## PHOTOPRODUCTION OF RHO MESONS FROM

 HYDROGEN AND DEUTERIUM FROM 9 GeV TO $16 \mathrm{GeV}^{*}$ROBERT FREDERICK GIESE<br>STANFORD LINEAR ACCELERATOR CENTER<br>STANFORD UNIVERSITY<br>Stanford, California 94305

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## Abstract

This is a study of rho photoproduction from hydrogen and deuterium for energies between 9 GeV and 16 GeV using a wire spark chamber spectrometer. Density matrix elements describing the decay angular distribution of the rho are presented. The momentum transfer dependence of these matrix elements confirm s-channel helicity conservation for momentum transfers out to $-0.3(\mathrm{GeV} / \mathrm{c})^{2}$ and over the full energy range covered by this experiment. The data are compared with the SXding model and all differential cross sections as well as mass distributions are presented within the context of this model, for all energies. The ratio of differential cross sections from deuterium and hydrogen is presented as a function of momentum transfer and is found to be consistent (at all energies) with no $I=1$ exchange. However a small amount of $I=1$ exchange is favored. The shape of the differential cross sections as a function of energy is consistent with Regge shrinkage. Finally, the differential cross sections are compared with pion-nucleon elastic scattering and with Compton scattering in the spirit of the quark model and vector meson dominance. While the rho and Compton differential cross sections are found to agree in shape only, the rho and pion-nucleon differential cross sections are found to agree in both shape and magnitude.

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## CHAPTER I

INTRODUCTION

Vector mesons were first discovered over a decade ago. The p meson was discovered first, in $1961^{1}$, and the $\omega$, $\Phi$, and $K^{*}$ mesons were found shortly thereafter. ${ }^{2}$ The existence of vector mesons had been postulated several years earlier to explain the form factors for the neutron and the proton. Nambu ${ }^{3}$ first postulated the existence of an isoscalar vector meson in 1957 to explain the charge distributions of the neutron and the proton, and in 1959 Frazer and Fulco ${ }^{4}$ predicted the existence of an isovector meson to describe the electromagnetic structure of nucleons. Assuming the existence of vector mesons allowed one to fit the nucleon form factors out to momentum transfers squared $\left(q^{2}\right)$ of $1-2(\mathrm{GeV} / \mathrm{c})^{2}$, the limit of then existing data. However, the physical $\rho$ mass differs from the predicted value. This difference in mass results in a poor fit to the isovector form fectors for the nucleon. Also, more recent measurements ${ }^{5}$ at much larger $q^{2}$ show a much faster falloff of the form factors than predicted by the above theories, and this rapid falloff is still unexplained.

Around the same time that vector mesons were being postulated to describe the nucleon form factors, other theorists who were attempting to construct a theory of strong interactions were led to similar predictions concerning the existence of vector mesons. In 1960 Sakurai ${ }^{6}$ formulated a field theory, based on conserved currents and universality in which an isovector and two isoscalar vector mesons played a key role. Slightly later Salam and Ward, ${ }^{7}$ Gell-Mann, ${ }^{8}$ and $\mathrm{Ne}^{\prime} \mathrm{eman}^{9}$ independently
extended the theory using unitary symmetry to include all of the vector mesons.

In 1961 the two approaches were unified by Gell-Mann and Zachariasen. ${ }^{10}$ They emphasized the concept of vector meson dominance (VDM) of all electromagnetic form factors of hadrons. They used a dispersion theory derivation as opposed to the field theory approach of Sakurai, but both approaches yielded the same final results. The theory was later extended to incluie all electromagnetic interactions with hadrons ${ }^{11,12}$.

The field-current identity ${ }^{11}$ expresses the electromagnetic hadron current, $j_{\mu}(x)$, as a linear combination of the neutral vector meson fields. Conventionally one assumes that the rho meson dominates the isovector component of $j_{\mu}(x)$, and the omega and phi mesons dominate the isoscalar component of $j_{\mu}(x)$. This can be summarized by:

$$
\begin{equation*}
f_{\mu}(x)=e\left[\frac{m_{\rho}^{2}}{2 \gamma_{\rho}} s_{\mu}^{\rho^{0}}(x)+\frac{m_{\omega}^{2}}{2 \gamma_{\omega}} s_{\mu}^{\omega}(x)+\frac{m_{\phi}^{2}}{2 \gamma_{\phi}} s_{\mu}^{\Phi}(x)\right] \tag{1.1}
\end{equation*}
$$

where the $s_{\mu}^{v_{1}} s$ are the vector meson fields, the $\gamma_{v}{ }^{\prime}$ s are the coupling constants, and the $m_{v}$ 's are the vector meson masses. Using identity (1.1) one can write the amplitude $F_{\gamma A B}\left(q^{2}\right)$ for a reaction: $\gamma+A \rightarrow B$ as: ${ }^{11,13}$

$$
\begin{equation*}
F_{\gamma A B}\left(q^{2}\right)=\langle A| j_{\mu}|B\rangle=r_{v} \frac{e m_{v}^{2}}{2 \gamma_{v}}<\frac{\left.A\left|j_{\mu}^{v}\right| B\right\rangle}{q^{2}+m_{v}^{2}} \tag{1.2}
\end{equation*}
$$

where $j_{\mu}^{V}(x)$ is the vector meson source current and satisfies:

$$
\begin{equation*}
\left(\square+m_{v}^{2}\right) s_{\mu}^{v}(x)=j_{\mu}^{v}(x) \tag{1.3}
\end{equation*}
$$

If one now assumes that $<A\left|j_{\mu}^{v}\right| B>$ is a slowly varying function of $q^{2}$, the photon mass squared, then it can be replaced by a constant $F_{V A B}$ which
describes the reaction : $V_{t r} .+A \rightarrow B$, where $V_{t r}$ refers to the transverse component of the contributing vector meson. With this assumption (1.2) becomes:

$$
\begin{equation*}
F_{\gamma A B}\left(q^{2}\right)=\sum_{v} \frac{e m_{v}^{2}}{2 \gamma_{v}} \frac{m_{v}^{2}}{q^{2}+m_{v}^{2}} F_{V A B} \tag{1.4}
\end{equation*}
$$

This can be represented using Feymman diagrams as shown below. ${ }^{14}$


Rho photoproduction is an attractive raction with which to test VDM because (a) the cross section is large, and (b) the reaction can be studied as a function of $q^{2}$, the photon mass squared. The large cross section allows one to obtain good statistics, but this advantage is partially offset by the large rho width, $\Gamma_{\rho} \approx 130-170 \mathrm{MeV} / \mathrm{c}^{2}$. 15 . The large width means that background subtractions are very important, and the resonance theory is somewhat complicated. These effects result in the determination of the rho cross section being model-dependent as will be discussed later in this section.

The $q^{2}$ dependence of $\gamma_{\rho}{ }^{2} / 4 \pi$ can be compared with the behavior predicted by VDM. Inelastic electron scattering covers the $q^{2}<0$ region, while photoproduction studies $q^{2}=0$. The $q^{2}>0$ region can be studied
by doing electron-positron scattering. This is the only region that allows a direct measurement of $\gamma_{\rho}^{2} / 4 \pi$. However, this reaction is different from the previous two in that no proton is present.

This experiment studied $\gamma_{\rho}^{2} / 4 \pi$ at $q^{2}=0$ and was interested in comparing this value with the directly measured value of $\gamma_{\rho}{ }^{2} / 4 \pi$ found for $q^{2}>0$. For photoproduction $\gamma_{\rho}^{2} / 4 \pi$ can be determined in two ways. First one can relate Compton Scattering to photoproduction by equation (1.4). One obtains:

$$
\begin{equation*}
\frac{d \sigma}{d t}(\gamma p \rightarrow \gamma p)=\frac{\alpha}{4}\left|\sum_{v^{\circ}=\rho, \infty, \Phi} \sqrt{\frac{d \sigma}{d t}\left(\gamma p \rightarrow v^{\circ} p\right)\left(\frac{\gamma^{2}}{4 \pi}\right)^{-1}} e^{i \delta}\right|^{2} \tag{1.5}
\end{equation*}
$$

where $\delta_{\mathbf{v}}$ relates the phases of the various amplitudes. Unfortunately the existence of $\phi$ and $\omega$ mesons complicates extracting $\gamma_{\rho}^{2} / 4 \pi$ in this case. Secondly, one can use equation (1.4) at $q^{2}=0$ and the optical theorem to relate forward rho photoproduction to rho elastic scattering by:

$$
\begin{equation*}
\frac{d \sigma}{d t}\left(\gamma p \rightarrow \rho^{\circ} p\right) \quad \left\lvert\, \theta=0^{\circ}=\frac{\alpha}{4} \frac{\sigma^{2}}{16 \pi} \frac{1+\eta_{\rho}^{2}}{\gamma_{\rho}^{2} / 4 \pi}\right. \tag{1.6}
\end{equation*}
$$

where $\sigma_{\rho N}$ is the total rho nucleon cross section and $\eta_{\rho}$ is the ratio of real to imaginary part of the rho-nucleon elastic scattering amplitude. Due to the short lifetime ( $10^{-24} \mathrm{~s}$ ) of the rho meson, the rho nucleon cross section cannot be measured directly. However, there are two indirect ways of measuring $\sigma_{\rho N}$.

The first method uses the strongly interacting nature of rho mesons to measure absorption in complex nuclei, where the various nuclei act as absorbers of varying thickness. Since the rho and the photon have
the same quantum numbers, rho photoproduction should proceed coherently (see the section later in this chapter on diffractive scattering), provided that the longitudinal momentum transfer to the nucleus is kept small. ${ }^{16}$ Under these conditions the nucleus remains unchanged, and Glauber theory can be used to relate the t-channel isospin-conserving part of the amplitude $f\left(\gamma p \rightarrow \rho^{\circ} \mathrm{p}\right)$ to the amplitude $\mathrm{f}\left(\gamma \mathrm{A} \rightarrow \rho^{\circ} \mathrm{A}\right)$. Simple Glauber theory gives the following result: ${ }^{17}$

$$
\begin{align*}
f_{\gamma A \rightarrow \rho} o_{A}(\vec{q})= & f_{\gamma N \rightarrow \rho^{\prime}} o_{N}(0) \int d^{2} b \int_{-\infty}^{\infty} d z \rho(\stackrel{\rightharpoonup}{b}, z) \exp \left(i q_{\|} z+\overrightarrow{i q} \cdot \vec{b}\right) \\
& \exp \left(-\sigma_{\rho N}\left(1-i \eta_{\rho}\right) \int_{z}^{\infty} d z^{\prime} \rho\left(\vec{b}, z^{\prime}\right)\right. \tag{1.7}
\end{align*}
$$

where $\rho(\forall, z)$ is the nuclear density and $q_{\|}$is the longitudinal momentum transfer. (This can never be 0 due to the difference between the rho and photon masses.) However, this simple description must be modified slightly to give a more accurate description of the nucleus. One must include the effects of nuclear correlations, ${ }^{18}$ as well as incoherent background effects. As will be discussed later in this chapter, this effect is small. ${ }^{19}$ Also the value of the nuclear radius is somewhat uncertain. One expects a value somewhere between the electron scattering radius ${ }^{20}$ and the radius determined for strong interactions. ${ }^{21,22}$ It turns out that these nuclear corrections are less troublesome than the corrections due to strong-interaction physics. The two biggest complications are due to $\eta_{\rho}^{23}$ and the uncertainty of the rho shape. If one assumes that VDM holds, one expects $\eta_{\rho}=\eta_{\gamma}$, where $\eta_{\gamma}$ is the ratio of real part to imaginary part of the Compton scattering amplitude. This value, calculated using dispersion-relation fits ${ }^{24}$ to high energy,
photon total cross-section data, ${ }^{25}$ is -0.2 at 5 GeV and falls to -0.1 at 20 GeV . The second effect concerns the rho shape. It is noticed that rho-photoproduction-mass distributions are skewed toward low dipion masses compared to the storage-ring mass distributions. Several models $26,27,28$ have been put forth to describe this effect, and they result in different values for the rho photoproduction cross section. A discussion of these models will be presented later in the chapter. Fortunately the determination of $\sigma_{\rho N}$ depends more on the relative dependence of the cross-section values as a function of the atomic number of the nucleus than on the absolute cross section values.

The second method for determining $\sigma_{\rho N}$ also depends upon Glauber scattering. Here rho photoproduction is studied in the double scattering region off deuterium. The advantage of this method is that it is independent of $\eta_{\rho}$. The results of this measurement, ${ }^{29}$ the complex-nuclei results and the Orsay storage-ring results are presented in Table I. The agreement is impressive, indicating that the theory of high-energy, coherent, nuclear scattering is a precision instrument. This makes the use of complex nuclear targets very attractive, because the coherence eliminates many of the backgrounds that plague experiments on hydrogen. Furthermore, VDM appears to work well in relating rho photoproduction to elastic rho scattering. However, as will be discussed in Chapter VI, the similar VDM relation between rho photoproduction and Compton scattering does not appear to be satisfied. ${ }^{30}$

Rho photoproduction is interesting for other reasons than testing vector dominance. One expects this reaction to be an example of

| EXPERIMENTT | $\begin{aligned} & \sigma_{\hat{p N}} \\ & (\mathrm{mb}) \end{aligned}$ | $\gamma_{\rho}^{2} / 4 \pi$ | $\begin{gathered} \text { (Assumed) } \\ \eta_{\mathrm{oN}} \end{gathered}$ |
| :---: | :---: | :---: | :---: |
| DESY-MIT: at 6.6 GeV <br> H. Alvensleben, et al., Nuclear Physics B18, 333 (1970) <br> (1) B-W + D-S + Poly. <br> (2) B-W $\left(m / m_{\pi \pi}\right)^{4}+$ Poly. | $\begin{aligned} & 26.7 \pm 2.0 \\ & 27.1 \pm 1.7 \end{aligned}$ | $\begin{aligned} & 0.57 \pm 0.10 \\ & 0.59 \pm 0.08 \end{aligned}$ | $\begin{aligned} & -0.20 \\ & -0.20 \end{aligned}$ |
| Cornell: <br> G. Mcclellan, et al., Phys. Rev. D4, 2683 (1971) <br> (1) 8.8 GeV <br> (2) 6.5 GeV <br> (3) 6.1 GeV | $\begin{aligned} & 26.8 \pm 1.2 \\ & 30.1 \pm 1.5 \\ & 26.1 \pm 0.9 \end{aligned}$ | $\begin{aligned} & 0.68 \pm 0.04 \\ & 0.74 \pm 0.05 \\ & 0.58 \pm 0.03 \end{aligned}$ | $\begin{aligned} & -0.24 \\ & -0.27 \\ & -0.27 \end{aligned}$ |
| Rochester: 8.0 GeV <br> H.J. Behrend, et al., Phys. <br> Rev. Lett. 24, 336 (1970) <br> (1) | $26.8 \pm 2.4$ | $0.62 \pm 0.12$ | -0.20 |
| SLAC: Ref. 29 <br> (1) 6 GeV <br> (2) 12 GeV <br> (3) 18 GeV | $\begin{aligned} & 28.6 \pm 1.4 \\ & 28.5 \pm 0.5 \\ & 27.6 \pm 0.6 \end{aligned}$ | $\begin{aligned} & 0.61 \pm 0.06 \\ & 0.70 \pm 0.04 \\ & 0.70 \pm 0.03 \end{aligned}$ |  |
| SIAC: <br> S.H. Williams et al., (to be publishē̃) <br> S.H. Williams (thesis) Combined fit to all complex nuclear targets at 5,7 and 9 GeV | $28.2 \pm 5.0$ | $0.93 \pm 0.22$ | -0.2 |

diffractive scattering. Diffractive scattering is characterized by sharp forward peaking, energy-independent cross section, and a purely imaginary forward amplitude (see below).



Diffraction scattering also involves no change of quantum numbers; however, angular momentum can be picked up during the scattering. Thus, the spin-parity of the diffractive system may change: $0^{-} \rightarrow 0^{-}, 1^{+}, 2 \ldots$. or $1^{-} \rightarrow 1^{-}, 2^{+}, 3^{-}$. . This property leads to the coherence in complex nuclear targets and causes the suppression of processes which involve meson exchange, spin flip, and I-spin exchange.

Diffractive reactions do not have to be elastic. Diffraction dissociation reactions such as:

$$
\pi+T \rightarrow A_{1}+T \quad \text { or } \quad \gamma+T \rightarrow \rho^{0}+T
$$

are also diffractive and can be produced coherently. In this case where the particles have spin, the reaction must proceed in such a way as to take care of the angular-momentum balance without affecting the target nucleons. Hence, the $A_{1}$ must be aligned so that $m=0$, and the rho must be transversely polarized $(m= \pm 1)$ since the photon has no $m=0$ state.

Diffractive reactions can also be described in terms of high energy

Regge theory, where interactions proceed via exchanges of particle or Regge trajectories, as the exchange of a Pomeron. Phi photoproduction reaction is particularly suited to the study of the Pomeron since all other exchange contributions ( $\mathrm{P}^{\prime}, \mathrm{A}_{2}, \pi$, etc.) are expected to be negligible. ${ }^{31}$ The diagrams for Pomeron exchange are shown below:


In the formalism of Regge one expects shrinkage of the forward slope. Some reactions such as $\mathrm{K}^{+} \mathrm{p}$ and pp elastic scattering do show Regge shrinkage. As in phi photoproduction the Pomeron is expected to be totally dominant in these reactions. However, other reactions such as $K^{-} p$ and $\pi^{ \pm} p$ elastic scattering show little or no shrinkage.


In the case of $\pi p$ elastic scattering this has been recently explained by including a small $P^{\prime}$ and $\rho$ exchange contribution. ${ }^{32}$ From the quark model one expects rhos to behave like pions, only in this care one expects
small $P^{\prime}$ and $A_{2}$ exchange contributions to be present.
If rho photoproduction does proceed diffractively, then the angular distribution of rho mesons produced with photons (especially polarized photons) should tell us about the spin-dependence of diffractive scattering. If the photon helicity is conserved, then one expects $\lambda_{\rho}= \pm 1$ in this helicity-conserving frame, resulting in a $\sin ^{2} \theta$ distribution, while $\lambda_{\rho}=0$ gives a $\cos ^{2} \theta$ distribution. Another feature is that for natural parity exchange the pions emerge preferentially in the plane of the photon polarization $\left(w \approx 0^{\circ}\right)$, and for unnatural parity exchange they emerge perpendicular to it $\left(\Psi \approx 90^{\circ}\right)$ (see below).


The Gottfried-Jackson coordinate system for the analysis of the $\rho^{\circ}$ decay angular distribution.


$r p-p \pi^{+} \pi^{-}$
$\mathrm{E}_{\gamma}=4.7 \mathrm{GeV}$ $0.60<\mathrm{M}_{\pi \pi}<0.85 \mathrm{GeV}$ $0.02<1 \mathrm{H}<0.4 \mathrm{GeV}^{2}$ 1457 EVENTS


Reaction $\gamma p \rightarrow \rho^{\circ} \mathrm{p}$ at 4.7 GeV . Rho decay angular distribution in the helicity system without background subtraction. The curves are proportional to $\sin ^{2} \theta_{H}$ and $\left(1+p_{\gamma} \cos ^{2} \Psi_{H}\right)$. Fig. taken from Ref. 33 .

The results shown are from a SLAC experiment using a back-scattered laser beam to form a polarized photon beam. ${ }^{33}$ This experiment, as well as several other experiments ${ }^{34}$ using unpolarized photon beams, shows that s-channel helicity is conserved in rho photoproduction at energies up to $9 \mathrm{Gev},{ }^{35}$ and for $t \leq 0.3$.

Another check on the diffractive nature of rho photoproduction comes from comparing the differential cross section from hydrogen and from deuterium as a function of $t$. This allows one to extract the relative amounts of $I=0$ and $I=1$ t-channel exchange. (See Chapter If for a description of this procedure.) Previous experiments found substantial
$I=1$ exchange for low $s$. However, the amount of $I=1$ exchange was found to decrease to about zero at 9 GeV .35

Finally, the forward and total cross sections are slowly falling and appear to be leveling out at around 8 GeV . This behavior is very similar to that observed in elastic pion scattering and is characteristic of reactions having a large Pomeron contribution. However, the actual values for the cross sections and slopes are model-dependent.

The two most common models for the mass shape of the rho are the Ross-Stodolsky and the s8ding. The Ross-Stodolsky model considers the $p$-mass shift to be kinematical in origin, and it suggests that for small $t$ the rho Breit-Wigner should be multiplied by $\left(m_{\rho} / m_{\pi \pi}\right)^{4}$. This factor was studied by Moffeit et al. for variable exponent $n$, $\left(m_{\rho} / m_{\pi \pi}\right)^{n(t)}$, and it was found that $n$ varied quite rapidly with $t$. The model does not predict this $t$ dependence as required by the data. The s8ding model, which is used in this experiment, is discussed in detail in Chapter IV. This model not only predicts the mass shift but also describes (a) the change in mass shape as a function of $t$ and (b) certain moment $Y_{L}^{M}(\theta, \phi)$ distributions as a function of dipion mass. ${ }^{33}$

Below 9 GeV there is an abundance of good data on rho photoproduction, and the reaction is found to be essentially diffractive in nature. However, above 9 GeV there is no data with large statistics in which a big fraction of the angular-decay region is actually observed. This experiment, begun in 1967, was designed to study rho photoproduction in the energy range $9-16 \mathrm{GeV}$, obtain large statistics, and measure a large fraction of the angular-decay distribution. Hydrogen, deuterium, and also complex
targets were used. A high mass search for coherent production of vector mesons from beryllium was also conducted. This thesis concentrates on the hydrogen and deuterium experiments.

Chapter II details the experimental apparatus, whereas Chapter III describes the reconstruction programs and the data reduction. Chapter IV describes the fitting procedures, while Chapter V lists corrections that were applied to the data in order to obtain cross sections. Chapter VI presents the results. Specifically, the density-matrix elements are studied, as well as the mass and $t$ distributions. There is also a discussion of possible $I=0$ exchange contribution and the question of Regge shrinkage. Finally, a short discussion of the compatability of VDM with the results of this experiment and the possible existence of higher-mass vector mesons is presented.

## A. A General Description with Design Considerations

The original experimental motivation was to study rho photoproduction at various energies from several targets, including complex nuclei. Since rho photoproduction proceeds coherently, one expects exponential behavior in $d \sigma / \mathrm{dt} \approx \exp (B t)$. For example, one expects $B \approx 450$ for lead, which means that $90 \%$ of the differential cross section occurs for $|t| \leq .005(\mathrm{GeV} / \mathrm{c})^{2}$. In comparison $\mathrm{B} \approx 8$ for hydrogen, which means that only $4 \%$ of the differential cross section has $|t| \leq .005(\mathrm{GeV} / \mathrm{c})^{2}$.

To do this experiment one needs to be able to detect very small $t$ events with a very good resolution. One also wants large statistics in order to be able to study the reaction as a function of all its variables. These two considerations--good $t$ resolution at small t's and high statistics--necessitate the use of a triggerable spark-chamber spectrometer or similar apparatus, rather than a bubble chamber, to avoid accumulating a large amount of unwanted data. Because of their high data-rate capabilities, good spacial resolution, and easy readout, wire spark chambers with magnetostrictive readout were used. See Figure 1 for a plan view of the experimental setup.

High energy is desired since photons and rhos have different masses giving rise to a minimum momentum transfer which is $\sim-m^{4} / 4 K^{2}$. This minimum momentum transfer necessitates an extrapolation to $t=0$.



FIg. 1.--Plan layout of experimental apparatus
(For example, at 5 GeV the extrapolation at the rho mass is about a factor of 3 for Pb and under $10 \%$ for $\mathrm{H}_{2}$ and $\mathrm{D}_{2}$, while at 16 GeV this extrapolation is reduced to about $10 \%$ for Pb and is negligible for $\mathrm{H}_{2}$ and $D_{2}$.) Another reason for wanting high energy is that diffractive processes do not fall off with energy, while non-diffractive processes do fall off with energy. Since these non-diffractive processes form a type of background, going to higher energies allows an easier extraction of the diffractive part of the reaction. Being limited to about 10 GeV with the annihilation beam, we were forced to use a bremsstrahlung beam to get higher energies.

This bremsstrahlung photon beam emerged from a vacuum line at the end station wall and was incident upon a $1 \%$ aluminum converter located just before the final beam-sweeping magnet 2D4. This magnet in conjunction with two small scintillation counters located just downstream from it formed a small electron-pair spectrometer used to monitor the $\gamma$ beam intensity. From here the $\gamma$ beam proceeded into an enclosure housing our targets, either a 40-inch-long liquid hydrogen (or deuterium) vessel or a target wheel containing several nuclear targets. This area was heavily shielded to reduce backgrounds. The photon beam was buried in a 90 radiation length, $5^{\prime \prime}$ wide tungsten plug to prevent the large amount of electromagnetic radiation associated with photon beams from saturating the chambers. Rhos produced in the target decayed into a $\pi^{+}$and $\pi^{-}$which passed around the plug, were monentum analyzed by $2 D 5$, and then passed through two "picket fence" trigger hodoscopes A and B and through a set of four wire-spark chambers.

The size of the plug, the magnetic field value, the target position, the chamber positions, and the trigger requirements were chosen to produce a compromise among good acceptance, high resolution, and low background. To improve acceptance one wants a narrow plug, low field intensity, and the target, spark chamber, and hodoscope package close to the magnet. For high resolution one wants the magnetic field high and the chamber package (a) spread out to get good angular resolution and (b) placed far back from the magnet to get good momentum resolution. For low background one wants to eliminate electromagnetically-produced events. This means a wide plug and a tight trigger requirement. The final values were determined by a combination of actual running and Monte Carlo studies for each energy. The final configuration for 16 $\mathrm{GeV} \mathrm{H}_{2}$ and $\mathrm{D}_{2}$ running is shown below.

Target 2 position $=140^{\prime \prime}$ from magnet ceriter $\int \mathrm{B}_{\mathrm{y}} \mathrm{dZ}=860 \mathrm{Kg}$-inches ( 21.84 Kg -meters) Plug Wiath $=5^{\prime \prime}$

B Hodoscope Z Position $=173^{\prime \prime}$ from magnet center Trigger $=2$ or more A counters in coincidence with 2 or more $B$ counters

An IBM 1800 computer logged the data, monitored the system on line, and reconstructed events, time permitting. A detailed description of each part of the apparatus and a discussion of data-collection procedures now follows.

## B. Beam

A SIAC primary electron beam at energies up to 10 GeV was used to
produce a zero-degree bremsstrahlung photon beam. (See Figures 2, 3.) A momentum-analyzed electron beam was steered onto a thin, movable target, TC-30, (Figure 4) located upstream of the beam target room, by steering magnets AP 30 and AP 31. The target was $0.03^{\prime \prime}$ in diameter, $0.005^{\prime \prime}$ thick, and made of aluminum. It was suspended by three .0005-inch-diameter stainless steel wires and could be moved in and out of the beam remotely. The target size was determined by considering desired photon-beam intensity, beam dispersion due to multiple scattering, and ability of the target to dissipate heat from the beam. A thin target was also desired to prevent the necessity of making thick target corrections to the photon spectrum when doing beam normalizations. As it turned out, the beam line could be used simultaneously by another experimenter without removing $T C-30$ on a pulse-to-pulse basis since the target was thin enough to introduce negligible background into another beam. The phase space of the beam was such as to illuminate fully the whole target.

After passing through the target, the electron beam then passed through several pulsed magnets and one DC magnet, $B-38$, which slowly separated it from the $\gamma$ beam without causing a large amount of synchrotron radiation. Finally, a large bending magnet, $B-36$, carried the electron beam down to a water-cooled beam dump.

The photon beam line was filled with protection collimators and permanent magnets to prevent the primary electron beam from ever accidentally reaching our experimental area. The $y$ beam proceeded into End Station $B$ through $2 D 2$, which acted as a sweeping magnet and a protection

Fig. 2--Beam layout I

Fig. 3--Beam layout II


Fig. 4--TC-30 beam target
against accidental penetration of the primary electron beam. Located in $2 D 2$ was a series of 'lead sheets which acted as a $\gamma$ filter to reduce the number of very low energy ( KeV ) synchrotron $y^{\prime}$ s. An extensive amount of steel and concrete shielding occupied most of the area surrounding the beam, running all the way from $2 D 2$ to the far side of End Station B. This was to shield the beam from the electron dump. Located between the various sections of shielding was a pair of remotely controlled beam collimators 2Cl (V, H). These determined the size and shape of the photon beam. Following 2C1 was another sweeping magnet, 2D3, which was lined with lithium hydride to "harden" the beam by removing low energy (MeV) $y^{\prime} s$. The beam traveled in a vacuum from the target to the downstream wall of End Station B, except for a small section from 2 D 2 to 2D3, and in a helium bag from the End Station B wall to our target area. This was done to prevent a degradation of the photon spectrum near the endpoint. Outside the end station wall was a second remotely controlled collimator $2 \mathrm{C} 2(\mathrm{~V}, \mathrm{H})$ which was adjusted to eliminate the beam halo.

One of the problems with a neutral beam is seeing where it is. To set up our beam we used P32 and PR34 monitors to locate the electron beam on the target. Then the beam was tuned up by taking beam profiles with a small counter located after $2 D^{4} 4$. The counter was remotely controlled and had a calibrated, positional readout. We first opened up 2C1 and 2C2, placed the counter at the beam center, and maximized the counting rate using AP 30 and $A P$ 31. Then we closed down $2 C 1$ to $.12^{\prime \prime} x \cdot 12^{\prime \prime}$ and took beam profiles. Finally, we closed down 2 C 2 to $.24^{\prime \prime} \mathrm{x} .24$ " to
eliminate the beam halo. The beam parameters are shown below.

| Size | $.25{ }^{\prime \prime} \times .25^{\prime \prime}$ at $\mathrm{H}_{2}$ target |
| :---: | :---: |
| Intensity: | 300-600 equivalent quanta/pulse |
| Target ${ }_{\text {in }} /$ Target $_{\text {out }}$ : | 1000/1 |
| REP Rate: | 180 pps |
| $e^{-}$current: | 100-200 $\mu \mathrm{mp}$ |
| Beam Spill: | $2.5 \mu s$ |
| Angular Dispersion: | m $1.6 \mu r a d i a n s$ |

## C. Beam Intensity Monitors

Monitoring a neutral beam always presents a problem. In our case the intensity was so low that a quantometer could not be used. Since photons interact easily with matter to produce electrons and positrons, which then become a source of background and also eliminate the original photon from the beam, beam counters will not easily work. We finally decided to place a $1 \%\left(2 / 32^{\prime \prime}\right)$ aluminum converter in front of the final sweeping magnet $2 D^{4}$. $81.75^{\prime \prime}$ downstream from the magnet center and $5^{\prime \prime}$ above and below the beam we mounted two small counters, $2^{\prime \prime}$ by $2^{\prime \prime}$. The counters and $2 D 4$ acted $s s$ an electron-pair spectrometer, detecting coincident electron-positron pairs. The acceptance of this system was about 30 and, since the produced pairs were swept by 2 D 4 out of the beam, background was not a problem. In principle the monitor could be calibrated by lowering the beam intensity, removing the tungsten plug, disconnecting the four hodoscopes intercepting the beam (2A's and 2B's), and then using $2 D 5$ as an electron-positron pair spectrometer. Unfortunately, the severe electromagnetic background swamped the chambers, making it
difficult to use this method. This problem will be more fully discussed in the section on normalization. Finally, we put a scintillation counter in the tungsten plug and integrated its signal. This monitor and the 2D4 pair spectrometer tracked very well throughout our running and gave us confidence in our primary 2D4 electron-positron pair spectrometer.
D. Targets

The target for our experiment was a $40^{\prime \prime}$ long x $2^{\prime \prime}$ diameter cylinder of liquid hydrogen or deuterium (see Figure 5). The density of the liquid was monitored by two calibrated platinum resistors located in the cell and the vapor pressure in the outlet tube. The resistor readings were continuously recorded on a strip-chart recorder, and the vapor pressure was monitored regularly. The whole target assembly was mounted on rails and could be remotely moved. Furthermore, sensors on the rails gave us a digital position reading, which was recorded on tape periodically. For our solid-target running we had a wheel with six targets also mounted on rails and remotely sensed and controlled.

