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1. PHOTON TRANSITION AMPLITUDES PREDICTED BY THE TRANSFORMATION BETWEEN CURRENT AND CONSTITUENT QUARKS

2. SATURATION OF THE DRELL-HEARN-GERASIMOV SUM RULE

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Moim Rodzicom i Bratu

PREFACE

The $SU(6)_W$- group structure appears in both current algebra and in the spectroscopy of hadrons. Recently, considerable progress has taken place in relating these two $SU(6)_W$ structures. We investigate the consequences of the proposed correspondence, as it applies to real photon transitions, in Part 1 of this work. We show the general structure of such transitions and present a set of resulting selection rules for the multipole character of the photon amplitudes. Many specific amplitudes for both mesons and baryons are worked out and their signs and magnitudes are compared with available experimental data.

In Part 2 we investigate the saturation of the Drell-Hearn-Gerasimov sum rule for the forward spin-flip amplitude of nucleon Compton scattering. We study the sum rules' saturation using recent analyses of single pion photoproduction in the region up to photon laboratory energies of 1.2 GeV. The original sum rule is decomposed into separate sum rules originating from different isospin components of the electromagnetic current. The isovector-isovector sum rule, whose contributions are known best, is found to be nearly saturated, lending support to the assumptions underlying the sum rules. The failure of the isovector-isoscalar sum rule to be saturated is then presumably to be blamed on inadequate data for inelastic contributions.

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Part 1.

PHOTON TRANSITION AMPLITUDES PREDICTED BY THE TRANSFORMATION BETWEEN CURRENT AND CONSTITUENT QUARKS

I. INTRODUCTION

At present, symmetries observed in the interactions of hadrons play a somewhat different role than in those areas of physics where we believe the dynamics of interactions is understood. In the theory of gravitation or of electromagnetism symmetries simply reflect invariance of known interactions. When results of some processes are related by a symmetry in these theories, we can always obtain these results through explicit calculations, but we may directly use the symmetry properties of the theory, which often turns out to be a much simpler method. We do not need the details of the dynamics in such cases, but instead make use of the fact that the interaction belongs to a class of interactions invariant under some type of transformation. For example, the cross section for electron scattering by an unpolarized charge is independent of the polarization of the incident electron. We can obtain this result directly with some effort, calculating the cross sections for both polarizations of the electron, or or we can simply use the invariance of electromagnetic interactions under parity to arrive at the same result.

In the case of strong interactions and the structure of hadrons, we do not have an adequate dynamical theory at the present time. Observed symmetries of strong interactions have become therefore one of the most powerful tools for re-

lating various processes and making predictions.[1] They also provide important information by narrowing down the class of possible dynamical theories of strong interactions to those which are consistent with the experimentally observed symmetries and conservation laws.

Historically, one of the first symmetries discovered in this area was the "charge independence" of nuclear forces.[2] Strong interactions of the proton and neutron, as well as their masses (to order α), were found to be identical. The two particles were postulated to be members of a two dimensional irreducible representation (I.R.) of a symmetry group of strong interactions: the SU(2) of isospin.[3] This group has three generators I_1, I_2 and I_3. Their commutation relations are the same as those of the rotation group generators. The third generator, I_3, is linearly related to the electromagnetic charge.[3] Irreducible representations of this group are characterized by the value of the isospin, I, just as the representations of the O(3) group are characterized by the value of the angular momentum, J. The I.R. containing proton and neutron has I=1/2, with proton and neutron having I_3=1/2 and -1/2, respectively. Isospin invariance of strong interactions was further confirmed as other hadrons were discovered. They were found to form isospin multiplets of definite spin, mass and parity : three pions with I=1, J = 0 , three ρ-mesons of I=1, J =1; four $N^*(1236)$ states with I=3/2 and J =3/2, etc. The isospin properties of the photon have also been deter-

mined. The photon has $I_3=0$ and consists of $I=1$ and $I=0$ components.

Discovery of strangeness and its conservation in strong interactions, together with the discovery of baryon and meson resonances, led to a postulate that strong interactions are invariant under a still higher symmetry group, the "unitary symmetry" SU(3), suggested by Gell-Mann in 1961.[4] This symmetry, although not exact, again results in approximately degenerate multiplets of hadrons, corresponding to the irreducible representations of SU(3). The group has 8 generators, conventionally labelled λ^i, $i=1,..8$, of which the first three coincide with the isospin generators, so that the isospin SU(2) is a subgroup of the larger symmetry. Within a given multiplet (I.R.) the various members are characterized by their isospin, the third component of the isospin, and the hypercharge Y, defined as a sum of the baryon number and strangeness, $Y=B+S$. The I.R. itself is characterized by two quantities, the dimension of the representation and the highest value of the hypercharge, for example. Mesons of a given J^P seem to appear as singlets, or to form octets approximately degenerate in mass: the pion belongs to the $J^P=0^-$ octet of pseudoscalar mesons, the ρ-meson is a part of the $J^P=1^-$ octet of vector mesons, etc. Baryons appear in singlets, octets and decuplets. For example, the nucleon belongs to a $J^P=1/2^+$ octet, while the $N^*(1236)$ (the 3-3 resonance), three Y^*'s, two Ξ^*'s and the Ω^- form the $J^P=3/2^+$

decuplet.

The lowest non-trivial representation of the SU(3) group is three dimensional

$$(Y,I,I_3)_1 = (1/3,1/2,1/2)$$
$$(Y,I,I_3)_2 = (1/3,1/2,-1/2)$$
$$(Y,I,I_3)_3 = (-2/3,0,0) \quad .$$

The question of whether quarks, as the members of this multiplet have been christened by Gell-Mann, exist as physical states has been studied extensively since their introduction into hadron physics in 1964.[5] They have not been found, but a very suggestive picture results, when we express baryons as three quarks and mesons as a quark and an antiquark. The multiplet structure for both baryons and mesons is then identical with the one discussed above. The SU(3) representation corresponding to qqq reduces to a decuplet, two octets and a singlet; the one corresponding to $q\bar{q}$ reduces to an octet and a singlet. This, in addition to the results of experiments designed to probe the structure of hadrons, seems to indicate that hadrons behave as if they have "constituents", and that quarks are very strong candidates for these constituents.

This success of the constituent quark picture for the multiplet structure of hadrons makes it important to study in detail those consequences of the quark model which we can

compare with the available experimental data. This thesis is devoted to a detailed study of these consequences in the case of electromagnetic transitions.

Electromagnetic and weak interactions can be described in terms of "currents",[6] in analogy with the classical electric current interactions. The Lorentz, isospin and SU(3) transformation properties of these currents have been proposed so as to agree with experimental observations; e.g. the electromagnetic current is a Lorentz vector, has isospin components 0 and 1, and transforms as a sum of the 3 and 8 members of an octet under SU(3). Conventionally, these properties are represented by $V_\mu^3(\vec{x},t) + \frac{1}{\sqrt{3}} V_\mu^8(\vec{x},t)$. Weak currents involve both currents with the properties of a Lorentz vector and axial-vector, $A_\mu(\vec{x},t)$. We present the SU(3) commutation relations proposed for the axial-vector currents and for space integrals of their time components, the axial charges, in Section II of this paper. The commutation relations of weak and electromagnetic currents and charges lead to numerous sum rules when both sides of such commutation relations are sandwiched between hadron states. Evaluation of these sum rules poses the next question: how are the transformation properties of currents related to the transformation properties of hadrons and to the symmetries of strong interactions? There are many attempts to answer this important question. A formalism, proposed by Gell-Mann[5] and based on "current quarks", expresses currents as $\bar{q}(\vec{x},t)\, F\, q(\vec{x},t)$, where $q(\vec{x},t)$ is the

"current quark" field and F is an operator of appropriate algebraic (SU(3)) and Lorentz transformation properties.

A complete knowledge of the transformation from constituent to current quark states, together with the identification of the observed hadrons with simple (constituent) quark model states, would permit one to calculate all current induced transitions between hadrons. A major step in this direction has been achieved by Melosh,[7] who was able to formulate and explicitly calculate such a transformation in the free quark model. While the details of such a transformation certainly depend on strong interactions dynamics, it is possible that certain algebraic properties of the transformation abstracted from the free quark model may hold in general.

We shall assume that such a transformation does indeed exist and that some of its algebraic properties can be abstracted from the free quark model. For the case of the axial-vector charge, the many consequences of this assumption have already been extensively worked out and compared with experiment using the Partially Conserved Axial-Vector Current hypothesis.[8,9] Here we study the consequences of the same assumption for real ($q^2=0$) photon transitions.

In the next section we review the origin and the basic properties of the theory along with some of its immediate consequences for the classification of baryons under an algebra generated by currents. This subject is further dis-

cussed in Section III, where we attempt to expand the wave functions of hadrons in terms of current algebra representations by fitting experimentally known matrix elements. The general algebraic structure of photon amplitudes is discussed in Section IV, as well as the method of calculating specific matrix elements. We derive a set of selection rules which include, and generalize, the old SU(6) result that the transition from the nucleon to 3-3 resonance should be magnetic dipole in character.[10] This general discussion of the theory is completed by a comparison with other theories with a related algebraic structure.

In Section V the photon transitions between mesons are detailed, along with a comparison of the predictions with the available experimental data. A detailed exposition of baryon electromagnetic transitions is contained in Section VI, and in Section VII we compare both the sign and magnitude of the predicted amplitudes with experiment. The signs are testable through a multipole analysis of pion photoproduction $\gamma N \to N^* \to \pi N$, where the signs of the previously calculated pion decay[8,9] amplitudes also come into play. Experiment and theory are found to be consistent, including the relative signs of pion decay amplitudes obtained from analizing $\pi N \to N^* \to \pi \Delta$. A summary and some conclusions are found in Section VIII. The general outlook is very good, encouraging further study of the underlying dynamics and the extension to the $q^2 \neq 0$ region.

II. CURRENT AND CONSTITUENT ALGEBRA

Let us begin with a review of developements in current algebra which led to the introduction of the $SU(6)_W$ algebra of currents. The vector and axial-vector currents, $V_\mu^i(\vec{x},t)$ and $A_\mu^i(\vec{x},t)$, have been postulated to obey simple equal time commutation relations, which are to be exact as far as strong interactions are concerned.[4] The right hand side of these commutation relations generally consists of a term involving a delta function and terms involving gradients of delta functions. The gradient terms do not contribute to the three dimensional space integrals of the commutation relations, so that the 16 vector and axial-vector charges

$$Q^i(t) = \int d^3x \; V_o^i(\vec{x},t)$$
$$Q^i(t) = \int d^3x \; A_o^i(\vec{x},t) \qquad \text{(II-1)}$$

where i=1, ,8 is an SU(3) index, commute to form the algebra abstracted by Gell-Mann[4]

$$\left[Q^i(t), Q^j(t)\right] = i f^{ijk} Q^k(t) \qquad \text{(II-2a)}$$

$$\left[Q^i(t), Q_5^j(t)\right] = i f^{ijk} Q_5^k(t) \qquad \text{(II-2b)}$$

$$\left[Q_5^i(t), Q_5^j(t)\right] = i f^{ijk} Q^k(t) \qquad \text{(II-2c)}$$

f^{ijk} =structure constants of SU(3)

The Q^i's generate ordinary SU(3) . It is easy to show that Eqs.(II-2) define the algebra of chiral SU(3) SU(3), since they are equivalent to the statement that $Q^i \pm Q_5^i$ each form an SU(3) algebra and that the right-handed charges, $Q^i + Q_5^i$, commute with the left-handed charges, $Q^i - Q_5^i$. The last of the Eqs.(II-2) sandwiched between nucleon states moving with infinite momentum in the z-direction yields the Adler-Weisberger sum rule.[11]

When a world scalar density is adjoined to the vector and axial-vector current components and commutation of space integrals of such operators continued until a closed algebra is formed, the result is a U(12) of 144 generators.[12] The elements of this algebra are space integrals of current densities of the general form $q^+(\bar{x},t)\Gamma\lambda^i q(\bar{x},t)$, when expressed in terms of the current quark fields.

An $SU(6)_W$ subalgebra of this U(12), which consists of positive parity, "good", i.e. those with finite matrix elements between states with $p_z \to \infty$ operators,[13] which commute z-boosts, proves to be of particular interest.[12] The "W" stands for the W-spin: $W_x = \frac{1}{2}\beta\sigma_x$, $W_y = \frac{1}{2}\beta\sigma_y$, $W_z = \frac{1}{2}\sigma_z$, which acting on quarks are the same as S_x, S_y and S_z, but for antiquarks, since $\beta = -1$, are equal $-S_x$, $-S_y$ and S_z. The generators of the $SU(6)_W$ algebra are space integrals of the time component of the vector currents, z-component of the axial-vector currents and space integrals of antisymmetric tensors $F_{\mu\nu}$, where μ,ν =2,3 or 3,1. Explicit current quark model expressions for the 35 generators of the SU(6) of currents are

$$F^i = \int d^3x\, q^+(\bar{x},t)\, \frac{\lambda^i}{2}\, q(\bar{x},t) \qquad \text{(II-3a)}$$

$i=1, \quad ,8$

$$F_1^i = \frac{1}{2}\int d^3x\, q^+(\bar{x},t)\, \beta\sigma_x\, \frac{\lambda^i}{2}\, q(\bar{x},t) \qquad \text{(II-3b)}$$

$$F_2^i = \frac{1}{2}\int d^3x\, q^+(\bar{x},t)\, \beta\sigma_y\, \frac{\lambda^i}{2}\, q(\bar{x},t) \qquad \text{(II-3c)}$$

$$F_3^i = \frac{1}{2}\int d^3x\, q^+(\bar{x},t)\, \sigma_z\, \frac{\lambda^i}{2}\, q(\bar{x},t) \qquad \text{(II-3d)}$$

$i=0,1, \quad ,8$

where λ^i is an SU(3) generator ($\lambda^0 = \sqrt{\frac{2}{3}}\,\mathbb{1}$) and $q(\vec{x},t)$ is a current quark field.

To classify operators and to describe the transformation properties of irreducible representations (I.R.'s) we will use here two equivalent methods of labeling. The first one directly defines the properties of a given object under $SU(6)_W$, e.g. in

$$\left\{ \underline{M}, \underline{N}^{2W+1} \right\}$$

M is the dimension of the I.R. under the full $SU(6)_W$, N the dimension under the ordinary SU(3) (generated by Q^i's) subalgebra, and W is the value of the W-spin. A second method, which we will use predominantly throughout this paper, defines the transformation properties of considered states and operators under the full $SU(6)_W$ and its chiral SU(3)×SU(3) subalgebra. Under the chiral algebra,

$$(A,B)_{S_z}$$

transforms as an A-dimensional representation under $Q^i + Q^i_5$, as a B-dimensional representation under $Q^i - Q^i_5$, and S_z is the eigenvalue of Q^0_5, the singlet axial-vector charge.[13] When we use the second notation, the $SU(6)_W$ transformation properties of a discussed object will either follow from the context, or will be given explicitly. The "ordinary" SU(3) content (under Q^i) of (A,B) is just that of A×B. We will also use L_z <u>defined</u> as $L_z = J_z - S_z$, to complete the description of the transformation properties of states and operators, so in effect we work within an $SU(6)_W \times O(2)$ framework.

According to the classification described above, the axial-vector charge

$$Q^i_5 = \frac{1}{2}\left(Q^i + Q^i_5\right) - \frac{1}{2}\left(Q^i - Q^i_5\right)$$

transforms as $\left\{(8,1)_0 - (1,8)_0,\ L_z=0\right\}$, or equivalently as $\left\{\underline{35},\ 8^3,\ L_z=0\right\}$. In this work we will be concerned primarily with photon transitions induced by the dipole operator, $D^{em}_{\pm}$, the first moment of the electromagnetic vector current:

$$D^{em}_{\pm} = -i\int \frac{x \pm iy}{\sqrt{2}}\left[V_0^3(\vec{x},t) + \frac{1}{\sqrt{3}}V_0^8(\vec{x},t)\right]d^3x \qquad (11\text{-}4)$$

When $D^{em}_{\pm}$ is sandwiched between two hadron states at infinite momentum, $\langle H' \mid D^{em}_{\pm} \mid H \rangle$ is directly proportional to the amplitude for the photon transition $H' \to \gamma + H$. For $H'=\Delta$, H=N this matrix element is proportional to the strength of the electromagnetic coupling of the nucleon and the delta. For H'= H =N this matrix element gives $\frac{1}{\sqrt{2}}$ times the anomalous magnetic moment of the nucleon. $D^{em}_{\pm}$ transforms as $\left\{(8,1)_0 + (1,8)_0,\ L_z=\pm 1\right\}$, since under Q^i and Q^i_5

$$\left[Q^i, V_0^j(\vec{x})\right]_t = i f^{ijk} V_0^k(\vec{x})$$

$$\left[Q^i_5, V_0^j(\vec{x})\right]_t = i f^{ijk} A_0^k(\vec{x})$$

All representations of the chiral SU(3)×SU(3) of currents can be built up from $(3,1)_{\frac{1}{2}}$, $(1,3)_{-\frac{1}{2}}$, $(1,\bar{3})_{\frac{1}{2}}$ and $(\bar{3},1)_{-\frac{1}{2}}$, which we define to be the current quark and antiquark states with spin projection ± 1/2 in the z-direction. The quarks form a $\underline{6}$ and the antiquarks a $\underline{\bar{6}}$ of the full $SU(6)_W$ of currents.

At the same time as when the $SU(6)_W$ of currents was in-

troduced, Lipkin and Meshkov postulated another $SU(6)_W$ as an approximate symmetry of strong interactions,[14] deriving this result from SU(3) and spin symmetry of the amplitudes for collinear processes. Ordinary SU(6) ,involving SU(3) and S-spin, however, forbids such observed strong couplings as $\rho\pi\pi$ and $\Delta\pi N$. The W-spin generators , which were first introduced in this framework using intrinsic parity P_{int}:

$$W_x = \frac{1}{2} P_{int}\sigma_x$$
$$W_y = \frac{1}{2} P_{int}\sigma_y \qquad \text{(II-5a)}$$
$$W_z = \frac{1}{2} \sigma_z$$

or, relativistically,

$$W_x = \frac{1}{2}\beta\sigma_x$$
$$W_y = \frac{1}{2}\beta\sigma_y \qquad \text{(II-5b)}$$
$$W_z = \frac{1}{2}\sigma_z$$

commute with z-boosts and are "good" operators. It was the W-spin that led to an $SU(6)_W$ as an appropriate group corresponding to the approximate symmetry of strong interactions. This $SU(6)_W$ known as the $SU(6)_W$ of constituents classifies hadrons into its irreducible representations (see Appendix). Generators of this algebra have the same equal time commutation relations, charge conjugation and parity as the generators of the $SU(6)_W$ of currents. However, it was shown soon

after their introduction that the two algebras could not be identified with each other. Even in the free quark model most of the operators in the $SU(6)_W$ of currents do not commute with the kinetic energy term in the Hamiltonian, if $p_\perp \neq 0$, so that they are not conserved.[7] Additionally, such an identification gives wrong predictions for g_A/g_V and for magnetic moments. To summarize, current and charge operators $\mathcal{O}^\alpha$ have well defined transformation properties under the $SU(6)_W$ of currents, and hadron states may be classified into irreducible representations of the $SU(6)_W$ of constituents. A relation between the two $SU(6)_W$'s is necessary to define within one algebra the algebraic properties of both hadrons and operators $\mathcal{O}^\alpha$ occuring in matrix elements $\langle$ hadron' $|\ \mathcal{O}^\alpha |$ hadron $\rangle$.

