A SYSTEMATIC STUDY OF $\mathrm{K}^{+}$and $\mathrm{K}^{-}$CHARGE EXCHANGE AT 8.36 AND $12.8 \mathrm{GeV} / \mathrm{c}^{*}$

MURDOCK G. D. GILCHRIESE STANFORD LINEAR ACCELERATOR CENTER

STANFORD UNIVERSITY Stanford, California 94305

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## ABSTRACT

The results of a wire chamber spectrometer experiment at the Stanford Linear Accelerator Center to study kaon charge exchange reactions are reported. The salient experimental features include good relative normalization between the $\mathrm{K}^{+}$and $\mathrm{K}^{-}$charge exchange reactions and a large increase, with respect to previous experiments, in the number of events obtained for $\mathrm{K}^{+} \mathrm{n}$ charge exchange at the higher energy. Approximately 1500 events at $12.8 \mathrm{GeV} / \mathrm{c}$ and 250 events at $8.36 \mathrm{GeV} / \mathrm{c}$ have been obtained for each of the reactions $K^{+} n^{\prime} \rightarrow K^{0} p, K^{-} p \rightarrow \tilde{\mathbf{R}}^{0} n$, $\mathrm{K}_{\mathrm{P}}^{+}+\mathrm{K}^{\mathrm{o}}{ }^{+}{ }^{+}$and $\mathrm{K}_{\mathrm{n}}^{-}+\overline{\mathrm{K}}^{0} \Delta^{-}$.

The results of the experiment show that the $\mathrm{K}^{+}$charge exchange cross sections are larger than the $\mathrm{K}^{-}$cross sections at both energies. In particular we find that $\sigma_{\text {tot }}\left(K_{n}^{+}+K_{p}^{0}\right) / \sigma_{\text {tot }}\left(K_{p}^{-} \rightarrow \bar{X}_{n}{ }_{n}\right)$ is $1.37 \pm$ .22 at $8.36 \mathrm{GeV} / \mathrm{c}$ and $1.38 \pm .09$ at $12.8 \mathrm{GeV} / \mathrm{c}$. The ratio of these two reactions is also consistent with no momentum transfer dependence at either beam energy. Similarly we have determined that $\sigma_{\text {tot }}$ $\left(\mathrm{K}_{\mathrm{p}}^{+} \rightarrow \mathrm{K}_{\Delta^{++}}^{\mathrm{H}^{+}}\right) / \sigma_{\text {tot }}\left(\mathrm{K}_{\mathrm{n}}^{-} \rightarrow \overline{\mathrm{K}}_{\Delta}^{0}{ }^{-}\right)$is $1.05 \pm .16 \mathrm{at} 8.36 \mathrm{GeV} / \mathrm{c}$ and $1.56 \pm$ .08 at $12.8 \mathrm{GeV} / \mathrm{c}$. The ratio of these two reactions is also consistent with momentum transfer independence for both beam energies. These results are in clear conflict with the predictions of exchange degenerate Regge pole models.

[^0]
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CHAPTER 1. INTRODUCTION

The study of two-body and quasi-two-body reactions has been a major source of information about strong interaction dynamics. Two-body processes allow the investigation of interactions with many different quantum numbers over a large range of energies. The outstanding feature of all such reactions is their peripheral nature which is manifested in the form of forward, or backward, peaks in the differential cross section. One picture of this peripheral nature has been that the angular momentum states of the two particle system determine the partial waves through which the reaction occurs and hence the angular distribution. An alternative and more fruitful viewpoint, however, has been the concept of particle exchange. The long apparent range of the interaction responsible for the peripheral peaks, strongly suggested the association of scattering with the exchange of a particle (or particles) with the appropriate quantum numbers. For almost the last twenty years, this concept has been systematized within the framework of Regge models, wherein the scattering has been assumed to be dominated by the exchange of Regge poles. Many features of two-body scattering data have been successfully accounted for by applying Regge models. ${ }^{1}$

In its simplest form (just pole exchange) Regge theory implies that an amplitude for a two-body process may be expressed as:

$$
\begin{equation*}
A_{i}(s, t)=\frac{\beta_{i}(t)\left\{1 \pm e^{-1 \pi \alpha_{i}(t)}\right\}}{\sin \pi \alpha_{i}(t)} s_{i}(t) \tag{1}
\end{equation*}
$$

The values of the residue functions, $\beta_{i}(t)$, and the trajectory, $\alpha_{i}(t)$, depend upon the quantum numbers of the exchanged particle. For many

- 2 -
processes a large number of exchanges may be important and thus, in principle, knowledge of many different residues and trajectories would be necessary. Therefore theoretical or phenomenological mechanisms which tend to restrict the possible values of the residues or trajectories are useful. Duality is one such concept. ${ }^{2}$

Historically, the idea of duality was introduced via an examination of finite energy sum rules. ${ }^{3}$ The assumption of analyticity and asymptotic Regge pole behaviour of the scattering amplitude allowed the derivation of aum rules relating the low energy behaviour of the imaginary part of an amplitude to the extrapolation of the high energy Regge pole contributions. If, furthermore, one assumes that the low energy side of the sum rule is dominated by s-channel resonances, then one finds that the sum rule implies a complementary picture of the scattering process. The exchange of Regge poles in the t-channel determines the s-channel behaviour and vice versa. Both the s-channel and t-channel pictures are thus distinct, separate descriptions of the same physical process.

This basic, dual approach to explaining the scattering process was extended by introducing an additional, non-resonant component in the amplitude. 4 In addition to associating the s-channel resonances with $t$-channel exchanges of ordinary Regge poles ( $\rho, A_{2}, \omega$, etc.), the exchange of the Pomeron singularity was related to the non-resonant background in the s-channel. This assumption leads to a number of interesting results.

In particular this two component duality hypothesis leads to a prediction of exchange degenerate trajectories. Although the idea of exchange degenerate trajectories had arisen before duality was invented, the duality concept predicts exchange degeneracy in a simple way. 5 The absence of s-channel resonances in, say, $K^{+} p$ scattering implies that the

$$
\text { - } 3 \text { - }
$$

Imaginary part of the amplitude that corresponds to the resonance behaviour must vanish. According to duality however, that same amplitude may be described by allowed pole exchanges in the t-channel. If the t-channel description is also to vanish, then at least two trajectories must contribute and cancel each other. Again using $\mathrm{K}^{+} \mathrm{p}$ as an example, the total amplitude for this process may be written in terms of amplitudes labelled by t-channel exchanges:

$$
A\left(K^{+} p\right)=A_{P}+A_{P^{\prime}}+A_{\omega}+A_{A_{2}}+A_{\rho}
$$

From (1)

$$
\begin{aligned}
& \operatorname{Im}\left(A_{A_{2}}\right)=-\beta_{A_{2}}(t) s^{\alpha_{A_{2}}(t)} \\
& \operatorname{Im}\left(A_{\rho}\right)=\beta_{\rho}(t) s^{\alpha_{\rho}}(t)
\end{aligned}
$$

Thus, the amplitudes that are dual to the (nonexistent) s-channel resonances will cancel at all energies if

$$
\begin{aligned}
& -\beta_{A_{2}}(t)=\beta_{\rho}(t) \\
& \alpha_{A_{2}}(t)=\alpha_{\rho}(t)
\end{aligned}
$$

and similarly for $P^{\prime}$ and $\omega$. This requirement has become known as strong exchange degeneracy or if only the trajectories are equal, weak exchange degeneracy.

If the exchange degenerate prediction is correct, then it may be studied in two separate regions. First, for $t>0$, the observed meson and baryon spectrum suggests that the Regge trajectories do indeed coincide. The fact that the $\rho, \omega, f$ and $A_{2}$, for example, approximately lie in a straight line in the Chew-Frautschi plot implies that their trajectories are the same (see Fig. 1). In order to test weak exchange degeneracy for $t<0$, it is possible to select reactions that should isolate particular


Fig. 1. Chew-Frautschi plot which shows that the $p, g$ and $A_{2}$ mesons lie approximately on the same straight line.

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exchanges and compare the different trajectories. One may isolate $\rho$ exchange, for example, in the reaction $\pi^{-} p \rightarrow \pi^{\circ} n$ or $A_{2}$ exchange by studying $\pi-p \rightarrow n n$. This method suffers, however, from some problems regarding the parametrization of residues and the possible presence of secondary trajectories.

A more rewarding way of checking weak exchange degeneracy is by comparing processes that depend upon the difference of the trajectories. It is easy to see that opposite signature trajectories will always yield amplitudes which are $90^{\circ}$ out of phase if the trajectories are exchange degenerate. Therefore reactions which are described by the exchange of two opposite signature trajectories, vector and tensor exchange for example, are of great interest in checking the predictions of weak exchange degeneracy. The simplest class of such reactions are those related by a transformation from the s-channel to the $u$-channel, i.e., line reversed reactions.

In particular, consider the pair of reactions $K^{+} n \rightarrow K^{0} p$ and $K^{-} p+\bar{K}^{\circ} n$, which are indeed related by line reversal. The amplitude for each reaction may be expressed in terms of the $\rho$ and $A_{2}$ amplitudes as

$$
\begin{aligned}
& A\left(K^{+}{ }_{n \rightarrow K^{\circ}}{ }_{p}\right)=A_{A_{2}}+A_{\rho} \\
& A\left(K^{-} p \rightarrow \bar{K}_{n}^{\circ}\right)=A_{A_{2}}-A_{\rho}
\end{aligned}
$$

Note that the negative signature amplitude changes sign under s-u crossing. The difference between these two reactions is thus proportional to the interference term between the amplitudes of opposite signature. Weak exchange degeneracy would therefore predict equality of these two reactions.

At this point it is useful to remark about the assumptions that have been made to arrive at the prediction that $K^{+}$and $K^{-}$charge exchange cross sections should be equal. First, secondary trajectories at low energy could contribute to the reaction and the interference between the leading and secondary trajectories could result in a difference between the prom cesses. Second, and more fmportant, the duality picture based on Regge pole exchange alone may not be valid. It is well known that pure Regge pole models are not successful in fitting all two-body reaction data. ${ }^{1}$ Initial and final state rescattering of the interacting particles (absorption corrections) are necessary additions to the simple pole models. Such corrections are not well understood and their introduction into the duality framework has not been simply defined. Nevertheless, it still remains interesting to compare line reversed reactions over as large a kinematic range as possible.

In this thesis the reactions that have been studied are line reversed pairs at 8.36 and $12.8 \mathrm{GeV} / \mathrm{c}$. The reactions are

$$
\begin{align*}
& \mathrm{K}^{+} \mathrm{n}^{+\mathrm{K}^{\mathbf{o}}} \mathbf{p}  \tag{2}\\
& \mathrm{K}^{-} \mathrm{p}+\mathrm{K}_{\mathrm{K}}^{\mathrm{O}} \\
& \mathrm{~K}^{+}{ }_{\mathrm{p} \rightarrow \mathrm{~K}^{0} \Delta^{++}}  \tag{3}\\
& \mathbb{K}^{-}{ }^{n}+\bar{K}^{\circ} \Delta^{-}
\end{align*}
$$

Of particular interest will be the differences between the cross sections of each pair. Up to 1974, the existing data for pair (2) suggested that the $\mathrm{K}^{+}$and $\mathrm{K}^{-}$cross sections were different at low energies ( $<5 \mathrm{GeV} / \mathrm{c}$ ) but that they came together for $>5.5 \mathrm{GeV} / \mathrm{c} .^{6}$ The experimental problem, however, with such comparisons is one of relative normalization between different experiments. This particular problem was solved by results of two experiments that measured both $\mathrm{K}^{+}$and $\mathrm{K}^{-}$charge exchange in the same spectrometer. ${ }^{7,8}$ They found a substantial ( $\sim 30 \%$ ), energy Independent

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excess of $\mathrm{K}^{+}$charge exchange for $3<\mathrm{P}_{\mathrm{LAB}}<6 \mathrm{GeV} / \mathrm{c}$. The experimental situation for pair (3) is somewhat similar. Bubble chamber data from different experiments indicated that the $K^{+}{ }^{+}$reaction was $50-100 \%$ larger than the $\mathrm{K}^{-} \mathrm{n}$ process for $\mathrm{P}_{\mathrm{LAB}}<5 \mathrm{GeV} / \mathrm{C} .^{9}$ Data from Ref. 8 also showed a $60 \%$ excess of $\mathrm{K}^{+} \mathrm{p}$ over $\mathrm{K}^{-} \mathrm{n}$, apparently independent of $\mathrm{P}_{\mathrm{LAB}}$ from $4-6 \mathrm{GeV} / \mathrm{c}$. Given this situation it is clearly interesting to extend the energy range of such comparisons.

The data in this thesis extend the range of systematic comparisons of the reaction pairs (2) and (3) up to $12.8 \mathrm{GeV} / \mathrm{c}$. The same spectrometer was used to measure each reaction and thereby avoid problems of relative normalization. In particular the principal goals of the experiment were:

- to measure the $P_{\text {LAB }}$ dependence of the ratio of the $K^{+}{ }_{n+}+K_{p}^{0}$ and $\mathrm{K}^{-} \mathrm{p}+\overline{\mathrm{K}}^{\mathrm{o}} \mathrm{n}$ total and differential cross sections
- to similarly measure the $P_{\text {LAB }}$ dependence of the ratio of the $\mathrm{K}^{+} \mathrm{p} \rightarrow \mathrm{K}^{\mathrm{o}} \Delta^{++}$and $\mathrm{K}^{-} \mathrm{n}^{+} \overline{\mathrm{K}}_{\Delta^{-}}$total and differential cross sections
- to compare the $t$ dependence of the different reactions, and to compare the results of this experiment with lower energy data.

This thesis describes in some detail each facet of the experiment. A description of the various beams and of the spectrometer used to collect the data is given in Chapter 2. The reconstruction of events and kinematics are discussed in Chapter 3 . Chapter 4 contains a description of the raw data and of the procedure used to extract information about the reactions that are of interest. Normalization and corrections are outlined in Chapter 5. Finally, the total and differential cross section results are presented and discussed in Chapter 6.

CHAPTER 2. EXPERIMENTAL APPARATUS AND PROCEDURES

## A. Introduction

The apparatus used in this experiment was part of the Large Aperture Solenoid Spectrometer (LASS) facility at SLAC. ${ }^{10}$ The LASS spectrometer consists of two magnets and associated detectors. One magnet is a 4 m . long $x$ 2m. diameter superconducting solenoid and the other is a large but conventional water cooled dipole. At the time of this experiment, the superconducting solenoid was in piace in the experimental area but not operational. Additionally, the spark and proportional chambers that were to be placed in the solenoid were under construction or being tested and were not used in this experiment. The downstrean part of the spectrometer, however, was fully instrumented and capable of providing the necessary mass and momentum transfer resolution to adequately study kaon charge exchange.

In Fig. 2 we show a plan view of the experimental set-up. An RF separated beam was incident upon a liquid deuterium target at the beginning of the solenoid. The momentum of each incident beam particle was measured by a scintillation counter hodoscope at the third focus of the beam line (not shown in Fig. 2). The spatial trajectories of the beam particles were determined by a scintillation counter hodoscope ( $\theta-\phi$ ) and two proportional wire chambers before the deuterium target. Identification of the incident particles was provided by the appropriate combinations of the threshold Cerenkov counters $C_{\pi}$ and $C_{K}$. The incident beam was defined in time and space for trigger purposes by a set of coincidences of the counters $S E$ and $X Y$ and the desired combination of signals from $C_{\pi}$ and $C_{K}$.

Interactions of the beam in the deuterium target produced particles that were detected in the spark and proportional chambers of the spectrometer. Particles leaving the solenoid were detected by three large size magnetostrictive spark chambers between the solenoid and dipole. A proportional hodoscope with vertical wires only was positioned between the spark chamber package and the dipole. The excellent time resolution of this proportional chamber was important in reducing the number of spurious and out of time tracks found in the upstrean spark chamber package. Downstream of the dipole the track determining package consisted of another identical proportional hodoscope, two large and two very large magnetostrictive chambers and two arrays of scintillation counter hodoscopes. The hodoscopes were used as in-time information for downstream tracks and provided the basic trigger for the experiment in coincidence with an incident beam particle. The low duty cycle of the SLAC accelerator also required that deadening polyurethane plugs be inserted in the beam region of each spark chamber. Additionally, a hole in each of the triggering hodoscopes was left through which the beam could pass.

This experiment was the first operational test of the extremely complicated LASS data acquisition system. The large volume of data from many different types of devices together with the potential of a high event rate required a sophisticated data handling capability. The raw data from each device (sparks, proportional wires, hits in hodoscopes, etc.) were collected and sorted by individual control modules particular to each kind of detector. Collected data were then passed along to a central minicomputer (PDP 11/20) where they were compressed and buffered. For the major part of this experiment the compressed data were sent from
the minicomputer to an IBM SYS/7 which in turn passed the data to one IBM $370 / 168$ of the SLAC computer triplex.

All data were written onto high density magnetic tape and a sample of the data was subjected to on-line analysis. The speed and memory capabilities of a $370 / 168$ allowed the sampled data to be passed to an interactive terminal in realtime in order that spectrometer performance, as well as physics results, could be monitored. During those times when the $370 / 168$ system was inactive, data were passed from the PDP 11/20 to an IBM 1800 computer located in LASS control room which recorded the data on low density magnetic tape and performed a minimal set of monitoring functions.

In the following sections of this chapter, each component of the spectrometer and data acquisition system is described in detail.
B. Beam

Beamline 20-21 (see Fig. 3) at SLAC was used to transport a hadron beam to the LASS spectrometer. ${ }^{11}$ This beamline has a large acceptance (60 uster-\% for standard conditions), a dispersed first focus (F1) for momentum selection and an achromatic second focus (F2) at which collimation for particle separation is done. The large dispersion (. 48 inches/\%) at the third focus (F3) allows accurate momentum measurements. The final focus at F 4 is also achromatic. Each of the four stages of the beamline has a unit transport matrix so that the source distribution is reproduced, to first order, at F2 and F4. Remotely variable collimators at Fl determine the momentum bite of the beam and a lead filter, also at F1, removes electrons. There are two RF separators between F1 and F2 that make possible the separation of pions, kaons and protons up to $16 \mathrm{GeV} / \mathrm{c}$. Collimators at F2 may be changed by hand to increase or decrease the yield of
 separated particles.

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For this experiment, the primary electron beam was steered through the switchyard and focused onto a spot ( 1.0 mm vertical and 1.5 mm horizontal FWHM) at the production target. Primary beam steering was continually monitored by observing signals from secondary emission monitors (SEMs) mounted on the face of the target. Although a position resolution of .25 mm was possible, only qualitative observations were used to monitor correct beam steering onto the target. The production target itself was .85 radiation lengths of Be preceded by .22 radiation lengths of Cu and was water cooled. The secondary beam production angle was $1^{\circ}$. The electron beam power was dissipated in a water cooled dump behind the production target.

The two RF separators in the beamline vertically deflected particles according to their mass and thus produced a spatial separation at F2 for particles of different velocity. Small dipole magnets were then used to compensate for the RF deflection and divert the desired type of particle through the F2 collimator. Each RF separator was synchronized with the basic frequency ( 2856 MHz ) of the accelerator. Electrons would strike the production target in bunches that were 5 psec . long every 350 psec. and therefore secondary particles within the same 5 psec . bunch would have different velocities depending upon their mass when they arrived at an RF separator. Thus, they experienced different electric field strengths on the sinusoidal RF wave in the separator.

Each separator was individually tuned to obtain the maximum deflection between unwanted particles ( $\pi, p$ ) and the desired particle ( $K$ ). This was done by counting $\pi$ 's since the R rates are low except at high primary beam currents. After the $\pi$ peak had been found, the RF phase was rotated by a pre-calculated amount and the compensating dipole magnet tuned to

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bring the kaons through the F2 collimator hole. The combination of both separators resulted in $>80 \%$ pure kain beams for the beam momenta of this experiment. Separator tuning was checked over the course of the experiment by monitoring the number of K 's/pulse and the $\mathrm{K} / \pi$ ratio.

The beamline instrumentation consisted of scintillation counters, Cerenkov counters and proportional chambers. A pair of scintillation coumters, $S_{1}$ and $S_{2}$, were located fust downstream of the $F 2$ collimator. They were used in tuning the beamline magnets from the production target to $F 2$ as well as in adjusting the RF separators. The momentum of each particle in the beam was measured by the p-hodoscope located at F3 which contained six vertical scintillation counters. The six counters overlapped to form 11 different bins that were each .64 cm wide. This hodoscope provided a $\Delta \mathrm{P} / \mathrm{P}$ resolution of approximately $\pm .25 \%$. A remotely moveable .64 cm wide vertical scanning counter was also located at F3 and was used together with the central two elements of the p-hodoscope to tune the beamline magnets from F2 to F3.

The angles and position of the beam incident upon the deuterium target were measured by a scintillation hodoscope ( $\theta-\phi$ ) and two sets of proportional wire chambers. The $\theta-\phi$ consisted of a vertical hodoscope ( $\theta$ ) containing twelve elements and an identical horizontal hodoscope ( $\phi$ ). Each of the elements was a .16 cm thickness of NE110 scintillator and was 1.27 cm wide. The twelve counters did not overlap and were positioned so as to minimize the dead space between each counter as much as possible.

There were two sets of proportional wire chambers just upstream of the deuterium target. The upstream chamber consisted of a vertical plane of wires ( $x$ ), a horizontal plane ( $y$ ) and two angle planes rotated by $\pm 45^{\circ}$ (e and p). One meter downstream of this chamber, there was a similar

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chamber with $x$ and $y$ planes and an $x^{\prime}$ and $y^{\prime}$ plane each displaced by .5 mm from the $x$ and $y$ planes. A more complete description of these chambers and their performance is given in Section E.