## E. 2D5 Spectrometer Magnet and Tungsten Plug

The spectrometer gap was $15^{\prime \prime}$ high along the field direction (y) and $40^{\prime \prime}$ wide in the bend plane ( $x$ ) with $48^{\prime \prime}$ pole faces along the beam axis (z). Magnetic-mirror plates were installed at both the entrance and exit gap, separated by $80^{\prime \prime}$, in order to prevent high flux leakage into the spark chamber region and to improve the field shape. The field was very uniform over the whole aperture; in fact, $\int \mathrm{B}_{\mathrm{y}} \mathrm{dz}$ was constant to within $.5 \%$ over $3 / 4$ of the active area. This uniformly allowed the use


Fig. 5--Hydrogen/deuterium target
of an effective-length (dipole field) approximation in the track reconstruction. We also included a small fringe-field-focusing term at each end. The field integral ( $\int \mathrm{B}_{\mathrm{y}} \mathrm{dz}$ ) was set to 860 Kg -inches ( $21.84 \mathrm{Kg}-\mathrm{m}$ ) during the running. The field strength was monitored during the experiment by the computer readout of the field-excitation current. These readings were also periodically logged onto tape. The tungsten plug was located 36 " from the magnet center between the magnet-mirror plate and the magnet-entrance gap. It was built up from tungsten bricks $1^{\prime \prime} \times 4^{\prime \prime} \times 3^{\prime \prime}$ to a width of $5^{\prime \prime}$, a height of $15^{\prime \prime}$, and a length of $13^{\prime \prime}$ (~90 radiation lengths) (see Figure 6).

## F. Trigger Hodoscopes

The trigger was determined by two "picket fence" hodoscopes. The first one, called hodoscope A, consisted of twenty scintillation counters $2 \frac{1}{4}$ " wide, $\frac{1}{4}$ " thick, and $20^{\prime \prime}$ long and was located $81^{\prime \prime}$ from the magnet center. The second one, called hodoscope $B$, consisted of thirtysix scintillation counters $2 \frac{1}{4}$ " wide, $\frac{1}{4}$ " thick, and 32 " long and was located $173^{\prime \prime}$ from the magnet center. Each counter was attached to an Amperex 56 DVP photo-multiplier tube. Each counter also had a low voltage (~ 50 volts) Feranti light pulser attached to it. These were very useful in originally adjusting, timing, and checking cable continuity for each counter. Each counter was voltage plateaued before the beginning of the run using a $\mathrm{Ru}^{106}$ source and a coincidence telescope. $\mathrm{Ru}^{106}$ is a particularly convenient source for testing counters since it emits a high energy ( 3.5 MeV maximum) electron, and despite the

large amount of energy available for decay, it has a usable shelf life ( $t_{\frac{1}{2}}=1$ year). To prevent false coincidences due to the $\gamma^{\prime} \mathrm{s}$ also emitted by the source, we had to install a third telescope counter. This was a thick ( $3 / 8^{\prime \prime}$ ) counter in which the electron would be stopped. The test setup is shown below.


$$
\mathrm{Ru}^{106} \rightarrow \mathrm{Rh}^{106} \rightarrow \mathrm{~Pa}^{106}
$$

$$
\begin{array}{ll}
40 \mathrm{KeV} \cdot & 3.5 \mathrm{MeV} \\
\mathrm{e}^{-} & \mathrm{e}^{-}+\gamma
\end{array}
$$

$$
t_{\frac{1}{2}}=1 \mathrm{yr} \quad t_{\frac{1}{2}}=30 \mathrm{~s}
$$

Each counter had .001" aluminum and .009" tape covering on each side. This setup produced an electron very close to minimum ionizing going through the test counter (see Appendix A). Using this setup each counter with its discriminator was plateaued, reaching an efficiency greater than 99.5\%. Plateaued operating voltages ranged from 1900 volts to 2400 volts. Each counter had its own high voltage source as well as its own light pulser voltage source. Any or all of the light pulsers could be turned on simultaneously and read into the computer for test purposes.

## G. Magnetostrictive Wire Chamber Package

To optimize our acceptance we used large chambers. The two chambers closest to the magnet had an active area of $50^{\prime \prime} \times 18^{\prime \prime}$, and the two farthest from the magnet had an active area of $61^{\prime \prime} \times 38.5^{\prime \prime}$. With chambers of this size one worries about uneven sensitivity due to the
large, distributed inductances of the wire planes. ${ }^{36}$ In these chambers that problem was dealt with by including aluminized Mylar sheets (.008" Mylar, .0004" aluminum) 1/16" behind each wire plane and using terminating resistors. This introduced an additional capacity and made the chamber look like a terminated transmission line with a characteristic impedance of several ohms (4-8 ohms/gap). Each chamber consisted of four planes and two gaps (see Figure 7a). In three of the chambers there were an $x$ plane, a $y$ plane, and two planes at $\pm 30^{\circ}$ relative to the vertical direction. In the fourth chamber there were two $x$ planes and two y planes. The two crossed planes allowed one to resolve ambiguities in sparks.

The planes were wound with .004" - .008", hard, aluminum wires. This necessitated special caution to prevent wire breakage, which could only be fixed by cutting open the chamber and removing the broken wire. To prevent breakage one must limit the discharge currents to less than 10 بcoulombs/spark. This was done two ways.

The first way was by choosing an appropriately shaped high voltage pulse. One wants to supply a puise, uniformly distributed throughout the chamber, of sufficient amplitude and duration to cause sparking in the presence of ionized tracks but avoid spontaneous breakdown. Furthermore, one must have current flow for a long enough time to produce a sufficiently large magnetostrictive pulse, whose amplitude is proportional to the integral of the current over time. The pulsing system used for these chambers consisted of (1) a spark gap which acted as a. pulsing switch, (2) energy storage elements consisting of several

(b)

2132426

Fig. 7--Spark chamber and energy box construction
capacitors, and (3) a coaxial cable which acted as a delay line of the same characteristic impedance as the chambers. Such an arrangement is shown in Figure 7b. This system produces a uniform pulse of some 250 300 ns duration throughout the chamber.

The second method used to protect the wires was to introduce alcohol into a $90 \% \mathrm{He}-10 \% \mathrm{Ne}$ gas system to "quench" the spark formation. The correct alcohol gas mixture was obtained by incorporating an alcohol reservoir into a Berkeley gas filtering and pumping system. The gas was pumped over an alcohol reservoir maintained at $5^{\circ} \mathrm{C}$. The condition of the gas system was checked periodically throughout our running.

The sparks were detected by a Fe - Co magnetostrictive wire, in which the velocity of sound is approximately $2 \times 10^{5}$ inches/sec, and preamplified at the chambers. The wire spacing was .04 ", and the measured spark resolution was about . 03". A set of 20 Mega-Hertz, 13 bit scalers was used to record the spark coordinate information. ${ }^{37}$ At this frequency one count corresponds to 0.01 " and the 13 bits are sufficient to ensure complete readout for spark chamber dimensions up to $80^{\prime \prime}$.

The chambers were constructed so that the first and last wire in each plane always had a current flow. This resulted in a fiducial measurement for each plane for each event. Each plane had four scalers attached to it, which were all started by the first fiducial spark. Succeeding sparks turned off the scalers in order. If there were less than four sparks, the final scaler gave the last fiducial reading and allowed calibration of that wand (since the physical spacing of the fiducials was known). For about $2 / 3$ of the events during our runs this was the case. In the case of four sparks no fiducial information was obtained,
but all real sparks were recorded. If more than four sparks occurred, only the first four were recorded. Since this overflow condition could lead to event reconstruction inefficiency, the number of sparks found in a specific plane was monitored visually on a scaler. The beam intensity was then adjusted to keep the number of overflow events small.

## H. Fast Electronics

The fast electronics was used (a) to generate a master trigger for the apparatus on potentially good events, (b) to monitor the incident flux, and (c) to record data from the chambers and hodoscopes. All fast electronics was EG\&G* unless otherwise stated.

1. Generating Master Trigger (Figure 8)

Each hodoscope input was fed into a "leading edge" T140 threshold discriminator whose thresholds varied from 140 mv to 345 mv . To compensate for biases introduced due to the different threshold levels, each hodoscope counter was plateaued with its own discriminator. The output width was set at 10 ns , and this width was used throughout the system for coincidences. The units also had a resolving power of 13 ns . The output of the discriminators was sent to a strobed buffer unit and to an AN106 linear adder. Each AN106 input sharply limits at 600 mv , and hence the output pulse must be attenuated before being fed into a second adder. With a six $D B$ attenuation one was assured of being able to detect in the second adder the difference between when 0,1 , or 2 hodoscopes were on simultaneously in any of the first adders. The output of the second adders was fanned out to four threshold discriminators

[^1]

Fig. 8--Fast electronics trigger logic
(2 TR204's +2 TR104's). The TR2O4's were dead-time-less discriminators operated in the clipped mode and had adjustable thresholds. By varying the threshold one could trigger on more than $0,1,2$ or more hodoscopes. For actual running one of the TR2O4's was set with a threshold for one hodoscope counter and the other with a threshold for two hodoscope counters. This allowed us also to scale various types of accidental triggers.

The TRIO4's were used to measure dead-time effects in the system. They had adjustable thresholds only in increments of 100 mv rather than continuously. Since the output level of the AN106 units was $n * 150 \mathrm{mv}$, where $n$ is the number of on counters in coincidence, these discriminators could be adjusted for the same trigger requirements as the IR2O4's. These discriminators were set ap to be dead for 13 ns after each pulse and were otherwise identical to the TR2d's. The output of each of these discriminators was fed into a scaler.

The outputs from the various TR204's were fed in various combinations into Lecroy 162 coincidence units. The various combinations scaled were:

A1 • B1
Single Track
A1 $\cdot{ }^{B 1}$ delay
Accidental Track
(A1 • Bl) • (AI • BI $)_{\text {delay }}$
$\mathrm{A} 2 \cdot \mathrm{~B} 2$

A2 ${ }^{-12}$ delay
$2 \gamma$ showers
(AI • B1) $\cdot I_{\text {delayl }} \cdot{ }^{B l_{\text {delay2 }}} \quad$ Track $+($ Track $+\gamma$ shower $)$
The last three were several of the possible types of accidentals.

A $\gamma$ shower is used to indicate a low energy $\gamma$ converting to give an $\mathrm{e}^{+} \mathrm{e}^{-}$pair which triggers one of the arrays. A track is a "high" energy charged particle passing through both hodoscopes. Clearly the accidentals could be caused by processes other than those listed. However, I tried to list only the most likely source of each accidental, as determined from studies of our background. It turned out that the dominant form of background came from a sea of "soft" photons uniformly distaibuted across the magnet exit aperture.
2. Gating for System (Figure 9)

The event trigger pulse first went to a Tl00 unit which acted as a. single shot. The single shot was effected by using a GG2.OO gate generator to set the width of the Tl00 to about 2 ms . This allowed time to generate veto pulses and thus only accept 1 trigger/pulse. One of the outputs of the GG200 was used to send a 3.7 ms veto pulse to each of the TR2O4 and TRIOH discriminators. A complimented pulse was also used in coincidence with our beam monitor to measure only that beam for which our system was in the on state. The 3.7 ms veto gave enough time for the slower bin gates to gate off the units in the system. Two sets of gates were generated. The first consisted of an acceleratorgenerated, beam-gate pulse which was run through our control system. This was referred to as a non-veto gate since it depended only on the machine pulse gates. The second gate generated was referred to as a veto gate. It was formed by having an event-pulse from the TIOO trigger a GG200 gate generator to provide a 300 ms spark-chamber, dead-time signal, which was combined in a FGl00 gate fan as a veto to the non-veto


Fig. 9--Fast electronics gating
gate. This gate was applied to all fast electronics and scalers except the UD ungated, monitoring logic, to which the non-veto gate was applied.

The T100 was also used to send a 20 ns strobe-coincidence pulse to all of the hodoscope strobe units which were reset 3 ms after each event. Another trigger pulse from the T100 was sent through an amplifier and used to trigger the spark chambers; still another T100 pulse went to a DG102 unit which provided an interrupt to the IBM 1800 computer. There was also a switch which allowed the system to run in a test mode with the chambers off. In this mode only hodoscopes were read Into the computer, and the event dead time was 3.7 ms .

## 3. Beam Monitoring Logic

Since the Up and Down $e^{t} e^{-}$counters were small, we used 4 ns widths for all of the associated logic to keep accidentals down. Here we scaled two parallel systems--one only beam-gated and the other on only when the spark-chamber system was in the ready state. The charge monitor, which integrated the output from a piece of scintillator buried in our tungsten plug, had a 2 ms gate applied to it. This ensured that we fully integrated all of our pulses and, since pulses are separated by 2.8 ms at SIAC, only our own pulses. The charge monitor automatically reset upon accumulating a fixed amount of charge, and this reset pulse was also scaled.

All of the fast electronics logic was tested and timed using a. Tectronics 661 sampling scope in conjunction with a Hewlett Packerd Model 215A pulse generator. Delay curves were also done using beam particles, and these confirmed the timing done with the pulse generator and
sampling scope.

## I. Online IBM 1800 Computer

The primary task of data acquisition and transfer of data to magnetic tape was handled online by an IBM 1800 computer. The IBM 1800 used 32 K of 16-bit word memory, a 2 us memory cycle, and integer registers. The computer also had a nine-track, model 2401 tape drive, two 2310 disk drives, a 1442 card reader, an 1816 typewriter, and a 1443 line printer. Besides these standard computer features the IBM 1800 had direct digital inputs and output which sensed or set binary voltage levels, analog inputs and outputs, a process-interrupt system, and a 24-level priority-interrupt system. To make the programs as fast and as compact as possible, all coding was done in machine language, although a Fortran compiler was available for numerous off-line jobs. 38

Communication with the computer occurred by means of a set of control buttons--begin run, reset run, stop, end run, unload. There was also a typewriter inquiry station which allowed one to redefine constants or histograms, as well as ask the computer the status of various quantities. When one pushed the begin run control button, a new file was started on the tape unit, the TSI scalers were reset, the DVM's (digital volt meters) were read out and logged on tape, and all histograms were zeroed. (A record describing the constants of the run--chamber positions, field strength, energy, etc.--was also written onto tape.) After a ten-second delay the fast electronics was turned on and data acquisition began.

Upon receiving an interrupt indicating that a potentially good event had occurred, the computer read out the contents of the hodoscope buffers and the magnetostrictive scalers. The computer then used a restrictive algorithm to attempt to reconstruct the events. Only tracks which had a wire from each plane contributing to the track were reconstructed. A simple momentum calculation was performed, based op the assumption that the track originated at the target center. For dipion events momentum, dipion mass, and $t$ distributions were histogrammed. Histograms of spark deviations for each plane were also calculated. These histograms could be displayed during a run on a CRT scope or printed out on a line printer. See Figure 10 for a sample of available displays.

At ten-minute intervals a monitor program was called into execution. This program read out the TSI scalers and DVM's, wrote them on tape, and printed them out, as well as noting a DVM which had changed by more than a specified amount. Also, a summary of chamber statistics, including the number of tracks and dipions found and plane, chamber, track and dipion efficiencies, was printed out. This gave us a good indication of the chamber performance throughout the run. Besides the computer monitoring procedures, there were several functions monitored by the experimenters directly. Things like hodoscope voltage levels, chamber voltage levels, target pressures and target fullness were monitored periodically. The various power supplies also had alarm bells which sounded if the current of voltage varied outside of set limits. Checking the beam quality was also the responsibility of the experimenter.


Fig. 10-Online displays: (a) raw spark data, (b) reconstruction pair, (c) Chamber deviation in hundredths of inch, (d) dipion mass

## J. Data Taking

Data were taken during two one-month periods. The first was in July of 1968. During the previous month the system was checked out, and a major effort was made to reduce electro-magnetic background. During July we took data at $10 \mathrm{GeV}, 13 \mathrm{GeV}$, and 16 GeV bremmstrahlung endpoint energies with the liquid $\mathrm{H}_{2}$ and $D_{2}$ target. It is this data which will be presented here.

Then in January 1969 the rest of the data was taken. During this period we ran at 16 GeV and off solid Be and Pb targets. An equal number of events from Be and Pb (2000 rho events with energy between 14 and 16 GeV ) were obtained. The second half of the run was spent in a search for high-mass vector mesons produced coherently from Be. For these runs the target was moved as close to the magnet as possible ( $90^{\circ}$ from magnet center) to reduce the acceptance for rhos and to increase the acceptance for higher mass objects.

The output of the experiment consisted of about 100,9 track tapes written by the IBM 1800 computer. The data reduction consisted of four steps: (1) reducing the 100 tapes to 10 tapes and classifying the runs, (2) searching the chamber package for lines for each event, (3) fitting all events with at least two opposite-signed tracks to a dipion hypothesis, and (4) producing a data summary tape after subjecting all dipion events to a series of geometrical and kinematical tests.

## A. Reducing the Data Tapes

There were several reasons for this step. First of all, since the data was to be analyzed on an IBM 360, Model 91 computer, in a batch environment with no control over the mounting of tapes, we produced two backup sets of the raw data which we could use in case a tape was accidentally damaged. Secondly, the 100 tapes could easily be compressed into 101600 -bpi, heavily blocked, labeled tapes. Finally, the tapes had to be read through at least once to confirm what was on each tape by comparing the contents of the tape with the log book.

A machine language program was written to handle the tape copying. Each input run was read and all CONSTANT, SCALER, DVM, and COMMENT records were printed out on a line printer. Each input record was blocked and written out at 1600 -bpi. The program automatically kept track of the number of records written and the amount of tape used to
ensure full utilization of each tape. For each rux an entry was made into a catalog of volume, file, and run mmber to facilitate keeping track of several hundred runs. The printer output for each run was examined, and the constants were all checked for corxectress; each run was also checked for having terminated normally. (occasionally the 1800 program hung up and the run had to be ended manually without a final SCALER record being written onto tape. In that case the SCAIER record written out during the monitor execution was used.) Each run was given one of four classifications: (1) good run, (2) bad run, (3) needs special attention, or (4) ended abnormally and SCATER reading must be corrected.

## B. Line Fitting

The program used to find lines in the chambers wes set up automatically to find and mount the correct tape based on a run momer catalog. At the beginning of each run was a CONSTANT, SCAIER, and DVM record. The CONSTANT record contained the physical and geometrical constants applicable to that run. They included things like beam energy, target material, target position, field strength, and fiducial values, as well as the location of all chambers and hodoscopes. These constants were also updated to correct any errors found during the first pass through the data tapes. The $D V M$ readings were used to get a more accurate reading of target position and field strength and were always within $5 \%$ of the CONSTANT values.

The first step in finding lines was to process the wand scalers.

Each scaler reading was checked to see whether or not it was a second fiducial reading. To be a fiducial scaler whad to satisfy both of the following:
(1) w lies within 10 of the fiducial count $F$
(2) $w+1 \geq w$ is not a fiducial.

Since the chambers were not located in a temperature-controlled environment, the fiducial value could slowly change with time due to expansion and contraction of the chambers and wands. To compensate for this an average fiducial value was maintained and each new fiducial was averaged into the count. The average was conditioned to change slowly. FIDAVE $=$ FIDAVE $+($ FID $w-$ FIDAVE $) / 100$. This, plus the fact that $F$ was fixed for the entire run, prevented locking in on the wrong fiducial. If a scaler was not a fiducial, it was checked for (1) spark order, $w<w+1<w+2$, and (2) $w$ is not an overflow. If no spark came along to shut off the 20 MHz clocks, the scalers would overflow and end up with a count of about 10-15 in them. This overflow had to be recognized and not treated as a valid spark. All good scalers were then formalized to the fiducial value, scaled by the physical fiducial separation, and oriented in space (see Figure 11b). This orientation was accomplished using various constants from the CONSTANT record and describing wire orientation, chamber center relative to beam center, and wand length.

After scaler readings are translated into physical space, the next step is to correlate these wires into points (sparks) in each chamber. The approach in this experiment is to use a collapsed-chamber algorithm. This consists of assuming each of the four planes in a chamber is
located at the same $z$. One then takes three planes at one time, which is an overconstrained system for the ( $x, y$ ) position of the spark, and checks whether the system is consistent, All combinations of wires from one gap are chosen, and the value of wires in the other chamber gap is predicted. If a wire in the other gap is found to lie within error of the predicted value, then the set of three wires is taken to represent a valid spark and is entered into a point list as an ( $x, y, z$ ) coordinate, and a code word describing the two selected gap wires contributing to the spark is constructed. The error depends upon the wire resolution and a geometrical resolution due to the finite plane separation which is about $0.42^{\prime \prime}$ between two adjacent planes. After combinations of wires from the first gap have been tried, the roles of the two gaps are reversed. Using this algorithm, at most 32 points can be found in each chamber. In practice the use of the diagonal planes reduces this number substantially by reducing ambiguities (see Figures 11a, 12). While this algorithm permits finding each spark twice, it prevents the loss of a spark due to plane inefficiency when only 4 -wire fits are accepted.

The next step is to use this point list to find lines or tracks. The algorithm used here is to try $a l l$ combinations of points in chambers 2 and 4 and look for a point in chamber 1 or 3 lying within $50\left(0.15^{\prime \prime}\right)$ of the line joining the selected points in chambers 2 and 4 . Upon finding such a point, a search for 211 wires in all planes Iying within 50 of the line is carried out. The following fitting procedure is terminated if (a) there are less than eleven wires in the line, or (b) there are not at least three chambers having three wires or more contributing to the line. Test (b) ensures that at least two diagonal planes are included

(b)

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Fig. 11-Spark chambers: (a) spark amgiguity resolving, (b) definition of spark chamber terms


Fig. 12-Number of points found per chamber (typical run)
to resolve $x, y$ ambiguities. The fitting procedure consists of (a) least-square fitting the set of wires to a line with each wire receiving equal weight, (b) finding the best wire within 50 of this line in each plane, (c) re-least-square fitting this set of best wires to a line, (d) finding the best wire within 30 of this line in each plane, and (e) refitting this latter set of best wires to a line. Steps (d) and (e) are repeated until the set of wires being used stabilizes. This final line is considered to be a valid track if the chi-square/degree of freedom is less than 5.0. This is a very non-restrictive test (see Figure 13). The effects of the 30 test are shown in Figures 14, 15. After the line has passed this test, all points having both wires contributing to this track are deleted from the point list. This helps to prevent the same track from being found many times starting with different combinations of points on the track. Finally, this newly found $n^{\text {th }}$ track is checked to see if it is a duplicate to any already reconstructed $j^{\text {th }}$ track. This is done by forming for all $j^{\text {th }}$ tracks $x_{j}^{2}=\Sigma_{i}\left(s_{i}^{j}-s_{i}^{n}\right)^{2} /(6 \sigma)^{2}$ for all planes having the same wires used for the $j^{\text {th }}$ and $n^{\text {th }}$ track. Then if $x^{2} /$ degree of freedom is $\leq 5.0$ the tracks are considered duplicate and the line with best $\chi^{2} /$ degree of freedom is kept. If more than five tracks are found, the one with highest $\chi^{2} /$ degree of freedom is deleted. In practice very rarely are more than five tracks found or are duplicate tracks found (see Figure 16). After all combinations of points in chambers 2 and 4 have been tried, the roles of chambers 2, 4 and chambers 1,3 are reversed and the process repeated. In the above $\sigma=0.03^{\prime \prime}$ was used. This was based on a


Fig. 13-- $x^{2} /$ (degree of freedom) for line fitting (typical run)


Fig. 14--Chamber deviations in hundredths of inch (typical run)


Fig. 15-Maxinum deviation for fit lines (typical run)


Fig. 16--Number of initial lines found per event (typical run)
study of spark chamber deviations as well as a careful study of the program performance with different $\sigma^{\prime} s$. Finally, each track has its momentum calculated on the assumption that the track originated from the target center. See rigure $17 a$ and Appendix $C$ for a description of the algorithm used.

## C. Event Fitting

Since we only had chambers located downstream of the magnet and no information on the target side, reconstruction of an event was complicated. Essentially the fit consisted of taking pairs of oppositely signed momenta and imposing a set of four constrainte:
(1) Min (Distance between Track 1 and Track 2) $=0$
(2) $\left(x_{1}+x_{2}\right) / 2-x \operatorname{target}=0$
(3) $\left(y_{1}+y_{2}\right) / 2-y$ target $=0$
(4) $\left(z_{1}+z_{2}\right) / 2-z$ target $=0$

Where $x, y$, and $z$ are the values for Track 1 and Track 2 satisfying (1). Put into words, these four constraints demand that the two tracks intersect at a point which lies within the target. In actuality the intersection point of the two tracks is determined almost entirely in the $y z$ plane since in the $x z$ plane the intersection point can be moved to any place by changing the momentum of the two tracks. There is a small correlation that comes in through the fringe-field focusing (see Figure 17b and Appendix $C$ ), but this is very weak. At first we were hoping to get some constraints in the $x z$ plane by using a magnet chamber. However, the background intensity forced us to locate the magnet chamber near the

(a)



(b)

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Fig. 17-(a) Geometry for correct trajectory used in momentum algorithm, (b) fringe field calculation
magnet exit, and hence the lever arm for determining the $x z$ target position was very weak. Also, to use the magnet chamber an effective-length approximation of the field was no longer adequate, and at best we were only able to locate the target position to within $20^{\prime \prime}$. Since the amount of information added by using the magnet chamber was small compared to the computer time needed to include it, it was ignored.

The actual fitting procedure consisted of several steps. The first step was to try all combinations of opposite-signed tracks. Each track was least-square fit again, only this time each plane was weighted by $1 / \sigma(I)$.

Here each $\sigma(I)$ was initially chosen to be 0.03 and allowed to change as a result of getting a better estimate from fjt events. Upon fitting, we obtained 11 measured variables and an associated error matrix. The variables were:
$x_{0}=x$ at magnet exit for each track
$\ell=\mathrm{dx} / \mathrm{dz}$ in chambers for each track
$\mathbf{y}_{0}=\mathbf{y}$ at magnet exit for each track
$x_{\text {cham }}: \quad m=d y / d z$ in chamber for each track

$$
\begin{aligned}
& x_{t}=\text { target production } x \text { (initially target center) } \\
& y_{t}=\text { target production } y \text { (initially target center) } \\
& z_{t}=\text { target production } z \text { (initially target center) }
\end{aligned}
$$

We finally want to obtain the following 6 variables:

$$
\begin{aligned}
P & =\text { momentum of track (unsigned) } \\
P: \quad I & =d x / d z \text { target side } \\
M & =d y / d z \text { target side }
\end{aligned}
$$

These six variables plus ( $x_{t}, y_{t}, z_{t}$ ) comprise all of the information concerning each event that we are able to determine, where ( $x_{t}, y_{t}, z_{t}$ ) is taken to be measured at target center with errors assigned as $1 / 3$ target size. This assures that 2/3 of events fall within 2/3 of target (see Figure 18).

In actual practice it turns out to be easier to express the constraints in terms of the set of variables $X_{\text {tar }}$ :

$$
\begin{aligned}
X_{0} & =x \text { at magnet entrance } \\
I & =d x / d z \text { target side } \\
Y_{0} & =Y \text { at magnet entrance } \\
X_{\operatorname{tar}} \quad M & =d y / d z \text { target side } \\
X & =\text { target production } x \\
y & =\text { target production } y \\
z & =\text { target production } z
\end{aligned}
$$

The first step in constraining the events is then to calculate derivative matrices relating $X_{\text {cham }}$ and $X_{t a r}$. Then one linearizes the constraint equations and solves them in terms of $X_{\text {tar }}$. The quantity that is actually minimized is:

$$
\begin{aligned}
& M=C^{T} G_{M} C+2 \alpha_{f}^{T} \\
& C=X_{\text {cham }}-X_{\text {cham }}^{0} \\
& G_{M}=\text { inverse of error matrix for } X_{\text {cham }} \\
& f=\text { four constraint equations ( } 0 \text { when satisfied) } \\
& \alpha^{T}=\text { Lagrange Multiplier }
\end{aligned}
$$

An iterative procedure is used to minimize M. This procedure is terminated for any of the following reasons:


Fig. 18--Reconstructed target $z$ distribution for hydrogen
(1) The TARGET $Z$ comes out on chamber side of magnet
(2) Procedure fails to converge
(3) $X^{2}=M$ is greater than 100 at any step
(4) The geometry becomes unphysical, and the event can no longer be projected through the magnet
(5) More than 10 cut steps are needed.

An event which successfully makes it through the fit is considered to be a valid pair, and the variables $P$ and their error matrices are calculated.

Having successfully been fitted, these events are used to calculate information concerning the chambers. All 16 wire fits are used to calculate average chamber deviations. If an average deviation for any plane exceeds 0.01 ", a special routine is called to ralign the chamber package. This routine accumulates 100 tracks and then adds constant shifts to each plane in order to minimize the average deviations for each plane. In order to prevent uniform translation or rotation of the whole chamber package, four planes are kept fixed. A linear shift term is calculated, as well as a $\sigma(I)$ for each plane. It is this $\sigma(I)$ which is used for the weighted least-square fit.

Then the average deviations are zeroed for each plane, and the calculated terms are applied to each spark for the rest of the run unless the average deviation again exceeds 0.01 ".