Considerable progress resulted from the assumption that constituent and current quark states can be related by a unitary transformation V, so that

$$|\ \text{I.R. constituents}\rangle = V\ |\ \text{I.R. currents}\ \rangle\ . \qquad \text{(II-6)}$$

Using this assumption, Melosh[7] further defined a set of operators W

$$W^i = V^{-1} F^i V \qquad \text{(II-7a)}$$

and imposed a condition that

$$[H_{strong}, W^i] \simeq 0 \qquad \text{(II-7b)}$$

From these assumption it follows that the W^i's generate an $SU(6)_W$ under which hadrons transform as those irreducible representations which correspond to the simple constituent quark model. The approximate symmetry of strong interactions may then be identified with the $SU(6)_W$ generated by the W^i's.

A matrix element of a current or charge operator $\mathcal{O}^\alpha$ between two hadronic states can now be rewritten

$$\begin{aligned} &\langle \text{hadron}' \mid \mathcal{O}^\alpha \mid \text{hadron} \rangle = \\ &\quad = \langle \text{I.R.}'\text{constituents} \mid \mathcal{O}^\alpha \mid \text{I.R.constituents} \rangle \\ &\quad = \langle \text{I.R.}' \text{ currents} \mid V^{-1} \mathcal{O}^\alpha V \mid \text{I.R.currents} \rangle \end{aligned} \qquad \text{(II-8)}$$

with all quantities in the last line of the equation labeled by current algebra.

The transformation V must satisfy a number of conditions discussed in detail in Refs. 7 and 16. We need only to recall here that V takes "good" operators into "good" operators, and that V conserves the "ordinary" SU(3) (generated by Q^i's). In consequence, the ordinary SU(3) content of $V|(A,B)$ currents$\rangle$ is the same as that of $|(A,B)$ currents$\rangle$, although the transformed state may span many representations of the full $SU(6)_W$ or its chiral $SU(3)\times SU(3)$ subalgebra. We

illustrate this in Section III, where we make a phenomenological attempt to expand physical states in terms of the representations of current algebra.

In this context, if the vacuum of currents is defined in analogy with Eq.(II-6) as:

$$\begin{aligned} &|\text{vaccum currents}\rangle = V^{-1}|\text{vacuum constituents}\rangle \\ &\left(\mid \underline{1}^1 \text{ currents} \rangle = V \mid \underline{1}^1 \text{ constituents} \rangle \right) \end{aligned} \qquad \text{(II-9)}$$

where the state $|$vacuum currents$\rangle$ is annihilated by currents and $|$vacuum constituents$\rangle$ is the "physical" vacuum, the matrix element

$$\langle \text{physical pion} \mid Q_5^i \mid \text{physical vacuum} \rangle$$

can be rewritten, assuming that the pion and physical vacuum can be identified with the corresponding constituent states, as:

$$\begin{aligned} &\langle \pi, \text{constituents} \mid Q_5^i \mid \text{vacuum constituents} \rangle \\ &= \langle \pi, \text{currents} \mid V^{-1} Q_5^i V \mid \text{vacuum currents} \rangle \\ &= \langle \pi, \underline{35}, 8^3 \text{currents} \mid V^{-1} Q_5^i V \mid \underline{1}^1 \text{ currents} \rangle . \end{aligned} \qquad \text{(II-10)}$$

This matrix element need not vanish, since $V^{-1} Q_5^i V$ is not a generator of the $SU(6)_W$ of currents.

We will discuss in detail algebraic properties of the

matrix elements of transition inducing operators and apply these properties to real transitions in Section IV of this paper. Before that, we will complete the presentation of the relation between the two $SU(6)_W$ algebras with a phenomenological approach to the mixing occuring in the hadron states under current algebra.

III. PHENOMENOLOGICAL APPROACH TO THE TRANSFORMATION PROPERTIES OF HADRONS UNDER CURRENT ALGEBRA

We now turn to the classification of hadrons under the $SU(6)_W$ algebra of currents and under its chiral subalgebra, SU(3)×SU(3). Much effort was devoted to this subject after the introduction of current algebra and the Adler-Weisberger sum rule[11] (see Eq.(II-2c)). To illustrate again the need for mixing, let us consider the following examples.

If the nucleon and delta (1236), the lowest mass baryons, behaved as three current quark states with totally symmetric wave functions and internal quark angular momentum L=0, then using chiral SU(3)×SU(3) to classify states and operators, we find that in the helicity $\lambda = 1/2$ state they must transform as a pure $|\underline{56},(6,3)_{\frac{1}{2}},0\rangle$, while in the helicity $\lambda = -1/2$ state as a pure $|\underline{56},(3,6)_{-\frac{1}{2}},0\rangle$.

G_A and G^* are conventionally defined so that the Adler-Weisberger sum rule reads:

$$
\begin{aligned}
2\langle p|Q^3|p\rangle &= \langle p|[Q_5^+, Q_5^-]|p\rangle \\
&= |\langle p|Q_5^+|n\rangle|^2 \\
&\quad - [|\langle p|Q_5^-|\Delta^{++}\rangle|^2 - |\langle p|Q_5^-|\Delta^0\rangle|^2] + \ldots \\
&= G_A^2 - G^{*2} + \ldots
\end{aligned}
\qquad \text{(III-1)}
$$

where $Q_5^{\pm} = Q_5^1 \pm iQ_5^2$ and the normalization is such that $\langle p|p\rangle = 1$. The operators Q_5^{α}, as $SU(6)_W$ (and SU(3)×SU(3)) generators, can only connect the I.R. $|\underline{56}, (6,3)_{\frac{1}{2}}0\rangle$ to itself. Without representation mixing in nucleon states the Adler-Weisberger sum rule[11] must be saturated by the contributions from the neutron and $\Delta(1236)$ alone. Without mixing, $G_A = 5/3$ and $G^* = 4/3$, whereas $2\langle p|Q^3|p\rangle = 1$. Evaluation[11] of this sum rule shows that there must be contributions from many higher mass N^*'s.

The anomalous magnetic moment of the nucleon is given by the matrix element of the dipole operator D_+^{em} of Eq.(II-4) between the nucleon states at infinite momentum,

$$
\sqrt{2}\,\mu_A(N) = \langle N, \lambda = \tfrac{1}{2}|\, D_+^{em} \,|N, \lambda = -\tfrac{1}{2}\rangle_{p_z \to \infty}
\qquad \text{(III-2)}
$$

Without representation mixing this matrix element vanishes, since the initial and final nucleon wave functions have

$L_z = L = 0$ only, while $D^{em}_{\pm}$ changes L_z by ± 1 (see Eq.(II-4)).

These and similar arguments show that some representation mixing must be present in the classification of hadrons under the algebra of currents.[17] In this section we discuss how the mixing parameters can be sought phenomenologically. We limit our presentation to the nucleon and $\Delta(1236)$, but a similar method applies to mesons and higher mass baryon resonances. We consider mixing of chiral SU(3)×SU(3) representations; mixing under the full $SU(6)_W$ of currents can be studied in the same manner.

A three quark, helicity $\lambda = 1/2$, $J = 1/2$, nucleon state may transform most generally as a linear combination of six I.R.'s of the chiral SU(3)×SU(3) current algebra. We therefore may parametrize:

$$|N, \lambda = \tfrac{1}{2}\rangle = \cos\theta \left(\cos\eta \; |(6,3)_{\frac{1}{2}}, 0\rangle + \sin\eta \; |(3,6)_{-\frac{1}{2}}, +1\rangle \right)$$
$$+ \sin\theta \Big[\cos\varphi \left(\cos\psi \; |(3,\bar{3})_{-\frac{1}{2}}, +1\rangle + \sin\psi \; |(8,1)_{\frac{3}{2}}, -1\rangle \right)$$
$$+ \sin\varphi \left(\cos\nu \; |(\bar{3},3)_{\frac{1}{2}}, 0\rangle + \sin\nu \; |(1,8)_{-\frac{3}{2}}, +2\rangle \right) \Big] \qquad \text{(III-3)}$$

Similarly, for a three quark $\lambda = 1/2$, $J = 3/2$ delta state we may choose a parametrization

$$|\Delta, \lambda = \tfrac{1}{2}\rangle = \cos\alpha \left(\cos\beta \; |(6,3)_{\frac{1}{2}}, 0\rangle + \sin\beta \; |(3,6)_{-\frac{1}{2}}, +1\rangle \right)$$
$$+ \sin\alpha \left(\cos\gamma \; |(10,1)_{\frac{3}{2}}, -1\rangle + \sin\gamma \; |(1,10)_{-\frac{3}{2}}, +2\rangle \right) \qquad \text{(III-4)}$$

To determine the mixing parameters we shall fit G_A, G^*, the anomalous magnetic moments of the proton and neutron and the strength of the electromagnetic $\gamma N \Delta$ coupling using the assumed form of the wave functions (Eqs(III-3,4)).

We obtain for G_A and G^*:

$$G_A = \frac{5}{3} \cos^2\theta \cos 2\eta + \sin^2\theta \cos 2\varphi$$
$$G^* = \frac{4}{3} \cos\theta \cos\alpha \cos(\eta - \beta) \qquad \text{(III-5)}$$

We already have predictions that $G^* < 4/3$, which agrees with the experimental limits on G^* (.8 to 1.05). Note also that for $G_A \simeq 1.25$ we obtain bounds on the angles θ and η :

$$\theta \leqslant 52^\circ$$
$$\eta \leqslant 26.5^\circ \qquad \text{(III-6)}$$

The strength of the $\gamma N \Delta$ coupling, μ^*, is proportional to the matrix element of the $D^{em}_{\pm}$ between the nucleon and delta states at infinite momentum

$$\mu^* = \sqrt{2} \, \langle \Delta, \lambda = \tfrac{1}{2} | \, D^{em}_{+} \, | \, N, \lambda = -\tfrac{1}{2} \rangle_{P_z \to \infty} \qquad \text{(III-7)}$$

Moments given by the assumed baryon wave functions and the algebraic properties of D^{em}_{+}, $\{(8,1)_0 + (1,8)_0 \; , 1\}$, allow five independent reduced matrix elements in the expressions

for μ^* and the anomalous magnetic moments:

$$M_1 = \langle (6,3)_{\frac{1}{2}}, 0 \| \{(8,1)_0 + (1,8)_0, 1\} \| (6,3)_{\frac{1}{2}}, -1 \rangle$$
$$M_2 = \langle (6,3)_{\frac{1}{2}}, 0 \| \{(8,1)_0 + (1,8)_0, 1\} \| (\bar{3},3)_{\frac{1}{2}}, -1 \rangle$$
$$M_3 = \langle (\bar{3},3)_{\frac{1}{2}}, 0 \| \{(8,1)_0 + (1,8)_0, 1\} \| (\bar{3},3)_{\frac{1}{2}}, -1 \rangle$$
$$M_4 = \langle (8,1)_{\frac{3}{2}}, -1 \| \{(8,1)_0 + (1,8)_0, 1\} \| (8,1)_{\frac{3}{2}}, -2 \rangle$$
$$M_5 = \langle (10,1)_{\frac{3}{2}}, -1 \| \{(8,1)_0 + (1,8)_0, 1\} \| (8,1)_{\frac{3}{2}}, -2 \rangle \qquad \text{(III-8)}$$

The terms multiplying M_4 involve also another parameter, since 8×8 reduces to 8_A and 8_S. This parameter, the f/d ratio, would be given if we used the full $SU(6)_W$.

In terms of the mixing angles and the reduced matrix elements, the anomalous magnetic moments and μ^* are then:

$$\mu^* = \frac{2}{\sqrt{3}} \cos\alpha \sin\theta \left[\cos\beta \cos\varphi \cos\psi + \sin\beta \sin\varphi \cos\nu\right] M_2 \qquad \text{(III-9a)}$$
$$+ \sqrt{\frac{2}{3}} \sin\alpha \sin\theta \left[\cos\gamma \sin\varphi \sin\nu + \sin\gamma \cos\varphi \sin\psi\right] M_5$$

$$\sqrt{2}\,\mu_A^V = -\cos^2\theta \sin 2\eta\, M_1 \qquad \text{(III-9b)}$$
$$+ \frac{2}{\sqrt{3}} \sin 2\theta \left[\cos\eta \cos\varphi \cos\psi - \sin\eta \sin\varphi \cos\nu\right] M_2$$
$$+ \sin^2\theta \sin 2\varphi \cos\nu \cos\psi\, M_3$$
$$+ \sin^2\theta \sin 2\varphi \sin\nu \sin\psi\, M_4 \left(\sqrt{\frac{3}{5}} - f/d \sqrt{\frac{1}{3}}\right)$$

$$\sqrt{2}\,\mu_A^S = -\cos^2\theta \sin 2\eta\, M_1$$
$$+ \sin^2\theta \sin 2\varphi \cos\nu \cos\psi\, M_3$$
$$+ \sin^2\theta \sin 2\varphi \sin\nu \sin\psi\, M_4 \left(-\sqrt{\frac{1}{15}} - f/d \sqrt{\frac{1}{3}}\right) \qquad \text{(III-9c)}$$

where

$$\mu_A^V = \mu_A(p) - \mu_A(n)$$
$$\mu_A^S = \mu_A(p) + \mu_A(n) \qquad \text{(III-10)}$$

are the isovector and isoscalar anomalous moments.

Eqs.(III-5) and (III-9) can be satisfied in many ways, since the number of free parameters is greater than the number of experimental quantities we want to fit. We can however conclude that since $G_A > 1$, $\cos^2\theta \neq 0$ and $\cos 2\eta \neq 0$ ($\theta \neq \frac{\pi}{2}$ and $\eta \neq \frac{\pi}{4}$), i.e., the nucleon's wave function must contain the $|(6,3)_{\frac{1}{2}}, 0\rangle$ and/or $|(3,6)_{-\frac{1}{2}}, 1\rangle$ terms, and the ratio between the mixing parameters multiplying these two terms must be different from 1.

One of possible solutions, related to that proposed by Harari,[18] can be obtained under the assumption that the internal quark angular momentum in both the nucleon and delta is not greater than one. This eliminates the mixing angles $\nu = \gamma = 0$ (see Eqs.(II-3,4)). We then additionally assume that $\alpha = \theta$ and $\beta = \eta$, i.e.

$$|\Delta, \lambda = \tfrac{1}{2}\rangle = \cos\theta \left(\cos\eta\, |(6,3)_{\frac{1}{2}}, 0\rangle + \sin\eta\, |(3,6)_{-\frac{1}{2}}, +1\rangle\right)$$
$$+ \sin\theta\, |(10,1)_{\frac{3}{2}}, -1\rangle \qquad \text{(III-11a)}$$

$$|N, \lambda = \tfrac{1}{2}\rangle = \cos\theta\Big(\cos\eta\, |(6,3)_{\frac{1}{2}}, 0\rangle + \sin\eta\, |(3,6)_{-\frac{1}{2}}, +1\rangle\Big)$$
$$+ \sin\theta\Big[\cos\varphi\Big(\cos\psi\, |(3,\bar{3})_{-\frac{1}{2}}, +1\rangle + \sin\psi\, |(8,1)_{\frac{3}{2}}, -1\rangle\Big)$$
$$+ \sin\varphi\, |(\bar{3},3)_{\frac{1}{2}}, 0\rangle\Big]$$

(III-11b)

With these assumptions we obtain, for $G^* \cong 1$,

$$\sin^2\alpha = \sin^2\theta \simeq \frac{1}{4} \quad ; \quad \alpha = \theta \cong 30^\circ$$

$$\sin 2\beta = \sin 2\eta \simeq \frac{4}{5} \quad ; \quad \beta = \eta \cong 18.4^\circ \qquad \text{(III-12)}$$

Since the isoscalar anomalous magnetic moment $\mu_A^S \ll \mu_A^V$ and $\mu_A^S \ll \mu^*$, we further assume $\mu_A^S \simeq 0$. Using this and the experimental value of the ratio μ^*/μ_A^V [45] to determine the remaining mixing angles we are led to:

$$\sqrt{2}\,\mu_A^S = -\frac{9}{20} M_1 + \frac{1}{2}\sin 2\varphi \cos\psi\, M_3 \simeq 0$$

$$\sqrt{2}\left(\mu^*/\mu_A^V\right) = \frac{\cos\eta\cos\varphi\cos\psi + \sin\eta\sin\varphi}{\cos\eta\cos\varphi\cos\psi - \sin\eta\sin\varphi}$$

$$= 1 \qquad \text{(III-13)}$$

which we may satisfy choosing $M_1 = 0$, $\varphi = 0$. The wave functions resulting from this particular solution are given by

$$|N, \lambda = \tfrac{1}{2}\rangle = \cos 30^\circ\left[\cos 18.4^\circ\, |(6,3)_{\frac{1}{2}}, 0\rangle + \sin 18.4^\circ\, |(3,6)_{-\frac{1}{2}}, +1\rangle\right]$$
$$+ \sin 30^\circ\left[\cos\psi\, |(3,\bar{3})_{-\frac{1}{2}}, +1\rangle + \sin\psi\, |(8,1)_{\frac{3}{2}}, -1\rangle\right]$$

(III-14a)

and

$$|\Delta, \lambda = \tfrac{1}{2}\rangle = \cos 30^\circ\left[\cos 18.4^\circ\, |(6,3)_{\frac{1}{2}}, 0\rangle + \sin 18.4^\circ\, |(3,6)_{-\frac{1}{2}}, +1\rangle\right]$$
$$+ \sin 30^\circ\, |(10,1)_{\frac{3}{2}}, -1\rangle \qquad \text{(III-14b)}$$

Even this simplified mixing scheme with $L \leqslant 1$ has no unique solution, as $\cos\psi$ is still left as a free parameter, except for the condition that $\cos\psi \neq 0$, because that would give $\mu_A^V = \mu^* = 0$ here.

The analogous situation of no unique solution for the current algebra representation mixing in the wave functions is encountered in the expansion of higher mass resonances, each of which must be treated separately. The phenomenological approach then is not very helpful in a systematic evaluation of the current and charge matrix elements between hadron states. It indicates the complexity of hadrons' wave functions under current algebra. The approach based on a unitary transformation relating current and constituent quarks also predicts complicated algebraic properties for V| I.R.currents>, which correspond to the properties of hadrons under current algebra.[7] But a systematic evaluation of the matrix elements < hadron'| σ^α |hadron > in Eq.(II-8) can

be carried out in spite of the complexity of the wave functions. We will describe the alternate systematic method of calculating such matrix elements and its applications in the following sections.

IV. PHOTON TRANSITIONS OF HADRONS

Evaluation of the matrix element $\langle \text{hadron}' | \mathcal{O}^{\alpha} | \text{hadron} \rangle$ in Eq.(II-8) can be carried out in spite of the complexity of the state $V \mid \text{I.R. currents}\rangle$, provided that the "effective" operator $V^{-1}\mathcal{O}^{\alpha}V$ has definite and simple transformation properties under current algebra.

