of the extraction channels (both radially and vertically). The growth of the radial emittance of the extracted beam is small, but the vertical emittance is blown up considerably (by about a factor 2 for $f_t \sim 0.3$), which leads to losses in the external beam transport and increases the spot size on the external target.

In order to keep these effects tolerable, usually one chooses f_t not bigger than about 0.35 (fraction of the beam after all previous operations), which corresponds to about 20% to 30% of the total accelerated beam.

6. Acknowledgements

We would like to thank Mrs. D. Dumollard and D.J. Simon for their emittance measurements, and G. Cuisinier, Ch. Serre and Ch. Steinbach for their help during machine development sessions.

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EXPECTED ENERGY - SPREAD IN THE EXTRACTED BEAM OF SATURNE II

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Abstract.

Nuclear spectroscopy in the range of medium energy (≈ 1 GeV) needs a good duty cycle and extracted beam little dispersed in energy ($< 10^{-3}$). To meet these requirements it is possible to slowly extract the beam in a resonant mode so as to give an instantaneous momentum spread small compared to the stored one. For this purpose we propose to slowly push the stored beam into the resonance $3\nu = 11$ by means of a betatron acceleration in order to avoid the creation of structure. In the frame work of this method we develop a simple formula in good agreement with computer results giving the dispersion in energy of the beam as a function of the principal parameters of the extraction. A numerical example is given applicable to the new design of Saturne II (proton accelerator in the range 0.5 - 2.7 GeV).

A large part of the experimental program around Saturne II is based on spectrometry facilities for nuclear physics studies in the range of medium energy (0.5 to 2.7 GeV). The new synchrotron will have to provide experimental areas with high quality proton beams slowly extracted.

The beam will be extracted in a resonant mode excited by the 11th harmonic of sextupolar fields in the neighbourhood of $\nu_H = 11/3$.

At a given amplitude, the particles encounter a septum magnet and are extracted. This extraction septum must be outside the stored beam particularly at injection. Reciprocally, the injection septum must not interfere with the unstable particles before they enter the extraction channel. Generally one can get rid of this problem by a local closed orbit deformation¹ or by moving the extraction septum towards the damped beam. In the present case, it has been possible to choose a suitable location of sextupoles with respect to injection and both extraction points (fig. 1) so that particles do not strike injection septum before entering extraction magnet which is set radially outside the injected beam.

In this scheme, during the last three turns, particles reach large amplitudes beyond the "good field" region.

Extraction is achieved by 2 sextupoles of equal strength but different polarities, set diametrically opposite in the machine. Another pair of chromaticity sextupoles of equal strength and same polarities makes it possible to act on :

$$K_H = \frac{d\nu_H}{(dp/p)}$$

and consequently, because of the sensitivity of the separatrix size to deviation of ν_H , they allow for adjustment of the instantaneous momentum range in the extracted beam.

Debunched beams with small momentum spread are requested. Therefore, on the flat top, we plan to extract with ν_H highly dependant on momentum and to push the debunched beam towards the resonance by means of a betatron acceleration.

Typically, at 1 GeV, the stored beam is characterized by its emittances $\mathcal{E}_H \approx 10$ mm x mrd, $\mathcal{E}_v \approx 25$ mm x mrd and its momentum range $\Delta p/p \approx \pm 1.6 \cdot 10^{-3}$.

For a deviation $|\Delta\nu_H| = 3.3 \cdot 10^{-3}$ of ν_H value from its value at resonance the triangular separatrix encloses the beam. The corresponding extraction sextupole strength allows a jump $\sigma \approx 2.5$ cm at 8 cm from the axis during the last three turns.

Total emittance and instantaneous momentum spread in the output depend on chromaticity. A range

$$\left(\frac{\Delta p}{p}\right)_{\text{ext}} = \frac{|\Delta\nu_H|}{K_H}$$

is extracted at the same time. The corresponding total emittance is $\mathcal{E}_x = \sigma \Delta x'$ (fig. 2).

One can easily establish formulae for $(\Delta p/p)_{\text{ext}}$ and $\mathcal{E}_x$ with σ as a parameter. Using K. R. Symon's results one finds :

$$\left(\frac{\Delta p}{p}\right)_{\text{ext}} = \frac{3^{1/2} X_o \sigma \cos^2 \varphi}{9\pi K_H (X_s^2 - X_o^2 + X_s \sigma)}$$

$$\mathcal{E}_x = \frac{\sin(\varphi - \alpha)}{\beta_x \cos \varphi} \left\{ \sigma R \left[g^2 + (\alpha_x g + \beta_x g')^2 \right]^{1/2} - \frac{2\sqrt{2}}{3} K_H \times \right. \\ \left. \times (X_s^2 - X_o^2 + X_s \sigma) \frac{1}{\cos \theta (\sqrt{3} - \tan \theta)} \right\} \left(\frac{\Delta p}{p}\right)_{\text{ext}}$$

where :

$$X_o = \sqrt{\frac{\mathcal{E}_H \beta_x}{3^{1/4}}} \cos \varphi$$

X_s : distance from beam center to extraction septum

R : mean radius

α_x, β_x, g, g' : usual orbit parameters

φ, a are represented in figure 3

$\theta = \varphi - \alpha + \pi/3$

Numerical results are given in fig. 4.

However for increasing chromaticity values which correspond to smaller and smaller extracted momentum range, ν_H is more and more amplitude dependant and extraction is entirely suppressed by separatrix distortion.

It is well known that non resonant terms can perturb the extraction mechanism. In the present case, we have to deal with sextupolar and octupolar components in magnets (extraction is very sensitive to quadrupole fringing fields) and especially with non resonant components created by extraction sextupoles and strong chromaticity sextupoles.

Results from analytical derivations and numerical computation lead to the same chromaticity limitation 3 :

$$-12.5 \leq K_H \leq 3$$

(these two values are symmetrical with respect to the intrinsic K_H).

Consequently the instantaneous extracted momentum dispersion cannot be less than :

$$\left| \frac{\Delta p}{p} \right|_{\text{ext total}} \geq 3 \cdot 10^{-4} \text{ at } 1 \text{ GeV}$$

Octupolar zero harmonic components will be used for widening the range of allowed K_H values, but with large stored beam $\Delta p/p$ the ν spread within the beam could become excessive. In fact $\Delta p/p = \pm 1.5 \cdot 10^{-4}$ will be the possible minimum.

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Fig. 4

INJECTION AND TRAPPING OF THE BEAM AT 800 MeV IN THE CPS

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Summary

The principles of injection and trapping of the 800 MeV beam from the CERN PS Booster are briefly described. A local deformation produced by a set of four bumpers displaces the closed orbit on the edge of a septum magnet, while the incident beam is made to jump the septum thickness by a full aperture kicker. The PS buckets are matched to the incoming bunches; the normal accelerating conditions are then reached adiabatically. A short review of the equipment needed for inflection, longitudinal trapping, acceleration and beam measurements is made. Results obtained in running-in sessions and operation are summarized.

Introduction

The CPS improvement programme was essentially completed in the second half of 1973 by the successful increase of the accelerated proton intensity using the Booster injector.

This paper describes the injection equipment and operating procedures. Two kinds of system are used, a specific one for the 800 MeV PSB beam inflection, and a general-purpose one adjusted for this usage. This is necessary because the CPS has two modes of operation: one at 50 MeV from the Linac and one from the PSB. The computer has proved to be a great help for these applications.

Injection

Principle

The slow-cycling injector synchrotron (PSB) has been described elsewhere^{2,3} as well as its performance^{4,5,6}. The beam extracted from the 4 PSB rings is recombined and measured in a transfer line^{7,8,9}. Matching is also achieved there by a set of 6 quadrupoles and drift spaces.

The principle of monoturn injection¹⁰ is rather simple. A localized deformation of the CPS closed orbit allows using a fixed septum magnet, lying outside the synchrotron acceptance, as an input point in the machine. Four horizontal dipoles (septum bumpers) produce a forced closed orbit deformation over two magnet periods (1 betatron period = 8 magnet periods). Currents are calculated to produce the desired displacement with no residual deformation outside the inflection region. Local fluctuations of the focusing betatron function (β) were taken into account since a $2 \cdot 10^{-3}$ error on one deflection leads to a 1 mm residual deformation. The incoming beam is brought on the closed orbit by a fast deflection at half a wavelength away from the end of orbit deformation,

Initial inflection conditions are achieved horizontally by a dipole in the transfer line and the septum magnet and vertically by two dipoles before the CPS input. Energy adjustment is made by a synchronization between the two machines, based on a very accurate pulse train linked to the magnetic field of the CPS (10 μ T resolution with 3 μ T jitter)¹¹.

The aim of the procedure is to achieve the conditions given in Fig. 3. The emittance of the incoming beam, suitably matched, determines the jump to be made

at the level of the septum. Synchronization is fixed by the PSB energy; it is only necessary to adjust the local closed orbit deformation so as to avoid any residual betatron oscillation. The usual strong focusing synchrotron matrix calculations show that only the closed orbit angle and position at the septum level have to be adjusted. The use of 4 dipoles gives an additional degree of freedom which is used to cater for some possible future acceptance-limiting conditions (vertical stacking, two-turn radial injection). The hardware specifications allow incoming beam emittances (95% of the particles) of $E_H = 33 \pi \cdot 10^{-6}$ rad.m and $E_V = 12 \pi \cdot 10^{-6}$ rad.m, on closed orbits which at the input point can vary horizontally between + 15 and - 7.5 mm in position, ± 0.6 mrad in angle, and vertically between ± 5 mm and ± 0.25 mrad.

Fig. 1 Layout of inflection region

Fig. 2 General optics for 800 MeV injection

Control of magnetic state of the CPS

This system comprises the set of magnetic corrections of up to the 3rd order to control the CPS magnetic state up to about 6 GeV/c for both 50 and 800 MeV injections.

The general policy adopted for the upgrading of this system¹² was to provide independence by using one supply for each lens. The various compensating circuits (in particular harmonics) are achieved by computer programming.

The location of the various elements in the machine is such as to ease some compensation by avoiding dangerous harmonics and minimizing bothersome secondary effects. The corrections enable adjustment of the working point and to act on all stopbands up to the 3rd order in the given energy range. This is necessary

because of the working point chosen and the space charge effects observed^{13,14}. The effect of octupolar resonance has not yet been demonstrated at these low energies but has been studied above transition¹⁵. The low-energy magnetic corrections are summarized in the following table.

Lens type	No	Description	Maximum strength per lens	Power supply	Use and performance (at 800 MeV)
Horizontal dipoles	20	2-turn windings on the main magnet yoke	10^{-3} Tm (4 mm)	± 10 A 50 V	local + 6th, 7th harmonics radial closed orbit < 8 mm p.p.
Vertical dipoles	20	air core on araldite frame	$2.6 \cdot 10^{-3}$ Tm	± 10 A 30 V	local + 6th, 7th harmonics vertical closed orbit < 5 mm p.p.
Normal quadrupoles	40	air core (L = 15 cm) 20 at δ_{Hmax} locations 20 at δ_{Vmax} locations	$5.6 \cdot 10^{-2}$ T	± 10 A 40 V	zero harmonic: $\Delta Q_{\text{max}} = 0.35$ 13th harmonic, horizontal and vertical (stopbands $2Q_x = 13$)
Skew quadrupoles	40	idem on the same frame as the vertical dipoles	$3.5 \cdot 10^{-2}$ T	± 10 A	zero harmonic ($Q_x = Q_y$) and 13th harmonic ($Q_x + Q_y = 13$)
Normal sextupoles	16	air core, in the straight sections 8 at δ_{Hmax} locations 8 at δ_{Vmax} locations	$2 \text{ T} \cdot \text{m}^{-1}$	± 20 A 60 V (± 40 A for enlarged lenses)	zero harmonic (chromaticity variation ± 0.5) 19th harmonic (stopbands $3Q_x = 19$, $2Q_y + Q_x = 19$)
Skew sextupoles	16	idem on the same frame and concentric with the normal sextupoles	$2 \text{ T} \cdot \text{m}^{-1}$	idem (± 60 A for enlarged lenses)	zero harmonic, 19th harmonic (stopbands $3Q_y = 19$, $2Q_x + Q_y = 19$)

* located between the main magnet coil head

These lenses are excited by medium-power amplifiers steered by programmed voltages which can be time-dependent functions or linked to the magnetic field. These voltages are produced by computer-driven function generators¹⁶. All the parameters are digitized and acquired by the computer.

Inflection system

This whole set of elements^{17,18,19,20,21} was specifically designed for the injection of the beam coming from the Booster and is also entirely computer-controlled. The name and location of the various elements can be found on Fig. 1. Their characteristics are summarized below.

Type of element	Main characteristics	Power supply	Performance	Use and remarks
Transfer dipole (TDH3, TDV7, TDV8)	± 8 mrad	dc ± 20 A	stability: $\leq 10^{-2}$	determination of initial conditions
Septum magnet (TIS)	gap height: 60 mm gap width: 100 mm septum thickness: 6 mm 66 mrad	dc 1960 A	field homogeneity: $\pm 2 \cdot 10^{-3}$ stability: $\leq 10^{-4}$	located in a δ_{Hmin} straight section, adjustable in radial and vertical positions and in angle
Septum bumpers (SB)	gap height: 90 mm gap width: 212 mm on metallized ceramic chamber 23.2 mrad	condenser discharge half sine wave current ($f = 240$ Hz) 4000 A	accuracy: $\leq 10^{-3}$ stability: $3 \cdot 10^{-4}$	4 in CPS straight sections; the discharges are balanced to limit the residual closed orbit to a few 1/10 mm during the pulse
Fast kicker (TIK)	delay line kicker gap height: 54 mm gap width: 150 mm 3.7 mrad	60 kV delay line 2.5 μ s pulse length	stability: $2 \cdot 10^{-3}$ ripple: $2 \cdot 10^{-2}$ fall time: 50 ns	the discharge instant is stabilized by feedback

The general synchronization of the injection is derived from a master pulse linked to the main CPS magnetic field. This master pulse defines the injection timing. Pre or post pulses synchronize the PSB ejection transfer and the CPS injection. To synchronize the bunches, pulses linked to the PSB accelerating voltage are used (fine tuning of recombinations, TIK, etc.). Most of the adjustments are made via a computer.