At the end of a run the following set of quantities based on the entire run is calculated for each plane:
(1) Average deviation
(2) A constant chamber shift
(3) A linear chamber factor
(4) Errors on (2) and (3)
(5) $\sigma$ for each plane
(6) $\sigma_{\text {cor }}$ using (2) and (3) to improve lines
(See Table II).
In practice (1) and (6) should be very close together and (1), (2), and (3) should be small if the aligrment routine is working correctly. The last step before writing out a constrained-pair event is to calculate the kinematics for the rho event. There are several assumptions made here. The first is that we are really seeing pions. This is a good assumption because (a) the acceptance has been optimized for rhos, and rhos decay almost entirely to $\pi^{+} \pi^{-}$, and (b) electromagnetic reactions go forward and will be stopped by the plug. In particular we worried about wide-angle electron pairs, and a detailed study showed that they were not a problem. Since we only measure the pions and not the photon energy, we cannot separate elastic and inelastic events. If we assume that the event is elastic, then we can solve for the photon energy. The consequences of this will be discussed more fully in the section on normalization. Based only on the first assumption we can calculate $E_{\rho}, P_{\rho}, M_{\rho}$ and with the addition of the elastic assumption we get $t, E_{\gamma}$, and the decay angles in the JACKSON or HFUTCITY systems.

$$
\begin{aligned}
& E_{\gamma}=\left(M_{\operatorname{tar}} * E_{\rho}-0.5 * M_{\rho}^{2}\right) /\left(M_{\operatorname{tar}}+\left(P_{\rho}\right)_{z}-E_{\rho}\right) \\
& t=2.0 * M_{\operatorname{tar}} *\left(E_{\rho}-E_{\gamma}\right)
\end{aligned}
$$

In practice it turns out to be more useful to use $t^{\prime}=-t+t_{\min }$ which

## TABLE II

Below is a summary of the Chamber Statistics for each Plane

|  | Chamber | Plane | Average Deviation | Sigme of Deviation | Constant Chamber Shift | Linear Chamber Shift | $\begin{aligned} & \text { Sigma of } \\ & \text { Fit Deviations } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 1 | $0.006831 \pm(0.000882)$ | 0.034688 | $0.006850 \pm(0.000883)$ | $0.000084 \pm(0.000157)$ | 0.034696 |
|  | 1 | 2 | $0.003897 \pm(0.000587)$ | 0.023071 | $0.003984 \pm(0.000585)$ | $0.000175 \pm(0.000049)$ |  |
|  | 1 | 3 | $0.004322 \pm(0.000594)$ | 0.023356 0.014293 | $0.004510 \pm(0.000590)$ $0.002271 \pm(0.000364)$ | $-0.000264 \pm(0.000050)$ $-0.000041 \pm(0.00028)$ | $\begin{aligned} & 0.023158 \\ & 0.014282 \end{aligned}$ |
|  | 1 |  | $0.002300 \pm(0.000364)$ | 0.014293 | $0.002271 \pm(0.000364)$ | $-0.000041 \pm(0.000028)$ |  |
|  | 2 | 1 | $0.005702 \pm(0.000396)$ | 0.015580 | $0.005749 \pm(0.000396)$ | $-0.000088 \pm(0.000038)$ | 0.015557 |
|  | 2 | 2 | $0.002736 \pm(0.000605)$ | 0.023771 | $0.002774 \pm(0.000605)$ | $0.000101 \pm(0.000064)$ | 0.023760 |
|  | 2 | 3 | $0.003496 \pm(0.000578)$ $0.004388 \pm(0.000670)$ | $\begin{aligned} & 0.022721 \\ & 0.026275 \end{aligned}$ | $\begin{aligned} & 0.003572 \pm(0.000578) \\ & 0.004439 \pm(0.000669) \end{aligned}$ | $\begin{aligned} -0.000135 & \pm(0.000062) \\ 0.000251 & \pm(0.000133) \end{aligned}$ | $\begin{aligned} & 0.022693 \\ & 0.026303 \end{aligned}$ |
| 응 |  |  | $0.002550 \pm(0.000541)$ | 0.021283 | $0.002583 \pm(0.000542)$ | $0.000083 \pm(0.000065)$ | 0.021278 |
|  | 3 | 2 | $0.001981 \pm(0.000917)$ | 0.036056 | $0.002049 \pm(0.000917)$ | $0.000392 \pm(0.000204)$ | 0.086025 |
|  | 3 | 3 | $0.001522 \pm(0.000903)$ | 0.035513 | $0.001441 \pm(0.000903)$ | $-0.000465 \pm(0.000201)$ | 0.035464 |
|  | 3 | 4 | $-0.006675 \pm(0.000581)$ | 0.022853 | $-0.006701 \pm(0.000582)$ | $0.000065 \pm(0.000070)$ | 0.022854 |
|  |  |  | $-0.003752 \pm(0.000596)$ | 0.023433 | $-0.003799 \pm(0.000596)$ | $0.000180 \pm(0.000081)$ | 0.023404 |
|  | 4 | 2 | $0.004920 \pm(0.001147)$ | 0.045106 | $0.004279 \pm(0.001066)$ | $-0.004156 \pm(0.000264)$ | 0.041887 |
|  | 4 | 3 | $0.007321 \pm(0.000903)$ | 0.023150 | $0.007258 \pm(0.000588)$ | $0.000233 \pm(0.000090)$ | 0.023108 |
|  | 4 | , | $0.002079 \pm(0.000581)$ | 0.022111 | $0.002076 \pm(0.000563)$ | $0.000023 \pm(0.000085)$ | 0.022117 |

EVENT SUMMARY LIST

| NUMBER OF GOOD EVENTS | 2599 |
| :--- | ---: |
| NUMBER OF MULTIPLE EVENTS | 99 |
| CHI-GQUARE TOO LARGE | 647 |
| FAILED CUT TEST | 361 |
| FAILED MOMENTUM TEST | 550 |
| TOO MANY CUT-STEPS | 6 |
| FAILED TO CONVERGE | 12 |
| FAILED CONSR TEST | 0 |
| NEGATIVE Z | 503 |

can be more accurately calculated. $t^{\prime} \doteq 4 \mathrm{E}_{\gamma} * \mathrm{E}_{\gamma} * \theta * \theta$ where $\theta$ is the angle the rho makes w.r.t. the $\gamma$ direction in the lab frame, and $t_{\min } \doteq \cdots\left(M_{p}^{2} /\left(2.0 * E_{\gamma}{ }^{*} E_{\gamma}\right)\right) * * 2$. After the above kinematical quantities have been calculated, the event is written out onto tape.

The programs were tuned up by selecting a particular 16 GeV $\gamma \mathrm{Be} \rightarrow \rho^{\circ} \mathrm{Be}$ run. A solid target run was chosen to reduce target $Z$ effects caused by the large length of the $\mathrm{H}_{2}$ and $\mathrm{D}_{2}$ target. First the target errors were all set large, and the value of $\sigma$ was varied. Various quantities, related to the quality of tracks found, were studied, including (1) $\%$ of 16 -wire, two-track events found, (2) number of wires found for 2 -track events, and (3) the number of 2 -track fit events found. All of the above indicated a value of $0.03^{\prime \prime}$ which compares very well with the 0.02" - .03" resolution attributed to the chambers.

Next the $x$ and $y$ target errors were set by studying the number of fit events versus target error. Here a value of $\cdot 3^{\prime \prime}$ gave good results. This is bigger than the value suggested by our beam size, indicating the difficulty of propagating back through the magnet.

Then the programs were checked by writing a dummy tape. On this dummy tape pairs of events were generated, followed through the magnet, and projected into the chambers. At this point the positions were converted into scaler readings and perturbed according to the plane resolution. Several background readings were also randomly added to simulate background events. The dummy tape had 200 events, all of which were found and correctly reconstructed. Also a few extra tracks were found, but no extra pairs were found. At this stage we believed
that the programs were running as well as possible.
Finally, shifts for the chamber system were determined by aligning the chamber packages. Here we looked for left, right asymmetry and a finite opening angle of the electron pairs. This would indicate a possible chamber package misalignment. Only minor corrections had to be made to the surveyed chamber position using this method.

## D. Geometrical and Kinematical Tests

After all the runs had been passed through stages $A, B$, and $C$, they were grouped according to target material, photon endpoint energy, and spectrometer setting. This resulted in the data sets shown in Table III.

Within each data set a run-by-run yield study was done to check run consistency. This entailed calculating the spark chamber efficiency, the details of which will be presented in Chapter $V$ in the section on corrections applied to the data. For all "good" runs the yields were flat, well within statistics. Also a study of target $Z$ distribution and the distribution of $X$ at the plug position was made. These were used for making an empty-target subtraction which will also be discussed in the section on corrections. Each data set was then subjected to this series of data tests and cuts:

1. The chamber tracks were extrapolated through the two hodoscopes, and a demand that all four hodoscopes had fired was made.
2. A target $Z$ cut was made for $H_{2}$ and $D_{2}$ runs to eliminate events originating in target windows. The cut consisted of accepting
only those events within $20^{\prime \prime}$ of the target center (see Figure 18).
3. Each event was run through program PASS which projects each track through the spectrometer. An event was kept if both tracks (a) passed around the plug, (b) passed through the magnet fiducial area, (c) passed through the active chamber area, and (d) triggered the hodoscopes. The dimensions of the magnet aperture were reduced from their actual dimensions to reduce the possibility of accepting events which scattered off pole tips.
4. Finally, each energy was subdivided into two GeV bytes. Thus, the 16 GeV endpoint runs were split into three energy regions
(a) $(14.0,16.0) \mathrm{GeV}$
(b) $(12.0,14.0) \mathrm{GeV}$
(c) $(10.0,12.0) \mathrm{GeV}$

After all cuts had been made, each of the data sets was written out onto disk for easy access. The numbers of events found for each data set are shown in Table III.

TABLE III

Events in Each Data Sample

| Er | $\mathrm{H}_{2}$ | $\mathrm{D}_{2}$ |
| :--- | :---: | :---: |
| 9 GeV | $1691^{*}$ | $1225^{* *}$ |
| $14-16$ | 2312 | 2680 |
| $12-14$ | 2635 | 3171 |
| $10-12$ | 2657 | 3103 |
| $11-13$ | 1367 |  |
| $9-11$ | 1564 |  |
| $7-9$ | 1636 |  |
| Events with $0.5 \leq \mathrm{M}_{\pi \pi} \leq 100 \mathrm{GeV} / \mathrm{c}^{2}$ |  |  |
|  | $0.0 \leq \mathrm{t}^{\prime} \leq 0.3(\mathrm{GeV} / \mathrm{c})^{2}$ |  |
|  |  |  |
|  |  |  |
| $* 0 \leq \mathrm{t}^{\prime} \leq 0.14(\mathrm{GeV} / \mathrm{c})^{2}$ |  |  |
| $* * 0 \leq \mathrm{t}^{\prime} \leq 0.025(\mathrm{GeV} / \mathrm{c})^{2}$ |  |  |

One of the disadvantages of a spark chamber spectrometer compared to, say, a bubble chamber is a lack of $4 \pi$ geometry. This lack of $4 \pi$ geometry means that all of our distributions such as $\pi \pi$ mass, $t$, decay angles, and energy spectra are distorted. Thus, to extract accurate distributions one must somehow fold in one's detection efficiency. This can be done in either of two ways.

One can simply divide each event by the detection efficiency for that event. Conversely, one can take a theoretical distribution, multiply it by one's detection efficiency, and compare it to the observed distribution. Using the first method, one is multiplying by a number between 1 and infinity, and a statistical fluctuation in a region of low detection efficiency could totally dominate a distribution. Using the second method, one is multiplying by a number between 0 and 1 and is less affected by regions of low detection efficiency. For this reason we use the second method.

Our acceptance for $\pi \pi$ mass extended from about $.4 \mathrm{GeV} / \mathrm{c}^{2}$ to above $1.0 \mathrm{GeV} / \mathrm{c}^{2}$, peaking at about $.775 \mathrm{GeV} / \mathrm{c}^{2}$ (see Figure 19a). The cutoff at low mass is due to the plug, while the high-mass cutoff is due to the magnetic field. The $t$ ' acceptance falls off exponentially with increasing t', dropping about a factor of four from 0 to $.3(\mathrm{GeV} / \mathrm{c})^{2}$ (see Figure 19 b ). The acceptance for $\cos \theta_{H}$ of the decay in the HEJICITTY system looks somewhat like $\sin ^{2} \theta_{H}$ with the acceptance going to zero for $\left|\cos \theta_{H}\right| \geq 0.8$ (see Figures 19c, d). The $\phi_{H}$ decay acceptance is fairly flat (see Figure


Fig. 19--(a) $M_{\pi \pi}$ acceptance, (b) $t '$ acceptance, (c) $\cos \theta_{H}$ acceptance (large $t^{\prime}$ ), (d) $\cos \theta_{H}$ Acceptance (small t'), (e) $\phi_{H}$ acceptance

19e); however, there are holes in the ( $\theta, \Phi$ ) space.
As can be seen in the figures showing the acceptance, both weighted and unweighted distributions have the same general features. However, they differ in detail. I have plotted the fitting function here, rather than the raw data, since the former is less susceptible in a region of low detection efficiency to a statistical fluctuation's dominating a weighted distribution. As can be seen in the $m_{\pi \pi}$ distribution, there is a very strong rho signal in both weighted and unweighted distributions. However, the mass skewing--faster falloff at masses above the rho than below--is more evident in the weighted distribution. This will be discussed in more detail in the section on mass and $t$ fits. Also, the cross section is clearly exponential in $t$ or $t$, and essentially flat in $\phi_{H}$. The $\cos \theta_{H}$ distribution is also more nearly distributed as $\sin ^{2} \theta_{H}$ than as $\cos ^{2} \theta_{H}$ or flat. All of these general features are characteristic of rho photoproduction.

## A. Efficiency Calculation

As seen in the previous section, although our acceptance does not totally alter the distribution, accurate determination is needed in order to do detailed fits. This acceptance, or apparatus efficiency, is calculated using standard Monte Carlo techniques. Since we have no polarization information, we need only five variables to describe the reaction. We take the set $(E, m, t, \theta, \Phi)$. E is the incoming photon energy; $m$ is the invariant dipion mass; $t$ is the square of the fourmomentum transfer to the nucleon; and $\theta$ and $\phi$ are the polar and
azimuthal angles respectively of the outgoing $\pi^{+}$in the dipion rest frame.

The definition of $\theta$ and $\phi$ is clearly coordinate-dependent. By convention the $y$ axis is chosen in the direction of $\overrightarrow{I N} \times O \overrightarrow{U T}$ or, in our case, $\vec{P}_{\gamma} \times \vec{P}_{\rho}$. Thus, if one specifies a $z$ axis and demands a righthanded coordinate system, $\theta$ and $\phi$ are completely determined. Two particular z axes are usually used: the JACKSON system specified the $z$ axis as along the incident photon in the dipion rest frame, while the HELICITY system specifies the $z$ axis as along the direction opposite to the outgoing target particle in the dipion rest frame.

To specify an event completely one needs the additional variables $\phi_{R}$, which relates the plane of the reaction to the physical world, and $\vec{T}$, which is the interaction point within the target. Since our apparatus is not symmetrical about the beam axis, these variables cannot be ignored. However, since there is no physics value to $\phi_{R}$ or $\vec{T}$, we can average over them when calculating our detection efficiency.

The calculation of efficiency for a set of variables ( $E, m, t, \theta, \phi$ ) consists of generating a number of trial events by randomly choosing the target vertex coordinates and $\phi_{R}$. The momenta describing this event are then run through the program PASS, described earlier in the section on data cuts and tests. The efficiency for this set of variables is given by the ratio of the number of events satisfying PASS to the total number of trial events.

In actual practice the data is binned to reduce the susceptibility to resolution effects, as well as to average out efficiency effects.

To further simplify the fitting process, the data is first fit as a function of $\theta$ and $\Phi$ and averaged over $m, E$, and $t$. Then the results of this fit are used to average over $\theta$ and $\phi$ and fit for $m$ and $t$ (again averaging over $E$ ). This causes the above efficiency to be modified slightly. Instead of choosing one set of ( $\mathrm{E}, \mathrm{m}, \mathrm{t}, \theta, \phi$ ), a range corresponding to the bin size is chosen. Secondly, instead of giving each event unit weight, each event is weighted by a function describing the averaging process.

To simplify data handling, all events for a particular data set are read in off disk and sorted into the chosen bins. Then a record is written out onto disk containing the efficiency and number of events for each bin. An estimate of the error on the efficiency is also included.

Since computer time is very valuable, the program was designed to be self-continuing. If a given set of efficiencies took longer to calculate than the job time specified, the program would automatically stop shortly before time expired. Another job could then be read in to continue where the first job left off. This allowed one to run long jobs without wasting computer time either by underestimating the time required or by having to request far more time than actually needed.

## B. Fitting Program

The fitting program was based on UCRL's MINFUN ${ }^{39}$ program using a ravine-crossing minimization algorithm. The basic minimization routine was taken in total, but most of the other subroutines were either
omitted or heavily altered. This program was also designed to be selfcontinuing, writing out the current fitting values and relevant derivatives onto disk at the end of each job. Maximum likelihood was used rather than chi-square due to having, on the average, less than one event per bin. When chi-square was used in a test, the results were similar to those obtained using maximum likelihood. The probability of finding $\lambda_{i j}$ events in any given bin is assumed to follow a Poisson distribution with mean $n_{i j}$.

$$
\begin{equation*}
P\left(\lambda_{i j}\right)=\frac{\exp \left(-n_{i j}\right)^{*}\left(n_{i, j}\right)^{\lambda_{i j}}}{\left(\lambda_{i j}\right)!} \tag{4.1}
\end{equation*}
$$

where $\mathrm{n}_{\mathrm{ij}}$ is given by a fitting function whose parameters, $\alpha$, one is trying to find. The likelihood function is given by the product of the individual Poisson probabilities. Since $P\left(\lambda_{i j}\right)$ is correctly normalized, we can use $\mathscr{L}_{1}=\Pi_{i j} P\left(\lambda_{i j}\right) \cdot \mathscr{L}_{1}$ is then maximized by varying $\alpha$. Actually, it is more convenient to use $\mathscr{L}=-\ln \mathscr{R}$ and minimize $\mathscr{P}$. The errors for a given parameter were found by stepping on that parameter and reminimizing to allow the other variables to change. The error on a parameter was taken to be the step size needed to change chi-square by one or the value of $\mathscr{X}$ by one-half, using the above procedure. In the case that the likelihood function is parabolic, this corresponds to $1 \sigma$. As will be seen, especially for the $\rho_{i j}$, the errors were asymmetric and, hence, the space was not always parabolic.

## C. Density Matrix Elements

The ultimate goal of experimental, high energy physics is to
determine the production amplitudes for various reactions. Since the density matrix elements ${ }^{40}$ are defined as a bilinear combination of the production amplitudes, determining the density matrix elements is a first step toward this goal. Following the formalism of Jacob and Wick, ${ }^{41}$ the density matrix element $\rho_{\lambda c \lambda c}^{J J}$ can be written as:

$$
\begin{align*}
& \rho_{\lambda c \lambda c^{\prime}}^{J J \prime}(S, m, t)=\frac{1}{\left(2 S_{a}+1\right)} \frac{1}{\left(2 S_{b}+1\right)} \sum \lambda a, \lambda b, \lambda d  \tag{4.2}\\
& f_{\lambda c^{\prime} \lambda d ; \lambda a \lambda b}^{J \prime} \quad *(S, m, t) f_{\lambda c \lambda d ; \lambda a \lambda b}^{J}(S, m, t)
\end{align*}
$$

This describes the following reaction:

where $f_{\lambda c \lambda d}^{J} ; \lambda_{a} \lambda_{b}(S, m, t)$ is the amplitude for producing particle $c$ with mass $m_{c}$, momentum transfer $t$, spin $J$, and helicity $\lambda c$. $\lambda a, \lambda b$, and $\lambda d$ are the helicities for particles $a, b$, and $d$ respectively. $S_{a}$ and $S_{b}$ represent the spins of incoming particles a and b respectively, while $S$ is the total energy-squared in the center-of-mass system of particles $a$ and $b$. If one now takes $a$ and $b$ to be unpolarized, sums over final spin states $d$, and takes particles $e$ and $f$ to have zero spin and be in a p-wave state only, one can express the angular distribution probability of particle $e$ (or $f$ ) in the cest irame as:

$$
\begin{align*}
& W(E, m, t, \theta, \phi)=3 / 8 \pi\left[\sin ^{2} \theta+\rho_{00}\left(3 \cos ^{2} \theta-1\right)\right.  \tag{4.3}\\
& \left.-2 \rho_{1-1} \sin ^{2} \theta \cos 2 \phi-2 \sqrt{2} \operatorname{Re} \rho_{10} \sin 2 \theta \cos \phi\right]
\end{align*}
$$

where for simplification $\rho_{m m}=\rho_{m m}^{11}(S, m, t)$ has been used and the explicit ( $s, m_{c}, t$ ) dependence has been suppressed. Equation (4.3) also depends
upon two relations: (1) $\rho_{-m-m^{\prime}}=(-1)^{m-m^{\prime}} \rho_{m m}$, which depends upon parity conservation and choosing the $y$ axis normal to the production plane and (2) $\rho_{00}+\rho_{11}+\rho_{-1-1}=1$ which ensures that $\int \mathrm{W} \mathrm{d} \Omega=1$. What we actually observed is expressed in equation (4.4).

$$
\begin{equation*}
O(E, m, t, \theta, \phi)=\mathbb{N}(E, m, t) * W(E, m, t, \theta, \phi) * \epsilon(E, m, t, \theta, \phi) \tag{4.4}
\end{equation*}
$$

where $0=$ the number of observed events for given ( $\mathrm{E}, \mathrm{m}, \mathrm{t}, \theta, \Phi$ ),
$\mathrm{N}=$ the number of events produced at the target for a given ( $E, m, t$ ), and
$\bar{\epsilon}=$ detection efficiency ( $\mathrm{E}, \mathrm{m}, \mathrm{t}, \theta, \phi$ ).
Since the density matrix elements are functions of $S$ (or $E$ ), $m_{c}$, and $t$, it would be desirable to fit the distributions with respect to these variables. Lacking infinite statistics, as well as having to deal with an efficiency function, requires binning the data. This accomplishes two things: it ensures sufficient statistics, and it smooths out effects due to the acceptance. The density matrix elements are expected to vary slowly with respect to $m$ near the rho mass and also with respect to $E$. The variation in $t$ is expected to be more rapid. Thus, we choose one mass bin ( $0.67 \leq m_{\pi \pi} \leq 0.87 \mathrm{GeV} / \mathrm{c}^{2}$ ) centered at the rho and a two-GeV E bin. We fit over six t' bins covering the interval $(0, .30)(\mathrm{GeV} / \mathrm{c})^{2}$. The size of the $t$ ' bins was chosen to ensure roughly equal statistics in each bin. We chose to divide the $(\cos \theta, \phi)$ space into $20 \times 20$ equal-sized $\cos \theta$ bins and equal-sized $\phi$ bins. This ensures that $\epsilon$ and $W$ are slowly varying over any one bin. If equation (4.4) is integrated over $m$, $E$, the $k^{\text {th }} t$ bin, $i^{\text {th }} \cos \theta$ bin, and $j^{\text {th }} \phi$ bin, we get $n_{i j}^{k}$, the number of events expected for that bin as
a function of $\rho_{00}, \rho_{1-1}$, and $\operatorname{Rep}_{10}$.

$$
\begin{align*}
n_{i j}^{k} & =\int_{B i n}^{d t} \int_{\operatorname{Bin} i}^{d \cos } \int_{B i n}^{d \Phi} \int d m \int d E O(E, m, t, \theta, \phi)  \tag{4.5}\\
& =N_{0}{ }^{*} \epsilon_{i j}^{k} * W_{i j}^{k}\left(\bar{\rho}_{O O}^{k}, \bar{\rho}_{1-1}^{k}, \operatorname{Re}_{\rho_{10}}^{k}\right)
\end{align*}
$$

where $\epsilon_{i k}^{k}$ is the efficiency, weighted by $N(E, m, t)$ and averaged over the allowed $E, m, t, \theta$ and $\phi$ range; $N_{0}, \bar{\rho}_{00}^{k}, \bar{\rho}_{1-1}^{k}$, and $R e \bar{\rho}_{10}^{k}$ are fitting parameters; and $\frac{\rho_{m n}^{k}}{\rho_{m}}$ represents an average value of $\rho_{m n}^{k}$ over the selected $E, m$, and $t$ range.

For $N(E, m, t)$ we use an s-wave Breit-Wigner to describe the mass shape, an exponential for $t$, and the known bremsstrahlung spectrum for E. 42 While the $m$ dependence will be given by a somewhat different form when discussing the $\rho$ cross-section normalization, the difference introduced here over this limited mass interval is negligible.

The definition of the density matrix elements, as a bilinear sum of production amplitudes, imposes a set of constraints upon their values. A discussion of these constraints is presented in Appendix B while the results of the density matrix element fits are presented in Chapter VI.

## D. Mass and $t$ Distributions

Fitting the rho mass distribution has always been a problem, due in part to its large width. Another apparent aspect of the rho mass shape is that for photoproduction the shape is skewed, and the value
of the peak is shifted to lower masses than those observed in collidingbeam experiments. This behavior can be described with several models, but we choose to use the söding ${ }^{27}$ model since, as mentioned in the introduction, this model seems to give the best overall description of the reaction. The Feynman diagrams included in the model are shown below.


Diagram (a) describes rho photoproduction and is what we are hoping to extract from the data. It reduces to a p-wave Breit-Wigner for the mass dependence. The other four diagrams are nonresonantbackground contributions. Diagrams (b) and (c) are a Drell or Deck type background, while diagrams (d) and (e) are the same as (b) and (c) except with final state interactions present. These diagrams correct the so-called "double" counting effects ${ }^{43}$ and ensure that at the rho mass only diagram (a) contributes. In other words, the rho saturates unitarity. A priori there is no reason for this to be true. However, to be compatible with other groups using the S8ding model, we will use this assumption, too.

The mass skewing results from the interference of diagram (a) with the other four diagrams. This interference term changes sign from positive to negative in passing through the rho mass.

The model assumes that all of the reactions are mediated by pomeron exchange. Hence, the rho-target and pion-target scattering amplitudes should be purely imaginary and exponential in $t$ (see the next section for the modification to the $t$ dependence off deuterium) and have the other properties of elastic scattering. This results in the following form for hydrogen:

$$
\begin{align*}
& d^{2} \sigma / d m_{\pi \pi} d t=N_{0}^{*} q^{3}\left(m_{\pi \pi}^{2}\right) * \exp (B t) * I /\left[\left(m_{\rho}^{2}-m_{\pi \pi}^{2}\right)^{2}+\gamma^{2}\right] *  \tag{4.6}\\
& {\left[1.0+2 * B A C K *\left(m_{\rho}^{2}-m_{\pi \pi}^{2}\right) /\left(m_{\pi \pi}^{2}-t\right)+\left(B A C K *\left(m_{\rho}^{2}-m_{\pi \pi}^{2}\right) /\left(m_{\pi \pi}^{2}-t\right)\right)^{2}\right]}
\end{align*}
$$

where $N_{0}$ is a normalization parameter, $q\left(m_{\pi \pi}^{2}\right)=\sqrt{m_{\pi \pi}^{2}-4 *_{\pi}^{2}} / 2$ is the momentum of the pions in the rho rest frame, $B$ is the slope parameter,
$m_{\rho}$ is the rho mass parameter, BACK gives the amount of söding background needed, and $\gamma$ is the p-wave mass-dependent width times the rho mass.

$$
\begin{equation*}
\gamma=m_{\rho}^{*} \Gamma \rho^{*}\left[q\left(m_{\pi \pi}^{2} / / q\left(m_{\rho}^{2}\right)\right]^{3} *\left(m_{\rho} / m_{\pi \pi}\right)\right. \tag{4.7}
\end{equation*}
$$

The three terms--rho, interference, and Drell-are shown in Figure 20 for $t=0$. The actual parameters found for one of the fits were used to obtain the results plotted. While the interference term contributes little to the integrated cross section, it does account for the observed rho mass shift and the skewing of the $m_{\pi \pi}$ spectrum. The Ross-Stodolsky model ${ }^{26}$ also reproduces the mass skewing by multiplying the rho diagram by $\left(m_{\rho} / m_{\pi \pi}\right)^{4}$. However, it is clear from (4.6) that the s8ding model has additional $t$ dependence other than the exponential term, $\exp (B t)$. Since the syding model describes $d^{2} \sigma / d m_{\pi \pi} d t$ very well, ${ }^{33}$ to get comparable results with the Ross-Stodolsky model one must use a form ( $m_{\rho} / m_{\pi \pi}$ ) $n(t)$ where $n(t)$ averaged over $t$ is 4 . However, the model provides no form for $n(t)$, and we choose to use the SBding model, where the $t$ dependence is explicitly proscribed.

Finally, there is a problem with using the p-wave Briet-Wigner, and that is the dependence for large $m_{\pi \pi}$. As $m_{\pi \pi}$ gets large, $d^{2} \sigma / d m_{\pi \pi} d t \propto 1 / m_{\pi \pi}$, and thus the integral over $m_{\pi \pi}$ diverges. This complication will be discussed further in Chapter VI in discussing cross-section values.

The fitting procedure is the same as described in the previous section. The only differences are that roles of $(\theta, \phi)$ and ( $m, t$ ) are reversed. For $\mathbb{N}(\theta, \phi, E)$ we now use $\sin ^{2} \theta_{H}$ to describe the $(\theta, \phi)$ shape, the known bremsstrahlung spectrum for $E$, and the equation (4.6) for $W$.


Fig. 20--S8ding Model Contributions

We fit over the range $.50 \leq m_{\pi \pi} \leq 1.0 \mathrm{GeV} / \mathrm{c}^{2}$ and $0 \leq \mathrm{t}^{\prime} \leq 0.3(\mathrm{GeV} / \mathrm{c})^{2}$ for the five parameters: $m_{\rho}, \Gamma_{\rho}, B A C K, B$, and $N_{o}$. We divide the ( $m_{\pi \pi}$, $t^{\prime}$ ) space into 20 equal-sized $m_{\pi \pi}$ bins and 20 equal-sized $t^{\prime}$ bins. The results of the fits are presented in Chapter VI.

## E. Hydrogen and Deuterium Ratio

Except for small corrections due to the finite rho mass, rho photoproduction can be treated as an elastic process. Hence, if we are interested in studying rho photoproduction off deuterium, for small $t$, Glauber theory should be applicable. Since Glauber theory has already been shown to work for elastic pion-deuteron reactions, ${ }^{44}$ it should also give good results here. Basically Glauber theory describes reactions in complex nuclei in terms of successive two-body interactions, all of which leave the nucleus undisturbed.