In the free quark model Melosh has been able to construct an explicit form of V. Effective operators $V^{-1}\mathcal{O}^{\alpha}V$ for $\mathcal{O}^{\alpha} = Q_5^i$ and $D_{\pm}^{em}$, turn out to have simple transformation properties when this V is used. They must connect only single quark states to single quark states, and both have a general form

$$\int d^3x \; q^{+}(\vec{x},t)\, \mathcal{F}(\partial_i, \gamma_i)\; q(\vec{x},t).$$

Here $\mathcal{F}$ is some fuction of the derivatives (∂_i) and the gamma matrices (γ_i).

An explicit form of the function $\mathcal{F}$ was originally determined by Melosh,[7] while Eichten et al.[16] argued that a large class of such functions exists. Without a detailed dynamical theory we are unable to make use of an explicit form, even if it were given to us. The property which we abstract from the free quark model and assume to hold in general is that the <u>effective operators</u> $V^{-1}Q_5^iV$ and $V^{-1}D_{\pm}^{em}V$ <u>have the transformation properties of the most general linear combination of single quark operators consistent with SU(3) and Lorentz invariance.</u>

In the explicit quark model calculations the operator $V^{-1}Q_5^iV$ with $J_z = 0$ contains two terms which transform as $\left\{ (8,1)_0 - (1,8)_0, L_z = 0 \right\}$ and $\left\{ (3,\bar{3})_1, -1 \right\} - \left\{ (\bar{3},3)_{-1}, 1 \right\}$ components of two $\underline{35}$'s of the full $SU(6)_W$ of currents. To apply this to observed hadron transitions, as few axial-vector decays are measured, the Partially Conserved Axial-Vector Current hypothesis[19] (PCAC) can be used to relate the matrix elements of Q_5^i between states at infinite momentum to matrix elements of the pion field. Such an approach results in a theory of the algebraic structure of pion amplitudes.[20]

As the matrix elements of $D_{\pm}^{em}$ (Eq.(II-4)) are directly proportional to photon amplitudes, no additional assumption like PCAC is necessary. Furthermore, matrix elements of D_{+}^{em} are equal, up to a sign, to those of D_{-}^{em} (with reversed helicities of the external states) via parity conservation, so that we need only consider the properties of D_{+}^{em}. The operator $V^{-1}D_{+}^{em}V$, with $J_z = 1$, has slightly more complicated tran-

sformation properties than $V^{-1}Q^i_5 V$. In general, as pointed out by Hey and Weyers,[21] there are four possible terms: $\{(8,1)_0 + (1,8)_0, 1\}$, $\{(3,\bar{3})_1, 0\}$, $\{(\bar{3},3)_{-1}, 2\}$ and $\{(8,1)_0 - (1,8)_0, 1\}$, all components of four $\underline{35}$'s. It appears that all four occur in the operator $V^{-1}D^{em}_{+}V$ in the free quark model. However, the last term, which corresponds to qq in a net quark spin S=0, unnatural spin-parity state, has no analogue with any natural spin-parity (in particular vector meson) state of the quark model.[7,16] Moreover, under a generalized parity transformation, $Pe^{-i\pi J_y}$, which takes

$$\{(A,B)_{S_z}, L_z\} \longrightarrow \{(B,A)_{-S_z}, -L_z\}$$

the first three terms do not change sign while the last one does. For the longitudinal ($J_z = 0$) component of the current this would eliminate the possibility of such a term. While we will carry all four terms in calculating photon transition amplitudes here, we will indicate experimental limits on the size of the $\{(8,1)_0 - (1,8)_0, 1\}$ term's contributions and indicate what situation ensues if it is totally absent.

Assuming that the transformation properties of the effective operators $V^{-1}\mathcal{O}^{\alpha}V$ for $\mathcal{O}^{\alpha} = Q^i_5$ or $D^{em}_{\pm}$ can be abstracted from the free quark model, we are almost in a position to apply the theory to actual decays. To make contact with experiment we make one physical assumption, namely, we assume that we can <u>identify</u> the <u>observed</u> (non-exotic) <u>hadrons</u>

with <u>constituent quark states</u>. In other words we assume that a part of the physical Hilbert space is well approximated by the single particle states of the constituent quark model. For baryons and mesons we have candidates which fit very well into the $SU(6)_W \times O(3)$ representations $\underline{56}$ L=0; $\underline{70}$ L=1; $\underline{56}$ L=2 and $\underline{35}$ L=0; $\underline{1}$ L=0 and $\underline{35}$ L=1, respectively (see Appendix). As we assume that states with different values of quark spin as well as L_z are related according to the constituent quark model, we can relate different helicity states to each other.

With this physical assumption we know the algebraic properties of all terms in the matrix element in Eq.(II-8) under current algebra. Therefore we may use the Wigner-Eckart theorem and tables of Clebsch-Gordan coefficients[22,23] to carry out the calculation from this point onward. Note that $SU(6)_W$ invariance of the effective operators $V^{-1}\mathcal{O}^{\alpha}V$ is not assumed either under the algebra of currents or that of strong interactions - only the transformation properties of the various terms of the effective operators are needed in the calculations.

We will now turn to a detailed discussion of photon decays. Photon transition amplitudes are proportional to the matrix elements of the dipole operator, $D^{em}_{\pm}$, between hadron

states. In the narrow resonance approximation,

$$\Gamma(\text{Hadron}' \to \text{Hadron} + \text{photon})$$

$$= \frac{e^2 p_\gamma^3}{\pi(2J'+1)} \sum_\lambda |\langle \text{Hadron}', \lambda | D_+^{em} | \text{Hadron}, \lambda-1\rangle|^2 \qquad \text{(IV-1)}$$

Here e is the hadron charge, p_γ the photon momentum and the sum extends over all possible helicities λ. Matrix elements of D_-^{em} have been eliminated from Eq.(IV-1) via parity. This equation may also be obtained from consideration of the narrow resonance approximation of the Hadron' contribution to the Cabibbo-Radicati sum rule[24] on Hadron states. We have no arbitrary phase space factors.

We shall use the narrow resonance approximation, Eq.(IV-1), for photon decay widths to compare the theory with experiment. For broad resonances or for decays of resonances where the physically available phase space is small, such an approximation introduces non-negligible errors. However, we consider the present comparison to be sufficiently valid as a first test of the theory, particularly in view of the experimental errors for photon (as well as pion) decay widths. When the situation eventually warrants it, the values of $|\langle \text{Hadron}' | D_\pm^{em} | \text{Hadron}\rangle|^2$ should be determined irrespective of any approximation in terms of contributions to the Cabibbo-Radicati sum rule.[24]

The assumptions on the transformation properties of $D_\pm^{em}$ matrix elements between hadrons under current algebra allow one to express every photon amplitude as a linear combination of four terms, which appear in the effective operator $V^{-1}D_\pm^{em}V$. More explicitly, for a given matrix element, we first decompose the total J of the initial and final hadron (with $J_z = \lambda-1$ and λ, respectively), into definite S_z and L_z quark states. After transformation to the $SU(6)_W$ of currents basis, the matrix element of any particular term in $V^{-1}D_\pm^{em}V$ can be written, using the Wigner-Eckart theorem, as a reduced matrix element times the product of quark angular momentum, $SU(6)_W$, SU(3) and W-spin Clebsch-Gordan coefficients.[22,23] For example, the matrix element of the $\{(\bar{3},3)_{-1},2\}$ term in $V^{-1}D_+^{em}V$ between initial and final states with helicity $\lambda-1$ and λ, total angular momentum J and J', internal quark angular momentum L and L', quark spin S and S', SU(3) representation A and A' and $SU(6)_W$ representation R and R', respectively, is calculated in the following way:

$$\langle R', A', L', S', J', \lambda, \text{currents} | \{(\bar{3},3)_{-1} 2\} | R, A, L, S, J, \lambda-1, \text{currents}\rangle$$

$$= \underbrace{(J'\lambda | L'S'L'_z S'_z)(L\ S\ L_z S_z | J\ \lambda-1)}_{\text{quark angular momentum Clebsch-Gordan cofficients}} \underbrace{\langle R', A'^{2W'+1} | 35, 8 \ | \ R, A^{2W+1}\rangle}_{\text{SU(6) Clebsch-Gordan coefficient}}$$

$$\underbrace{\left\langle A'(Y', I', I'_3) \,|\, 8\left((010)+\frac{1}{\sqrt{3}}(000)\right) \,|\, A\,(Y, I, I_3)\right\rangle}_{\text{SU(3) Clebsch-Gordan coefficient}}$$

$$\underbrace{\langle W'W' | 1W-1W_z\rangle}_{\text{W-spin Clebsch-Gordan coefficient}} \qquad \langle R'L' \| (\bar{3},3)_{-1}, 2 \| R\ L\rangle$$

reduced matrix element

The signs which result from the conversion from quark spin to W-spin[25] in the operators are included in the reduced matrix elements.

The reduction of the other terms in $V^{-1}D_+^{em}V$ proceeds just as above and we need only recall that $(8,1)_0 + (1,8)_0$; $(8,1)_0 - (1,8)_0$ and $(3,\bar{3})_1$ transform as $W=W_z=0$; $W=1, W_z=0$ and $W=1, W_z=1$ objects, respectively. Note that since total J_z is conserved and the net value of $W_z=S_z$ must also be conserved by the W-spin Clebsch-Gordan coefficient in Eq.(IV-2) and its analogues, it follows that $L_z = J_z - S_z$ must also be conserved between the initial and final state (including the photon operator). We then observe that the reduced matrix elements depend on the $SU(6)_W$ multiplet, the L_z' and L_z values of the external states components, and the particular term in $V^{-1}D_+^{em}V$.

If L is zero, as is the case in essentially all cases of physical interest at the present time, then of course $L_z=0$ and the L_z' dependence of the $SU(6)_W$ reduced matrix element becomes trivial due to conservation of L_z. In such a case (L=0), all photon decays from one $SU(6)_W$ multiplet to another are related to the same four reduced matrix elements (dropping the trivial L_z labels):

$$\langle R'\ L' \parallel (8,1)_0 + (1,8)_0 \parallel R\ L=0 \rangle$$

$$\langle R'\ L' \parallel (3,\bar{3})_1 \parallel R\ L=0 \rangle$$

$$\langle R'\ L' \parallel (\bar{3},3)_{-1} \parallel R\ L=0 \rangle$$

and $$\langle R'\ L' \parallel (8,1)_0 - (1,8)_0 \parallel R\ L=0 \rangle$$

some of which may be zero or may have zero coefficients due to selection rules in particular decays.

This algebraic structure of photon matrix elements does in fact lead to interesting and powerful selection rules. Consider the $\{(8,1)_0 + (1,8)_0, 1\}$ term in $V^{-1}D_+^{em}V$, which has W-spin zero. The W-spin Clebsch-Gordan coefficient in the analogue of Eq.(IV-2) implies

$$\vec{W}' = \vec{W} \qquad \text{(IV-3)}$$

which is the same as

$$\vec{S}' = \vec{S} \qquad \text{(IV-4)}$$

Now, for Hadron' and Hadron states we have

$$\vec{J}' = \vec{L}' + \vec{S}' \qquad \text{(IV-5)}$$

and

$$\vec{J} = \vec{L} + \vec{S} \qquad \text{(IV-6)}$$

while angular momentum conservation for the total decay demands

$$\vec{J}' = \vec{J} + \vec{j}_\gamma \qquad \text{(IV-7)}$$

where j_γ is the net angular momentum carried by the photon and determines the multipole character of the decay. Combining Eqs.(IV-4) to (IV-7) we obtain

$$|L - L'| \leqslant j_\gamma \leqslant |L + L'| \qquad \text{(IV-8)}$$

and in the case L=0

$$j_\gamma = L' \qquad \text{(IV-9)}$$

Thus decays through the $\{(8,1)_0 + (1,8)_0, 1\}$ term in $V^{-1}D_+^{em}V$ to L=0 baryons and mesons always have $j_\gamma = L'$ of the decaying

hadron. As the parity change is $(-1)^{L'-L}=(-1)^{L'}=(-1)^{j_\gamma}$, this always corresponds to an electric 2L'-pole transition in the usual multipole notation.

For the $\{(3,\bar{3})_1,0\}$, $\{(\bar{3},3)_1,2\}$ and $\{(8,1)_0-(1,8)_0,1\}$ terms in $V^{-1}D_+^{em}V$, all of which have W-spin one, Eq.(IV-4 is modified to[26]

$$\vec{S}' = \vec{S} + \vec{1} \qquad \text{(IV-10)}$$

and as a result one finds in place of Eq.(IV-9) that

$$\big|\,|L'-L|-1\,\big| \leqslant j_\gamma \leqslant |L'+L|+1\,| \qquad \text{(IV-11)}$$

so that

$$j_\gamma = L'-1,\ L',\ L'+1\ . \qquad \text{(IV-12)}$$

As the parity change is again $(-1)^{L'}$, these correspond to magnetic 2(L'-1)-pole, electric 2L'-pole and magnetic 2(L'+1)-pole, respectively.

The actual correspondence between reduced matrix elements and a set of multipole amplitudes can also be proven using Racah coefficients to rewrite Eq.(IV-2) and its analogues. For example, baryon transitions from R, L=0 to R',L' can be described in terms of multipole amplitudes

$$M\,(j_\gamma=L') = \langle R'L'\|\,(8,1)_0+(1,8)_0\,\|\,R\ L=0\rangle \qquad \text{(IV-14a)}$$

$$\begin{aligned} M\,(j_\gamma=L'-1,L',L'+1) &= (1,L',1,0|j_\gamma,1)\langle R'L'\|\,(3,\bar{3})_1\,\|R\ L=0\rangle \\ &+ (1,L',0,1|j_\gamma,1)\langle R'L'\|\,(8,1)_0-(1,8)_0\,\|R\ L=0\rangle \\ &+ (1,L',-1,2|j_\gamma,1)\langle R'L'\|\,(\bar{3},3)_{-1}\,\|R\ L=0\rangle. \end{aligned} \qquad \text{(IV-14b)}$$

Note ,of course, that one only has $j_\gamma \geqslant 1$. Thus for $L'=0\rightarrow L=0$ only $j_\gamma=1$ is allowed. This is just the old result that the nucleon to 3-3 resonance transition is magnetic dipole in character.

Note that for values $L'\geqslant 3$, not only does the theory limit values of j_γ to $j_\gamma \leqslant L'+1$, but also non-trivially forbids values of j_γ less than L'-1,[27] which are otherwise kinematically allowed , and even favored by angular momentum barrier arguments. The transition of a $J^P=3/2^-$ baryon resonance in a $\underline{70}$ L'=3 multiplet into a nucleon plus a photon with $j_\gamma=1$ is forbidden, for example, even though this is the lowest allowed multipole on spin-parity grounds.

The algebraic structure of the theory of photon transitions presented above is closely related to various quark model calculations, both non-relativistic[28] and relativistic,[29] done in the past. They may be put into one to one correspondence if the $\{(8,1)_0+(1,8)_0,1\}$ term in $V^{-1}D_+^{em}V$ is identified with the photon interacting with the quark convection current, and the $\{(3,\bar{3})_1,0\}$ term identified with the photon interacting with the quark magnetic moment. The $\{(\bar{3},3)_{-1},2\}$ and $\{(8,1)_0-(1,8)_0,1\}$ terms are absent in

these quark models.[28,29] However, the assumption of a "potential" and the resulting wave function for the bound states in the quark model calculations yield definite predictions for the reduced matrix elements themselves, as they depend on masses and other parameters of the models. This is something we do not obtain, since we consider only the algebraic structure.

Closely related to the quark model results are those following from various versions of $SU(6)_W$ (of strong interactions) invariance. The assumption of $SU(6)_W$ conservation plus vector dominance for photon transitions is equivalent to keeping only the $\{(3,\bar{3})_1,0\}$ term in $V^{-1}D^{em}_{+}V$.

As we will soon see, this is totally contradicted by the data. As a result, various broken $SU(6)_W$ schemes were developed.[30] Some of these are very similar to the present theory in algebraic structure, particularly for decays to L=0 hadrons. For vector meson decays, and via vector dominance for photon decays, one such scheme[31] corresponds in algebraic structure to the one presented here if the reduced matrix elements of $\{(\bar{3},3)_{-1},2\}$ vanish and those of $\{(8,1)_0+(1,8)_0,1\}$ and $\{(8,1)_0-(1,8)_0,1\}$ terms are equal.

V. PHOTON TRANSITIONS OF MESONS

Now that the basic properties of the theory and the manner of its application to actual hadrons have been spelled out, we begin the discussion of detailed predictions with radiative decays of mesons. We limit our listing of amplitudes to those corresponding to non-strange mesons; the extension to transitions involving strange mesons is easily accomplished using SU(3).

Let us begin with the photon transitions from L'=0 to L=0 mesons, i.e., among the members of the $SU(6)_W$ $\underline{35}$ and $\underline{1}$, whose non-strange members are ρ, ω, ϕ, π, η and (presumably) X^0. As $L'_z=L_z=0$ for the external states since L'=L=0, only the term with $L_z=0$ and transforming as $\{(3,\bar{3})_1,0\}$ in $V^{-1}D^{em}_{+}V$ can contribute. The selection rule in Eq.(IV-12) immediately gives the result that $j_\gamma=1$ only. This is already non-trivial, as $j_\gamma=2$ transitions are possible from $\rho^{\pm}$ to $\rho^{\pm}$ in general, and the theory then predicts zero electric quadrupole moment for the ρ-meson.

Since W-spin zero octets and singlets belong to the $\underline{35}$ and $\underline{1}$ representations of $SU(6)_W$, respectively, decays involving meson states which are mixtures of W=0 SU(3) octets and singlets may be used to fix the ratio of the $\langle \underline{1}\ L'=0 \parallel (3,\bar{3})_1 \parallel \underline{35}\ L=0\rangle$ and $\langle \underline{35}\ L'=0 \parallel (3,\bar{3})_1 \parallel \underline{35}\ L=0\rangle$. In particular, for this purpose we use Zweig's rule[32] to forbid the decay $\phi\to\gamma\pi$, where the ϕ is assumed to be the usual

ideal mixture of singlet and octet so as to be composed of purely strange quarks. All amplitudes are then multiples of a single magnetic dipole amplitude, or alternatively, are proportional to the single reduced matrix element

$$\langle \underline{35}\ L'=0 \parallel (3,\bar{3})_1 \parallel \underline{35}\ L=0\rangle.$$

One observed transition then fixes all other decays.[33] The result of the computation of transition matrix elements are given in Table V.A, where the η and X° are assumed to be SU(3) octet and singlet, respectively, while the ω and ϕ are ideal-mixtures of octet and singlet:

$$\omega = \cos\theta\ \omega^{(1)} + \sin\theta\ \omega^{(8)}$$

$$\phi = -\sin\theta\ \omega^{(1)} + \cos\theta\ \omega^{(8)} \qquad \text{(V-1)}$$

where

$$\sin\theta = \frac{1}{\sqrt{3}}$$

Table V.B contains the corresponding predictions for all the $L'=0 \to L=0$ radiative decay widths using $\Gamma(\omega \to \gamma\pi)$ =890 Kev as an input.[34] The sparse experimental data[34,35] are also given. Note that the predictions in the first column are for unmixed pseudoscalar mesons. Taking a mixing angle[36] $\theta_P = -10.5°$, as suggested by a quadratic mass formula, gives the second column. The predicted width for $\phi \to \gamma\eta$ is reduced to 179 KeV, agreeing with experiment within errors.[37] The corresponding prediction in this case for $\Gamma(X^\circ \to \gamma\rho)$ is 120 KeV. Assuming that $X^\circ \to \gamma\pi^+\pi^-$ is dominated by $X^\circ \to \gamma\rho$, and taking the

branching ratio[34] for this mode to be 26%, we find a total X° width of 460 KeV. This is also consistent with the X° width obtained from the branching ratio[34] for $X^\circ \to \gamma\gamma$ plus SU(3) and the new value[38] of $\Gamma(\eta \to \gamma\gamma)$. The overall situation for $L'=0 \to L=0$ decays is thus quite satisfactory, although many pieces of information are absent in comparing theory and experiment.