Transverse beam observation and measurements²²

Beam position is checked by a few TV screens but mainly by electrostatic PU electrodes (two at the end of the transfer). The usual CPS trajectory measurement system²³ is the main tool for fine tuning of the inflection, thanks to the 1st turn synchronization in a given bunch.

Independently of the data given by the PSB instrumentation²⁴, a set of current transformers²⁵, located at the end of the transfer line and in the CPS, measure the beam intensity. The signals are used for analog observation and are afterwards integrated and digitized (for elaborate numerical display). The frequency range of interest is covered by passive (60 MHz) or active transformers with various integration ranges (with the possibility of independent digitizing of the first 6 turns).

Signals of 6 beam loss monitors with ACEM-type photo-multipliers²⁶ are used to detect the likely cause of an inflection loss: analog observation and integration digitization, gated to choose the timing of the measurement (n turns after hth).

The most sophisticated apparatus is for emittance and matching measurements. A set of 10 secondary emission wire monitors (SEM grids²⁷) is used with a double monitor on the incoming and circulating beam at the septum exit, and 3 monitors at $3 \lambda/8$, $5 \lambda/8$ and $6 \lambda/8$ from the inflection point in both transverse planes. The measuring grid is made of 32 0.1 mm Cu-Be wires, spaced by 2 mm. The charge on each wire is integrated over one CPS turn and digitized. The data are processed by computer to reconstruct the beam profile and with 3 profiles the injected beam emittance can be computed and its matching checked. The double monitor is also used to check whether the injection is correct.

SEM grid and its mechanical positioning device

Measured beam profiles. Left: circulating beam. Right: both incident/circulating beams

Longitudinal Trapping

Principle

The PSB bunches have to be matched to the CPS RF buckets by modifying the main magnet B and the RF voltage, and normal conditions are then reached rapidly but adiabatically. Several methods were possible, depending upon the dilation allowed. The following was chosen as the best compromise between various requirements (accuracy of synchronization from magnetic field

Fig. 3 Inflection trajectories in a normalized phase diagram

derived pulses among others): $\dot{B} = 0.22 \text{ Ts}^{-1}$, initial bunch length 145° ; RF voltage: 37.5 kV, stable phase angle 15° , which give a theoretical 5% dilation after matching. B is kept constant for 20 ms, then raised linearly to 2.3 Ts^{-1} in 20 ms, which is compatible with the adiabaticity condition (synchrotron period of 0.4 ms). The RF voltage is simultaneously adjusted to prevent the bunch length decreasing faster than $(\beta \gamma^2)^{-1}$ in order not to increase the transverse space charge forces.

Moreover, the accelerating voltage is adjusted so that the buckets are almost filled in order to prevent possible instabilities by Landau damping.

a = main magnet B, b = RF voltage envelope, c = wide-band pick-up signal (detected)

Equipment and measurements

The general phase and radial CPS beam control system²⁹ has not been modified. The usual adjustments are used and are not described here. The only elements specific to the 800 MeV injection are the initial accelerating voltage programming and the synchronization with the RF of the PSB.

The latter is achieved before injection proper by replacing the PS phase PU signal by one coming from the reference oscillator of the PSB, which is properly phase matched. The usual observations related to the acceleration (radial position error, stable phase, etc.) and to the feedback loops, are used. Beam behaviour is observed with wide-band (1 GHz) electrostatic PU elec-

trodes which enable the evolution of the longitudinal density and its stability during acceleration to be followed. Fast sampling and digitization of these signals is also possible³⁰.

Operator Interaction

All controls are made using an IBM 1800 computer and its data transmission system³¹, with the exception of those for longitudinal capture and acceleration. In consequence all operator access is concentrated at one console³², which is centred on an alpha-numeric and graphical display. Adjustments and beam observations are made by using the display with the aid of a keyboard and four shaft encoders. The basic software tool is an interpretive syntax, ISAAC (Interpretive System for Automated Accelerator Control³³). This makes it possible to establish a setting-up procedure employing the calculations mentioned above, and displaying theoretical parameters to the operator in normalized units and in a decoupled form. This has been a significant factor in the success of this injection. For example, the closed orbit is adjusted relative to the injection septum, while keeping the resultant deformation automatically at zero, without the operator having to concern himself with the actual currents in the various dipoles. In addition, it is possible to store reference measurements on the computer's disk store, and by subtracting the present values to display in real time such features as the residual betatron oscillation at the injection. Since this constitutes the difference between the closed orbit and the actual trajectory, its minimization via the console is, in fact, the fine tuning of the injection.

The injection console

A complete description of the console is outside the scope of this paper. It is however worth noting that all the relevant analog signals are under computer control, providing centralized multiplexing, selection and identification of 225 signals up to 2 MHz and 225 more up to 30 MHz. The simultaneous availability of a large number of analog, numeric and graphical data has been of incomparable help in the understanding, running-in and operation of the injection from the PSB.

Results

After the usual equipment checks for the running-in of a system of this complexity, study sessions (12 hours per month on average) gave the following results in 1973.

"Nominal" beam (from a single PSB ring)

Initial conditions at transfer: intensity $\sim 10^{12}$ p⁺/p, $E_H = 21 \cdot 10^{-6} \pi$ rad.m, $E_V = 9 \cdot 10^{-6} \pi$ rad.m, obtained by target collimation; radial and vertical positions ≤ 2 mm, longitudinal emittance 9 mrad ($\Delta p/m_0 c$, RF rad). Transverse trapping: $97 \pm 2\%$, longitudinal trapping 100%, residual oscillation < 1 mm. Matching: at its best leads to a dilation of about 10% horizontally and 20% vertically (controlled by SEM grid at injection and targets at 10 GeV/c). Loss-less acceleration of $\sim 10^{12}$ p⁺/p. Transition crossed without losses or longitudinal blow-up, thanks to the use of the y-transition jump³⁴. Longitudinal instabilities avoided by careful optimization of the accelerating voltage³⁵. Longitudinal density twice as large as a 50 MeV injected beam. Transverse instabilities damped by octupoles³⁶.

oscillation
(first turn)

losses and
efficiencies

some instructions
of current use

Example of use of alpha-numeric and graphic display

5 bunches (one PSB ring) at end of transfer (a)
and two first turns in CPS (b)

"Intermediate intensity" (4 PSB rings)

PSB beams without collimation give: $E_H = 24$ to $30 \cdot 10^{-6} \pi$ rad.m; $E_V = 14$ to $20 \cdot 10^{-6} \pi$ rad.m; intensity = 1 to $1.5 \cdot 10^{12}$ p⁺ by ring, depending on the ring. Losses at inflection $\sim 15\%$. Longitudinal trapping 100%. Loss-less acceleration giving about $5 \cdot 10^{12}$ p⁺/p at 26 GeV/c. In addition to the dilation mentioned Above, an initial blow-up occurs due to differences between the rings; after this, no noticeable blow-up appears until 10 GeV/c. However, from 10 to 26 GeV/c, there is an unexplained vertical emittance increase by a factor ≤ 2 . Successful tests of fast extraction

(26 GeV/c, efficiency $\geq 97\%$) and slow extraction (6 1/3 resonance, 200 ms spill at 24 GeV/c, efficiency $\geq 90\%$) preceded operational use of the system,

"Slow" transformer signals: a = with acceleration
b = coasting beam.

Operation

The "intermediate intensity" has been used several times essentially for the neutrino channel. During these periods the overall PSB - CPS reliability proved to be reasonably good (same failure rate as usual). The peak intensity reached was $6.5 \cdot 10^{12}$ p⁺/p and $5.58 \cdot 10^{12}$ p⁺/p was averaged over two weeks.

Near future

Some improvements are planned to achieve reliable operation at 10^{13} p/p:

- to obtain this intensity within the nominal emittances and to refine the matching, avoiding the initial losses at inflection
- to improve the stabilization process of intense beams to maintain high density
- to complete computerization^{37,38} providing automatization of procedures and monitoring
- to complete density distribution measurements and studies of their perturbations from injection to medium energies³⁹.

Acknowledgements

This work, and in particular the hardware, is the result of the teamwork of many people from the CERN MPS Division

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Summary and Introduction

At the 6th International Conference (1967), higher energy linacs and booster synchrotrons, cycling fast or slow, were discussed as a means for raising the intensity of the main proton synchrotron. Today one machine of each of these types is in operation: the 200 MeV A6 Linac; the 8 GeV NAL Booster; and the 800 MeV slow-cycling four-ring CERN PS Booster (PSB). It is still too early to assess the full PSB capabilities, but at present the design expectations seem, on the whole, to be realizable, as far as intensity (10^{13} ppp), machine systems performance, and reliability are concerned. Not unexpectedly, obtaining and preserving beam density has been more difficult, and we are almost a factor of 2 below the design values.

This report consists of two parts. First, the major design options are assessed in the light of the experience gained over the last two years. Secondly, the efforts to obtain and to preserve beam density are reported; in particular, optimization of multiturn injection, choice of the working point in the Q_H , Q_V diagram, compensation of stopbands, adiabatic RF trapping, and avoidance or damping of various instabilities. The concluding remarks contain data on the latest performance as a CPS injector, and an outlook into the future.

Assessment of Some Major Design Choices

For 50 MeV injection, the CPS intensity is space-charge limited to about 2×10^{12} ppp. The purpose of the new CPS injector is to raise the intensity to about 10^{13} ppp with only moderate increase in transverse and longitudinal emittances. Major choices concerning this injector (Fig. 1) and their assessment are listed in Table 1. While good progress was made during the last two years^{21,8}, resulting in successful runs for neutrino physics, the original performance aims have not yet been reached completely. (Because of the reduction in resources available, it was decided to operate the PSB for part of the CPS time only, and at an intermediate intensity level of about 5×10^{12} ppp, until higher intensity is required by the SPS.) Thus some of these assessments may need to be updated and/or eventually completed.

Multiturn Injection^{8,21}

Extensive computational studies²² showed that horizontal multiturn injection with stacking in transverse phase space is a promising method for obtaining beams of high transverse density in the PSB. With a stable Linac beam of good quality it was found that the measured efficiency values, typically in the region of 30 to 50% (depending on such parameters as number of turns, Linac beam emittance, acceptable PSB beam emittance, value of Q_H , etc.) agree remarkably well (to within 5%) with the predicted figures. Unfortunately, lack of usable observation equipment for the trajectory at injection led to rather lengthy setting-up procedures by trial and error. With a Linac beam of 50 mA (half the design intensity), 8 to 9×10^{12} p (total of the four rings) are now routinely injected into the nominal horizontal emittance ($\epsilon_H = 130\pi \cdot 10^{-6}$ rad m at 50 MeV). Recent experimental and theoretical studies²³ demonstrated that for small values of $|Q_H - Q_V|$ a substantial improvement of the efficiency (up to 60%) can be achieved by means of a linear coupling introduced by zero-harmonic skew quadrupoles. After reinventing²⁴ this technique for the PSB, we used it in operation for

some time. Although leading to an unwanted vertical emittance increase (due to $\epsilon_V < \epsilon_H$) it is nevertheless considered an interesting way of obtaining higher beam intensities for making machine experiment, for instance on beam loading.

Choice of Working Point, Stopbands

At the design stage it was unclear whether the working point should be above 4.5 or below. On the one hand, transverse resistive wall instabilities are more serious above. (Ionic instabilities are more serious below, but because of the low pressure of $< 10^{-7}$ Torr, they were not expected to occur.) On the other hand, for a Q -value below 4.5, there is not much working room available for accommodating larger (incoherent) Q -shifts. Furthermore, the space-charge driven fourth-order difference resonance $2Q_H - 2Q_V = 0$ was expected to lead to beam blow-up for $|Q_H - Q_V| < 0.2$ ²⁵.

The final choice was $Q_H = Q_V = 4.5$ for zero quadrupole trim currents and to rate the trim supplies such that the entire area $4.0 < Q_H, V < 5.0$ and some outside regions could be reached. On account of the high current, high proton density goal, slightly more weight was, however, given to providing room for the Q -shifts. Hence, the first lenses for stopband narrowing were planned with $4.5 < Q_H, V < 5.0$ in mind and the running-in started with working points in that area.

While the PSB could then be made to accelerate beams of good intensity, their vertical emittance was up to two or three times larger than the design value ($\epsilon_V = 9\pi \cdot 10^{-6}$ rad m at 800 MeV). The most likely explanation is that a combination of the Laslett Q -shifts with the synchrotron oscillations causes repeated crossing of several stopbands. When a working point was found in this area that avoids stopbands, denser beams could be accelerated but then the beam became unstable transversely. Head-tail modes 1, 2, and 3 were observed for the first time²⁶. Moderate to strong octupole fields cured this instability, but widened some stopbands as expected²⁷.

Most of these problems were overcome by moving the working point to the region $Q_H \approx 4.18$, $Q_V \approx 5.24$ for the following reasons²⁷: i) both Q -values are above integers, hence resistive-wall instability growth rates are slower; ii) the difference line $2Q_H - 2Q_V = -2$ was expected to be narrower than the $2Q_H - 2Q_V = 0$ line since both space-charge forces and Landau damping octupoles [which can anyway be weaker because of (i)] are essentially zero harmonic; iii) because of (ii), more room for Laslett shifts should be available even without narrowing of stopbands. These points were confirmed by experiments, and for the time being this is the usual working point leading to the transverse densities reported in Table 1.

The measured widths of the stopbands agree remarkably well (to within 20% in many cases)²⁸ with those computed from the actual azimuthal distribution of the measured magnet errors.

Though exploring and narrowing stopbands is tedious (and needs more correction lenses than we have at present) we may have to come back to the area $4.5 < Q_H, V < 5.0$ for accommodating larger Q -shifts. (At present it is only used to produce high intensity beams when a vertical blow-up can be accepted exceptionally.)

RF Trapping and Acceleration

For the adiabatic voltage rise used²⁹ trapping efficiencies above 90% are expected theoretically^{29,30}.

Experimental results³¹ are shown in Fig. 2. The values obtained for a beam of 1.2×10^{12} p/ring thus agree with the estimates. Corresponding figures (lower efficiencies for larger Linac beam energy spreads) apply to 2.5×10^{12} p/ring, as long as no instabilities occur (see below). Acceleration and synchronous transfer into the CPS buckets³² are almost lossless, i.e. total loss about 5%, since the time that the beam control and synchronization system³³ became fully operational³⁴. Under normal conditions the bunch area of a 6×10^{12} ppp beam is 9 mrad in units of $(\Delta p/m_0 c) \times RF$ radians.