Since we do not detect the deuteron, we cannot separate the reactions $\gamma \mathrm{d} \rightarrow \rho^{\circ} \mathrm{pn}$ and $\gamma \mathrm{d} \rightarrow \rho^{\circ} \mathrm{d}$. For this reason we use a slight modification, which includes both of the above reactions in the formalism. If we ignore spin, we can then write: 45,35

$$
\begin{equation*}
\left(\frac{d \sigma}{d \Omega}\right)_{D}=4\left|a_{0}(t)\right|^{2}\left[\frac{1+F(t)}{2}+G(t)\right]+4\left|a_{1}(t)\right|^{2} \frac{(1-F(t))}{2} \tag{4.8}
\end{equation*}
$$

where $a_{0}(t)$ and $a_{1}(t)$ are the respective isoscalar and isovector $t$ channel exchange amplitudes off nucleons, $F(t)$ is the deuteron form factor, and $G(t)$ is the Glauber scattering correction which takes into account the shadowing of one nucleon by another. For $F(t)$ we use the results of electron scattering, ${ }^{46} F(t)=\exp (56 t)$. Similarly,

$$
\begin{equation*}
\left(\frac{d \sigma}{d \Omega}\right)_{H}=\left|a_{0}(t)+a_{1}(t)\right|^{2}=\left|a_{0}(t)\right|^{2}+\left|a_{1}(t)\right|^{2}+2 \operatorname{Re}\left(a_{0}(t) a_{1}^{*}(t)\right) \tag{4.9}
\end{equation*}
$$

$$
\begin{align*}
& \text { Then } \\
& R_{D H}^{(t)}\left(\frac{d \sigma}{d t}\right)_{D} /\left(\frac{d \sigma}{d t}\right)_{H}=4\left(\left(1-2 A_{I}(t)\right) *\left(\frac{1+F(t)}{2}+G(t)\right)-A_{2}(t) *(F(t)+G(t))\right) \tag{4.10}
\end{align*}
$$

where

$$
\begin{aligned}
& A_{1}(t)=\frac{\operatorname{Re} a_{0}(t) a_{1}^{*}(t)}{\left|a_{0}(t)+a_{1}(t)\right|^{2}} \\
& A_{2}(t)=\frac{\left|a_{1}(t)\right|^{2}}{\left|a_{0}(t)+a_{1}(t)\right|^{2}}
\end{aligned}
$$

Since for each $t$ value we only measure one quantity, $R(t)_{D H}$, and since we have two unknowns, we must make some simplifying assumption to get any farther. If we assume both $a_{0}(t)$ and $a_{1}(t)$ have the same $t$ dependence, then $A_{1}(t)$ and $A_{2}(t)$ become constants and can be solved for. For $G(t)$ we use the calculations of Ogren: ${ }^{49}$

$$
\begin{equation*}
G(t)=\frac{-\sigma_{\rho N}\left\langle r^{-2}\right\rangle}{4 \pi} \exp (-b t / 4)+\left(1+\eta_{\rho}^{2}\right)^{\sigma_{\rho N}^{2}\left\langle r^{-2}\right\rangle} \exp (-b t / 2) \tag{4.11}
\end{equation*}
$$

where $\sigma_{\rho N}$ is the rho-nucleon total cross section, $\left\langle r^{-2}\right\rangle$ is the expectation value of $1 / r^{2}$ for the deuteron wave function, $\eta_{\rho}$ is the ratio of real part to imaginary part of the rho-nucleon amplitude, and $b$ is the slope parameter applicable to rho photoproduction off hydrogen. If we choose $\sigma_{\rho N}^{\cdot}=28 \mathrm{mb},\left\langle\mathrm{r}^{-2}\right\rangle=.03 \times 10^{-27} \mathrm{~cm}^{2}\left(.03(\mathrm{mb})^{-1}\right), \eta_{\rho}=-.2$, and $\mathrm{b}=8(\mathrm{GeV} / \mathrm{c})^{-2}(3.11 \mathrm{mb})$, we obtain:

$$
\begin{equation*}
G(t)=-0.0668 \exp (-2 t)+0.00624 \exp (-4 t) \tag{4.12}
\end{equation*}
$$



Fig. 2l--Glauber terms for rho photoproduction off deuterium

The two functions, $\frac{1+F(t)}{2}+G(t)$ and $F(t)+G(t)$, are shown in Figure 21. It is clear that the two functions are sufficiently different so as to allow the extraction of $\mathrm{A}_{1}$ and $\mathrm{A}_{2}$ and, consequently, the fraction of isovector exchange amplitude, $\left|a_{1}\right| /\left|a_{0}+a_{1}\right|$. Fuxthermore, we can solve for $\left|a_{1} / a_{0}\right|$ and $\cos \phi$ where $\phi$ is the relative phase between $a_{1}$ and $a_{0}:\left|\frac{a_{1}}{a_{0}}\right|=\sqrt{A_{2} /\left(1-2 A_{1}-A_{2}\right)}$ and $\cos \phi=\frac{A_{1}}{A_{2}} \sqrt{A_{2} /\left(1-2 A_{1}-A_{2}\right)}$.

Since $R_{D H}(t)$ only depends upon the ratio of cross sections, any error in the normalization of the UD counters or in the acceptance calculation cancels. For this reason we compare unweighted distributions directly after making the appropriate target empty corrections and all other corrections which are target dependent, including number of UD counts for each target, different target densities, absorption corrections, and hodoscope and spark chamber efficiencies. These corrections are presented in Chapter $V$ while the results of the fits are presented in Chapter VI.

## CHAPTER V

## CROSS-SECTION DETERMINATION

To measure cross sections four quantities must be determined: (1) the photon flux incident upon the target, (2) the number of atoms $/ \mathrm{cm}^{2}$ in the target, (3) the fraction of the observed mass and $t$ spectra actually due to rho production versus background, and (4) corrections to the observed distribution for inefficiencies due to pion and photon absorption, spark chamber inefficiency, geometrical acceptance, etc.

## A. Flux Determination

Our run-by-run monitor was the 204 pair spectrometer described earlier. Originally we had hoped to calibrate the flux monitor by converting 2D5 into an electron-pair spectrometer and actually counting the $e^{+} e^{-}$pairs in the spark chambers. The intense electromagnetic background permitted only a $20 \%$ calibration using this method. Hence, to get a more accurate calibration we relied upon accurately calculating the bremsstrahlung spectrum, the angular-pair production spectrum, and the acceptance of the 2D pair spectrometer. With this method we were able to obtain a more accurate calibration of $\tilde{Q}=\left(\frac{1}{E_{\max }} \int_{0}^{E_{\max }} K \frac{d N}{d K} d K\right)$ to the number of coincidence counts, UD, in the pair spectrometer.

1. Photon Spectrum

Since the bremsstrahlung beam was formed in a 0.0014 radiation-length A target, a thin target bremsstrahlung representation was used for the
beam, and after several modifications this gave an accurate description of the beam. The first modification was to take the finite solid angle of the $\gamma$ beam into account, and the second was to take into account the multiple scattering of the beam. The bremsstrahlung spectrum was finally described by the formula: ${ }^{47}$

$$
\begin{equation*}
d \mathrm{~N} / \mathrm{dK}=Q / \mathrm{K}\left[\mathrm{~V}^{2}+\mathrm{C}^{*} \cdot(1-\mathrm{V})\right] \tag{5.1}
\end{equation*}
$$

where $Q$ is the normalization constant which one is trying to find, and to within $10 \% Q=\tilde{Q} ; V$ is the ratio of the photon energy to the primary electron energy; and $C$ is a constant depending on the electron beam energy, multiple scattering, and finite solid angle of the bremsstrahlung beam.

## 2. Electron-Pair Production Spectrum

The electron decay spectrum is given by: ${ }^{47}$

$$
\begin{equation*}
d N / d K^{+}=T^{*}\left[(V-1 / 2)^{2}+B\right] \tag{5.2}
\end{equation*}
$$

where $\mathrm{K}^{+}$is the momentum of the positron, $V$ is the ratio of the positron momentum to the momentum of the photon pair producing the electron-positron pair, and $B$ is a known constant depending only on the $Z$ of the target material.

$$
\begin{gather*}
B=(24 * A-1) /[4 *(12 * A+1)]  \tag{5.3}\\
A=\ln \left(183 * z^{1 / 3}\right) * Z /(Z+1)+\ln \left(1440 * Z^{-2 / 3}\right) * 1 /(Z+1)
\end{gather*}
$$

Since we only need the shape of the decay spectrum at this point, $T$ can be ignored.
3. Acceptance of 2D4 Pair Spectrometer

If one uses a small-angle approximation and an effective-length
approximation for tracking the electron-positron pair through the magnet, the acceptance, $a(K \gamma)$, can be easily calculated as:

$$
\begin{align*}
& \frac{(1-\mathrm{KMIN} / \mathrm{K})^{3}+12 * \mathrm{~B}^{*}(1-\mathrm{KMIN} / \mathrm{K})}{12 * \mathrm{~B}+1}: \mathrm{KMIN} \leq \mathrm{K} \leq \mathrm{KM} \\
& a(K)=  \tag{5.4}\\
& \frac{(\mathrm{KMAX} / \mathrm{K}-1)^{3}+12^{*} \mathrm{~B}^{*}(\mathrm{KMAX} / \mathrm{K}-1)}{12^{*} \mathrm{~B}+1}: \quad \mathrm{KM} \leq K \leq \mathrm{KMAX}
\end{align*}
$$

where:
KMIN $=$ Minimum detectable $\gamma$ energy
KMAX $=$ Maximum detectable $\gamma$ energy
$K M=(K M I N+K M A X) / 2$
These values depend only on the counter separation and the magnetic field strength of 2 D 4 . For 16 GeV running the spectrometer was set to be sensitive to photons between 9.85 GeV and 14.8 GeV , while for 13 GeV running the spectrometer was sensitive to photons between 7.96 GeV and 11.94 GeV . The field strength and counter separation were determined so that the spectrometer was sensitive to a relatively flat part of the pair spectrum away from the endpoint. This reduced effects due to shifting of the $2 D 4$ current. The current was monitored continuously and found to be constant to a fraction of a percent throughout the running.
4. Determining Q/UD

Q is given by the formula:

$$
\begin{equation*}
1 U D=Q * A C C * P C * M T \tag{5.5}
\end{equation*}
$$

where $A C C=\frac{1}{Q} \int_{0}^{\cdot E_{M A X}}(\mathrm{dN} / \mathrm{dK}) * a(\mathrm{~K} \gamma) \quad \mathrm{dK} \gamma$. ACC is the fraction of electron-positron pairs created in the converter in front of 204 which
actually give coincidences in the pair spectrometer, weighted by the photon spectrum. The error on ACC is determined by varying the magnetic field strength of $2 D 4$ and the position of the counters relative to the beam over a reasonable range of values. PC is the probability of converting a photon in a $1 / 16^{\prime \prime}$ Al converter into an electron-positron pair; the error on PC is due to an uncertainty of $1 / 3 \mathrm{mil}$ in the thickness of the converter. MT is a correction for pairs not created in the converter. This correction is determined by comparing rates with the AI converter in the beam and with it removed from the beam. See below for a summary of these quantities as obtained for both energies.

|  | 13 GeV | 16 GeV |
| :--- | :---: | :---: |
| SB di of 284 | $395 \mathrm{Kg}-$ in $(9.78 \mathrm{Kg}-\mathrm{m})$ | $477 \mathrm{Kg}-\mathrm{in}(12.1 \mathrm{Kg}-\mathrm{m})$ |
| KMIN | 7.96 GeV | 11.94 GeV |
| KMAX | 9.85 GeV | 14.77 GeV |
| PC a 1/16" Al | $0.0139 \pm 0.00074$ | $0.0139 \pm 0.00044$ |
| ACC | $0.0331 \pm 0.0013$ | $0.0330 \pm 0.0013$ |
| BACK | $1.29 \pm 0.02$ | $1.29 \pm 0.02$ |
| Q/UD | $1685 \pm 90$ | $1690 \pm 90$ |
| C | 1.522 | 1.513 |

Thus, we were able to obtain a $5 \%$ measurement of the normalization using the 204 pair spectrometer.
B. Data Corrections

The biggest correction, that of the 2D5 acceptance, has already been discussed. It is very difficult to assign an error to such a Monte

Carlo acceptance calculation. However, after a comprehensive study of the problem, an error of $3 \%$ was assigned to the calculation. Below the numerous smaller correction and their errors are discussed.

## 1. Hodoscope Inefficiency

As had already been mentioned, each individual counter was plateaued before the run and had an efficiency greater than $99.5 \%$. However, during the run we had a 1 " Pb sheet in front of the B hodoscope. This, plus the fact that counter efficiency can change with time, made it necessary to calculate the hodoscope efficiency during the run itself.

Since the experiment had no other timing information for an event, some assumptions had to be made in order to calculate the efficiency, The first assumption was that it is sufficient to calculate an average efficiency for each hodoscope array. Since the individual counters had very similar efficiencies when originally plateaued, and since checks were made periodically during the run to ensure that no counter had died, this was probably a reasonable assumption. The second assumption was that given that a dipion pair (1) has triggered a counter for each track in one hodoscope array but (2) has failed to trigger one or more of the counters in the second hodoscope array through which the tracks have passed, the event would still have triggered the system. This means that on the average each hodoscope array has more than two counters on for each event. Table IV shows the number of $0,2,2,3,4$, and 5 track events found, as well as the number of events having $0,1,2,3,4$, or 5 hodoscopes on for the $A$ and the $B$ hodoscope array. It is clear that there are far more hodoscopes on than there are tracks. This is
probably due to the heavy electromagnetic background present, and this makes the second assumption appear reasonable.

With these two assumptions the average hodoscope efficiency can be calculated. First the set of all events satisfying all the tests listed under geometrical and kinematical tests in Chapter III, except the one demanding that the four hodoscopes on the tracks have fired, is selected. Then, to calculate the efficiency of the first hodoscope array, all events having two hodoscopes on in the second hodoscope array are selected. Third, the number of those events having 0,1 , and 2 hodoscopes on in the first array is determined. By fitting these numbers (with maximum likelihood) to a binomial distribution, the average efficiency of the first hodoscope array is obtained. At first the fitting was done as a function of 2 GeV energy cuts, but, since the results were independent of the cuts, to improve statistics a larger energy cut of $10-16 \mathrm{GeV}$ was used for the 16 GeV data, and a $7-13 \mathrm{GeV}$ cut was used for the 13 GeV data. Because four hodoscopes were demanded for all good pairs, the hodoscope efficiency for each event is the product of the $A$ and $B$ hodoscope efficiency squared. The results are listed below:

|  | $16 \mathrm{GeV} \mathrm{H}_{2}$ | 16 GeV D |  |
| :---: | :---: | :---: | :---: |
| A: | 13 GeV H |  |  |
| B: | $0.9993 \pm .0003$ | $0.9984 \pm .0003$ | $.997 \pm .001$ |
| $\mathrm{CF}_{\text {HODOSCOPE }}:$ | $0.973 \pm .009$ | $0.969 \pm .008$ | $.951 \pm .010$ |
|  | $(1.06 \pm .02)$ | $(1.07 \pm .02)$ | $(1.11 \pm .02)$ |

As can be seen, the A hodoscope efficiency agrees well with the plateaued values, while the $B$ hodoscope shows a lower efficiency due to the $1 " \mathrm{~Pb}$ sheet placed in front of it. It is hard to estimate

## TABLE IV

|  | $\mathrm{N}=$ | 0 | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NUMBER OF EVENTS WITH N TRACKS |  | 4498 | 4681 | 3900 | 270 | 20 | 2 |
| NUMBER OF EVENTS WITH N HODOSCOPE A |  | 2 | 39 | 9887 | 2094 | 678 | 671 |
| NUMBER OF EVENTS WITH N HODOSCOPE B |  | 2 | 51 | 5616 | 2657 | 1589 | 3456 |

THERE WERE 13371 EVENTS
quantitatively the effect of the Pb since even if the pion does not make it through the Pb to trigger the counter, some charged particles probably will.

The target empty hodoscope efficiencies were in all cases consistent with the respective target full results listed above, and the above hodoscope efficiencies were used for all subsequent target empty calculations.

## 2. Spark Chamber Efficiency

Spark chamber inefficiency has three sources: (1) the misfiring of the energy box, (2) the gas discharge not taking place within the gap, and (3) the magnetostrictive wand discriminator not firing for weak signals. These are henceforth referred to as energy-box, gap, and plane inefficiencies respectively.

The algorithm was chosen to generate all possible combinations (6837) of the above inefficiencies consistent with the track reconstruction requirements (at least eleven wires with three or more wires in three or more chambers). These, together with the track information associated with the set of all good rho events, were used to calculate the above three types of inefficiency. Since the spark chamber inefficiencies varied with time, the efficiency was calculated on a run-by-run basis. The efficiencies were not a function of the energy cut, so to improve statistics an energy cut of 10-16 GeV was used for the 16 GeV data, and an energy cut of $7-13 \mathrm{GeV}$ was used for the 13 GeV data.

In determining the energy-box, gap, and plane efficiency for any one chamber, only those tracks which had at least three wires in the
other three chambers contributing to the track were used. The plane efficiency was calculated for only those tracks in which the other plane in the gap had contributed, to ensure that the gap had fired Similarly, the gap efficiency was calculated only for those tracks in which the othex gap in the chamber had contributed at least one wire to the track. This ensured that the energy box had fired. This gep efficiency was then corrected for the case where the gap fired but both planes failed to fire. Finally, the energy-box efficiency was calculated as the fraction of events having at least one plane in that chamber contributing to the track. This efficiency was then corrected for the case of no planes firing due to gap and plane inefficiencies.

Having determined all of the individual plane, gap, and energy-box efficiencies, we used these to calculate the probability of detecting each of 6837 possible allowed combinations of wires and, at the same time, to generate a predicted set of wire distributions. These were compared with the observed spark distributions, and in practically every case the agreement was good. The track-finding efficiency was then the sum of the above probabilities, while the pair efficiency was the square of the single-track efficiency.

A few runs had very low pair efficiencies and were deleted. These runs occasionally occurred when the gas system became poisoned. As a further check the yield, the number of pairs in the "rho region" corrected for spark chamber efficiency/the number of $U D$ counts, was plotted for each target material and photon endpoint energy as a function of run. Even when the efficiency varied, the yield remained constant. To simplify future calculations an average spark chamber pair efficiency was
calculated, being weighted by the number of events contributing to the calculation for each run. The results are listed below:

$$
\begin{array}{llllll}
16 \mathrm{GeV} \mathrm{H} \\
2 & 16 \mathrm{GeV} \mathrm{D} & 16 \mathrm{GeV} \mathrm{MT} & 13 \mathrm{GeV} \mathrm{H} & 13 \mathrm{GeV} \mathrm{MT} \\
& & 1.11 \pm .02 & 1.06 \pm .02 & 1.21 \pm .13 & 1.06 \pm .02
\end{array} 1.05 \pm .05
$$

3. Pion and Photon Absorption in the Target

Since the target was fairly long, both the incoming photon and outgoing $\rho\left(\pi^{+} \pi^{-}\right)$were absorbed in it. The only other sources of absorption were the target windows, the A hodoscope, the four chambers, and the air. All of these put together constituted less than $5 \%$ of the effect due to the target alone and could be ignored when doing a $10 \%$ experiment. The absorption for a rho, produced $t$ radiation lengths into the target, is given by:

$$
\begin{equation*}
\mathrm{R}_{\text {absorption }}=\exp (-7 / 9 * t) * \exp (-(T-t) * A) \tag{5.6}
\end{equation*}
$$

where $T$ is the target length in radiation-lengths and $A$ is the dipion absorption factor in inverse radiation lengths. The value of $A$ was determined from experimental, pion-absorption cross sections. ${ }^{48}$ The variation of $\mathrm{R}_{\text {absorption }}$ was between $10 \%$ and $15 \%$ for both hydrogen and deuterium and, hence, an average correction for the whole target was made. The results are listed below.

|  | 16 GeV H 2 | 16 GeV D 2 | 13 GeV H 2 |
| :---: | :---: | :---: | :---: |
| T | 0.124 | 0.145 | 0.124 |
| $A^{*}{ }^{\text {T }}$ | $0.189 \pm 0.035$ | $0.445 \pm 0.080$ | $0.189 \pm 0.035$ |
| $\mathrm{CF}_{\text {ABSORPTION }}$ | $0.16 \pm 0.02$ | $1.32 \pm 0.050$ | $1.16 \pm 0.02$ |

4. Pion Decay

Since pions decay into muons, a pion might decay and we would actus.lly observe the muon and not know the difference. Since the muon and pion will have very nearly the same trajectory, due to the high energy of the pion, very few such events will be actually lost. The major effect will be to slightly smear out our resolution. This correction was studied using a Monte Carlo approach and was found to be $C F_{\text {DECAY }}=1.01 \pm$ 0.01 for pions actually lost.
5. TARGET Empty and TARGET Z Cut Corrections
a. TARGET Empty Correction

A target empty subtraction, while very small due to the thinness of the Al windows, is essential because rho photoproduction is a coherent process. Thus, all the events coming from the window will occur at very low t's. This will result in a coherent peak in $d \sigma / d t$ at low $t$. This peak was in fact observed if no target empty correction was made. It was also observed that almost all of the empty events came from the downstream side of the target (see Figure 22).

Three different methods were used to determine this correction, and all gave consistent results. In each case a scale factor for the empty runs is being sought. This number accounts for the difference in the amount of full and empty running time.

The first method calculated the scale factor by the following formula:

$$
\begin{equation*}
\mathrm{R}_{\text {Theory }}=\frac{\text { Number } \gamma_{\mathrm{Full}}}{\text { Number } \gamma_{\mathrm{MT}}} * \frac{\left(\mathrm{CF}_{\mathrm{S} . \mathrm{C}}\right)_{\mathrm{MT}}}{\left(\mathrm{CF}_{\mathrm{S} . \mathrm{C} .}\right)_{\mathrm{Full}}} * \exp \left(\frac{-7 t}{9}\right) \tag{5.7}
\end{equation*}
$$



Fig. 22--Reconstructed target $Z$ distribution for target empty
where $\mathrm{CF}_{\mathrm{S}} . \mathrm{C}$. refers to the spark chamber efficiency correction, $t$ is the target thickness in radiation-lengths, and the ratio of photons is replaced by the ratio of UD's. The exponential factor comes from (1) the difference in the absorption of photons for the target full and empty cases and (2) the fact that most of the empty events occur at the downstream window.

The second method calculated the scale factor by the formula:

$$
\begin{equation*}
R_{\text {TARGZ }}=\frac{\left(\text { Number events with TARGZ } \leq 64^{\prime \prime}\right)_{\text {Full }}}{\left(\text { Number events with TARGZ } \leq 64^{\prime \prime}\right)_{\text {Empty }}} \tag{5.8}
\end{equation*}
$$

These events came from the plug region and, hence, took into account experimentally the difference in UD counts, spark-chamber efficiencies, and photon absorption.

The third method calculated the ratio of plug events based on the $\mathbf{x}$ distribution of events at the plug.

$$
\begin{equation*}
R_{\text {PLUGx }}=\frac{\left(\text { Number events with xPLUG } \leq 1.5^{\prime \prime}\right)_{\text {Full }}}{\left(\text { Number events with xPLUG } \leq 1.5^{\prime \prime}\right)_{\text {Empty }}} \tag{5.9}
\end{equation*}
$$

Since the three values gave consistent results, a best value (in the least-square sense) was calculated from them.

$$
\begin{array}{llll} 
& 16 \mathrm{GeV} \mathrm{H} \\
2 & 16 \mathrm{GeV} \mathrm{D} & 13 \mathrm{GeV} \mathrm{H} \\
2
\end{array}
$$

b. WARGET Z Cut Correction
 cemter, nost of the empty events would be omitted, making a bin-by-bin empty subtraction unnecessary. However, since the experimental TARGET $Z$ resolution was not good enough, it was necessary to correct for events
actually coming from the target that were lost due to this cut.
$\mathrm{CF}_{\text {TARGET }}=\frac{\text { Full Events }-\mathrm{R}_{\mathrm{MT}}{ }^{*}(\mathrm{MT} \text { EVENTS })_{\text {NO CUT }}}{\text { Full Events }-\mathrm{R}_{\mathrm{MT}}{ }^{*}(\mathrm{MT} \text { EVENTS })_{\text {TARGZ CUT }}}$

$$
\begin{array}{llll} 
& 16 \mathrm{GeV} \mathrm{H} & 17 \mathrm{GeV} \mathrm{D}_{2} & 13 \mathrm{GeV} \mathrm{H} \\
\mathrm{CF}_{\text {TARGET }} & 1.13 \pm 0.02 & 1.12 \pm 0.02 & 1.15 \pm 0.02
\end{array}
$$

## 6. Inelastic Production Correction

Since we only measure the two outgoing pions, we can only calculate the photon energy inducing the reaction by assuming that the event is elastic. This means that on an event-by-event basis we cannot tell whether or not an event is elastic. However, we can measure a large fraction of the calculated photon-energy spectrum and compare this with the true bremsstrahlung spectrum; this allows us to determine the amount of inelastic contamination in any given energy bin.

$$
\frac{\mathrm{d} \mathrm{~N}^{\text {obs }}}{\mathrm{dK}}=\frac{\mathrm{dN}}{\mathrm{dK}} \gamma^{*}\left(\sigma_{\text {elastic }}(\gamma \mathrm{p} \rightarrow \rho \mathrm{p})+\sigma_{\text {inelastic }}\left(\gamma \mathrm{p} \rightarrow \pi^{+} \pi^{-} \mathrm{p}+\text { anything }\right)(5.1 \mathrm{I})\right.
$$

If we assume that $\sigma_{\text {elastic }}$ is constant, an assumption which is good to several percent for this reaction at high energies*, then deviations of the observed spectrum with respect to the bremsstrahlung spectrum measure the $\sigma_{\text {inelastic }}$ term. We fit $\frac{d N^{\circ b s}}{d K}$ to:

$$
\begin{equation*}
\frac{\mathrm{d} \mathrm{~N}^{\mathrm{Obs}}}{\mathrm{dK}}=\mathrm{A} \frac{\mathrm{dN}}{\mathrm{dK}} \gamma+\mathrm{B} *(E M A X-K)+C *(E M A X-K)^{2} \tag{5.12}
\end{equation*}
$$

[^2]We used only events satisfying all data cuts as listed in Chapter IV (i.e., $.5 \leq m_{\pi \pi} \leq 1.0 . \mathrm{GeV} / \mathrm{c}^{2}$ and $0 \leq t^{\prime} \leq 0.3(\mathrm{GeV} / \mathrm{c})^{2}$ ). In all cases $B$ turned out to be zero.

To obtain confidence in this method, we also fit our data at 16 GeV off Pb and Be targets. Since rho production in complex nuclei is coherent, the forward amplitude should go as $A^{4 / 3}$. (The $A^{4 / 3}$ dependence rather than an $A^{2}$ dependence is due to rhos being strongly absorbed, and thus only surface nucleons can contribute.) On the other hand, the inelastic production is not coherent and thus the forward amplitude should go as $A^{1 / 3}$. Thus the inelastic contribution should be greatly suppressed relative to the elastic contribution for higher A nuclei. This trend is certainly clear as seen in the table below. However, it is somewhat surprising that the Be fit has such a large inelastic contribution. This can be explained by observing that:

$$
\begin{equation*}
\left(\frac{d \sigma}{d t}\right)_{\text {elastic }} \sim e^{-45 t^{\prime}} \text { and }\left(\frac{d \sigma}{d t}\right)_{\text {inelastic }} \sim e^{-8 t^{\prime}} \tag{5.13}
\end{equation*}
$$

Hence, even though the elastic contribution is much larger at $t^{\prime}=0$, it falls off much faster with $t$ ', and its contribution over the $t$ ' interval ( $0, .3$ ) is relatively reduced. See Figure 23 for the fits off the various targets. 16 GeV Pb 16 GeV Be $16 \mathrm{GeV} \mathrm{D}_{2} \quad 16 \mathrm{GeV} \mathrm{H}_{2} \quad 16 \mathrm{GeV} \mathrm{Be}$

| t. | $0-.3$ | $0-.3$ | $0-.3$ | $0-.3$ | $0-.06$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{c} / \mathrm{A} \times 10^{3}$ | $0 \pm 0.3$ | $0.59 \pm .17$ | $1.0 \pm .20$ | $1.06 \pm .09$ | $.22 \pm .07$ |
| \% inelastic (14,16) | 0 | 1.5 | 2.3 | 2.3 | 0.5 |
| \% inelastic (12,14) | 0 | 7.0 | 11.7 | 12.0 | 2.7 |
| \% inelastic (10,12) | 0 | 14.8 | 23.2 | 23.7 | 6.1 |
| Confidence Level | 0.70 | 0.83 | 0.24 | 0.55 | 0.74 |



Fig. 23--E $\mathrm{E}_{\pi \pi}\left(\mathrm{E}_{\gamma}\right)$ spectrum. Solid curve is fit to the hypothesis of photon spectrum (elastic) and inelastic
(a) $\gamma \mathrm{Pb} \rightarrow \pi^{+} \pi^{-} \mathrm{Pb}, 16 \mathrm{GeV}$, (b) $\gamma \mathrm{Be} \rightarrow \pi^{+} \pi^{-} \mathrm{Be}, 16 \mathrm{GeV}$,
(c) $\gamma \mathrm{D}_{2} \rightarrow \pi^{+} \pi^{-} \mathrm{D}_{2}, 16 \mathrm{GeV}$, (d) $\gamma \mathrm{H}_{2} \rightarrow \pi^{+} \pi^{-} \mathrm{H}_{2}, 16 \mathrm{GeV}$,
(e) $\gamma \mathrm{H}_{2} \rightarrow \pi^{+} \pi^{-} \mathrm{H}_{2}, 13 \mathrm{GeV}$

Equation (5.12) was used in the fitting function to determine the expected number of events for a given energy cut, and only those events associated with the first term were considered to represent the eiastic contribution.

