APPENDIX I

The Supersymmetric Lagrangian

I.1 BASICS

Superspace and Superfields

We remind the reader of some fundamental aspects of supersymmetric theories. We extend normal space-time to "super-space" which includes a number of complex anti-commuting (Grassman) coordinates $\theta_a, \overline{\theta}_a$ $(\overline{\theta} \equiv \theta^\dagger)$. Superfields are functions of these superspace coordinates

$$\phi = \phi(x^\mu; \theta_i, \overline{\theta}_i) \qquad \mu = 1, \ldots, D \quad a = 1, \ldots K$$

where D is the dimensionality of space-time and we refer to this as an "$n = K$" supersymmetry. In this paper we will restrict ourselves to $D = 4$ and $K = 1$ which corresponds to the simplest supersymmetry .

We shall take θ^α and $\overline{\theta}^{\dot{\alpha}}$ to be two-component anti-commuting objects (akin to Weyl spinors):

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} \qquad \overline{\theta} = \begin{pmatrix} \overline{\theta}_1 \\ \overline{\theta}_2 \end{pmatrix} . \tag{I.1}$$

Thus

$$\{\theta_i, \theta_j\} = \{\overline{\theta}_i, \overline{\theta}_j\} = \{\theta_i, \overline{\theta}_j\} = 0 \qquad i, j = 1, 2 . \tag{I.2}$$

In particular

$$\theta_1^2 = \theta_2^2 = \overline{\theta}_1^2 = \overline{\theta}_2^2 = 0 . \tag{I.3}$$

For short we write

$$\theta^2 = \theta\theta = \tfrac{1}{2}\,\epsilon^{\alpha\beta}\theta_\alpha\theta_\beta = \theta_1\theta_2 \ . \tag{I.4}$$

Note that $\theta^3 = 0 = \overline{\theta}^3$ and likewise for higher powers of θ.

Supertransformations and the Supersymmetry Generators

We examine a superfield under the action of an infinitesimal translation in superspace ("supertransformation")

$$\theta \to \theta + \epsilon \qquad \overline{\theta} \to \overline{\theta} + \overline{\epsilon} \qquad x^\mu \to x^\mu + \delta x^\mu \tag{I.5}$$

where ϵ and $\overline{\epsilon}$ are infinitesimal, two-component, anti-commuting c-numbers. We desire to construct δx^μ from the anti-commuting coordinates θ and $\overline{\theta}$ subject to the constraint that x^μ must be a real coordinate which commutes, not anti-commutes. Since $\chi\psi$, the product of two Grassman objects, is a commuting object we must construct δx^μ from products of ϵ's and θ's.

The two simplest transformations which retain the Lorentz structure are

$$\begin{aligned} x^\mu &\to x^\mu + \epsilon^\alpha \sigma^\mu_{\alpha\dot{\beta}} \overline{\theta}^{\dot{\beta}} + \theta^\alpha \sigma^\mu_{\alpha\dot{\beta}} \overline{\epsilon}^{\dot{\beta}} \\ x^\mu &\to x^\mu + i\left(\theta^\alpha \sigma^\mu_{\alpha\dot{\beta}} \overline{\epsilon}^{\dot{\beta}} - \epsilon^\alpha \sigma^\mu_{\alpha\dot{\beta}} \overline{\theta}^{\dot{\beta}}\right) \ . \end{aligned} \tag{I.6}$$

Note that using $\epsilon_{\alpha\beta}$ and $\epsilon^{\dot{\alpha}\dot{\beta}}$ these may be recast into equivalent forms. The second equation in (I.6) is the traditional choice. These forms follow because σ^μ (and $\overline{\sigma}^\mu$) are the only dimensionless constants with a single Lorentz index. Note that here $\epsilon, \overline{\epsilon}$ are c-numbers and we are dealing with rigid (global) supersymmetry. In a local theory (i.e. supergravity) $\epsilon = \epsilon(x)$ and $\overline{\epsilon} = \overline{\epsilon}(x)$ but $\epsilon \neq \epsilon(\theta_i)$.

Representing these supertransformations by (ϕ is a superfield)

$$\phi \to \phi + \delta_\epsilon \phi \tag{I.7}$$

where

$$\delta_\epsilon \equiv \epsilon Q + \overline{\epsilon}\overline{Q} \tag{I.8}$$

we can find the supersymmetry generators Q and $\overline{Q}$.