Instabilities and their Cure

At the present level of operational beam intensity, instabilities, both transverse and longitudinal, are not harmful with either working point, provided the Linac beam energy spread is $\Delta E \approx \pm 150$ keV (design value) and that appropriate zero-harmonic octupole fields are applied. In machine experiments where different working points, other magnet cycles, and/or smaller energy spreads are used, various types of instabilities were studied. An example of a horizontal instability occurring with a 50 MeV coasting beam is shown in Fig. 3. Note the presence of a longitudinal structure, as yet unexplained. Bunched beam instabilities have already been referred to when discussing the working point; they are reported elsewhere in this Conference²⁶. These instabilities appear to be driven by the long-range resistive-wall forces³⁵. In general, the growth rates are as expected from theory, but the threshold intensities tend to be higher than those calculated, possibly because of neutralization (for coasting beams) or the additional frequency spreads introduced by the strong non-linear space-charge forces³⁶.

For large intensities and Linac beam energy spreads ± 150 keV, bunch position and bunch shape oscillations occur soon after injection; so far, studies have brought out neither pure modes nor exponential growth. Instrumentation for more detailed observation is being developed³¹.

Conclusions and Outlook

During the two neutrino physics runs in 1973 with the PSB-CPS beam³⁷, the average number of protons delivered to the external target was about 5×10^{12} ppp, thus fully meeting the intermediate intensity goal. The beam was remarkably stable (Fig. 4) and operation reliable. All studies made so far have not disclosed any difficulties of a basic nature which would prevent us from attaining the original performance aims. Hence the major design options chosen appear to have been sound.

Besides the completion of the beam observation apparatus and the expansion of the computer control system already mentioned, we list among the necessary improvements: i) more protons from the Linac (a new 50 MeV accelerator is under construction³⁹); ii) extra multipoles¹² for stopband compensation (being ordered); iii) refined RF voltage amplitude control¹² (under way); and iv) a faster Linac beam distributor to facilitate trajectory studies at injection and equal filling of the four rings (being built).

The anticipated capacity for accommodating new requirements has already shown up in two ways: i) the planned use of the PSB as pulse-to-pulse intensity modulator when the CPS will supply sequentially various users (25 GeV physics, ISR, SPS) with intensities differing by a substantial factor; and ii) simultaneous and independent use of each of the four rings for machine experiments, operator training, and hardware and software tests.

Acknowledgements

We are indebted to many members of the PS Department, past and present, who were not, or no longer are, members of the PSB staff, in particular to H.G. Hereward who participated actively in the PSB studies, and to G. Brianti, who led the Construction (SI) Division. We would also like to thank the other CERN Divisions for their collaboration, in particular SB Division. Finally, we gratefully acknowledge the contribution of European Industry, without whose competence and enterprising spirit the PSB could never have been built.

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Fig. 1 View into PSB tunnel showing (from right to left) long straight section, bending magnet (with vacuum pumps), triplet, bending magnet, long SS.

Fig. 3 Horizontal instability observed with a 10^{12} p per ring beam coasting at 50 MeV.

AE Linac = +60 keV; $Q = 4.18$, $Q_V = 5.24$;
time scale 20 msec per division.
Upper trace: difference signal from electro-
static pick-up electrode.
Lower trace: sum signal.

Fig 2 Measured efficiency of RF trapping η_{tr} of a 1.2×10^{12} p/ring beam using an adiabatic voltage rise from 1 kV to final RF voltage indicated ($K = 1$ means rise-time equal to sychrotron oscillation period T), a) as a function of RF voltage (for two Linac beam' energy spreads); b) as a function of K .

Fig. 4 Histogram (for 100 acceleration cycles) of Linac current and PSB intensities. (At injection 100 mA circulating correspond to $\sim 10^{12}$ p). From top to bottom: Linac current, intensities accepted by PSB before trapping (rings one to four from left to right), intensities trapped, and intensities accelerated to 800 MeV and synchronized. The figures indicate mean values and standard deviations, respectively. The left-hand column gives the average of the total intensities (of the four rings). The PSB was not optimized when this photograph was taken; during the neutrino runs the intensities were about 20% higher (with the same distribution).

Table 1

Assessment of some major PSB design choices

Aim	Choice, including reasons given in 1966-1969	Consequences of choice	Particular actions taken	Assessment in March 1974
10^{13} ppp	800 MeV slow-cycling circular injector: for about the same cost as a 200 MeV Linac, higher beam quality; spare capacity for contingencies and improvements', ² ($\beta\gamma_{800 \text{ MeV}}/\beta\gamma_{200 \text{ MeV}} \approx 3.5$)	Need to study, select, and design the first slow-cycling "Booster" synchrotron ³ , a fairly complex machine in which collective effects were expected to be significant, particularly in longitudinal phase space.	Taking into account beam intensity effects from the start ⁴ ; emphasis on simple design and easy, reliable operation.	Final cost indeed very comparable to that of a 200 MeV Linac; too early for comparing performance limits; with a 50 mA Linac beam $\sim 8 \times 10^{12}$ ppp were accelerated to 800 MeV ($\sim 5 \times 10^{12}$ ppp within design emittances),
High phase-space density, adjustable within some range	a) Four rings: comparatively modest extrapolation of transverse phase-space density beyond the 1966 state of the art; favourable numerology including the possibility of combining two beams for better ISR performance', ⁵ .	Need to compare horizontally interlaced version with vertically stacked version; higher complexity and probably cost than with two or three rings; need to distribute the Linac beam and to recombine the four beams prior to transfer into the CPS.	Vertical stacking whenever possible; use of combining (magnet yokes, pumping of vacuum systems etc.) and multiplexing (beam observation, data treatment, and display, etc.); emphasis on simple and reliable design for beam distribution and recombination.	Distribution of the beam to the four PSB levels and precision recombination present no particular problems <i>per se</i> , but the load of running in and setting up four almost independent rings was underestimated; vertical stacking is definitely beneficial.
	b) Lattice with 16 periods and no super-periods; separate function magnet: no systematic resonances for $4.0 < Q_H, V < 5.3$; ease of moving the working point; good probability of providing a magnet system of high quality, notably through the use of a computer simulation program'.	Larger number of magnet units than in the case of a combined function magnet; occupation of a larger fraction of the machine circumference in the present case of relatively low peak magnet fields.	No remedy possible or needed in view of the relatively small total number of magnet units and the relatively large PSB circumference, determined independently as $4R_{\text{PSB}} = R_{\text{CPS}}$.	Performance aims achieved; distortions of the uncorrected closed orbit do not exceed a few millimetres peak to peak; half integer stopband widths were measured as $\delta Q \approx 0.01$, and the four rings were found* to be within the narrow tolerances specified'.
	c) Provision of a number of correction lenses from the beginning: faster running in; improved final lens order.	Need to anticipate most wanted types of lenses and their strength without much quantitative experience available on which to base oneself.	Extensive calculations and machine measurements; provision of two extra lenses of a different type for experiments.	On the whole successful approach (see text); the need for skew sextupole corrections was underestimated.
	d) Decision to design equipment from the start with a low beam-coupling impedance: faster running in; better final phase-space density.	Need to specify acceptable impedance values without much quantitative experience for judging beforehand the effect on beam; this led in some cases to awkward decisions whether or not to change a given prototype design.	Calculations ^g , ¹ and measurements" on prototypes and final equipment (RF cavities, fast kicker and septum magnets, quick-connect vacuum couplings, beam detectors, etc.); assessment and modifications to the best of one's knowledge.	Since so far no beam interaction with a fixed-frequency parasitic resonator has been observed, and the impedances cannot be adjusted, assessment is not easy; in view of the difficulty of later changes the course of action adopted appears justified.

	Choice, including reasons given in 1966-1969	Consequences of choice	Particular actions taken	Assessment in March 1974
	a) Magnet power supply fed directly from the electricity mains: no rotating machinery (i.e. less wear, no continuous manning of power house); model for SPS.	Lengthy negotiations with the local Electricity Authority including simulated tests with the grid; electrical coupling with other users inside and outside CERN.	Provision of adequate a.c. compensating and filtering networks ¹³ , appropriate regulation of power supply.	Supply gives entire satisfaction from all points of view: stability, reproducibility, ease of control, freedom from creating perturbations (outside tolerances).
	b) Triplet focusing: resulting beam cross-section comparable to that in the CPS in the long straight sections, hence use of some proven existing CPS designs and even equipment; reduced energy storage in PSB magnets.	Sixteen extra lens units (compared to FODO lattice).	(None possible.)	Exact value somewhat difficult to assess; main advantage probably reduction of energy exchange with electricity mains; also appreciated: possibility to install borrowed CPS equipment in PSB.
	c) Air-cooled RF accelerating cavity having only final amplifier stage in machine tunnel: low failure rate.	New development ¹⁴ based on restricted experience; possibility of thermal runaway of ferrites ¹⁵ .	Adoption of sufficiently low working temperature; individual measurement and stacking of ferrite rings according to properties.	So far entirely satisfactory performance from all points of view.
	d) Standardization and modularization: general economy; less spare parts; easier training of operating and maintenance staff.	Unless standardization is based on a full knowledge of all cases to be covered, difficulties arise; standardization may adversely affect individual inventiveness.	Appropriate organization of Project Division, including Parameter and Technical Coordination Committees.	Unqualified success where applied (injection and ejection lines, power supplies, kicker system ¹⁶ , vacuum components ¹⁷ , naming of PSB components from drawings to software, etc.).
	e) No electronics or power supplies in machine tunnel (unless indispensable): no irradiation; spread of maintenance in time.	Degradation of signal quality and power loss due to longer cable transmission.	Construction of auxiliary tunnel and equipment rooms above machine tunnel; appropriate equipment design.	Aims achieved; easy addition of extra equipment appreciated.
	f) Centralized controls, located in the CPS main control room (MCR): easier running-in; faster take over by CPS operating team.	i) Lack of space and specialized facilities during running-in. ii) MCR and PSB systems activities separated by 400 m distance.	i) Basically none, in the future possibly rebuilding of MCR. ii) Intercom and public address, TV and other communication links.	Completion of the centralized controls was clearly a major progress compared to the earlier (temporary) stage of distributed controls; CPS team has not yet taken over PSB operation.
	g) Powerful beam observation apparatus ¹⁸ : faster running-in; better beam quality; easier fault diagnosis.	i) Largish cost item (~ 10% of total). ii) Dilemma of what priority to give: not required before main machine components but much less useful if not ready from the start-up	i) Multiplexing wherever possible (e.g. one camera for looking at two screens, etc.); cheap solutions where possible (e.g. beam loss monitors) ii) Building by industry (e.g. measurement targets).	Nearly all facilities for the measurement of beam position, beam current, bunch shape, emittance, energy spectrum, closed orbit, and betatron frequency meet expectations and are in use almost continually.
	h) Computer assistance for data acquisition and control: faster measurements; improved beam quality; better reproducibility; easier operation ¹⁹ , hardware tests ²⁰ .	i) Limitations due to existing IBM 1800 (used because of available capacity and time pressure). ii) No manual back-up foreseen (to save cost).	i) Short term: change over to MPX, addition of core memory and a satellite computer for on-line work; long term: new system. ii) Diagnostic means for fault finding.	Data acquisition, treatment and display even more powerful than anticipated; helpful for control after teething troubles ²⁰ were overcome; too early to assess full capability for reducing PSB down time.

A Review Paper

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I should like to start by paraphrasing some notes I circulated at the start of the European 300 GeV project, now known as the SPS. In these notes I said that there were four main purposes a control system for an accelerator has to serve.

1. Requirements

An accelerator control system has to provide facilities for :-

- (i) Normal operation of the accelerator
- (ii) Interaction with the experimenters
- (iii) Experiments on the accelerator (machine development)
- (iv) Keeping records

There is no longer any argument about the advantages of adding computers to control systems, but there is some conflict between the requirements for these different facilities, and it is clear that in the past the designers of some systems have concentrated more on one aspect than on the others.

For normal operation of the accelerator, we want a system that is simple to operate and that allows the normal setting up or changing of operating conditions to be carried out with the minimum possibility of error. It is quite possible to provide a system that, once the desired operating conditions had been inserted, could run up an accelerator from cold, but this would be unnecessarily complicated, and could lead to a system that is too inflexible to serve the other purposes. It seems sufficient to provide a system that leads the operator through the correct steps to run up the accelerator, and tells him what actions to take in the event of some failure.

For the interaction with the experimenters, the first requirement is a good means of communication, together with provision for the transfer of the control of some elements in the beam lines to these, when this is permissible. It is also advantageous to be able to transfer some data periodically from the accelerator control system to the experimenters' data acquisition systems, for subsequent use in the analysis.

For machine development the utmost flexibility is required, and it is in this area that the largest amount of computing power is likely to be required. This is shown particularly in the case of the AGS. Some control of the machine was already carried out using PDP 8 computers at the time when the purchase of a PDP 10 was justified mainly on the effect it could have in speeding up machine development.

Lastly, for record keeping we want selection. It is all too easy to record masses of data that no one looks at.

2. Control Actions

Turning from the overall areas of control to the control actions that can be assisted by computers, I think the most important actions include :

- (1) Complicated setting up
- (2) Supervisory control
- (3) Multiple acquisition and treatment
- (4) Closed-loop control
- (5) Function generation

The first thoughts of most people in the early days was that one of the main contributions that computers would make was in the area of closed-loop control, and the earliest application was in this field. This was the closed-loop control of the Bevatron guide field. It had been pointed out previously, that some people who tried closed-loop control found that by the time the system concerned had been analysed in sufficient detail for the loop to be closed by a computer, the causes of instability had been found and eliminated, and so the closed-loop control became unnecessary. This is not always true, and closed-loop control can improve significantly the performance, or drastically reduce the setting up time, as in the case of the injection steering in the ISR. In other cases, such as the r.f. control for the Bevatron, it would be more correct to call it self-adaptive function generation.

In most cases where a computer was added to an existing accelerator, its first duty was supervisory control, by which I mean the checking of status or

values against those demanded, and providing some warning of unwanted changes. Other facilities were added later. One of the most useful of these is the one I have called complicated setting up. The facility to set up a system of many power supplies either to predetermined values from a table, or according to some mathematical relationship, has saved the operators a lot of tedious work, and has enabled some things that would not otherwise be possible, such as acceleration of the stored beams in the ISB, and the β and tune control on **SPEAR**, where a very considerable amount of computing is required.