For the 13 GeV data we had only a hydrogen target, and, hence, we could not study the effect as a function of target material. However, the results were similar to the 16 GeV hydrogen results.

$$
\mathrm{C} / \mathrm{A} \times 10^{3} \begin{array}{ccc}
\text { \% inelastic } & \text { \% inelastic } \% & \text { inelastic } \\
(11-13) & (9-11) & (7-9)
\end{array}
$$

$\begin{array}{llllll}13 \mathrm{GeV} \mathrm{H} \\ 2 & 1.87 \pm .23 & 3.3 & 15.7 & 28.1 & 0.35\end{array}$
C. Fraction of $\pi^{+} \pi^{-}$Background

As has been mentioned in the previous chapter, the amount of $\pi^{+} \pi^{-}$ background in a given mass and $t$ distribution depends heavily upon the model used to describe the distribution. Furthermore, one has to make an assumption concerning the rho width. If one uses a p-wave Breit-Wigner to describe the data as we did, then one must impose some sort of high mass cutoff since without a cutof'f, the integral over $\pi \pi$ masses diverges. One way around this is to use the method suggested by Yennie:

$$
\begin{equation*}
\left.\frac{d \sigma}{d t}\left(\gamma p \rightarrow \rho^{\circ} p\right) \equiv \frac{\pi \Gamma}{2} \frac{d^{2} \sigma}{d m_{\pi \pi} d t} \right\rvert\, m_{\pi \pi}=m_{\rho} \tag{5.14}
\end{equation*}
$$

The above determination of the cross section is sensitive to the choice of the rho mass, and it is equivalent to using an $S$-wave Breit-Wigner. However, until the rho shape is better understood, this method does offer three advantages: (1) it eliminates the divergence; (2) since the sbding terms go to zero at the rho mass, it eliminates the need to worry about the exact shape of the sbding over the whole mass range; (3) it offers
a simple method of comparing various experiments independent of the model used to fit the distribution.
D. Atoms $/ \mathrm{cm}^{2}$ for the Target

As was mentioned in Chapter II, Section D, the vapor pressure of the target was continuously monitored. The pressure remained relatively constant, and an average value of the vapor pressure was used. The target was kept at one atmosphere and at the boiling point; hence, the vapor pressure was a sensitive check on this condition. The average density used was $0.0708 \pm .0015 \mathrm{gm} / \mathrm{cm}^{3}$ for hydrogen and $0.163 \pm .035 \mathrm{gm} / \mathrm{cm}^{3}$ for deuterium. This gave:

$$
\begin{equation*}
\text { Atom } / \mathrm{cm}^{2} \times 10^{-23} \quad 43.0 \pm 0.9 \quad 50.0 \pm 1.15 \tag{5.15}
\end{equation*}
$$

## E. Summary

The table below summarizes the results of this chapter. The overall correction does not include the acceptance factor or the factor mentioned in the previous section concerning mass distributions. The error on the overall correction does include a $4 \%$ error due to uncertainty in the bremsstrahlung shape (f), another $3 \%$ due to uncertainty in the acceptance calculation (g), a $2 \%$ uncertainty due to other absorption in the system (h), and a $1 \%$ track-finding algorithm uncertainty (i), as well as contributions due to the flux ( $j$ ) and target density ( $k$ ) errors. All errors have been combined in quadrature.

Event Loss (Gain)
Correction Factor
$16 \mathrm{H}_{2} \quad 16 \mathrm{D}_{2} \quad 13 \mathrm{H}_{2}$

| (a) Hodoscope Inefficiency | $1.06 \pm 0.02$ | $1.07 \pm 0.02$ | $1.11 \pm 0.02$ |
| :--- | :---: | :---: | :---: |
| (b) Spark Chamber Inefficiency | $1.11 \pm 0.02$ | $1.06 \pm 0.02$ | $1.06 \pm 0.02$ |
| $\quad$ (Fmpty) | $(1.21 \pm 0.13)$ | $(1.21 \pm 0.13)$ | $(1.05 \pm 0.05)$ |
| (c) Target $\gamma$ and Pion Absorption | $1.16 \pm 0.02$ | $1.32 \pm 0.05$ | $1.16 \pm 0.02$ |
| (d) Pion Decay | $1.01 \pm 0.01$ | $1.01 \pm 0.01$ | $1.01 \pm 0.01$ |
| (e) TARGET Z CUT and | $1.13 \pm 0.02$ | $1.12 \pm 0.02$ | $1.15 \pm 0.02$ |
| $\quad$ MT Subtraction | $1.00 \pm 0.04$ | $1.00 \pm 0.04$ | $1.00 \pm 0.04$ |
| (f) Bremsstrahlung Shape | $1.00 \pm 0.03$ | $1.00 \pm 0.03$ | $1.00 \pm 0.03$ |
| (g) Acceptance Calculation | $1.00 \pm 0.02$ | $1.00 \pm 0.02$ | $1.00 \pm 0.02$ |
| (h) Other Source of Absorption | $1.00 \pm 0.01$ | $1.00 \pm 0.01$ | $1.00 \pm 0.01$ |
| (i) Track-Finding Algorithm | $1.00 \pm 0.05$ | $1.00 \pm 0.05$ | $1.00 \pm 0.05$ |
| (j) Flux Normalization | $1.56 \pm 0.14$ | $1.69 \pm 0.16$ | $1.58 \pm 0.14$ |

Below the number of UD counts and $\sigma$, where $\sigma *$ WEIGHT is the cross section equivalent to one event in the chambers in a given energy byte of the photon spectrum, are given for each target and endpoint energy.

Number of UD counts $\times 10^{-6}$ :

| $(14-16):$ | $5.74 \pm 0.51$ | $10.4 \pm 1.0$ | - |
| :--- | :---: | :---: | :---: |
| $(12-14):$ | $4.63 \pm 0.42$ | $8.37 \pm 0.79$ | - |
| $(10-12):$ | $3.91 \pm 0.35$ | $7.06 \pm 0.67$ | - |
| $(11-13):$ | - | - | $6.87 \pm 0.61$ |
| $(9-11):$ | - | - | $5.31 \pm 0.47$ |
| $(9-7):$ | - | - | $4.14 \pm 0.37$ |

$\sigma \times 10^{4} \mu b$
$16 \mathrm{H}_{2}$
3.18
$16 \mathrm{D}_{2}$
1.64
$13 \mathrm{H}_{2}$
2.16
$4.14 \pm 0.37$

In this chapter the results of the various fits described in Chapter IV are presented. All results given here are in the context of the Sరding model description of the non-resonant dipion background. Because of the model-dependent nature of the results, cross-sections for the total reaction $\gamma p \rightarrow \pi^{+} \pi^{-} p$ are also presented whenever possible.

The results of the fits to $d^{2} \sigma_{\pi \pi} / d M_{\pi \pi} d t$ are presented in Table VI. The top line for each entry gives the "best" fit values. The parameter errors represent a change of 0.5 in the likelihood function while allowing the other fit parameters to readjust. The error on the forward cross section includes the error due to the normalization parameter in the fit, as well as uncertainties due to the various corrections listed in Table V. However, the errors do not include uncertainties due to the model-dependent nature of the results.

It is clear from the fit parameters that for a given bremsstrahlung spectrum the rho width increases as one moves away from the endpoint. This is most likely due to an inelastic contribution which increases as one moves away from the endpoint. This inelastic contribution has not been included in the model used to fit the data and could influence the results. Even near the endpoint energy the widths tend to be somewhat large compared to the world average. For this reason fits were made constraining the width and mass to the "accepted" values. There is some disagreement as to what the rho parameters in fact are, and they may vary from reaction to reaction. The Particle Data Group list gives

TABLE VI

| Energy Range GeV | $\begin{aligned} & \mathrm{Mp} \\ & \mathrm{MeV} / \mathrm{c}^{2} \end{aligned}$ | $\begin{aligned} & \Gamma_{p} \\ & \mathrm{MeV} / \mathrm{c}^{2} \end{aligned}$ | $\begin{aligned} & \text { SLOPE } \\ & (\mathrm{GeV} / \mathrm{c})^{-2} \end{aligned}$ | BACK | NORM | $\begin{gathered} \mathrm{d} \sigma^{0} / \mathrm{dt} \\ \mu \mathrm{~b} /(\mathrm{GeV} / \mathrm{c})^{2} \\ \hline \end{gathered}$ | $\chi_{m}^{2}$ | $x_{t}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 14-16 | $780 \pm 5$ | $185 \pm 8$ | $8.57 \pm 0.34$ | $1.79 \pm .10$ | $129 \pm 8$ | $104 \pm 11$ | 18.5 | 25.9 |
| $\mathrm{H}_{2}$ | 765 | 145 | $8.73 \pm 0.34$ | $1.65 \pm .06$ | $104 \pm 3$ | $104 \pm 10$ | 48.2 | 25.9 |
| 1.2-14 | $779 \pm 5$ | $193 \pm 8$. | $8.00 \pm 0.32$ | $1.72 \pm .10$ | $134 \pm 8$ | $104 \pm 11$ | 27.8 | 17.8 |
| $\mathrm{H}_{2}$ | 765 | 145 | $8.17 \pm 0.33$ | $1.72 \pm .05$ | $102 \pm 3$ | $103 \pm 10$ | 71.8 | 18.2 |
| 10-12 | $790 \pm 5$ | $220 \pm 9$ | $6.10 \pm 0.33$ | $1.67 \pm .10$ | $133 \pm 9$ | $94 \pm 11$ | 17.9 | 20.6 |
| $\mathrm{H}_{2}$ | 765 | 145 | $6.52 \pm 0.33$ | $1.62 \pm .05$ | $92.8 \pm 3$ | $95 \pm 9$ | 114.5 | 21.2 |
| 11-13 | $771 \pm 5$ | $166 \pm 8$ | $7.40 \pm 0.46$ | $1.68 \pm .14$ | $112 \pm 9$ | $99 \pm 12$ | 24.6 | 13.0 |
| $\mathrm{H}_{2}$ | 765 | 145 | $7.50 \pm 0.46$ | $2.68 \pm .07$ | $98.2 \pm 4$ | $99 \pm 10$ | 29.2 | 13.0 |
|  | $783 \pm 6$ | $206 \pm 11$ | $7.41 \pm 0.46$ | $1.68 \pm .13$ | $140 \pm 12$ | $104 \pm 13$ | 13.7 | 29.9 |
| $\mathrm{H}_{2}$ | 765 | 145 | $7.76 \pm 0.45$ | $1.67 \pm .06$ | $102 \pm 4$ | $104 \pm 10$ | 51.1 | 30.7 |
| 7-9 | $794 \pm 8$ | $231 \pm 13$ | $7.70 \pm 0.50$ | $1.81 \pm .16$ | $153 \pm 15$ | $105 \pm 14$ | 30.8 | 23.3 |
| $\mathrm{H}_{2}$ | 765 - | 145 | $8.34 \pm 0.50$ | $1.85 \pm .06$ | $102 \pm 4$ | $106 \pm 10$ | 44.9 | 23.4 |
| 14-16 | $777 \pm 7$ | $186 \pm 7$ | $8.70 \pm 0.31$ | $1.56 \pm .10$ | $505 \pm 28$ | $403 \pm 45$ | 20.7 | 30.8 |
| $\mathrm{D}_{2}$ | 765 | 145 | $8.81 \pm 0.32$ | $1.50 \pm .05$ | $394 \pm 11$ | $394 \pm 40$ | 53.3 | 30.6 |
| 12-14 | $780 \pm 4$ | $198 \pm 7$ | $6.69 \pm 0.29$ | $1.69 \pm .08$ | $490 \pm 27$ | $369 \pm 41$ | 34.4 | 24.7 |
| $\mathrm{D}_{2}$ | 765 | 145 | $6.85 \pm 0.28$ | $1.69 \pm .04$ | $365 \pm 10$ | $364 \pm 36$ | 96.4 | 25.0 |
| 10-12 | $781 \pm 4$ | $203 \pm 8$ | $5.83 \pm 0.29$ | $1.51 \pm .08$ | $\begin{array}{ll}480 \\ 346 & \pm 26\end{array}$ | $352 \pm 39$ 345 | 30.1 | 27.7 26.7 |
| $\mathrm{D}_{2}$ | 765 | 145 | $6.08 \pm 0.29$ | $1.54 \pm .04$ | $346 \pm 10$ | $345 \pm 35$ | 96.9 | 26.7 |

TABLE VI

| Energy <br> Range <br> GeV | $\begin{aligned} & \mathrm{M} \mathrm{\rho} \\ & \mathrm{MeV} / \mathrm{c}^{2} \end{aligned}$ | $\Gamma \rho$ $\mathrm{MeV} / \mathrm{c}^{2}$ | $\begin{gathered} \text { Slope } \\ (\mathrm{GeV} / \mathrm{c})^{-2} \end{gathered}$ | Back | Norm | $\begin{gathered} \mathrm{d} \sigma^{\mathrm{o}} / \mathrm{dt} \\ \mu \mathrm{~b} /(\mathrm{GeV} / \mathrm{c})^{2} \end{gathered}$ | Con. Level |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{H}_{2}$ | $765 \pm 5$ | $\begin{aligned} & 136 \pm 9 \\ & 140 \end{aligned}$ | $\begin{aligned} & 9.3 \pm 1.1 \\ & 9.3 \end{aligned}$ | $1.81 \pm 0.24$ $1.90 \pm 0.13$ | $111 \pm 15$ $110 \pm 5$ | $\begin{aligned} & 118 \pm 18 \\ & 113 \pm 10 \end{aligned}$ | $\begin{array}{r} .99 \\ .25 \end{array}$ |
| $\mathrm{D}_{2}$ | $769 \pm 3$ 772 | $\begin{aligned} & 155 \pm 13 \\ & 140 \end{aligned}$ | $\begin{aligned} & 30.6 \pm 4.9 \\ & 30.6 \end{aligned}$ | $1.79 \pm 0.26$ $1.98 \pm 0.14$ | $452 \pm 59$ $391 \pm 23$ | $373 \pm 55$ $378 \pm 34$ | $\begin{aligned} & .98 \\ & .40 \end{aligned}$ |

$\stackrel{M \rho}{\sim} \quad=$ rho mass in $\mathrm{MeV} / \mathrm{c}^{2}$
u $\quad \Gamma_{\rho}=$ rho width at $M \rho$ in $\mathrm{MeV} / \mathrm{c}^{2}$
Slope $=$ exponential slope in $(\mathrm{GeV} / \mathrm{c})^{-2}$
Back $=$ amount of sbding required
$\mathrm{d} \sigma^{\circ} / \mathrm{dt}=$ forward differential cross section in $\mu \mathrm{b} /(\mathrm{GeV} / \mathrm{c})^{2}$
Norm $\propto d \sigma^{\circ} / \mathrm{dt}$
Con. Level $=$ confidence level for the fit
$\chi^{2} \mathrm{~m}=$ chi-square for mass projection (20 bins)
$x^{2} t=$ chi-square for $t$ projection ( 20 bins)
*The $D_{2} t$ distribution is fit to an exponential times the Glauber form factor. This is the slope for the exponential.


Fig. 24--Reaction $\gamma p \rightarrow \pi^{+} \pi^{-} p$ : the differential cross section $\mathrm{do} / \mathrm{dt}$, is shown for $.5 \leq \mathrm{M}_{\pi \pi} \leq 1.0 \mathrm{GeV} / \mathrm{c}^{2}$. The solid line is the result of the soding fit. (a) $14-16 \mathrm{GeV}$, (b) $12-14 \mathrm{GeV}$, (c) 111-13 GeV, (d) 9-11 GeV


Fig. $25-\pi^{+} \pi^{-}$mass spectrum for the reaction $\gamma p \rightarrow \pi^{+} \pi^{-} p$. The solid curve is the result of the s 8 ding fit. (a) $0 \leq t^{\prime} \leq 0.15(\mathrm{GeV} / \mathrm{c})^{2},(\mathrm{~b}) 0 \leq \mathrm{t}^{\prime} \leq 0.3(\mathrm{GeV} / \mathrm{c})^{2}$
$m \rho=770 \pm 5 \mathrm{MeV} / \mathrm{c}^{2}$ and $\Gamma \rho=146 \pm 10 \mathrm{MeV} / \mathrm{c}^{2}$.
As has been seen, fixing the rho width and mass has almost no effect on the forward rho cross section and only a small effect on the $d \sigma_{\rho} / \mathrm{dt}$ slope parameter. Figures 24 and 25 show the projections of $d^{2} \sigma_{\pi \pi} / d M_{\pi \pi} d t$ onto the $d \sigma_{\pi \pi} / d t$ and $d \sigma_{\pi \pi} / d M_{\pi \pi}$ axes. The solid line represents the fit values for the fixed width and mass. Figures 26 and 27 show the comparison between the "best" fit and the "constrained" fit for the reaction $\gamma p \rightarrow \pi^{+} \pi^{-} p$ within the momentum byte $14-16 \mathrm{Gev} / \mathrm{c}$. The "rho" differential cross section values are obtained by using the fitting function to determine the rho fraction of each event.

Since only a fraction of the phase space is covered--dipion mass range $0.5-1.0 \mathrm{Gev} / \mathrm{c}^{2}$ and $t^{\prime}$ range $0.0-0.3(\mathrm{Gev} / \mathrm{c})^{2}$--obtaining cross section values requires a correction to be made for the restricted phase space. For this reason two pseudo cross sections, $d \tilde{\sigma} / d M_{\pi \pi}$ and $\mathrm{d} \tilde{\sigma} / \mathrm{dt}$, are defined.

$$
\begin{align*}
& \frac{d \tilde{\sigma}}{d m_{\pi \pi}}=\int_{0}^{0.3}\left(\frac{d^{2} \sigma}{d m_{\pi \pi} d t}\right) d t  \tag{6.1}\\
& \frac{d \tilde{\sigma}}{d t}=\int_{0.5}^{1.0}\left(\frac{d^{2} \sigma}{d m_{\pi \pi} d t}\right) d m_{\pi \pi} \tag{6.2}
\end{align*}
$$

For the dipion case these quantities are particularly useful in that they are model-independent and can be compared directly to other experiments. For the rho case, the cross section values $d \sigma_{\rho} / d m_{\pi \pi}$ are obtained from these pseudo cross sections by using the slope parameter to correct for the finite $t$ range, while $d \sigma / d t$ is obtained by normalizing to $(d \sigma / d t)_{t=0}$ as defined in Chapter $V$ (5.13). These values are meaningful only in the context of the s8iing model used in the fit. Tables VII-XI


Fig. 26--The effects on $d \sigma / \mathrm{dt}$ ' due to constraining the $\rho$ mass and width are shown. (a) the width and mass of the rhg fixed to nominal values of $\Gamma_{\rho}=145 \mathrm{MeV} / \mathrm{c}^{2}$ and $M_{\rho}=765 \mathrm{MeV} / \mathrm{c}^{2}$, (b) the width and the mass allowed to be completely free


Fig. 27--The effects on $\alpha \sigma / \partial M_{\pi \pi}$ due to constraining the $\rho$ mass and width are shown. (a) the width and mass of rho fixed, (b) rho wirth and mass lefic mon

Table VII
$\gamma+\mathrm{p} \rightarrow \pi^{+} \pi^{-}+\mathrm{p}$ at 16 GeV

| $\begin{gathered} t^{\prime} \mathrm{BIN} \\ (\mathrm{GeV} / \mathrm{c})^{2} \end{gathered}$ | $14-16$ | $\begin{gathered} \mathrm{dt} t^{\prime} \mu \mathrm{b} /(\mathrm{GeV} \\ 12-14 \end{gathered}$ | 10-12 |  | $14-16$ | $\begin{gathered} \mathrm{dt} \cdot \mu \mathrm{~b} /(\mathrm{Ge} \\ 12-14 \end{gathered}$ | $\begin{aligned} & \text { c) })^{2} * \\ & 10-12 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.000-0.015 | $105.9 \pm 10.8$ | $117.6 \pm 11.7$ | $116.3 \pm 11.9$ |  | $83.6 \pm 9.2$ | $84.5 \pm 9.9$ | $82.2 \pm 10.0$ |
| $0.015-0.030$ | $79.6 \pm 8.4$ | $85.4 \pm 9.0$ | $97.2 \pm 10.2$ |  | $66.1 \pm 7.8$ | $67.4 \pm 8.5$ | $70.1 \pm 8.9$ |
| 0.030-0.045 | $69.4 \pm 7.7$ | $75.1 \pm 8.1$ | $75.0 \pm 8.4$ |  | $54.1 \pm 6.7$ | $61.9 \pm 8.0$ | $53.8 \pm 7.3$ |
| 0.045-0.060 | $56.6 \pm 6.6$ | $70.4 \pm 7.8$ | $88.1 \pm 9.7$ |  | $51.8 \pm 7.1$ | $48.7 \pm 6.5$ | $60.0 \pm 7.5$ |
| 0.060-0.075 | $58.8 \pm 7.0$ | $56.7 \pm 6.6$ | $74.2 \pm 8.7$ |  | $46.7 \pm 6.0$ | $46.1 \pm 6.4$ | $52.6 \pm 7.5$ |
| 0.075-0.090 | $45.8 \pm 5.9$ | $54.9 \pm 6.7$ | $64.0 \pm 7.9$ |  | $38.9 \pm 5.8$ | $44.8 \pm 6.9$ | $45.9 \pm 6.9$ |
| 0.090-0.105 | $43.5 \pm 5.8$ | $43.5 \pm 5.7$ | $60.3 \pm 7.7$ |  | $37.5 \pm 5.4$ | $40.1 \pm 6.8$ | $40.1 \pm 6.0$ |
| 0.105-0.120 | $33.1 \pm 4.8$ | $44.9 \pm 6.0$ | $54.0 \pm 7.3$ |  | $34.7 \pm 6.0$ | $48.5 \pm 8.5$ | $39.9 \pm 6.3$ |
| 0.120-0.135 | $31.6 \pm 4.8$ | $35.2 \pm 5.2$ | $43.3 \pm 6.4$ |  | $29.9 \pm 5.3$ | $37.1 \pm 6.8$ | $34.4 \pm 6.7$ |
| 0.135-0.150 | $29.1 \pm 4.5$ | $29.4 \pm 4.8$ | $41.7 \pm 6.6$ |  | $32.8 \pm 6.3$ | $23.1 \pm 4.2$ | $31.6 \pm 5.6$ |
| 0.150-0.165 | $19.3 \pm 3.5$ | $25.1 \pm 4.4$ | $37.1 \pm 6.4$ |  | $23.1 \pm 4.9$ | $22.5 \pm 4.5$ | $27.8 \pm 5.4$ |
| 0.165-0.180 | $18.7 \pm 3.7$ | $23.9 \pm 4.4$ | $35.0 \pm 6.2$ |  | $14.5 \pm 3.1$ | $17.8 \pm 3.7$ | $30.2 \pm 6.6$ |
| 0.180-0.195 | $19.3 \pm 3.7$ | $21.6 \pm 4.3$ | $42.2 \pm 7.7$ |  | $25.8 \pm 5.8$ | $14.3 \pm 3.2$ | $32.5 \pm 6.9$ |
| 0.195-0.210 | $14.0 \pm 3.2$ | $22.3 \pm 4.7$ | $29.5 \pm 6.6$ |  | $12.7 \pm 3.2$ | $22.0 \pm 5.7$ | $23.1 \pm 6.2$ |
| 0.210-0.225 | $16.9 \pm 3.8$ | $24.4 \pm 5.0$ | $26.9 \pm 6.4$ |  | $17.2 \pm 4.1$ | $25.4 \pm 6.2$ | $23.9 \pm 6.4$ |
| 0.225-0.240 | $24.0 \pm 3.5$ | $24.8 \pm 3.9$ | $20.8 \pm 5.9$ |  | $9.9 \pm 2.6$ | $10.7 \pm 3.2$ | $14.4 \pm 4.2$ |
| 0.240-0.255 | $11.4 \pm 3.1$ | $13.6 \pm 4.0$ | $25.7 \pm 6.9$ |  | $11.6 \pm 4.0$ | $12.6 \pm 4.8$ | $16.1 \pm 4.9$ |
| 0.255-0.270 | $13.1 \pm 3.5$ | $13.9 \pm 4.2$ | $12.5 \pm 4.9$ |  | $12.5 \pm 3.9$ | $9.5 \pm 3.2$ | $9.6 \pm 3.9$ |
| 0.270-0.285 | $2.3 \pm 1.4$ | $10.2 \pm 3.9$ | $11.4 \pm 4.7$ |  | $2.5 \pm 1.5$ | $9.0 \pm 3.6$ | $12.1 \pm 5.5$ |
| 0.285-0.300 | $10.8 \pm 3.4$ | $10.3 \pm 4.3$ | $16.1 \pm 5.9$ |  | $11.2 \pm 4.0$ | $7.2 \pm 3.9$ | $18.7 \pm 8.0$ |
| ${ }^{*} d \sigma_{\rho} / d t^{\prime}=c m^{*} d \tilde{\sigma}_{\rho} / d t^{\prime}$ |  |  |  | cm | 1.255 | 1.255 | 1.255 |

Table VIII
$\gamma+p \rightarrow \pi^{+} \pi^{-}+p$ at 16 GeV

| $\begin{gathered} \left.\mathrm{m}_{\pi \pi} \mathrm{BIN}^{\mathrm{GeV}} / \mathrm{c}^{2}\right) \end{gathered}$ | $\begin{aligned} & d \tilde{\sigma}_{\pi \pi} / \mathrm{c} \\ & 14-16 \end{aligned}$ | $\begin{gathered} \pi \pi^{\mu \mathrm{b}} /(\mathrm{GeV} \\ 12-14 \end{gathered}$ | 10-12 |  | $14-16$ | $\begin{gathered} m_{\pi r}: 1 b /(\mathrm{Ge} \\ 12-14 \end{gathered}$ | $\begin{aligned} & \left./ \mathrm{c}^{2}\right) * \\ & 10-12 \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.500-0.525 | $15.0 \pm 3.0$ | $12.4 \pm 2.2$ | $16.8 \pm 2.9$ |  | $2.2 \pm 0.6$ | $1.6 \pm 0.3$ | $2.0 \pm 0.5$ |
| 0.525-0.550 | $14.3 \pm 2.8$ | $17.1 \pm 2.6$ | $25.0 \pm 4.5$ |  | $2.7 \pm 0.6$ | $2.6 \pm 0.5$ | $3.9 \pm 0.8$ |
| 0.550-0.575 | $14.1 \pm 2.6$ | $19.2 \pm 3.0$ | $21.5 \pm 3.5$ |  | $3.1 \pm 0.7$ | $3.9 \pm 0.7$ | $3.8 \pm 0.8$ |
| 0.575-0.600 | $15.3 \pm 2.6$ | $18.6 \pm 2.7$ | $23.9 \pm 3.8$ |  | $4.0 \pm 0.8$ | $4.1 \pm 0.8$ | $4.9 \pm 1.0$ |
| 0.600-0.625 | $23.1 \pm 3.3$ | $24.0 \pm 3.5$ | $30.0 \pm 4.7$ |  | $7.3 \pm 1.1$ | $6.7 \pm 1.1$ | $7.7 \pm 1.4$ |
| 0.625-0.650 | $24.9 \pm 3.4$ | $27.3 \pm 2.7$ | $34.6 \pm 5.0$ |  | $9.4 \pm 1.3$ | $9.1 \pm 1.4$ | $10.6 \pm 1.8$ |
| 0.650-0.675 | $27.9 \pm 3.5$ | $36.1 \pm 4.4$ | $38.8 \pm 5.2$ |  | $12.6 \pm 1.6$ | $14.5 \pm 1.9$ | $14.2 \pm 2.1$ |
| 0.675-0.700 | $33.3 \pm 3.9$ | $45.0 \pm 5.2$ | $46.1 \pm 5.9$ |  | $18.0 \pm 2.2$ | $21.8 \pm 2.6$ | $20.2 \pm 2.8$ |
| 0.700-0.725 | $45.3 \pm 5.1$ | $49.7 \pm 5.7$ | $50.0 \pm 6.0$ |  | $30.0 \pm 3.4$ | $29.3 \pm 3.4$ | $26.0 \pm 3.3$ |
| 0.725-0.750 | $48.7 \pm 5.2$ | $53.6 \pm 6.1$ | $66.2 \pm 7.6$ |  | $38.7 \pm 4.2$ | $38.2 \pm 4.4$ | $41.4 \pm 4.9$ |
| 0.750-0.775 | $40.6 \pm 4.6$ | $52.5 \pm 5.9$ | $62.0 \pm 7.2$ |  | $38.8 \pm 4.4$ | $45.3 \pm 5.1$ | $46.4 \pm 5.4$ |
| 0.775-0.800 | $38.4 \pm 4.5$ | $36.0 \pm 4.4$ | $48.8 \pm 6.3$ |  | $44.3 \pm 5.2$ | $37.9 \pm 4.7$ | $43.8 \pm 5.6$ |
| 0.800-0.825 | $28.1 \pm 3.7$ | $33.1 \pm 4.3$ | $32.5 \pm 4.7$ |  | $39.0 \pm 5.0$ | $42.5 \pm 5.5$ | $35.3 \pm 5.0$ |
| 0.825-0.850 | $18.5 \pm 2.7$ | $17.2 \pm 2.6$ | $28.2 \pm 4.3$ |  | $31.5 \pm 4.5$ | $27.4 \pm 4.2$ | $36.6 \pm 5.4$ |
| 0.850-0.875 | $8.0 \pm 1.4$ | $12.3 \pm 2.2$ | $18.5 \pm 3.4$ |  | $16.8 \pm 3.0$ | $24.1 \pm 4.1$ | $28.8 \pm 5.0$ |
| 0.875-0.900 | $6.4 \pm 1.3$ | $6.1 \pm 1.6$ | $11.8 \pm 2.4$ |  | $16.0 \pm 3.4$ | $14.2 \pm 3.4$ | $22.4 \pm 4.3$ |
| $0.900-0.925$ | $5.2 \pm 1.2$ | $4.2 \pm 1.3$ | $9.6 \pm 2.2$ |  | $16.4 \pm 3.7$ | $12.4 \pm 3.3$ | $21.8 \pm 4.7$ |
| 0.925-0.950 | $3.6 \pm 1.0$ | $3.0 \pm .9$ | $7.6 \pm 1.8$ |  | $13.0 \pm 3.6$ | $11.6 \pm 3.4$ | $22.0 \pm 5.0$ |
| 0.950-0.975 | $2.3 \pm 0.8$ | $3.6 \pm 1.0$ | $4.5 \pm 1.6$ |  | $10.6 \pm 3.6$ | $17.5 \pm 4.9$ | $15.6 \pm 4.9$ |
| 0.975-1.000 | $2.9 \pm 0.9$ | $4.8 \pm 1.4$ | $5.8 \pm 2.0$ |  | $15.7 \pm 5.0$ | $27.5 \pm 7.3$ | $23.7 \pm 7.1$ |
| ${ }^{*} \mathrm{~d} \sigma_{\rho} / \mathrm{dm} \mathrm{~m}_{\pi \pi}=\mathrm{CT} \mathrm{~T}^{*} \mathrm{~d} \gamma_{\rho} / \mathrm{dm} \mathrm{~m}_{\pi \pi}$ |  |  |  | CT | 1.079 | 1.094 | 1.165 |