When we go to $L'=1 \to L=0$ decays, there is no experimental information available, although there are both many allowed transition amplitudes and predictions. Of the four terms generally present in $V^{-1}D_+^{em}V$, only $\{(\bar{3},3)_{-1},2\}$ cannot contribute here, since it changes L_z by two units. The selection rules of Section IV show that the $\{(8,1)_0 + (1,8)_0, 1\}$ term in $V^{-1}D_+^{em}V$ leads to purely electric dipole ($j_\gamma=1$) transitions, and only $j_\gamma = 1$ and 2 can arise from the $\{(3,\bar{3})_1, 0\}$ and $\{(8,1)_0 - (1,8)_0, 1\}$ terms. In fact it is possible to express linear combinations of their reduced matrix elements as electric dipole and magnetic quadrupole amlitudes, multiples of which occur in all decays from $L'=1$ to $L=0$ mesons.

All posible radiative decay amplitudes for non-strange $L'=1 \to L=0$ mesons are given in Table V.C in terms of the reduced matrix elements

$$\langle \underline{35}\ L'=1 \parallel (8,1)_0 + (1,8)_0 \parallel \underline{35}\ L=0\rangle,$$

$$\langle \underline{35}\ L'=1 \parallel (3,\bar{3})_1 \parallel \underline{35}\ L=0\rangle,$$

and

$$\langle \underline{35}\ L'=1 \parallel (8,1)_0 - (1,8)_0 \parallel \underline{35}\ L=0\rangle\ .$$

Matrix elements of $SU(6)_W$ singlet states are related to those of $\underline{35}$'s using Zweig's rule,[32] as was done above for $L'=0 \rightarrow L=0$ decays. The η and H are assumed to be purely octet members, while f, D, σ, and ω are all taken to be ideal mixtures of singlets and octets, so as to be composed of only non-strange quarks. Note that in the decay $2^+ \rightarrow \gamma 1^-$, e.g., in $A_2 \rightarrow \gamma\rho$, an electric octupole could be present in principle, as well as electric dipole and magnetic quadrupole amplitudes. However, the selection rule limiting j_γ to 1 or 2 eliminates the octupole amplitude and results in the linear relation

$$A_{\lambda=2}(A_2 \rightarrow \gamma\rho) = 2\sqrt{2}\, A_{\lambda=1}(A_2 \rightarrow \gamma\rho) - \sqrt{6}\, A_{\lambda=0}(A_2 \rightarrow \gamma\rho). \tag{V-2}$$

among the three helicity amplitudes for $2^+ \rightarrow \gamma 1^-$. Almost any experimental information on these decays would be helpful in sorting out the relative importance of the various (three) possible amplitudes, and testing the theory.

VI. PHOTON TRANSITIONS BETWEEN BARYONS

The electromagnetic transitions of baryons provide a second and very rich area of predictions for the theory. As before, we restrict our attention to non-strange baryons decaying into L=0 states, this being by far the main area for experimental comparison. In this section we will enumerate

the possible decay amplitudes, deferring an experimental comparison to the next section.

The case of transitions from L'=0 to L=0, i.e., within the L=0 baryon multiplet, is particularly simple. As for mesons, only magnetic dipole transitions are allowed by the theory and all amplitudes are proportional to a single reduced matrix element, that of the term transforming as $\{(3,\bar{3})_1, 0\}$ in $V^{-1}D_+^{em}V$. The results are presented in Table VI.A for the three possible transitions, $N \rightarrow N$, $N \rightarrow \Delta$, and $\Delta \rightarrow \Delta$. It can be explicitly checked that all transitions are magnetic dipole in character, as demanded by the selection rule (Eq.(IV-12)), including those for $\Delta \rightarrow \Delta$, where both electric quadrupole and magnetic octupole transitions are allowed by spin and parity.

For decays from the next identified baryon multiplet, the 70 L'= 1 to the ground state $\underline{56}$ L=0 we have the three possible reduced matrix elements:

$$\langle \underline{70}\ L'=1 \parallel (8,1)_0 + (1,8)_0 \parallel \underline{56}\ L=0 \rangle$$

$$\langle \underline{70}\ L'=1 \parallel (3,\bar{3})_1 \parallel \underline{56}\ L=0 \rangle$$

and $$\langle \underline{70}\ L'=1 \parallel (8,1)_0 - (1,8)_0 \parallel \underline{56}\ L=0 \rangle.$$

The matrix elements of D_+^{em} for decays into both γN and $\gamma\Delta$ are enumerated[40] in Table VI.B in terms of these reduced matrix elements.

By the selection rules of Section IV, the $(8,1)_0 + (1,8)_0$ term in $V^{-1}D_+^{em}V$ acts as an electric dipole transition opera-

tor, while the two remaining terms act as a combination of electric dipole ($j_\gamma = 1$) and magnetic quadrupole ($j_\gamma = 2$). According to the discussion around Eq.(IV-14) in Section IV we can in fact write amplitudes,

$$
\begin{aligned}
E1' &= \langle \underline{70}\ L'=1 \parallel (8,1)_0 + (1,8)_0 \parallel \underline{56}\ L=0 \rangle \\
E1 &= \sqrt{1/2}\ \langle \underline{70}\ L'=1 \parallel (3,\bar{3})_1 \parallel \underline{56}\ L=0 \rangle \\
&\quad - \sqrt{1/2}\ \langle \underline{70}\ L'=1 \parallel (8,1)_0 - (1,8)_0 \parallel \underline{56}\ L=0 \rangle \\
M2 &= \sqrt{1/2}\ \langle \underline{70}\ L'=1 \parallel (3,\bar{3})_1 \parallel \underline{56}\ L=0 \rangle \\
&\quad + \sqrt{1/2}\ \langle \underline{70}\ L'=1 \parallel (8,1)_0 - (1,8)_0 \parallel \underline{56}\ L=0 \rangle ,
\end{aligned}
$$

which are electric dipole and magnetic quadrupole amplitudes in terms of which all the helicity amplitudes given in Table V.B may be alternately expressed. Note that $N^*(J^P=5/2^-) \to \gamma N$, for example, could in general go via $j_\gamma=2$ or 3, but only $j_\gamma=2$ (magnetic quadrupole) is allowed by the theory. Similarly, $N^*(5/2^-) \to \gamma \Delta$ could proceed with $j_\gamma=1,2,3$ or 4 in general, but only $j_\gamma=1$ and 2 are allowed by the selection rules. Note also that the Moorhouse quark model selection rule[41] forbidding $\gamma p \to N^*$, where N^* has quark spin S=3/2, is reflected in Table VI.B.

For $\underline{56}$ L'=2 decays to $\underline{56}$ L=0 we have reached a high enough value of L' that all four terms in $V^{-1} D^{em} V$ can contribute to decay amplitudes. In this case the $(8,1)_0 + (1,8)_0$ term is electric quadrupole in character, while linear combinations of the other three terms act as $j_\gamma = 1,2$ and 3 transitions:

$$E2' = \langle \underline{56}\ L'=2 \parallel (8,1)_0 + (1,8)_0 \parallel \underline{56}\ L=0 \rangle$$

$$
\begin{aligned}
M1 &= \sqrt{1/10}\ \langle \underline{56}\ L'=2 \parallel (3,\bar{3})_1 \parallel \underline{56}\ L=0 \rangle \\
&\quad - \sqrt{3/10}\ \langle \underline{56}\ L'=2 \parallel (8,1)_0 - (1,8)_0 \parallel \underline{56}\ L=0 \rangle \\
&\quad + \sqrt{3/5}\ \langle \underline{56}\ L'=2 \parallel (\bar{3},3)_1 \parallel \underline{56}\ L=0 \rangle
\end{aligned}
$$

$$
\begin{aligned}
E2 &= \sqrt{1/2}\ \langle \underline{56}\ L'=2 \parallel (3,\bar{3})_1 \parallel \underline{56}\ L=0 \rangle \\
&\quad - \sqrt{1/6}\ \langle \underline{56}\ L'=2 \parallel (8,1)_0 - (1,8)_0 \parallel \underline{56}\ L=0 \rangle \\
&\quad - \sqrt{1/3}\ \langle \underline{56}\ L'=2 \parallel (\bar{3},3)_{-1} \parallel \underline{56}\ L=0 \rangle
\end{aligned}
$$

$$
\begin{aligned}
M3 &= \sqrt{6/15}\ \langle \underline{56}\ L'=2 \parallel (3,\bar{3})_1 \parallel \underline{56}\ L=0 \rangle \\
&\quad + \sqrt{8/15}\ \langle \underline{56}\ L'=2 \parallel (8,1)_0 - (1,8)_0 \parallel \underline{56}\ L=0 \rangle \\
&\quad + \sqrt{1/15}\ \langle \underline{56}\ L'=2 \parallel (\bar{3},3)_{-1} \parallel \underline{56}\ L=0 \rangle
\end{aligned}
$$

The various amplitudes for resonances in the $\underline{56}$ L'=2 to decay into γN are listed[40] in Table VI.C. The $\gamma\Delta$ amplitudes presented in Table VI.D are not experimentally testable as yet. Again, the selection rules derived in Section IV have clear and direct consequences: $\Delta^*(7/2^+) \to \gamma N$, for example, which could go via $j_\gamma=3$ or 4 is restricted to be purely magnetic octupole ($j_\gamma=3$).

Decays from higher L' multiplets are easily computable, but little in the way of experimental tests is available at present. For $\underline{56}$ L'=0, $\underline{70}$ L'=1 and $\underline{56}$ L'=2 photon transition amplitudes, which we have enumerated, however, photo-

production data permit many direct experimental comparisons. To these we now turn.

VII. EXPERIMENTAL TESTS OF BARYON AMPLITUDES

The predictions for transitions within 56 L=0 multiplet are already testable using the magnetic moments of the neutron and proton, for a direct evaluation of D_+^{em} between nucleon states at infinite momentum gives

$$\mu_A(N) = \sqrt{\tfrac{1}{2}}\,\langle N, \lambda = \tfrac{1}{2} \mid D_+^{em} \mid N, \lambda = -\tfrac{1}{2}\rangle \qquad \text{(VII-1)}$$

where $\mu_A(N)$ is the anomalous magnetic moment of the nucleon. However, a careful evaluation of $V^{-1}D_+^{em}V$ between one nucleon states at infinite momentum gives a result which has the transfomation properties of the four terms discussed in Section IV minus a term which is exactly the Dirac moment. Adding the Dirac moment to the anomalous moment, we see that the four terms in $V^{-1}D_+^{em}V$ discussed before should be interpreted as being proportional to the total moment when taken between the same initial and final state. Thus, the matrix elements in Table VI.A are to be interpreted as predicting

$$\mu_T(n)/\mu_T(p) = -\frac{2}{3} \qquad \text{(VII-2)}$$

the SU(6) result,[42] which is within 5% of the experimental value of (-1.91/2.73) = -0.70.

For the transitions from Δ to N the ratio of $\sqrt{3}$ between the λ=3/2 and λ=1/2 matrix elements corresponds to a pure magnetic dipole transition, as we already know must occur due to the selection rules discussed in the last section. All photoproduction analyses[43] agree that the electric quadrupole amplitude is at most a few percent of the magnetic dipole amplitude for the excitation of the 3-3 resonance. Using the conventional definition of μ^*, Eq.(II-7), we obtain from the results in Table VI.A that

$$\mu^*/\mu_T(p) = \frac{2\sqrt{2}}{3} \qquad \text{(VII-3)}$$

An older phenomenological analysis[44] of the data for pion photoproduction gave a result for $\mu^*/\mu_T(p)$ which is 1.28 ± 0.03 times the right hand side of Eq.(VII-3) by finding a residue at the Δ-pole in $\gamma N \to \pi N$. By considering the contribution[45] of the Δ to the Cabibbo-Radicati sum rule we find a value of $\mu^*/\mu_T(p)$ which is 0.9 ± 0.1 times the right hand side of Eq.(VII-3), in quite satisfactory agreement with the theory. While the sign of $\mu^*/\mu_T(p)$ cannot be measured, the product of the γN and πN couplings of the nucleon can be compared with that of the 3-3 resonance in pion photoproduction. As the theory also predicts the relative sign of πN couplings,[8,9] it makes an unambiguous prediction of the sign of the reso-

nance excitation amplitude relative to the nucleon Born terms. This sign is correctly given by the theory.[46]

For the transitions from Δ to Δ, which is also purely magnetic dipole in character, we should again interpret the results in Table VI.A as being for the <u>total</u> moment. The relation between matrix elements of D_+^{em} and the conventional anomalous magnetic moment of the Δ, μ_A^{**}, is

$$\mu_A^{**} = -\sqrt{\tfrac{3}{2}}\langle \Delta, \lambda = \tfrac{3}{2} \mid D_+^{em} \mid \Delta, \lambda = \tfrac{1}{2}\rangle_{p_z \to \infty} \qquad \text{(VII-4)}$$

From this we see that we have from Table VI.A

$$\begin{aligned} \mu_\pi^{**}(\Delta^{++})/\mu_\pi(p) &= 2 \\ \mu_\pi^{**}(\Delta^{+})/\mu_\pi(p) &= 1 \\ \mu_\pi^{**}(\Delta^{0})/\mu_\pi(p) &= 0 \\ \mu_\pi^{**}(\Delta^{-})/\mu_\pi(p) &= -1 \end{aligned} \qquad \text{(VII-5)}$$

As with Eqs.(VII-2) and (VII-3), all these are standard SU(6) results,[42] as is to be expected since the $\{(3,\bar{3})_1, 0\}$ term in $V^{-1}D_+^{em}V$ has the same transformation properties as the magnetic moment used in SU(6).

The transitions from the <u>70</u> L'=1 to the ground state <u>56</u> L=0 provide a much richer set of amplitudes for comparison of the theory and experiment. Rather than carry out a statistical "best fit" to all data, in Table VII.A we have fixed the possible reduced matrix elements allowed by the

theory in terms of some relatively well determined amplitudes for the process $\gamma N \to D_{13}(1520) \to \pi N$.

The quantities in the table are the matrix elements of D_+^{em} taken between identified resonant states[47] in the <u>70</u> with $J_z = \lambda$ and nucleon states with $J_z = \lambda - 1$. The signs are those found in the specific processes $\gamma p \to N^{*+} \to \pi^+ n$ and $\gamma n \to N^{*0} \to \pi^- p$. To make a theoretical prediction of these signs we need a theory of both the γNN^* and πNN^* vertices. The γNN^* couplings are taken from Table VI.B while for <u>70</u> L'=1 $\to$ <u>56</u> L=0 pion transitions we may express the reduced matrix elements as linear combinations of amplitudes S and D, corresponding to $\ell = 0$ and 2:[48]

$$\begin{aligned} \langle \underline{70}\ L'=1 \parallel (8,1)_0 - (1,8)_0 \mid \underline{56}\ L=0\rangle &= (S + 2D)/3 \\ \langle \underline{70}\ L'=1 \parallel (3,\bar{3})_1 - (\bar{3},3)_{-1} \mid \underline{56}\ L=0\rangle &= (S - D)/3 \end{aligned} \qquad \text{(VII-6)}$$

S=+D if only the $(8,1)_0 - (1,8)_0$ term in $V^{-1}D_+^{em}V$ is present, while S=-2D if only $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ is present. While an earlier phase shift solution[49] to the $\pi N \to \pi\Delta$ data disagreed with the signs predicted for pion transitions, a new solution agrees completely[50] and shows that the signs of S and D are opposite, i.e., it appears that the $(3,\bar{3})_1 - (\bar{3},3)_{-1}$ reduced matrix element is dominant for <u>70</u> L'=1 $\to$ <u>56</u> L=0 pion decays. In constructing Table VII.A we have taken the πNN^* couplings from Table V of Ref.9, and have assumed opposite signs for S and D in calculating the πNN^* vertex. Mi-

xing between the two S_{11} or two D_{13} states in the $\underline{70}$ has been neglected in computing the predicted amplitudes.

The "data" is taken from a recent analysis of electromagnetic couplings of N^* resonances from single pion photoproduction data.[51] In terms of amplitudes A_λ for $\gamma N \to N^*$ of that analysis,[51] matrix elements of D_+^{em} are

$$\langle N^*, \lambda \mid D_+^{em} \mid N, \lambda-1 \rangle = \left(\frac{M_N}{2\pi\alpha M_{N^*} p_\gamma} \right)^{\frac{1}{2}} A_\lambda \qquad \text{(VII-7)}$$

where p_γ is the photon momentum in the N^* rest frame, and λ can take the values 1/2 and 3/2. The results of Ref.51 generally agree well with those of another recent analysis,[52] although the "errors" on the amplitudes quoted in the latter are much larger. Judging from the differences between succesive or independent analyses, we would opt for larger "errors" than those in Ref.51 , which are reproduced in Table VII.A.

As a first comparison, we set the reduced matrix element $\langle \underline{70}\ L'=1 \parallel (8,1)_0 - (1,8)_0 \parallel \underline{56}\ L=0 \rangle$ equal to zero, so that we are left with only two terms in $V^{-1} D_+^{em} V$, which are those present in quark model calculations.[28,29] The well determined amplitude for $\gamma p \to D_{13}(1520)$ with $\lambda=3/2$ then determines the reduced matrix element $\langle \underline{70}\ L'=1 \mid (8,1)_0 + (1,8)_0 \parallel \underline{56}\ L=0 \rangle$ directly and fixes an overall free sign. The $\lambda=1/2$ transition to the same resonance then determines the other reduced

matrix element, $\langle \underline{70}\ L'=1 \parallel (3,\bar{3})_1 \parallel \underline{56}\ L=0 \rangle$. In fact, the smallness of the $\lambda=1/2$ amplitude means that

$$\langle \underline{70}\ L'=1 \parallel (8,1)_0 + (1,8)_0 \parallel \underline{56}\ L=0 \rangle$$
$$\simeq 2 \langle \underline{70}\ L'=1 \parallel (3,\bar{3})_1 \parallel \underline{56}\ L=0 \rangle \qquad \text{(VII-8)}$$

All the well determined signs of the resulting amplitudes agree with experiment (9, in addition to input). However, the magnitudes of a number of the predicted amplitudes are not in such great agreement with experiment. The $\lambda = 3/2$ amplitude for $\gamma n \to D_{13}^0(1520)$ is too large. Mixing, at least with the small mixing angles otherwise suggested,[53] will not cure this, although it could well help to improve the situation with regard to the poorly known $D_{13}(1700)$ amplitudes.

For the two S_{11} states, a fairly large mixing angle is known to be necessary from other considerations,[53] and such an angle would give $S_{11}(1700)$ amplitudes which agree with experiment in sign. The predicted $S_{11}(1535)$ amplitudes would still be much too large, however. The amplitudes predicted for S_{31} and D_{33} also are too large, and no mixing (within the $\underline{70}$ L=1) is possible in these cases. A fit to all the data would of course scale down the reduced matrix elements, making the agreement better for the magnitudes of S_{31}, D_{33} and S_{11} amplitudes, at some cost to those of $D_{13}(1520)$.