$$\begin{aligned}\phi(x^\mu, \theta, \overline{\theta}) &\to \phi + \delta_\epsilon \phi = \phi(\theta + \epsilon, \overline{\theta} + \overline{\epsilon}, x^\mu + i[\theta\sigma^\mu\overline{\epsilon} - \epsilon\sigma^\mu\overline{\theta}]) \\ &= \phi + \epsilon \frac{\partial\phi}{\partial\theta} + \overline{\epsilon} \frac{\partial\phi}{\partial\overline{\theta}} + i \frac{\partial\phi}{\partial x^\mu} (\theta\sigma^\mu\overline{\epsilon} - \epsilon\sigma^\mu\overline{\theta}) \\ &= \phi + \epsilon \left(\frac{\partial}{\partial\theta} - i\sigma^\mu\overline{\theta} \frac{\partial}{\partial x^\mu} \right) + \overline{\epsilon} \left(\frac{\partial}{\partial\overline{\theta}} - i\theta\sigma^\mu \frac{\partial}{\partial x^\mu} \right)\end{aligned} \tag{I.9}$$

where the final minus sign comes from anticommuting $\overline{\epsilon}$ through $\theta\sigma^\mu$. Note that $\partial/\partial\theta$ and $\partial/\partial\overline{\theta}$ are two-component anticommuting operators. Comparing (I.7), (I.8) and (I.9) we have

$$\begin{aligned} Q &= \frac{\partial}{\partial\theta} - i\sigma^\mu\overline{\theta} \frac{\partial}{\partial x^\mu} \\ \overline{Q} &= \frac{\partial}{\partial\overline{\theta}} - i\theta\sigma^\mu \frac{\partial}{\partial x^\mu} \, . \end{aligned} \tag{I.10}$$

Note that the definition of $\overline{Q}$ differs by a sign when compared with those of Wess and Bagger.[1] This arises because they use $\overline{\theta}_{\dot{\alpha}}$ and we use $\overline{\theta}^{\dot{\alpha}} = \epsilon^{\dot{\alpha}\dot{\beta}}\theta_{\dot{\beta}}$ "=" $-\overline{\theta}_{\dot{\alpha}}$ where "=" means "which has the numerical magnitude".

Summarizing, under the supersymmetry transformation δ_ϵ

$$\begin{aligned} \theta &\to \theta + \epsilon \\ \overline{\theta} &\to \overline{\theta} + \overline{\epsilon} \\ x^\mu &\to x^\mu + i(\theta\sigma^\mu\overline{\epsilon} - \epsilon\sigma^\mu\overline{\theta}) \end{aligned} \tag{I.11}$$

the superfield $\phi(x^\mu, \theta, \overline{\theta})$ transforms as

$$\begin{aligned}
&\phi \to \phi + \delta_\epsilon \phi \\
&\delta_\epsilon = \epsilon Q + \overline{\epsilon}\overline{Q} \\
&Q = \frac{\partial}{\partial \theta} - i\sigma^\mu \overline{\theta}\, \frac{\partial}{\partial x^\mu} \\
&\overline{Q} = \frac{\partial}{\partial \overline{\theta}} - i\theta\sigma^\mu\, \frac{\partial}{\partial x^\mu}\ .
\end{aligned} \tag{I.12}$$

Relationship with Hamiltonian

It can be shown that $[\delta_\epsilon, \delta_\xi] \sim (\epsilon\sigma^\mu\overline{\xi} - \xi\sigma^\mu\overline{\epsilon})\partial_\mu$ and so two successive supersymmetry transformations lead to a Lorentz transformation (in a local theory this will necessitate a torsional term). In a sense the supersymmetry transformations are "square roots" of Lorentz transformations. In particular $\{Q_\alpha, \overline{Q}_{\dot{\beta}}\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu$. Multiplying by $(\overline{\sigma}^0)^{\dot{\beta}\alpha}$ (i.e. taking the trace) leads to

$$Q_1\overline{Q}_1 + Q_2\overline{Q}_2 + h.c. = P_0 = \mathcal{H} \tag{I.13}$$

and so the Hamiltonian is sum of the squares of the supersymmetry generators and is always positive semi-definite. This Q is like the "square root of $\mathcal{H}$".

Covariant Derivative

We desire a supersymmetric derivative which commutes with ϵQ and $\overline{\epsilon}\overline{Q}$ so that if $\phi \to \phi + \delta_\epsilon\phi$ then $D\phi \to D\phi + \delta_\epsilon D\phi$ (i.e. if ϕ transforms as a superfield then so does $D\phi$). It is easily checked that if we let

$$\begin{aligned}
&D = \frac{\partial}{\partial \theta} + i\sigma^\mu \overline{\theta}\, \frac{\partial}{\partial x^\mu} \\
&\overline{D} = \frac{\partial}{\partial \overline{\theta}} + i\theta\sigma^\mu\, \frac{\partial}{\partial x^\mu}
\end{aligned} \tag{I.14}$$

then $[\epsilon Q, \xi D] = \{Q, D\} = 0$ and similarly

$$\{Q, \overline{D}\} = \{\overline{Q}, \overline{D}\} = \{\overline{Q}, D\} = 0 \; . \tag{I.15}$$

Note that D is like Q with a change of sign. (I.15) implies that $\delta_\epsilon D\phi = D\delta_\epsilon\phi$ using the definition of δ_ϵ in (I.12).