Table IX
$\gamma+\mathrm{p} \rightarrow \pi^{+} \pi^{-}+\mathrm{p}$ a.t 13 GeV

| $\begin{gathered} \mathrm{t}^{\prime} \mathrm{BIN} \\ (\mathrm{GeV} / \mathrm{c})^{2} \end{gathered}$ | $11-13{ }^{\mathrm{d} \tilde{\sigma}_{,}}$ | $\begin{gathered} / \mathrm{dt} \cdot \mu \mathrm{~b} /(\mathrm{GeV} / \\ 9-11 \end{gathered}$ |  |  | 11-13 | $\begin{gathered} d t^{\prime} \quad \mu \mathrm{b} /(\mathrm{GeV} \\ 9-11 \end{gathered}$ | $c)^{2} *$ $7-9$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.000-0.015 | $93.6 \pm 10.1$ | $106.6 \pm 11.4$ | $123.9 \pm 13.6$ |  | $72.0 \pm 9.2$ | $86.3 \pm 11.2$ | $96.1 \pm 16.8$ |
| 0.015-0.030 | $81.4 \pm 9.2$ | $101.0 \pm 11.0$ | $125.3 \pm 14.0$ |  | $64.6 \pm 8.3$ | $83.2 \pm 11.5$ | $78.8 \pm 10.9$ |
| 0.030-0.045 | $65.9 \pm 8.0$ | $83.0 \pm 9.6$ | $92.7 \pm 11.2$ |  | $59.9 \pm 8.7$ | $62.9 \pm 9.0$ | $62.5 \pm 10.1$ |
| 0.045-0.060 | $64.5 \pm 8.0$ | $83.2 \pm 10.0$ | $81.2 \pm 10.5$ |  | $58.8 \pm 9.5$ | $60.6 \pm 8.3$ | $59.3 \pm 11.3$ |
| 0.060-0.075 | $63.3 \pm 8.2$ | $51.4 \pm 7.1$ | $78.1 \pm 10.3$ |  | $56.1 \pm 9.1$ | $33.8 \pm 5.7$ | $58.3 \pm 11.4$ |
| 0.075-0.090 | $44.9 \pm 6.5$ | $46.1 \pm 6.9$ | $71.2 \pm 10.3$ |  | $43.4 \pm 7.5$ | $35.9 \pm 65$ | $51.9 \pm 11.7$ |
| 0.090-0.105 | $42.3 \pm 6.5$ | $60.3 \pm 8.5$ | $62.0 \pm 9.8$ |  | $38.0 \pm 6.7$ | $53.4 \pm 9.1$ | $38.4 \pm 7.7$ |
| 0.105-0.120 | $34.8 \pm 6.0$ | $38.6 \pm 6.6$ | $44.0 \pm 8.3$ |  | $28.3 \pm 5.7$ | $26.7 \pm 5.1$ | $42.3 \pm 14.7$ |
| 0.120-0.135 | $36.3 \pm 6.2$ | $38.5 \pm 7.0$ | $40.5 \pm 8.5$ |  | $34.6 \pm 6.7$ | $40.7 \pm 10.2$ | $28.2 \pm 7.8$ |
| 0.135-0.150 | $27.5 \pm 5.7$ | $38.5 \pm 7.4$ | $40.7 \pm 9.3$ |  | $22.2 \pm 5.0$ | $24.6 \pm 5.6$ | $36.8 \pm 14.1$ |
| 0.150-0.165 | $27.1 \pm 5.9$ | $37.7 \pm 7.7$ | $41.5 \pm 9.8$ |  | $25.1 \pm 7.0$ | $28.5 \pm 6.6$ | $28.2 \pm 8.5$ |
| 0.165-0.180 | $25.5 \pm 5.9$ | $17.2 \pm 5.3$ | $20.3 \pm 6.9$ |  | $22.7 \pm 6.5$ | $13.3 \pm 4.5$ | $12.4 \pm 5.2$ |
| 0.180-0.195 | $19.3 \pm 5.1$ | $20.3 \pm 5.9$ | $47.0 \pm 12.2$ |  | $21.3 \pm 6.7$ | $25.0 \pm 8.8$ | $45.7 \pm 21.5$ |
| 0.195-0.210 | $13.1 \pm 4.5$ | $24.9 \pm 8.1$ | $27.8 \pm 9.9$ |  | $10.3 \pm 3.7$ | $14.9 \pm 4.9$ | $30.1 \pm 13.4$ |
| 0.210-0.225 | $13.9 \pm 4.6$ | $16.4 \pm 6.2$ | $21.2 \pm 9.5$ |  | $10.3 \pm 3.4$ | $17.1 \pm 7.8$ | $10.9 \pm 6.2$ |
| 0.225-0.240 | $18.1 \pm 6.0$ | $24.5 \pm 7.8$ | $3.0 \pm 4.7$ |  | $22.1 \pm 8.4$ | $20.0 \pm 6.5$ | $2.6 \pm 4.6$ |
| 0.240-0.255 | $23.5 \pm 7.5$ | $33.2 \pm 10.5$ | $17.6 \pm 9.7$ |  | $15.6 \pm 5.3$ | $26.0 \pm 8.3$ | $8.1 \pm 6.2$ |
| $0.255-0.270$ | $18.8 \pm 6.2$ | $18.4 \pm 7.7$ | $20.7 \pm 13.3$ |  | $20.7 \pm 7.8$ | $19.6 \pm 9.4$ | $9.8 \pm 7.5$ |
| 0.270-0.285 | $5.0 \pm 4.1$ | $13.7 \pm 6.9$ | $0.0 \pm 5.1$ |  | $4.0 \pm 3.3$ | $14.1 \pm 8.4$ | $0.0 \pm 5.2$ |
| 0.285-0.300 | $13.9 \pm 6.7$ | $8.4 \pm 6.9$ | $0.0 \pm 6.6$ |  | $11.5 \pm 5.6$ | $6.4 \pm 5.1$ | $0.0 \pm 6.2$ |
| $* \mathrm{do} \sigma_{\rho} / \mathrm{dt}{ }^{\prime}=\mathrm{cm}^{*} \mathrm{~d} \tilde{\sigma}_{\mathrm{p}} / \mathrm{dt}$, |  |  |  | cm | 1.257 | 1.257 | 1.257 |

Table X
$\gamma+p \rightarrow \pi^{+} \pi^{-}+p$ at 13 GeV

| $\begin{aligned} & M_{\pi \pi} B I N \\ & \left(\mathrm{GeV} / \mathrm{c}^{2}\right) \end{aligned}$ | ${ }_{11-13}^{\mathrm{d} \tilde{\sigma}_{\pi}}$ | $\begin{gathered} \mathrm{m}_{\pi \pi}{ }^{\mu \mathrm{b} /(\mathrm{Ge}} \\ 9-11 \end{gathered}$ | $\left.c^{2}\right)$ |  |  | $\begin{gathered} m_{\pi \pi} \\ \\ \\ \\ 9 \mathrm{~b} / \mathrm{Cl} \end{gathered}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.500-0.525 | $12.8 \pm 3.1$ | $16.9 \pm 4.4$ | $19.6 \pm 5.0$ |  | $2.1 \pm 0.7$ | $2.4 \pm 0.8$ | $1.9 \pm 0.7$ |
| 0.525-0.550 | $7.6 \pm 1.8$ | $21.6 \pm 4.4$ | $26.9 \pm 8.1$ |  | $1.2 \pm 0.5$ | $3.5 \pm 1.0$ | $3.5 \pm 1.3$ |
| 0.550-0.575 | $20.1 \pm 4.3$ | $22.3 \pm 4.9$ | $29.4 \pm 3.8$ |  | $4.6 \pm 1.2$ | $4.3 \pm 1.3$ | $2.6 \pm 9.9$ |
| 0.575-0.600 | $15.1 \pm 3.3$ | $22.1 \pm 4.5$ | $21.2 \pm 5.2$ |  | $3.9 \pm 1.1$ | $5.1 \pm 1.3$ | $3.6 \pm 1.1$ |
| 0.600-0.625 | $24.2 \pm 4.5$ | $23.4 \pm 4.2$ | $30.2 \pm 6.3$ |  | $7.8 \pm 1.6$ | $6.3 \pm 1.5$ | $6.6 \pm 1.7$ |
| 0.625-0.650 | $20.3 \pm 3.6$ | $34.5 \pm 6.9$ | $43.3 \pm 7.0$ |  | $7.5 \pm 1.5$ | $12.0 \pm 2.7$ | $12.3 \pm 2.2$ |
| 0.650-0.675 | $31.0 \pm 4.5$ | $37.5 \pm 6.3$ | $45.2 \pm 7.3$ |  | $13.8 \pm 2.2$ | $15.3 \pm 2.9$ | $14.8 \pm 2.8$ |
| 0.675-0.700 | $45.9 \pm 6.3$ | $39.6 \pm 5.8$ | $46.2 \pm 7.1$ |  | $25.0 \pm 3.6$ | $18.9 \pm 2.8$ | $18.3 \pm 3.2$ |
| $0.700-0.725$ | $54.0 \pm 7.2$ | $50.0 \pm 6.6$ | $51.5 \pm 7.3$ |  | $35.6 \pm 4.8$ | $29.2 \pm 4.0$ | $24.7 \pm 3.9$ |
| 0.725-0.750 | $49.2 \pm 6.3$ | $59.7 \pm 7.7$ | $62.2 \pm 8.5$ |  | $38.8 \pm 4.9$ | $41.8 \pm 5.5$ | $36.8 \pm 5.2$ |
| 0.750-0.775 | $50.7 \pm 6.4$ | $53.5 \pm 6.9$ | $64.6 \pm 8.7$ |  | $48.3 \pm 6.1$ | $45.1 \pm 5.8$ | $47.0 \pm 6.5$ |
| 0.775-0.800 | $29.5 \pm 4.1$ | $39.6 \pm 5.8$ | $46.6 \pm 7.5$ |  | $34.2 \pm 4.8$ | $40.5 \pm 5.7$ | $41.9 \pm 6.7$ |
| 0.800-0.825 | $29.8 \pm 4.5$ | $29.6 \pm 4.9$ | $21.2 \pm 4.2$ |  | $41.9 \pm 6.3$ | $36.6 \pm 5.9$ | $24.2 \pm 4.6$ |
| 0.825-0.850 | $18.3 \pm 3.4$ | $23.9 \pm 4.3$ | $23.9 \pm 5.4$ |  | $32.2 \pm 5.6$ | $36.0 \pm 6.3$ | $33.7 \pm 7.1$ |
| 0.850-0.875 | $13.5 \pm 2.9$ | $12.6 \pm 3.1$ | 19.1 $\pm 5.1$ |  | $28.0 \pm 5.8$ | $22.7 \pm 5.5$ | $34.0 \pm 8.2$ |
| 0.875-0.900 | $6.6 \pm 2.2$ | $10.3 \pm 2.6$ | $10.1 \pm 3.2$ |  | $16.1 \pm 4.8$ | $23.2 \pm 5.6$ | $23.0 \pm 6.7$ |
| 0.900-0.925 | $4.7 \pm 1.4$ | $5.2 \pm 1.6$ | $9.9 \pm 4.0$ |  | $16.1 \pm 4.6$ | $15.0 \pm 4.4$ | $28.1 \pm 9.5$ |
| 0.925-0.950 | $2.2 \pm 1.3$ | $5.1 \pm 1.7$ | $6.3 \pm 2.7$ |  | $7.0 \pm 4.2$ | $18.5 \pm 5.6$ | $26.2 \pm 9.3$ |
| 0.950-0.975 | $2.2 \pm 1.0$ | $7.8 \pm 2.8$ | $7.2 \pm 4.0$ |  | $10.5 \pm 4.8$ | $29.6 \pm 9.8$ | $32.0 \pm 16.1$ |
| 0.975-1.000 | $2.0 \pm 1.1$ | $1.6 \pm 1.0$ | $0.5 \pm 1.1$ |  | $11.3 \pm 5.9$ | $9.5 \pm 5.1$ | $5.8 \pm 5.9$ |
| $*_{\rho} d \sigma_{\rho} / \mathrm{dm}_{\pi \pi}=C T^{*} d \tilde{\sigma}_{\rho} / \mathrm{dm} m_{\pi \pi}$ |  |  |  | CT | 1.118 | 1.108 | 1.089 |

Table XI
$\gamma \mathrm{d} \rightarrow \pi^{+} \pi^{-} \mathrm{pn}$

| $\begin{gathered} \mathrm{t}^{\prime} \mathrm{Bin} \\ (\mathrm{GeV} / \mathrm{c})^{2} \end{gathered}$ | $\mathrm{d} \tilde{\sigma}_{\pi \pi} / \mathrm{dt}{ }^{\prime} \mu \mathrm{b} /(\mathrm{GeV} / \mathrm{c})^{2}$ |  |  | $\mathrm{d} \tilde{\sigma}_{\pi \pi} / \mathrm{dM} \mathrm{m}_{\pi \pi} \mu \mathrm{b} /\left(\mathrm{GeV} / \mathrm{c}^{2}\right)$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 14-16 | 12-14 | 10-12 | $\begin{aligned} & M_{\pi \pi} \operatorname{Bin} \\ & \left(\mathrm{GeV} / \mathrm{c}^{2}\right) \end{aligned}$ | 14-16 | 12-14 | 10-12 |
| 0.000-0.015 | $253 \pm 26$ | $274 \pm 28$ | $287 \pm 29$ | 0.500-0.525 | $18.8 \pm 4.2$ | $21.9 \pm 3.7$ | $33.6 \pm 5.4$ |
| 0.015-0.030 | $180 \pm 20$ | $192 \pm 20$ | $192 \pm 22$ | 0.525-0.550 | $19.1 \pm 3.8$ | $31.3 \pm 4.8$ | $43.2 \pm 7.3$ |
| 0.030-0.045 | $142 \pm 16$ | $164 \quad \pm 18$ | $164 \quad \pm 18$ | 0.550-0.575 | $26.5 \pm 4.6$ | $36.5 \pm 5.0$ | $36.3 \pm 6.0$ |
| 0.045-0.060 | $104 \pm 12$ | $123 \pm 14$ | $151 \quad \pm 17$ | 0.575-0.600 | $36.8 \pm 5.5$ | $39.6 \pm 5.9$ | $48.7 \pm 8.2$ |
| 0.060-0.075 | $87.1 \pm 11$ | $105 \pm 12$ | $115 \quad \pm 14$ | 0.600-0.625 | $35.2 \pm 5.1$ | $51.1 \pm 6.4$ | $50.2 \pm 7.7$ |
| 0.075-0.090 | $77.8 \pm 10$ | $84.7 \pm 10$ | $111 \pm 14$ | 0.625-0.650 | $41.8 \pm 5.5$ | $52.2 \pm 6.5$ | $54.7 \pm 7.8$ |
| 0.090-0.105 | $57.6 \pm 8.2$ | $72.0 \pm 9$ | $106 \quad \pm 14$ | 0.650-0.675 | $53.9 \pm 6.6$ | $73.8 \pm 8.8$ | $74.5 \pm 9.3$ |
| 0.105-0.120 | $59.1 \pm 8.5$ | $94.9 \pm 12$ | $83.0 \pm 11$ | 0.675-0.700 | $60.9 \pm 7.2$ | $67.7 \pm 8.2$ | $87.6 \pm 10$ |
| 0.120-0.135 | $47.7 \pm 7.3$ | $57.1 \pm 8.6$ | $90.3 \pm 13$ | 0.700-0.725 | $83.0 \pm 9.3$ | $93.9 \pm 11$ | $112 \pm 13$ |
| 0.135-0.150 | $34.2 \pm 6.3$ | $56.5 \pm 8.7$ | $81.6 \pm 12$ | 0.725-0.750 | $86.8 \pm 9.3$ | $115 \pm 13$ | $116 \pm 13$ |
| 0.150-0.165 | $27.7 \pm 5.6$ | $41.9 \pm 7.4$ | $56.1 \pm 10$ | 0.750-0.775 | $81.1 \pm 9.2$ | $109 \pm 12$ | $116 \pm 13$ |
| 0.165-0.180 | $24.2 \pm 5.2$ | $44.8 \pm 8.1$ | $35.4 \pm 7.9$ | 0.775-0.800 | $63.7 \pm 7.5$ | $67.3 \pm 8.3$ | $89.3 \pm 11$ |
| 0.180-0.195 | $30.0 \pm 6.0$ | $44.6 \pm 8.5$ | $55.8 \pm 11$ | 0.800-0.825 | $47.8 \pm 6.1$ | $55.6 \pm 7.3$ | $60.6 \pm 8.1$ |
| 0.195-0.210 | $25.1 \pm 5.6$ | $41.8 \pm 8.3$ | $45.2 \pm 10$ | 0.825-0.850 | $34.9 \pm 4.8$ | $32.8 \pm 5.1$ | $38.5 \pm 5.9$ |
| 0.210-0.225 | $26.4 \pm 6.0$ | $37.4 \pm 8.3$ | $47.0 \pm 11$ | 0.850-0.875 | $16.3 \pm 3.1$ | $27.0 \pm 4.6$ | $35.0 \pm 6.0$ |
| 0.225-0.240 | $21.2 \pm 5.4$ | $26.2 \pm 6.8$ | $31.9 \pm 9.3$ | 0.875-0.900 | $14.9 \pm 2.8$ | $21.2 \pm 4.0$ | $21.4 \pm 4.3$ |
| 0.240-0.255 | $10.8 \pm 4.0$ | $18.7 \pm 6.0$ | $52.7 \pm 13$ | 0.900-0.925 | $7.2 \pm 1.8$ | $12.8 \pm 3.1$ | $23.7 \pm 4.6$ |
| 0.255-0.270 | 19.2土 5.5 | $16.0 \pm 6.0$ | $25.0 \pm 9.0$ | 0.925-0.950 | $7.8 \pm 2.1$ | $8.1 \pm 2.0$ | $16.7 \pm 4.1$ |
| 0.270-0.285 | $15.3 \pm 6.0$ | $22.1 \pm 7.2$ | $30.8 \pm 9.3$ | 0.950-0.975 | $7.4 \pm 2.0$ | $5.0 \pm 1.6$ | $8.3 \pm 2.6$ |
| 0.285-0.300 | $3.2 \pm 2.7$ | $24.8 \pm 7.7$ | $17.5 \pm 8.3$ | 0.975-1.000 | $3.9 \pm 1.2$ | $5.9 \pm 1.8$ | $9.2 \pm 2.7$ |

give the pseudo cross section values for dipions and rhos, as well as the two corrections to be applied for rhos to account for the finite phase space accepted.

Figures 28 and 29 show the forward cross-section $\operatorname{do} /\left.\operatorname{dt}\left(\gamma \mathrm{p} \rightarrow \rho^{\circ} \mathrm{p}\right)\right|_{t=0}$ and the slope parameter as functions of energy. Only results obtained by using the Sbding model were included. This was done since both slope and cross-section values are sensitive to the model used. The forward cross section appears to fall with energy and levels of $f$ around 8 GeV . Here we use only our bremsstrahlung points near the endpoint to reduce inelastic effects as much as possible. Further, as mentioned earlier, an inelastic correction has been made in determining our total cross sections.

The situation with the slope parameter as a function of energy is even more interesting. If one omits our data points, it appears that rho photoproduction exhibits no Regge shrinkage. Since Pomeron exchange is expected to dominate this reaction, one expects behavior similar to other pomeron-dominated reactions. Several such reactions, like $\mathrm{K}^{+} \mathrm{p}$ and $p p$ elastic scattering, do show shrinkage; while others, like $\pi^{ \pm} p$, $K^{-} p$ elastic scattering, show little or no shrinkage. This apparent contradiction has been resolved in the case of $\pi p$ elastic scattering by including a small amount of $P^{\prime}$ and $\rho$ exchange. ${ }^{32}$ A similar analysis is applicable to rho photoproduction. With the addition of the results from this experiment the data now seem to favor shrinkage; however, the error bars are large enough to allow other interpretations. 63 It must be noted that most of the low energy data are from bubble-chamber experiments which have events mainly with $t^{\prime} \geq 0.02(\mathrm{GeV} / \mathrm{c})^{2}$, while most of


Fig. 28-- The reaction $\gamma p \rightarrow \rho^{\circ} p$ : the forward differential cross section as a function of lab photon energy. - Cornell, (35) - SBT, ${ }^{(33)}$ םSWT, ${ }^{(56)} \mathrm{X} \mathrm{SLAC}$ (this experiment), $\triangle$ DESY-MIT ${ }^{(57)}$


Fig. 29--The reaction $\gamma p \rightarrow \rho^{\circ} \mathrm{p}$ : the forward slope parameter as a function of lab photon energy. Cornell, (35) ${ }^{(35 S T}$, (33)
a SWT, ${ }^{\text {(56) }} \mathrm{X}$ SLAC (this experiment)
the high energy experiments are "counter" type and cover a $t$ ' range of $0.0-0.2(\mathrm{GeV} / \mathrm{c})^{2}$. As observed in other reactions, a difference in $t$, range could give slightly different results for the slope. It will be interesting to see what happens at still higher energies like 100 GeV at NAL.

Next the results of the density matrix element fits are presented. Figures 30 and 31 give the results for production off hydrogen at 13 and 16 GeV respectively, while figure 32 gives the results off deuterium at 16 GeV . The results of fits from our previous experiment at 9 GeV using a quasi-monochromatic photon beam ${ }^{49}$ are given in figures 33, 34 and 35, 36 off hydrogen and deuterium respectively. The predicted values which are shown as $X$ on the figures are the values of the $\rho_{i j}$ as predicted from the $\rho_{i j}$ found in the other frame. This was done to ensure that a false solution was not accidentally found during the fitting procedure. Several conclusions can easily be drawn from the five figures.

First, the behavior of the $\rho_{i j}$ 's in either the HELICITY or the JACKSON frames is almost completely energy-independent. This is somewhat remarkable when one considers that on an event-by-event basis we have no way of separating elastic and inelastic reactions. As discussed earlier, the energy distributions indicate an increasing inelastic contribution as one accepts events farther from the endpoint of the bremsstrahlung spectrum. This lack of energy-dependence in the $\rho_{i j}$ seems to indicate that both elastic and inelastic reactions in the rho region proceed by a similar production process. However, as was shown in the case of the mass spectra, this is not completely the case.


Fig. 30--Reaction $\gamma \mathrm{p} \rightarrow \rho^{\circ} \mathrm{p}$ : the density matrix elements as a function of $t$ ' in the HELICITY and JACKSON frames for $0.67 \leq M_{\pi \pi} \leq 0.87 \mathrm{GeV} / \mathrm{c}^{2}$. The results are shown for the energy bins $(9,11)$ and $(11,13)$ from the 13 GeV endpoint bremsstrahlung beam. The predicted values for the $\rho_{i j}$ in one frame are based on the values for $\rho_{i f}$ as obtained from the fit in the other frame. The 9 GeV points are the same as in Figures 33 and 34
$\gamma p-\rho^{\circ} p$
HELICITY

- data
JACKSON
$\times$ PREDICTED VALUE











Fig. 31--Reaction $\gamma \mathrm{P} \rightarrow \rho^{\circ} \mathrm{p}$ : same as Fig. 30 except for the energy bins which are $(12,14)$ and $(14,16)$ from the 16 GeV endpoint bremsstrahlung beam.


Fig. 32--Reaction $\gamma \mathrm{d} \rightarrow \rho^{o_{d}}+\rho^{\circ} \mathrm{pn}$ : same as Fig. 31

$\overline{141482}$
Fig. 33--Reaction $\gamma \mathrm{p} \rightarrow \rho^{\circ} \mathrm{p}$ : the density matrix elements as a function of $t$ for $0.67 \leq M_{\pi \pi} \leq 0.87 \mathrm{GeV} / \mathrm{c}^{2}$ using the 9 GeV quasi-monochromatic proton beam. JACKSON FRAME

$\overline{141481}$

Fig. 34--Same as Fig. 33 for HELICITY FRAME

$$
\begin{aligned}
& \gamma d \rightarrow \rho^{\circ} d 9 \mathrm{GeV} \\
& \text { JACKSON SYSTEM }
\end{aligned}
$$





Fig. 35-Reaction $\gamma \mathrm{d} \rightarrow \rho^{\circ} \mathrm{d}+\rho^{\circ} \mathrm{pn}$ : same as Fig. 33
$\gamma d \rightarrow \rho^{\circ} d$
HELICITY SYSTEM


Fig. 36 --Reaction $\gamma \mathrm{d} \rightarrow \rho^{\circ} \mathrm{d}+\rho^{\circ} \mathrm{pn}$ : same as Fig. 34

Another interesting question which can be answered by looking at the $\rho_{i j}$ is in which frame, if any, helicity is conserved between the photon and the rho. In this preferred frame all of the $\rho_{i j}$ 's are expected to be zero, and the decay angular distribution of the rho is expected to be $\sin ^{2} \theta$. It is clear from the figures that for the energy range covered by these experiments ( $9 \mathrm{GeV}-16 \mathrm{GeV}$ ) and for $t$ ' out to 0.3 $(\mathrm{GeV} / \mathrm{c})^{2}$ helicity is conserved in the HELICITY frame and not in the JACKSON frame. Thus, over the range covered by these experiments schannel helicity is conserved for rho photoproduction.

Thirdly, the density-matrix elements for rho photoproduction have the same behavior off deuterium and off hydrogen. Experimenters have always assumed s-channel helicity conservation for tho photoproduction off complex nuclei, and this adds confidence to that assumption.

Finally, several of the constraints due to the physical nature of the density matrix elements (see Appendix B) are violated on the order of $1 \sigma-2 \sigma$. The worst offender is $\rho_{00}$ which must be non-negative. A careful study of the data, our acceptance, and the fitting procedure indicated that the preferred negative values for $\rho_{00}$ were due to regions of $\cos \theta$ at the limits of our acceptance. Thus one could assume that small error in the acceptance calculation could be causing the problem. However, we have carefully studied the acceptance and believe we understand it. Another interesting observation is that bremsstrahlung photoproduction experiments in both bubble and streamer chambers also show to a lesser extent this same effect for small t'. In these experiments there are no possible acceptance problems. Possibly this effect is due to some form of inelastic contamination. However, in that case

Table XII

| $\gamma \mathrm{P} \rightarrow \mathrm{P}^{\circ} \mathrm{P}$ Density Matrix Elements (unconstraines) |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\pm$ tis | : | 0.0-0.01 | $0.01-0.0225$ | $\underline{0.0225-0.04}$ | $0.04-0.07$ | $0.07-0.23$ | 0.13-0.30 |
| $14 \leq E_{\gamma} \leq 26 \mathrm{GeV}$ <br> HELICITY FRAME |  |  |  |  |  |  |  |
| $\rho_{00}$ (meas.) | : | 0.027 +0.203 | -0.318 +0.245 | $0.060+0.180$ | $-0.233+0.211$ | $-0.223 \pm 0.200$ -0.300 | $-0.079+0.179$ |
| $0_{1-1}$ (meas.) | : | -0.006 +0.048 | -0.024 +0.065 | $-0.040+0.045$ | -0.049 +0.055 | 0.032 +0.102 | 0.073 +0.092 |
| $\mathrm{ReO}_{10}$ (meas.) | : | 0.051 | 0.019 +0.073 | 0.119+0.075 | ${ }^{-0.077}+0.095$ | $-0.066+0.073$ | $-0.086+0.098$ |
| $\chi^{2}: \operatorname{cosen}_{u w}{ }^{*}$ | : | 18.8/18 | 14.4/16 | 18.4/18 | 19.3/18 | 23.2/18 | 7.4/18 |
| $x^{2}$ : *uw* | : | 23.9/20 | 25.9/20 | 21.1/20 | 11.5/20 | 15.1/20 | 16.6/20 |
| Confidence Level: |  | 0.12 | 0.44 | 0.34 | 0.62 | 0.61 | 0.29 |
| Number of Events: |  | 235 | 233 | 257 | 304 | 305 | 215 |
| Jackson frame |  |  |  |  |  |  |  |
| $\rho_{00}$ (meas.) | : | $0.163+0.168$ | -0.275 | $0.164+0.170$ -0.211 | $0.023+0.042$ | -0.123 +0.178 | $\begin{array}{r}0.364+0.097 \\ \hline 0.235\end{array}$ |
| $\mathrm{D}_{1-1}$ (meas.) | : | 0.025 +0.0443 | 0.037 +0.065 | 0.075 +0.036 | $0.049 \pm 0.042$ | 0.273 +0.063 | $\begin{aligned} 0.212 & +0.067 \\ & 0.064\end{aligned}$ |
| $\mathrm{ReO}_{10}$ (meas.) | : | 0.087 +0.049 | 0.268 +0.074 | $0.228+0.047$ -0.050 | 0.284+0.112 | $0.388+0.104$ | 0.253 +0.166 |
| $x^{2}=\operatorname{cosen}{ }^{4}{ }^{*}$ | : | 8.1/16 | 18.3/16 | 12.9/16 | 22.3/18 | 18.1/28 | 26.9/18 |
| $x^{2}: 4{ }^{\text {a }}$ | : | 33.7/20 | 29.6/20 | 33.1/20 | 15.9/20 | 18.4/20 | 13.1/20 |
| Confiderce Level |  | 0.36 | 0.29 | 0.09 | 0.28 | 0.60 | 0.09 |
| Number of Events: |  | 235 | 233 | 257 | 304 | 305 | 215 |
| $12 \leq \Sigma_{\gamma} \leq 14 \mathrm{GeV}$ <br> HELICITY FRANE |  |  |  |  |  |  |  |
| $8_{00}$ (mess.) |  | -0.092 +0.181 | -0.669 +0.329 | -0.088 +0.152 | $-0.255 \pm 0.186$ | 0.035 +0.066 | $-0.326+0.091$ |
| $\rho_{1-1}$ (meas.) |  | $-0.004+0.041$ | -0.027 +0.083 | 0.021 +0.043 | $-0.027+0.056$ | $-0.046 \pm 0.041$ | -0.063 +0.070 |
| $\mathrm{ReO}_{10}$ (meas.) |  | $-0.030+0.047$ | ${ }^{-0.090}+0.087$ | 0.082 +0.049 | $-0.002+0.066$ | $0.091+0.032$ | $0.005+0.035$ |
| $x^{2}: \cos \theta_{u w}{ }^{\prime \prime}$ | : | 17.5/18 | 18.8/16 | 28.7/18 | 17.2/18 | 12.0/18 | 10.1/16 |
| $x^{2}$ : *uw | : | 28.4/20 | 9.8/20 | 27.1/20 | 15.3/20 | 18.7/20 | 41.8/20 |
| Confidence Level: |  | 0.07 | 0.41 | 0.06 | 0.52 | 0.18 | 0.0 |
| Number of Events: |  | 259 | 221 | 298 | 303 | 337 | 220 |
| JaCKSon frave |  |  |  |  |  |  |  |
| $0_{0}$ (meas.) |  | ${ }^{-0.122}+\begin{aligned} & 0.181 \\ & -0.209\end{aligned}$ | -0.546 +0.342 | 0.142 +0.169 | 0.255 +0.132 | 0.432 +0.056 | $0.415 \pm \begin{aligned} & 0.081 \\ & 0.083\end{aligned}$ |
| $0_{\text {2-1 }}$ (meas.) | : | $0.021+0.045$ | 0.060 -0.075 | 0.114 +0.052 | 0.087 +0.040 | $0.150+0.035$ | 0.329 +0.057 |
| $\mathrm{ReO}_{10}$ (meas.) | : | $0.059+\begin{aligned} & 0.047 \\ & -0.047\end{aligned}$ | $0.207+0.077$ | 0.215 +0.059 | 0.272 +0.047 | $0.181+0.044$ | 0.316 +0.051 |
| $x^{2}: \cos \theta_{\text {uw }}{ }^{*}$ |  | 21.3/16 | 11.3/17 | 20.7/18 | 12.2/18 | 26.4/18 | 17.0/18 |
| $x^{2}$ : "uw" |  | 15.5/20 | 9.6/20 | 23.2/20 | 27.0/20 | 14.3/20 | 40.5/20 |
| Conftdence Level |  | 0.41 | 0.41 | 0.05 | 0.20 | 0.18 | 0.34 |
| Number of Events |  | 251 | 212 | 241 | 292 | 322 | 220 |