The incident beam particles were identified by two threshold Cerenkov counters, $C_{\pi}$ and $C_{K}$, in beamline 21. The $\pi$-Cerenkov was 5 m long and 20 cm in diameter and was filled with hydrogen at 25 psig to yield light only for pions. The K -Cerenkov was 1.0 m long and was filled with $\mathrm{CO}_{2}$ at 55 psig to trigger on both kaons and pions. Thus a $\pi$ was identified by $\mathrm{C}_{\pi} \cdot \mathrm{C}_{\mathrm{K}}$, a K by $\overline{\mathrm{C}}_{\pi} \cdot \mathrm{C}_{\mathrm{K}}$ and a proton by $\overline{\mathrm{C}}_{\boldsymbol{T}} \cdot \overline{\mathrm{C}}_{\mathrm{K}}$.

Two other scintillation counters existed in the beamine to define the incident beam for trigger purposes. The first of these was SE, a . 64 cm thick, $10 \mathrm{~cm} \times 10 \mathrm{~cm}$ counter located at the exit of the F 4 collimator. This counter served to precisely tag each beam particle in time. The XY-RING counters were located immediately upstream of the deuterium target and ensured that the incident beam particle would hit the target. The XY counter consisted of four .64 cm thick, $1.91 \mathrm{~cm} \times 1.91 \mathrm{~cm}$ counters joined to form a $3.82 \times 3.82 \mathrm{~cm}$ square. Each element was optically distinct from its neighbors and had its own phototube. Surrounding the $X Y$ counter was the RING counter which was viewed by four phototubes and which acted to veto halo particles in the beam. The performance and characteristics of the XY and RING counters will be described in the Beam Logic section of this chapter.

The beamline 20/21 magnet currents were controlled by a computer system (YARDMUX) for this experiment. ${ }^{12}$ A Data General Nova computer could separately adjust each beamline magnet current through a CAMAC controlled interface by means of digital-to-anetog converters. Magnet values for a particular beam momentum were stored in memory and the computer could

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scan the magnet currents and automatically make adjustments if the currents deviated by more than a preset tolerance. A CRT display of the stored and actual values was available at all times. Polarity reversal of the entire beamline could be accomplished under computer control and generally was completed within five minutes.

Beam tuning for both the 12.8 and $8.36 \mathrm{GeV} / \mathrm{c}$ beams was done in steps to maximize the flux at F2, F3 and F4. After the primary beam was optimized, iterative tuning was done on the magnets in the target to F2 leg of the beamine. Rates for $S_{1} \cdot S_{2}$ in an unseparated beam were measured as first the dipoles (20D1 and 20D2) and then the quadropoles (20Q2/5, 20Q1 and 20Q3/4) were adjusted. Tuning to F3 was monitored by counting coincidences between the central two elements of the p-hodoscopes and/or the vertical scanning counter. The dipoles and quadropoles in the F3 to F4 leg were adjusted to optimize the flux counted by the XY counter and to minimize the RING/XY ratio. Next, the RF separators and their compensating magnets were individually tuned to yield the maximum K flux and best $\mathrm{K} / \pi$ ratio. Operating values and typical rates are given in Table 1 for the final 12.8 and $8.36 \mathrm{GeV} / \mathrm{c}$ beams. It should be noted that the $8.36 \mathrm{GeV} / \mathrm{c}$ part of the experiment was conducted when the SLAC accelerator was operating at low energy to conserve power. In order to obtain the desired kaon flux, the F 2 collimators were opened more than would have been necessary at the full linac energy, which resulted in a small increase in spot size at F4.
c. Liquid Deuterium Target

The 1iquid deuterium target used in this experiment was of the condensing type and measured 91 cm long and 5 cm in diameter. The target cell was fabricated of two concentric mylar cylinders and the $\mathrm{LD}_{2}$ was

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table 1

| TABLE 1 |  |  |  |
| :--- | :--- | :--- | :--- |
| Momentum | -8.36 | $12.8 \mathrm{GeV} / \mathrm{c}$ |  |
| Primary $\mathrm{e}^{-}$Beam Energy | -15.0 | 20.5 GeV |  |
| Electron Beam Current (Maximum) | - | 50 | 40 ma |
| F1 Collimator - Horizontal | - | 4 | $3 \%$ |
| F1 Collimator - Vertical | - | .25 | .2 inches |
| F2 Collimator | - | .4 | .25 inches |
| Kaons/Pulse | $-4-5$ | $4-5$ |  |
| $\mathrm{~K}^{-} / \pi^{-}$ | -8 | 6 |  |
| $\mathrm{~K}^{+} / \pi^{+}$ | -12 | 18 |  |
| Spot Size ( $\sigma$ ) at Focus - Horizontal | -5.4 | 3.7 mm |  |
| Spot Size ( $\sigma$ ) at Focus - Vertical | -5.3 | 3.6 mm |  |

continuously forced down the inner cylinder by a pump and returned via the space between cylinders. The density of the $\mathrm{LD}_{2}$ was monitored by two vapor pressure bulbs, one at the inlet of the cell and one at the outlet. These two pressures and the overall cell pressure were recorded continually during the data taking. During the experiment, the circulating pump that forced the $L D_{2}$ into the target cell occasionally experienced considerable difficulty and was replaced twice. Data from the runs during these periods were not analyzed. The boil off deuterium gas was analyzed a number of times during the experiment to measure the $H_{2}$ and $H D$ contamination in order to allow corrections to be made as will be discussed in Chapter 5.

## D. Magnetostrictive Spark Chamber System

The heart of the track determining part of the spectrometer consisted of seven double gap magnetostrictive spark chambers. There were three $300 \mathrm{~cm} \times 150 \mathrm{~cm}$ chambers upstream of the dipole and two downstream together

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with two $400 \mathrm{~cm} \times 200 \mathrm{~cm}$ "super" chambers also downstream of the dipole. With the exception of the "super" chambers, these chambers have been used in previous spectrometers and are described in detail in Ref. 13.

Each plane of a chamber was composed of an aluminum wire-nylon filament mesh with 11.22 wires $/ \mathrm{cm}$. One gap of each chamber consisted of mutually orthogonal aluminum wires such that $x$ (horizontal) and $y$ (vertical) coordinates could be defined. In the other (e-p) gap of each chamber, the wires were rotated plus and minus 30 degrees from the vertical. The gap spacing was 1 cm in each case and the separation between gaps ( 6.2 cm ) was kept small to minimize the difference in the $x-y$ coordinates measured in each gap. This allowed match points to be more easily found.

After an event trigger had been defined by the fast logic, high voltage pulses were applied to each chamber gap using a thyraton switch to discharge capacitors in the form of 30 meter long charging cables. The resulting square pulse was typically 300 nsec. long and five to six kilovolts high. A D.C. clearing field of 50 volts and a pulsed field of 200 volts were employed to clear out old sparks and residual ionization.

The chambers were run at rates of up to 30 triggers/sec. with a deadtime of 14 msec. , imposed to prevent another event from occurring before the chambers could be efficiently re-fired. The gas used in the chambers was $90 \%$ neon and $10 \%$ helium and was recirculated at a rate of about one chamber volume every four hours. The long memory time of spark chambers combined with the SLAC duty cycle required small deadening polyurethane plugs to be placed in the beam region in each gap. These plugs were 8.4 cm in diameter in the upstream chambers and 15.2 cm in diameter in the downstream chambers.

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For each of the 28 chamber planes, a single magnetostrictive wand and amplifier was used to detect the sparks. The fringe field of the dipole did not affect the bias of the wands. After the wand signals were amplified and shaped, zero-cross discriminators were used to locate the spark centers as well as the readout from pulsed fiducials. The discriminated signals were sent to a time digitizing system described in detail in Ref. 14. Up to 15 sparks-per-wand could be recorded

At the beam intensities of this experiment ( $4-5$ kaons/spill) there were typically 6-7 sparks per event in the chamber planes upstream of the dipole. Average multiplicities in the downstream planes were less, being approximately 5-6 sparks per event. There were no essential differences between spark multiplicitfes found for incident $K^{-}$or $K^{+}$beams or substantial differences between 8.36 and $12.8 \mathrm{GeV} / \mathrm{c}$ beam momentum. All of the chambers were capable of supporting multispark events while retaining high efficiency as may be seen in Fig. 4, which shows typical gap efficiencies for different numbers of sparks in the plane. The efficiency also did not significantly depend upon the location of the spark within a chamber. The resolution of the chambers was also investigated. Typical plots of the residual difference between a fitted track coordinate and the spark coordinate are shown in Figs. $5 a, b, c$ and $d$ for representative $x, y, e$ and $p$ planes. The typical standard deviation of such distributions was . 4 -. 6 man. The resolution did not significantly depend on spark position within a chamber plane.

## E. Proportional Chamber System

The proportional wire chambers used in this experiment wexe the first of the future $>15,000$ wire LASS proportional chamber system. ${ }^{15}$ The poor duty cycle at $S L A C$ makes it desirable to supplement the $\sim 1 \mu s e c$ resolving


Fig. 4. Typical gap efficiencies of the magnetostrictive spark chambers for different number of sparks in the gap.


Fig. 5. Typical residual differences for the magnetostrictive chambers a) $x-p l a n e$; b) y -plane; c) e-plane; d) p-plane.
time of the spark chambers used in the spectrometer with devices of superior time resolution. Proportional chambers combine good time resoIution and accurate space resolution and were very useful in reducing the number of track possibilities with respect to a pure spark chamber system. The chambers used in the experiment were two sets of beam chambers to measure the trajectories of the incident particles and two vertical wire hodoscopes that were used to corroborate tracks found in the upstream and downstream spark chamber packages. In the next few paragraphs the chamber mechanical features, chamber electronics and chamber performance are described.

A detailed description of the mechanical construction of the beam chambers is given in Ref. 16 and Ref. 17 contains a description of the construction techniques used in fabricating the proportional hodoscopes. Only a brief discussion will be presented here.

Each of the beam chamber packages contained four sense wire planes. There were 64 wires-per-plane spaced 1.016 mm apart. The sense wires were $20 \mu$ diameter gold plated tungsten and the gap between the wires and high voltage planes was 4 mm . The four planes of the upbeam package were an $x$ plane to measure the horizontal coordinate, a $y$ plane to measure the vertical and two planes ( $e$ and $p$ ) rotated by $\pm 45^{\circ}$ to provide corroboration for those cases with multiple coordinates in the $x$ and $y$ planes. The downeam package contained two sets of $x$ and $y$ planes displaced by one half wire spacing from each other. The $x$ and $y$ resolution, therefore, was a factor of two better than in the upbeam chamber.

The proportional hodoscopes were each large enough to cover the entrance or exit aperture of the dipole magnet. They each contained 512
vertical wires, 4 mm apart. The wire diameter was $50 \mu$ and the sense plane to high voltage plane spacing was also 4 man

The proportional chamber electronica used in the experiment represented only a small part of the complete LASS system. A more complete description of the electronics has been published in Refs. 18 and 19 and only a brief outline will be given here. The design goals of the electronics were to build a system that performed well under the high instantaneous flux conditions experienced at SLAC and to maximize detector area while increasing overall reliability by removing the electronics from the chambers. There were two basic divisions within the system: (1) amplifying the raw signals and storing them; and (2) reading the data out and formatting it for the data acquisition system.

The signals from the proportional chambers were negative going pulses of amplitude of a few tenths to tens of millivolts. In the LASS system these signals were conducted from the chamber to the amplifying electronics by 40 feet of $95 \Omega$ coaxial cable (see Fig. 6). Sixteen amplifier channels were arranged on one physical module to read out 16 wires. The amplified signals from each wire were shaped, discriminated and stored in the equivalent of a 40 MHz shift register until the decision to accept an event had been made. Once an event decision had been made, the stored information for each wire was shifted out, compacted and sent to the central data acquisition minicomputer used in the experiment. There were essentially no deadtime losses in the read out system which was an important consideration considering the large instantaneous fluxes in the beam.

The chambers were typically operated with a discrimination threshold of $250 \mu \mathrm{~V}$. All eight planes of the beam chambers were $\mathbf{> 9 8 \%}$ efficient over the duration of the experiment. The proportional hodoscopes were on


Fig. 6. The proportional chamber electronics and read-out system.
average $\underset{\sim}{ } 94 \%$ efficient during the experiment. The small inefficiencies were caused by intermittent electronics malfunctions and an occasional degradation of the gas purity in the chambers.

## F. Spectrometer Dipole Magnet

The dipole used in this experiment was a large conventional magnet capable of providing a field integral of $30 \mathrm{kG}-\mathrm{m}$ at full power with a maximum field of 16 kG . The magnet field region was 2 m wide, 2.4 m along the beam direction and had a gap of 1 m . For this experiment the dipole could only be energized to .7 of full field because of the unavailability of a large enough power supply.

Prior to the experiment the magnetic field was measured by a computer controlled mapper. Three coils measured the induced voltages as the mapper was moved in the field. The mapper moved in 2.54 cm increments along the beam ( $z$ ) direction and the three components of fields were measured at $170 \mathrm{x}-\mathrm{y}$ points at each z value. The field components were measured at a total of 249 z points. The computer controlling the mapper recorded the induced voltages on magnetic tape and, at the completion of the mapping, the voltages were converted to magnetic field values using a precise reference. In order to reduce the 42,330 data points to a manageable form, the field was fitted at a particular $z$ value to a sixth order polynomial in $x$ and $y$. The field was uniform enough in $z$ that a fit to all 249 values was unnecessary. A number of different partitions of the field were investigated and it was found that a representation at eight $z$ values was adequate to describe the field for momentum determination purposes. Therefore, a polynomial fit was done at these $z$ values and the resulting coefficients were stored for later use to determine the momentum of particles crossing through the magnet.

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G. Scintillation Hodoscopes HA and HB

The trigger for the experiment was provided by a coincidence between an incident beam particle and two or more hits in both of the hodoscopes HA and HB. The HA hodoscope array (Fig. 7a) consisted of two groups of 21 counters 33 inches long arranged symmetrically with respect to the horizontal beam plane. Hodoscope HB (Fig. 7b) similarly consisted of two groups of 38 counters. For each array, the counters were rigidly mounted within a framework so as to minimize cracks between counters. The fast electronics for the hodoscopes is discussed under trigger logic in Section H .
H. Electronics and Fast Logic

The purpose of the electronics and fast logic in this experiment was fourfold:

- to identify and count the incident beam particles
- to provide one part of the basic trigger for the experiment
- to buffer and store the relevant information from the scintillation counters
- to interface the trigger logic with the data acquisition system. In the following sections beam logic, trigger logic, buffering and scaling, and gating logic are described.

1. Beam logic

The beam logic identified the incoming beam particles. Coincidences (and anticoincidences) between the counters $S E, X Y$ and RING together with the appropriate combination of signals from the Cerenkov counters $C_{\pi}$ and $C_{K}$ served to tag each beam particle. Fig. 8 shows a diagram of the beam logic.


Fig. 8. A logic diagram of the beam fast electronics.

Fig. 7. The trigger hodoscopes a) Hodoscope A; b) Hodoscope B.

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Anode signals from each of the four $X Y$ quadrants were discriminated and shaped to 15 nsec. wide pulses. A logical $O R$ ( $\Sigma X_{\geq} \geq 1$ ) was formed from the four signals to indicate a hit in at least one of the quadrants. This signal and a narrow, 5 nsec. Wide pulse from the $S E$ counter were required to be coincident for a good beam trigger. Additional vetoes, however, were imposed to reduce or eliminate two problems. First, halo particles coincident with a good beam particle were eliminated by making the coincidence $\Sigma X Y \geq 1 \cdot S E \cdot \overline{\Sigma R}$ where $\overline{\Sigma R}$ indicates that no RING counter had fired. Second, a veto pulse was formed if two or more $X Y$ quadrants had simultaneous signals. Additionally, if two $\Sigma X Y \geq 1$ signals occurred within 20 nsec. of each other, a veto was formed. This veto was not perfect and the correction to counting the proper number of beam particles is discussed in Chapter 5. A final coincidence called BEAM was therefore formed by SE• $\Sigma \mathrm{XY} \geq 1$ • $\overline{\Sigma R} \cdot \overline{\Sigma X Y} \geq 2$.

The particles in the beam were identified by a coincidence of BRAM and the two Cerenkov counters. In particular, pions were identified by the combination $B E A M \cdot C_{\pi} \cdot C_{K}$, kaons by $B E A M \cdot \bar{C}_{\pi} \cdot \mathrm{C}_{\mathrm{K}}$ and protons by BEAM $\cdot \overline{\mathrm{C}}_{\pi} \cdot \overline{\mathrm{C}}_{\mathrm{K}}$. The final output of the beam logic was a signal indicating that a kaon was incident upon the target. This signal was called BEAMTRIG and was defined as BEAMTRIG $=\mathrm{SE} \cdot \Sigma \mathrm{XY} \geq 1 \cdot \overline{\Sigma \mathrm{R}} \cdot \overline{\Sigma \mathrm{XY} \geq 2} \cdot \overline{\mathrm{C}}_{\pi} \cdot \mathrm{C}_{\mathrm{K}}$.

## 2. Trigger logic

The trigger for the experiment was designed to select those interactions of the incident kaon that caused two or more particles to pass through the dipole magnet of the spectrometer. This was done by requiring two or more hits in both of the hodoscopes HA and HB. Although, in principle, the $\mathrm{K}^{\mathrm{O}} \mathrm{s}$ from the change exchange reactions of interest should have created two and only two hits in HA and HB, a less restrictive trigger was

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used. Accidental coincidences, halo particles and delta rays produced in the last part of the spectrometer could have created additional coincidences in $H A$ and $H B$ and thus veto a real event if an exactly-two trigger had been used. There were no scintillation counters lining the inside of the dipole to veto particles interacting in the magnet iron since this may also have resulted in vetoing events of interest. Finally, decays of the incident beam within the spectrometer were in no way suppressed and accounted for a large fraction of the event triggers.

A schematic of the trigger electronics is shown in Fig. 9. Each of the counter elements of $H A$ and $H B$ was carefully plateaued and the counters were timed to $\pm 2 \mathrm{nsec}$. with respect to each other. The discriminated outputs of each element were mixed and a final linear OR of each hodoscope was formed. A variable discriminator attached to this final mixing stage allowed the selection of the multiplicity in each hodoscope. Discriminator curves were done to select the level for $H A \geq 2$ and $H B \geq 2$. The master trigger was then formed by a coincidence of BEAMTRIG•HA>2•HB>2.
3. Buffer strobes and scalers

The information from the scintillation and Cerenkov counters in the beam and spectrometer was stored in two ways. First, buffer strobe modules retained the counter data on an event by event basis. These modules contained a simple latch for each coumter element that was set if a narrow strobe pulse ( 5 nsec. wide) was coincident with the counter signal. Thus for each event, a strobe pulse was generated and the pattern of counter elements that were on for that event was stored. The second way of retaining information was by accumulating individual counter rates and coincidence rates, i.e. by using fast scalers.

Two different classes of scalers were used in the experiment. These were event gated scalers and run gated scalexs. Event gated scalers were arranged such that they counted until an event trigger was generated by the trigger logic. Thus, these scalers counted the number of fncident kaons, pions, etc. from the beginning of the beam spill until an event occurred. These scalers were the ones used in calculating, for example, the absolute normalization for the experiment. Considerable care was taken to accurately time (to $\pm 2 \mathrm{nsec}$.) the input signals to these scalers so that nothing would be counted after an event had occurred.

The second type of scalers (run gated scalers) used were gated such that they counted for the entire $1.6 \mu \mathrm{sec}$. long beam spill. These scalers were useful in determining various absolute rates such as the number of kaons/beam pulse. The contents of all the scalers were sent to the central minicomputer of the data acquisition system after every 256 event triggers.
4. Gating logic

The PDP $11 / 20$ that collected the data from the experiment interacted with the external logic via the gating system shown in Fig. 10. The most important part of the gating logic was the EVENT GATE generation. An EVENT GATE was required to allow an event trigger which fired the spark chambers and initiated the data collection cycle. The EVENT GATE was opened by a pre-spill pulse (B2NU) from the accelerator and closed when the event trigger reset the 11 OK Flip Flop. Another EVENT GATE was generated on the next accelerator pulse unless the PDP $11 / 20$ was busy or unless the 14 msec . long spark chamber deadtime pulse was still on. This part of the gating, therefore, prevented another event from occurring if the computer or spark chambers were not ready.

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## I. Data Collection

A substantial part of the complete lass data acquisition system was used for the first time in this experiment. ${ }^{20}$ This system was designed to handle high trigger rates together with a large amount of data from each event. Additionaliy, the complexity of the LASS experimental apparatus made it mandatory to include a facility for sophisticated on-1ine monitoring of the experimental instrumentation.

An overall view of the computer arrangement is showil in Fig. 11. The raw data from the apparatus (spark chambers, proportional chambers and counters) were collected by individual controllers. Information then passed from the controllers by direct memory access transfer to the central minicomputer (PDP $11 / 20$ ) in the system. The PDP 11/20 interacted with the experiment via a control panel and with the gating logic as described in the previous section. The on1y function of the $11 / 20$ was to buffer and compress the data for transfer to the next computer in the data chain. Event buffers could be routed from the $11 / 20$ to an IBM 1800 computer 10cated in the LASS control room and/or to an IBM $370 / 168$ computer via an IBM SYS/7 interface over a $1 / 2$ mile long coaxial cable. The SYS/7 acted as a high speed buffering and formatting device to make data from the $11 / 20$ compatible with the $370 / 168$. The SYS/7 also communicated directly with the IBM 1800 which allowed the experimenter to send data from the $370 / 168$ to a line printer attached to the 1800. The 370/168, containing three million bytes of real memory, was part of the SLAC computer triplex. This computer complex contained two identical 370/168's and one 360/91. Only the 168 's were available to accept on-line data and write it onto high density magnetic tape.

Data managing of the event buffers was controlled by a real-time program resident in the LBM $370 / 168 .{ }^{21}$ A simplified diagram of this realtime network is shown in Fig. 12. Various subtasks existed within the real-time framework and set of routes and nodes (or destinations) were defined upon initialization of the job. Nodes could be defined as sample or no-sample. Thus, for example, the spooler node was defined to be a nosample node such that all of the data would be written onto 6250 bpi magnetic tape. The analysis node, however, which could find tracks, compute efficiencies, etc. was usually only required to be in the sample node. If a no-sample node could not respond to a communication from the message router (a tape not ready for example), a message was sent through the SYS/7 to the PDP $11 / 20$ which prevented data taking from continuing via the gating logic.