Differentiation and Integration

Differentiation and integration over Grassman variables is largely a matter of definition. We define (ξ is a single component)

$$\frac{\partial}{\partial \xi} A = 0 \qquad \frac{\partial}{\partial \xi}\, \xi = 1 \tag{I.16}$$

when A is independent of ξ and could be commuting, anti-commuting, or a combination.

Note that

$$\frac{\partial}{\partial \xi} A\xi = \begin{cases} A & \text{if } A \text{ is commuting} \\ -A & \text{if } A \text{ is anti-commuting} \end{cases} \tag{I.17}$$

For integrations we invoke three axioms

(i) Linearity

$$\int (AF(\xi) + BG(\xi))d\xi = A\int F(\xi)d\xi + B\int G(\xi)d\xi \; . \tag{I.18}$$

Note that $\int d\xi$ is a definite integral over "all values of ξ" and that $\int A d\xi = -\int d\xi A$ if A is anti-commuting.

(ii) Shifting variable of integration

$$\int F(\xi + \chi)d\xi = \int F(\xi)d\xi \; . \tag{I.19}$$

(iii) If $\chi\xi = 0 \quad \forall \quad \chi$ then $\xi = 0$.

Consider:

$$\int (\xi + \chi) d\xi = \int \xi d\xi + \int \chi d\xi \qquad \text{by linearity}$$

$$\int (\xi + \chi) d\xi = \int \xi d\xi \qquad \text{by shifting}$$

$$\Rightarrow \int \chi d\xi = 0 \tag{I.20}$$

$$= \chi \int d\xi \qquad \forall \chi$$

$$\Rightarrow \int d\xi = 0 \ .$$

Since $\xi^2 = 0$ (ξ a single-component object) any function of ξ has a trivial expansion of the form

$$F(\xi) = A + B\xi \tag{I.21}$$

and the most general function is a linear function.

Then

$$\int F(\xi) d\xi = \int A d\xi + B \int \xi d\xi = B \int \xi d\xi$$

and unless we want integration to be totally trivial (all integrals vanish) we must have $\int \xi d\xi \neq 0$. For simplicity we select

$$\int \xi d\xi \equiv 1 \ . \tag{I.22}$$

We see that the rules for differentiation and integration are identical with $\partial_\xi = \int d\xi$ acting like projection operators, projecting out only those terms with a ξ.

Multiple integrals work the same way (keeping track of anti-commuting $d\xi\, d\chi$, etc.). Thus for a two-component object $\theta = (\theta_1 \quad \theta_2)^T$

$$\int \theta^2 d^2\theta = \int \theta_1 d\theta_1 \int \theta_2 d\theta_2 = 1 \tag{I.23}$$

and all other combinations vanish. Note again that $\partial F/\partial\theta^2 \equiv \int F\, d\theta^2$.

Component Fields

Let us consider the expansion of a superfield $\phi(x,\theta,\overline{\theta})$ where $\theta,\overline{\theta}$ are two-component anti-commuting (coordinates) parameters

$$\begin{aligned}\phi(x,\theta,\overline{\theta}) &= s(x) + (\overline{\theta}\psi_1 + \overline{\psi}_2\theta) + (a_1\theta\theta + a_2^*\overline{\theta\theta}) \\ &\quad + A_\mu\overline{\theta}\sigma^\mu\theta + \lambda_1\overline{\theta}\theta\theta + \overline{\theta\theta}\theta\overline{\lambda}_2 + D\theta\theta\overline{\theta\theta}\ .\end{aligned} \tag{I.24}$$

All other Lorentz invariant combinations, such as $B_{\mu\nu}\theta\sigma^{\mu\nu}\theta$ vanish due to $\theta_i^2 = 0$. $\overline{\theta}\sigma^\mu\theta\overline{\theta}\sigma_\mu\theta$ is proportional to $\theta\theta\overline{\theta\theta}$ and so is not included. If ϕ is to be real then s, A_μ and D are real and $\psi_1 = \psi_2, a_1 = a_2$ and $\lambda_1 = \lambda_2$.

This represents *many* degrees of freedom. Just as a massless gauge field, A^μ, has two, not four, physical degrees of freedom. To reduce the multitude of non-physical degrees of freedom a number of "supergauge-like" constraints are used. Although it is not readily apparent, a hallmark of supersymmetric theories is that the number of physical fermionic and bosonic degrees of freedom are equal. First it should be realized from (I.24) that *any function of a superfield is another superfield.*

This can be explicitly checked by testing that it transforms as a superfield under δ_θ. For instance, since $D\phi$ and $\overline{D}\phi$ are superfields, $\phi^3\overline{D}\phi e^{i\phi}D\phi$ would also be a superfield. For our constraint we could choose $f(\phi) = 0$ which would be a manifestly supersymmetric constraint. In practice the two which are usually chosen are $D\overline{\phi} = 0$ and $\phi = \phi^*$.