Table XIII
$r p \rightarrow D^{\circ} p$ Density Matrix Elements (unconstrained)

| $t{ }^{+} \mathrm{BIN}$; | $: \quad 0.0=0.01$ | 0.01-0.0225 | $0.0225-0.04$ | $0.04-0.01$ | $\underline{0.07 \cdot 0.13}$ | $0.13-0.30$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $11 \leq E_{\gamma} \leq 13 \mathrm{GeV}$ |  |  |  |  |  |  |
| HESICITY FRAME |  |  |  |  |  |  |
| $P_{\infty}$ (mese.) : | 0.019 +0.196 | $-0.458+0.135$ -0.142 | 0.026+0.198 | -0.292+0.248 | 0.218 +0.084 | $-0.409+0.306$ -0.445 |
| $0_{1-1}$ (meas.) : | 0.001+0.062 | $-0.149+0.090$ -0.088 | -0.027+0.060 | $0.042+0.093$ <br> 0.073 | $-0.110+0.045$ $=0.043$ | $0.074+0.160$ -0.111 |
| $\operatorname{Rep}_{10} \text { (meas.) }$ | : $0.027+0.063$ | $\begin{array}{r}0.036+0.068 \\ \hline 0.200\end{array}$ | 0.054+0.064 | $-0.072+0.097$ -0.132 | 0.165+0.030 | $\begin{aligned} &-0.070+0.132 \\ &-0.240\end{aligned}$ |
| $x^{2}: \cos \theta_{\text {us }}{ }^{*}$ : | 19.9/18 | 11.8/16 | 22.0/18 | 19.9/28 | 12.8/18 | 14.0/16 |
|  | 22.0/20 | 52.8/20 | 84.4/20 | 16.8/20 | 28.9/20 | 20.5/20 |
| Confidence Level: | : 0.34 | 0.58 | 0.06 | 0.29 | 0.49 | 0.46 |
| Sumber of Events: | : 128 | 131 | 132 | 178 | 185 | 129 |
| Jackson frame |  |  |  |  |  |  |
| $0_{\infty}$ (meat.). : | 0.042+0.299 | $-0.212+0.119$ $=0.139$ | $0.281+0.175$ -0.234 | $-0.017+0.239$ | 0.610+0.060 | 0.423 +0.184 |
| $\mathrm{p}_{1-1}$ (meat.) : | $:-0.002+0.061$ | $-0.066+0.079$ $=0.089$ | 0.035+0.054 | $\begin{array}{r}0.244+0.070 \\ \hline 0.061\end{array}$ | $0.107+0.036$ -0.033 | $0.348+0.094$ -0.09 |
| $\mathrm{ReO}_{10}$ (meas.) : | 0.050+0.066 | 0.265 +0.098 | 0.139 +0.081 | 0.232+0.111 | 0.115*0.052 | 0.256 +0.148 |
| $x^{2}: \operatorname{cosen}{ }_{\text {ux }}{ }^{\prime \prime}$ : | : 12.9/16 | 33.3/18 | 25.2/18 | 25.8/18 | 21.8/18 | 12.0/18 |
| $x^{2}$ : "uv" | 20.1/20 | 13.4/20 | 24.5/20 | 32.3/20 | 15.8/20 | 16.3/20 |
| Confidence Level: | $: 0.38$ | 0.57 | 0.09 | 0.15 | 0.02 | 0.64 |
| Number of Evente: | : 128 | 131 | 132 | $17^{8}$ | 185 | 129 |

$9 \leq \mathbf{I}_{\gamma} \leq 11 \mathrm{GeV}$
HETICITY PRANE

| ${ }^{\circ} \times$ (mean.) | : -0.187 ${ }^{+} 0.214$ |
| :---: | :---: |
| $0_{1-1}$ (menc.) | $=-0.065+0.065$ |
| $\mathrm{ReO}_{10}$ (mear.) | $: 0.021+0.060$ |
| $\chi^{2}: \cos \theta_{u v}$ | : $10.54 / 16$ |
| $x^{2}$ : ${ }^{\text {ux }}$ | : $16.24 / 20$ |

Confldence Level: 0.5
Number of Eventa: 14

| $0_{\infty}$ (meas.) | : $-0.085+0.218$ |
| :---: | :---: |
| $0_{1-1}$ (mees.) | : $-0.056+0.063$ |
| $\mathrm{ReO}_{20}$ (mear.) | : $0.060+0.060$ |
| $x^{2}: \cos \theta_{u s}$ | : 13.2/17 |
| $x^{2}: " u v * ~_{\text {" }}$ | : 15.0/20 |
| Confldence Level: | : 0.45 |
| Number of Events: | 142 |


| -0.082 | +0.220 |
| ---: | :--- |
|  | $=0.290$ |
| 0.034 | +0.065 |
|  | $=0.062$ |
| 0.117 | $=0.082$ |
|  | $=0.073$ |
| $27.0 / 18$ |  |
| $28.9 / 20$ |  |
| 0.06 |  |

JACKSON FEAME
"Thim is cht-square for the unweighted, projected ifstribution/the namber of norzers bins
$\gamma d \rightarrow \rho^{\circ} \mathrm{pn}$ Density Matrix Elements (unconstrained)

| $\underline{t}^{\prime} \mathrm{BIN}$ | $0.0-0.01$ | $\underline{0.01-0.0225}$ | $0.0225-0.04$ | $0.04-0.07$ | $0.07-0.13$ | 0.13-0.30 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $14 \leq \mathrm{E}_{\gamma} \leq 16 \mathrm{GeV}$ |  |  |  |  |  |  |
| HETICITY FRAME |  |  |  |  |  |  |
| $0_{00}$ (meas.) : | : $-0.550+0.371$ | -0.624 +0.400 | $-0.012+0.149$ | $-0.021+0.128$ -0.125 | -0.082 +0.144 | 0.124 +0.154 |
| $\rho_{1-1}$ (meas.) : | : -0.114+0.058 | $0.060+0.066$ -0.064 | $-0.013+0.044$ -0.039 | $-0.080+0.044$ -0.042 | -0.076 +0.047 | -0.039 +0.052 |
| $\mathrm{Rec}_{20}$ (reas.) : | $:-0.101+0.073$ | $-0.082+0.066$ | $0.024+0.057$ -0.081 | 0.056+0.056 | 0.049+0.064 | $0.070+0.074$ -0.116 |
| $\chi^{2}: \cos \theta_{u w}{ }^{*} \quad$ : | : 15.1/16 | 17.2/16 | 26.1/18 | 7.2/18 | 20.9/18 | 13.9/18 |
| $\chi^{2}:{ }_{u w}{ }^{\prime \prime} \quad$ : | 17.5/20 | 16.5/20 | 22.3/20 | 21.7/20 | $14.8 / 20$ | 21.1/20 |
| Confidence Level: | : 0.20 | 0.15 | 0.11 | 0.31 | 0.47 | 0.49 |
| Number of Events: | $: 343$ | 331 | 318 | 307 | 313 | 199 |
| JACKSON FRAME |  |  |  |  |  |  |
| $0_{00}$ (meas.) | $=-0.583+0.370$ | -0.378 $+\begin{aligned} & +0.332 \\ & -0.441\end{aligned}$ | $-0.042+0.212$ -0.298 | 0.261 +0.137 | 0.34 +0.143 | $0.514+0.063$ -0.103 |
| $o_{1-1}$ (meas.) | $:-0.094+\begin{aligned} & 0.053 \\ & -0.069\end{aligned}$ | 0.132 +0.062 | 0.076+0.045 | $0.052+0.037$ -0.034 | 0.146+0.042 | $0.283+0.053$ $=0.050$ |
| $\mathrm{ReO}_{10}$ (meas.) | $: 0.081+0.078$ | $\begin{aligned} & 0.278+0.105 \\ &-0.086\end{aligned}$ | $0.190+0.063$ -0.060 | 0.224+0.048 | 0.267 +0.329 | $0.093+0.063$ -0.057 |
| $x^{2}: \cos \theta_{\text {uk }}{ }^{\prime \prime}$ | : $24.2 / 16$ | 10.5/26 | 12.5/17 | 8.9/18 | $8.0 / 18$ | 13.2/18 |
|  | : 18.5/20 | 12.9/20 | 14.3/20 | 19.5/20 | 16.9/2 | 23.5/20 |
| Confidence Level: | : 0.30 | 0.38 | 0.21 | 0.57 | 0.66 | 0.46 |
| Number of Events: | : 343 | 332 | 318 | 307 | 313 | 199 |

$12 \leq \mathbf{E}_{\gamma} \leq 14 \mathrm{GeV}$ HENICITTY PRRAME

| $p_{00}$ (reas.) | : $-0.154+0.153$ |
| :---: | :---: |
| $\mathrm{D}_{1-1}$ (meas.) | : $-0.044+0.040$ |
| $\mathrm{Rep}_{10}$ (meas.) | $: 0.004+0.043$ |
| $x^{2}: \cos \theta_{u w}{ }^{*}$ | : 19.2/18 |
| $x^{2}$ : *uw | : 23.4/20 |
| Confidence Level | : 0.10 |
| number of Events | : 382 |
| $0_{00}$ (meas.) | $:-0.175+0.168$ |
| $c_{1-1}$ (meas.) | $\begin{aligned}=-0.112\end{aligned}+0.041$ |
| $\mathrm{Rec}_{10}$ (meas.) | $=0.113+0.043$ |
| $\chi^{2}: \cos \theta_{u w}{ }^{*}$ | : 13.2/16 |
| $x^{2}:{ }^{\text {a }}$ u** | : 26.8/20 |

Conftience Level: 0.44
Number of Events: 382
$-0.167+0.155$
-0.198
$-0.006+0.045$
-0.044
$-0.029+0.051$
-0.048
$8.1 / 18$
$7.2 / 20$
0.78
340
$-0.067^{+0.145}$ $0.012+0.041$
$0.065+0.039$
46.6/18
$24.8 / 20$
0.03

322
TACXSON FRAME

| $-0.075+0.169$ | $0.036+0.169$ |
| :---: | :---: |
| $0.046+0.043$ | $0.106+0.037$ |
| -0.043 | 0.215 |
| $0.154+0.050$ | $0.242+0.051$ |
| -0.048 |  |
| $13.4 / 17$ | $21.8 / 18$ |
| $8.0 / 20$ | $22.7 / 20$ |
| 0.20 | 0.49 |
| 340 | 322 |

$-0.376+0.238$
$-0.008+0.064$
$-0.070+0.090$
-0.112
27.9/18
14.3/20
0.27

363
$0.304+0.012$
+0.137
$0.167+0.044$
+0.052
$0.140+0.032$

| $16.1 / 18$ | $9.4 / 18$ | $23.3 / 19$ |
| :--- | :--- | :--- |
| $42.0 / 20$ | $16.5 / 20$ | $13.9 / 20$ |
| 0.12 | 0.27 | 0.10 |
| 325 | 363 | 254 |

325
363
$0.528+0.062$
-0.095
$0.187+0.046$
$=0.061$
$0.160+0.069$
$=0.048$
$0.528+0.062$
-0.095
$0.187+0.046$
$=0.061$
$0.160+0.069$
$=0.048$
$0.528+0.062$
-0.095
$0.187+0.046$
$=0.061$
$0.160+0.069$
$=0.048$
$-0.041+0.077$
-0.124
$-0.103+0.052$
-0.047
$0.084+0.031$
-0.046
$26.9 / 20$
34.3/20
0.00

254
23.3/19
13.9/20

254
"This is chi-square for the unweighted, projected distribution/the number of nonzerc bins

| $y p \rightarrow \rho^{\circ} \mathrm{p}$ Density Matrix Elements (constrained) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $14 \leq \mathrm{E}_{\gamma} \leq 16 \mathrm{GeV}$ |  |  |  |  |  |  |
| HEHICITY FPAME |  |  |  |  |  |  |
| $0_{00}$ (meas.) : | $: 0.064+0.182$ | 0.028 +0.068 | $\begin{array}{r}0.086\end{array}+0.087$ | 0.002+0.071 | $0.000+0.034$ +0.000 | $\begin{array}{r}0.010\end{array}+0.14$ |
| $0_{1-1}$ (mess.) : | $=-0.006+0.076$ | $-0.026+0.045$ -0.045 | $-0.040+0.045$ -0.028 | -0.056+0.019 | $0.013+0.027$ -0.030 | $\begin{array}{r}0.044+0.058 \\ \hline 0.055\end{array}$ |
| $\mathrm{ReO}_{10}$ (meas.) : | $: 0.047+0.051$ | $0.061+0.005$ +0.019 | 0.120 | 0.004+0.017 | -0.000+0.010 0.009 | $-0.033+0.087$ -0.045 |
| $\chi^{2}: \cos \theta_{u w}{ }^{*}$ | 19.2/18 | 18.8/18 | 18.9/18 | 18.8/18 | 14.2/18 | 7.4/18 |
| $x^{2}:{ }^{\text {a }}$ " | 24.0/20 | 25.9/20 | 21.1/20 | 11.5/20 | 24.0/20 | 16.6/20 |
| Confidence Level: | : 0.17 | 0.79 | 0.37 | 0.85 | 0.87 | 0.32 |
| Number of Events: | : 235 | 233 | 257 | 304 | 305 | 215 |
| JACKSON FRAME |  |  |  |  |  |  |
| $0_{\infty}$ (meas.) | 0.08+0.12 | $0.212+0.090$ -0.083 | $\begin{aligned} 0.264 & +0.085 \\ & -0.088\end{aligned}$ | $\begin{array}{r}0.281+0.077 \\ \hline 0.074\end{array}$ | 0.257 +0.081 | $0.360+0.093$ -0.107 |
| $\rho_{1-1}$ (mess.) : | $0.016+0.042$ -0.043 | $\begin{array}{r}0.046 \\ +0.037 \\ \hline\end{array}$ | 0.075 +0.034 | 0.056+0.030 | $0.138+0.030$ -0.032 | $0.195+0.030$ -0.029 |
| $\mathrm{Reg}_{10}$ (meas.) : | $=0.084+0.029$ | $0.176+0.023$ $=0.022$ | $0.179+0.017$ -0.025 | $0.197+0.007$ $=0.003$ | $0.154+0.014$ $=0.015$ | $0.106+0.030$ -0.020 |
| $x^{2}=\operatorname{cose}{ }_{\text {uw }}{ }^{*}$ | 8.9/16 | 31.5/16 | 17.0/17 | 8.6/18 | 15.0/18 | 20.0/18 |
| $x^{2}:{ }_{\text {ux }}{ }^{\prime \prime}$ | $22.1 / 20$ | 20.5/20 | $36.2 / 20$ | 16.4/20 | 18.1/20 | 13.3/20 |
| Confidence Level: | : 0.55 | 0.80 | 0.07 | 0.46 | 0.76 | 0.35 |
| Number of Eventa: | : 253 | 248 | 263 | 324 | 331 | 243 |
| $12 \leq \mathrm{E}_{7} \leq 14 \mathrm{GeV}$ |  |  |  |  |  |  |
| HELICITY FRANE |  |  |  |  |  |  |
| $\mathrm{P}_{\infty}$ (meas.) | $=0.002+0.098$ | 0.002+0.052 | $0.053+0.075$ $=0.040$ | $0.026 * 0.055$ <br> 0.026 | $0.050+0.056$ <br> 0.024 | $\begin{array}{r}0.042+0.038 \\ \hline 0.023\end{array}$ |
| $\mathrm{p}_{1-1}$ (meas.) | $=-0.005+0.045$ | -0.018 +0.048 | 0.007+0.043 | -0.037+0.040 | -0.046+0.014 | $-0.069+0.052$ |
| $\mathrm{ReO}_{10}$ (meal.) | : $-0.016+0.032$ | $0.005+0.010$ $=0.012$ | 0.087* +0.009 | $-0.053+0.006$ | 0.093 +0.042 | 0.088 +0.060 |
| $x^{2}: \cos \theta_{u * * *}{ }^{*}$ | : 18.0/28 | 27.0/18 | 19.4/18 | 19.6/18 | 12.1/18 | 13.8/18 |
| $x^{2}: *{ }^{*}$ | : 18.5/20 | 9.7/20 | 27.2/20 | 15.4/20 | 28.6/20 | $41.0 / 20$ |
| Confidence Level: | : 0.12 | 0.85 | 0.11 | 0.88 | 0.20 | 0.0 |
| Number of Events: | : 259 | 221 | 258 | 303 | 337 | 220 |
| JACKSON FRAME |  |  |  |  |  |  |
| $\rho_{00}$ (meas.) | $=0.013+0.034$ | $0.073+0.069$ -0.039 | $0.260+0.084$ -0.085 | 0.288 +0.088 | 0.465 +0.069 | $\begin{array}{r}0.544 \pm 0.066 \\ \hline 0.077\end{array}$ |
| $0_{1-1}$ (meas.) | $=0.011+0.045$ | $0.038+0.047$ | $0.096+0.037$ -0.037 | $\begin{array}{r}0.083+0.033 \\ \hline 0.033\end{array}$ | 0.138 +0.024 | 0.178 ${ }^{+}+0.017$ |
| $\mathrm{ReO}_{10}$ (meas.) | $=0.050+0.015$ | $\begin{array}{r}0.103\end{array}+0.045$ | 0.173+0.015 | 0.257 +0.029 | $\begin{aligned} & \\ & 0.157+0.015 \\ &=0.088\end{aligned}$ | $0.103+0.007$ -0.010 |
| $x^{2}: \operatorname{cose}^{4 w}{ }^{*}$ | : 11.8/16 | 13.6/17 | 22.7/18 | 12.2/28 | 26.9/18 | 15.0/18 |
| $x^{2}:$ ux $^{*}$ | : 15.5/20 | 9.5/20 | 23.3/20 | 27:2/20 | 14.2/20 | 40.1/20 |
| Confidence Level: | : 0.56 | 0.80 | 0.25 | 0.26 | 0.20 | 0.59 |
|  | 251 | 212 | 241 | 298 | 322 | 220 |

[^3]| $\gamma_{P} \rightarrow \rho^{\circ} \mathrm{p}$ Density Matrix Elements (constrained) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $11 \leq \mathrm{E}_{\gamma} \leq 13 \mathrm{GeV}$ |  |  |  |  |  |  |
| HETICITY FRAME |  |  |  |  |  |  |
| $8^{\infty}$ (meas.) | 0.026+0.189 | $\begin{array}{r}0.000 \\ +0.041 \\ \hline\end{array}$ | $0.030+0.148$ -0.030 | 0.002+0.079 | $\begin{array}{r}0.150\end{array}+0.097$ | 0.027 +0.075 -0.027 |
| $\rho_{1-1}$ (mess.) : | - $0.001+0.056$ | -0.094 +0.062 | $-0.027+0.062$ -0.055 | 0.014 +0.057 | -0.108+0.045 | $-0.006+0.022$ -0.067 |
| $\mathrm{ReO}_{10}$ (meas.) | 20.025+0.060 | 0.001 +0.057 | $0.054+0.065$ -0.050 | $0.005+0.023$ -0.018 | $0.167+0.042$ -0.042 | $\begin{array}{r}0.041\end{array}+0.035$ |
| $x^{2}: \operatorname{cose} \theta_{u w}{ }^{*}$ | 19.9/18 | 20.2/18 | 12.0/18 | 21.2/18 | 13.0/18 | $22.1 / 18$ |
| $x^{2}:{ }^{\text {a }}$ | 22.0/20 | 46.3/20 | 84.2/20 | 16.7/20 | 28.9/20 | 20.8/20 |
| Confidence Level: | : 0.34 | 0.90 | 0.06 | 0.48 | 0.56 | 0.53 |
| Number of Events: | - 128 | 131 | 132 | 178 | 185 | 129 |
| JACKSON FRAME |  |  |  |  |  |  |
| $\rho_{\infty}$ (meas.) . : | 20.078+0.151 | $0.068+0.079$ -0.042 | $\begin{aligned} 0.289\end{aligned}+0.139$ | 0.206+0.096 | $0.615+0.068$ -0.074 | $0.562+0.080$ -0.078 |
| $p_{1-1}$ (meas.) | -0.012 +0.057 | $-0.066+0.059$ -0.060 | $0.034+0.049$ -0.052 | 0.215+0.046 | $0.105+0.029$ $=0.025$ | $\begin{array}{r}0.188\end{array}+0.028$ |
| $\mathrm{ReO}_{10}$ (meas.) | $0.069+0.046$ -0.053 | $0.121+0.022$ -0.028 | $0.135+0.064$ $=0.067$ | $0.257+0.021$ | $0.109+0.063$ -0.030 | $\begin{array}{r}0.079\end{array}+0.034$ |
| $x^{2}: \cos \theta_{u w}$ | 15.4/17 | 31.3/18 | 25.2/18 | 27.1/18 | 21.7/18 | 13.2/18 |
| $x^{2}={ }^{\text {a }}$ " | 22.4/20 | 10.5/20 | $24.5 / 20$ | $32.1 / 20$ | 15.8/20 | 15.7/20 |
| Confidence Level: | : 0.36 | 0.85 | 0.07 | 0.25 | 0.02 | 0.76 |
| Number of Events: | - 135 | 132 | 132 | 178 | 185 | 129 |
| $9 \leq \mathrm{E}_{\gamma} \leq 11 \mathrm{GeV}$ |  |  |  |  |  |  |
| HELICITY FRAME |  |  |  |  |  |  |
| ${ }^{0} 0$ (mesm.) | 0.011 +0.101 | $\begin{array}{r}0.006+0.072 \\ \hline 0.006\end{array}$ | $0.007+0.075$ -0.007 | 0.093 +0.069 | $0.020+0.056$ -0.020 | $0.001+0.211$ -0.001 |
| $0_{\text {1-1 }}$ (mess.) | :-0.055+0.059 | $0.003+0.048$ +0.055 | $-0.026+0.055$ $=0.049$ | $-0.119+0.034$ -0.041 | 0.037 +0.011 | -0.104 +0.078 |
| $\mathrm{ReO}_{10}$ (meas.) | $0.019+0.053$ -0.038 | -0.034 +0.020 | 0.011 + 0.020 | 0.131 +0.070 | $0.040+0.003$ $=0.003$ | $0.001+0.069$ -0.029 |
| $x^{2}: \cos \theta_{\text {ux }}{ }^{\prime \prime}$ | 11.8/16 | 30.1/17 | 24.4/17 | 21.5/16 | $31.0 / 17$ | 8.2/16 |
| $x^{2}$ : "uw" | 16.3/20 | 17.6/20 | 25.7/20 | 15.1/20 | $24.2 / 20$ | 22.0/20 |
| Confidence Level: | : 0.75 | 0.78 | 0.68 | 0.12 | 0.10 | 0.77 |
| Number of Events: | : 142 | 158 | 162 | 205 | 218 | 116 |
| JACKSON FRAME |  |  |  |  |  |  |
| $0_{00}$ (mess.) | $0.027+0.120$ $-0 . c 26$ | $0.042+0.058$ 0.034 | $\begin{array}{r}0.189+0.085 \\ \hline 0.072\end{array}$ | $0.429+0.076$ -0.082 | $0.318+0.082$ -0.082 | $0.493+0.124$ -0.129 |
| $\mathrm{D}_{1-1}$ (meas.) | $=-0.050+0.057$ | 0.027+0.054 | $0.063+0.046$ -0.048 | 0.058 +0.034 | $0.170+0.034$ $=0.036$ | 0.094 +0.059 |
| $\mathrm{ReO}_{10}$ (meas.) : | $\begin{aligned} 0.044 & +0.043 \\ & =0.048\end{aligned}$ | $0.087+0.008$ $=0.024$ | 0.159 +0.016 | $0.192+0.025$ $=0.042$ | $\begin{array}{r}0.244+0.002 \\ \hline 0.003\end{array}$ | 0.172 -0.049 |
| $x^{2}: \cos \theta_{\text {um }}{ }^{\prime \prime}$ | 13.0/17 | 26.9/18 | 16.8/18 | 16.2/18 | 14.6/18 | 14.1/19 |
| $\chi^{2}:{ }^{\text {a }}$ * | : $15.1 / 20$ | 28.9/20 | 23.1/20 | $11.8 / 20$ | 20.8/20 | 18.4/20 |
| Confijence Level: | : 0.56 | 0.09 | 0.61 | 0.58 | 0.06 | 0.62 |
| Number of Events: | : L C | 158 | 161 | 205 | 218 | 116 |

*This is chi-square for the unweighted, projected distribution/the number of nonzero bins

Table XVII

| $\gamma \mathrm{d} \rightarrow 0^{0} \mathrm{pn}$ Density Metrix Elements (constrained) |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| E'BIN | 0.0-0.01 | 0.01-0.0225 | $0.0225-0.04$ | 0.04-0.07 | 0.07-0.13 | 0.13-0.30 |
| $14 \leq \mathrm{E}_{y} \leq 16 \mathrm{GeV}$ |  |  |  |  |  |  |
| HREICITY FTRAME |  |  |  |  |  |  |
| Pro (meas.) | $: 0.006+0.059$ | 0.012+0.054 | 0.018+0.122 | 0.032 +0.074 | $0.049+0.064$ $=0.032$ | $0.144+0.112$ $=0.144$ |
| $0_{1-1}$ (meas.) | $=-0.079+0.033$ | 0.032 +0.042 | -0.014 +0.023 | $\begin{aligned} &-0.080+0.021 \\ &-0.031\end{aligned}$ | $\begin{aligned}-0.081\end{aligned}+0.036$ | $-0.047+0.041$ -0.039 |
| $\mathrm{Rep}_{10}{ }^{\text {(meas . ) }}$ | $:-0.043+0.017$ | 0.030+0.015 | 0.027 ${ }^{+}+0.014$ | $0.064+0.005$ +0.005 | 0.075 +0.046 | $0.085-0.067$ -0.085 |
| $x^{2}: \cos \theta_{u v}{ }^{\prime \prime}$ | 18.1/18 | 21.1/18 | 26.3/18 | 7.4/18 | 19.9/18 | 13.7/18 |
| $x^{2}: u^{*}$ | 17.5/20 | 26.4/20 | 22.3/20 | 21.6/20 | 14.9/20 | 21.1/20 |
| Contidence Level: | : 0.75 | 0.66 | 0.18 | 0.45 | 0.64 | 0.51 |
| Number of Eveats: | : 343 | 331 | 318 | 307 | 313 | 199 |
| JACKSON FRAME |  |  |  |  |  |  |
| $p_{00}$ (meas .) | $=0.010+0.040$ | 0.135 +0.071 | 0.243+0.077 | $0.337+0.090$ -0.089 | 0.67: +0.073 | $0.483+0.083$ -0.095 |
| $0_{1-1}$ (meas.) | $\begin{aligned} &:-0.064+0.040 \\ & 0.038\end{aligned}$ | 0.086 +0.035. | $0.064+0.033$ $=0.037$ | $0.044+0.030$ +0.032 | 0.114+0.025 | $0.165+0.030$ -0.025 |
| $\mathrm{ReO}_{10}$ (mees.) | $: 0.035+0.064$ | 0.131 +0.023 | $0.146+0.023$ -0.033 | $0.186+0.018$ $=0.031$ | 0.174 +0.014 | $0.109+0.013$ $=0.030$ |
| $x^{2}: \operatorname{cosen}^{4 *}$ | : 20.4/16 | 12.0/16 | 13.0/17 | 9.3/18 | 7.6/18 | 15.0/18 |
| $x^{2}: *{ }^{\prime \prime}$ | : 18.6/20 | 15.1/20 | 14.4/20 | 19.4/20 | 16.~", | 27.2/20 |
| Contlitence Level: | : 0.80 | 0.88 | 0.37 | 0.68 | 0.68 | 0.30 |
| Number of Events: | ; 343 | 342 | 318 | 307 | 313 | 211 |
| $12 \leq \mathrm{E}_{7} \leq 14 \mathrm{GeV}$ |  |  |  |  |  |  |
| mincity frame |  |  |  |  |  |  |
| $0_{00}$ (meas.) | $=0.00 e^{+}+0.061$ | $0.000+0.063$ -0.000 | ( $0.045+0.050$ | 0.028+0.036 | 0.019+0.048 | 0.093+0.056 |
| $\rho_{1-1}$ (meas.) | : -0.041 +0.033 | -0.007+0.015 | $0.009+0.039$ $=0.038$ | $0.012+0.003$ $=0.013$ | -0.042 +0.015 -0.037 | $-0.111+0.062$ -0.047 |
| Rep ${ }_{10}$ (mear.) | $=0.003+0.039$ -0.023 | $0.000+0.007$ -0.013 | 0.083 +0.063 | $0.054+0.041$ -0.039 | $\begin{array}{r}0.042+0.005 \\ \hline 0.006\end{array}$ | 0.117 +0.027 |
| $x^{2}: \cos \theta_{\text {ux }}{ }^{\prime \prime}$ | : $21.0 / 18$ | 9.4/18 | 47.7/18 | 14.8/18 | 30.4/18 | 14.5/18 |
|  | : 23.4/20 | 7.4/20 | $24.5 / 20$ | 14.5/20 | 14.0/20 | 34.7/20 |
| Confldence Level: | : 0.28 | 0.96 | 0.08 | 0.09 | 0.58 | 0.08 |
| Number of Events: | : 382 | 340 | 322 | 325 | 363 | 254 |
| JAgKson franc |  |  |  |  |  |  |
| $0_{00}$ (meas.) | $: 0.053+0.053$ | 0.102+0.061 | 0.269 +0.071 | 0.305 +0.073 | 0.417+0.066 | $0.544+0.069$ -0.078 |
| $0_{2-1}$ (meas.) | $\begin{aligned}:-0.011\end{aligned}+0.033$ | $0.038+0.037$ +0.037 | $0.094+0.028$ -0.031 | $\begin{array}{r} \\ 0.211\end{array}+0.030$ | 0.112+0.024 | $0.161+0.009$ -0.018 |
| $\mathrm{Reg}_{10}$ (meas.) | $: 0.077+0.006$ | $0.110+0.068$ $=0.029$ | $0.165+0.007$ $=0.012$ | $0.170+0.008$ -0.019 | 0.177 +0.013 | $\begin{array}{r}0.120 \\ \hline\end{array}$ |
| $x^{2}: \cos \theta{ }^{4}$ | : 13.6/16 | 14.1/17 | 23.4/18 | $14.9 / 18$ | 9-3/18 | 23.8/19 |
| $x^{2}:$ uw" $^{\prime \prime}$ | : 26.7/20 | 8.0/20 | 22.7/20 | 42.2/20 | 16.6/20 | 13.9/20 |
| Confldence Level: | : 0.77 | 0.42 | 0.83 | 0.53 | 0.47 | 0.13 |
| Numker of Events: | 382 | 340 | 322 | 325 | 363 | 254 |

[^4]one would expect the effect to be more prominent as one moves away from the endpoint of the bremmstrahlung spectrum, and this does not appear to be true.