The experimenter could extract information from the analysis task via the graphics and command tasks. These tasks allowed one to interactively examine and change variables within the analysis program, define and look at histograms and scatter plots and conveniently display chamber efficiencies and other relevant finformation about the current performance of the spectrometer on an IBM 2250 graphics terminal located in the LASS control room. The analysis task and performance monitoring package evolved during the data taking as new and improved code was added. The large amount of available computer memory ( 1200 kilobytes) allowed the analysis done to be as sophisticated as desired. Not only were chamber efficiencies, spark multiplicities and counter distributions monitored, but physics quantities of interest (e.g., effective mass, missing mass and $t$ distributions) were also available for plotting in realtime. This was of considerable use in determining how well the experiment was progressing in texms of physics results.

When the $370 / 168$ was unavailable, data were written onto tape by the IBM 1800 using tape drives located in the LASS control room. A rudimentary analysis package existed on the IBM 1800 to monitor efficiencies but no physics analysis could be done. Approximately $15 \%$ of data were recorded in this way.


Fig. 12. A simplified representation of the real-time data managing and analyzing network.

## CHAPTER 3. TRACK RECONSTRUCTION AND KINEMATICS

## A. Introduction

The data for this experiment was accumulated from March through May of 1975 . A total of 12.8 million events were written onto magnetic tape during this period. Five million event triggers for an incident $K^{-}$ beam and five million triggers for an incident $\mathrm{K}^{+}$beam were recorded at a beam momentum of $12.8 \mathrm{GeV} / \mathrm{c}$. At $8.36 \mathrm{GeV} / \mathrm{c}, 1.4$ million events each were taken for incident $\mathrm{K}^{+}$and $\mathrm{K}^{-}$beams. The vast majority of these events (95\%) were recorded with full target and with the $\geq 2$ forward trigger described in Chapter 2. Empty target data (3\%), alignment (1\%), and special efficiency runs (1\%) were also taken.

Events were reconstructed in the spectrometer by a multistage process (see Fig. 13). Independent track segments in the upstream and downstream spark chamber packages were found first. These track segments were then paired across the dipole magnet and the known magnetic field used to calculate a signed momentum associated with the track. Vertices were determined by finding the point of closest approach of the upstream tracks. The magnitude and direction of the momentum of the incident beam particle were defined by the p-hodoscope and a beam track found by the $\theta-\phi$ hodoscope and beam proportional chambers. Finally, the relevant kinematic variables (e.g. effective mass, missing mass, etc) were calculated. The last stage of processing was to write a data sumary tape (DST) for the events of interest.

The remainder of this chapter is devoted to a description of the track finding and analysis process. Alignment and constants are discussed in Section B. Track reconstruction and momentum determination are described in some detail in section $C$. The calcuation of kinematic


Fig. 13. The overall data processing chain.
variables, including the effects of Fermi motion, is outlined in section $D$.

## B. Alignment and Constants

In order to obtain precise mass and angular measurements, it was necessary to check the alignment of each part of the spectrometer and the relative alignment between the different sections. The starting point for the alfgnment procedure was an accurate survey of the $x, y$ and $z$ position of the chambers and hodoscopes. This was done both as the chambers were installed and at the conclusion of the experiment. The survey values were then fine tuned by internally aligning the upstream and downstream spark chamber packages. Within each package the residual distributions from the $x$ and $y$ planes of each chamber were forced to have zero mean (within . 1 mm ) by slightly shifting ( $\leq 1$ mm or so) the nominal $x$ and $y$ positions of each chamber. In general it took only a few iterations to accomplish this.

Once the chamber packages were internally consistent, it was possible to align them with respect to the target and dipole. Downstream tracks were crossed through the dipole (see section $C$ for a more complete description of the crossing procedure) and compared in space and angle with the independently found upstream tracks. At the same time a vertex search for events with two or more upstream tracks was done. The means of the crossed track difference and the $x$ and $y$ vertex distribution were forced toward zero by rotating and shifting each spark chamber package (some care was taken to require good beam steering into the $\mathrm{LD}_{2}$ target for these particular runs so as not to introduce a systematic bias.) The rotations and shifts needed to accomplish this process were small ( $\leq 2$ mrad and $\leq 2 \mathrm{~mm}$ ). The beam defining devices
-42-
( $\Theta-\phi$ and the two proportional chambers) were also required to be internally consistent. Alignment with respect to the rest of the spectrometer was accomplished by comparing the difference at a fixed $z$ between the beam track and the incident kaon reconstructed from its $3 \pi$ ( $\tau$ ) decay in the spectrometer. All of these major constants were checked whenever the spectrometer was re-arranged in any way e.g. when the momentum of the beam was changed. Finally constants, once determined, were permanently saved and were recalled during the track processing to analyze a specific block of data.

In addition to the lengthy procedure described above, the internal constants of each spark chamber package were checked more frequently. Every time the dipole magnet was reversed and/or about every 20 or 30 runs, the magnetostrictive chamber residuals were plotted and chamber alignment examined. Small changes were eliminated by finding new constants that were then used to analyze that particular block of $20-30$ runs.

The momentum determining hodoscope (p-hodoscope) at F3 was also calibrated and the effective dispersion determined. Special runs at low incident flux were taken with the spark chambers displaced by a fixed amount such that the deadening plugs were out of the beam. In principle this allowed the determination not only of the magnitude of the beam momentum, but also of the $\Delta p / p$ corresponding to each $p$-hodoscope bin. The special runs were done at $12.8 \mathrm{GeV} / \mathrm{c}$ where the error in the momentum determination was dominated by errors in the dipole systems. The determination of $\Delta \mathrm{p} / \mathrm{p}$ so obtained was $.0038 \pm .0008$. A more accurate determination was later made by reconstructing the beam momentum from the tau decays of the incident kaons as analyzed in the spectrometer.

This method resulted in a value of $.0044 \pm .0004$ and was the one finally used. Although the magnitude of the incident momentum was also measured from the special runs and tau decays, the final determination was made by requiring the missing mass to the forward $K^{\circ}$ to be a nucleon mass. The results of the different methods were in good agrement and the beam momenta found to be $8.36 \pm .01$ and $12.80 \pm .015 \mathrm{GeV} / \mathrm{c}$.

The calibration of the dipole magnet was checked by comparing the measured $K^{\circ}$ mass, from its two pion decay mode, with the accepted world value. The two pion effective mass distribution was fitted to two gaussians and a linear background for each beam charge and at each beam energy. In each case, fits were done to the entire sample of reconstructed $K^{\circ}$ 's and also to the $K^{0}$ mass distribution after a vertex cut was made to select $K^{0}$ 's that had decayed beyond the liquid deuterium target. A typical fit is shown in Fig. 14. The results of all the fits are given in Table 2, and the mean masses show good agreement with the accepted value of .4977 GeV for the $\mathrm{K}^{\circ}$ mass. The mass resolution determined from the vertex cut data sample was found to be better than that found from the complete data sample as would be expected from multiple scattering of the pions in liquid deuterium.

## c. Track Reconstruction

Events in the spectrometer were analyzed using a set of computer routines to find tracks and the vector momenta of particles that passed through the dipole magnet. Although the forward two particles trigger was designed to select interesting (f.e. forward $\mathrm{K}_{\mathrm{s}}^{\circ} \rightarrow \pi \pi$ ) events, additional filtering was employed in the track finding programs to minimuze the number of completely analyzed triggers. This significantly reduced the average time to analyze an event which was a serious


Fig. 14. A typical fit (two gaussians + linear background) to the $\mathbb{R}^{0}$ mass distribution at $12.8 \mathrm{GeV} / \mathrm{c}$.

|  |  |  | -45- <br> table 2 |  | $\mathrm{X}^{2} / \mathrm{NDF}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | $\begin{gathered} \text { Mean } \\ \text { Mass }(\mathrm{GeV}) \end{gathered}$ | $\begin{gathered} \text { Resolution } \\ (\mathrm{GeV}) \\ \hline \end{gathered}$ |  |
| 8.36 | $\mathrm{K}^{\circ}$ | (a11 data) | . 4968 | . 0068 | 1.17 |
| " | $\mathrm{K}^{\circ}$ | (vertex cut) | . 4976 | . 0045 | . 83 |
|  | $\overline{\mathrm{K}^{\circ}}$ | (all data) | . 4970 | . 0051 | . 86 |
| " | $\overline{\chi^{\circ}}$ | (vertex cut) | . 4980 | . 0040 | . 83 |
| 12.8 | $\mathrm{K}^{0}$ | (a11 data) | . 4980 | . 0069 | . 87 |
| " | $\mathrm{K}^{\circ}$ | (vertex cut) | . 4986 | . 0054 | 1.15 |
|  | $\overline{\mathrm{K}^{\mathbf{o}}}$ | (all data) | . 4978 | . 0069 | 1.26 |
| " | $\overline{\mathrm{K}^{0}}$ | (vertex cut) | . 4984 | . 0062 | 1.52 |

consideration due to the large volume of data. The reconstruction programs were written exclusively in FORTRAN and, although loosely based on previous experience, were completely new and were tested for the first time on data from this experiment. The computer codes constituted one part of the more general set of reconstruction and monitoring programs for the LASS spectrometer. ${ }^{22}$ In the following sections, downstream tracking, upstream tracking, momentum determination and vertex finding are discussed.

1. Downstream tracking

Tracking in the downstream spark chamber package was restricted to events with greater than two but less than or equal to four hits in the scintillation hodoscopes HA and HB. This substantially reduced the average time to process an event and was found to have no effect on the number of reconstructed $\mathrm{K}^{\boldsymbol{O}^{\prime}} \mathrm{s}$. Raw data from the event were also required to be successfully unpacked before tracking finding comenced.
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The track finding scheme was to find match points in two pivot spark chambers and require enough points in the other chambers to lie on the Ine defined by the match points. Association with the proportional hodoscope and scintillation counters could also be required to define the track. Match points were found in the spark chambers in two steps. First, all possible $x-y$ points in a chamber were used to calculate a rotated coordinate (e or $p$ coordinate). The calculated coordinate was then compared with the real e (and/or p) coordinate, and a three (or four) wire match point was found if the real spark was within a specified tolerance of the calculated coordinate. The tolerance was defined by the distance between the $x-y$ and e-p gaps tan (maximum track angle of interest). For downstream tracks the maximum angle was $45^{\circ}$. After all the $x-y$ candidates had been explored, e-p pairs where used to predict $x$ or $y$ points and the same criteria were imposed to select more match points. Duplicate match points were eliminated by removing $x$ and $y$ points from the appropriate storage banks during the first stage.

After match points had been found in the two pivot chambers, a trial line was "drawn" between the two points. Corroboration with the proportional hodoscope and the two scintillation arrays was also required but with a relaxed tolerance. Tracks that satisfied these cuts were then extrapolated into the other two spark chambers. The closest spark in each of the $x, y, e$ and $p$ planes of the two chambers was found. If the spark was within 1 cm of the projected line it was assumed to be on the track. A total of four points (of a possible 16) were allowed to be missing and still retain a valid line segment. For an acceptable segment, the spark coordinates were subjected to a least
-47-
squares fit in three dimensions to find the slope and intercept parameters of the fitted line. The resulting chi-squared was required to be reasonable before proceeding.

The next step in the track finding was to repeat everything but with somewhat tighter tolerances. Proportional hodoscope and counter hodoscope association could be required again for the fitted line. Spark points were compared with the fitted line and accepted if they were within .7 cm of the line. A minimum of 12 coordinates were again required to lie on the line and if enough points were found, they were least squares fitted and final line parameters calculated. A chisquared cut was imposed to finally retain the track.

Once the track was accepted, information about the track was stored for later use. A computer code word with one bit for each spark chamber plane was filled in if a particular plane had a spark on the track and this was later used to compute efficiencies for the chambers. Residuals for each plane on the track were also kept. A computer word was formed indicating which element of the counter hodoscopes was associated with the track. Finally, the sparks on the track were "poisoned" out of the spark banks so that a large number of duplicate tracks were not found.

The tracking procedure described above was performed for three different sets of pivot chambers and only the combination of the two inner chambers was excluded from the procedure. Although poisoning sparks that had been assoclated with lines was designed to eliminate duplicate tracks, further checks were made. The $x$ and $y$ slopes and intercepts of the different tracks were compared and if they were
similar within a tolerance, the track with the best chi-square was kept and the others eliminated.

## 2. Upstream tracking

Track finding in the upstream spark chamber package was done only If two or more downstream lines had been found. The reconstruction process was essentially the same as for downstrean tracks but with slightly different tolerances. Track slopes were restricted to be less than $27^{\circ}$ and only two planes from the three chambers could be missing from the line. All three possible combinations were used as pivot chambers. Corroboration of a track by the upstream proportional hodoscope was also required.
3. Momentum determination

The magnitude of the momentum of a track was compsted by an iterative routine CRISS. Before computing the momentum, however, various cuts were made to restrict the pairing of upstream and downstream tracks. The difference between the $y-z$ slopes of the upstream and downstream 1 ines was required to be less than 200 mrad . Additionally, the difference between the average of upstream and downstreatn $y-z$ slopes and the $y-z$ slope determined by the last upstream point and first downstream was required to be less than 100 mrad . Finally, the intersection of the upstream and downstream lines was required to be approximately within the magnet aperture. Only if these criteria were satisfied, was CRISS used to find the momentum.

The known field of the dipole and the upstream and downstream track segments were used to calculate the momentum of the particle. As described in Chapter 2, the dipole field components were fitted to a polynomial in $x$ and $y$ which allowed the components to be reliably
calculated at any $x$ and $y$ within the aperture. A trial momentum was found for an upstream-downstream pair from

$$
P_{o}=\frac{B}{\theta_{\text {up }}-\theta_{\text {down }}}
$$

where, $B=21.22 \mathrm{kG}-\mathrm{m}\left[1+7 \times 10^{-6}\left(x_{\mathrm{o}}^{2}+y_{o}^{2}\right)\right] / 33.356$ and $\theta_{\mathrm{up}}$ and $\theta_{\text {down }}$ were the upstream and downstream angles; $x_{0}$ and $y_{0}$ were coordinates found by projecting the downstream line to the center of the magnet, and represent a slight modification to a uniform field.

Given this first estimate of the momentum, the downstream track was stepped through the dipole using the polynomial representation of the field. The integration was not continuous but was done in eight steps at which the magnetic field was calculated. A new field integral was calculated and a new momentum computed. The integration procedure was repeated and a final (signed) momentum determined by the last field integral.

The downstream track that was crossed through the dipole as described above was compared with an upstream track at a fixed $z$ value. The difference in space ( $x$ and $y$ ) and angle ( $\theta_{x}$ and $\theta_{y}$ ) were computed. Typical distributions for $\Delta x$ and $\Delta \theta y$ are shown in Fig. 15a and 15b. The shape is consistent with the approximately 1 mrad. angular accuracy of the tracking chambers. Generous cuts, as shown, were applied to these distributions to ultimately accept the track pair. The vector momenta of these crossed tracks was found from the upstream track vectors.
4. Beam tracks

The magnitude of the incident beam momentum was found for each good beam track from the p-hodoscope information. It was


Fig. 15. a) The difference in $x$ between an upstream track and a downstream track crossed through the dipole. b) The difference in $y-z$ slope between an upstream track and a downstream track crossed through the dipole.
discovered after the experiment that improper timing of the p-hodoscope elements had resulted in a $15-30 \%$ rate of ambiguous events with two or more possible values of the momentum. These events were removed from the final data sample.

The vector components of the incident beam particle were found by reconstructing tracks in the $\Theta-\phi$ hodoscope and the two beam proportional chambers. A number of different modes of track finding were employed. First, a large fraction of the events ( $060 \%$ ) were such that only single points were found in each of the two beam chambers. These events were handled separately and the beam track immediately found. Events with multiple hits in the chambers were analyzed differently. Tracks were found separately in the $x-z$ and $y-z$ planes and then required to pass through the same match point in the upstream chamber. If three points were available in the plane a least squares fit was done to find the line parameters but lines defined by two points were accepted if necessary. Tracks found in this manner were required to pass through the correct quadrant of the $X Y$ counter. Up to three possible beam tracks were kept and the typical track finding efficiency was 85-90\%. 5. Vertex determination

Vertices were found by determining the coordinates of the distance of closest approach between pairs of upstream tracks. Only tracks for which a momentum had been found (crossed tracks) were considered. A histogram of the distance of closest approach is shown in Fig. 16 and a vertex was defined if this distance was less than 3 cm .

In the case of reconstructed $\mathrm{K}^{\circ}$ 's the vertex found as above was the $K^{\circ}$ decay point i.e. the secondary vertex. The primary vertex was found by associating a beam track with the reconstructed $K^{\circ}$ track.


Fig. 16. a) The distance of closest approach between pairs of upstream tracks. b) The distance of closest approach between a beam track and the reconstructed $\mathrm{K}^{\circ}$ track.

Again a distance of closest approach was computed and the resulting distribution is shown in Fig. 17. A cut at 2 cm was applied to define a good primary vertex point.
D. Kinematics

For those events wherein a two track vertex had been found, the corresponding particles were assumed to be pions and the following kinematic variables computed assuming a stationary nucleon target:

- $M_{\pi \pi}$ - the two pion effective mass
- $M^{2}$ - the missing mass squared to the forward going twopion system
- $t^{\prime}$ - the four momentum transfer to the two-pion system
- $\operatorname{Cos} \theta_{H}$ - the polar decay angle in the two pion center of mass frame.

The effective mass for oppositely charged tracks was determined assuming that both particles were pions. The backgrounds resulting from the assumption of pion masses are discussed in the next chapter. The effects of the nature of the deuteron were neglected in computing the missing mass squared to the forward going $K^{\circ}$. It will be shown that neglecting Fermi motion is a valid assumption in the last part of this section.

The four momentum transfer ( $t$ ) between the $K^{\circ}$ and the incident beam particle was also computed and thereby $t^{\prime}=t-t_{\min }$. Although the exact value of the minimum momentum transfer, $t_{\text {min }}$, depends upon $M^{2}$, the small difference between the charged and neutral kaon masses makes $t_{\min }$ negligible ( $<1 \times 10^{-4} \mathrm{GeV}^{2}$ ) at the center of mass energies of this experiment. In the helicity reference frame in which the two pion angular distribution was calculated, the $Z$ axis was defined by the $X^{\circ}$


Fig. 17. The spectator momentuil distribution as calculated from the Hulthen wave function.
direction in the overall $C M$ system and $\theta$ by the direction of the positive pion track with respect to the $Z$ axis in the $K^{\circ}$ rest frame.

Of the four variables discussed so far, only the missing mass squared may be influenced by Fermi motion within the deuteron. In the next few paragraphs, the simplest deuteron wave function is introduced and used to determine the $M M^{2}$ smearing caused by the bound nature of the deuteron.

In the impulse approximation, the amplitude for a particular interaction on a deuteron may be considered to be the sum of the individual processes on a free neutron and proton. The usual procedure is to make this assumption (e.g. in computing missing mass as in the preceding section) and then make small corrections to the data. Multiple scattering and screening effects, the contribution from coherent production and the restrictions imposed by the Pauli Exclusion Principle are the corrections that need to be understood to extract free neutron (or proton) cross sections from data taken with a deuterium target. These effects will be discussed in Chapter 5; a short description of the deutron wave function and the fmportance of Fermi motion in smearing the effective incident beam momentum is given here.

The simplest wave function that attempts to describe the deuteron is the Hulthen function which assumes that the deuteron is a pure s-wave state of the two nucleon system. More complicated descriptions are available but would not significantly alter the results presented here and therefore they have not been discussed. ${ }^{23}$ The Hulthén wave function may be written as

$$
\begin{equation*}
\Psi(\vec{r})=\frac{N}{x}\left[e^{-\alpha r}-e^{-\beta r}\right] \tag{1}
\end{equation*}
$$

which yields the following momentum distribution for the nucleon inside the deuteron:

$$
\begin{equation*}
\frac{d N}{d p}=C p^{2}\left[\frac{1}{p^{2}+\alpha^{2}}-\frac{1}{p^{2}+\beta^{2}}\right]^{2} \tag{2}
\end{equation*}
$$

where

$$
\begin{aligned}
& C=\text { normalization constant } \\
& p=\text { magnitude of nucleon } 3 \text {-momentum } \\
& \alpha=.046 \mathrm{GeV} \\
& B=5.65 \alpha
\end{aligned}
$$

This distribution is plotted in Fig. 18.
The Hulthén momentum distribution may be used to compute the smearing of the incident beam momentum and the resulting missing mass distribution.

Consider the reactions

$$
\begin{equation*}
\mathrm{K}^{+}{\mathrm{d} \rightarrow \mathrm{~K}^{\circ}}_{\mathrm{pp}}^{\mathrm{s}} \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
K^{+}{ }_{n+K^{0}}^{p} \tag{4}
\end{equation*}
$$

Energy-momentum conservation for (3) implies

$$
p_{k}+p_{d}=p_{o}+p_{p}+p_{s}
$$

or

$$
\mathrm{p}_{\mathrm{k}}+\left(\mathrm{p}_{\mathrm{d}}-\mathrm{p}_{\mathrm{s}}\right)=\mathrm{p}_{\mathrm{o}}+\mathrm{p}_{\mathrm{p}}
$$

Defining $p_{v} \equiv p_{d}-p_{s}$ the center of mass energy squared for (3) is

$$
s=m_{k}^{2}+m_{v}^{2}+2 E_{k} E_{v}-2\left|p_{k}\right|\left|p_{s}\right| \cos \theta
$$

Denoting the center of mass energy squared for (4) as $s_{k}$ it is easy to show in the high energy limit ( $p_{k} \gg$ masses) and for $\left|p_{s}\right| / m$ ( $m \equiv$ nucieon mass) small that

$$
\begin{equation*}
s-s_{k} \approx-\left.2\left|p_{s}\right|\right|_{p_{k}} \mid \cos \theta \tag{5}
\end{equation*}
$$

or

$$
\begin{equation*}
\frac{p-p_{k}}{p_{k}} \approx-\frac{\left|p_{s}\right|}{m} \cos \theta \tag{6}
\end{equation*}
$$

where $p_{k}$ is the nominal beam momentum and $\left|p_{s}\right|$ the magnitude of the proton spectator momentum given by the Hulthén distribution.