However, it is in the multiple acquisition and treatment of data from the accelerator that the most spectacular (literally!) advances have been seen. The ability to acquire masses of data from the beam instrumentation system, manipulate it and produce sophisticated displays of emittance etc. has been shown at a number of laboratories, of which Argonne was amongst the pioneers, and it was said at one of the recent Linac conferences that a linac that did not have an on-line emittance display was way behind the times.

Thus we have seen a tendency away from direct closed-loop control, except in the function generator, and a concentration on the supervisory control and complicated setting up to assist the operator in normal operation, and on the acquisition and presentation of data to assist the machine development.

3. Hardware

Turning from the types of operations, let us now look at the development of the hardware and software needed to meet these requirements, and see if there are any trends from which we can make predictions. Taking the hardware first, we naturally start off with the computers themselves.

3.1 Computers

When computers were first applied to control accelerators they were relatively expensive and so a lot of effort was expended in making small machines do the maximum amount of work. As prices came down, or the control budget went up, larger machines could be afforded, notably the **CDC 924** at the **ZGS**, and the general idea seemed to be to couple as many things as possible to the largest single computer one could afford. However, it was found that a number of routine tasks were taking up an unreasonable amount of **CPU** time, and, with the advent of the so-called mini-computer, the idea of using a dedicated processor for performing some routine task became practicable. A number of systems that started with a single computer have had small dedicated processors added to them, such as the function generators at the **AGS** and **CPS**.

The next stage was that the satellites became larger and more powerful so that they could be used to operate some part of the equipment under the command of the main computer, and may also have some stand-alone capability. This increase in the size of the system can be seen by quoting a few examples.

The **AGS** started off with 2 **PDP 8** computers and later added a **PDP 10** and more **PDP 8's**. This is the exception to my generalization - in this case a large

main computer was added after the satellites had been operating in a stand-alone fashion for some time. This system is described in one of the papers to be presented at this session².

The **Bevatron** started with a single **PDP 5** and now has added **PDP 8's** and a **PDP 9**.

The **CPS** started with an **IBM 1800**, added a **Varian 620** and is now in the process of adding 3 **PDP 11's**, with the prospect of further additions later.

SLAC started with a **PDP 9** for the machine, which was then coupled up to the **SDS 975** controlling the beam lines and now 9 **PDP 8's** are being added³,

Even at **LAMPF**, which started with one of the largest single computer configurations of any machine, a **NOVA 620** and a **PDP 11** have been added for special purposes.

More recent accelerator systems have been planned to use a number of computers from the start, but even then the number of computers has increased. For example, **NAL** started with two **Sigma 2** and eight **Mac 16**, and the latest plans include four **Sigma 2** and over twelve **Mac 16**.

Almost all these systems are arranged so that a control computer acts as master, the others being satellites. The next stage, which we have adopted in the **SPS**, is to split the duties of the central computer amongst a number of processors, of which none is absolute master. In order to do this, one has to have an effective message transfer system to sort out the intercommunication problems. This system is outlined in a paper to be printed in the proceedings⁴.

Summarizing the progress, we have proceeded from single computers to the addition of dedicated satellites, then the stand-alone satellites to a true multicomputer computer system. The next stage seems to be the addition of dedicated processors to the multicomputer system. These could be the so-called micro processors, but more likely will be peripherals with some processing power such as the **CAMAC** units described below.

This development can be paralleled with the **Business ADP** scene. Here also, a single computer, as big as possible, was used to provide a service to a number of "dumb" terminals. The single computer was increased in size until it began to suffer from organizational indigestion. The dumb terminals were then replaced by "intelligent" terminals and now proposals are being made that the big computer should be replaced by a network of mini-computers, some acting as "super-intelligent" terminals, and others dividing the duties of the control computer between them, working on a common base.

Thus the picture on the computer side shows an increase in the number of processors, and the consequent need for a good means of inter-computer communication. However, it is also seen that as the number of computers increases, the number of tasks to be carried out by each computer does not decrease - the total load increases to use up all the additional power! This can be illustrated by the case of the **SPS**. We started out with the idea of having a large number of very small computers, each doing a relatively simple task and having the very

minimum operating system. We have ended up with the same number of computers, but now these are fairly powerful processors with a multiprogramming operating system in each, when all the requirements have been taken into account.

3.2 Interface to the equipment

The next most important thing after the computers is the interface system. In the earlier systems either the manufacturers standard interface, or some special system developed in the laboratory concerned, was used. I think we can now say that we are well on the way towards standardization on CAMAC, with NAL, Triumph, Los Alamos, Daresbury, Rutherford and the CERN machines, amongst others, using it for at least part of the interfaces. CAMAC may not be the ideal interface for control use, but it is a recognized standard that has been accepted by the experimental physics community, and is being used more and more in control outside this field, including medical and some industrial applications. Sufficient different modules are now available to cover almost all requirements.

The main disadvantages of CAMAC are that the miniature plugs and sockets, which have to be used because of the narrow panels, cannot be used with normal sized cables, so that, for transmission over distances of more than a few metres, junction boxes have to be used, and relays, transformers or opto-couplers inserted to minimize pick-up where standard CAMAC modules serve several different pieces of apparatus. There are also problems in having the CAMAC crates any appreciable distance from the computer with most existing systems, but this will be solved when the serial highway alternative gets going.

On a large accelerator, these disadvantages can be overcome, without losing too many of the advantages, by using a second level of multiplexing for acquisition and control where the timing is not too critical. The system for the SPS, which uses CAMAC as the primary interface, is outlined in a paper in these proceedings⁴.

I mentioned earlier the trend towards putting some processing power into the peripherals. The SPS system has a CAMAC autonomous function controller that can be loaded with a simple program to cause it to perform a repetitive action. An example of its use is for status checking via the multiplex system. When initialized, it can scan a list of addresses and desired status in the core of the host computer, via DMA, and check the actual status of the equipment connected to the multiplex system. If the status is correct, it will step through the whole table. The CPU is not involved, unless an error is found and in such a case an interrupt is raised and the address of the offending unit is passed to an alarm program. Other simple processors in CAMAC are under development in other laboratories.

3.3 Interface to the operator

Another area where there have been some developments is that of the computer/operator interface. Gone are the days when all action was through a teletype. More and more systems are using visual displays with various means of interaction. The light pen found a few friends, and we were told about the LAMPF card-programmable buttons and the SLAC cross-wire touch-screen

at the National Accelerator conference last year^{5,6}. A further development on this theme is the capacitive-change touch-screen described in a paper in these proceedings⁷. Various systems have used knobs connected to incremental encoders that could be hooked to control variables in the system, and the latter paper also describes a knob with feedback to the operator, as it can have variable resistance to being turned, or can become a multiway switch, or a rate-control, spring loaded to zero.

4. Software

I have taken up rather a lot of my space with the hardware, but this does not mean that I think it is more important than the software. In fact, it is in the software that I think the accelerator community has made the greatest contributions to computer technology.

The earlier machines were small, and so programming had to be carried out in assembly language. Following the practice of the computer engineers, the programs were written to carry out a particular function and the specialized handlers for the hardware were considered as part of the program. Gradually, as more than one program acted on the same variables, the handlers were made re-entrant and concentrated data bases were introduced. There are certainly some advantages in using a single massive data base with a system using a single computer, as in the system for the CPS Booster described in one of the papers presented at this conference⁸. In the case of a multi-computer system, I think that to concentrate the data into a single area will result in too great a traffic on the data links, and so with a distributed computer system the active data base should be distributed, but there should be a master copy kept centrally, which is updated relatively infrequently, for reference or in case of accident.

The next stage, where computers increased in size somewhat, was the introduction of higher level languages for the production of applications programs, mainly on pressure from the machine physicists. The further development on this theme was the use of a real time interpreter, with a suitable command language, so the operator could interact in a simple way with the control system, and changes could be made on-line. These ideas were first developed in the Rutherford Laboratory in England, and some parallel work was going on in the early days of NAL.

I shall return to the subject of interpreters in the second part of my talk.

Rapporteur's Summing up

In the first part of my talk I traced the evolution of computer control of accelerators up to the present, and now let us see if we can look into the future.

On the hardware side, there seems to be general agreement that, except for the smaller systems, a multicomputer arrangement is the best, with some satellites carrying out the true real-time functions tied to the accelerator time scale, and one or more central computers working in the operator time scale. Whether we should use a single large computer like at Brookhaven, or LAMPF, or many small ones, like at the SPS, is more controversial, and so a point for further discussion.

On the interface side, obviously laboratories which have already developed successful systems for interfacing, such as the DATACOM at Brookhaven, will continue to use them, but for those starting from scratch on future accelerators, I think the pressure to standardize on CAMAC for at least the primary interface will be very strong. A point of discussion could be how much of the routine processing should be carried out by such devices as the autonomous function controller in CAMAC.

Turning to the operator-computer interaction, I think that the accelerator control community is in the forefront of this field, and so I hesitate to predict the next stage. The idea of an accelerator being controlled by verbal instructions from the operator is not too far beyond present technology, but I cannot see any justification for putting any effort into it.

It is on the software side that the biggest differences exist - whether one should use only compiled or assembled programs, or should use an interpreter. For those not steeped in the computer jargon, I should explain that an interpreter is a program that takes statements in a command language and forms the links between the appropriate subroutines at the time of execution. This allows changes to be made without going through the stages of compiling or assembling, linking and loading normally needed.

Thus the operator can interact fully with the system, even programming on-line, seeing the effect of each statement before going on to the next, protected by the interpreter from doing things fatal to the system.

One pays for these advantages by slower execution, but this has not proved a major problem so far. One can have very simple subroutines and a relatively long command language program, which gives the maximum flexibility but slow execution, or more complicated subroutines and a shorter command program, which is less flexible but runs faster.

For the SPS we are basing our software very heavily on an interpreter for a very powerful control language, NODAL, which is mentioned in one of the papers to which reference has already been made⁴. Coupled with this is the use of a separate data base for each fundamental type of equipment, with its specialized handler, which we have called an equipment subroutine.

As a simple example of the power of this system, the complete NODAL program given in Table 1 causes a secondary emission monitor to traverse through a beam, one step at a time from any initial position to any final position, takes a measurement at a particular time in the cycle, and types out a table of positions and values. I would like to draw your attention to the second half of statement 2.1. This EXECUTE command is used to obtain the execution of a group (in this case group 3) of the program in another computer (number 8) using the present value of some variable (P). The REMIT command in line 3.1 causes the return of the value of A to the original program.

In the paper on the CPS Booster system, Dr. Schindl pointed out the difficulties they had run into trying to commission the machine and the software at the same time. An interpreter helps a lot in such circumstances, as the subroutines can be tested with the prototype equipment and difficulties sorted out before the whole machine is finished. Almost all our technicians can program in NODAL, and as there are quite a few computers around already, they are using them to test out both prototype and production units.

Thus we will get quite a bit of experience with practical use of the software even before the accelerator is assembled.

There was some discussion on the relative merits of interpreters at the last conference of this series, and some of the participants thought that interpreters were fine for machine development, but that programs for the regular running of the machine should be compiled, so that the operator would not be tempted to change anything.

I wonder if there has been any change in this opinion since then. The Brookhaven system seems to come half way between the two. The assembly language systems in the PDP 8's with their command lists and execute lists are like rather complicated subroutines that could be called by compiled Fortran statements. However the powerful PDP 10 allows rapid recompilation and so a lot of the advantages of an interpreter are obtained, but this can only be done with a comparatively large central computer.

Incidentally, it has been pointed out to me that you can usually tell if a control man started his career in the computer field or as an engineer. If he is a supporter of interpreters it is highly likely that he started as an engineer!

Summing up, I will stick my neck out and prophesy that the next generation of accelerators will have control systems based on multicomputer layouts, using CAMAC for an appreciable part of the interface. The operator will communicate with the system through touch screens, knobs and tracker balls, and the software will be based on the use of an interpreter for a high level control language,

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1.1 ASK "INITIAL POSITION=" IP "FINAL POSITION=" FP
1.2 SET P=IP; EXECUTE (8) 3.2 P
1.3 FOR P=IP+1,FP; DO 2
1.4 END

2.1 WAIT EVENT(4);EXECUTE (8) 3 P
2.2 TYPE "POSITION=" P-1 "CHARGE=" A

3.1 SET A=MINSNC(2); REMIT A
3.2 SET MINSNC(2,ØPOS)=P

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TABLE I

DISCUSSION

Bruce L. White (University of British Columbia): How much accelerator down time can be attributed to either computer failures or software failures as a proportion of the total down time?

Crowley-Milling: Some of the papers do have some information on this and it varies quite considerably from laboratory to laboratory but in most cases it is a very, very small proportion of the total time once the initial teething troubles have been overcome. As you've heard, in some cases there are considerable teething troubles, in other cases there are relatively minor ones, but I don't think anyone can get away without teething troubles.

Lee Teng (NAL): At NAL, our experience is that down time due to computer hardware and software failure is less than 10% of down time.

Peter Wolstenholme (CERN): It ranges from about 2.2% for the ISR to about 10%.

Alfred Maschke (BNL): I'm personally in favor of interpreters but they have one drawback in that they don't interface well with an enormous amount of the existing FORTRAN sub-routines. Have you a solution for that problem?

Crowley-Milling: This is one of the problems for laboratories that have built up large FORTRAN libraries, or are you meaning the computational programs?

Mnschke: The most difficult programs to write, frequently the I/O things that when you buy a display unit, the manufacturer will provide you with a quarter-million-dollar FORTRAN program to display it.

Crowley-Milling: Well, one of the advantages of using CAMAC

for your main interface from the system is that you can approach the interface problems in a uniform way and this solves quite a lot of problems.

Shigeki Mori (NAL): How many cables carry other than digital signals?

Crowley-Milling: The multiplex system I described has an analog highway and for analog measurements where the highest precision is not required, the multiplex switches the analog signals onto the highway where they are fed into an ADC in the CAMAC and then they become digital from there. The other type of analog signal is the analog wave form signal where we have a switching system under computer control for switching the various analog signals from the various distant buildings to the central control area. So there's that analog system and there's also the analog measurement system.

Gregory Loew (SLAC): In your future system at CERN, what happens if one of the computers that's controlling part of the ring fails?