Because some of the density matrix elements are non-physical, they were also fit using the constraint equations. The results of both fits are presented in Tables XII - XVII. As in the cross-section parameters, the errors represent a 0.5 change in the likelihood function while allowing all other parameters to vary.

If one compares the ratio, $\mathrm{R}_{\mathrm{DH}}$, of $\left(\frac{\mathrm{d} \sigma}{\mathrm{dt}}\right)_{\mathrm{D} 2} /\left(\frac{\mathrm{d} \sigma}{\mathrm{dt}}\right)_{\mathrm{H} 2}$ one can by the use of Glauber shadowing theory extract the amount of $I=1$ exchange. The results of such a fit are presented in Table XVIII. While the amount of $I=1$ exchange is not zero, for all but the $12-14 \mathrm{GeV}$ fit, the values are less than one standard deviation from zero. This is shown in Figure 37 where the solid line represents the case of no $I=1$ exchange. Probably part of the difficulty in extracting the amount of $I=1$ is that this amplitude seems to be almost $90^{\circ}$ out of phase with the $I=0$ amplitude. This results effectively in our seeing only $\left|a_{1} / a_{0}\right|^{2}$ which is quite small even for non-negligible $\left|a_{1} / a_{0}\right|$. The similarity of the results as a function of energy cut also shows that, as in the case of the density matrix elements, the inelastic background in the region of the rho exhibits rho-like behavior.

Finally, we make a few observations concerning vector dominance, VDM. Without going through all the ideas presented in Chapter I, for convenience we repeat some of the results and predictions. Starting with the field-current identity:


Fig. 37--The ratio of differential cross sections for $\gamma \mathrm{p} \rightarrow \rho^{\circ} \mathrm{p}$ and $\gamma \mathrm{d} \rightarrow \rho^{\circ} \mathrm{d}+$ $\rho^{\circ} \mathrm{pn}$ as a function of t '. The $10-12 \mathrm{GeV}, 12-14 \mathrm{GeV}$, and $14-16 \mathrm{GeV}$ data is from the 16 GeV endpoint bremsstrahlung beam and the 7.9-9.9 cut is for the quasimonochromatic 9 GeV beam. A mass cut of $0.67 \leq \mathrm{M} \leq 0.87 \mathrm{GeV} / \mathrm{c}^{2}$ has been made. The solid curve is the prediction for no $I=\frac{\pi}{1}$ exchange in the $t$-channel

Table XVIII

Fit to $\mathrm{d} \sigma / \mathrm{dt}\left(\gamma \mathrm{d} \rightarrow \rho^{\circ} \mathrm{pn}\right) / \mathrm{d} \sigma / \mathrm{dt}\left(\gamma \mathrm{p} \rightarrow \rho^{\circ} \mathrm{p}\right)$

| Energy Byte | $\left\|a_{1} / a_{0}\right\|$ | $\operatorname{Cos} \Phi$ | $A 1$ | $A 2$ | $x^{2}$ | Degrees <br> Freedom | Confidence <br> Level |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $14-16 \mathrm{GeV}$ | $0.004 \pm$0.22 <br> 0.004 | $0.75 \pm$0.25 <br> 0.75 | -0.0093 | 0.0355 | 16.3 | 31 | 0.99 |
| $12-14 \mathrm{GeV}$ | $0.27 \pm$0.12 <br> 0.17 | $-0.02 \pm$0.12 <br> 0.09 | -0.0051 | 0.0686 | 15.3 | 31 | 0.99 |
| $10-12 \mathrm{GeV}$ | $0.19 \pm$0.15 <br> 0.19 | $-0.05 \pm$0.30 <br> 0.36 | 0.0030 | 0.0000 | 28.8 | 31 | 0.58 |

$$
\begin{aligned}
& \mathrm{A} 1=\left|a_{1} / a_{0}\right| * \operatorname{COS} \Phi / \text { DENOM } \\
& \mathrm{A} 2=\left|a_{1} / a_{0}\right|^{2} / \text { DENOM } \\
& \text { DENOM }=1+\left|a_{1} / a_{0}\right|^{2}+2 *\left|a_{1} / a_{0}\right| * \operatorname{Cos} \Phi
\end{aligned}
$$

$$
\begin{equation*}
j_{\mu}(x)=-\Sigma_{v} \frac{m_{v}^{2}}{2 \gamma_{v}} v_{\mu}^{0}(x) \tag{6.3}
\end{equation*}
$$

one is led to the following relation between photon amplitudes $f_{\gamma}$ and transverse vector meson amplitudes $f_{v}$ :

$$
\begin{equation*}
f_{\gamma}=e \sum_{v} \frac{f_{v}}{2 \gamma_{v}} \tag{6.4}
\end{equation*}
$$

The Orsay colliding-beam experiment determines the values of the coupling constants for $q^{2}=m_{v}^{2}>0$. For $\gamma^{2} \rho / 4 \pi$ they obtain $0.64 \pm 0.06$. Photoproduction, on the other hand, uses vector dominance for $q^{2}=0$, and, hence, is a test of the independence of the rho coupling constant on $q^{2}$. Using $V D M$ and the optical theorem one can write:

$$
\begin{equation*}
\frac{d \sigma^{\circ}}{d t}\left(\gamma p \rightarrow \rho^{\circ} p\right)=\frac{\alpha}{4}\left(\frac{\gamma_{\rho}^{2}}{\frac{\rho}{\pi}}\right)^{-1}\left(\frac{1+\eta_{\rho}^{2}}{16 \pi}\right) \sigma_{\rho N}^{2} \tag{6.5}
\end{equation*}
$$

where $\sigma_{\rho N}$ is the total rho-nucleon cross-section and $\eta_{\rho}$ ( $\sim$.15) is the ratio of real to imaginary parts of the amplitude. If one uses $27.6 \pm 0.6$ for the value of $\sigma_{\rho N}$ as obtained from an experiment on the rho photoproduction off deuterium in the double scattering region at $18 \mathrm{GeV}^{28}$ and the Orsay value ${ }^{15}$ for $\gamma_{\rho}^{2} / 4 \pi$, one obtains a predicted forward cross section for hydrogen of $113.5 \pm 11.5 \mu \mathrm{~b} /(\mathrm{GeV} / \mathrm{c})^{2}$ in good agreement with our observed value at 16 GeV . If one also uses the quark model to relate rho-nucleon to pion-nucleon scattering, one predicts: ${ }^{50}$

$$
\begin{equation*}
\frac{d \sigma}{d t}\left(\gamma p \rightarrow \rho^{\circ} p\right)=\frac{\alpha}{4}\left(\gamma_{\rho}^{2} / 4 \pi\right)^{-1}\left[\frac{1}{2} \sqrt{\frac{d \sigma}{d t}\left(\pi^{+} p \rightarrow \pi^{+} p\right)}+\frac{1}{2} \sqrt{\frac{d \sigma}{d t}\left(\pi^{-} p \rightarrow \pi^{-} p\right)}\right]_{(6.6}^{2} \tag{6.6}
\end{equation*}
$$

The predicted rho photoproduction differential cross section is plotted together with the observed differential cross section in Figure 38. Here we have used the data of Foley ${ }^{51}$ to obtain the pion-nucleon cross


Fig. 38--The differential cross section as a function of $t$ for reactions: (a) ם $\gamma \mathrm{p} \rightarrow \gamma \mathrm{p},{ }^{(52)}$ (b) $\Delta \pi^{ \pm} \mathrm{p} \rightarrow \pi^{ \pm} \mathrm{p},{ }^{\text {(51) ( }}$ (c) $\cdot$ $\gamma p \rightarrow \rho^{0} \mathrm{p}$ (this experiment). All reactions occur in the energy range $14-17 \mathrm{GeV}$
sections. The agreement is really quite good. Bolstered by our success here, we also try to relate Compton scattering to rho photoproduction.

VDM predicts the following relation:

$$
\begin{equation*}
\frac{d \sigma}{d t}(\gamma p \rightarrow \gamma p)=\left.\left.\frac{\alpha}{4}\right|_{=p, \omega, \phi} ^{v^{\Sigma}} \sqrt{\frac{d \sigma}{d t}\left(\gamma p \rightarrow v^{\circ} p\right)\left(\frac{\gamma^{2} v}{4 \pi}\right)^{-1}} e^{i \delta_{v}}\right|^{2} \tag{6.7}
\end{equation*}
$$

where $\delta_{v}$ is the amplitude phase. Due to the low $\omega$ and $\phi$ cross-sections and the difficulty of detecting $\omega^{\prime}$ s, there is almost no data on these reactions at 16 GeV . However, since the cross sections for these reactions are so much smaller than for $\rho$ photoproduction, and since the coupling constants are much bigger, the $\rho$ contribution to the right hand side of equation (6.7) dominates. The cross section predicted by Compton scattering ${ }^{52}$ as given by equation 6.7 is also shown in Figure 38. It is clear from the figure that the shape is correct, but the magnitude is wrong. Even if one includes the best estimates for $\omega$ and $\phi$ cross-sections ${ }^{53}$ at 16 GeV , one still needs a value of 0.40 for $\gamma_{\rho}^{2} / 4 \pi$ to obtain agreement. Figure 39 shows relations 6.6 and 6.7 plotted for the forward differential cross section as a function of energy. Here again we have used the results of Foley for the pion-nucleon crosssections. For the rho photoproduction cross sections we have used only the counter-type experiments, and we have used the DESY and SLAC Compton scattering results. ${ }^{22,54}$ Again there is agreement in shape and magnitude for pion-nucleon and rho photoproduction, but there is only agreement for shape between Compton scattering and rho photoproduction. One explanation for this disagreement would be the existence of higher mass vector mesons ( $\rho^{\prime}, \rho^{\prime \prime}$, etc.) which couple strongly to the photon. The


Fig. 39--The forward differential cross section as a function of lab momentum for reactions: (a) $\gamma \mathrm{p} \rightarrow \gamma \mathrm{p}:$ DENY, ${ }^{(54)}$. SLAG (Anderson, et al.), ${ }^{(52)} \triangle$ SIAC (Boyarski, et al.). ${ }^{(52)}$ (b) $\gamma \mathrm{p} \rightarrow \rho^{\circ} \mathrm{p}$ : $\diamond$ Cornell, ${ }^{(35)}{ }^{(3)}$ DESY-MIT, ${ }^{(57)}$ - this experiment, (c) $\pi^{ \pm} p \rightarrow \pi^{ \pm} p: X B N L$
strength of these mesons would have to be:

$$
\sum_{v=p^{\prime}, \rho^{\prime} \cdot \ldots\left(\gamma_{v}^{2} / 4 \pi\right)} \frac{1}{d t}(\gamma p \rightarrow V p)_{t=0} \approx 10-30 \mu b
$$

The existence of these mesons had been predicted by the Veneziano Model The first two had predicted masses of $\mathrm{mo}^{\prime}=1.3 \mathrm{GeV} / \mathrm{c}^{2}$ and $\mathrm{mo}^{\prime \prime}=1.5$ $\mathrm{GeV} / \mathrm{c}^{2}$. Also, recent analysis of the nucleon form factor ${ }^{56}$ predicts the existence of several new isovector and isoscalar vector mesons with masses above $1.5 \mathrm{GeV} / \mathrm{c}^{2}$. Several searches for such higher mass vector meson have been made in the last few years covering the mass range $1.0-2.0 \mathrm{GeV} / \mathrm{c}^{2}$. (See Table XIX)

Three different techniques were used to search for such a $\rho^{\prime}$ :
(1) Studying final state hadronic reactions. In particular

$$
\begin{aligned}
& \gamma \mathrm{p} \rightarrow \pi^{+} \pi^{-} \mathrm{p} \\
& \gamma \mathrm{p} \rightarrow \pi^{+} \pi^{-} \pi^{+} \pi^{-} \mathrm{p}
\end{aligned}
$$

(2) Using a missing mass spectrometer and integrating over all decay modes.
(3) Studying lepton pair spectra.

A SIAC experiment using the apparatus described in this thesis studied $\gamma \mathrm{Be} \rightarrow \pi^{+} \pi^{-} \mathrm{Be} .57$ If such a $\rho^{\prime}$ exists one would expect it to be coherently produced and readily observable if $\rho^{\prime} \rightarrow \pi^{+} \pi^{-}$is a significant decay mode. No narrow structure (width $\lesssim 200 \mathrm{MeV}$ ) was seen up to 2 GeV . Assuming $\sin ^{2} \theta_{\pi}$ distribution for a $\rho^{\prime}$ with mass $\approx 1500^{\circ} \mathrm{MeV}$

$$
\frac{d \sigma}{\rho^{\prime}}<10^{-3} \frac{d \sigma}{d t} \frac{\Gamma_{\rho}^{2}}{\Gamma_{\rho}^{\Gamma} \rho^{\prime} \rightarrow 2 \pi} .
$$

This same result was obtained by several experiments, ${ }^{58}$ including a very high statistics experiment by the DESY-MIT group.

## TABLE XIX



Studies of $\gamma p \rightarrow \rho^{\prime}(4 \pi) p$ gave more positive results. ${ }^{60}$ In this case a total cross section $\sim .9 \mu \mathrm{~b}$ is found for a $\rho^{\prime} \rightarrow 4 \pi$ with $m \rho^{\prime} \sim 1568$ $\mathrm{MeV}, \Gamma^{\prime}{ }^{\prime} \sim 300 \mathrm{MeV}$.

A high statistics missing mass experiment done at $t=-.2(\mathrm{GeV} / \mathrm{c})^{2}$ and $17 \mathrm{GeV} / \mathrm{c}$ photon energy ${ }^{61}$ which because of the low $t$ and high energy should have been very sensitive to the existence of a $\rho$ ' with a mass up to 2000 MeV , saw little if any structure. They found

$$
\frac{d \sigma}{d t} \rho^{\prime}<\frac{.025 \Gamma \rho^{\prime}}{\Gamma \rho} \frac{d \sigma}{d t}
$$

When compared with the $4 \pi$ result one sees that the $4 \pi$ decay must account for almost the entire cross section.

Thus, there is evidence for a $\rho^{\prime}$ (high mass vector meson). The cross sectional value is related to $\gamma_{p}^{2}$, by

$$
{\frac{\gamma_{\rho^{\prime}}}{\gamma_{\rho}}}^{2}=\frac{\sigma(\gamma p \rightarrow \rho p)}{\sigma\left(\gamma p \rightarrow \rho^{\prime} p\right)} \frac{\Gamma_{\rho^{\prime}}^{\prime} \rightarrow 4 \pi}{\Gamma_{\rho^{\prime}}} \lesssim 15
$$

Thus $\frac{1}{\left(\gamma \rho^{\prime} / 4 \pi\right)} \frac{d \sigma}{d t}\left(\gamma p \rightarrow \rho^{\prime} p\right) \leq 1 \mu \mathrm{~b} /(\mathrm{GeV} / \mathrm{c})^{2}$, which is not nearly enough to satisfy relation 6.7. Since the existence of $\rho^{\prime}$ has been shown, there is no reason not to believe in a whole series of $\rho^{\prime \prime}, \rho^{\prime \prime}$ etc. Although the existence of such a series of resonances would satisfy VDM, it would greatdly reduce the simplicity of the theory and make it much more difficult to use.

It will be interesting to see if an extension to higher energies at NAL does reveal the existence of a series of $\rho^{\prime}$ required to satisfy VDM. At high energies, presumably pomeron exchange will be even more dominant,
and the interpretation of the data easier. It will also be interesting to see if the present indication of Regge shrinkage continues. Even at present energies, rho photoproduction seems to be well described as a diffractive process conserving s-channel helicity and is very similar (except for magnitude of cross sections) to pion-nucleon elastic scattering.

## APPENDIX A

This appendix is devoted to a discussion of the hodoscope testing telescope. It will be shown that the test setup resulted in minimum ionizing electrons passing through each test counter.

First consider the equation for energy loss by an electron passing through matter. If the material has an average atomic number $A$ and $Z$ electrons and if the average ionization potential is given by $I$, then the energy loss $d \equiv / d x$ in $\mathrm{MeV}-\mathrm{cm}^{2} / \mathrm{g}$ is given by equation Al . ${ }^{62}$

$$
\begin{equation*}
-d E^{e} / d \mathbf{x}=\frac{4 m e^{c^{2}}}{\beta^{2}} * \frac{Z}{A} *\left[\operatorname{In}\left(\frac{\pi m e^{c^{2}}}{I} *(\gamma-1) *(\gamma+1)^{\frac{1}{2}}\right)-1.45 \beta^{2}\right] \tag{A1}
\end{equation*}
$$

For scintillator $Z / A=\frac{1}{2}$, and thus everything in $A I$ is known except $I$.
To determine $I$, the experimental results for heavy, charged particles passing through scintillator are used. Equation A2 gives the energy loss for heavy charged particles. ${ }^{47}$ This contains the same $I$ and hence $A 2$ can be used to determine the ionization potential.

$$
\begin{equation*}
-d E^{H} / d x=\frac{4 m e^{c^{2}}}{\beta^{2}} * \frac{Z}{A} *\left[\operatorname{In} \frac{2 m e^{c^{2}}}{\left(1-\beta^{2}\right) I}-\beta^{2}\right] \tag{A2}
\end{equation*}
$$

Experimentally $\left.-d \mathrm{~F}^{\mathrm{H}} / \mathrm{dx}\right)_{\min }=2.03\left(\mathrm{MeV}-\mathrm{cm}^{2} / \mathrm{g}\right)$ in scintillator. Thus by solving $A 2$ for its minimum I can be eliminated. Then the minimum value can be used to determine $\beta_{\min }^{2}$. Finally $\beta_{\min }^{2}$ can be used to find $I$.
When this is done, $\beta_{\min }^{2}=0.924$ and $I=23.3 \mathrm{eV}$. Using this value for $I$, Al becomes A3.

$$
\begin{equation*}
-\frac{d E^{\mathrm{e}}}{d x}=\frac{0.1553}{\beta^{2}}\left[11.138+\operatorname{In}(1-\gamma)+0.5 \operatorname{In}(\gamma+1)-1.45 \beta^{2}\right] \tag{A3}
\end{equation*}
$$

This equation is plotted in figure 40 as a. function of $\gamma=\mathrm{E} / \mathrm{m}_{\mathrm{e}} \mathrm{c}^{2}$. A3 has a minimum $\left.\left(-\mathrm{dE}^{\mathrm{e}} / \mathrm{dx}\right)_{\min }=-1.91 \mathrm{MeV}-\mathrm{cm}^{2} / \mathrm{g}\right)$ for $\mathrm{B}^{2}=0.9375(\gamma=4.00$, $E=2.04 \mathrm{MeV}$ ). For $\gamma$ between 2 and 10 the curve is essentially flat $\left(+1.95 \mathrm{MeV}-\mathrm{cm}^{2} / \mathrm{g}\right)$ while for $\gamma$ between 1 and 2 the curve can be approximated (dashed line) as $-\mathrm{dF}^{\mathrm{e}} / \mathrm{dx}=1.2 / \beta^{2}+0.8 \mathrm{MeV}-\mathrm{cm}^{2} / \mathrm{g}$. The test setup is repeated below for convenience.


Test Counter

$$
\begin{gathered}
\mathrm{Ru}^{106} \rightarrow \mathrm{Rh}^{106} \rightarrow \mathrm{Pd}^{106} \\
40 \mathrm{KeV} \\
\mathrm{e}^{-} \quad 3.5 \mathrm{MeV} \\
\mathrm{e}^{-} \quad+\gamma
\end{gathered}
$$

$$
t_{\frac{1}{2}}=1 \mathrm{yr} \quad t_{\frac{1}{2}}=30 \mathrm{~s}
$$

The dimensions above are the "effective" amount of scintillator and include the aluminum foil and tape used to wrap the counters.

Using our approximations, A3 can be integrated to obtain $E_{I}=$ $2.64+E_{4}$, where $E_{I}$ is the initial electron energy and $F_{4}$ is the energy deposited in counter $C_{4}$. Let $E_{1}$ be defined as the energy of the electron entering $E$ and $E_{2}$ the energy of the electron exiting $T$. Then using the fact that $E_{I} \leq 3.5 \mathrm{MeV}$ due to the source and $E_{4}>0$ to trigger $C$, limits can be placed on $\mathrm{E}_{1}$ and $\mathrm{E}_{2}$. These limits are given by A 4 .

$$
\begin{align*}
& 2.670<\mathrm{E}_{1}<3.175 \mathrm{MeV}  \tag{A4}\\
& 1.695<\mathrm{E}_{2}<1.875 \mathrm{MeV}
\end{align*}
$$

These limits are shown in Figure 40. Clearly, to within several per cent the entire range of energies seen by the test counter is minimumionizing.


Fig. 40--dE/dX electrons plot for thin scintillation counter telescope used to test hodoscopes. T represents counter being tested. Solid curve shows $d E / d X(\gamma)$, while dashed line gives $d E / d X\left(1 / \beta^{2}\right)$. The test counter range is shown to cover the minimum ionizing region of the $\mathrm{dE} / \mathrm{dX}$ curve

## APPENDIX B

In this section the three constraint equations, B12, B13 and B14, governing the density matrix elements will be derived. These equations depend upon the Schwartz inequality relation B 9 and several other relations which are a consequence of the density matrix defining relation, Equation 4.3 which is repeated for convenience as B1.

$$
\begin{equation*}
\rho_{i j}=\frac{1:}{\left(2 s_{a}+1\right)} \frac{1}{\left(2 s_{b}+1\right)} \Sigma_{\lambda a, \lambda b, \lambda d} f_{j \lambda d ;}^{*} ; \lambda a b f_{i \lambda d ; \lambda a \lambda b} \tag{B1}
\end{equation*}
$$

Three properties ( $B 2, B 3$, and $B^{4}$ ) of the $\rho_{i j}$ 's follow directly from B1.

$$
\begin{align*}
& \rho_{i j}=\rho_{j i}^{*}  \tag{B2}\\
& \rho_{i i} \geq 0  \tag{B3}\\
& \operatorname{Det}\left(\rho_{i j}\right) \geq 0 \tag{4}
\end{align*}
$$

Relation B 2 immediately reduces the number of independent complex variables from nine to three comples ( $\rho_{10}, \rho_{1-1}, \rho_{-10}$ ) and three real variables ( $\rho_{00}, \rho_{11}, \rho_{-1-1}$ ).

Two more density matrix relations (B5 and B6) reduce the number of independent variables to two real ( $\rho_{00}$ and $\rho_{1-1}$ ) and one complex ( $\rho_{10}$ ).

$$
\begin{gather*}
\rho_{i j}=(-1)^{i-j} \rho_{-i-j}  \tag{B5}\\
\rho_{00}+\rho_{11}+\rho_{-1-1}=1 \tag{B6}
\end{gather*}
$$

Relation B5 depends upon parity conservation and the choice of the $y$ axis as normal to the production plane, while relation $B 6$ depends upon integrating $B 7$, the angular distribution probability (w) over all angles.

$$
\begin{equation*}
W=\Sigma_{i j} \rho_{i j} D_{i j}(\theta, \phi) \tag{B7}
\end{equation*}
$$

In practice only $\rho_{00}, \rho_{1-1}$ and $R e \rho_{10}$ can be measured using a nonpolarized photon beam (see 4.3). Using B5 to get $\rho_{11}=\rho_{-1-1}$ we can then use relation B 6 to obtain B 8 .

$$
\begin{equation*}
\rho_{11}=\rho_{-1-1}=\frac{1}{2}\left(1-\rho_{00}\right) \tag{B8}
\end{equation*}
$$

The constraint equations depend upon B3, B4, B8 and the Schwartz inequality Bg .

$$
\begin{equation*}
\left|\rho_{i j}\right|^{2} \leq \rho_{i i} \rho_{j j} \tag{B9}
\end{equation*}
$$

Using B 9 we obtain two relations Bl and B 2 (plus the trivial relation $\left|\rho_{00}\right|^{2} \leq \rho_{00}{ }^{2}$ ). Using B3 and B8 we obtain a constraint B13 on $\rho_{00}$. Finally by expanding $\mathrm{B}^{4}$ we get B1O.

$$
\begin{equation*}
2 \frac{1-\rho_{00}}{2}+\rho_{1-1} \frac{\rho_{00}}{2}\left(1-\frac{\rho_{00}}{2}-\rho_{1-1}\right)+\left|\rho_{10}\right|^{2} \geq 0 \tag{B10}
\end{equation*}
$$

Constraint B12 ensures that the first factor be non-negative and hence so must the second. This leads to the final constraint B14. Clearly constraint B14 on $\mathrm{Re}_{10}$ is stronger than B1l; and, therefore, the set of constraints used to fit the data was B12, B13, and B14.

$$
\begin{gather*}
\left(\operatorname{Re}\left(\rho_{10}\right)\right)^{2} \leq\left|\rho_{10}\right|^{2} \leq \rho_{11} \rho_{00}=\rho_{00}\left(1-\rho_{00}\right) / 2  \tag{B1I}\\
\left|\rho_{1-1}\right|^{2} \leq \rho_{11} \rho_{-1-1}=\left(1-\rho_{00}\right)^{2} / 4  \tag{B12}\\
0 \leq \rho_{00} \leq 1  \tag{B13}\\
\left(\operatorname{Re} \rho_{10}\right)^{2} \leq\left|\rho_{10}\right|^{2} \leq \rho_{00}\left(1-\rho_{00}-2 \rho_{1-1}\right) / 4 \tag{B14}
\end{gather*}
$$

## APPENDIX C

The determination of the momentum of each track is based on the assumptions that the trajectory can be described as (a) a helix within the magnetic field, (b) a straight line outside the magnetic field, and (c) that the fringe field can be represented by a simple lens at magnet entrance and exit which results in a change of the dip angle at each lens (see figures $17 a$ and $b$ ).

The measured quantities in our experiment are (a) the track on the exit side of the magnet and (b) a point corresponding to the interaction point within the target. There is only one trajectory through the magnet which will satisfy these measurements, but it must be calculated by iteration. Figure 17a shows the geometry of the correct trajectory. For that trajectory, triangles $\triangle A C D$ and $\triangle B C D$ are congruent right triangles. Thus

$$
\begin{equation*}
B D=A D \equiv \lambda \tag{Cl}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{BE}+\mathrm{FA}=\mathrm{L} \text { (magnet length) } \tag{c2}
\end{equation*}
$$

Then if one introduces the parameter $x$ defined in C3,

$$
\begin{equation*}
\mathbf{x}=\mathrm{BE} / \mathrm{L}=\lambda \cos \beta \tag{c3}
\end{equation*}
$$

one finds that

$$
1-x=F A / L=\lambda \cos \alpha .
$$

If we define the function $f(x)$ as in $C 4$,

$$
\begin{equation*}
f(x)=(1-x)^{2} \frac{\cos ^{2} \beta}{\cos ^{2} \alpha}-x^{2} \tag{c4}
\end{equation*}
$$

then $f(\bar{x})=0$ for the true trajectory. The iteration consists of
selecting $\dot{x}=0.5$, calculating point $D$ from $x$ and the straight line track $D A$, and then calculating the angle $\beta$ for the track on the target side of the magnet. An improved estimate for $\bar{x}$ is determined according to relation C6.

$$
\begin{gather*}
f(\bar{x}) \equiv f(x-\delta)=0=f(x)-f^{\prime}(x) \delta  \tag{c6}\\
\delta=f(x) / f\left(x^{\prime}\right)
\end{gather*}
$$

In practice three iterations results in $\delta<10^{-5}$ inches. This
final $\overline{\mathrm{x}}$ is then used to calculate the angle $\beta$ and the momentum.
The fringe field focusing is shown in flgure 17 b . The radious of the trajectory in the $x z$ plane is calculated from the angles $\alpha$ and $\beta$ (positive as shown in 17a) according to $\mathrm{C7}$.

$$
\begin{equation*}
R=L /(\sin \alpha+\sin \beta) \tag{c7}
\end{equation*}
$$

Then each focal length $f_{1}$ (magnet entrance) and $f_{2}$ (magnet exit) is given as in C8.

$$
\begin{equation*}
f_{1}=R / \operatorname{TANB} \text { and } f_{2}=R / \operatorname{TAN} \alpha \tag{c8}
\end{equation*}
$$

If the dip angle $\delta$ is defined as in $C 9$

$$
\begin{equation*}
\tan \delta=(d y / d z) / \sqrt{1+(d x / d z)^{2}} \tag{c9}
\end{equation*}
$$

then the change in dip angles at the magnet entrance and exit is given as in ClO .

$$
\begin{equation*}
\delta_{2}=\delta_{1}-y_{1} / f_{1} \text { and } \delta_{3}=\delta_{2}-y_{2} / f_{2} \tag{c10}
\end{equation*}
$$

Thus this algorithm takes as input the track seen in the chamber package along with the assumed interaction point within the target and calculates the track momentum and incoming track parameters.

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[^0]:    *Ph.D. dissertation, May 1974.

[^1]:    *Edgerton, Germeshausen, and Grier, Inc.

[^2]:    * See the cross section values near the tip of the bremsstrahlung spectrum at 9, 13, and 16 GeV as presented in Chapter VI. Near the tip these cross section values are almost free from inelastic contributions.

[^3]:    *This is chi-square for the unweighted projected istricution/the number of nonzero bins

[^4]:    - Thie is chi-square for the unweightel, projected distribution/the number of nonzero bins