The Hulthén function peaks at $\quad \mathbf{5 0} \mathrm{MeV} / \mathrm{c}$ which implies a width of about $5 \%$ for the effective beam momentum distribution. The result of the calculation without approximation is shown in Fig. 19 for $P_{k}=8.36 \mathrm{GeV} / \mathrm{c}$.

Although Fig. 19 shows the equivalent momentum distribution for (4), it does not take into account the energy dependence of the cross section for reaction (4) nor does it include the flux factor effects of what may be considered to be (in a modest way) a colliding beam process. The inclusion of these effects however does not substantially change the broadening of the incident momentum. 24

The effect of the momentum distribution on the calculation of missing mass squared may also be computed. With the approximation that the recoil momentum $p_{r}$ and the spectator momentum $p_{s}$ are $\operatorname{small}(\ll m)$, the missing mass squared is

$$
M^{2} \approx m^{2}-\left.2\left|p_{s}\right|\right|_{p_{r}} \mid \cos
$$



Fig. 19. The smearing of the $M^{2}$ distribution as a result of Fermi motion.

This is independent of beam momentum and for fixed $\left|p_{s}\right|$ depends only on the momentum transfer $t$ which is given in the same approximacion by

$$
\mathrm{t} \tilde{\sim}\left|\mathrm{P}_{\mathbf{s}}\right|^{2}+\left|\mathrm{p}_{\mathbf{r}}\right|^{2}+2\left|\mathrm{p}_{s}\right|\left|\mathrm{p}_{\mathrm{r}}\right| \cos \theta
$$

The actual missing mass squared distribution without any approximations Is shown in Fig. 20 for $|t|=.1$ and $|t|=.5$. For this $t$ range, the experimental resolution (. $13 \mathrm{GeV}^{2}$ at $8.36 \mathrm{GeV} / \mathrm{c}$ ) overshadows the Fermi motion effect.


Fig. 20. Two-pion mass distribution for all $\mathrm{MM}^{2}$ and all $t^{\prime}$. a) 8.36 $\mathrm{GeV} / \mathrm{c}$; b) $12.8 \mathrm{GeV} / \mathrm{c}$.
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## CHAPTER 4. DATA ANALYSIS

## A. Introduction

The final result of the data processing described in the previous chapter was a data summary tape containing the physics variables of interest. These data, however, could not be simply used to obtain the raw number of events for the charge exchange processes that are of concern. As will be shown, the missing mass distribution to the forward $K^{0}$ contained backgrounds from non- $K^{0}$ events as well as a non-resonant background which were required to be eliminated to extract a clean recoil nucleon or nucleon isobar $\Delta$ (1236) signal. The procedure that was employed to do this is discussed in detail in section $C$. Section B contains a brief description of the raw data distribution with particular emphasis placed on the differences (or similarities) between events from $\mathrm{K}^{+}$and $\mathrm{K}^{-}$incident beams, as well as energy differences.

## B. Raw Data Distributions

1. Two pion mass

The two pion effective mass was computed for events with two oppositely charged tracks that passed through the dipole and that formed a good vertex. The resulting distributions are shown in Figs. 21a and 21 b for the $12.8 \mathrm{~K}^{+}$and $8.36 \mathrm{~K}^{-}$beams respectively (the $8.36 \mathrm{~K}^{+}$and $12.8 \mathrm{~K}^{-}$distributions are similar). The small number of events at large $\pi \pi$ mass results from the rapidly falling acceptance of the spectrometer for masses greater than about $600 \mathrm{MeV} / \mathrm{c}^{2}$. The very large accumulation of events at low $m_{\pi \pi}$ comes in part from the $3_{\pi}$ decay ( $\tau$ decay) of the incident kaon. The $\tau$ decay characteristics are such that one of the three pions often passes through the deadening plugs in the upstream (or downstream) spark chambers and is therefore lost.


Fig. 21. The two-pion mass showing the $\mathrm{K}^{\mathrm{o}}$ and background sample regions.

Reconstruction of the remaining two pions yields a distribution strongly peaked near threshold.

The other obvious feature of the $m_{\pi \pi}$ distribution is the $K^{o}$ peak. Fig. 22 shows the $m_{\pi \pi}$ distribution on a finer scale in $1 \mathrm{MeV} / \mathrm{c}^{2}$ bins. The interval $.4777<\mathrm{m}_{\pi \pi} .5177 \mathrm{GeV} / \mathrm{c}^{2}$ was defined to be the region of the $\mathrm{K}^{\mathrm{o}}$ (for both beam momenta). Not all the events in this interval are real $\mathrm{K}^{0} \mathrm{~s}$. Approximately $10 \%$ of the events come from non $\pi \pi$ events i.e. $K \pi$ production for both $K^{+}$and $K^{-}$beams and forward $\Lambda^{\circ} \rightarrow \pi p$ production for the incident $K^{-}$beams. The lack of kaon or proton identification resulted in confusion of the kaon or proton with a pion. 2. Two pion decay angular distribution

The most obvious indication of these background processes may be seen in a scatter plot of the helicity $\operatorname{cosine}\left(\cos \theta_{H}\right)$ against two pion mass $\left(m_{\pi \pi}\right)$. These plots for the four different beams are shown in Figs. 23a, b, c, and d. A true two pion state is a vertical band in such a plot and non $\pi \pi$ states have a mass vs angle dependence. The production of the $K^{*}(890)$ may be clearly observed for both $K^{+}$and $K^{-}$ beams (see Fig. 24 as a guide). For incident negative kaons, the production of forward $\Lambda^{\circ} '_{s}$ encloses a very well defined region in the $\mathrm{m}_{\pi T}-\cos \theta_{\mathrm{H}}$ plot. An indication of $\phi \rightarrow \mathrm{K}^{+} \mathrm{K}^{-}$production may also be seen as a boomerang shaped enhancement at low $\pi \pi$ mass in the $K^{-\quad}$ data.
3. $K^{\circ}$ vertex distribution and lifetime

The primary vercex distribution along the beam direction (Zdirection) for events in the $K^{\circ}$ mass region is shown in Fig. 25. For events at small $t^{\prime}\left(\left|t^{\prime}\right|_{\sim} .02 \mathrm{GeV}^{2}\right)$ the resolution in the $Z$ direction is degraded such that the primary vertex position appears to be outside the liquid deuterium target boundaries. Interactions in the air,


Fig. 22. The angular distribution vs. two-pion mass. a) $8.36 \mathrm{GeV} / \mathrm{c} \mathrm{K}^{-}$; b) $8.36 \mathrm{GeV} / \mathrm{c} \mathrm{K}^{+}$; c) $12.8 \mathrm{GeV} / \mathrm{c} \mathrm{K}^{-}$; d) $12.8 \mathrm{GeV} / \mathrm{c} \mathrm{K}^{+}$.


Fig. 23. Guide to understanding Fig. 22. a) $\mathrm{K}^{-}$; b) $\mathrm{K}^{+}$.


Fig. 24. The $K^{o}$ primary $Z$ vertex distribution at $12.8 \mathrm{GeV} / \mathrm{c} \mathrm{K}^{+}$.


Fig. 25. a) $\mathrm{K}^{0}$ primary $x$ vertex distribution at $12.8 \mathrm{GeV} / \mathrm{c} \mathrm{K}^{+}$; b) $\mathrm{K}^{0}$ primary y at $12.8 \mathrm{GeV} / \mathrm{c} \mathrm{K}^{+}$; b) $\mathrm{K}^{0}$ primary y
vertex distribution at $12.8 \mathrm{GeV} / \mathrm{c}^{+}$.
target walls, etc. only account for a small fraction of the events outside the target iimits. The primary $Z$ vertex was required to be between -10 . cms and +110 cms to be included in the final data sample. Similarly a generous cut ( $\pm 4 \mathrm{~cm}$ ) was applied to the vertex distribution transverse to the beam direction (see Figs. $26 a$ and 26b).

For events in the $K^{0}$ mass band, the decay length was computed if a primary vertex had been determined. Knowing the momentum of the $\mathrm{K}^{\boldsymbol{0}}$ one could then calculate the lifetime of $K^{\circ}$. The results for both beam momenta are shown in Figs. 27a, b, c, d where the units are in time $x$ velocity of light $=$ cms. The dashed curved has been normalized to the total number of events in the plot and is the distribution expected from the accepted value of 2.68 cm for the $\mathrm{K}^{\circ}$ lifetime. The agreement at both momenta is somewhat fortuitous in that approximately $10 \%$ of the events are not real $\mathrm{K}^{\circ}{ }^{\prime} s$ and since no acceptance or other corrections have been made. As will be shown in the following chapter, a simulation of the spectrometer characteristics would predict a decrease in the number of detected events at short ifetime below the expected curve. The non- $K^{0}$ background events are peaked at short lifetime and thus "fill in" the anticipated hole. One should also notice that, within the statistical error, there is no difference between the lifetime distributions for opposite charges at the same beam energy or between the different energies.
4. Missing mass to the $K^{\circ}$

The missing mass squared $\left(M^{2}\right)$ to the forward going $K^{\circ}$ was computed for those events with an unambigious momentum for the incident beam particle. The resulting distributions for all four momentum transfer values to the $\mathrm{R}^{\circ}$ are shown in Figs. $28 \mathrm{a}, \mathrm{b}, \mathrm{c}$ and d . At $8.36 \mathrm{GeV} / \mathrm{c}$ the


Fig. 26. The $\mathrm{K}^{0}$ Iffetime. The curve is the expected distribution.


Fig. 27. The $M^{2}$ to the $K^{0}$ for all $t^{\prime}$. a) $8.36 \mathrm{GeV} / \mathrm{c} \mathrm{K} \mathrm{K}^{-}$; b) $8.36 \mathrm{~K}^{+}$; c) $12.8 \mathrm{~K}^{-}$; d) $12.8 \mathrm{~K}^{+}$.


Fig. 28. The $M \mathbb{M}^{2}$ for the background sample on either side of $K^{0}$ for all $t^{\prime}$. a) $8.36 \mathrm{GeV} / \mathrm{c} \mathrm{K}^{-}$; b) $8.36 \mathrm{~K}^{+}$; c) $12.8 \mathrm{~K}^{-}$; d) $12.8 \mathrm{~K}^{+}$.
resolution is sufficient to completely separate the nucleon and $\Delta$ peaks, while at $12.8 \mathrm{GeV} / \mathrm{c}$ the nucleon appears as a shoulder on the larger $\Delta$ signal. There is no obvious evidence for the production of resonant states with mass greater than that of the $\Delta$ at either energy.

## C. Extracting a Signal from the $M^{2}$ Distributions

In order to obtain the actual number of nucleon and $\Delta$ events. produced for each beam charge and momentum, the $M^{2}$ distributions for the different beam charges and momenta were subjected to a least squares fitting procedure using the program MINUIT. ${ }^{23}$ The data were divided into $9 t^{\prime}$ slices ( 8 at $8.36 \mathrm{GeV} / \mathrm{c}$ ) and the $\mathrm{MM}^{2}$ distribution in the interval $\mathrm{MM}^{2}<3.0 \mathrm{GeV}^{2}$ for each slice was fit by a smooth function. This function comprised four components:
1). a resolution function describing the nucleon peak
2). a p-wave Breit-Wigner to describe the $\Delta$ shape
3). a simple parameterization of the non-resonant (i.e. nonnucleon or $\Delta$ ) events at higher $M^{2}$
4). a fixed representation of the non- $K^{\circ}$ background events. The remainder of this section is devoted to a detailed description of each of the above components, the resulting overall fits to the data and a discussion of errors. Finally, the number of events obtained for each of the processes


1. The nucleon peak

Although the nucleon peak was ultimately represented by a single gaussian function, a number of checks were made to ensure consistency of this simple function with the data. A two gaussian parametrization.
of the peak with identical means and different standard deviations to represent different contributions to the resolution function was also investigated but was found to be unnecessary.

For each of the four sets of data, the $\mathrm{MM}^{2}$ distribution summed over all $t$ ' was used to check that the mean value $(\bar{M})$ of the gaussian was as expected. The value of $\bar{M}$ as determined by the fitting program was consistent with a value of $.88\left(\mathrm{GeV} / \mathrm{c}^{2}\right)^{2}$ within error at both beam momenta. A change of only $10 \mathrm{MeV} / \mathrm{c}$ in the nominal beam momentum was sufficient to measurably shift the value of $\bar{M}$. Any possibly $t$ ' dependence of $\vec{M}$ was investigated by repeating the above analysis for four broad $t^{\prime}$ slices. Again, within the fit errors, there was no $t$ ' dependence of the value of $\bar{M}$ at either momentum or for either beam charge.

The second parameter of the gaussian that was determined was the standard deviation ( $\sigma$ ). Again the data summed over $t^{\prime}$ were used to find the values of $\sigma$ that yielded the best fits ( $\bar{M}$ was fixed at $.88 \mathrm{GeV}^{2}$ ). There was no appreciable difference in the value of $\sigma$ between $K^{+}$and $K^{-}$beams at the same momentum. Also no significant $t^{\prime}$ dependence was found for the values of $\sigma$. The values for $\sigma$ that were used in the final fits were fixed to be $.13\left(\mathrm{Gev} / \mathrm{c}^{2}\right)^{2}$ at $8.36 \mathrm{GeV} / \mathrm{c}$ and $.215\left(\mathrm{GeV} / \mathrm{c}^{2}\right)^{2}$ at $12.8 \mathrm{GeV} / \mathrm{c}$ for all $\mathrm{t}^{\prime}$ slices. Thus the only variable parameter used to describe the nucleon peak in the final fitting procedure was a normalization coefficient ( $N_{1}$ ).
2. $\Delta$ parametrization

The $\Delta$ line shape was described by a p-wave Breit-Wigner multiplied
by phase space and smeared by the resolution function described in (1). The Breit-Wigner used was of the form: ${ }^{24}$
-75-

$$
\begin{equation*}
B W(M)=N_{2} \frac{M}{P} \frac{r(M)}{\left(M_{0}^{2}-M^{2}\right)^{2}+M_{0}^{2} r^{2}(M)} \tag{1}
\end{equation*}
$$

$\mathrm{N}_{2}$ - normalization constant
M - missing mass
$M_{0}$ - nominal mass of the $\Delta$
$\Gamma(M)$ - mass dependent width
The expression used for the width was:

$$
\begin{equation*}
r(M)=\Gamma_{0}\left(\frac{P}{P_{0}}\right)^{3} \frac{\rho(M)}{\rho\left(M_{0}\right)} \tag{2}
\end{equation*}
$$

「o - nominal $\Delta$ width
p - momentum of $\Delta$ decay products for mass $M$ in the $\Delta$ rest frame
$P_{0} \quad$ - momentum of decay products for mass $M_{0}$
$\rho(M)$ - empirical form factor
A form for $\rho(M)$ was used as given by Gell-Mann and Watson ${ }^{25}$

$$
\begin{equation*}
\rho(M)=\left\{a m_{\pi}{ }^{2}+p^{2}\right\}^{-1} \quad a=1.3 \tag{3}
\end{equation*}
$$

This parametrization was found to yield an adequate representation of the $\Delta$ shape.

Checks were made on the values of the nominal $\Delta$ mass and width in the same manner as described in (1). The a priori unknown shape of the background under the $\Delta$ tends to make a detailed extraction of $M_{0}$ and $F_{0}$ from these data somewhat difficult. However, the values of 1.23 GeV for $M_{o}$ and .109 GeV for $\Gamma$ were found to yield good fits to the missing mass squared distributions. The results were also consistent with no $t^{\prime}$, beam momentum or beam charge dependence of $M_{o}$ or $\Gamma_{o}$.
3. $\mathrm{K}^{\mathrm{o}}$ background

The third component of the fitting function represented the production of higher mass nucleon isobars and/or multiparticle production at the upper vertex e.g. $K^{\circ} \pi$ production. A number of possible parametrizations were tried: (1) phase space (PS) for $K N-K N \pi$; (2) a polynomial x PS and (3) a "triple Regge" $x$ PS form. The criteria for picking the best form were, of course, a good fit but also smoothness and consistence as a function of $t$ ' with a minimum number of free parameters. For this reason the polynomial $x$ PS was eliminated although it could be made to give good fits to the data. Simple KNT phase space was found to fit the data poorly at larger $t^{\prime}\left(\left|t^{\prime}\right| \gtrsim .3 \mathrm{Gev}^{2}\right.$ ) for $\mathrm{MM}^{2} \gtrsim^{2} \mathrm{Gev}^{2}$. The remainder of this section describes the function that was found to be successful.

The "triple Regge" form used was

$$
\begin{equation*}
N_{3} \times \operatorname{PS} \times\left(\frac{M^{2}-M_{t h}^{2}}{S}\right)^{1-2 \alpha\left(t^{\prime}\right)} \times A\left(M^{2}, t^{\prime}\right) \tag{4}
\end{equation*}
$$

where
$N_{3}$ - normalization constant
$P S$ - KNT phase space
$M^{2}$ - missing mass squared
$M_{t h}^{2}$ - threshold missing mass squared for $K \pi N$
$s-$ square of center of mass energy
$\alpha\left(t^{\prime}\right)=\alpha_{0}-\alpha^{\prime}\left|t^{\prime}\right|$ where $\alpha_{0}$ and $\alpha^{\prime}$ are fixed
$A\left(M^{2}, t^{\prime}\right)-a$ fixed acceptance correction factor

Although this form of the background was motivated by the history of successful parametrization of inclusive particle data, no claim is made that it describes the physics of inclusive $\mathrm{X}^{0}$ production at the
small values of $\mathrm{MM}^{2}$ that are being considered. It is simply a smooth function with consistent $t$ ' dependence and only one free parameter that fits the data quite well at both beam momenta.

The phase space $f$ tor was found to be necessary for smooth behavior near threshold and to obtain a good fit in the $\Delta$ region. An acceptance correction factor was necessary because the geometrical acceptance of the spectrometer at fixed $t^{\prime}$ depended strongly on the $K^{\circ}$ momentum. As the $\mathbb{M M}^{2}$ is increased the $K^{\circ}$ momentum is decreased, and hence fewer events are found at larger $M^{2}$. It was determined that, for each $t^{\prime \prime}$ slice, the $M^{2}$ acceptance correction could be adequately described by a Inear function in $M^{2}$. For $\left|t^{\prime}\right| \lesssim .3 \mathrm{GeV}^{2}$ the difference in acceptance between $M^{2}=1 \mathrm{GeV}^{2}$ and $M^{2}=3.0 \mathrm{GeV}^{2}$ was $5-15 \%$. At the largest $\left|t^{\prime}\right|$ interval for which we had data, the difference was about $40 \%$ in acceptance. This acceptance correction was crucial in obtaining reasonable fits, particularly for $\left|t^{\prime}\right| \geqslant .25 \mathrm{GeV}^{2}$.

Although the effective trajectory, $\alpha$ in the triple Regge formula was fixed for the final fits to the data, it was rigorously examined beforehand for consistency. Attempts were made to determine the slope ( $\alpha^{\prime}$ ) and intercept ( $\alpha_{0}$ ) of the effective trajectory by finding those values that maximized the quality of the fit to the data. In doing so it became apparent that, if both $\alpha_{0}$ and $\alpha^{\prime}$ were allowed to vary, consistent values for all $t$ ' were unobtainable. Furthermore, for a reasonable value of $\alpha_{0}(\underset{\sim}{\sim} .5)$, the quality of the fits was not particularly sensitive to changes in $\alpha^{\prime}$ of $\pm .2$ units from a value of 1.0 for all $t^{\prime}$. Although the naively expected values of $\alpha_{0}=.5$ and $\alpha^{\prime}=1.0$ did yield reasonable fits to all four sets of data, slightly different values were chosen. The procedure that was followed was to fix $\alpha^{\prime}$ and
find the best value of $\alpha_{0}$ to minimize the chi-squared of the fit. This was repeated until a value of $\alpha^{\prime}$ was found such that $\alpha_{0}$ was approximately the same, within errors, for each $t$ ' slice i.e. and $t '$ such that all the explicit $t^{\prime}$ dependence was contained in the effective trajectory. This procedure was followed for all four data sets and the end result was that values of $\alpha_{0}=.45$ and $\alpha^{\prime}=.85$ were found to give good values for chi-squared for both beam charges and both momenta.
4. Non- $K^{\circ}$ background

Not all of the events in the missing mass squared distributions that are to be fit come from $K N \rightarrow K^{\circ}+X$. An inspection of the $\pi \pi$ mass plot of Fig. 22 shows that approximately $10 \%$ of the events are not true $\mathrm{K}^{\mathrm{O}} \mathrm{g}$. As was shown in Section $\mathrm{B}-2$, most of these background events are from $K^{\circ}$ * production where $K^{\circ}{ }^{\circ}+K_{\pi}$ and the $K$ has been assumed to be a pion as there was no particle identification in the experiment. For the negative beams the process $K^{-} p \rightarrow \Lambda^{\circ}+X$ also contributed background events. Non-resonant $K N \rightarrow K \pi+N$ production made a small additional contribution. Multiparticle reactions where two of three or more particles produced at the upper vertex, were not, in general, of interest since the $M^{2}$ for such events was generally greater than $3 \mathrm{GeV}^{2}$

In order to understand this background in the $K^{\circ}$ region (.4777 $-.5177 \mathrm{GeV})$, events in two sideband regions, (.4527-.4727 and .5227 - . 5427 GeV ) each $1 / 2$ the width of the $\mathrm{K}^{\circ}$ mass cut, were subject to analysis (see Fig. 22). It was assumed that the characteristics of the background varied smoothly in $\pi \pi$ mass and therefore that an adequate representation of the background under the $K^{\circ}$ could be obtained by summing events on either side of the $K^{\circ}$.