Scalar Superfields

We call superfields satisfying

$$\overline{D}\phi = 0 \tag{I.25}$$

chiral (or scalar) superfields and their conjugates

$$D\phi^\dagger = 0 \tag{I.26}$$

anti-chiral superfields, or superfields of the opposite chirality. These equations may be trivially solved. Recalling that

$$\overline{D}_{\dot\alpha} = \frac{\partial}{\partial\overline{\theta}^{\dot\alpha}} + i\,\theta^\beta\sigma^\mu_{\beta\dot\alpha}\,\frac{\partial}{\partial x^\mu}$$

we note that

$$\overline{D}_{\dot\alpha}(x^\mu + i\theta\sigma^\mu\overline{\theta}) = \overline{D}_{\dot\alpha}\theta^{\dot\alpha} = 0 \tag{I.27}$$

and therefore

$$\phi = \phi(x^\mu + i\theta\sigma^\mu\overline{\theta}, \theta) \tag{I.28}$$

is a solution of $\overline{D}\phi = 0$. This follows since

$$\frac{\partial}{\partial\overline{\theta}}\,\phi(x^\mu + i\theta\sigma^\mu\overline{\theta}, \theta) = -i\theta\sigma^\mu\frac{\partial\phi}{\partial x}$$

(remember to anti-commute $\partial/\partial\overline{\theta}$ past θ) and

$$\frac{\partial}{\partial x^\mu}\,\phi(x^\mu + i\theta\sigma^\mu\overline{\theta},\theta) = \phi'$$

$$\left(\frac{\partial}{\partial\overline{\theta}} + i\theta\sigma^\mu\,\frac{\partial}{\partial x^\mu}\right)\,\phi = 0 = \overline{D}\phi\ .$$

Similarly $D\overline{\phi} = 0$ has solution

$$\phi = \overline{\theta}(x^\mu - i\theta\sigma^\mu\overline{\theta},\overline{\theta}) \tag{I.29}$$

is an "anti-chiral" field.

Since we can write

$$\begin{aligned}\phi &= \phi(y,\theta)\\ \overline{\phi} &= \overline{\phi}(y^\dagger,\overline{\theta})\\ y^\mu &= x^\mu + i\theta\sigma^\mu\overline{\theta}\end{aligned} \tag{I.30}$$

we see that ϕ is not an explicit function of $\overline{\theta}$ and that $\overline{\phi}$ is not an explicit function of θ. This is the principle reason that these particular constraints were chosen. The expansion of $\phi(y,\theta)$ is now simplified considerably

$$\phi = A(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y) \tag{I.31}$$

where the $\sqrt{2}$ is chosen to make the closure of the algebra simple. Since $\phi(y,\theta)$ is independent of $\overline{\theta}$ we see that **any function of $\phi(y,\theta)$ will also be a chiral superfield**, assuming that there is no $\overline{\theta}$ explicitly in the function definition, because $f(\phi)$ will also be independent of $\overline{\theta}$. Note that $A(y)$ is a scalar field and $\psi(y)$ a spinor. This closure is easily seen by considering the effect of the supersymmetry generators Q and $\overline{Q}$ on A, ψ and F which are called the "component fields" of the superfield ϕ.

Transformation of Scalar Superfield Components

$$\begin{aligned}\delta_\epsilon A &= (\epsilon Q + \bar{\epsilon}\,\overline{Q})A \\ \delta_\epsilon \psi &= (\epsilon Q + \bar{\epsilon}\,\overline{Q})\psi \\ \delta_\epsilon F &= (\epsilon Q + \bar{\epsilon}\,\overline{Q})F \ .\end{aligned} \tag{I.32}$$

Now consider the dimensionality of the quantities involved. Since $Q \sim \sqrt{H} \sim \sqrt{E} \Rightarrow$

$$[Q] = m^{1/2}$$

Similarly

$$[\theta] = m^{-1/2} \quad [d\theta] = m^{1/2} \quad [A] = m \quad [\psi] = m^{3/2} \quad [F] = m^2 \quad [\overline{Q}] = m^{1/2} \ . \tag{I.33}$$