Crowley-Milling: We have, wherever possible, tried to make the hardware on the principle that it carries on doing what it was told to do previously until it's told to do something different. So that in many cases, it's a case of if a computer went out, that part of the system would continue to run although, of course, you wouldn't get warning signals back or your surveillance system wouldn't be in operation. But there are other regions, for example the computer controlling the main power supply where it is feeding out information every ten milliseconds, where if that goes down, you're off.

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Summary and Introduction

The CERN PS Booster' (PSB) was designed to be completely computer-controlled and no manual back-up was foreseen. The physical construction of the four-ring PSB necessitates simultaneous operation of different parts of the accelerator and therefore requires a multi-entry software facility. A significant feature of this facility is the data base, which links the software to the hardware of the accelerator and also leads to an increase in the versatility of tests and operation. The evolution of the system from its conception, through the phases of commissioning and running-in, each with its different needs, to the present operational stage is described. Attention is drawn to the lessons that have been learnt during the first two years of exploitation. The initial phases were made more difficult because it was impossible to separate completely effects due to the commissioning of the controls system and the first tests of the accelerator. However, the investment in this first phase gave rise to substantial benefits as soon as basic settings for the PSB were found. The current phase of operation typifies some of the problems involved with simultaneous development of a machine and its computer-controls system. There is a certain conflict between those who have to operate the machine as it is and those who want to develop the control system further. Most aspects of the system have worked satisfactorily from the start, but some have been less reliable. However, it is hoped that the effort which is being given to the hardware will allow more sophisticated tasks, such as closed-loop control, to be carried out.

Computer-Controls System

The basic computer configuration¹ ³ for the Linac, PSB, and CPS (Fig. 1) includes an IBM 1800 with 48K of core memory (16-bit words) operating under MX (Multi-programming System), and several peripheral devices (two PDS-1 minicomputers to drive alphanumerical and graphical displays and a Varian 620i satellite to generate analog functions). A 4K partition is dedicated to each accelerator where application programs are housed. Core resident programs go into higher level partitions called Special Areas, shared with other users. A larger partition of 6.5K, called Variable Core (VCORE) is used for background programming and for execution of long or special programs called on interrupt (Interrupt Core Loads) when requested by a partition. The more important application programs in use are listed in Table 1. The total length of programs is about 350K words; 20 man-years were required to define, write, debug, and optimize these programs. Most programs refer to the data base' (~ 700 process variables) which was introduced to avoid conflicting control from the various operator consoles: three midi-consoles (Fig. 2) (for the injection line, RF, and transfer line), a central maxi-console (Fig. 3) concentrating control of all subsets, and a mobile console, used for hardware tests in the equipment rooms (Fig. 4). The control and data acquisition of the four PSB rings are largely independent of each other but are synchronized with the Linac and the CPS. The essential components of the control system are assessed in Table 2.

Problems During Running-In

At present the system works as specified. While certain parts were satisfactory from the start, others had teething troubles, though not more than would be expected for a system of such newness and complexity. Two types of problems are discussed, those which turned out to be more serious than anticipated and those which had not been fully foreseen.

When deciding that there would be no manual back-up from the Main Control Room (MCR), it was realized that if one of the major components of the control system failed, the accelerator could no longer be controlled. (In view of the expense of full manual back-up, the risk was nevertheless taken.) Thus the situation of sometimes not being able to change a parameter setting by computer was a major handicap until reliability improved sufficiently.

Trouble was caused by the front-end equipment (transmission system, converters, power supplies) in the early stages of running-in. This was mainly due to trying to start up the accelerator at the scheduled time although some of the construction staff had moved to another project before they had reached the testing stage of their equipment. It was only with the first beam in the machine that the control system could be fully tested out, and therefore it had to be accepted that the last part of the running-in of the software coincided with the running-in of the PSB.

As for the multi-user computer system itself, computer crashes caused by program development occasionally disturbed the PSB program under execution. Since less program development occurs now, this is less noticeable. Besides this, the data base on disk was sometimes erroneously overwritten. The situation has been improved by introducing protection in the form of a software indicator. A long-standing puzzle has been solved only recently. An injection line magnet changed its value when no-one was working with it. It shared, by error, its control address with an ejection dipole (controlled from a different midi-console). There is no protection against this, but a checking program has been written.

Three problems arose in connection with the function generator, which has been in heavy use since the beginning. Firstly, some operators found it clumsy to use (possibly because of insufficient contact during the design). As a remedy, for some functions (choice of Q-values during cycle) an automatic setting procedure has since been implemented. Secondly, up till now, there has been no digital acquisition of a complete function. This will be rectified. Finally, if the Varian computer breaks down, the PSB stops. A back-up Varian has recently arrived.

Finally, the human element played its role in determining the speed of running-in. To the operators, not only the accelerator but also the control system was completely new, and they were unaware of the full capabilities of computer-assisted control. For some of them, accustomed to controlling a machine by hand, confrontation with a control system, supposedly easier but in practice more difficult during the early stages of running-in, led to psychological problems.

Among the problems not fully foreseen were the following. Initially, the information on the status of the elements was not displayed continuously. When a power supply tripped, the operator would notice this through its effect on the beam but there was no immediate information from the control system on such an event. This has been overcome with the introduction of the Status Display and the Repetitive Varilog on the midi-consoles (see Fig. 2 and below).

The injection line quadrupoles which are controlled by stepping motors (incremental control) were lengthy to set by hand or by program. Recently a closed loop iteration procedure program has been used successfully to set up the injection line focusing.

While the bulk of the measurement, data treatment, and display programs (which were ready at start-up) fulfilled their requirements, some modifications and additions were, on occasion, needed rapidly. This presented no particular problems, notably thanks to the close collaboration between machine and software specialists.

Aspects of Special Interest to the Users

i) During optimization from the midi-console, the operator can compare the current machine set-up with a reference set-up without, however, losing the current values (stored in a buffer on the disk-based data base). Owing to the slowness of the computer disk accesses, these subset operations are used much less frequently than had been hoped. Such an operation takes about 40 seconds. As often only a few parameters vary between different settings, the operator does not take much longer to make a comparison by tuning only these parameters. However, a subset operation regularly used is the setting from the maxi-console, for which 40 seconds is very short compared with the time for a manual setting of some hundred parameters.

ii) The voltages of the four accelerating cavities are controlled by high-speed generators. The parameters of the given mathematical function are controlled by program through the RF midi-console. This allows one to obtain a mathematically described function from the digitally controlled voltage generator just like any ordinary process variable. At present the procedure to enter changes is slow owing to limitations of the present system. A program orientated more towards advanced machine experiments is being developed and will include a facility to reduce the voltage slowly during the cycle to obtain a constant bucket area.

iii) Watchdog programs fall into three categories. Firstly, whenever a status changes, the number of parameters that are OFF (power supply disconnected) and NOT READY (down) is displayed on the midi-console (Fig. 2). Secondly, there are programs called at regular intervals: a) Vacuum Survey displays an alarm message forewarning vacuum specialists of possible breakdowns; b) RF Voltage Generator Survey checks the memories of the Voltage Generator and displays and resets those that have varied; c) Repetitive Varilog warns the operators of parameters drifting away from reference values. A problem with the Varilog is that only one reference value exists on disk and different operators require different reference values -- a fixed theoretical value, a value gained from experience, or a value taken when the beam was running well. Finally a statistics program compiles beam histograms'.

iv) The Booster electronic equipment is located in the Booster building (about 400 metres from the MCR), and it is there that various calibrations and adjustments have to be made. Originally the results could be observed only on the maxi-console in the MCR, which implied, in addition, communication between the MCR and the operator working on the electronics. It was decided to introduce a second, mobile, maxi-console in the Booster building. It can be connected, through a single cable, to the computer network from any one of seven different points. This allows the operator to move the console close to the equipment being tested. The mobile console offers the same facilities as the maxi-console in the MCR, so the 'computer-control-acquisition' loop can be thoroughly checked locally. The console is regarded as an extremely useful tool by those responsible for PSB systems.

v) The data base may be updated and modified via the interactive display by means of a special program. Users quickly learned how to check or modify characteristics such as conversion factors, status bit coordinates, maximum values, permissible variations, etc. They had the possibility of fitting the relevant data bank parameters to reality, masking a status bit, etc., within a few minutes. Whole runs might otherwise have been lost: Thus we found that the risk of having vital parameters changed by inexperienced operators is small compared to the advantages of a flexible data base for program development and accelerator operation.

Conclusions and Outlook

At this stage the correctness of the major options (use of existing IBM 1800, addition of a satellite for real-time work, choice of multi-access consoles, introduction of a data base, no manual back-up, addition of the mobile console) are confirmed by our good experience. The midi-consoles remain an easy to handle setting-up tool. The maxi-console is used for more complicated tasks. We are less sure whether we should not have already introduced an interpreter facility (not available when the control system was defined). We expect this to be particularly useful for machine experiments. While nearly all PSB parameters are computer controlled and/or acquired, there remain a few (important) exceptions. We expect to complete their computerization in the framework of the expansion of the PS computer system. At the same time we plan to expand the PSB system's monitoring and fault analysis facilities. Like any other system, the computer control system needed running-in. This progressed more quickly and easily where there was close co-operation between the control engineers, PSB system specialists, the programmers, and the operations team. Most of the authors were newcomers to the field; now that we are familiar with computer controls we are looking forward to using the extended capabilities of the expanded system under construction.

Acknowledgements

We are greatly indebted to the former and present members of the CO/CCI Group, in particular E. Asseo, H. van der Beken and J. Bosser, who designed and built most of the PSB computer control system and are now working on its expansion. C. Bovet has defined the machine physics content of a large number of application programs and contributed to their implementation. Constructive criticism by the PSB operating team has led to numerous developments, K.H. Reich suggested substantial improvements to the manuscript.

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Fig. 1 On-line operation facilities at the PS Booster

Fig. 2 A midi-console

Fig. 3 The maxi-console

Fig. 4 The mobile console

TABLE 1 COMPUTER PROGRAMS

No.	Program name and description	Frequency of call/week	Length of time/call (C=continuous)	Further uses (see below)	Input media (see below)	Output media (see below)	Refers to data base	Length (K word: on disk)
<u>GENERAL OPERATION PROGRAMS</u>								
1	MIDI-CONSOLE (see Table 2)	300	5 sec	HT	M,D	M	YES	15
2	KNOBS	50	15 min	HT	P,KNOBS	P	YES	27
3	STATISTICS. PSB intensity	21	8 h		P	L,D	NO	10
4	LOG. parameter print-out	50	2 min	ME,HT,ST	P	L	YES	16
5	VARILOG, Display of parameters outside tolerance	10	3 min	ME,HT,ST	P	P	YES	10
6	STATUS DISPLAY on PDS-1	40	30 sec	ME,HT	P	P	NO	6
7	GENERAL DISPLAY (see Table 2)	LOO	C	ME	Push-buttons	Nixies	NO	3
8	REPETITIVE VARILOG. Watchdog	10 min	30 sec	ME	P	M,D,P	NO	6
9	TRANSFER LINE PICK-UPS CALIB.	2	3 min	HT,ME	P	P,T,D	NO	5
10	SETTING, of 12 PSB sub-sets	10	30 sec	HT	P	P	YES	57
11	PARAMETER behaviour display	20	5 min	ME,HT	P	P,T	(YES)	9
12	VACUUM, Past and present state of pressure in pumps	30	10 sec	HT	P,Vacuum panel	P,L	NO	15
13	RF VOLTAGE GENERATOR	5	10 min	ME	M	P,M	YES	16
14	FUNCTION GENERATOR (see Table 2)	20	10 min	ME	F.G. Console	Oscilloscope	NO	1
<u>BEAM OBSERVATION PROGRAMS</u>								
15	LINAC SPECTROMETER	30	30 sec		P	Television	(YES)	7
16	LINAC EMITTANCE	20	30 sec		P	Television	NO	4
17	Q-MEASUREMENT in both planes	30	10 min	ME	P,Q-meas panel	P,T,L	NO	5
18	ORBIT DISPLAY (with dipoles control and Q-calculation)	50	10 min	ME	P,RU-panel	P,D,L, Cards	YES	38
19	IONIZATION BEAM SCANNER (IBS)	15	5 min	ME,HT	P,IBS-panel	P,L,D	NO	33
20	Evaluation of beam profile	10	10 min	ME	P,800 MeV-panel	P,T, Cards	YES	16
<u>CONTROL SYSTEM DIAGNOSTICS (HARDWARE AND SOFTWARE)</u>								
21	UPDATE DATA BANK	15	2 min	OP	P,D	P,D	YES	7
22	READ DISK	20	30 sec		P,D	P	NO	2
23	REPETITIVE RF VOLTAGE SURVEY	2 hours	2 min		P	P,T	YES	3
24	RING PICK-UPS HARDWARE TEST	1	10 min	OP	P,RU-panel	P,RU-panel	NO	3
75	MIDI-CONSOLE HARDWARE TEST	1	5 min		P,M	M	NO	2
26	KNOBS HARDWARE TEST	1	5 min		P,KNOBS	P	NO	1
<u>MISCELLANEOUS FAULT RECOVERY</u>								
27	RESTART PARTITION	1	30 sec		P,IBM switches	IBM 1800	NO	1
28	RESTART PDS-1	4	30 sec		PRU button	P	NO	1
29	ENABLE FUNCTION GENERATOR	2	10 sec		PRU button	F.G.	NO	1
30	COLD START MIDI-CONSOLE	2	30 sec		PRU button	M,P	NO	1