The missing mass squared distribution for events in the sideband regions are shown in Figs. 29a, $b, c$, $d$. The nucleon and $\Delta$ peaks are from the processes $K N \rightarrow K^{\circ}{ }^{*}$ and $K N \rightarrow K^{\circ}$. One should note that the mean mass values of the .uc eon and $\Delta$ are slightly shifted to higher values of $M M^{2}$. This comes about from the mislabelling of an actual kaon as a pion and hence an incorrect computation of $\mathrm{MM}^{2}$. An additional feature to observe is the small difference between $K^{+}$and $K^{-\quad}$ distributions. Relative to the amount of $\Delta$ signal, there is more background at larger missing mass for the $K^{-}$beams as would be expected from the production of $\Lambda^{\circ}$. The final salient observation to make concerns the $t^{\prime}$ distribution for the side band events shown in Fig. 30 for the $8.36 \mathrm{~K}^{-}$beam (the other energy and charge are similar). Comparing this with that for the $\mathrm{K}^{\circ}$ region (also Fig. 30) one notices that the side band events are relatively more peaked at smaller $t^{\prime}$. Therefore one expects a larger background subtraction in the forward direction which may be further accentuated since the real signal may tend to disappear at very small $t$ '.

In order to make a quantitative representation of the background for each slice in $t^{\prime}$ the side band data was parametrized by functions as described in (1), (2), and (3) but with relaxed parameters. In particular the mean and sigma of the resolution function and the mean and width of the $\Delta$ were allowed to vary. The value of the fitted number of events in each $M^{2}$ bin was permanently retained and used later for each $t^{\prime}$ interval fit to $K^{o}$ region data.
5. Final fitting procedure and error analysis

The missing mass squared data in each $t^{\prime}$ slice were fitted to the four component function described above using MINUIT. The total fitting


Fig. 29. The $t^{\prime}$ distribution for $M M^{2}<3 \mathrm{GeV}^{2}$ for $\mathrm{K}^{\mathrm{o}}$ events (solid Iine) and background sample events (dashed line).


Fig. 30. Typical fits to the $\mathrm{mm}^{2}$ distribution at $8.36 \mathrm{GeV} / \mathrm{c}$ : dotted line-- resolution function; dashed line -- $\Delta$ line shape; open-circle-dashed line -- multiparticle background; dot dash line -- non- $\mathrm{K}^{0}$ background; solid line -- overall function.
-82-
function, $\mathrm{F}_{\text {TOT }}$, was computed at the center of each $0.05\left(\mathrm{MeV} / \mathrm{c}^{2}\right)^{2}$ bin in $M M^{2}$ and chi-squared ( $X^{2}$ ) was computed by

$$
x^{2}=\sum_{1}^{60} \frac{\left(N-F_{\mathrm{TOT}}\right)^{2}}{\sigma^{2}}
$$

where $N$ is the number of real events in a particular bin and $\sigma$ is the statistical ( $=\sqrt{\mathrm{N}}$ ) error for that bin. MINUIT would attempt to minfmize $\chi^{2}$ by varying the three parameters $N_{1}, N_{2}$ and $N_{3}$. After the minimization program had converged, the parameters were changed by fixed increments and $X^{2}-\chi^{2}$ min was computed and displayed to check if a local or true minimum had been found. In all cases it was possible to find the true minimum in $\chi^{2}$-space and thereby attain reasonable values for $\chi^{2}$ per degree of freedom. Some typical fits are shown in Figs. 31a, $b$, and $c$ and $32 a, b$, and $c$ for the 8.36 and $12.8 \mathrm{GeV} / \mathrm{c}$ data respectively.

For each $t$ ' slice, the errors on the three fitting parameters were also determined by MINUIT in two ways. First, the diagonal. elements of the error matrix were used to compute the uncertainty in each parameter. These errors were reasonable, however, only when the behavior of the $X^{2}$. surface was parabolic in the neighborhood of the minimum for each parameter. The validity of this assumption was checked by employing the subroutine MINOS in the MINUIT package. This routine was capable of estimating the error in each parameter even if the $\chi^{2}$ surface was not truly parabolic in that parameter. Although a more reliable error estimate was obtained in this way, it was found that the errors determined by MINOS agreed very well with the
-83-


Fig. 31. Typical fits to the $\mathrm{mm}^{2}$ distribution at $12.8 \mathrm{GeV} / \mathrm{c}$.
uncertainties found from the assuming parabolic behavior for all fits. It is therefore believed that the errors that were found are indeed rellable.

In order to extract the number of nucleon and $\Delta$ events in each $t$ ' interval fit, cuts in $\mathrm{MM}^{2}$ were applied to the fitted function. At 8.36 $\mathrm{GeV} / \mathrm{c}$ the nucleon cuts were $.52-1.24\left(\mathrm{GeV} / \mathrm{c}^{2}\right)^{2}$ and $.22-1.4\left(\mathrm{GeV} / \mathrm{c}^{2}\right)^{2}$ for $12.8 \mathrm{GeV} / \mathrm{c}$. The lower $\mathrm{MM}^{2}$ cut for the $\Delta$ region was chosen to be well below the minimum mass of the $\Delta$ with smearing included. The upper $\mathrm{MM}^{2}$ cut was taken to be $2.0\left(\mathrm{GeV} / \mathrm{c}^{2}\right)^{2}$ although the Breit-Wigner description of the $\Delta$ line shape extends to higher $M^{2}$. The value of 2.0 was chosen since the validity of a pure p-wave Breit-Wigner description of the $\Delta$ at large $M_{M}^{2}$ is in doubt. If the upper cut were $3.0\left(\mathrm{GeV} / \mathrm{c}^{2}\right)^{2}$, the number of raw $\Delta$ events would increase by approximately $15 \%$ independent of energy and $t '$. The number of nucleon and $\Delta$ events extracted using the above cuts is given in Table 3 together with the estimated errors and $\chi^{2}$ value of each fit.

Additional checks were made to enhance the credibility of the fitting process and the extraction of the nucleon and $\Delta$ signals. First, for the $8.36 \mathrm{GeV} / \mathrm{c}$ data, the better $\mathrm{MM}^{2}$ resolution allowed a simple mass cut to be used in each $t^{\prime}$ bin to extract a nucleon signal. Allowing for the non- $K^{\circ}$ background and the small tail of the $\Delta$, this method yielded essentially the same result (and the same errors) as the fitting procedure. Second, at both energies and for both the nucleon and $\Delta$, the $M^{2}$ distributions were replotted requiring a secondary vertex outside the target region. Such a cut removed the non-K ${ }^{\circ}$ background (except for a residual $\Lambda^{0}$ signal for the $K^{-}$beams). The "vertex cut" data were binned in $M^{2}$ and $t$ ' exactly as the full data
table 3

| $\left\|t^{\prime}\right\|$ | $8.36 \mathrm{~K}^{-}$ |  |  | $8.36 \mathrm{~K}^{+}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Nucleon Events | $\begin{gathered} \Delta \\ \text { Events } \end{gathered}$ | $\mathrm{x}^{2} / \mathrm{NDF}$ | Nucleon Events | $\underset{\text { Events }}{\Delta}$ | $\mathrm{x}^{2} / \mathrm{NDF}$ |
| 0-. 04 | $38 \pm 10$ | $44 \pm 16$ | 1.0 | $60 \pm 12$ | $47 \pm 19$ | . 9 |
| . $04-.08$ | $60 \pm 9$ | $108 \pm 13$ | 1.1 | $99 \pm 11$ | $113 \pm 18$ | 1.8 |
| . 08 - . 12 | $47 \pm 8$ | $92 \pm 13$ | 1.0 | $76 \pm 9$ | $102 \pm 12$ | 1.2 |
| . $12-.16$ | $35 \pm 7$ | $58 \pm 9$ | . 6 | $41 \pm 7$ | $86 \pm 12$ | . 9 |
| . 16 - . 22 | $31 \pm 7$ | $56 \pm 10$ | . 9 | $64 \pm 8$ | $70 \pm 12$ | . 5 |
| . 22 - . 30 | $26 \pm 6$ | $35 \pm 8$ | . 8 | $30 \pm 6$ | $46 \pm 8$ | . 5 |
| . $30-.50$ | $17 \pm 6$ | $32 \pm 8$ | . 5 | $24 \pm 6$ | $27 \pm 7$ | . 3 |
| . $50-1.0$ | $1 \pm 1$ | $1 \pm 1$ | -- | $1 \pm 1$ | $1 \pm 1$ | -- |
| Total | $255 \pm 20$ | $427 \pm 30$ |  | $394 \pm 23$ | $492 \pm 35$ |  |


| $\left\|t^{\prime}\right\|$ | $12.8 \mathrm{~K}^{-}$ |  |  | $12.8 \mathrm{~K}^{+}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Nucleon Events | $\begin{gathered} \Delta \\ \text { Events } \end{gathered}$ | $\mathrm{x}^{2} / \mathrm{NDF}$ | Nucleon Events | $\begin{gathered} \Delta \\ \text { Events } \end{gathered}$ | $\mathrm{x}^{2} / \mathrm{NFD}$ |
| 0-. 02 | $23 \pm 9$ | $24 \pm 20$ | 1.0 | $44 \pm 10$ | $59 \pm 25$ | 1.0 |
| . $02-.04$ | $82 \pm 12$ | $80 \pm 20$ | 1.5 | $109 \pm 13$ | $92 \pm 23$ | 1.0 |
| . 04 - . 08 | $232 \pm 18$ | $243 \pm 25$ | 1.1 | $309 \pm 20$ | $364 \pm 32$ | 1.3 |
| . 08 - . 12 | $168 \pm 15$ | $274 \pm 24$ | . 9 | $264 \pm 19$ | $405 \pm 30$ | 1.5 |
| . 12 - . 16 | $172 \pm 16$ | $227 \pm 22$ | 1.0 | $226 \pm 18$ | $419 \pm 28$ | 1.1 |
| . 16 - . 22 | $199 \pm 16$ | $234 \pm 21$ | 1.4 | $288 \pm 19$ | $415 \pm 28$ | 1.0 |
| . 22 - . 30 | $151 \pm 14$ | $180 \pm 20$ | 1.3 | $237 \pm 17$ | $277 \pm 24$ | 1.5 |
| . $30-.50$ | $168 \pm 15$ | $189 \pm 20$ | 1.4 | $217 \pm 16$ | $355 \pm 23$ | 1.5 |
| . $50-1.0$ | $21 \pm 8$ | $65 \pm 12$ | 1.1 | $51 \pm 9$ | $111 \pm 12$ | . 9 |
| Total | $1215 \pm 42$ | $1517 \pm 62$ |  | $1745 \pm 48$ | $2492 \pm 77$ |  |

sample and the identical fitting procedure was employed to get the number of nucleon and $\Delta$ events. For each $t$ ' interval the ratio of the cut to non-cut data was then computed. If the non- $\mathrm{K}^{\circ}$ background had been incorrectly handled, then this ratio would have a $t$ ' dependence as the background events were comparatively more peaked at low $t^{\prime}$. In fact it was found that the cut to uncut ratio was independent of $t^{\prime}$, within the computed errors, for each of the four sets of data. It is therefore belleved that the non- $\mathrm{X}^{0}$ background was correctly parametrized in the fits.

CHAPTER 5. NORMALIZATION AND CORRECTIONS
A. Introduction

In order to obtain absolute differential cross sections, the raw number of events obtained as described in the previous chapter were corrected for a number of effects. A correct count of the number of incident kaons and an accurate knowledge of liquid deuterium density was also necessary to compute the normalization for the experiment. This chapter covers in some detail the correct counting of the number of incident beam particles and contains a description of geometrical and other acceptance correction factors. Efficiency corrections are discussed and the magnitude of the empty target subtraction is determined. Finally, the various nuclear physics effects of the deuteron are discussed in terms of their impact on deterwining the free nucleon cross sections.

## B. Normalization

The number of kaons incident upon the $\mathrm{LD}_{2}$ target was counted by the beamline Cerenkov and scintillation counters as described in the Beam Logic section of Chapter 2. One should be reminded that the particle types were identified by:

$$
\begin{aligned}
\pi & =\text { BEAM } \cdot \mathrm{C}_{\pi} \cdot \mathrm{C}_{K} \\
\mathrm{~K} & =\mathrm{BEAM} \cdot \bar{C}_{\pi} \cdot \mathrm{C}_{K} \\
\mathrm{P} & =\mathrm{BEAM} \cdot \overline{\mathrm{C}}_{\pi} \cdot \bar{C}_{K}
\end{aligned}
$$

$$
\text { BEAM }=\mathrm{SE} \cdot \Sigma \mathrm{XY}>1 \cdot \Sigma \overline{\mathrm{R}} \overline{\Sigma \mathrm{XY}>2}
$$

The raw number of pions, kaons or protons counted during the experiment was not an exact measure of the true number of incident particles.

In particular, for the number of incident kaons, the following corrections must be made:

1) Cerenkov inefficiencies
2) Delta ray production that simulates a kaon
3) Logic deadtime
4) Decay of a real incident kaon before it could interact in the target.

Although each of these corrections is small ( $<6 \%$ ), they shall be considered in detail in the following sections.

1. Cerenkov inefficiencies

The normalization of the experiment is affected by Cerenkov efficiencies only if the inefficiency results in kaon trigger. In particular if $C_{\pi}$ has an efficiency $\varepsilon_{\pi}$ and if $\pi / K$ is the ratio of pions to kaons in the beam, then a factor of $\pi /{ }_{K}\left(1-\varepsilon_{\pi}\right)$ is needed to correct the counted number of kaons. More precisely if $N_{K}$ is the number of raw counted kaons and $R_{K}$ the actual number, then

$$
R_{K}=N_{K}\left(1-\frac{\pi}{K}\left(1-\varepsilon_{\pi}\right)\right)
$$

It was impossible to monitor the value of $\varepsilon_{\pi}$ during the experiment with only two Cerenkov counters present in the beam. However, tests with pion beams indicated that $\varepsilon_{\pi}$ was $\geq 99 \%$. Since $\pi / K$ varied from .1 to .2 , the maximum correction would be $.2 \%$ which is insignificant in comparison to other corrections. Therefore the $C \pi$ inefficiency correction has been ignored. An inefficiency in $C_{K}$ does not influence the normalization as it is required to get an event trigger.

## 2. Delta rays

A proton (or antiproton) going through $C_{K}$ has some probability of making a knock-on electron that gives light in the counter and therefore counts as a kaon. An approximate formula for the number of delta rays produced per cm in a gas of density $\rho$, atomic number $Z$ and atomic weight A is

$$
\frac{\mathrm{dN}}{\mathrm{~d} \ell} \approx \cdot 3 p \frac{\mathrm{Z}}{\mathrm{~A}} \frac{\mathrm{M}_{\mathrm{e}}}{\mathrm{E}_{\mathrm{TH}}}
$$

where $M_{e}$ is the mass of the electron and $E_{T H}$ in the Cerenkov threshold energy in the gas. Given the length and density of the $\mathrm{CO}_{2}$ in $\mathrm{C}_{\mathrm{K}}$, the above formula yields a value of about $2 \times 10^{-3}$ for the probability of producing a delta ray for each incident proton. The correction formula then looks like:

$$
R_{K}=N_{K}\left(1-2 \times 10^{-3}\left\{\frac{P}{K}\right\}\right)
$$

where $P / K$ is the proton/kaon ratio in the beam. As $P / K$ was at least $1 / 10$, this very small correction was also neglected.
3. Logic deadtime

The beam logic did not accurately count beam particles if the time separation between two particles was too short. The precise time, $\Delta t$, during which a given counter was dead was difficult to estimate. However, it was reasoned that 20 nsec . represented a reasonably accurate estimate of the maximum deadtime assaciated with any of the three relevant counters ( $\mathrm{XY}, \mathrm{SE}$ or $\mathrm{C}_{\mathrm{K}}$ ). For $4-5$ particles/pulse and an accelerator spill length of $1.6 \mu \mathrm{sec}$, the probability of getting a second particle within 20 nsec . is then approximately $5 \%$. As the XY counter was segmented, $5 \%$ does not indicate the total percentage of
particles that escaped detection. The assumption was made that the beam was uniformly distributed over the four XY quadrants and therefore only particles going through the same $X Y$ quadrant were not counted. The total loss was then estimated to be $.25 \times .05$, or about $1.25 \%$. The error on this calculation was taken to be about . 5\%. Finally, this estimate ignores the possibility of the second particle not being a kaon or of the second particie decaying, both of which were small corrections to a small correction.
4. Decays of the beam particles

The largest correction factor to the number of incident kaons came from decays in the region from $C_{\pi}$ to the target. The decay losses were estimated to be $5 \pm 2 \%$ at $12.8 \mathrm{GeV} / \mathrm{c}$ and $6 \pm 2 \%$ at $8.36 \mathrm{GeV} / \mathrm{c}$.

## 5. Sunmary of corrections

As determined above, the significant corrections to the measured number of incident kaons come from beam logic deadtime and beam particle decays. Table 4 shows the number of measured kaons for the different beams and the corrected number of beam kaons. The total systematic error associated with uncertainties in the dead time and decay correction was about $2 \%$.
6. Liquid deuterium density

The temperature and vapor pressure monitors in the target were used to compute the effective $\mathrm{LD}_{2}$ density. The average density for each of the four beam conditions was computed after weighting each run by the number of incident kaons recorded during that run. The results are given in Table 4 and include small corrections ( $\langle 3 \%$ ) for the presence of $H_{2}$ and HD. It was found that the density was essentially constant (to $\pm 1 \%$ ) during the experiment.

## TABLE 4

|  |  | $8.36 \mathrm{~K}^{-}$ | $8.36 \mathrm{~K}^{+}$ | $12.8 \mathrm{~K}^{-}$ | $12.8 \mathrm{~K}^{+}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1) | Uncorrected Number of Kaons $\left(\times 10^{6}\right)$ | 33.74 | 36.21 | 95.16 | 102.82 |
| 2) | Corrected Number of Kaons ( $\times 10^{6}$ ) | $32.11 \pm .70$ | 34.46土.75 | $91.53 \pm 1.96$ | $98.90 \pm 2.12$ |
|  | $\mathrm{LD}_{2}$ Density (gm/cc) | . $1675 \pm .002$ | $.1675 \pm .002$ | $.1675 \pm .002$ | $.1675 \pm .002$ |
|  | $\mu b / e v e n t$ for unity acceptance and efficiency | $\begin{gathered} 1.975 \pm .049 \\ \times 10^{-2} \end{gathered}$ | $\begin{gathered} 1.839 \pm .046 \\ \times 10^{-2} \end{gathered}$ | $\begin{array}{r} 6.92 \pm .17 \\ \times 10^{-3} \end{array}$ | $\begin{gathered} 6.408 \pm .157 \\ \times 10^{-3} \end{gathered}$ |
|  | Beam track with good momentum efficiency | $.690 \pm .01$ | $.673 \pm .009$ | $.773 \pm .009$ | . $779 \pm .008$ |
| 6) | Two Track Efficiency | $.761 \pm .084$ | . $781 \pm .086$ | . $741 \pm .082$ | . $723 \pm .080$ |
| 7) | (5) $\times$ (6) | $.525 \pm .058$ | $.526 \pm .058$ | $.573 \pm .064$ | $.563 \pm .062$ |
| 8) | Efficiency Corrected $\mu \mathrm{b} /$ event | $\begin{gathered} 3.759 \pm .428 \\ \times 10^{-2} \end{gathered}$ | $\begin{gathered} 3.498 \pm .398 \\ \times 10^{-2} \end{gathered}$ | $\begin{gathered} 1.209 \pm .137 \\ \times 10^{-2} \end{gathered}$ | $\begin{gathered} 1.138 \pm .129 \\ \times 10^{-2} \end{gathered}$ |
| 9) | Empty target subtraction | . $02 \pm .01$ | . $02 \pm .01$ | $.02 \pm .01$ | . $02 \pm .01$ |
| 10) | Final $\mu \mathrm{b} /$ event | $\begin{gathered} 3.683 \pm .421 \\ \times 10^{-2} \end{gathered}$ | $\begin{gathered} 3.428 \pm .392 \\ \times 10^{-2} \end{gathered}$ | $\begin{gathered} 1.185 \pm .135 \\ \times 10^{-2} \end{gathered}$ | $\begin{gathered} 1.115 \pm .127 \\ \times 10^{-2} \end{gathered}$ |

7. Absolute normalization

Given the corrected number of incident kaons, the target density and target length, the number of $\mu \mathrm{b} /$ event for unity acceptance may be calculated (see Table 4). The numbers in line 4 of Table 4 also include a correction for unseen decay modes of the $K^{\circ}$ using the accepted value of .6867 for the branching ratio of $\mathrm{K}_{\mathrm{B}}^{0} \rightarrow \pi^{\dagger} \pi^{-} .26$
C. Acceptance and Other Corrections

In order to determine final cross sections it was necessary to correct the observed number of events for the acceptance and efficiency of the spectrometer. These corrections included:

1) geometrical acceptance
2) nuclear absorption
3) decays in flight
4) resolution
5) chamber and track finding efficiencies
6) empty target subtraction

The corrections (1-4) were made within the framework of a Monte Carlo simulation of the spectrometer characteristics and are described in detail below. Efficiencies and the empty target data are discussed later.

1. Geometrical acceptance

The geometry of the spectrometer was simulated by generating fake events in the target which were traced through the apparatus. In particular, the reactions $K \leadsto \rightarrow K^{\circ} N$ or $K N \rightarrow K^{\circ} \Delta$ or $K N \rightarrow K^{\circ}+M^{2}$ were simulated using the computer program SAGE ${ }^{27}$. This program used as inputs the incident beam momentum vector, the mass of the $K^{0}$ and recoil particle and returned the $\mathbb{R}^{\circ}$ momentum vector and the momentum transfer to the $\mathbb{R}^{0}$
according to a fixed exponential distribution. SAGE was also used to generate the momentum vectors of the two pions from the $\mathrm{K}^{\mathbf{0}}$ decay. The incident beam direction and momentum were smeared using the known values of the beam momentum and angular resolution. The $X$ and $Y$ points of the primary vertex were generated according to a gaussian distribution using the measured mean and standard deviation for each of the four different beams. The primary $Z$ coordinate was distributed in the target according to the known $\mathrm{K}^{+}$or $\mathrm{K}^{-}$total cross sections. The accepted value of 2.68 cm for the $\mathrm{K}^{\mathrm{o}}$ lifetime was used to determine the secondary vertex.