In constructing a supermultiplet with a left-handed Weyl spinor ϕ we might try $QA \sim \psi$ on dimensional grounds and since $[Q\psi] = m^2$ we could invent a state with dimension m^2 or we can have $Q\psi \sim \partial_\mu A$. In general when we operate with Q we will obtain a state of higher dimension by $m^{1/2}$ or the derivative of a state of lower dimension. It can be shown that $\overline{Q}$ adds $\frac{1}{2}$ a unit of right-handed helicity while Q adds $\frac{1}{2}$ a unit of left-handed helicity (helicity here means s_z). As always adding $j = \frac{1}{2}$ may result in states of $j_z \pm \frac{1}{2}$ corresponding to adding $\left|\frac{1}{2}, \frac{1}{2}\right\rangle$ or $\left|\frac{1}{2}, -\frac{1}{2}\right\rangle$. Thus operating with δ_ϵ results in both states which have been increased and decreased by $j = \frac{1}{2}$. Since we are considering a multiplet with a left-handed spin we take $\overline{Q}A = 0$ (since it would give a right-handed Weyl spinor). Take

$$QA = \sqrt{2}\psi \ . \tag{I.34}$$

(The $\sqrt{2}$ is conventional.) Similarly

$$Q\psi = \sqrt{2}F \qquad \overline{Q}\psi = i\sqrt{2}\sigma^\mu\partial_\mu A \ . \tag{I.35}$$

Since there is no field with θ^3 coefficient F had better transform entirely into $\psi's$. (Also $F \sim Q^2A$ and so $QF = 0$ since $Q^3 = 0$ being anti-commuting) thus

$$QF = 0 \qquad \overline{Q}F = i\sqrt{2}\partial_\mu\sigma^\mu\psi \tag{I.36}$$

or

$$\begin{aligned}
&\delta_\epsilon A = \sqrt{2}\epsilon\psi \\
&\delta_\epsilon\psi = i\sqrt{2}\sigma^\mu\overline{\epsilon}\partial_\mu A + \sqrt{2}\epsilon F \\
&\delta_\epsilon F = i\sqrt{2}\overline{\epsilon}\sigma^\mu\partial_\mu\psi = \partial_\mu\{i\sqrt{2}\overline{\epsilon}\sigma^\mu\psi\} \ .
\end{aligned} \tag{I.37}$$

Note that $\delta_\epsilon F$ = a total divergence. This is of *critical* significance and is always true of the "top-most" component field of a superfield (i.e. that which is multiplied by the most powers of θ's and $\overline{\theta}$'s). The reason is that if we use the F-term as a Lagrangian density then $L = \int dx\, F$ will transform under a supersymmetric shift δ_ϵ by $\sim \int \partial_\mu\psi \sim \int \psi \cdot dS$ which is a surface term at infinity and vanishes for topologically trivial systems (such as all that will be considered here). Thus $\delta_\epsilon L = 0$ and the Lagrangian will be invariant under supersymmetry transformations. More on this in Section I.2.

Vector Superfields

The previous sections introduced the scalar or chiral superfield which contains a Weyl spinor, a scalar and an object "F" called an auxiliary field. This was subject to the constraint $\overline{D}\phi = 0$. Here we introduce the multiplet which contains gauge fields. The constraint used is that the superfield is real. We saw from (I.24) and the subsequent discussion that the expansion of such a superfield into its component fields is

$$V = s(x) + \overline{\theta}\psi + \overline{\psi}\theta + \theta^2 a + \overline{\theta}^2 a^* + \theta\sigma^\mu\overline{\theta}A_\mu \\ + \lambda\overline{\theta}\theta\theta + \overline{\theta}\,\overline{\theta}\theta\overline{\lambda} + D\theta\theta\overline{\theta}\,\overline{\theta}\ . \tag{I.38}$$

Any superfield of the form $\overline{\phi}\phi$ is a real superfield ($\overline{\phi} = \phi^\dagger$) and similarly any terms like $\phi + \overline{\phi}$ in a Lagrangian will be real. In particular ϕ could be a scalar superfield.

Super Gauge Transformations

In order to expose the physical degrees of freedom of the vector superfield, V, we will have to foreshadow our discussion of Lagrangian construction of the next section.

The kinetic term of a supersymmetric Lagrangian can be written as ($d^4\theta \equiv d^2\theta d^2\overline{\theta}$)

$$\mathcal{L} = \int d^4\theta\ \overline{\phi}(y^\dagger, \overline{\theta})\phi(y, \theta) \tag{I.39}$$

where

$$y^\mu = x^\mu + i\theta\sigma^\mu\overline{\theta} \qquad y^{\mu\dagger} = x^\mu - i\theta\sigma^\mu\overline{\theta} \tag{I.40}$$

recalling that

$$\overline{D}\phi = 0 \qquad D\overline{\phi} = 0\ . \tag{I.41}$$

This may seem like a strange kinetic term since it has no derivatives in it! (One might expect $\int d^4\theta\overline{D}\,\overline{\phi}D\phi$ or some such similar expression.) Remember that ϕ

and $\overline{\phi}$ are at *different points in superspace* $(x^\mu \pm i\theta\sigma^\mu\overline{\theta})$ and so we have a *difference* instead of a derivative.