A second comparison of the theory with experiment is also found in Table VII.A where all three possible reduced matrix elements are allowed to be non-zero, and fixed by the transitions $\gamma p \to D_{13}^+(1520)$ with $\lambda=1/2$ and 3/2, and by

$\gamma n \to D^{o}_{13}(1520)$ with $\lambda = 3/2$. Again, all the well determined signs agree with experiment, although the predicted (and poorly determined experimentally) signs for $D_{13}(1700)$ and $S_{11}(1700)$ are opposite to those discussed above.

There still is trouble in this case with the magnitudes of various amplitudes. The $\lambda = 1/2$, $\gamma n \to D^{o}_{13}(1520)$ amplitude is too small, as is the amplitude for $\gamma p \to S^{+}_{11}(1535)$. Mixing only hurts here, as the γp transition to the other S_{11} is forbidden, resulting in an even smaller prediction for $\gamma p \to S^{+}_{11}(1535)$ and too small a result as well for $\gamma p \to S^{+}_{11}(1700)$. Although the D_{33} amplitude predictions now agree well with experiment, that for S_{31} is still much too large.

It is interesting to note that for this second fit we have

$$\langle \underline{70}\ L'=1 \parallel (8,1)_o + (1,8)_o \parallel \underline{56}\ L=0\rangle$$
$$\simeq \langle \underline{70}\ L'=1 \parallel (8,1)_o - (1,8)_o \parallel \underline{56}\ L=0\rangle \qquad \text{(VII-9)}$$
$$\langle \underline{70}\ L'=1 \parallel (3,\bar{3})_1 \parallel \underline{56}\ L=0\rangle \simeq 0.$$

Equality of the first two reduced matrix elements is exactly what is forced by vector dominance plus the scheme of Petersen and Rosner[31] for vector meson decays. The reason why $\langle \underline{70}\ L'=1 \parallel (3,\bar{3})_1 \parallel \underline{56}\ L=0\rangle$ should be small, which in the fit is forced by the smallness of the amplitude $\gamma p \to D^{+}_{13}(1520)$ with $\lambda = 1/2$, is possibly an interesting theoretical problem.

At the present time, given the uncertainties we feel exist in the electromagnetic couplings of the N^{*}'s, either set of predictions should be regarded as in fair agreement with experiment as far as magnitudes are concerned. The signs in either case are a triumph of the theory for both pion and photon transitions and verify that S and D amplitudes have opposite signs.

For transitions from the $\underline{56}$ $L'=2$ to the ground state $\underline{56}$ $L=0$ we also have in principle a large set of allowed amplitudes for comparison with experiment. In practice the amplitudes are less well known, as shown in Table VII.B. The quantities in the Table, as in the previous one, are matrix elements of D^{em}_{+} with signs appropriate for $\gamma p \to N^{*+} \to \pi^{+} n$ and $\gamma n \to N^{*o} \to \pi^{-} p$. For the $\pi N N^{*}$ vertex we express the two reduced matrix elements for $\underline{56}$ $L'=2 \to \underline{56}$ $L=0$ pion decays as[48]

$$\langle \underline{56}\ L'=2 \parallel (8,1)_o - (1,8)_o \parallel \underline{56}\ L=0\rangle = (2P + 3F)/5$$
$$\langle \underline{56}\ L'=2 \parallel (3,\bar{3})_1 - (\bar{3},3)_{-1} \parallel \underline{56}\ L=0\rangle = \sqrt{3}(P - F)/5\ , \qquad \text{(VII-10)}$$

where the amplitudes P and F correspond to $\ell = 1$ or 3 pion angular momenta, respectively. The relative signs of P and F are the same (opposite), if the $(8,1)_o - (1,8)_o$ ($(3,\bar{3})_1 - (\bar{3},3)_{-1}$) matrix element dominates. The reaction[48,50] $\pi N \to \pi \Delta$ indicates that P and F have the same sign, and we use this together with Table VI of Ref.9 in constructing Table VII.B. The "data" is again from Ref.51.

To compare theory and experiment, we simplify the situation for the photon vertex by setting

$$\langle \underline{56}\ L'=2 \parallel (\bar{3},3)_{-1} \parallel \underline{56}\ L=0\rangle = 0$$

and

$$\langle \underline{56}\ L'=2 \parallel (8,1)_0 - (1,8)_0 \parallel \underline{56}\ L=0\rangle = 0,$$

This leaves only the $\langle \underline{56}\ L'=2 \parallel (8,1)_0 + (1,8)_0 \parallel \underline{56}\ L=0\rangle$ and $\langle \underline{56}\ L'=2 \parallel (3,\bar{3})_1 \parallel \underline{56}\ L=0\rangle$ reduced matrix elements, as would be the case in most quark model calculations.[28,29] Rather than making a fit to all the amplitudes, we use the well measured $\gamma p \to F_{15}^{+}$ amplitudes to fix the two reduced matrix elements, and then calculate the remaining amplitudes.

All the experimentally well determined signs, with the possible exception of the F_{35} amplitude with $\lambda = 3/2$, agree well with the theory. In a previous analysis,[54] both the F_{35} amplitudes also agreed. The signs of the $P_{33}(2000)$ amplitudes among the p-wave πN resonances, provide some (marginal) support for the P and F amplitudes at the pion vertex to have the same sign, as the $\pi N \to \pi\Delta$ analysis shows much more definitely.[43,50]

The magnitudes of the predicted amplitudes are in fair agreement with what is observed. There is no need to allow $\langle \underline{56}\ L'=2 \parallel (8,1)_0 - (1,8)_0 \parallel \underline{56}\ L=0\rangle$ and $\langle \underline{56}\ L'=2 \parallel (\bar{3},3)_1 \parallel \underline{56}\ L=0\rangle$ to be non-zero. In fact, fitting all four reduced matrix

elements to $\gamma p \to F_{15}^{+}$ with $\lambda=1/2$ and $3/2$, $\gamma n \to F_{15}^{0}$ with $\lambda=3/2$, and $\gamma p \to P_{33}^{+}$ with $\lambda=1/2$ results in essentially the same predictions; the two additional reduced matrix elements have values more than an order of magnitude smaller than either $\langle \underline{56}\ L'=2 \parallel (3,\bar{3})_1 \parallel \underline{56}\ L=0\rangle$ or $\langle \underline{56}\ L'=2 \parallel (8,1)_0 + (1,8)_0 \parallel \underline{56}\ L=0\rangle$. The smallness of the $\lambda=3/2$ amplitude for $\gamma n \to F_{15}^{0}$ by itself assures the strong constraint on the two additional reduced matrix elements,

$$-\frac{4}{45}\langle \underline{56}\ L'=2 \mid (\bar{3},3)_{-1} \mid \underline{56}\ L=0\rangle \simeq +\frac{4\sqrt{2}}{45}\langle \underline{56}\ L'=2 \parallel (8,1)_0 - (1,8)_0 \parallel \underline{56}\ L=0\rangle. \qquad \text{(VII-11)}$$

There is thus fairly good evidence in this case that only the two reduced matrix elements found in the quark model are present at a significant strength, and, in particular, that equality of $\langle \underline{56}\ L'=2 \parallel (8,1)_0 + (1,8)_0 \parallel \underline{56}\ L=0\rangle$ and $\langle \underline{56}\ L'=2 \parallel (8,1)_0 - (1,8)_0 \parallel \underline{56}\ L=0\rangle$ is ruled out.

Finally we examine the transitions from a "radially excited" $\underline{56}\ L'=0$ back to the ground state $\underline{56}\ L=0$. The $\underline{56}\ L'=0$ includes the Roper resonance $P_{11}(1470)$ and $P_{33}(1718)$. We fit the one possible reduced matrix element, $\langle \underline{56}\ L'=0 \parallel (3,\bar{3})_1 \parallel \underline{56}\ L=0\rangle$ to the amplitude for $\gamma p \to P_{11}(1470)$ and predict the other amplitudes in Table VII.C using the $\underline{56}\ L'=0 \to \underline{56}\ L=0$ matrix elements from Table VI.A. Again the signs are those in $\gamma p \to \pi^{+} n$ and $\gamma n \to \pi^{-} p$. The experimental results of both the Berkeley[51] and Lancaster[52] analyses are shown,

there being some discrepancy between the two.

VIII. SUMMARY AND CONCLUSIONS

The operator V, which by definition takes us from a current to constituent quark basis, contains in principle all the information about matrix elements of the weak and electromagnetic currents when taken between hadron states, assuming that the hadrons can be treated as if constructed out of (constituent) quarks. Knowing V, we would know the exact transformation properties of hadrons under current algebra. We have shown in Section III that these properties are complicated, and that a phenomenological attempt to determine them by fitting experimentally known transitions does not lead to a unique solution, as the number of parameters involved is greater than the number of available experimental quantities. Lacking a complete knowledge of V, we have abstracted only certain of its algebraic properties from the free quark model and assumed that they hold in the real world. In particular, we have abstracted here properties of the operators $V^{-1}D^{em}_{\pm}V$, which correspond to those which induce real photon transitions between hadrons.

In our case, abstraction from the free quark model leads to $V^{-1}D^{em}_{+}V$ being assumed to be the sum of four terms which transforms as: $\{(8,1)_0 + (1,8)_0, 1\}$, $\{(3,\bar{3})_1, 0\}$, $\{(8,1)_0 - (1,8)_0, 1\}$ and $\{(\bar{3},3)_{-1}, 2\}$, all of which belong to $\underline{35}$'s of the full $SU(6)_W$ of currents. In Section IV we have shown how matrix elements of $D^{em}_{\pm}$ are related to real photon amplitudes and how they may be expressed as as sum of Clebsch-Gordan coefficients times at most four independent reduced matrix elements, once initial and final $SU(6)_W$ multiplets are specified. We have shown that the theory leads to multipole selection rules, a particular example of which is the old SU(6) result that the transition from the nucleon to 3-3 resonance is magnetic dipole in character.[10] In fact, we may generally express the four reduced matrix elements for transitions between L=0 and any other multiplet in terms of four multipole amplitudes, two electric (of the same j_γ) and two magnetic. These selection rules yield very interesting predictions, which may be subject to a qualitative experimental test in that low values of j_γ (and ℓ for pions) are forbidden for $L' \geqslant 3 \rightarrow L=0$ transitions, even though they are otherwise allowed by spin-parity considerations and even favored by angular momentum barrier arguments.

When applied to mesons there are many amplitudes which are related, but little to compare with experiment besides transitions between vector and pseudoscalar mesons, both

of which lie in $\underline{35}$ and $\underline{1}$ with L=0. The available data are consistent with the theory, but little else can be said at the moment.

For baryons on the other hand, we have years of experimental effort that has been devoted to pion photoproduction in the resonance region, from which baryon electromagnetic couplings may be extracted by phase shift analysis. For the $\underline{70}$ L'= 1 baryon states, not only do we find agreement of all the experimentally well determined signs with the theory, but also the photopion matrix elements, which contain information on both γNN^* and πNN^* vertices, indicate that the S and D wave amplitudes at the pion vertex have opposite sign. This is in agreement with the results[49,50] from the reaction $\pi N \to \pi\Delta$. For the $\underline{56}$ L'=2 baryon resonances, again all signs agree with the theory, except for possibly one of the $\gamma p \to F_{35}^{+}$ amplitudes. There is also an indication from $\gamma N \to \pi N$ that the P and F amplitudes at the πNN^* vertex have the same sign, in agreement with the results[49,50] from $\pi N \to \pi\Delta$. While the signs check very well, the magnitudes, particularly for the $\underline{70}$ L'=1 $\to$ $\underline{56}$ L=0 transitions, leave something to be desired. Given the uncertainties in the experimental analyses, however, we feel that the present situation is fairly satisfactory.

The general outlook is then quite good. Between the phase shift analyses of $\pi N \to \pi\Delta$ and $\gamma N \to \pi N$, more than 25 signs predicted by the theory agree with experiment. For

the first time we have some good evidence that not only is the multiplet structure of the quark model found in Nature, but further that the wave functions of the states resemble those of the consituent quark model in that the relative signs (and more roughly, magnitudes) are correctly predicted with such an assumption. However, neither the results for the γNN^* or the πNN^* vertex correspond to the hypothesis of $SU(6)_W$ conservation; the most direct and powerful evidence being the signs and magnitudes of amplitudes for $\underline{70}$ L'=1 baryon resonances to decay into πN, $\pi\Delta$ and γN. The predictions resulting from the quark model,[28,29] where the reduced matrix elements are explicitly calculable, are wrong in places also - in particular in the signs of pion transition amplitudes for $\underline{56}$ L'=2 $\to$ $\underline{56}$ L=0 baryons.

With the success of the theory, it may now be used as a tool to help classifying new resonances into multiplets by using information on their signs in $\pi N \to \pi\Delta$ and $\gamma N \to \pi N$. What is still needed is a dynamics, or possibly an even higher symmetry, which will correctly give the magnitude and sign of the reduced matrix elements. This, and the extension to $q^2 \neq 0$, remain as important problems for the future.

APPENDIX

Baryons

Listed are non-strange members of the baryon multiplets. The names of the resonances conventionally used in phase shift analysis are also given.[47] For baryons, since they are composed of quarks only, W=S. No mixing is assumed.

$SU(6)_W$	L	$SU(3)^{2W+1}$	J^P	$N^*(\Delta^*)_{2I,2J}$ (mass, MeV)
56	0	8^2	$1/2^+$	$P_{11}(940)$
		10^4	$3/2^+$	$P_{33}(1236)$
56	0	8^2	$1/2^+$	$P_{11}(1470)$
		10^4	$3/2^+$	$P_{33}(1718)$
70	1	8^2	$1/2^-$	$S_{11}(1535)$
			$3/2^-$	$D_{13}(1520)$
		8^4	$1/2^-$	$S_{11}(1700)$
			$3/2^-$	$D_{13}(1700)$
			$5/2^-$	$D_{15}(1700)$
		10^2	$1/2^-$	$S_{31}(1650)$
			$3/2^-$	$D_{33}(1670)$

Baryons (cont'd)

56	2	8^2	$3/2^+$	$P_{13}(1770)$
			$5/2^+$	$F_{15}(1688)$
		10^4	$1/2^+$	$P_{31}(1860)$
			$3/2^+$	$P_{33}(2000)$
			$5/2^+$	$F_{35}(1860)$
			$7/2^+$	$F_{37}(1920)$

Mesons

Mesons are composed of a quark and an antiquark, and since $W_\perp = \frac{1}{2}\beta\sigma_\perp$, in calculating the $SU(6)_W$ matrix elements (specifically, the W-spin Clebsch-Gordan coefficients), we must remember the W-S correspondence:

$$|W,W_z\rangle = |1,1\rangle \sim |S,S_z\rangle = |1,1\rangle$$
$$|W,W_z\rangle = |1,0\rangle \sim |S,S_z\rangle = -|0,0\rangle$$
$$|W,W_z\rangle = |1,-1\rangle \sim |S,S_z\rangle = -|1,-1\rangle$$
$$|W,W_z\rangle = |0,0\rangle \sim |S,S_z\rangle = -|1,0\rangle.$$

A simple way to obtain these relation is to look at the action of $W_\pm$ and $S_\pm$ on the states, e.g.

$$W_+|q,\ 1/2,-1/2\rangle = S_+|q,\ 1/2,\ 1/2\rangle = |q,1/2,\ 1/2\rangle$$
$$W_+|\bar{q},\ 1/2,-1/2\rangle = -S_+|\bar{q},\ 1/2,\ 1/2\rangle = -|\bar{q},1/2,\ 1/2\rangle$$

Building states from qq:

$$|S,S_z\rangle = |1,0\rangle = \frac{1}{\sqrt{2}}|1/2,1/2\rangle|1/2,-1/2\rangle + \frac{1}{\sqrt{2}}|1/2,-1/2\rangle|1/2,1/2\rangle,$$

Acting on this state, S gives

$$\frac{1}{\sqrt{2}}|1/2,1/2\rangle|1/2,1/2\rangle + \frac{1}{\sqrt{2}}|1/2,1/2\rangle|1/2,1/2\rangle,$$

which is the same as $\sqrt{2}$ times $|S,S_z\rangle = |1,1\rangle$. The W_+ operator, however, when acting on the $|S,S_z\rangle=|1,0\rangle$ state, gives $-\sqrt{\frac{1}{2}}\,|1/2,1/2\rangle|1/2,1/2\rangle + \sqrt{\frac{1}{2}}\,|1/2,1/2\rangle|1/2,1/2\rangle = 0$, so that the $|S,S_z\rangle = |1,0\rangle$ state corresponds to the $W=0$ state.

For L=0 (ground state) mesons we list both the S-spin and W-spin assignment. For L=1 the W-spin assignment becomes too involved, as many mesons have components with both W=1 and W=0; so for L=1 mesons we list the S-spin only.

Given are the I=1 and I=0 members of meson multiplets.[47] The I=0 physical states are mixtures of SU(3) singlets and octets (see Section V).

$SU(6)_W$	L	$SU(3)^{2W+1}$	J^P	I=0, I=1 state
1	0	1^1	1^-	longitudinal $\omega^{(1)}$
35	0	8^3	0^-	$\eta^{(8)}$, π
			1^-	transverse $\omega^{(8)}$, ρ
		8^1	1^-	longitudinal $\omega^{(8)}$, ρ
		1^3	0^-	$\eta^{(1)}$

SU(6)	L	$SU(3)^{2S+1}$	J^{PC}	I=0, I=1 state
1	0	1^1	0^-	$\eta^{(1)}$
35	0	8^1	0^-	$\eta^{(8)}$, π
		$[8+1]^3$	1^-	$\omega^{(8)}$, $\omega^{(1)}$, ρ

Mesons (cont'd)

35	1	$[8+1]^3$	2^{++}	f, f', A_2
			1^{++}	?, ?, A_1
			0^{++}	S?, σ, δ
		8^1	1^{+-}	H ?, B
1	1	1^1	1^{+-}	?

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TABLE V.A

Matrix elements for photon transitions among $\underline{35}$ and $\underline{1}$ L = 0 states. The ω and ϕ are assumed to be ideally mixed, while the η and X^o are taken as the SU(3) octet and singlet pseudoscalar mesons (see text).

Transition	Coefficient of $\langle \underline{35}\ L' = 0 \| (3,\bar{3})_1 \| \underline{35}\ L = 0 \rangle$
$\omega \to \gamma\pi$	$\sqrt{3}/6$
$\rho \to \gamma\pi$	$\sqrt{3}/18$
$\phi \to \gamma\pi$	0
$\rho \to \gamma\eta$	1/6
$\omega \to \gamma\eta$	1/18
$\phi \to \gamma\eta$	$-\sqrt{2}/9$
$X^o \to \gamma\rho$	$\sqrt{2}/6$
$X^o \to \gamma\omega$	$\sqrt{2}/18$
$\phi \to \gamma X^o$	1/9

TABLE V.B

Predicted and experimental widths for radiative transitions among $\underline{35}$ and $\underline{1}$ L = 0 mesons.