OP = Operation, M = Machine experiments, HT = Hardware Test, ST = Software Test

T = Typewriter, L = Lineprinter, M = Midi-console, P = PDS-1, D = Disk

Table 2 Survey of the PSB control system

Item	Purpose and reasons for choice	Performance expected	Assessment in March 1974
IBM 1800 (in MCR)	Process control computer (already used by CPS). Use by PSB implied i) no delay in making choice and ordering a new computer; ii) profit from CPS knowledge and experience.	i) Almost all elements effectively controllable and acquirable from MCR. ii) Watchdog and statistics programs. iii) Hardware tests' and calibration facilities.	All performance aims attained. However, sharing the existing computer led to some constraints, particularly program length.
Alphanumerical and graphical display (in MCB and at PSB)	Centralized interactive console driven by IMLAC PDS-1 mini-computer.	i) To allow fast on-line interaction with programs. ii) To display results of programs in an easily understandable form.	Good for alphanumerics but should be improved for graphical displays. Hard copy facility very much appreciated.
Varian 620i (at PSB) with special console (in MCR)	Flexible generation of up to 48 analog functions in real time.	Interactive creation and modification of functions from the console.	i) Performs as expected. ii) Sampling of function incomplete.
Three midi-consoles (in MCR)	To allow simultaneous control of three PSB sub-sets without syntax handling; simultaneous control of two selected parameters of a sub-set (increase, decrease, inversion of polarity, ON/OFF).	i) Possibility of linear coupling of variables. ii) Common operation on all parameters of the entire sub-set. iii) Display of information on parameters under display and watchdog programs.	i) Very useful in setting-up periods. ii) Watchdog programs are a useful diagnostic tool. iii) Slow in operation, owing to repetition rate (twice per acceleration cycle) and disk access time on IBM. iv) Too many buttons (120) for operator comfort.
KNOBS (in MCR and at PSB)	Four incremental encoders with which one can control most of the PSB parameters from a central point.	i) Ability to form special couplings. ii) Ability to work alongside other programs, e.g. beam quality measurement programs.	Very useful during steady operation and for machine experiments.
General display (in MCR)	Twenty Nixie displays distributed over the control room. Already used by CPS.	Cycle-to-cycle display of manually selected parameters such as Q-values, efficiencies, mean radial position.	Gives an over-all view of the machine. Name of displayed parameter difficult to read.
Program Request Unit (in MCR and at PSB)	Push-button matrices from which i) programs can be called into core; ii) options can be exercised in running programs. Already used by CPS.	i) Queueing of programs. ii) Easy to handle by operators. iii) Flashing light to show which program is running.	Performance expectations attained. With increasing number of programs, limited number of buttons led to: i) regrouping of several programs in one button; ii) too rigid nomenclature for the option buttons.
BCER console (at PSB)	Alphanumerical and graphical display and Program Request Unit. A facility for local hardware testing in the Booster building.	To give most of the facilities of the central console.	Turned out to be indispensable for maintenance and calibration. Shares computer access with central console.
Data base	Up-to-date image of the current accelerator state covering most PSB variables (~ 700) with their characteristics.	Easy to use for setting and other application programs. Automatic updating of current control values. Easy modification of parameter specifications.	Its flexibility has been greatly appreciated, especially during the running-in period, but misuse possible.
STAR	16-bit parallel data transmission system for acquisition and control. Already used by the CPS. Designed for use over long distances.	i) Reliable transmission in range of 20 to 100 K words per sec. ii) Random addressing of up to 1024 process variables per call.	i) In general, meeting requirements. ii) Connection to specific electronic systems is very simple. iii) Diagnostic equipment to speed up fault localization is being improved.

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Summary

A general purpose real-time data acquisition and control system implemented on a PDP-10 time shared computer and interfaced to devices via PDP-8 peripheral processors and a serial transmission scheme has been constructed. The design objectives of Fortran coded time-shared access to the accelerator have been achieved under the manufacturer's monitor with the addition only of shareable device handlers. The time-sharing monitor has allowed the development and operation of alternative control systems in parallel with continued use of an earlier system. Program development has been facilitated by the convenient access provided by the operating system and the multiple terminals available for programming and testing.

Objectives

Computer control and monitoring of the Brookhaven AGS dates from 1965. A system of control terminals interfaced to dual PDP-8's with disc storage was developed and served usefully in accelerator operation and physics studies of the machine. With the completion of the AGS conversion, it became apparent that exploitation of the machine could be greatly aided by suitable computerization of instrumentation and control of the accelerator. At this time it was also recognized that such computerization requires very significant efforts in manpower if effective results are to be achieved in a reasonable period of time. Two ways in which this burden could be alleviated were proposed 1) implementation of the control and monitoring programs to a great degree in Fortran, 2) implementation of the system in a time-sharing environment.

The first of these points is not seriously disputed at this time. No matter how efficient the individual, experienced analyst, the range of talent which can usefully be brought to bear on a problem and the speed of implementation are both greatly increased by use of the high level and universally spoken language. The second point is only slightly more subtle. The many and varied activities around the accelerator are obviously facilitated by independent access to computational power but the total integration of the system and interchange of information are simplified in a single machine. Both possibilities are provided by a medium scale general-purpose time-shared computer. This paper will discuss the implementation of an accelerator control system based on such a machine.

Configuration

The time-shared computer selected as the basis for the operating system is a Digital Equipment Corp. PDP-10. Among other reasons for the selection of this machine were principally the amply demonstrated capability of the time-shared operating system and the prior existence of two machines of this class in the Accelerator Department. A more difficult decision, given the objectives which included control of the accelerator complex from a single location, was the selection of direct device

communication from the PDP-10 or interfacing to the hardware devices via a small computer. It was decided to interpose the small computer for the following reasons:

1. The PDP-10 manufacturer's time-shared operating system is designed around time responses of the order of terminal needs. To impose on this monitor the real-time needs of the AGS, while not impossible, would nullify many of the time sharing advantages. By interposing a small computer or computers operating synchronously with the AGS it would be possible to allow the PDP-10 to operate asynchronously under its normal scheduling algorithm.
2. The totality of the PDP-10 monitor is impressive in both bulk and complexity. By divorcing the real-time code procedures from the PDP-10 it was possible to develop these independently of the monitor. The alternative of "user mode" I/O development or actual incorporation of the necessarily complex service routine into monitor development versions would be both laborious and hazardous to the system. There would be no time-sharing for the system developers: Definition of a reasonably simple service routine for data communication with the small computer allowed development of the small machine code without "bombing" the PDP-10 monitor.
3. Demanding real-time requirements in one area of the AGS would preclude the provision of such responses to other areas if operation were from a single processor. Such conflicts are rather unpredictable in some areas of accelerator operation and at some periods in the AGS cycle. This leads to the desirability of increased processor power which is conveniently allowed by multiple small computers assigned to specific hardware groupings.
4. An extension of the arguments of 3) is the dedicated processor assigned to function generation or fast switching as a functional part of accelerator hardware as distinct from monitoring or supervisory control. The possibility of implementation of dedicated processors controlled by the PDP-10 is included by adopting similar approaches for all input-output.
5. As will be described later, hardware facilities for communication within the system are by means of relatively slow, serial pulse trains. Any device requiring high speed computer response would require an alternative parallel interface. For consistency with the above arguments this should not be directly on the PDP-10 but requires an intermediate processor which could then be located near to the data source or sink.
6. Some consideration has been given to hardware redundancy. Spare or redundant small processors can be provided but this is hardly an advantage for a system which includes them over one without them! Operational systems to continue during a PDP-10 failure are possible using the small processors but must be at a much reduced scale. Also, a significant effort is required to produce this system for what is likely to be a small reward. Thus although this offers some advantage in principle we believe this to be slight in practice.

The sum of the above arguments has persuaded us that the "peripheral processor" approach has sufficient merit to justify its selection over a directly interfaced design. The peripheral processor selected is the PDP-8E. This decision was based to some extent on historical precedent but has justification in the price, modern physical design and local expertise in coding, cross assemblers, etc.

The remainder of the selected configuration consists of a serial transmission scheme using a single co-axial cable connection to distributed locations for control of hardware devices. The principle is that electronics be located as close to the physical control point as allowed by the radiation environment but long traditions of multiple parallel cables have only been overcome to a modest degree.

The overall configuration is shown in Fig. 1.

PDP-8E Monitor

In order to implement a system with the desired properties in the configuration described above, it was necessary to impose severe restrictions upon the processing allowed to be performed within the PDP-8. To preserve the programmer's flexibility it was necessary that all logical decisions as well as all arithmetic calculation be made in the PDP-10. Thus the PDP-8's function only as sophisticated input/output devices rather than processors. Nevertheless, to make these devices useful in a dynamic environment required that input and output buffers and I/O sequences be completely dynamic. An attempt has been made to generalize the

input/output operation in the only dimension by which real-time differs from batch or time-share ie. time. Input/Output operations are characterized by a device address, a list, a time and a priority. The time is specified in milliseconds after the start of an AGS pulse. The priority is one of four values representing the real time criticality of the operation and is used by the monitor to resolve conflicts in I/O requests. In particular the lowest priority causes the monitor to ignore the time and perform the operations as soon as higher priority requests permit.

One further complexity was introduced to improve the efficiency of repeated operations. A request to the PDP-8 is divided into two operations. The first step in an I/O operation is to define a COMMAND LIST which is transmitted to the PDP-8. This list is stored until overwritten or explicitly deleted. The commands within a list may then be performed by transmitting an EXECUTE LIST specifying a command list and a time and priority at which to execute it. More than one command list may be executed as defined in one execute list or the same command list may be repeated at two or more times. On completion of the operations specified in an execute list, the PDP-10 is called and the service routine within the PDP-10 schedules the calling job to run again. Multiple (currently 4) user access to the PDP-8's is provided to allow flexibility in the assigning of devices to PDP-3 channels.

Higher Level Organization

Presently a number of Fortran time sharing programs have been written which access machine data via the system described. The Fortran interface to machine data is quite convenient and allows programs to be written without concern for the real-time aspects of data acquisition. When the execute list has been transmitted to the PDP-3, the program can issue an input request and pass into an I/O wait state, or preferably suspend until awakened by the completion of the requested operations.

Operations within the PDP-8 are currently restricted to a single AGS pulse with longer time scale operations integrated in the PDP-10. Design is now proceeding on a system to provide file organized data and operations control for major areas of the machine from a single operations console. It is planned that an operator will maintain general control of machine operations with additional diagnostic activities and studies conducted from other terminals.

System Extension

The present system allows operations initiated by the PDP-10 and completed in the peripheral processor during one AGS pulse. It is envisaged that some repetitive functions in the machine will be allowed by repeating operations within the PDP-8 from pulse to pulse. Such tasks might include function generation, elementary monitoring functions, etc. They would be initiated by a program in the PDP-10 and would notify the program of problems beyond the elementary level. They could be coded to continue if the PDP-10 stopped. It is not planned to extend the PDP-8 capabilities appreciably beyond this level to avoid placing restrictions on the access and decision making functions. However, some extension of the list processing facilities will be required to interface the PDP-8 monitor to some non-standard devices in existing machine equipment. This will be done by defining additional list formats.

Hardware

The control system equipment which connects the PDP-8's to controlled devices is called collectively DATAON2. The basic data transmission scheme is a transformer coupled phase encoded bipolar signal on a single co-axial line. Each transmission consists of a frame pulse followed by a key bit, 32 data bits and a parity check bit sent to devices. The data is self-clocked. An addressed device responds, after some delay allowed for internal processing, with a similar pattern but phase reversed to distinguish a reply from a message. Remote device? contain high impedance re-

ceivers of which 100 may be bridged on the line. Repeaters are required every 2000 feet and allow the number of receivers to be increased to the addressing limit of 256 stations per line. Signal details are given in Fig. 2.

FIG. 2-DATAON2 WAVEFORMS AND WORD FORMATS

The single line technique minimizes hardware and cabling while giving a data rate of 600K bits/s which is a good match to an interrupt driven I/O service routine in the PDP-8E. The frame pulse serves as a "clear and initialize" to all receivers. The key bit which is always logical one indicates completion of a transmission when it reaches the end of a shift register. The parity bit is used for conventional data checking. The system provides for a suitable signalling rate with high noise immunity and excellent isolation.

The same transmission technique has been adapted to couple the PDP-10 to PDP-8 computers. Two bi-directional lines are used to provide full duplex transmissions with interrupt response control. TTL pulse trains are used for distances up to a few feet. Beyond this distance the phase encoded format is adopted using the drivers and receivers developed for the device control system.

The general purpose remote receiver is capable of communicating with up to 64 device controllers. As distinct from other systems using fast parallel data transmission at remote locations and serial transmission between locations, the DATAON2 system uses serial transmission throughout. The remote receiver functions to perform some system overhead of address decoding, parity checking, etc., to convert the phase encoded signal to TTL levels and to generate a clock signal. The data is then transmitted, still as a serial train, to the device control cards where it is clocked into shift registers. After a few microseconds for settling an ACCEPT pulse is generated. The addressed device, which lies at the intersection of two lines generated by two 3 bit to 8 line decoders in the remote receiver, processes data and returns a REPLY pulse. The "crate controller" then generates a REPLY CLOCK and strobes data from the device cards, converts it to phase encoded form and transmits it on the line. This approach, while not fully bussed within a remote crate, minimizes the intercard wiring and is almost free of cross talk problems. A non-trivial checkout

advantage of the serial scheme is that very few lines need be checked to identify a problem.

This system is used as a general purpose means of device control and monitoring. For some large power supplies where serious noise problems and isolation requirements were encountered, a dedicated receiver approach in which the only external connection is via the coupling transformer has been developed. Excellent isolation is achieved allowing control voltages and monitored signals to be referenced to the local ground.

About a dozen standard device control modules have been developed for the common applications of self-contained device control and subsystems. These include power supply controllers, predetermined timers, scalars, and stepping motor controllers. The standard devices also include so-called system components, A/D converter and multiplexers, which permit monitoring of analog signals either associated with controlled equipment or independently generated.

All monitoring within the system is performed on a polling basis. Extension to interrupt operation would be straightforward utilizing a second co-axial line but has not been found useful at this point.

Acknowledgements

Significant contributions to the hardware system have been made by D. Easler, V. Kovarik, R. Scheetz and G. Smith.

DISCUSSION

Is the Datacom transmission system a commercial system?

Barton: No, it was developed at Brookhaven. I believe the specifications are in a paper presented in a previous National Conference. I'm not sure.

James Halbig (LASL): Recently we have **run** into an interesting problem where we were sending out a command and asking for data in the same area, and since they were running into each other too fast, it bombed the command and we were having very weird things happening. I notice with only three PDP-8's, have you **run** into this problem yet or are your command ingestion processes fast enough so that you haven't run into these situations?

Barton (rephrasing the question): Do we have a problem with overloading of the Datacom rates and overlapping of commands? No, the system is free of that. It can't happen.

Halbig: Is it protected by the system, you say?

Barton: No transmission **can** go out until you've got the reply from the previous one. The minicomputer software is run on the interrupt system so that it can't try to send another one until it's got the previous one back. There's also a time-out so that if you fail to get a reply at all, it won't hang up; it will give you an error bit in the system and for that type of problem, it's worked out very well.