Consider an Abelian theory for now. In a standard theory matter fields transform under the group rotation

$$\psi \to e^{\Lambda(x)}\psi \tag{I.42}$$

where we have absorbed the charge into the definition of $\Lambda(x)$. When $\Lambda(x) = \Lambda$, i.e. the rotation is rigid (global), the kinetic term $\mathcal{L}_{KIN} = \overline{\psi}\ \partial\!\!\!/\psi$ is invariant under (I.42). When $\Lambda(x)$ is spatially dependent

$$\overline{\psi}\ \partial\!\!\!/\psi \to \overline{\psi}e^{-\Lambda(x)}\ \partial\!\!\!/[e^{i\Lambda(x)}\psi] = \overline{\psi}\ \partial\!\!\!/\psi + i\overline{\psi}(\partial\!\!\!/\Lambda(x))\psi\ .$$

To compensate we make the standard "minimal substitution"

$$\partial^\mu \to D^\mu = \partial^\mu + ieA^\mu \tag{I.43}$$

where A^μ is a new field, called a gauge field, subject to the transformation

$$A^\mu \to A^\mu - \partial^\mu\Lambda(x) \tag{I.44}$$

when ψ is subject to (I.42). "D" is the "covariant derivative". The "covariant" Lagrangian (i.e. invariant under the gauge transformation (I.44)) is

$$\mathcal{L}_{KIN} = \overline{\psi}\ D\!\!\!\!/\psi = \overline{\psi}(\partial\!\!\!/ + ieA\!\!\!/)\psi$$

transforms as

$$\begin{aligned}\overline{\psi}\ D\!\!\!\!/\psi &\to \overline{\psi}e^{-\Lambda(x)}(\partial\!\!\!/ + ieA\!\!\!/ + ie\ \partial\!\!\!/\Lambda(x))\, e^{i\Lambda(x)}\psi \\ &= \overline{\psi}(\partial\!\!\!/ + ieA\!\!\!/)\psi \\ &= \overline{\psi}\ D\!\!\!\!/\psi\ .\end{aligned} \tag{I.45}$$

We desire a similar realization for a supersymmetric version of an Abelian gauge theory. Let us promote $\Lambda(x)$ to be a chiral (scalar) superfield

$$\overline{D}\Lambda(x,\theta,\overline{\theta}) = 0 \qquad D\overline{\Lambda}(x,\theta,\overline{\theta}) = 0 \ . \tag{I.46}$$

The matter field is also a scalar superfield, i.e. ψ is replaced by $\phi(x,\theta,\overline{\theta})$ and $\overline{\psi}$ by $\overline{\theta}(x,\theta,\overline{\theta})$. The kinetic term (I.39) becomes

$$\overline{\phi}\phi \to \overline{\phi}(y^{\dagger},\overline{\theta})\, e^{-i\overline{\Lambda}(y^{\dagger},\overline{\theta})} e^{i\Lambda(y,\theta)} \phi(y,\theta) \neq \overline{\phi}\phi \tag{I.47}$$

where we have used (I.30). We can amend this in the same fashion as before by introducing a (vector) gauge superfield:

$$\mathcal{L}_{KIN} \to \overline{\phi} e^{V} \phi \tag{I.48}$$

where

$$V \to V + i(\overline{\Lambda} - \Lambda) \tag{I.49}$$

and so

$$\begin{aligned} \overline{\phi} e^{V} \phi &\to \overline{\phi} e^{-i\overline{\Lambda}} e^{(i\overline{\Lambda} + V - i\Lambda)} e^{i\Lambda} \phi \\ &= \overline{\phi} e^{V} \phi \ . \end{aligned} \tag{I.50}$$

The non-Abelian case is similar except that

$$\phi \to e^{iT^a \Lambda_a} \qquad \overline{\phi} \to e^{-i\overline{\Lambda}_a T^{a\dagger}} \tag{I.51}$$

and the gauge transformation take the highly non-linear form

$$e^{V} \to e^{+i\overline{\Lambda}_a T^{a\dagger}} e^{V} e^{-iT^a \Lambda_a} \ . \tag{I.52}$$

Note that in such a transformation *we must maintain the constraint on the superfields*, that is chiral superfield remain chiral and vector (real) superfields remain so. Recalling from the section on chiral superfields that any function of chiral superfields is itself a chiral superfield (and similarly for anti-chiral fields) we see that $e^{i\Lambda(x)}$ is chiral and so is $\phi e^{i\Lambda(x)}$ so that the scalar nature is retained under this transformation. From (I.49), since $\overline{\Lambda} = \Lambda^{\dagger}$, we see that the vector nature of V is also retained.