With a knowledge of the $\mathrm{K}^{\circ}$ decay point and the momentum vectors of each pion, the two pions could be tracked through the spectrometer. The known position and size of the chambers, hodoscopes and the dipole magnet aperture were used to decide if the particle had successfully passed through the spectrometer. The acceptance in $t^{\prime}$ was found to be approximately exponential in shape except at larger $t^{\prime}\left(\geq .3 \mathrm{GeV}^{2}\right)$ where the large distance between the target and the dipole caused a rapid decrease in acceptance. In the forward direction the acceptance was slightly reduced by the presence of the deadening plugs in both the upstream and downstream chambers.

## 2. Absorption

Corrections due to the nuclear absorption of the $\mathrm{K}^{0}$ and the two decay pions in the material of the spectrometer and target were also determined. The known charged kaon and pion cross sections were used to calculate the probability of the interaction of the $\mathrm{K}^{\mathrm{O}}$ and the decay pions. For a $\mathrm{K}^{\mathrm{o}}$ decay outside the target, the probability of losing either pion was approximately $2 \%$. If the $\mathrm{K}^{\mathrm{o}}$ decayed within the target, the average probability of absorption of either pion increased to 7\%.
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The average probability to lose the $\mathrm{K}^{\mathrm{o}}$ in the target was about $5 \%$ corresponding to a mean distance of travel in the $\mathrm{LD}_{2}$ of approximately 30 cm . The total average absorption probability of either the $\mathrm{K}^{0}$ or a pion was found to be 13\%. Although the numbers given above are for $12.8 \mathrm{GeV} / \mathrm{c}$ incident momentum, the results at $8.36 \mathrm{GeV} / \mathrm{c}$ were not greatly different as the kaon and pion total cross sections are approximately constant over the momentum range of interest.
3. Pion decay

The Monte Carlo simulator of the spectrometer also determined the losses from the decay $\pi \rightarrow \mu \nu$. For each pion, the path length before decay was calculated and the direction vector of the $\mu$ found if a decay within the spectrometer was indicated. The muon was then tracked from the decay point through the remainder of the apparatus. The same geometrical cuts were applied to the muon as to pions. For the few decays within a spark chamber package ( $1.5 \%$ of all generated tracks), additional track-finding like cuts were applied. The decay probability per track was found to be about $4 \%$ of which $1.5 \%$ were immediately rejected by the geometrical and track finding cuts. Some of the remaining events were lost if the calculated two pion mass was found to be outside the $\mathrm{K}^{\mathrm{o}}$ limits. Combining the results of the geometrical, absorption and decay corrections, the spectrometer acceptance as a function of $t^{\prime}$ shown in Fig. 32 for the reaction $K N \rightarrow K^{\circ} N$. The acceptance was found to be approximately $55 \%$ and $30 \%$ in the forward direction at 12.8 and $8.36 \mathrm{GeV} / \mathrm{c}$ respectively. The acceptance for $\mathrm{KN} \rightarrow \mathrm{K}^{\circ} \Delta$ was typically $\langle 5 \%$ lower over the same $t$ ' range.

The expected $\mathrm{K}^{\mathrm{o}}$ lifetime distribution was also computed. The result at $12.8 \mathrm{GeV} / \mathrm{c}$ is shown in Fig. 33 (the result at $8.36 \mathrm{GeV} / \mathrm{c}$ is


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Fig. 32. The spectrometer acceptance for $K N \rightarrow K^{\circ} N$ at 8.36 and $12.8 \mathrm{GeV} / \mathrm{c}$.


Fig. 33. The expected $\mathrm{K}^{0}$ lifetime distribution as predicted by the spectrometer simulation program at $12.8 \mathrm{GeV} / \mathrm{c}$.
similar). One notices a decrease below the expected result at small lifetimes. This was primarily caused by an increased absorption probability if the $K^{o}$ decayed early. For such decays, the probability of absorbing either of the decay pions is larger than the probability of absorption of a single $K^{0}$. The slightly decreased geometrical acceptance for short $\mathrm{K}^{\circ}$ path length also made a small contribution to the decrease at small lifetime.
4. Resolution

The angular and position resolution of the spectrometer were simulated by calculating the multiple scattering and measurement errors for each generated event. As the $K^{\circ}$ mass and missing mass limits were chosen to accept all true events with much less than $1 \%$ loss, no correction was made for the resolution broadening of these distributions. However, corrections were computed for the broadening of the $Z$ vertex resolution at small $t^{\prime}$. The $K^{o}$ primary vertex was recalculated using the track and momentum vectors after allowing for resolution smearing. A correction factor was computed corresponding to the percentage of events for a given $t^{\prime}$ interval inside the final target cut ( $-10<2<110 \mathrm{cms}$ ). The correction factor corresponded to about a $30 \%$ loss of events in the extreme forward direction and smaller losses for $\left|t^{\prime}\right| \geqslant .02 \mathrm{GeV}^{2}$.

## 5. Efficiency corrections

The measured efficiencies of the chambers and hodoscopes of the spectrometer were used to determine the overall probability to reconstruct a good event. The important factors were: the efficiency for finding a beam track with a well-defined momentum from the p-hodoscope; the spark chamber and proportional hodoscope efficiencies; the counter hodoscope efficiencies, and the track finding algorithm efficiency. In
totality these factors were substantial and are subject to larger errors than any of the corrections that have been previously discussed.

The number of beam tracks with a well-defined momentum was found by simply determining the percentage of events with both one or more beam tracks and a correct p-hodoscope combination. The results for the different beams are given in Table 4 and account for a $32 \%$ and $23 \%$ loss of events at 8.36 and $12.8 \mathrm{GeV} / \mathrm{c}$ respectively. It should be noticed that the difference between opposite charges for the same incident momentum is small.

The spark chamber efficiencies that had been measured during the course of the experiment were averaged for each of the four different beam configurations. The average values were then used to compute a track finding efficiency for both the upstream and downstream chamber packages. Typical values were $97 \%$ for the upstream and $99 \%$ for the downstream packages.

The proportional hodoscopes were handled in a similar manner and their efficiencies were found to be $94-96 \%$. The overall efficiencies of the scintillation counter hodoscopes HA and HB were determined by weighting the individual counter efficiencies with the spatial distribution of $\mathrm{K}^{\circ}$ events found from the Monte Carlo simulation. Both hodoscopes were found to be efficient ( $\mathrm{H} A>99 \%$ and $\mathrm{HB} \geq 96 \%$ ). Combining all these numbers the overall track finding efficiency was typically 8590\% per track. The efficiency to find two tracks is the square of the single track probability and is given in Table 4.

Possible failures in the track finding algorithms were investigated by visually inspecting a computer reconstruction of a small sample of events. The track finding code was written such that one could "draw
in" obvious tracks between two chamber points and obtain an explanation why the track had not been found. In all cases that were examined, the loss of the track was the result of an insufficient number of points on the track i.e. chamber inefficiency. The data sample was small ( $\sim 100$ events) however, and therefore the absolute determination of the tracking efficiency is subject to some error. The results in Table 4 include a determination of the overall error on reconstructing two tracks in the spectrometer. This was estimated to be approximately $11 \%$ of the probability of finding two tracks.
6. Empty target subtraction

The experimental runs with empty target conditions were analyzed with the identical procedures and cuts as the full target data. For the class of events that are of interest, the empty target rate was $2 \pm 1 \%$ of the full target rate and was independent of $t^{\prime}$. This number was used to correct the final normalization for the full target data.
D. Corrected Absolute Normalization, Relative Normalization and Errors

The $t^{\prime}$ independent efficiency corrections from the previous section were combined to find the corrected absolute normalization. The results for the different beams are given in Table 4. The errors in the normalization shown in Table 4 primarily come from the uncertainty of the chamber and track finding efficiencies. They also include smaller uncertainties from counting the number of incident beam particles and from computing the average target density. The final systematic error was estimated to be about $11.5 \%$.

The reliability of the spectrometer efficiency error estimates was checked by comparing the number of reconstructed $\tau$ decays for $\mathbb{K}^{+}$
-100-
and $\mathrm{K}^{-}$beams at each energy. An inspection of Table 1 will show that the spectrometer efficiency corrections were equal within $1 \%$ for $\mathrm{K}^{+}$ and $\mathrm{K}^{-}$beams at the same energy. One would therefore expect that the ratio of the number of $\mathrm{K}^{+} \tau$ decays $\left(\tau_{+}\right)$to the $\mathrm{K}^{-} \tau$ decays ( $\tau_{-}$) would be close to one after correcting for the difference in the number of incident kaons and the absorptive differences between $\mathrm{K}^{-}$and $\mathrm{K}^{+}$and $\pi^{+}$and $\pi^{-}$. Ignoring the absorptive differences for the moment, the ration $\tau+I_{\tau-}$ vs the $Z$ position of the decay is plotted in Fig. 34 for $8.36 \mathrm{GeV} / \mathrm{c}$ incident momentum after correcting for the different number of incident kaons. The ratio is approximately one in the region before the target and slowly increases until a 15-20\% difference is reached after the target region. The latter is an indication of the expected 20\% difference between the $\mathrm{K}^{+}$and $\mathrm{K}^{-}$total cross sections on deuterium at $8.36 \mathrm{GeV} / \mathrm{c}$. Suming the number of events in the region $-100<\mathrm{Z}<0 \mathrm{cms}$ and making a small correction ( $\sim 1 \%$ ) for the $\pi^{+}$and $\pi^{-}$absorption differences, a ratio of $\tau+\tau_{-}=1.04 \pm .03$ was obtained. A similar analysis at $12.8 \mathrm{GeV} / \mathrm{c}$ yielded a value for $\tau+\tau_{-}$of $1.11 \pm .03$. Although the result for $12.8 \mathrm{GeV} / \mathrm{c}$ is somewhat high, these ratios do lie within the estimated systematic normalization errors. Furthermore, these ratios give a direct measure of the relative $K^{+} / K^{-}$normalization Independent of chamber and track finding efficiencies.

## E. Deuterium Corrections

The final correction to be considered concerns the extraction of free nucleon cross sections from the deuterium data. As a particular example we want to relate the measured cross section for $K^{+}{ }_{d} \rightarrow K^{\circ}{ }_{p p}$ s to the charge exchange (CE) reaction $K^{+} n^{+} K^{\circ} p$. Within the framework of the impulse approximation these two reactions may be related as


Fig. 34. Corrected ratio of the number of $K^{+} \tau$ decays ( $\tau$ ) to $\mathrm{K}^{-} \tau$ decays ( $\tau$ ) vs, the position of the decay at $8.36 \mathrm{GeV} / \mathrm{c}$.
$\frac{d \sigma}{d t^{\prime}}\left(K^{+} d K_{p p_{g}}^{0}\right)=\left\{1-S\left(t^{\prime}\right)\right\}\left(\frac{d \sigma}{d t^{\prime}}\right)_{\text {non-flip }}^{C E}+\left\{1-\frac{l}{3} S\left(t^{\prime}\right)\right\}\left(\frac{d \sigma}{d t^{\prime}}\right)_{\text {flip }}^{C E}$
where $S\left(t^{\prime}\right)$ is the $S$-wave or spherical form factor of the deuteron ${ }^{28}$. This simple expression ignores a number of effects such as the $D$-wave component of the deuteron and double scattering and interference terms as calculated from the Glauber formalism which are known to be small in the $t$ ' region that is covered by the dat $a^{29,30}$. This expression does include the effects of the Pauli Exclusion Principle. At low $t$ ' it is impossible to tell which final state proton was the spectator and which the initial neutron. The proper symmetrization of the two proton final state leads to the equation (1) which supresses the number of events at small t'. The suppression will occur for $t^{\prime}$ small enough such that the recoil proton has a momentum similar to that of the spectator. The spectator momentum distribution, as calculated from the Hulthén wave function, is negligible above $300 \mathrm{MeV} / \mathrm{c}$ and therefore one would expect a substantial suppression only in the region $\left|t^{\prime}\right| \lll \mathrm{GeV}^{2}$.

The deuteron form factor, $S\left(t^{\prime}\right)$, may be computed from the Hulthén wave function as given in Chapter 3 to gield:

$$
\begin{equation*}
S\left(t^{\prime}\right)=\frac{2 \alpha \beta(\alpha+\beta)}{(\beta-\alpha)^{2} \sqrt{\left|t^{\prime}\right|}}\left[\tan ^{-1} \sqrt{\frac{\left|t^{\prime}\right|}{2 \alpha}}+\tan ^{-1} \frac{\sqrt{\left|t^{\prime}\right|}}{2 \beta}-2 \tan ^{-1} \sqrt{\frac{\left|t^{\prime}\right|}{\alpha+\beta}}\right] \tag{2}
\end{equation*}
$$

This function falls rapidly with $t$ and approximately as $e^{-22|t '|}$ for sma11 $\left|t^{\prime}\right|$.

The expression (1) may be rewritten in a more convenient form as

$$
\begin{equation*}
\frac{d \sigma}{d t^{\prime}}\left(K^{+} d+K^{0}{ }_{p p s}\right)=\frac{\left(1-S\left(t^{\prime}\right)\right)+\left(1-\frac{1}{3} S\left(t^{\prime}\right)\right) R\left(t^{\prime}\right)}{1+R\left(t^{\prime}\right)}\left(\frac{d \sigma}{d t^{\prime}}\right)^{C E} \tag{3}
\end{equation*}
$$

where $R\left(t^{\prime}\right)$ is the charge exchange flip/non-flip ratio. The value of $R\left(t^{\prime}\right)$ is, of course, not known except at $t^{\prime}=0$ at which point kinematics requires $R(0)=0$. Since the correction factor is substantial only at small $t^{\prime}$, we have assumed that $R(t)=R(0)=0$. Therefore (3) may be written

$$
\begin{equation*}
\left(\frac{d \sigma}{d t^{\prime}}\right)^{C E} \approx \frac{1}{1-S}\left(t^{\prime}\right) \frac{d \alpha}{d t^{\prime}}\left(R^{+} d^{d}+R_{p p s}^{o}\right) \tag{4}
\end{equation*}
$$

The value of ( $1 / 1-\mathrm{S}\left(\mathrm{t}^{\prime}\right)$ ) is shown in Fig. 35 and was used to correct the measured $\mathrm{K}^{+} \mathrm{d}+\mathrm{K}^{0} \mathrm{pp}_{s}$ and $\mathrm{K}^{-} \mathrm{d}^{+}+\mathrm{K}^{0} \mathrm{nn}_{s}$ differential cross sections.


Fig. 35. A plot of the deuteron correction factor $1 /\left(1-5\left(t^{\prime}\right)\right)$.

## CHAPTER 6. RESULTS AND CONCLUSIONS

## A. Introduction

The raw number of nucleon and delta events obtained from the fitting procedure described in Chapter 4 and the normalization and corrections from Chapter 5 are combined in this chapter to yield differential and total cross sections. The cross sections are discussed and compared with data from previous experiments in part $B$ of this chapter. The kaon charge exchange data are compared to pion charge exchange via $\mathrm{SU}(3)$ sum rules in part C. Section D contains a detailed comparison of the energy, momentum transfex and missing mass dependence of the $K^{+}$and $K^{-}$reactions. A brief comparison with other sets of line reversed reactions, particularly hypercharge exchange data, is also made in section D. Finally, a few qualitative comments about the applicability of various Regge models to line reversal breaking and exchange degeneracy will be made.
B. Total and Differential Cross Sections

1. The reactions $K^{+}{ }_{n} \rightarrow K_{p}^{o}$ and $K^{-} p \rightarrow \bar{K}_{n}^{o}$

Table 5 contains the final corrected results for the processes
 errors do not include the normalization or efficiency correction uncertainties, and therefore reflect only the fitting and statistical errors on the number of events for a given $t^{\prime}$ interval. The total cross sections result from an integration over the interval $0 \leq\left|t^{\prime}\right| \leq 1.0 \mathrm{GeV}^{2}$ and do include the systematic errors.
a. $\mathrm{K}^{-} \mathrm{p} \rightarrow \overline{\mathrm{K}}^{\mathrm{O}} \mathrm{n}$ total cross sections

The total cross sections as measured in previous experiments and the results from this experiment for the $\mathrm{K}^{-1} \mathrm{p} \rightarrow \bar{K}^{\circ} \mathrm{n}$ total cross section in the range $3<\mathcal{P}_{L A B}<34.6 \mathrm{GeV} / \mathrm{c}$ are given in Table 6.32 The data have not been
corrected for different $t^{\prime}$ Ifmits. In general, however, such corrections would be within the measurement errors. The total cross section values are plotted in Fig. 36 and show that the results of this experiment are in good agreement with previous data. A least squares fit to the data with the form $A P_{L A B}^{-n}$ has also been done. The values $A=3089 \pm 185 \mu b$ and $n=1.74$ $\pm .04$ were found to give the best fit with a $\chi^{2}$ of 39 for the 25 data points. Total cross sections in Regge pole exchange models are predicted to have an energy dependence of the form $P_{L A B}{ }^{2<\alpha>-2}$ where $<\alpha>$ is the average value of the trajectory of the exchange. For $\rho$ or $A_{2}$ exchange, $\langle\alpha\rangle \tilde{\sim}, 2$, which leads to a $\sim_{P_{L A B}}-1.6$ energy dependence in approximate agreement with the observed value.

## b. $K^{+}{ }_{n \rightarrow R^{\circ}}$ p total cross sections

Most of the existing worlds data for the range $3<\mathrm{P}_{\mathrm{LAB}}<12.8 \mathrm{GeV} / \mathrm{c}$ for $\mathrm{K}^{+}$charge exchange are summarized in Table 7 and plotted in Fig. 37. ${ }^{33}$ The $12.8 \mathrm{GeV} / \mathrm{c}$ result 1 s the highest energy point so far available. These data have also been fit to an $A P_{L A B}^{-n}$ dependence and the values $A=5922 \pm 624 \mu \mathrm{~b}$ and $n=1.95 \pm .07$ with a $x^{2}$ of 24 for 11 data points have been obtained. Comparing this value for $n$ with that from $\mathrm{K}^{-}$charge exchange would tend to indicate that the cross sections are converging as $P_{\text {LAB }}$ increases and would intersect at about $22 \mathrm{GeV} / \mathrm{c}$. However, as will be shown in section D , the uncertainty of the relative normalization of the various experiments probably makes this conclusion incorrect.

## c. $\quad \mathrm{K}^{-} \mathrm{p} \rightarrow \overline{\mathrm{K}}^{\mathbf{0}} \mathrm{n}$ differential cross sections

The differential cross sections for $\mathrm{K}^{-}$charge exchange are plotted in Figs. 38 a and 38 b for 8.36 and $12.8 \mathrm{GeV} / \mathrm{c}$ respectively. Both the magnitude and shape of the result at $12.8 \mathrm{GeV} / \mathrm{c}$ are in excellent agreement with a recent high statistics experiment at $13.0 \mathrm{GeV} / \mathrm{c} .^{32}$ Although the

table 7

| $\underline{K}^{+}{ }_{\underline{n}+\mathrm{K}^{0}{ }_{p}}$ |  |
| :---: | :---: |
| $\mathrm{P}_{\text {LAB }}(\mathrm{GeV} / \mathrm{c})$ | $\sigma_{\text {tot }}(\mu \mathrm{b})$ |
| 3.0 | $697 \pm 54$ |
| 3.8 | $567 \pm 41$ |
| 4.0 | $403 \pm 20$ |
| 4.6 | $230 \pm 29$ |
| 5.5 | $173 \pm 20$ |
| 6.0 | $188 \pm 10$ |
| 8.36 | $98 \pm 14$ |
| 12.0 | $45 \pm 5$ |
| 12.8 | $50 \pm 6$ |

Fig. 36. $\sigma_{\text {TOT }}\left(\bar{K}^{-} \mathrm{p} \rightarrow \overline{\mathrm{R}}^{\mathrm{O}} \mathrm{n}\right)$ vs. 1aboratory momentum.


Fig. 37. $\sigma_{\mathrm{TOT}}\left(\mathrm{K}_{\mathrm{n}}^{+} \rightarrow \mathrm{K}_{\mathrm{p}}^{\mathrm{o}}\right)$ vs. laboratory momentum.


Fig. 38. Differential cross sections for $K^{+} p \rightarrow \bar{K}_{n}^{\circ}$ and $X^{+}{ }_{n} \rightarrow K^{o}$.
errors are large at small $\left|t^{\prime}\right|$, both cross sections definitely show a change of slope or turnover in the region $\left|t^{\prime}\right|<.05 \mathrm{Gev}^{2}$, which is indicative of the presence of a substantial spin flip amplitude. A comparison with lower (or higher) energy data shows no obvious change in the magnitude of the forward turnover.