Decay	Predicted Width (KeV) (no mixing)	Predicted Width (KeV) ($\theta_P = -10.5^o$)	Experimental Width (KeV)[34,35]
$\omega \to \gamma\pi$	890 (input)	890 (input)	890 ± 90
$\rho \to \gamma\pi$	94	94	< 730
$\phi \to \gamma\pi$	0	0	< 14
$\rho \to \gamma\eta$	37	57	< 160
$\omega \to \gamma\eta$	5	7	< 49
$\phi \to \gamma\eta$	230	170	126 ± 46
$X^o \to \gamma\rho$	160	120	0.26 $\Gamma(X^o \to$ all)
$X^o \to \gamma\omega$	15	11	
$\phi \to \gamma X^o$	0.5	0.6	

TABLE V C

Photon transition amplitudes from non-strange mesons[39] with $L' = 1$ and $J_z = \lambda$ to those with $L = 0$ and $J_z = \lambda - 1$. The ω, f, D, and σ are assumed to be ideal mixtures of singlets and octets, so as to be composed purely of non-strange quarks; the η and H are purely octet, and the X^0 a pure singlet. Zweig's rule[32] is used to relate $SU(6)_W$ $\underline{35}$ and $\underline{1}$ reduced matrix elements (see text), and forbids decays like A_2, A_1, δ, f, D, $\sigma \to \gamma\phi$ and $f' \to \gamma\rho$ or $\gamma\omega$.

a	b	c	d
$A_2^+ \to \gamma\pi^+$, $\lambda = 1$	0	$\sqrt{3}/8$	$\sqrt{6}/12$
$A_1^+ \to \gamma\pi^+$, $\lambda = 1$	0	$-\sqrt{3}/8$	$\sqrt{6}/12$
$B \to \gamma\pi$, $\lambda = 1$	$\sqrt{6}/24$	0	0
$B \to \gamma\eta$, $\lambda = 1$	$\sqrt{2}/8$	0	0
$B \to \gamma X^0$, $\lambda = 1$	1/4	0	0
$H \to \gamma\pi$, $\lambda = 1$	$\sqrt{2}/8$	0	0
$H \to \gamma\eta$, $\lambda = 1$	$-\sqrt{6}/24$	0	0
$H \to \gamma X^0$, $\lambda = 1$	$\sqrt{3}/12$	0	0
$A_2 \to \gamma\rho$, $\lambda = 0$	1/24	-1/12	$-\sqrt{2}/36$
$\lambda = 1$	$\sqrt{3}/24$	$-\sqrt{3}/24$	0
$\lambda = 2$	$\sqrt{6}/24$	0	$\sqrt{3}/18$
$A_1 \to \gamma\rho$, $\lambda = 0$	$\sqrt{3}/24$	0	$-\sqrt{6}/36$
$\lambda = 1$	$\sqrt{3}/24$	$\sqrt{3}/24$	0
$\delta \to \gamma\rho$, $\lambda = 0$	$\sqrt{2}/24$	$\sqrt{2}/24$	-1/18
$B \to \gamma\rho$, $\lambda = 0$	0	$-\sqrt{6}/24$	0
$\lambda = 1$	0	0	$\sqrt{3}/6$

TABLE V. C (cont'd)

$A_\lambda[A_2 \to \gamma\omega] = 3A_\lambda[A_2 \to \gamma\rho]$

$A_\lambda[f \to \gamma\rho] = 3A_\lambda[A_2 \to \gamma\rho]$

$A_\lambda[f \to \gamma\omega] = A_\lambda[A_2 \to \gamma\rho]$

$A_\lambda[f' \to \gamma\phi] = -2A_\lambda[A_2 \to \gamma\rho]$

$A_\lambda[A_1 \to \gamma\omega] = 3A_\lambda[A_1 \to \gamma\rho]$

$A_\lambda[D \to \gamma\rho] = 3A_\lambda[A_1 \to \gamma\rho]$

$A_\lambda[D \to \gamma\omega] = A_\lambda[A_1 \to \gamma\rho]$

$A_\lambda[\delta \to \gamma\omega] = 3A_\lambda[\delta \to \gamma\rho]$

$A_\lambda[\sigma \to \gamma\rho] = 3A_\lambda[\delta \to \gamma\rho]$

$A_\lambda[\sigma \to \gamma\omega] = A_\lambda[\delta \to \gamma\rho]$

(a). Transition

(b). Coefficient of $\langle \underline{35}\ L' = 1 \| (8,1)_0 + (1,8)_0 \| \underline{35}\ L = 0 \rangle$

(c). Coefficient of $\langle \underline{35}\ L' = 1 \| (3,\bar{3})_1 \| \underline{35}\ L = 0 \rangle$

(d). Coefficient of $\langle \underline{35}\ L' = 1 \| (8,1)_0 - (1,8)_0 \| \underline{35}\ L = 0 \rangle$

TABLE VI. A

Photon amplitudes for transitions from $\underline{56}$ $L' = 0$ states with $J_z = \lambda$ to $\underline{56}$ $L = 0$ states with $J_z = \lambda - 1$.

Transition		Coefficient of $\langle \underline{56}\ L' = 0 \| (3, \bar{3})_1 \| \underline{56}\ L = 0 \rangle$
$N^+(1/2^+) \to \gamma N^+$,	$\lambda = 1/2$	$(-2/15)\sqrt{5}$
$N^0(1/2^+) \to \gamma N^0$,	$\lambda = 1/2$	$(4/45)\sqrt{5}$
$\Delta^+(3/2^+) \to \gamma N^+$,	$\lambda = 1/2$	$(-2/45)\sqrt{10}$
	$\lambda = 3/2$	$(-2/45)\sqrt{30}$

$$A_\lambda[\Delta^+ \to \gamma N^+] = A_\lambda[\Delta^0 \to \gamma N^0]$$

$\Delta^{++}(3/2^+) \to \gamma \Delta^{++}$,	$\lambda = -1/2$	$(-4/45)\sqrt{15}$
	$\lambda = 1/2$	$(-8/45)\sqrt{5}$
	$\lambda = 3/2$	$(-4/45)\sqrt{15}$

$$A_\lambda[\Delta^+ \to \gamma \Delta^+] = (1/2)\, A_\lambda[\Delta^{++} \to \gamma \Delta^{++}]$$

$$A_\lambda[\Delta^0 \to \gamma \Delta^0] = 0$$

$$A_\lambda[\Delta^- \to \gamma \Delta^-] = -(1/2)\, A_\lambda[\Delta^{++} \to \gamma \Delta^{++}]$$

TABLE VI. B

Photon amplitudes for transitions from $\underline{70}$ $L' = 1$ states with $J_z = \lambda$ to nucleon and delta states in the $\underline{56}$ $L = 0$. States are labelled by J^P and [SU(3) multiplet]$^{2S+1}$ where S is the quark spin.

		a	b	c	d
$N^*(3/2^-) \to \gamma N^+$,	$\lambda = 1/2$	$-\sqrt{2}/12$	$\sqrt{2}/6$	$\sqrt{2}/12$
$[8]^2$	$\lambda = 3/2$	$-\sqrt{6}/12$	0	$-\sqrt{6}/12$
$\to \gamma N^0$,	$\lambda = 1/2$	$\sqrt{2}/12$	$-\sqrt{2}/18$	$-\sqrt{2}/36$
	$\lambda = 3/2$	$\sqrt{6}/12$	0	$\sqrt{6}/36$
$\to \gamma \Delta^+$,	$\lambda = -1/2$	0	$\sqrt{3}/9$	0
	$\lambda = 1/2$	0	$1/9$	$-1/9$
	$\lambda = 3/2$	0	0	$-\sqrt{3}/9$
$N^*(1/2^-) \to \gamma N^+$,	$\lambda = 1/2$	$-1/6$	$-1/6$	$+1/6$
$[8]^2$ $\to \gamma N^0$,	$\lambda = 1/2$	$1/6$	$+1/18$	$-1/18$
$\to \gamma \Delta^+$,	$\lambda = -1/2$	0	$\sqrt{6}/18$	0
	$\lambda = 1/2$	0	$-\sqrt{2}/18$	$-\sqrt{2}/9$
$\Delta^*(1/2^-) \to \gamma N^+$,	$\lambda = 1/2$	$-1/6$	$+1/18$	$-1/18$
$[10]^2$ $\to \gamma N^+$,	$\lambda = -1/2$	0	$-\sqrt{6}/18$	0
	$\lambda = 1/2$	0	$\sqrt{2}/18$	$\sqrt{2}/9$

TABLE VI. B (cont'd)

a		b	c	d
$\Delta^*(3/2^-) \to \gamma N^+$,	$\lambda = 1/2$	$-\sqrt{2}/12$	$-\sqrt{2}/18$	$-\sqrt{2}/36$
$[10]^2 \to$	$\lambda = 3/2$	$-\sqrt{6}/12$	0	$\sqrt{6}/36$
$\to \gamma \Delta^+$,	$\lambda = -1/2$	0	$-\sqrt{3}/9$	0
	$\lambda = 1/2$	0	$-1/9$	$1/9$
	$\lambda = 3/2$	0	0	$\sqrt{3}/9$
$N^*(5/2^-) \to \gamma N^+$,	$\lambda = 1/2$	0	0	0
$[8]^4$	$\lambda = 3/2$	0	0	0
$\to \gamma N^o$,	$\lambda = 1/2$	0	$\sqrt{5}/30$	$\sqrt{5}/30$
	$\lambda = 3/2$	0	$\sqrt{10}/30$	$\sqrt{10}/30$
$\to \gamma \Delta^+$,	$\lambda = -1/2$	$-\sqrt{30}/60$	$\sqrt{30}/30$	$\sqrt{30}/60$
	$\lambda = 1/2$	$-\sqrt{10}/20$	$\sqrt{10}/15$	$\sqrt{10}/60$
	$\lambda = 3/2$	$-\sqrt{5}/10$	$\sqrt{5}/15$	$-\sqrt{5}/30$
	$\lambda = 5/2$	$-\sqrt{3}/6$	0	$-\sqrt{3}/6$
$N^*(3/2^-) \to \gamma N^+$,	$\lambda = 1/2$	0	0	0
$[8]^4$	$\lambda = 3/2$	0	0	0
$\to \gamma N^o$	$\lambda = 1/2$	0	$-\sqrt{5}/90$	$2\sqrt{5}/45$
	$\lambda = 3/2$	0	$-\sqrt{15}/30$	$+\sqrt{15}/45$
$\to \gamma \Delta^+$	$\lambda = -1/2$	$-\sqrt{30}/30$	$\sqrt{30}/90$	$\sqrt{30}/30$
	$\lambda = 1/2$	$-\sqrt{10}/15$	$-\sqrt{10}/45$	$\sqrt{10}/45$
	$\lambda = 3/2$	$-\sqrt{30}/30$	$-\sqrt{30}/30$	$-\sqrt{30}/90$

TABLE VI. B (cont'd)

a		b	c	d
$N^*(1/2^-) \to \gamma N^+$,	$\lambda = 1/2$	0	0	0
$[8]^4 \to \gamma N^o$,	$\lambda = 1/2$	0	$-1/18$	$+1/18$
$\to \gamma \Delta^+$,	$\lambda = -1/2$	$-\sqrt{6}/12$	$-\sqrt{6}/18$	$\sqrt{6}/12$
	$\lambda = 1/2$	$-\sqrt{2}/12$	$-\sqrt{2}/9$	$\sqrt{2}/36$

$$A_\lambda(\Delta^{*+} \to \gamma N^+) = A_\lambda(\Delta^{*o} \to \gamma N^o)$$

$$A_\lambda(N^{*+} \to \gamma \Delta^+) = A_\lambda(N^{*o} \to \gamma \Delta^o)$$

$$A_\lambda(\Delta^{*++} \to \gamma \Delta^{++}) = 2 A_\lambda(\Delta^{*+} \to \gamma \Delta^+)$$

$$A_\lambda(\Delta^{*o} \to \gamma \Delta^o) = 0$$

$$A_\lambda(\Delta^{*-} \to \gamma \Delta^-) = -A_\lambda(\Delta^{*+} \to \gamma \Delta^+) \quad .$$

(a) Transition

(b) Coefficient of $< \underline{70}\ L' = 1 \| (8,1)_0 + (1,8)_0 \| \underline{56}\ L = 0 >$

(c) Coefficient of $< \underline{70}\ L' = 1 \| (3,\bar{3})_1 \| \underline{56}\ L = 0 >$

(d) Coefficient of $< \underline{70}\ L' = 1 \| (8,1)_0 - (1,8)_0 \| \underline{56}\ L = 0 >$

TABLE VI. C

Photon amplitudes for transitions from $\underline{56}$ L' = 2 states with $J_z = \lambda$ to nucleon states in the $\underline{56}$ L = 0 with $J_z = \lambda - 1$. States in the $\underline{56}$ L' = 2 are labelled by J^P and [SU(3) multiplet]$^{2S+1}$ where S is the quark spin.

a		b	c	d	e
$N^*(5/2^+) \to \gamma N^+$,	$\lambda = 1/2$	$\frac{2}{15}$	$-\frac{2}{15}\sqrt{3}$	0	$-\frac{2}{15}$
$[8]^2$	$\lambda = 3/2$	$\frac{2}{15}\sqrt{2}$	0	$\frac{2}{15}$	$\frac{2}{15}\sqrt{2}$
$\to \gamma N^0$,	$\lambda = 1/2$	0	$\frac{4}{45}\sqrt{3}$	0	$\frac{4}{45}$
	$\lambda = 3/2$	0	0	$-\frac{4}{45}$	$-\frac{4\sqrt{2}}{45}$
$N^*(3/2^+) \to \gamma N^+$,	$\lambda = 1/2$	$\frac{1}{15}\sqrt{6}$	$\frac{2}{15}\sqrt{2}$	0	$-\frac{\sqrt{6}}{15}$
$[8]^2$	$\lambda = 3/2$	$-\frac{1}{15}\sqrt{2}$	0	$\frac{4}{15}$	$-\frac{\sqrt{2}}{15}$
$\to \gamma N^0$,	$\lambda = 1/2$	0	$-\frac{4}{45}\sqrt{2}$	0	$\frac{2\sqrt{6}}{45}$
	$\lambda = 3/2$	0	0	$-\frac{8}{45}$	$\frac{2\sqrt{2}}{45}$
$\Delta^*(7/2^+) \to \gamma N^+$,	$\lambda = 1/2$	0	$-\frac{4\sqrt{7}}{105}$	$-\frac{2\sqrt{42}}{315}$	$-\frac{8\sqrt{21}}{315}$
$[10]^4$	$\lambda = 3/2$	0	$-\frac{4\sqrt{105}}{315}$	$-\frac{2\sqrt{70}}{315}$	$-\frac{8\sqrt{35}}{315}$
$\Delta^*(5/2^+) \to \gamma N^+$,	$\lambda = 1/2$	0	$\frac{2\sqrt{42}}{315}$	$-\frac{4\sqrt{7}}{105}$	$-\frac{2\sqrt{14}}{63}$
$[10]^4$	$\lambda = 3/2$	0	$\frac{4\sqrt{21}}{105}$	$-\frac{8\sqrt{14}}{315}$	$-\frac{4\sqrt{7}}{315}$
$\Delta^*(3/2^+) \to \gamma N^+$,	$\lambda = 1/2$	0	$\frac{2\sqrt{2}}{45}$	$-\frac{4\sqrt{3}}{45}$	0
$[10]^4$	$\lambda = 3/2$	0	$-\frac{2\sqrt{6}}{45}$	$-\frac{4}{45}$	$\frac{4\sqrt{2}}{45}$

TABLE VI. C (cont'd)

a		b	c	d	e
$\Delta^*(1/2^+) \to \gamma N^+$,	$\lambda = 1/2$	0	$-\frac{2\sqrt{2}}{45}$	$-\frac{4\sqrt{3}}{45}$	$\frac{2\sqrt{6}}{45}$
$[10]^4$					

$$A_\lambda[\Delta^{*+} \to \gamma N^+] = A_\lambda[\Delta^{*0} \to \gamma N^0]$$

---

a — Transition

b — Coefficient of $\langle \underline{56}\ L' = 2 \| (8,1)_0 + (1,8)_0 \| \underline{56}\ L = 0 \rangle$

c — Coefficient of $\langle \underline{56}\ L' = 2 \| (3,\bar{3})_1 \| \underline{56}\ L = 0 \rangle$

d — Coefficient of $\langle \underline{56}\ L' = 2 \| (\bar{3},3)_{-1} \| \underline{56}\ L = 0 \rangle$

e — Coefficient of $\langle \underline{56}\ L' = 2 \| (8,1)_0 - (1,8)_0 \| \underline{56}\ L = 0 \rangle$

TABLE VI. D

Photon amplitudes for transitions from $\underline{56}$ $L' = 2$ states with $J_z = \lambda$ to delta states in 56 $L = 0$ with $J_z = \lambda - 1$. States in the $\underline{56}$ $L' = 2$ are labelled by J^P and $[\text{SU(3) multiplet}]^{2S+1}$ where S is the quark spin.

		a	b	c	d	e
$N^*(5/2^+) \to \gamma\Delta^+$ $[8]^2$	$\lambda = -1/2$	0	$-\frac{4}{30}\sqrt{2}$	0	0
	$\lambda = 1/2$	0	$-\frac{4}{90}\sqrt{6}$	0	$\frac{4}{45}\sqrt{2}$
	$\lambda = 3/2$	0	0	$-\frac{2}{45}\sqrt{2}$	$\frac{8}{45}$
	$\lambda = 5/2$	0	0	$-\frac{2}{45}\sqrt{30}$	0
$N^*(3/2^+) \to \gamma\Delta^+$ $[8]^2$	$\lambda = -1/2$	0	$-\frac{4}{45}\sqrt{3}$	0	0
	$\lambda = 1/2$	0	$\frac{4}{45}$	0	$\frac{4}{45}\sqrt{3}$
	$\lambda = 3/2$	0	0	$-\frac{4}{45}\sqrt{6}$	$-\frac{4}{45}$
$\Delta^*(7/2^+) \to \gamma\Delta^+$ $[10]^4$	$\lambda = -1/2$	$-\frac{2\sqrt{14}}{105}$	$-\frac{2\sqrt{42}}{105}$	0	$-\frac{2\sqrt{14}}{105}$
	$\lambda = 1/2$	$-\frac{2\sqrt{42}}{105}$	$-\frac{4\sqrt{14}}{105}$	$\frac{2\sqrt{21}}{315}$	$-\frac{2\sqrt{42}}{105}$
	$\lambda = 3/2$	$-\frac{2\sqrt{70}}{105}$	$-\frac{2\sqrt{210}}{315}$	$\frac{4\sqrt{35}}{315}$	$\frac{2\sqrt{70}}{315}$
	$\lambda = 5/2$	$-\frac{2\sqrt{70}}{105}$	0	$\frac{2\sqrt{70}}{105}$	$\frac{2\sqrt{70}}{105}$