F. Beck

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The new CERN accelerator is controlled through a large computer network with the help of a specially designed real-time interpretive language. This has enabled the problem of designing an operator interface to be separated from that of producing control mechanisms.

The paper discusses the appearance of this interface, and some of the facilities which it will provide for machine operators.

Introduction

The new accelerator to be built by CERN will be almost exclusively computer-controlled. That is, all remotely-controllable or remotely-monitored functions of the machine can be accessed by executing suitable interpretive statements in one more central computers. It is not the purpose of this paper to describe the mechanism of this control (see paper by T. Hyman in these proceedings), but rather to explain how it has enabled us to make certain very desirable innovations at the operator interface.

Purpose of the Control Room

The control room is used for two distinct functions when the accelerator is running :

One is to supervise the machine when it is running and to make small operational adjustments. At such times there may well be only one operator in the control room. The other use of the control room is for the so-called machine physics studies, where several groups of workers may be operating more or less independently on different parts of the accelerator. A third use of the control room is when the accelerator is temporarily shut down. Safety requirements dictate the need for some hard-wired circuits, but these can be augmented by computer control to give extra flexibility when the computer is available.

The Consoles

The machine physics and supervision functions can be fulfilled by supplying a number of consoles with controls and displays on them. Each console is interfaced to the control system by means of its own computer, and the consoles are interchangeable in every way, taking their individual functions on a temporary basis from the control program currently loaded. This interchangeability means that the breakdown of one console or its computer reduces the flexibility of the rest of the system, but leaves it workable.

The commitment to identical facilities on each console places a premium on multi-purpose equipment, and strictly excludes any item specifically dedicated to one piece of hardware on the accelerator. The number of consoles is not difficult to decide. In view of the price of having a computer per console, it should be as small as possible. The requirement to have several

teams working at once implies several consoles. Three was decided on as a practical compromise.

Control-room layout

Thus we arrive at a central control complex with three identical consoles and a personnel access desk, and a computer room next door containing the central part of the control equipment, including eight mini-computers (three console computers, a display computer, a library computer, a computer to handle alarms, a service computer, and one which does message-switching for the whole control system). The control room itself, except for a silent printer and some wall displays, contains nothing else. Fig. 1 gives the general layout.

Design of the Console

The console itself is designed to allow access to everything by a seated operator. There is a nearly vertical surface for displays, a nearly horizontal surface for controls, and a table-top. Everything is in 19-inch rack units for convenience, and the console is arranged in a gentle curve with 20° between successive rack elements. With five units in the curve an operator sitting at the middle unit can see and reach everything and two or three people can stand behind him, or he can move over for a colleague to sit beside him. It was proved to be possible to design the console so that the table-top can clear the operator's knees, and for him simultaneously to see wall-displays and other consoles over the displays on his own. Fig. 2 shows the general form of the console which will be used.

Two multi-purpose devices enable us to keep the number of controls on the console small. One is a general purpose knob which can be changed under program control to one of various types of control devices and the other is a set of touch-buttons with programmable legends which can be used to choose from a programmable selection of possibilities with a touch of the finger.

Displays

The general-purpose displays are standard TV monitors, some in colour and some in black and white. All pictures are in the form of 625-line, 50 frame-per-second raster signals, and are driven from an external synchronisation line. Addition of certain pictures is possible, up to three pictures contributing to any monitor input. Possible sources of pictures are CCTV, dot-pictures on 1024 x 625 mesh from scan converters, a computer-driven cursor and 64 x 24 character arrays in up to four colours. Picture-sources and monitors are connected to a computer-controlled video crossbar with summing amplifiers, and the effect of this is to enable the operator to allocate any screen for the output of his program. This picture can subsequently be moved or copied on operator request to any other screen in the system, as well as to remote lines and to a printer (See Fig. 3).

The program which does the video switching is driven by the programmable touch buttons, but the monitors are identified by a single button under each. Thus on touching the button marked "move", the operator is presented with the question "which picture do you want to move?" and then by the question "where do you want to move this picture?". In each case identification is done, if possible, by pressing the identifying button under the screen in question. When this is not possible, as with moves to remote buildings or to a printer, a touch-button is temporarily dedicated to each such purpose.

Hard Copy

Hard copy is obtained by channeling the chosen signal to a special device which traps each TV line sequentially in a 1024 - bit buffer and uses the next TV frame time to read it into a computer, which prints it on a xerographic dot-printer. In the absence of buffering, which is not currently envisaged, only one picture at a time may be requested on the general-purpose printer, which is silent. A frame will take about 10 seconds to print.

Wall Displays

In addition to the displays on the console, displays are also provided on the wall. Projection TV has been considered, but requires a rather low ambient light level as well as having a rather restricted life. At the moment large TV monitors are being considered. These can be in colour, and by supplying two or three on the front wall and one especially for visitors looking in on the double height control room from windows on the top floor, it may be possible to avoid the use of projection TV. Information which changes only rarely may be displayed on indicators of the airport "flap" type, which can now be obtained in a relatively silent version with a full 64-character facility. We are still looking into alternative methods of displaying large characters.

Analog Signals

The general layout of the consoles and the electrical interconnections between the components is shown in Fig. 3. The four-beam oscilloscopes available on each console can be connected to chosen remote lines by a computer-controlled switch, the operator can make such connections by means of a touch panel dedicated to this purpose, and a display gives the names of the signals currently connected.

Ambient Conditions

Acoustical disturbances to the operator's concentration will be avoided where possible. The air conditioning system is isolated from the control room by concrete walls without doors, except for the necessary under-floor and over-ceiling ducts. By careful use of the pressure difference from this system it is hoped to dispense with the use of cooling fans in most, if not all of the equipment in the consoles. A xerographic printer has been found which is silent. The typing machine on the safety desk will be chosen with this characteristic in view. The walls and floor of the control room will have finishes chosen for sound absorption.

Lighting will in general be at a low level, in

order to allow the displays to have their maximum effect. A light-lock system using circularly polarized light and suitable filters over the display screens is being investigated. This, if successful would enable the desk tops and the printer to be adequately illuminated without destroying the contrast of the displays.

Use of the System

The major test of the control room and its system is - will it allow an accelerator to be controlled? The answer, judging from experience of systems of this kind at NAL, SLAC, LAMPF and elsewhere, is that the degree of control available can be made to approach that of a hard-wired system but not to equal it.

Using the knob, the ball, the touch-buttons and the graphics system, a program for the control and monitoring of a subsystem can be written quite quickly. Such a program will be written in interpretive language, usually by a worker engaged with the subsystem, rather than by a central controls programmer. A general selection program makes all such subsystems available at the touch of a button.

The real-time monitor in each console computer makes it practical to launch a monitoring or data-collecting program which will run while the operator performs other operations, reporting its results on one of the many screens in the system. At all times an alarm computer processes reports of unusual conditions and transmits them to a dedicated screen on each console. Thus the operator can have an overview of the entire accelerator while he is studying one subsystem.

Will the system work?

Most individual components of the control-room system have now been tested singly. The interpretive language and its hardware interfaces work. All components of the graphics system have been demonstrated. The message transfer system connecting the console computer to the rest of the system, including the library and graphics facilities which are properly part of it, is the only component which is at the current state of the art, but its specification is very conservative.

The greatest risk inherent in a system of this kind is the basic one that the overall reliability of such a lot of electronic hardware may be below a tolerable level. This risk is known, and attempts have been made to reduce it by various features of the design. There are undoubtedly those who regret the passing of the old type of control room, with its racks of specialised controls each dealing with a subsystem. With a machine of this size, even if it were economically realisable, the indications are that a control system of this type will in the end actually prove to be more convenient for the operator seeking either an overview of the whole machine or control of particular parts of it.

Fig. 1 General Layout of Control Room

Fig. 2 General Form of Console

Fig. 3 General Layout of Consoles a & b Electric 1 Interconnects between Components

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Summary

The aim in the design of the control system for the SPS has been to give the operators and machine physicists an integrated system capable of providing sophisticated control facilities but yet retaining simplicity of use for normal operations.

Twenty four minicomputers are used in the system with CAMAC input/output and a fast multiplex system to provide the interface to the SPS equipment. The inter-computer communication is over serial data links through a store and forward message transfer system.

The software is based on the use of an interpreter in conjunction with a small real-time executive. A comprehensive addressing scheme, with segmented data tables is used in preference to a large monolithic structure.

Introduction

Two factors influenced the design of the computer control system, namely, the physical size of the machine - with the large number of pieces of equipment to be controlled and the determination to liberate the power of modern computers from the past software barrier at the user level. Figure 1 shows the physical layout of the site and gives an idea of the distributed nature of the control system.

Experience elsewhere had demonstrated that the use of Interpretive software (where source statements are interpreted directly at run-time and not pre-compiled or assembled) would do much to provide the flexibility needed for the commissioning, machine development and operational phases of the machine¹. By careful consideration of all the steps involved in the software, it has been possible to provide an extremely powerful but easy to learn and use control language - NODAL, which retains good real-time response through the use of assembly language programs for critical operations.

In all, 24 modern 16 bit computers (NORD-10 manufactured by Norsk Data Elektronikk, Oslo) are distributed around the site. Each is connected via a fast serial data link to a packet switching message transfer system employing a further NORD-10 computer. Some of the computers perform specific functions while others provide general control and surveillance facilities. A simplified diagram of the system is shown in figure 2 and the functions listed in Table 1. The hardware and the software have been developed very much in parallel to the benefit of both the two disciplines and the rate at which implementation has been possible.

Computer	Core(k)	Drum(k)	Other Periphs.	Function
GP1 - 6	16	64	TTY, VDU	General Acquisition and control in BA's
Extraction North/West	24	128	TTY, VDU PT-R/P MT cassette	Control of Extraction systems
Power Supply	24	128	TTY, VDU PT-R/P MT cassette	Control of Power Supply & Function Generators
Radio Frequency	24	128	TTY, VDU PT-R/P MT cassette	RF system
Beam Line North/West BA80/WRB2 Injection	16	-	TTY	Control of Primary Beams
BA81	16	64		Control of North Expt. Area. User Interface
CPS	16			Interface to CPS
Console	24			Console and Operator Interface
Display	16	-	Special Character Controllers Printer	Display Generation
Alarm	16	256	-	Analysis of Alarms
Service	64	2 x 512	2 X Discs TTY, VDU PT-R/P Printer M. Tape	Number Crunching Program Development
Library	16	2 x 512	2 X Discs	System Library Data Bank

Hardware

Several features of the NORD-10 make the computer particularly suitable for the SPS application².

- i) fast microprogram including all operator communications and floating point operations
- ii) availability in a range of sizes from 8K up to 64K words of core store
- iii) separate internal and fully buffered external input/output busses - the latter with full interrupt facilities
- iv) full set of operating registers for each of the 16 hardware interrupt levels - context switching in 1.5 μ s.
- v) the ability to have a large number (512 theoretically!) of true cycle-stealing DMA channels (1 M word/second interleaved throughput)
- vi) good well proved software (compatible with the earlier NORD-1)

These computers are interfaced to the SPS equipment in the manner shown in figure 3. The primary interface is provided by CAMAC using a crate controller interface developed by CERN in collaboration with the computer manufacturers³. The controller provides direct vector addressing of interrupts, DMA (with an additional module) and an Autonomous Function Control (allowing traffic on the CAMAC crate dataway independent of the computer). Some SPS equipment is controlled and read directly by the CAMAC, notably the fast beam monitors. All interrupts from the SPS equipment pass via the CAMAC system. Both commercially available CAMAC modules and specially designed ones are used.

All other equipment is connected via a secondary interface in the form of a General Purpose Multiplexer, designed by CERN, and controlled itself by four CAMAC modules as shown in figure 3⁴. A basic concept of the multiplexer is that of having one module to provide all of the setting and monitoring facilities necessary for the full control of the piece of equipment connected. By careful choice of the number of different on/off controls, digital acquisition words, analogue acquisition channels and status check bits for each module design, the number of different types has been kept below 20. The modules use photo-coupler or reed-relay outputs to eliminate unwanted earth loops. As distinct from the CAMAC system, where the plug-in module address is associated with its position in the crate, the multiplex modules carry their own local (i.e. within a crate) address decoders. Serial transmission at a rate of approximately 1 M Baud over twisted pairs is used between the MPX Control Units and the remote stations. The distribution of general timing signals (derived from SPS Event + N ms delay) is also conveniently performed through the G.P. MPX⁵.

The only control circuits between the various buildings, apart from those required for synchronisation between the SPS and its injector (the CERN PS) and those for the power supply, are provided by the Data Links and Message Transfer System. To provide adequate security in the transmission both cross parity checks within each word and longitudinal parity (checksum) techniques are employed. The data link for each

computer uses two quads (a video pair + an audio pair in each direction) of a 6 quad or pod cable (of the type used for domestic television distribution). Data is carried in serial form over the video pairs, leaving and entering the computers via DMA. There is a handshake for each block of data together with other control information over the audio pairs using P/I/O computer entry and interrupts. Facilities are provided in the system, using the hardware operator communication of the NORD-10 micro processor, to bootstrap the remote computers from the central library system.

In addition to the hardware described above, the operator interface in the Main Control Room uses some special devices. The operator interface is described in a separate paper presented at the Conference⁶.

Software

Very great store was placed on the availability of good basic software when the choice of computer was being made. Specific requirements for the system were stated as:-

- 1) A sophisticated monitor supporting assemblers, interactive editor, linker etc.
- 2) A small real-time executive capable of accepting input for scheduling tasks from programs running under its control.

These two requirements are satisfied by the NORD-10 software in the form of TSS - a multi-user time sharing system and the SINTRAN II real-time executive, which is capable of working with a drum system or in a core-only environment. All of the relevant SINTRAN commands are available from NODAL so that program scheduling can be carried out on-line.

NODAL itself has been developed at CERN and has a powerful set of commands⁷. These can be used either:-

- a) Immediately -

SET MBV(2) = 154

which will set the current in Vertical Bending Magnet 2 to 154 amps.

- b) Programmed -

1.1 FOR INJPHS = 5,15;DO 2

1.2 END

2.1 TYPE INJPHS

2.2 TYPE BCT(4)!

which will type the reading of Beam Current Transformer 4 over a range of INJection PHase. Comprehensive mathematical functions are available and special commands exist which allow one computer to send a NODAL program to another computer and for this latter to send back the results of running the program received.