The data in the region. $12<\left|\mathrm{t}^{\prime}\right|<1.0 \mathrm{GeV}^{2}$ have been fit with the form $\mathrm{Ae}^{\mathrm{Bt}}{ }^{\prime}$ and the resuits are given in Table 8. The behaviour of the cross section in this region is well fit by an exponential and shows no indication of structure. There is no evidence for a flattening of the distribution at large $|t '|$ within the errors of the measurement. A comparison of the slopes obtained at 8.36 and $12.8 \mathrm{GeV} / \mathrm{c}$ shows definite indications of shrinkage (see Fig. 39a).
d. $K^{+}{ }^{n}+K_{p}^{0}$ differential cross sections

The $t^{\prime}$ distributions for $\mathrm{K}^{+}$charge exchange at 8.36 and $12.8 \mathrm{GeV} / \mathrm{c}$ are as shown in Figs. 38a and 38b. These results also show a change in slope or turnover for $\left|t^{\prime}\right|_{\lesssim} .05 \mathrm{Gev}^{2}$ of approximately the same magnitude as in $\mathrm{K}^{-}$charge exchange, again implying the presence of a significant spin flip amplitude. Fits to an exponential in the region $.12<\left|t^{\prime}\right|<1.0 \mathrm{GeV}^{2}$ have been done and the resuits are given in Table 8. Again shrinkage is observed and the slope values are consistent with the trend of previous data as is shown in Fig. 39b. The slopes are also consistent with being the same as those for the $K^{-}$charge exchange reaction at both energies. 2. The reactions $K^{+} n^{+} K^{0} \Delta$ and $K_{n}^{-}+\bar{K}^{0} \Delta$

The data for the reactions $K^{+} n+K^{\circ} \Delta$ and $K^{-} n+\bar{K}^{0} \Delta$ are given in Table 9 where $K^{0} \Delta$ and $\bar{K}^{\circ} \Delta$ represent the total signal obtained from the $K^{+}$and $\mathbb{K}^{-}$ interactions respectively. Once again the error for a particular $t^{\prime}$ interval only includes the statistical and fitting uncertainties whereas the

Table 9

|  | $\mathrm{d} / \mathrm{dt}^{\prime}\left(\mu \mathrm{b} / \mathrm{Gev}^{2}\right)$ |  | $8.36 \mathrm{GeV} / \mathrm{c}$ |  |  | $\mathrm{d} / \mathrm{dtt}{ }^{\prime}\left(\mu \mathrm{b} / \mathrm{Gev}^{2}\right)$ |  | $12.8 \mathrm{GeV} / \mathrm{c}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\left\|\mathrm{t}^{\prime}\right\|\left(\mathrm{GeV}^{2}\right)$ | $\mathrm{K}^{-1}+\overline{\mathrm{K}}^{\text {o }}$, | $\mathrm{K}^{-} \mathrm{n}+\overline{\mathrm{K}}^{\Delta^{-}}$ | $\mathrm{K}^{+} \mathrm{d} \rightarrow \mathrm{K}^{\circ} \mathrm{\Delta}$ | $\mathrm{K}^{+}{ }_{\mathrm{P}} \rightarrow \mathrm{K}^{\text {O }} \Delta^{++}$ | $\|\mathrm{t} \cdot\|\left(\mathrm{GeV}{ }^{2}\right)$ | $\mathrm{K}^{-} \mathrm{d}+\overline{\mathrm{R}}^{\circ} \Delta$ | $\mathrm{K}^{-} \mathrm{n}+\overline{\mathrm{K}}^{\circ} \Delta^{-}$ | $\mathrm{K}^{+} \mathrm{d} \rightarrow \mathrm{K}^{\circ} \Delta$ | $\mathrm{K}^{+}{ }_{\mathrm{P}}+\mathrm{K}^{0} \Delta^{++}$ |
| 0-. 04 | $192 \pm 68$ | $144 \pm 51$ | $191 \pm 76$ | $144 \pm 57$ | 0-. 02 | $41 \pm 35$ | $31 \pm 26$ | $84 \pm 40$ | $63 \pm 30$ |
| . $04-.08$ | $453 \pm 58$ | $340 \pm 40$ | $439 \pm 69$ | $329 \pm 52$ | . $02-.04$ | $100 \pm 25$ | $75 \pm 19$ | $109 \pm 28$ | $82 \pm 21$ |
| . 08 - . 12 | $473 \pm 65$ | $355 \pm 49$ | $490 \pm 58$ | $367 \pm 44$ | . $04-.08$ | $147 \pm 15$ | $110 \pm 11$ | $208 \pm 18$ | $156 \pm 14$ |
| . 12 - . 16 | $362 \pm 56$ | $271 \pm 42$ | $495 \pm 72$ | $372 \pm 54$ | . $08-.12$ | $162 \pm 14$ | $122 \pm 11$ | $226 \pm 17$ | $169 \pm 12$ |
| . 16 - . 22 | $297 \pm 55$ | $202 \pm 41$ | $342 \pm 56$ | $257 \pm 42$ | . 12 - . 16 | $137 \pm 14$ | $103 \pm 10$ | $238 \pm 16$ | $179 \pm 12$ |
| . 22 - . 30 | $194 \pm 43$ | $146 \pm 33$ | $236 \pm 41$ | $177 \pm 31$ | . 16 - . 22 | $98 \pm 9$ | $74 \pm 7$ | $164 \pm 10$ | $123 \pm 8$ |
| . $30-.50$ | $130 \pm 32$ | $98 \pm 24$ | $100 \pm 27$ | $75 \pm 20$ | . 22 - . 30 | $61 \pm 7$ | $46 \pm 5$ | $89 \pm 8$ | $67 \pm 6$ |
| . $50-1.0$ | $25 \pm 25$ | $18 \pm 18$ | $23 \pm 23$ | $17 \pm 17$ | . $30-.50$ | $34 \pm 4$ | $26 \pm 3$ | $60 \pm 4$ | $45 \pm 3$ |
|  |  |  |  |  | . $50-1.0$ | $9 \pm 2$ | $7 \pm 1$ | $14 \pm 2$ | $11 \pm 1$ |
| $\sigma_{\text {tot }}(\mu \mathrm{b})$ | $131 \pm 21$ | $97 \pm 16$ | $136 \pm 21$ | $102 \pm 16$ |  | $43 \pm 5$ | $32 \pm 4$ | $67 \pm 8$ | $50 \pm 6$ |
| $0<\left\|\mathrm{t}^{\prime}\right\| \leq 1 \mathrm{Gev}^{2}$ |  |  |  |  |  |  |  |  |  |


total cross section error also contains the systematic uncertainties. An error that has not been included involves the assuaptions that were made regarding the background under the $\Delta$ signal. Although it has been demonstrated that the parametrization of this background that has been used yields consistent and good fits to the data, it should be remembered that there may be an additional, systematic error in the assumptions that have been made. This error is estimated to be approximately $15 \%$.

Isospin conservation has been assumed to extract the individual cross sections from the total $\mathrm{K}^{0} \Delta$ and $\overline{\mathrm{K}}^{\mathrm{O}} \Delta$ data. The reactions $\mathrm{K}+\mathrm{N}+\mathrm{K}+\Delta$ are pure $I=1$ in the $s-c h a n n e l$ for allowed charge states and therefore:

$$
\begin{aligned}
& \sigma\left(\mathrm{K}_{\mathrm{p}}^{-} \rightarrow \overline{\mathrm{K}}^{\mathrm{o}} \Delta^{\mathrm{o}}\right)=\frac{1}{3} \sigma\left(\mathrm{~K}_{\left.\mathrm{n}+\overline{\mathrm{K}}^{\mathrm{O}} \Delta^{-}\right)}\right. \\
& \sigma\left(\mathrm{K}^{+} \mathrm{n}_{\mathrm{n}} \mathrm{~K}^{\mathrm{o}} \Delta^{+}\right)=\frac{1}{3} \sigma\left(\mathrm{R}^{+}{ }_{\mathrm{p} \rightarrow \mathrm{R}^{\mathrm{o}} \Delta^{++}}\right)
\end{aligned}
$$

Thus we have defined:

$$
\begin{align*}
& \mathrm{K}^{+}{ }_{\mathrm{p} \rightarrow \mathrm{~K}^{\circ} \Delta^{++}}=\frac{3}{4} \mathrm{~K}^{+}{ }_{\mathrm{d}+\mathrm{K}^{\circ}}{ }_{\Delta}  \tag{1}\\
& \mathrm{K}^{-} \mathrm{n}+\overline{\mathrm{K}}^{\circ} \Delta^{-} \equiv \frac{3}{4} \mathrm{~K}^{-} \mathrm{d}+\overline{\mathrm{K}}^{\mathrm{O}} \Delta  \tag{2}\\
& K^{+}{ }_{n \rightarrow K^{0}} \Delta^{+} \equiv \frac{1}{4} K^{+} d \rightarrow K^{0} \Delta  \tag{3}\\
& K^{-} \mathrm{p}+\overline{\mathrm{K}}^{\circ} \Delta^{\circ} \equiv \frac{1}{4} \mathrm{~K}^{-} \mathrm{d} \rightarrow \overline{\mathrm{R}}^{\circ} \Delta \tag{4}
\end{align*}
$$

The data for reactions (1) and (2) are also included in Table 9.

## a. Total cross sections

Before examining each individual cross section, a brief discussion of the difficulties associated with obtaining a standard $\Delta$ result is necessary. The diversity of parametrizations and cuts associated with $\Delta$ reactions makes standardization very difficult. In general results from previous experiments have been lowered by $10-15 \%$ to agree with the cuts used in this experiment. Such data points are indicated by open circles. When
a correction has been impossible to compute, open triangles have been used to plot the data.

## 1. $\mathrm{K}^{+}{ }_{\mathrm{p} \rightarrow \mathrm{K}^{\mathrm{o}} \Delta^{++}}$

This reaction has been extensively studied in the energy range
$3<\mathrm{P}_{\mathrm{LAB}}<16 \mathrm{GeV} / \mathrm{c}$. A representative sample of data has been plotted in Fig. $400^{34}$ The results from this experiment are in reasonable agreement with the trend of previous data. A fit to the form $A P_{\text {LAB }}{ }^{-n}$ yields values of $3915 \pm 552 \mu \mathrm{~b}$ for $A$ and $1.68 \pm .07$ for $n$ with $a X^{2}$ of 5.8 for 12 data points.
2. $\mathrm{K}^{-} \mathrm{n}^{+}+\mathrm{K}^{0} \Delta^{-}$

Prior to this experiment, this reaction had been studied only up to $P_{\text {LAB }}=6 \mathrm{GeV} / \mathrm{c}^{35}$ Both the available corrected and uncorrected data have been plotted in Fig. 41 for $3<P_{\text {LAB }}<12.8 \mathrm{GeV} / \mathrm{c}$. All data points have been used to fit the $P_{\text {LAB }}$ dependence and the results $A=2213 \pm 428 \mu \mathrm{~b}$ and $n=1.66 \pm .11$ with $x^{2}=14.1$ for 7 data points were obtained. This value for $n$ would indicate an energy dependence similar to that for $K^{+}{ }_{p} \rightarrow K^{0} \Delta^{++}$.

$$
\text { 3. } K-p+\overline{\mathbb{K}}^{\circ} \Delta^{\circ}
$$

The data for this reaction have been plotted in Fig . 42 for $3<\mathrm{P}_{\mathrm{LAB}}$ $<15.7 \mathrm{GeV} / \mathrm{c} .{ }^{36}$ The cross sections of this experiment are in fair agreement with previous data, and the energy dependence has been determined using all data points. A value of $2013 \pm 573$ for $A$ and $2.0 \pm .14$ for $n$ were obtained.

## b. Differential cross sections

Only the differential cross sections for $\mathrm{K}^{+} \mathrm{p} \rightarrow \mathrm{K}^{0} \Delta^{++}$and $\mathrm{K}^{-} \mathrm{n}^{-} \overline{\mathrm{K}}^{-0} \Delta^{-}$will be discussed. The assumption of isospin conservation that has been made makes an examination of the $t^{\prime}$ shapes of $\mathrm{K}^{-} \mathrm{p} \rightarrow \overline{\mathrm{K}}^{0} \Delta^{0}$, or $\mathrm{K}^{+} \mathrm{n}^{\prime} \mathrm{K}^{0} \Delta^{+}$redundant.

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Fig. 41. $\sigma_{T O T}\left(\mathrm{~K}^{-} \mathrm{n} \rightarrow \overline{\mathrm{K}}^{\mathbf{0}} \Delta^{-}\right)$vs. $\mathrm{P}_{\mathrm{LAB}}$.
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Fig. 42. $\sigma_{T O T}\left(\bar{K}^{-}{ }^{-} \rightarrow \bar{K}^{\circ} \Delta^{\circ}\right)$ vs. $P_{\text {LAB }}$.

## 1. $\mathrm{K}^{+}{ }_{\mathrm{p}+\mathrm{K}^{\mathrm{o}} \Delta^{++}}$

The values of do/dt' for this reaction at 8.36 and $12.8 \mathrm{GeV} / \mathrm{c}$ are plotted in Figs. 43a and 43c. The data at both momenta show a substantial forward turnover possibly indicative of a relatively larger spin filp contribution than in the $\mathrm{K}^{+} \mathrm{n}^{\prime} \mathrm{K}^{0} \mathrm{p}$ or $\mathrm{K}^{-} \mathrm{p} \rightarrow \overline{\mathrm{K}}_{\mathrm{n}}^{\mathrm{o}}$ reactions. In this regard, it should be remembered that double flip as well as single flip amplitudes are allowed for the $\Delta$ reactions. ${ }^{37}$ The shapes of the distributions also show some regular features, a maximum in the region $\left|\mathrm{t}^{\prime}\right| \approx, 1 \mathrm{GeV}^{2}$ and a forward dip to about $1 / 3$ the value of the maximum for $\left|t^{\prime}\right| i_{n}^{n} .01-.02 \mathrm{GeV}^{2}$. Lower energy data at 4 and $6 \mathrm{GeV} / \mathrm{c}$ are in excellent agreement with these qualitative features. High statistics data at $13 \mathrm{GeV} / \mathrm{c}$ also agree with the position of the maximum but do not show as substantial a forward dip. ${ }^{32}$

The data at both energies have been fit to the form $A e^{B t^{\prime}}$ for $.12<\left|t^{\prime}\right|$ $<1.0 \mathrm{GeV}^{2}$ and the results are given in Table 8. Although shrinkage is not indicated, the substantial errors at $8.36 \mathrm{GeV} / \mathrm{c}$ make definite conclusions difficult. The fitted slopes from this experiment together with previous data are plotted in Fig. 44a.
2. $\bar{K}^{-} \mathrm{n}^{-} \overline{\mathrm{K}}^{\mathrm{O}} \Delta^{-}$

The differential cross sections for this reaction are shown in Figs. 43 b and 43 d at 8.36 and $12.8 \mathrm{GeV} / \mathrm{c}$ respectively. Essentially the same qualitative features as in the $K^{+}{ }_{p}+\mathrm{K}^{\mathrm{o}} \Delta^{++}$data may be seen in these results; there is a substantial forward dip and a maximum at $\left|t^{\prime}\right| \approx \cdot 1 \mathrm{GeV}^{2}$. These data also agree with the behaviour seen at lower energies.

The results of fitting to an exponential form are given in Table 8. In this case, shrinkage is observed and the slope values are consistent with an extrapolation of lower energy data (see Fig. 44b). The $t$ ' dependence may also be seen to be approximately the same as for $\mathrm{K}^{+} \mathrm{p}^{\rightarrow} \mathrm{K}^{0} \Delta^{++}$.


Fig. 43. Differential cross sections for $K^{+} p \rightarrow K^{0} \Delta^{++}$and $K^{-} n \rightarrow \bar{K}^{0} \Delta^{-}$.
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Fig. 44. Exponential sfope parameters for $|\mathrm{t}| \geqslant .1 \mathrm{GeV}^{2}$. a) $\mathrm{K}^{+} \mathrm{p} \rightarrow \mathrm{K}_{\Delta}^{\mathrm{o}} \mathrm{Cl}^{+}$; b) $\mathrm{K}^{-} \mathrm{n} \rightarrow \overline{\mathrm{K}}^{\mathrm{o}} \Delta^{-}$.
C. Symmetry Relations

1. Vertex relations

It is possible to relate the differential cross sections for $K N+K^{\circ} N$ scattering to the $K N \rightarrow \mathbb{K}^{0} \Delta$ reactions. In particular if one assumes $S U(6)_{w}$ symmetry then the $\rho N \bar{N}$ and $\rho N \bar{\Delta}$ vertices may be related to yield: ${ }^{38}$

$$
\begin{align*}
& \frac{\left(d \sigma / d t^{\prime}\right)\left(K^{+} p+K^{o} \Delta^{++}\right)}{\left.\left(d \sigma / d t^{\prime}\right)\left(K^{+} n+K^{0} p\right)\right|_{f l i p}}=C  \tag{5}\\
& \frac{\left(d \sigma / d t^{\prime}\right)\left(K^{-} n \rightarrow \bar{K}^{o} \Delta^{-}\right)}{\left.\left(d \sigma / d t^{\prime}\right)\left(K_{p}^{+}+\bar{K}^{o} n\right)\right|_{f l i p}}=C \tag{6}
\end{align*}
$$

The $\operatorname{SU}(6)_{w}$ value for $C$ is $24 / 25=.96$. An alternative viewpoint uses $\gamma \overline{N N}$ and $\gamma N \Delta^{-}$couplings together with vector meson dominance and finds a velue of $C=1.5 .^{39}$ It is important to remember that only the flip part of the charge exchange cross section enters into the ratio. Although one expects the $f 1 i p$ amplitude to dominate the charge exchange cross section, the nonvanishing polarization in $K^{-}$charge exchange, and various attempts at amplitude analysis, would indicate that the non-filp contribution to the cross section is not zero. ${ }^{40,41}$ one would not, therefore, expect the relations (5) and (6) to be valid, especially at small $t$ ' where the non-flip amplitude may become important.

The predictions have been tested using the measured charge exchange cross section since the flip part is unknown. The results from this experiment are shown in Fig. 45. The behaviour is substantially the same at both energies. The ratio is small for $\left|t^{\prime}\right|_{i} .05-.1 \mathrm{GeV}^{2}$ and increases to a constant value that iles within the range of either the $\operatorname{SU}(6)_{w}$ or vector dominance predictions.


Fig. 45. Tests of $\operatorname{SU}(6)_{w}$ relation between $K N \rightarrow K^{\circ} N$ and $K N \rightarrow K^{\circ} \Delta$.
2. SU(3) sum rules

The $\rho$ and $A_{2}$ exchanges in kaon charge exchange may be related to $\rho$ exchange in $\pi^{-} p^{\prime}+\pi^{0} n$ and to $A_{2}$ exchange in $\pi^{-} p+n n$ by $\operatorname{SU}(3)$ symmetry. ${ }^{42}$ In particular, an $\operatorname{SU}(3)$ consideration of the $\rho \pi \pi, \rho K \bar{K}, A_{2} \eta \pi$ and $A_{2} K \bar{K}$ couplings, leads to the following sum rules:

$$
\begin{equation*}
\left(\frac{d \sigma}{d t},\right)\left(K^{+}{ }_{n \rightarrow K^{0}}^{p}\right)+\left(\frac{d \sigma}{d t},\right)\left(K^{-} p+\bar{K}^{0} n\right)=\left(\frac{d \sigma}{d t},\right)\left(\pi^{-} p \rightarrow \pi^{0} n\right)+\gamma\left(\frac{d \sigma}{d t}\right)\left(\pi^{-} p \rightarrow \eta n\right) \tag{7}
\end{equation*}
$$


The value of the constant $\gamma$ in (7) and (8) is 3 if the $\eta$ is pure SU(3) octet. For $\gamma=3$, the sum rules have been tested using the results of this experiment and the data cited in Ref. 43. The agreement between the LHS and RHS of (7) is very good at $12.8 \mathrm{GeV} / \mathrm{c}$ (Fig. 46 b ). At $8.36 \mathrm{GeV} / \mathrm{c}$, the agreement is also reasonable except for $\left|t^{\prime}\right|<.1 \mathrm{GeV}^{2}$ where the LHS is larger (Fig. 46a). The sum rule (8) has been tested at $12.8 \mathrm{GeV} / \mathrm{c}$ (Fig. 47). The sum rule is approximately satisfied except for $\left|t^{\prime}\right|<.1$ $\mathrm{GeV}^{2}$ where the RHS is larger. The availability of higher statistics $\pi^{0} \Delta^{++}$and $\Pi \Delta^{++}$data would allow a better comparison to be made.

The experimental and theoretical difficulties of the sum rule comparisons are threefold. First, the uncertainties in the relative normalization between two (or three) different experiments make precise comparisons difficult. Second, the sum rules have been derived assuming pure pole exchange which may be an invalid assumption. Finally, in an analysis of the available data on $2^{+} \rightarrow 0^{-} 0^{-}$decays, a pseudoscalar mixing angle of about $-10^{\circ}$ was obtained. 44 This would suggest that the value of the constant $\gamma$ in (7) and (8) should be 1.8-2 rather than $3 .{ }^{45}$ If this is the case then the sum rule predictions are badly violated and the agreement for $\gamma=3$ may be considered to be a fortunate accident.


Fig. 46. Tests of Barger-C1ine $\operatorname{SU}(3)$ sum rule at $8.36 \mathrm{a} . \mathrm{d} 12.8 \mathrm{GeV} / \mathrm{c}$.


Fig. 47. $\mathrm{SU}(3)$ sum rule for $\Delta$ reactions at $12.8 \mathrm{GeV} / \mathrm{c}$.

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D. A Comparibon of the $\mathrm{K}^{+}$and $\mathrm{K}^{-}$Induced Reactions

Before discussing the differences between the $K^{+}$and $K^{-}$reactions,
a few words about relative normalization errors are in order. In comparing, for example, the $K^{+}{ }_{n} \rightarrow \mathrm{~K}^{0} \mathrm{p}$ and $\mathrm{K}^{-} \mathrm{p}^{-} \overline{\mathrm{K}}^{-0} \mathrm{n}$ total cross sections, the full systematic errors have not been used in calculating the error on the ratio of the two cross sections. Instead a relative normalization error has been added in quadrature with the statistical and fit
uncertainties to compute the final error. We estimate that the relative normalization error is at most $4 \%$ and we therefore have used this value when comparing total cross sections.