TABLE VI. D (cont' d)

		a	b	c	d	e
$\Delta^*(5/2^+) \to \gamma\Delta^+$ $[10]^4$	$\lambda = -1/2$	$-\frac{\sqrt{21}}{35}$	$-\frac{2\sqrt{7}}{105}$	0	$-\frac{\sqrt{21}}{35}$
	$\lambda = 1/2$	$-\frac{\sqrt{7}}{21}$	$\frac{4\sqrt{21}}{315}$	$\frac{2\sqrt{14}}{105}$	$-\frac{\sqrt{7}}{63}$
	$\lambda = 3/2$	$-\frac{\sqrt{14}}{105}$	$\frac{2\sqrt{42}}{105}$	$\frac{16\sqrt{7}}{315}$	$\frac{\sqrt{14}}{315}$
	$\lambda = 5/2$	$\frac{\sqrt{210}}{105}$	0	$\frac{4\sqrt{105}}{315}$	$-\frac{\sqrt{210}}{105}$
$\Delta^*(3/2^+) \to \gamma\Delta^+$ $[10]^4$	$\lambda = -1/2$	$-\frac{2}{15}$	$\frac{2}{45}\sqrt{3}$	0	$-\frac{6}{45}$
	$\lambda = 1/2$	0	$\frac{4}{45}$	$\frac{2}{45}\sqrt{6}$	0
	$\lambda = 3/2$	$\frac{2}{15}$	$-\frac{2}{45}\sqrt{3}$	$\frac{4}{45}\sqrt{2}$	$-\frac{2}{45}$
$\Delta^*(1/2^+) \to \gamma\Delta^+$ $[10]^4$	$\lambda = -1/2$	$-\frac{1}{15}$	$\frac{2}{45}\sqrt{3}$	0	$-\frac{1}{15}$
	$\lambda = 1/2$	$\frac{\sqrt{3}}{15}$	$-\frac{4}{45}$	$-\frac{2}{45}\sqrt{6}$	$\frac{3}{45}$

$$A_\lambda(N^{*+} \to \gamma\Delta^+) = A_\lambda(N^{*0} \to \gamma\Delta^0)$$
$$A_\lambda(\Delta^{*++} \to \gamma\Delta^{++}) = 2A_\lambda(\Delta^{*+} \to \gamma\Delta^+)$$
$$A_\lambda(\Delta^{*0} \to \gamma\Delta^0) = 0$$
$$A_\lambda(\Delta^{*-} \to \gamma\Delta^-) = -A_\lambda(\Delta^{*+} \to \gamma\Delta^+)$$

---

a - - Transition

b - Coefficient of $\langle \underline{56}\ L' = 2 \parallel (8,1)_0 + (1,8)_0 \parallel \underline{56}\ L = 0 \rangle$

c - Coefficient of $\langle \underline{56}\ L' = 2 \parallel (3,3)_1 \parallel \underline{56}\ L = 0 \rangle$

d - Coefficient of $\langle \underline{56}\ L' = 2 \parallel (3,3)_{-1} \parallel \underline{56}\ L = 0 \rangle$

e - Coefficient of $\langle \underline{56}\ L' = 2 \parallel (8,1)_0 - (1,8)_0 \parallel \underline{56}\ L = 0$

TABLE VII. A

Comparison of matrix elements of $D_+^3 + (1/\sqrt{3})\, D_+^8$ for $\underline{70}$ $L' = 1 \rightarrow \underline{56}$ $L = 0$ photon transitions with experiment.[51] Nucleon resonances are identified as in Ref. 18 with the quark model states, which are labelled by their quantum numbers J^P and [SU(3) multiplet]$^{2S+1}$, where S is the quark spin. The signs of amplitudes are those in $\gamma p \rightarrow \pi^+ n$ and $\gamma n \rightarrow \pi^- p$, with the S and D amplitudes at the πNN^* vertex taken to have opposite sign (see text).

Transition		$\langle N^*, \lambda \| D_+ \| N, \lambda-1 \rangle$ Experiment[51] (1/GeV)	$\langle N^*, \lambda \| D_+ \| N, \lambda-1 \rangle$ Predicted with $\langle \underline{70} \| (8,1)_0 - (1,8)_0 \| \underline{56} \rangle = 0$	$\langle N^*, \lambda \| D_+ \| N, \lambda-1 \rangle$ Predicted with $\langle \underline{70} \| (8,1)_0 - (1,8)_0 \| \underline{56} \rangle \neq 0$
$D_{13}(1520)$	$\rightarrow \gamma p$, $\lambda = 1/2$	$-.10 \pm .04$	$-.10$ (input)	$-.10$ (input)
$3/2^-$, $[8]^2$	$\lambda = 3/2$	$+.91 \pm .06$	$+.91$ (input)	$+.91$ (input)
	$\rightarrow \gamma n$, $\lambda = 1/2$	$+.41 \pm .03$	$+.32$	$+.23$
	$\lambda = 3/2$	$+.64 \pm .05$	$+.91$	$+.64$ (input)
$S_{11}(1535)$	$\rightarrow \gamma p$, $\lambda = 1/2$	$+.30 \pm .10$	$+1.18$	$+.07$
$1/2^-$, $[8]^2$	$\rightarrow \gamma n$, $\lambda = 1/2$	$+.27 \pm .03$	$+0.89$	$+.30$
$S_{31}(1650)$	$\rightarrow \gamma p$, $\lambda = 1/2$	$+.16 \pm .07$	$+.59$	$+.53$
$1/2^-$, $[10]^2$				
$D_{33}(1670)$	$\rightarrow \gamma p$, $\lambda = 1/2$	$+.36 \pm .04$	$+.73$	$+.36$
$3/2^-$, $[10]^2$	$\lambda = 3/2$	$+.32 \pm .04$	$+.91$	$+.38$

TABLE VII. A (cont'd)

Transition		$\langle N^*, \lambda \| D_+ \| N, \lambda-1 \rangle$ Experiment[51] (1/GeV)	$\langle N^*, \lambda \| D_+ \| N, \lambda-1 \rangle$ Predicted with $\langle \underline{70} \| (8,1)_0 - (1,8)_0 \| \underline{56} \rangle = 0$	$\langle N^*, \lambda \| D_+ \| N, \lambda-1 \rangle$ Predicted with $\langle \underline{70} \| (8,1)_0 - (1,8)_0 \| \underline{56} \rangle \neq 0$
$D_{15}(1670)$	$\rightarrow \gamma p$, $\lambda = 1/2$	$+.06 \pm .07$	0	0
$5/2^-$, $[8]^4$	$\lambda = 3/2$	$+.07 \pm .04$	0	0
	$\rightarrow \gamma n$, $\lambda = 1/2$	$+.20 \pm .03$	+.20	+.13
	$\lambda = 3/2$	$+.33 \pm .14$	+.28	+.19
$D_{13}(1700)$	$\rightarrow \gamma p$, $\lambda = 1/2$	$-.07 \pm .18$	0	0
$3/2^-$, $[8]^4$	$\lambda = 3/2$	$+.14 \pm .18$	0	0
	$\rightarrow \gamma n$, $\lambda = 1/2$	$+.16 \pm .18$	+.07	-.19
	$\lambda = 3/2$	$-.11 \pm .11$	+.34	-.19
$S_{11}(1700)$	$\rightarrow \gamma p$, $\lambda = 1/2$	$+.26 \pm .08$	0	0
$1/2^-$, $[8]^4$	$\rightarrow \gamma n$, $\lambda = 1/2$	$+.07 \pm .16$	-.15	+.12

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TABLE VII. B

Comparison of matrix elements of $D_+^3 + (1/\sqrt{3})\, D_+^8$ for $\underline{56}\ L' = 2 \to \underline{56}\ L = 0$ photon transitions with experiment.[51] Nucleon resonances are identified as in Ref. 47 with the quark model states, which are labelled by their quantum numbers J^P and $[SU(3)\ \text{multiplet}]^{2S+1}$, where S is the quark spin. The signs of amplitudes are those in $\gamma p \to \pi^+ n$ and $\gamma n \to \pi^- p$, with the P and F amplitudes at the πNN^* vertex taken to have the same sign (see text).

Transition		$\langle N^*, \lambda \| D_+ \| N, \lambda\text{-}1 \rangle$ Experiment[51] (1/GeV)	$\langle N^*, \lambda \| D_+ \| N, \lambda\text{-}1 \rangle$ Predicted with $\langle \underline{56} \| (\bar{3}, 3)_{-1} \| \underline{56} \rangle$ and $\langle \underline{56} \| (8,1)_0 - (1,8)_0 \| \underline{56} \rangle = 0$
$F_{15}(1688) \to \gamma p$,	$\lambda = 1/2$	$-.07 \pm .06$	$-.07$ (input)
$5/2^+$, $[8]^2$	$\lambda = 3/2$	$+.44 \pm .03$	$+.44$ (input)
$\to \gamma n$,	$\lambda = 1/2$	$-.11 \pm .02$	$-.26$
	$\lambda = 3/2$	$0 \pm .08$	0
$P_{13}(1770) \to \gamma p$,	$\lambda = 1/2$	$-.02 \pm .14$	$-.70$
$3/2^+$, $[8]^2$	$\lambda = 3/2$	$-.03 \pm .13$	$+.22$
$\to \gamma n$,	$\lambda = 1/2$	$-.06 \pm .06$	$-.21$
	$\lambda = 3/2$	$+.03 \pm .11$	0
$F_{37}(1920) \to \gamma p$,	$\lambda = 1/2$	$-.27 \pm .05$	$-.17$
$7/2^+$, $[10]^4$	$\lambda = 3/2$	$-.30 \pm .04$	$-.22$
$F_{35}(1860) \to \gamma p$,	$\lambda = 1/2$	$+.17 \pm .06$	$-.07$
$5/2^+$, $[10]^4$	$\lambda = 3/2$	$-.09 \pm .08$	$-.30$
$P_{33}(2000) \to \gamma p$,	$\lambda = 1/2$	$-.12 \pm .07$	$-.11$
$3/2^+$, $[10]^4$	$\lambda = 3/2$	$+.05 \pm .03$	$+.18$
$P_{31}(1860) \to \gamma p$,	$\lambda = 1/2$	$+.04 \pm .05$	$-.11$
$1/2^+$, $[10]^4$			

TABLE VII. C

Comparison with experiment of matrix elements of $D_+^3 + (1/\sqrt{3})\, D_+^8$ for photon transitions from resonances in a radially excited $\underline{56}\ L' = 0$ multiplet to the nucleon in $\underline{56}\ L = 0$. Amplitude signs are those in $\gamma p \to \pi^+ n$ and $\gamma n \to \pi^- p$.

Transition		Predicted Matrix Element	Experimental Matrix Element Ref. 51	Ref. 52
$P_{11}(1470) \to \gamma p$,	$\lambda = 1/2$	-0.37 (input)	-0.37 ± 0.04	-0.55 ± 0.13
$\to \gamma n$,	$\lambda = 1/2$	-0.25	0 ± 0.07	-0.51 ± 0.32
$P_{33}(1718) \to \gamma p$,	$\lambda = 1/2$	-0.18	$+0.01 \pm 0.07$	$+0.07 \pm 0.25$
	$\lambda = 3/2$	-0.31	-0.15 ± 0.10	$+0.33 \pm 0.29$

Part 2.

SATURATION OF THE DRELL-HEARN-GERASIMOV SUM RULE

I. INTRODUCTION

The Drell-Hearn-Gerasimov sum rule[1] for the spin-flip amplitude in forward Compton scattering rests on two very basic and simple assumptions. These are the low energy theorem for the spin-flip amplitude[2] and the validity of an unsubracted dispersion relation for this amplitude in Compton scattering.

Since these are very basic and well accepted assumptions, it is of interest to look into the validity of the Drell-Hearn-Gerasimov sum rule in the light of presently available experimental data, which have much improved in the last few years.

An unsubtracted dispersion relation for the forward spin-flip nucleon Compton amplitude f_2,

$$\mathrm{Re}\, f_2(\nu) = \frac{2\nu}{\pi} \int_{\nu_0}^{\infty} \frac{d\nu'}{\nu'^2 - \nu^2}\, \mathrm{Im}\, f_2(\nu') \qquad (1)$$

gives the Drell-Hearn-Gerasimov[1] sum rule

$$\frac{2\pi^2\alpha}{M^2}\mu_A^2 = \int_{\nu_0}^{\infty} \left[\sigma_{3/2}(\nu') - \sigma_{1/2}(\nu')\right] \frac{d\nu'}{\nu'} \qquad (2)$$

when the low energy theorem is applied to $f_2(\nu)$. Here μ_A is the nucleon's anomalous magnetic moment, M the nucleon's mass, ν_0 the threshold energy for a single pion photoproduc-

tion, which in the laboratory is

$$\nu_0 = \frac{m_\pi^2}{2M} + m_\pi \simeq 150\ \mathrm{MeV} \tag{3}$$

and $\sigma_{3/2}$ ($\sigma_{1/2}$) is the total cross section for the process: photon + nucleon $\longrightarrow$ hadrons in the net helicity state 3/2 (1/2 . These cross section enter the sum rule when we use the optical theorem (to order α) to relate the imaginary part of the forward scattering amplitude to the total cross section into the intermediate states.

In their original paper, Drell and Hearn[1] did attempt to investigate the validity of the sum rule for a proton target by using an isobar model of single pion photoproduction. Their results were generally encouraging but some important contribution from high energy (greater than 1 GeV) seemed to be likely. Somewhat later, Chan et al.[3] extended the examination of the proton sum rule using an analysis of single pion photoproduction through the second resonance region. They found good agreement without any high energy contribution. Finally, in the course of an analysis of many sum rules, Fox and Freedman[4] have considered the Drell-Hearn Gerasimov sum rule, using Walker's partial wave analysis[5] of pion photoproduction. They found the somewhat suprising

result that while the sum rule involving only the isovector part of the electromagnetic current appeared well satisfied, the sum rule involving one isovector and one isoscalar current, equivalent to the difference of proton and neutron sum rules, was badly violated.

Since that time, there has been a considerable improvement in both the pion photoproduction data and their analysis. In particular, relatively good neutron data are becoming available and have been incorporated in the recent results of Pfeil and Schwela,[6] and Moorhouse and Oberlack.[7]

Given this changed situation, we reexamine in this work the Drell-Hearn-Gerasimov sum rule for both proton and neutron targets, with particular attention to their difference. In the next section we give the relevant definitions and the contributions to the sum rule using several recent analyses of pion photoproduction. The third section contains some conclusions.

II. ANALYSIS OF CONTRIBUTIONS

The sum rule in Eq.(2) can be decomposed into three sum rules of different isospin character. We will show that the three sum rules provide more detailed and sensitive tests than the proton (neutron) sum rule alone.

The isovector and isoscalar magnetic moments of the

nucleon are defined as

$$\mu_A^V = \mu_A(p) - \mu_A(n)$$
$$\mu_A^S = \mu_A(p) + \mu_A(n) \qquad (4)$$

where $\mu_A(p)(\mu_A(n))$ is the proton (neutron) anomalous magnetic moment. The Drell-Hearn-Gerasimov sum rule[1] may then be separated into three distinct sum rules,

the isovector-isovector sum rule:

$$I_{VV} \equiv \int_{\nu_0}^{\infty} [\sigma^{VV}_{3/2}(\nu') - \sigma^{VV}_{1/2}(\nu')] \frac{d\nu'}{\nu'} = \left(\frac{\mu_A^V}{2}\right)^2 \frac{2\pi^2\alpha}{M^2} = 218.5\mu b \qquad (5a)$$

the isoscalar-isoscalar sum rule:

$$I_{SS} \equiv \int_{\nu_0}^{\infty} [\sigma^{SS}_{3/2}(\nu') - \sigma^{SS}_{1/2}(\nu')] \frac{d\nu'}{\nu'} = \left(\frac{\mu_A^S}{2}\right)^2 \frac{2\pi^2\alpha}{M^2} = 0.3\mu b \qquad (5b)$$

and the "interference" sum rule:

$$I_{VS} \equiv \int_{\nu_0}^{\infty} [\sigma^{VS}_{3/2}(\nu') - \sigma^{VS}_{1/2}(\nu')] \frac{d\nu'}{\nu'} = \frac{\mu_A^V \mu_A^S}{2} \frac{2\pi^2\alpha}{M^2} = -14.7\mu b \qquad (5c)$$

Here σ^{VV} (σ^{SS}) corresponds to the interaction of the isovector (isoscalar) component of the photon with the nucleon, and σ^{VS} corresponds to the difference between photon-proton and photon-neutron interactions.

Presently, the data limit direct study of the saturation of these sum rules to the contribution (to the total cross sections $\sigma_{1/2}$ and $\sigma_{3/2}$) of the nucleon and one pion final hadronic state. Resonance dominance then allows one to estimate also a part of the inelastic contributions. Still, one may well hope that the largest contributions come from not too far above threshold (particularly since possible higher energy contributions to the $\sigma_{3/2} - \sigma_{1/2}$ difference are multiplied by a $1/\nu$ factor), so that the nucleon plus one pion state will at least provide an indication of these sum rules saturation.

Consider then the contribution of the nucleon plus one pion state. Single pion photoproduction amplitudes have a simple isospin decomposition:[5]

$$M^+_{\gamma p \to \pi^+ n} = \sqrt{\tfrac{1}{3}}\left[M^{(3)} - \sqrt{2}\left(M^{(1)} - M^{(0)}\right)\right] \qquad (6a)$$

$$M^0_{\gamma p \to \pi^0 p} = \sqrt{\tfrac{2}{3}}\left[M^{(3)} + \sqrt{\tfrac{1}{2}}\left(M^{(1)} - M^{(0)}\right)\right] \qquad (6b)$$

$$M^-_{\gamma n \to \pi^- p} = \sqrt{\tfrac{1}{3}}\left[M^{(3)} - \sqrt{2}\left(M^{(1)} + M^{(0)}\right)\right] \qquad (6c)$$

$M^{(3)}$ corresponds to the isospin 3/2 and $M^{(1)}$ to the isospin 1/2 s-channel amplitudes of the isovector component of the photon current; $M^{(0)}$ describes the interaction of the nucleon with the isoscalar component of this current.

The one pion photoproduction cross sections of definite isospin character are then proportional to the following combinations of amplitudes:

$$\sigma^{VV} \propto |M^{(3)}|^2 + |M^{(1)}|^2$$

$$\sigma^{SS} \propto |M^{(0)}|^2 \qquad (7)$$

$$\sigma^{VS} \propto - M^{(0)*}M^{(1)} - M^{(0)}M^{(1)*}$$

Additionally, we will make use of the decomposition of definite helicity cross sections for this process in terms of amplitudes of definite angular momentum and parity:[7,8]

$$\sigma_{1/2} = \frac{8\pi q}{k} \sum (n+1) \left(|A_{n+}|^2 + |A_{(n+1)-}|^2\right)$$

$$(8)$$

$$\sigma_{3/2} = \frac{8\pi q}{k} \sum \frac{n(n+1)(n+2)}{4} \left(|B_{n+}|^2 + |B_{(n+1)-}|^2\right)$$

$A_{n\pm}$ and $B_{n\pm}$ correspond to a state with final pion orbital angular momentum n, definite parity $P = (-1)^{n-1}$ and the total angular momentum $j = n \pm 1/2$.

The above relations (Eqs.(5), (6), (7) and (8)), allow us

to consider the isovector-isovector, isoscalar-isoscalar, and interference sum rules separately, to insert the single pion plus nucleon intermediate state cross sections into the integrals, and to separate the contributions of each partial wave. For example, the single pion plus nucleon part of the isovector-isovector cross section in the net helicity state $\lambda = 3/2$ is

$$\sigma_{3/2}^{VV} = \frac{8\pi q}{k} \sum \frac{n(n+1)(n+2)}{4} \left(|B^{(3)}_{n+}|^2 + |B^{(3)}_{(n+1)-}|^2 + |B^{(1)}_{n+}|^2 + |B_{(n+1)-}|^2\right) \qquad (9)$$

Since we can extract the contributions of definite isospin and of definite total and orbital angular momentum, we explicitly have the contribution due to each resonance and using the known inelasticity we are subsequently able to evaluate the corresponding part of the inelastic contribution to the integrals.