Functions of chiral fields, ϕ_i with $\overline{D}\phi_i = 0$, are chiral because

$$\overline{D}F(\phi_i) = \frac{\partial F}{\partial \phi_i}\,\overline{D}\phi_i = 0 \tag{I.53}$$

and similarly $DF(\overline{\phi}_i) = 0$. *Products of chiral and anti-chiral superfields are, in general, neither chiral nor anti-chiral.*

Wess-Zumino Gauge

We are going to make use of our gauge freedom in the selection of a particular $\Lambda(y,\theta)$ to reduce the degrees of freedom (i.e. component fields) in V as much as possible. From (I.30) and (I.31)

$$\phi = \widetilde{\psi}(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)$$

$$y = x + i\theta\sigma\overline{\theta}$$

substituting and expanding about $x = y$ as a power series allows us to express ϕ in terms of x^{μ}:

$$\begin{aligned}\phi(x,\theta,\overline{\theta}) &= \widetilde{\psi}(x) + \sqrt{2}\theta\psi(x) + \theta^2 F(x) + i\theta\sigma^{\mu}\overline{\theta}\partial_{\mu}\widetilde{\psi}(x) \\ &\quad - \frac{i}{\sqrt{2}}\,\theta^2\partial_{\mu}\psi(x)\sigma^{\mu}\overline{\theta} + \frac{1}{4}\,\theta^2\overline{\theta}^2\partial_{\mu}\partial^{\mu}\widetilde{\psi}(x)\end{aligned} \tag{I.54}$$

with a similar expression for $\overline{\phi}$ (the $\widetilde{\psi}$ is a **bosonic** field [we formerly called it

$A(x)]$) whereas, from (I.38)

$$V(x,\theta,\overline{\theta}) = s(x) + \overline{\theta}\chi + \overline{\chi}\theta + \theta^2 a + \overline{\theta}^2 a^* + \theta\sigma^\mu\overline{\theta}A_\mu \\ + \lambda\overline{\theta}\theta\theta + \overline{\theta}\overline{\theta}\theta\overline{\lambda} + D\theta\theta\overline{\theta}\overline{\theta} \ . \tag{I.55}$$

Thus for the gauge transformation $V \to V + i(\overline{\Lambda} - \Lambda)$ we can select $\widetilde{\psi}_\Lambda + \overline{\widetilde{\psi}}_\Lambda$ (i.e. $Re\,\widetilde{\psi}_\Lambda$) to cancel $s(x)$; kill "a" using F_Λ and "a^*" using $F_\Lambda^\dagger$ (from $\overline{\Lambda}$); and cancel off χ with ψ_Λ (the "Λ" means "from expanding $\Lambda(x,\theta,\overline{\theta})$ as we did $\phi(x,\theta,\overline{\theta})$ in (I.54)"). Thus we are free to set $s, \chi, \overline{\chi}, a$ and a^* to zero in (I.55). This is akin to using the gauge $A_0 = 0$ in QED. We are left with the relatively simple set of component fields:

$$V(x,\theta,\overline{\theta}) = -\theta\sigma^\mu\overline{\theta}A_\mu(x) + i(\theta^2\overline{\theta}\,\overline{\lambda}(x) - \overline{\theta}^2\theta\lambda(x)) + \tfrac{1}{2}\,\theta^2\overline{\theta}^2 D(x) \ . \tag{I.56}$$

We have altered the coefficients to conform with the definitions used in Wess and Bagger. This cancellation can be seen from writing out the transformations of the components explicitly. For instance

$$s(x) \to s(x) + \widetilde{\psi}_\Lambda(x) + \overline{\widetilde{\psi}}_\Lambda(x)$$

which leads to our choice of setting $s(x)$ to zero.

This special choice of gauge is called the **Wess-Zumino Gauge.** We shall use it henceforth at all times. By working in a particular gauge our calculations will lose manifest supersymmetry invariance.

We have not exhausted our gauge freedom yet. Since $Re\,\widetilde{\psi}_\Lambda$ was used to cancel $s(x)$ we still have $Im\,\widetilde{\psi}_\Lambda$ to play with. This implements the standard $U(1)$

gauge transformations (this is all for the Abelian case):

$$A_\mu \to A_\mu + \partial_\mu (Im\, \widetilde{\psi}_\Lambda) \ . \tag{I.57}$$

We are left with a gauge vector boson A_μ, an associated Weyl gauge fermion (λ) and an auxiliary field which transforms into a total derivative (just as F did) under a supersymmetry transformation δ_ϵ.

Aside from eliminating so many non-physical fields, the Wess-Zumino gauge also possesses the useful property of being of quadratic or higher powers in $\theta, \overline{\theta}$ in all of its components. For this reason $V^3 = 0$ and any function of V terminates after only 3 terms. In particular

$$e^V = 1 + V + \frac{1}{2} V^2 \tag{I.58}$$

a welcome simplification in the kinetic term $\mathcal{L} = \overline{\phi} e^V \phi$.

I.2 ANATOMY OF A LAGRANGIAN

In the previous section we have established the machinery to construct supersymmetric Lagrangians.