In the examples, MBV, INJPHS and BCT are the names of system variables and are used by NODAL to find entry into the appropriate Equipment Subroutines. These subroutines are written in assembly language and each contains its own data table (with entries for property of interest i.e. nominal value, current value, status etc). One equipment subroutine can be used for

all equipment of the same type. By avoiding a large monolithic data base, it is possible to add new equipment to the system with the minimum of trouble and without changing what already exists and is proved.

Considerable thought has been given to the detailed way in which access is achieved to the required piece of equipment. An equipment numbering scheme has been developed where information redundant to later addressing operations is replaced successively by further data. This scheme has made possible full unambiguous addressing of all equipment throughout the SPS within the 16 bits available. Data table accesses are reduced to simple indexed operations⁸. The equipment subroutine approach and the addressing scheme are shown in outline in figure 4.

To provide the necessary program priority, two main task buffers are provided in the NODAL-SINTRAN SYSTEM. The lowest level buffer is provided for interactive use while the second is for system programs. A third small high priority buffer is provided for immediate commands passed down the data links. In addition to programs written in NODAL, assembly language programs can be run directly under SINTRAN with, in computers with drums, swapping of core-loads.

Two schemes will be used for surveillance of the SPS. The first of these will be the normal periodic scan which will be implemented as time initiated tasks (either NODAL or assembly language) under SINTRAN. The second will use a special DMA scan facility built into the G.P. MPX. This latter scan will soak up any idle time the computers have. Any errors discovered from either of the scans - or from program errors detected by the NODAL interpreter will be noted in an Error "Bucket". This will be sent to the Alarm computer once per machine cycle (or when full).

The Message Transfer System software is designed to be completely transparent to the users and to allow any computer to communicate with any other⁹. Messages will be divided into 64 word blocks to facilitate coupling to the overall and local filing schemes. Each block will be preceded by a 4 word leader as shown below.

Source Computer	Destination
Priority	Word count
Directive *	
Program Name	

* Directive gives information to destination computer as to type of message.

The whole MTS works on a store and forward principle and will look to each satellite computer rather like a mag-tape unit (with about the same data rate). The MIS software will run under SINTRAN and programs will be available for traffic analysis.

The software for the operator interface has been designed in such a way that the maximum use is made of the NODAL-SINTRAN system used in the remote satellite computers. The "drivers" required for the special interface hardware are implemented as either NODAL functions or as CALLS to assembly language subroutines.

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FIG. 3 COMPUTER / SPS EQUIPMENT INTERFACE

FIG. 4 EQUIPMENT SUBROUTINE APPROACH

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Summary

The ISR control system has now developed to a state in which it relies heavily for both routine operation and machine development on the control computer. This gives significant benefits in security of operation, as well as easing the task of controlling a very complex installation. The computer has enabled the machine development team to concentrate on beam studies, without worrying overmuch about calibrations and reliability of individual components, so that they have been able in three years to surpass the original design objectives.

1. Initial Objectives

The basic design document for the Intersecting Storage Rings was the design study published in 1964. Planning of the control computer system started in 1967, at about the same time as the excavation work in the ring tunnel.

At that date, although some process plants were being controlled by computer, the experience in accelerator laboratories of computer control was not very encouraging. Small computers were only just changing in technology from discrete to integrated circuits, and the high reliability we not take for granted had not been proved.

Furthermore, the large proton storage rings would be unique, and it was feared that a long and arduous commissioning period might be needed to achieve any sort of useful performance. The computer system could not be allowed to present its own commissioning problems at the same time.

The computer system was therefore planned as a complement to the manual facilities, to be installed in parallel and used whenever its performance was superior.

In designing control systems for both computer and manual control, a number of compromises had to be accepted. For example, digital-to-analog converters for control of beam transfer line power supplies were made to accept decimal settings. Extensive general purpose control systems were ruled out, in favour of individual solutions for particular groups of hardware. To minimise the cost of the control system and to avoid undue duplication, the design principle adopted was of data transmission systems linking the control room to remote equipment buildings, having both a computer connection and a manual control panel in the control room. The bulk of the expense was in the command link and in the remote equipment, and as these were common to the manual and computer controls, they could be developed and tested in parallel with the computer system.

The equipment initially selected for computer control was that in which sheer quantity appeared likely

to overwhelm operators, and in which individual monitoring or adjustment was inappropriate or excessively laborious. The main examples selected were power supply adjustment, in which nearly 300 power units had to be set, beam observation (e.g. of the closed orbit) and vacuum system supervision. This latter choice surprised some people at the time, but turned out to be very important indeed. The vacuum system had 300 pumps and a similar number of gauges, making an over-all appreciation of the vacuum situation difficult without reasonably sophisticated data reduction and presentation.

Flexibility of the computer software was taken very seriously and an elaborate file structure was adopted for all data, even when it seemed fixed, rather than incorporating coefficients into the programs. In this way, very drastic changes to machine operating conditions could be introduced without revisions to programs. This approach contrasted markedly with the suggestion made in the ISR design study for a function generator controlling tune through the pole-face windings, and is one of the key elements in the success of the computer system.

The mixture of manual and computer controls precluded a control room layout based on two or three computer consoles, and the solution adopted was to insert computer terminals in three positions in the operating area, which was nevertheless kept fairly small.

A long-term objective of complete computer control was agreed, to provide a framework for gradual development of the system, and all controls and instrumentation were designed with the future computer connection in mind.

In 1968, the problems and costs of setting up a network of computers appeared considerable. The gradual implementation coupled with a requirement for flexibility to meet unforeseen requirements caused us to choose a single computer of reasonably high power: a 24-bit machine with a 1-microsecond core store cycle. A fast-access disc store and a graphic display system were also specified. To permit continuous hardware and software development, as well as to assure a high level of serviceability, a second computer was included. The order was placed in November 1968 and the computers delivered 12 months later.

2. Early Experience

Tests of the first part of the beam transfer line from the CPS started in May 1970, and at the end of that month the Storage Ring Control Room (SRC) was first used. The computer had been installed some six months previously, and was used to take data from pick-up electrodes and compute beam positions. This work continued for the rest of the year. As the beam transfer system was commissioned, the power supply

setting and checking programs were put into operation, and in September when the TTI transfer line was first tested over its 350 m length, the magnet currents were set by computer, and a STEER command, requesting the computer to adjust the beam position at a particular luminescent screen, made the next proton pulse pass right through the line.

In November, some tests were performed on one storage ring, and in the following January both rings were put into operation. Initial tests were at low momentum (15 GeV/c) at which the magnet profile should have given the design conditions. During the next few months, working points in the Q_H - Q_V diagram were tested, followed by working lines. By April 1971 there was data for ten lines stored in the computer for 15 GeV/c and rather fewer for other momenta. After quite an amount of trouble with the power supply control system, it settled down to become reliable and the development of working lines went ahead smoothly. A suite of programs was made available for manipulation of the machine tune, and development progressed rapidly¹.

During the same period, orbit measurement and correction facilities were put into operation. It was realised that the orbit measurements could not be used as a basis for setting up collision of the beams at the intersections, and a procedure was established for sweeping the two beams vertically across each other at the intersection, and monitoring the interaction rate. This permitted a steering resolution to better than 0.5 mm and also provided a measurement of effective beam height and hence of luminosity.

From the very beginning, steering in the beam transfer system was performed through the computer, which was told to make certain corrections at particular monitor positions, and worked out which correction element to use and by how much to change it. At the injection point to a storage ring the adjustments are very delicate, as the residual betatron oscillations about the closed orbit must be minimised. As the manual estimation of injection errors was rather subjective, and because of the four adjustments required per ring (angle and position, referred to a standard point, in radial and vertical senses) an automatic procedure was instituted, performing in a few minutes an operation which initially could take up to an hour. This closed-loop adjustment by the computer remains a major task at present, and efforts are continuing to improve its accuracy, which limits ISR luminosity directly.

Later in the year, currents of a few amps were stored in the ISR, and the vacuum was found to be very critical in limiting the current achievable. The computer had been connected to monitor pump currents and gauges because of the immense complexity and size of the vacuum system, and because the calculation of true pressure for each Bayard-Alpert u.h.v. gauge required an elaborate calibration sequence. Soon, programs which had been expected to run a few times a day were in operation every 15 seconds, monitoring pressure rises at the beams were stacked, and the programs had to be revised to allow them to run even faster. The computer terminal installed by the vacuum racks in a corner of the control room became the centre of attention.

By the end of 1971 the computer was performing many of the operational and machine-development functions, and since then its use has been extended and consolidated.

3. Beam Control

Many essential functions of the computer may be grouped under this one heading, which will be discussed as an illustration of our current position.

Periodically, after a long shut-down or a re-alignment, a major orbit correction is needed. The procedure for orbit measurement is assigned to a set of push-buttons, and the measurement is made on five successive pulses from the CPS and averaged. If a correction is thought necessary a data file is sent to the CERN central computers for analysis, and a list of corrections is sent back to the ISR for application by the control computer. A further check of the corrected orbit is made, and this is normally satisfactory².

At the start of each run, when beams are to be set up for physics, the orbit is again checked and the small errors in vertical position at each intersection are taken out. This establishes a reasonably accurate collision of the beams when they are later stacked.

The main magnets and the correction windings have been set up to standard values selected from many alternative tables stored in the computer, and provided that injection has been optimised, the stacks can be built up. If a high luminosity is required, the "8C" working line may have been selected: this calls for adjustment of the working line at intervals while the stack is being established, so as to compensate the curvature introduced by space charge³.

Alternatively, the stack may be established at 26 GeV/c, and then accelerated (by phase-displacement) to 31.4 GeV/c. In this case a dialogue is established between the programs which progressively alter the main magnetic field and correction winding currents, and the radio-frequency system, the whole procedure being automatic⁴.

A luminosity check is commonly required, and in this case we either perform a high-accuracy calibration at one intersection region, or a steering adjustment at about six of our intersection points simultaneously. At first, the orbit bumps were introduced by means of equal energisation of two magnets (per ring) placed approximately one quarter-wavelength upstream and downstream of each intersection, but now we use four-magnet bumps which have minimum second-order effects outside the intersection of interest, and we also correct the coefficients from our knowledge of the actual betatron wavelengths at central orbit. The bump amplitude can be perturbed by space-charge effects, and these are also compensated for.

When it became necessary to minimise background by altering the angle at which the beams passed through the intersection, a set of coefficients was calculated and the existing program was able to handle the manoeuvre, replacing millimetres by milliradians.

By means of the standard procedures outlined above, we have been able to establish beams of the required characteristics, with low decay rates, at any required energy, and to place them exactly in the intersection region, to achieve high luminosity, as a routine operation. To illustrate the effort this represents, it can be remarked that there are 35 programs for dealing with the set of 200 ISR power supplies (excluding beam transfer and experimental magnets) and there are 87 data files of settings for these power supplies stored in the

computer memory. To relate this to the over-all project, the total number of programs available to run on the control computer exceeds 400.

4. Faults

When dealing with an intricate machine, the question of fault-finding is important. In the computer system, all control programs have been made to follow a rigorous protocol. First, status signals are examined, to see whether the requested actions may be performed. If conditions are incorrect, the operator is warned. Next, it is usual to check on present settings and to calculate the new settings: the details vary with the system. The correction is then made, and after a short delay a new series of measurements and checks is initiated. Sometimes a fine adjustment is required, and this can be automatic, but if it seems excessive the operator is warned of a possible calibration problem.

This may seem very obvious, but it is one of the major differences between digital computer control and pure hardware control, in which checking of outputs is rare. There are several available techniques for achieving the sort of coordinated adjustment of multiple outputs which we often need, the digital differential analyser being admirably suited to present-day technology, but the computer retains the great advantage of reporting on failures in the external world and becoming obviously paralysed by internal failures.

In a project such as the Intersecting Storage Rings, where a simple test of new operating conditions may easily require two hours, security of operation is of the greatest importance in facilitating machine development. One must be quite certain that perturbations introduced are exactly those desired and that their amplitudes are recorded.

It is fair to state that, although in such a complex machine as the ISR we have had our share of hardware faults, the computer has very often signalled and identified these at once, so avoiding waste of time checking a large mass of equipment to localise the fault, or worse still continuing in blissful ignorance.

A further aspect of computer operation is the routine supervision of equipment, with the aim of reporting on failures and drifts. At the ISR we have about 30 programs dealing with various classes of equipment, which run at appropriate intervals during the standby periods, filling, physics experimental runs, and machine development. These programs report to an alarm printer. This is a little unconventional, in that control computers in accelerators or nuclear power stations often use cathode-ray display tubes for alarm presentation. At the ISR, perhaps on account of the very long time cycle of refilling the machine, we have chosen to print all alarm messages and to keep them for some time.

5. Operator Communication

At present, we have four computer terminals in the control room. Each has a key-board, a small printer and a text display screen. These facilities are supplemented by a graphics display system with two screens, four fast printers (for alarms, logging of routine happenings, vacuum and general purposes) and various data processing peripherals.

At each terminal, so as to ease the task of the operator, there is a set of 12 buttons: four of these are for special purposes such as stopping the program currently executing at that terminal, and eight are general-purpose buttons to which can be assigned any named file of eight commands. At present we have 165 such files of commands in the computer store, and the system is intensively used.

One noticeable omission is the knob. We have no digital computer-linked knobs at present. The characteristics of the storage ring seem to make knob-twiddling undesirable: there are very few control knobs in the entire control room. When an adjustment is desired, it is usually necessary to calculate its magnitude and then to apply it exactly, and it may well be an elaborate quantity involving setting of 40 parameters in a smoothly coordinated fashion.

6. Conclusion

At the ISR, computer control is well established, to the extent that the simpler routine operations would be virtually impossible without it, and complex operations now common would be ruled out altogether.

As our knowledge of the machine improves, and as the machine performance is pushed ever higher, the flexibility and security achieved by computer techniques becomes ever more important.

In three years of operation, the storage rings have been brought to performance levels exceeding the design objectives, and the operational procedures now handled as routine are in advance of early predictions. The computer has played a vital role in permitting the accelerator physicists to concentrate on studying the beams without being confused by irregular or unknown behaviour of the hardware. The development time saved by adopting this tool can only be guessed, but must certainly be appreciable.

7. Acknowledgements

I must express my appreciation of the hard work and continued enthusiasm, despite many early discouragements and difficulties, of the staff of what is now the ISR Computer Group, and also for the co-operation and trust of the entire staff of the ISR Division.

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