A. The isovector-isovector sum rule.

Table I presents the results of the analysis for the isovector-isovector, I_{VV}, sum rule. We have assumed for those partial waves which receive contributions from resonances that the resonant contributions are dominant, and on this basis we evaluated the corresponding inelastic contributions to the sum rule. The analysis extends in photon laboratory energy, ν_{LAB},

from just above the threshold to 1.2 GeV. We have evaluated the contributions for $\nu_{LAB} \leqslant 0.45$ GeV and $\nu_{LAB} \geqslant 0.45$ Gev separately, since the prominent 3-3 resonance is below this energy and higher resonances above, so available analyses[6] make this a natural division in ν_{LAB}.

For ν_{LAB} below 0.45 GeV we find good agreement between the results obtained with the analyses of Refs. 6, 7 and 8. Since the Pfeil-Schwela fit extends to the lowest energy, we have chosen to list the contributions obtained with their analysis in this region, where possible.[10] For ν_{LAB} above 0.45 GeV the values resulting from Refs. 6 and 8 also agree well and we list only the contributions obtained with the recent Moorhouse -Oberlack fit.

The inelastic part is evaluated as a sum of inelastic contributions of $N^*(1520)$, $N^*(1670)$ and $N^*(1688)$ (D_{13}, D_{15} and F_{15}, respectively).

Fig.1 shows the behaviour of one pion plus nucleon contribution to $\sigma^{vv}_{3/2} - \sigma^{vv}_{1/2}$ as a function of energy. One can easily recognize in this graph the dominant features of a big negative non-resonant s-wave contribution and a large positive contribution from $P_{33}(1236)$ and two other resonances.

B. The Isoscalar-Isoscalar Sum Rule.

Table II presents the results for the isoscalar-isoscalar Drell-Hearn-Gerasimov sum rule. There are serious discrepan-

cies between the results obtained with different fits, which extend even to a disagreement on the signs of various contributions. This difficulty has been previously noted by Fox and Freedman.[4] The isoscalar-isoscalar amplitudes cannot be extracted directly from the data. Instead, as can be seen from Eqs.(6) and (7), they must be obtained indirectly from the sum and difference of the proton and neutron data. This results in relatively large errors, as the isoscalar-isoscalar amplitudes are small compared to those of the proton or neutron.

The behaviour of the one pion plus nucleon contribution to $\sigma^{ss}_{3/2} - \sigma^{ss}_{1/2}$ in Fig.2 combines the Pfeil-Schwela[6] and the Moorhouse-Oberlack[7] results. To observe some sysytematic trend and to be able to conclusively evaluate the isoscalar-isoscalar sum rule, we must wait for more accurate and higher energy data.

C. The Interference Sum Rule

Table III shows the present state of the isovector-isoscalar sum rule. The combination of amplitudes involved here corresponds to I=1 exchange in the t-channel, or in other words corresponds to the difference between the proton and neutron Drell-Hearn-Gerasimov sum rules. We expect a negative value for the integral I_{vs}, since it is proportional to

$$(\mu_A(p))^2 - (\mu_A(n))^2$$

We find agreement between the results obtained with diffe-

rent analyses where they overlap. The Pfeil-Schwela fit[6] was used to calculate contributions to I_{VS} for $\nu_{LAB} \leq 0.45$ Gev, and the Moorhouse-Oberlack fit[7] for $\nu_{LAB} \geq 0.45$ GeV. Each of these fits results in a big, non-resonant, s-wave contribution with a positive sign. This contribution is canceled almost entirely by the non-resonant (I=1/2) part of the 1^+ partial wave in the Moorhouse-Oberlack analysis.[7] The second and third resonance cause the total result for this sum rule to have the wrong, positive sign. Figure 3 shows this behaviour in terms of the one pion plus nucleon contributions to $\sigma^{VS}_{3/2} - \sigma^{VS}_{1/2}$.

III. DISCUSSION AND CONCLUSIONS

To discuss the results we have obtained let us recall that the studied sum rules rest on two assumptions: The low energy theorem for nucleon Compton scattering[2] and the validity of an unsubtracted dispersion relation for the f_2 (spin-flip) forward Compton amplitude. The first is quite general and has a very solid theoretical basis. Therefore, the validity of the sum rules is presumably dependent on the second assumption, which is equivalent to the absence of a fixed J-plane singularity ($J^P = 1^+$) in the f_2 amplitude.[13]

This hypothesis receives strong support from the evaluation of the contributions to the isovector-isovector sum rule, as shown in Table I. This sum rule, which has large and well

determined contributions, seems to be rather well saturated using results obtained from the single pion plus nucleon data, complemented by some estimate of the inelastic contributions to $\nu_{LAB} = 1.2$ GeV. Even more pleasing is that the saturation occurs in a non-trivial way. We observe strong cancellations, mainly between the large negative non-resonant s-wave contributios and the positive $P_{33}(1236)$ contributions (see Fig.1). The large non-resonant s-wave contribution is of interest in itself, however, as it violates <u>local</u> two-component duality. The imaginary part of a non-diffractive amplitude, like $f_2(\nu)$, should contain <u>only</u> s-channel <u>resonances</u>. This contribution may be cancelled by higher energy non-resonant contributions to the isovector-isovector sum rule, so as to satisfy <u>global</u> duality.

Our total numerical result for the isovector-isovector sum rule is in general agreement with previous analyses.[4,5] While contributions from still higher values of ν_{LAB} need not be small, we expect the contributions listed in Table I to be the largest individual ones, particularly since the sum rule integrand involves the $1/\nu$ factor.

The isoscalar-isoscalar sum rule present some problems. Because of the sensitivity of the isoscalar amplitude to relatively small differences between the neutron and proton photoproduction data, it is very difficult to achieve reliable values for the isoscalar-isoscalar contributions. Furthermore, as can be seen from the results for this sum rule collected in

Table II, individual non-resonant partial waves make (canceling) contributions. In such a situation, small shifts in the data or the inelastic contributions may easily remove the present disagreement between the total value predicted for I_{ss} in Eq.(5b) and the total value shown in Table II.

In this light, the behaviour of the interference sum rule is puzzling. Since the isovector-isovector sum rule is almost saturated, we have every reason to expect the validity of the assumptions underlying the derivation of the sum rules. The total value for I_{vs} obtained from the same energy region which almost saturates the isovector-isovector sum rule, is of the wrong, positive, sign. This general difficulty has been previously noted by Fox and Freedman[4] using an earlier analysis of pion photoproduction. More detailed analysis of the contributions presented in Table III shows that there are large non-resonant contributions in the 0^+ and 1^+ partial waves, which tend to cancel. The second and third resonances contribute to the sum rule with wrong sign. This again is of interest in itself as it violates local two-component duality. Global duality would seem to be satisfied for this part of $f_2(\nu)$, since various non-resonant partial waves are canceling in the full amplitude. This leads one to believe that if the sum rule is to work it may well be the contributions of poorly known quite inelastic resonances, many in low partial waves, that saturate the sum rule. Unfortunately, the determination of these contributions is very difficult.

To summarize, we find no reason to doubt the validity of the Drell-Hearn-Gerasimov sum rule,[1] and therefore the validity of the unsubtracted dispersion relation for the $f_2(\nu)$. The near saturation of the isovector-isovector sum rule, which has the largest and best determined contributions, even furnishes direct evidence of support. There seems little reason to be alarmed at the non-saturation of the isoscalar-isoscalar and interference sum rules at the present stage of photoproduction analysis, as presently poorly determined contributions could well give good agreement. What is needed is the more direct experimental determination of $\sigma_{3/2}(\nu)$ and $\sigma_{1/2}(\nu)$, using a polarized beam and target, something which is now becoming a real possibility.

References

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8. See also, for example, F.J. Gilman, "Photoproduction and Electroproduction," Physics Reports 4C, 95 (1972).
9. I_{vv} for $V_{LAB} \leqslant 0.45$ Gev due to $P_{33}(1236)$ has the following values resulting from different analyses: 260μb with Walker's fit, 210μb with the Moorhouse-Oberlack fit, and 237μb with the Pfeil-Schwela fit (Refs.5, 7, and 6, respectively).
10. The Pfeil-Schwela analysis is limited to the first resonance region and to 0^+, 1^+ and 1^- partial waves.

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Table I

The Isovector-Isovector Contributions to the Drell-Hearn-Gerasimov Sum Rule[12]

Partial Wave	$.18\,\text{GeV} \le \nu_{lab} \le .45\,\text{GeV}$ (μb)	$.45\,\text{GeV} \le \nu_{lab} \le 1.2\,\text{GeV}$ (μb)	Total (μb)
0^+	-115^a	-51^b	-166
1^-	-2^a	-9^b	-11
1^+ I = 3/2; includes $P_{33}(1236)$	$+237^a$	$+2^b$	$+239$
1^+ I = 1/2	$+12^a$	$+17^b$	$+29$
2^- I = 3/2	$+7^b$	$+5^b$	$+12$
2^- I = 1/2; includes $D_{13}(1520)$	$+13^b$	$+41^b$	$+54$
2^+ I = 3/2	$+1^b$	-2^b	-1
2^+ I = 1/2; includes $D_{15}(1670)$	$+2^b$	$+1^b$	$+3$
3^- I = 3/2	$+1^b$	$+1^b$	$+2$
3^- I = 1/2; includes $F_{15}(1688)$	$+2^b$	$+7^b$	$+9$
Inelastic	$+12$	$+37^b$	$+49$
Total	$+170$	$+49^b$	$+219$

a - Pfeil and Schwela, Ref. 3.

b - Moorhouse and Oberlack, Ref. 7.

Table II

The Isoscalar-Isoscalar Contributions to the Drell-Hearn-Gerasimov Sum Rule[12]

Partial Wave	$.18\,\text{GeV} \le \nu_{lab} \le .45\,\text{GeV}$ (μb)	$.45\,\text{GeV} \le \nu_{lab} \le 1.2\,\text{GeV}$ (μb)	Total (μb)
0^+	-1.07^a		
	$-.80^c$	$-.50^c$	-1.30^c
	-1.93^b	-1.06^b	-2.99^b
1^-	$-.33^a$		
	$-.29^c$	$-.34^c$	$-.63^c$
	$-.26^b$	-1.61^b	-1.87^b
1^+	$+.30^a$		
	$-.28^c$	$-.60^c$	$-.88^c$
	$+1.77^b$	$+2.62^b$	$+4.39^b$
2^- (includes $D_{13}(1520)$)	$+.30^c$	$-.21^c$	$+.09^c$
	$+1.30^b$	-1.75^b	$-.45^b$
2^+ (includes $D_{15}(1670)$)	$+.01^c$	$+.04^c$	$+.05^c$
	$-.10^b$	$+.25^b$	$+.15^b$
3^- (includes $F_{15}(1688)$)	$+.25^c$	$+1.38^c$	$+1.63^c$
	$+.01^b$	$+2.49^b$	$+2.50^b$
Inelastic	$+.41^c$	$+.62^c$	$+1.03^c$
	$+.95^b$	$+.24^b$	$+1.19^b$
Total	$-.40^c$	$+.39^c$	$-.01^c$
	$+1.74^b$	$+1.18^b$	$+2.92^b$

a - Pfeil-Schwela analysis, Ref. 3.

b - Moorhouse-Oberlack analysis, Ref. 7

c - Walker analysis, Ref. 5.

Table III

The Isovector-Isoscalar Contributions to the Drell-Hearn-Gerasimov Sum Rule[12]

Partial Wave	$.18\,\text{GeV} \le \nu_{lab} \le .45\,\text{GeV}$ (μb)	$.45\,\text{GeV} \le \nu_{lab} \le 1.2\,\text{GeV}$ (μb)	Total (μb)
0^+	$+17^a$	$+5^b$	+22
1^-	$+1^a$	$+2^b$	+3
1^+	-6^a	-17^b	-23
2^- (includes D_{13} (1520))	-3^b	$+16^b$	+13
2^+ (includes D_{15} (1670))	0	$+1^b$	+1
3^- (includes F_{15} (1688))	0	$+8^b$	+8
Inelastic	-2	+17	+15
Total	+7	+32	+39

a - Pfeil-Schwela analysis, Ref. 6

b - Moorhouse-Oberlack analysis, Ref. 7.

Figure captions

Figure 1. Single pion photoproduction contribution to the difference between the photon-nucleon cross sections in the net helicity states 3/2 and 1/2 ($\sigma_{3/2} - \sigma_{1/2}$) for the isovector photons.

Figure 2. As in Fig.1 but for isoscalar photons.

Figure 3. As in Fig.1 but for the interference term between the isoscalar and isovector photons.

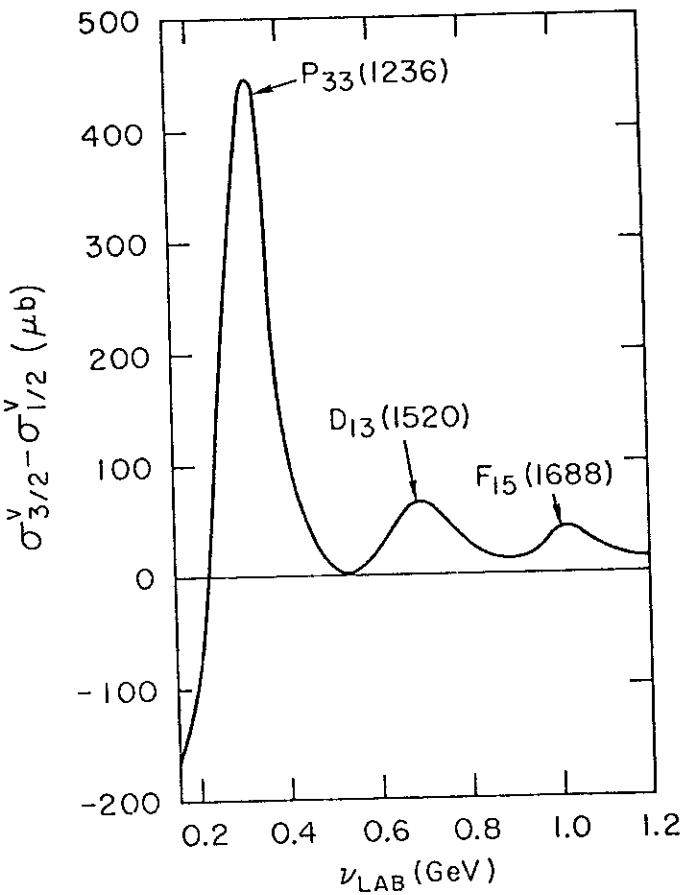
500
400
300
200
100
0
-100
-200
0.2
0.4
0.6
0.8
1.0
1.2
$P_{33}(1236)$
$D_{13}(1520)$
$F_{15}(1688)$
$\sigma^{v}_{3/2}-\sigma^{v}_{1/2}$ (μb)
ν_{LAB} (GeV)

FIG. 1

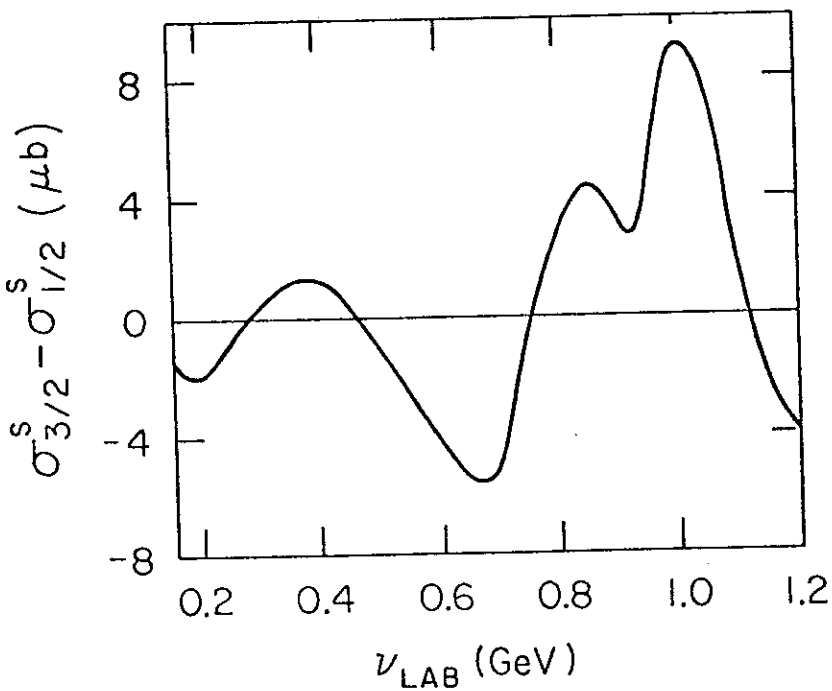
8
4
0
-4
-8
0.2
0.4
0.6
0.8
1.0
1.2
$\sigma^{s}_{3/2}-\sigma^{s}_{1/2}$ (μb)
ν_{LAB} (GeV)

FIG. 2

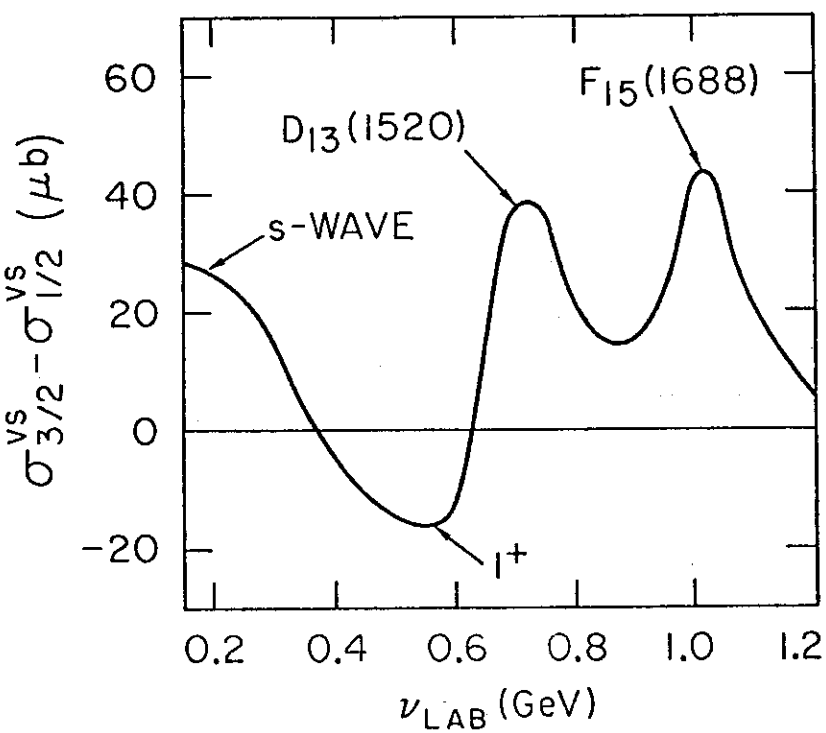
60
40
20
0
-20
$\sigma^{VS}_{3/2} - \sigma^{VS}_{1/2}$ (μb)
F_{15}(1688)
D_{13}(1520)
s-WAVE
1^+
0.2
0.4
0.6
0.8
1.0
1.2
ν_{LAB} (GeV)

FIG. 3