1. $\mathrm{K}^{+} \mathrm{n}_{\mathrm{K}} \mathrm{O}_{\mathrm{n}}$ and $\mathrm{K}^{-} \mathrm{p} \rightarrow \mathrm{K}^{-} \mathrm{n}$

In addition to predicting the equality of these two cross sections, exact exchange degeneracy implies that the amplitude for $\mathrm{K}^{+} \mathrm{n}_{\mathrm{n}} \rightarrow \mathrm{K}_{\mathrm{p}}{ }_{\mathrm{p}}$ is purely real and that for $K^{-} p \rightarrow \bar{R}_{n}^{0}$ has a phase $\sim e^{-1 \pi \alpha}$. This may be approximately tested by comparing the forward cross section obtained in this experiment with that expected from the imaginary part of the amplitude obtained via the optical theorem and isospin conservation. In particular we have

$$
\begin{equation*}
\left\{\operatorname { I m } \left(\mathrm{K}_{\left.\left.\mathrm{n} \rightarrow \mathrm{~K}_{\mathrm{p}}^{0}\right)\right\}_{\mathrm{t}=0}=\frac{1}{16 \pi}\left\{\sigma_{\mathrm{TOT}}\left(\mathrm{~K}_{\mathrm{p}}^{+}\right)-\sigma_{\mathrm{TOT}}\left(\mathrm{~K}_{\mathrm{n}}^{+}\right)\right\}^{2}, ~}^{2}\right.\right. \tag{9}
\end{equation*}
$$

and for $\bar{K}^{-} \mathrm{p}+\mathrm{K}^{-} \mathrm{n}$

$$
\begin{equation*}
\left\{\operatorname{Im}\left(K^{-} p+K^{-0} n\right)\right\}_{t=0}^{2}=\frac{1}{16 \pi}\left\{\sigma_{T O T}\left(K_{n}\right)-\sigma_{T O T}\left(K^{-}\right)\right\}^{2} \tag{10}
\end{equation*}
$$

The extrapolated forward cross sections from this experiment and the RHS of (9) and (10) are plotted in Fig. 48. As may be seen, the RHS of (9) is consistent with a value of zero which implies that the


Fig. 48. da/dt' at $t^{\prime}=0$ for $K^{+} n \rightarrow K^{\circ} p$ and the contribution of the imaginary parts obtained from the total cross section measurements.
$K^{+}{ }_{n \rightarrow K^{\circ}}{ }_{p}$ amplitude is purely real (at $t^{\prime}=0$ ) as predicted by exchange degeneracy. The amplitude for $\bar{K}^{-} \mathrm{p}^{-\mathrm{K}^{-0}} \mathrm{n}$, is seen to be almost purely imaginary in the forward direction, as would be expected for a degenerate $\rho-A_{2}$ trajectory with an intercept of 0.5 .
 total cross section values given in Table 5 . The values $R_{1}=1.37 \pm .22$ at $8.36 \mathrm{GeV} / \mathrm{c}$ and $\mathrm{R}_{1}=1.38 \pm .09$ at $12.8 \mathrm{GeV} / \mathrm{c}$ have been determined. These results are in obvious contradiction to the predictions of a weak exchange degenerate Regge models. The ratio $R_{1}$ from this experiment and the previous data at 3,4 , and $6 \mathrm{GeV} / \mathrm{c}$ have been plotted in Fig. 49.46 These results strongly suggest that $\mathrm{R}_{1}$ is constant at a value of approximately 1.3 for $3<\mathrm{P}_{\mathrm{LAB}}<13 \mathrm{GeV} / \mathrm{c}$. An alternative way of presenting this result is shown in Fig. 50 where the total cross sections for each reaction has been $p l o t t e d$. The total cross sections for both $K^{+} n_{n}+K_{p}$ and $\mathrm{K}^{-} \mathrm{p}+\mathrm{K}^{\mathrm{O}} \mathrm{n}$ were fit by an $\mathrm{AP}_{\mathrm{LAB}} \dot{\mathrm{n}}^{\mathrm{n}}$ form. A value of $1.85 \pm .08$ for n was obtained in each case, which also indicates that the $K^{+}{ }_{n \rightarrow K^{\circ}}{ }^{0}$ and $\mathrm{K}^{-} \mathrm{p} \rightarrow \overrightarrow{\mathrm{K}}^{\circ} \mathrm{n}$ reactions have essentially the same energy dependence.

The behaviour described above not only rules out the simple weak exchange degenerate pure pole models but may also constrain some of the more common additions to pure leading pole exchange models. The exchange of secondary trajectories, for example, has been suggested as a possible cause of the breaking of line reversal equality. ${ }^{47}$ The difference in energy dependence, however, of such exchanges is predicted to be about $P_{\text {LAB }}{ }^{-2}$ and the interference between leading and secondary exchanges to be about $P_{\text {LAB }}{ }^{-3 / 2}$. 48 Such rapid energy behaviour would appear to exclude the exchange of secondary trajectories as the sole source of the observed line reversal breaking.


Fig. 49. The ratio $\left\{\sigma_{\mathrm{TOT}}\left(\mathrm{K}^{+} \mathrm{n}_{\mathrm{n}} \rightarrow \mathrm{K}_{\mathrm{p}}^{\mathrm{o}}\right) / \sigma_{\mathrm{TOT}}\left(\mathrm{K}_{\mathrm{p}}^{-} \rightarrow \overline{\mathrm{K}}_{\mathrm{n}}^{\mathrm{n}}\right)\right\}$ at $3,4,5,8.36$ and $12.8 \mathrm{GeV} / \mathrm{c}$ for $0<\left|\mathrm{t}^{\prime}\right|$ § $1 . \mathrm{O}^{\mathrm{GeV}}{ }^{2}$.


Fig. 50. The energy dependence of $\sigma_{T O T}\left(K_{n}^{+} \rightarrow K_{p}^{0}\right)$ and $\sigma_{T O T}\left(X_{p}^{-}+\widetilde{R}_{n}{ }_{n}\right)$ as determined by experiments with good relative normalization.

It is also possible to introduce a small difference between the $\rho$ and $A_{2}$ trajectories to account for line reversal breaking. 48 If one assumes that the effective trajectories extracted from $\pi^{-} p+\pi^{\circ} n$ and $\pi^{-} p+\eta n$ data actually correspond to the $\rho$ and $A_{2}$ trajectories respectively, then $\Delta(t)=\alpha_{\rho}(t)-\alpha_{\rho_{2}}(t) \approx .11$ at $t=0.49$ For pure pole exchange it is then possible to write

$$
\begin{equation*}
\frac{\sigma_{+}\left(K^{+} n \rightarrow K^{o} p\right)-\sigma_{-}\left(K^{-} p \rightarrow \bar{K}^{\circ} n\right)}{\sigma_{+}\left(K^{+} n \rightarrow K^{0} p\right)+\sigma_{-}\left(K^{-} p \rightarrow \bar{K}^{0} n\right)}=\frac{1}{f(t) s^{\Delta(t)}+g(t) s^{-\Delta(t)}} \tag{11}
\end{equation*}
$$

This suggests, at fixed $t$ and large $s$, that the two cross sections are coming together as $\sim_{S}^{-\Delta(t)}$. If this is the case, then a value of $\sigma_{+} / \sigma_{-}=1.35$ at $3 \mathrm{GeV} / \mathrm{c}$ would imply a value of $\sigma_{+} / \sigma_{-} \mathcal{i} 1.3 \mathrm{at} 13 \mathrm{GeV} / \mathrm{c}$. We therefore conclude that from energy dependence alone, it is not possible to rule out exchange degeneracy breaking of this form as a cause of line reversal inequality.

Regge models with cut corrections (absorptive corrections) to pure pole exchange have also been offered as an explanation for line reversal breaking. 50 A large number of different parametrizations of the absorptive corrections have been attempted but quantitative predictions of the magnitude of line reversal breaking are in general difficult to obtain from the existing literature. It has been claimed, however, that all models which start with exchange degenerate poles will require that the ratio $\sigma_{+} / \sigma_{-}$approach unity as $s$ increases. 51 on the other hand, models which break exchange degeneracy for the pole terms may be able to account for the observed amount of line reversal breaking. ${ }^{51}$ In particular, the strong absorption model as described in Ref. 51 predicts that the "real" reaction ( $K^{+} n \rightarrow K^{\circ} p$ ) should be larger than the "rotating"

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reaction ( $\overline{\mathrm{X}}^{-} \mathrm{p}+\mathrm{K}^{\circ} \mathrm{n}$ ) for all a although some slight energy dependence is predicted in the $5-10 \mathrm{GeV} / \mathrm{c}$ region. A quantitative comparison of the data in this thesis with such a model would be interesting.

The $t$ ' dependence of the ratio
$R_{2}=\left[\left(d \sigma / d t^{\prime}\right)\left(K^{+} n+K^{0} p\right) /\left(d \sigma / d t^{2}\right)\left(K^{-} p+\bar{K}^{0} n\right)\right]$ has also been investigated. The results are shown in Fig. 51a and Fig. 51 b at 8.36 and $12.8 \mathrm{GeV} / \mathrm{c}$ respectively (the $4 \%$ relative normalization error has not been included In these results as the statistical errors are large.) The ratio is consistent with no $t^{\prime}$ dependence for $0<\left|t^{\prime}\right|<1 \mathrm{GeV}^{2}$ at both energies. Similar behaviour has been observed at 3,4 and $6 \mathrm{GeV} / \mathrm{c} .^{46}$ Strong absorption models would predict a steeper slope for $K^{+}{ }_{n} \rightarrow K^{0} p$. We see no substantial evidence for this effect.
2. $\mathrm{K}^{+}{ }_{\mathrm{p}} \rightarrow \mathrm{K}^{0} \Delta^{++}$and $\mathrm{K}^{-} \mathrm{n}+\mathrm{K}^{-0} \Delta^{-}$

The ratio $R_{3}=\sigma_{T O T}\left(K^{+}{ }_{p} \rightarrow K^{0^{++}}\right) / \sigma_{T O T}\left(K^{-}{ }_{\left.n \rightarrow \mathbb{R}^{0} \Delta^{-}\right)}\right.$was determined to be $1.05 \pm .16$ at $8.36 \mathrm{GeV} / \mathrm{c}$ and $1.56 \pm .08$ at $12.8 \mathrm{GeV} / \mathrm{c}$. These results and the ratios at 4 and $6 \mathrm{GeV} / \mathrm{c}$ have been plotted in Fig. 52. The values of $R_{3}$ at 4,6 and $12.8 \mathrm{GeV} / \mathrm{c}$ would suggest that the magnitude of line reversal breaking seen in this reaction pair is also approximately independent of energy. Our result at $8.36 \mathrm{GeV} / \mathrm{c}$, however, conflicts somewhat with this assumption. As will be shown in part (3) of this section, an examination of the raw $M^{2}$ distributions would tend to support the apparent equality of the two reactions at $8.36 \mathrm{GeV} / \mathrm{c}$. In other words,
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 a) $8.36 \mathrm{GeV} / \mathrm{c}$; b) $12.8 \mathrm{GeV} / \mathrm{c}$.


Fig. 52. The value of $\left\{\sigma_{T O T}\left(\mathrm{~K}_{\mathrm{p}}^{+} \rightarrow \mathrm{K}^{\mathrm{O}} \Delta^{++}\right) / \sigma_{\mathrm{TOT}}\left(\mathrm{K}_{\mathrm{n}}^{-} \rightarrow \mathrm{K}_{\Delta}^{\mathrm{o}} \Delta^{-}\right)\right\}$at $4,6,8.36$ and $12.8 \mathrm{GeV} / \mathrm{c}$ for $0<\left|\mathrm{t}^{\prime}\right|$ § $1.0 \mathrm{GeV}^{2}$.
we do not believe that the result may be explained by an improper parametrization in the fitting procedure used to extract the $\Delta$ signal. It is possible, however, that the result is a statistical fluctuation at about the $2 \sigma$ level.

The ratin $\left[\left(d \sigma / d t^{\prime}\right)\left(K^{+} p \rightarrow K^{0} \Delta^{++}\right) /\left(d \sigma / d t^{\prime}\right)\left(\mathbb{K}^{-} n \rightarrow \bar{K}_{\Delta}^{0}\right)\right]$ is shown in Fig. 53 a and Fig. 53 b at 8.36 and $12.8 \mathrm{GeV} / \mathrm{c}$ respectively. The ratio is consistent with no $t$ ' dependence. This same behaviour has been observed at 4 and $6 \mathrm{GeV} / \mathrm{c} .46$
3. A comparison of $K^{+} d \rightarrow K^{o}+M M^{2}$ and $K^{-} d+K^{-}+M M^{2}$

The $M M^{2}$ dependence of the ratio $R=\left(d N / d M M^{2}\right)\left(K^{+} d \rightarrow K^{\circ} X\right) /\left(d N / d M M^{2}\right)$ ( $K^{-} \mathrm{d} \sim \bar{R}^{\circ}+X$ ) has also been investigated. The $M M^{2}$ distributions for $0<\left|t^{\prime}\right| \leq 1.0 \mathrm{GeV}^{2}$ were corrected for non- $\mathrm{K}^{\circ}$ background and for the relative number of incident kaons. The relative efficiency corrections were also made. Our results are shown in Fig. 54 a at $12.8 \mathrm{GeV} / \mathrm{c}$ and Fig. 54 b and $8.36 \mathrm{GeV} / \mathrm{c}$. The resuits at $12.8 \mathrm{GeV} / \mathrm{c}$ show that R is about 1.3 in the region of the nucleon and increases to approximately 1.5 in the $\Delta$ mass interval. The results are also consistent, however, with a constant ratio of approximately $1.4 \pm .2$ for the range $\mathrm{MM}^{2}<3.0 \mathrm{GeV}^{2}$. This comparison is independent of the minimization procedure employed to determine the nucleon and $\Delta$ cross sections and therefore provides a more direct measure of the difference between $K^{+}$and $K^{-}$cross sections. The ratio at $8.36 \mathrm{GeV} / \mathrm{c}$ has similar behaviour to the $12.8 \mathrm{GeV} / \mathrm{c}$ result in the region of the nucleon but a value near 1.0 was determined near the $\Delta$. This is in agreement with the ratio of the $\Delta^{++}$and $\Delta^{-}$cross sections found in the previous sections. The ratio at larger $\mathrm{Mm}^{2}$ at 8. $36 \mathrm{GeV} / \mathrm{c}$ is also somewhat greater than at $12.8 \mathrm{GeV} / \mathrm{c}$.
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 a) $8.36 \mathrm{GeV} / \mathrm{c}$; b) $12.8 \mathrm{GeV} / \mathrm{c}$.


Fig. 54. The ratio $\left\{\left(d N / d M M^{2}\right)\left(K^{+} d \rightarrow K^{0}+X\right) /\left(d N / M M M^{2}\left(K^{-} d \rightarrow \bar{K}^{o}+X\right)\right\}\right.$.
4. Summary of results and conclusions

The results from this experiment at 8.36 and $12.8 \mathrm{GeV} / \mathrm{c}$ together with the data at 3,4 and $6 \mathrm{GeV} / \mathrm{c}$ allow one to make the following conclusions:

- the ratio $\left[\sigma_{T O T}\left(K^{+} n+K^{\circ} p\right) / \sigma_{T O T}\left(K^{-}{ }^{-}+\bar{K}^{0} n\right)\right]$ has Iittle or no energy dependence from 3 to $13 \mathrm{GeV} / \mathrm{c}$ and has a value of about 1.3 in this energy range
- the ratio $\left[\left(d \sigma / d t^{\prime}\right)\left(K^{+} n+K_{p}^{o}\right) /\left(d \sigma / d t^{\prime}\right)\left(K^{-} p+\bar{K}^{0} n\right)\right]$ is consistent with being independent of $t^{\prime}$ for $0<\left|t^{\prime}\right| \leq 1 \mathrm{GeV}^{2}$ for $3<\mathrm{P}_{\mathrm{LAB}}<13 \mathrm{GeV} / \mathrm{c}$
- the ratio $\left[\sigma_{T O T}\left(\mathrm{~K}^{+} \mathrm{p}^{+} \mathrm{K}^{\mathrm{o}} \Delta^{++}\right) / \sigma_{\mathrm{TOT}}\left(\mathrm{K}^{-} \mathrm{n}^{\left.\left.-\bar{K}^{0} \Delta^{-}\right)\right] \text {apparently has little }}\right.\right.$ or no energy dependence from 4 to $13 \mathrm{GeV} / \mathrm{c}$ and has a value of about 1.6
- the ratio $\left[\left(d \sigma / d t^{\prime}\right)\left(\mathrm{K}^{+}{ }^{p} \rightarrow \mathrm{~K}^{0} \Delta^{++}\right) /\left(d \sigma / d t^{\prime}\right)\left(\mathrm{K}^{-} \mathrm{n} \rightarrow \overline{\mathrm{K}}^{0} \Delta^{-}\right)\right]$is consistent with being independent of $t^{\prime}$ for $0<\left|t^{\prime}\right| \leq 1 \mathrm{Gev}^{2}$
- the ratio $\left[\left(\mathrm{dN} / \mathrm{dMM}^{2}\right)\left(\mathrm{K}^{+} \mathrm{d} \rightarrow \mathrm{K}^{\mathrm{o}} \mathrm{X}\right) /\left(\mathrm{dN} / \mathrm{dMM}^{2}\right)\left(\mathrm{K}^{-} \mathrm{d} \rightarrow \overline{\mathrm{K}}^{\mathrm{O}} \mathrm{X}\right)\right]$ is approximately independent of $\mathrm{MM}^{2}$ for $\mathrm{MM}^{2}<3.0 \mathrm{GeV}^{2}$.

These results, when taken together, strongly suggest that the naive assumptions that led to exact (weak) exchange degeneracy and to the equality of itne reversed reactions are in serious difficulty in kaon charge exchange.

A similar situation also exists in other line-reversal-related reactions. In particular, data with good relative normalization now exist for the hypercharge exchange (HYCEX) reaction pairs

$$
\begin{array}{ll}
\pi^{+}{ }_{p \rightarrow K^{+} \Sigma^{+}} & \mathrm{K}^{-} \mathrm{p} \mathrm{\rightarrow} \mathrm{\pi}^{-} \Sigma^{+} \\
\pi^{+}{ }_{p \rightarrow K^{+} \Sigma^{+}(1385)} & \mathrm{K}^{-}{ }^{+} \rightarrow \pi^{-} \Sigma^{+}(1385)
\end{array}
$$

at $10.1 \mathrm{GeV} / \mathrm{c}^{52}$ These results are characterized by high statistics, good acceptance at small $t^{\prime}\left(\left|t^{\prime}\right|<.3 \mathrm{GeV}^{2}\right)$ and no normalization problems as all reactions were measured in the same spectrometer. The magnitude
of line reversal breaking observed in these HYCEX reactions may be compared to the results of this thesis for charge exchange by comparing ratios of total cross sections integrated up to $|t|=.3 \mathrm{GeV}^{2}$. We find

$$
\text { HYCEX ( } 10.1 \mathrm{GeV} / \mathrm{c}) \quad \sigma\left(\mathrm{K}_{\left.\mathrm{p} \rightarrow \pi^{-} \Sigma^{+}(1385)\right) / \sigma\left(\pi^{+} \mathrm{p}^{-} \mathrm{K}^{+} \Sigma^{+}(1385)\right)=2.0 \pm .220 .2}\right.
$$

$$
\operatorname{CEX}(12.8 \mathrm{GeV} / \mathrm{c}) \quad \sigma\left(\mathrm{K}^{+}{ }_{P \rightarrow K^{0}}^{0_{\Lambda}^{++}}\right) / \sigma\left(\mathrm{K}^{-} \mathrm{n} \cdot{\left.\overline{R^{\circ}} \Delta^{-}\right)}^{\mathrm{C}^{-}} \quad=1.51 \pm .12\right.
$$

The results are strikingly similar. For both HYCEX and CEX, the "real"
 cross sections than the "rotating" reactions ( $\pi^{+}{ }_{p} \rightarrow K^{+} \Sigma^{+}, \pi^{+}{ }_{p \rightarrow K^{+}} \Sigma^{+}(1385)$, $\mathrm{K}^{-} \mathrm{p}^{-} \overline{\mathrm{K}}^{\mathrm{O}} \mathrm{n}$ and $\mathrm{K}_{\mathrm{n}}^{-} \rightarrow \bar{K}_{\Delta}^{0} \Delta^{-}$). Furthermore, the $\Sigma(1385)$ reactions show evidence for greater line reversal breaking than the $\Sigma$ reactions just as the breaking is larger for $\Delta$ production in kaon charge exchange. The result of these comparisons indicates that the observed breaking is approximately SU(3) symmetric. ${ }^{53}$

Additional d. sa exists which indicates that line reversal symmetry is broken and that exact exchange degeneracy is violated. The reactions $n p \rightarrow p n$ and $\bar{p} p \rightarrow \bar{n} n$, which involve $\pi$ exchange as well as other exchanges, have been studied in the $40 \mathrm{GeV} / \mathrm{c}$ region (in different spectrometers) and significant line reversal breaking apparently still exists at that energy. ${ }^{54}$ Weak exchange degeneracy also predicts that the reactions of a line re~ versed pair should have equal but opposite sign polarization. Hypercharge exchange data in the $4 \mathrm{GeV} / \mathrm{c}$ region do not agree with this prediction. 55

$$
\begin{aligned}
& \operatorname{CEX}(8.36 \mathrm{GeV} / \mathrm{c}) \quad \sigma\left(\mathrm{K}^{+}{ }_{\left.\mathrm{n} \rightarrow \mathrm{~K}^{0} \mathrm{p}\right) / \sigma\left(\mathrm{K}^{-} \mathrm{p} \rightarrow \overline{\mathrm{~K}}^{0} \mathrm{n}\right)=1.43 \pm .16}\right. \\
& \operatorname{CEX}(12.8 \mathrm{GeV} / \mathrm{c}) \quad \sigma\left(\mathrm{K}^{+} \mathrm{n} \rightarrow \mathrm{~K}^{0} \mathrm{p}\right) / \sigma\left(\mathrm{K}_{\left.\mathrm{p} \rightarrow \mathrm{X}^{0}{ }_{\mathrm{n}}\right)}\right)=1.38 \pm .09
\end{aligned}
$$

The strong EXD prediction of vanishing polarization is known to be badly (even maximaliy) violated even at higher energies (except perhaps near $t^{\prime}=0$ ). 55

The conclusion is that line reversal symmetry and exact exchange degeneracy are significantly violated in many processes and not only in kaon charge exchange.

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