Supersymmetry Invariance

In addition to all the invariance properties we normally expect from a Lagrangian (gauge, Lorentz) we additionally require that $\mathcal{L}$ be invariant under supersymmetry transformations. If $\mathcal{L}$ is a Lagrangian density then the full Lagrangian, L, is obtained by integrating $\mathcal{L}$ over all non-timelike coordinates

$$L = \int d^3x\, d^4\theta\;\mathcal{L}(x^\mu, \theta, \overline{\theta})\ . \tag{I.59}$$

Since $\int d\theta_i = 0$ and $\int \theta_i d\theta_i = 1$ we see that (I.59) has no anti-commuting character left ($L = L(t)$) and that (I.59) merely projects out the $\theta^2\overline{\theta}^2$ ($\equiv \theta^4$) component.

When we have only scalar fields, $\phi(y,\theta)$ and $\overline{\phi}(y^\dagger,\overline{\theta})$, as terms in a Lagrangian (and no terms involving both) we see that

$$L = \int d^3y \left[\int d^2\theta\, F(\phi) + \int d^2\overline{\theta}\, F^*(\overline{\phi})\right] \tag{I.60}$$

where (since $\overline{\theta} = \theta^\dagger$) L is now manifestly real and we have used the assumed translational property of the differentials $d^3x = d^3y$. For vector superfields we must use (I.59)

$$L = \int d^3x \int d^4\theta [F(V) + F^*(V)] \tag{I.61}$$

and no conjugation of V is needed since V is real.

Superpotentials

The term which corresponds to the potential in an ordinary field theory is the **superpotential**. Denoted $W[\phi_i]$ it is a function of the chiral (but not anti-chiral) superfields in the theory and is thus itself a chiral superfield. It appears in $\mathcal{L}$ as

$$\mathcal{L}_{POT} = \int W[\phi]\, d^2\theta + h.c. \tag{I.62}$$

where $h.c. = \int W^*[\overline{\theta}] d^2\overline{\theta}$ ensures a real Lagrangian. Such terms are responsible for masses and Yukawa couplings. This is known as the "F" sector since the highest (most θ's) component fields is of the "F" type (see (I.31)).

From (I.33) we have $[\phi] = m$ and $[d^2\theta] = m$ so that the most general *renormalizable* term of the type (I.62) is

$$W[\phi] = C + \lambda_i\phi_i + m_{ij}\phi_i\phi_j + g_{ijk}\phi_i\phi_j\phi_k \,. \tag{I.63}$$

We will ignore C which could only play a role in supergravity. Note that there is no quartic term so one might suppose that four boson couplings cannot occur. Since the "F" term always appears without a derivative in $\mathcal{L}$ (once it has been expanded into its component fields) we can trivially obtain its equation of motion via Lagrange's equations

$$\frac{\delta\mathcal{L}}{\delta F} = \partial_\mu \frac{\delta\mathcal{L}}{\delta(\partial F/\partial x^\mu)} = 0 \,. \tag{I.64}$$

When F is solved for and resubstituted into $\mathcal{L}$ we might obtain quartic scalar component field couplings (once the other terms in $\mathcal{L}$ have been added). Note that in (I.63) there may be many different scalar fields.

Why we need more Higgs bosons

The Higgs-scalar interactions which generate quark masses originate in the cubic terms. It is because we can only use ϕ, and not $\overline{\phi}$, that the supersymmetric standard model requires two Higgs doublets. In the standard model Lagrangian we have

$$\mathcal{L} = \ell^{ij} H \ell_{L_i} \overline{\ell}_{R_j} + d^{ij} H q_{L_i} \overline{d}_{R_j} + u^{ij} H^* q_{L_i} \overline{u}_{R_j} + h.c. \tag{I.65}$$

where ℓ^{ij} is diagonal and $\langle H \rangle = v$. The supersymmetric version would be

$$\mathcal{L} = \ell^{ij} \hat{\mathbf{H}} \hat{\mathbf{L}}_i \hat{\mathbf{R}}_j + d^{ij} \hat{\mathbf{H}} \hat{\mathbf{Q}}_i \hat{\mathbf{D}}_j + u^{ij} \hat{\mathbf{H}}^\dagger \hat{\mathbf{Q}}_i \hat{\mathbf{U}}_j \tag{I.66}$$

and since $\hat{\mathbf{H}}^\dagger = \overline{\hat{\mathbf{H}}}^{\mathbf{T}}$ cannot appear in W we must define a second $\hat{\mathbf{H}}$, $\hat{\mathbf{H}}_2$, with the electroweak quantum numbers of $\hat{\mathbf{H}}^\dagger$.

Components of the Superpotential

In terms of component fields ($\widetilde{\psi}$ scalar, ψ a left-handed Weyl spinor, F a bosonic auxiliary):

$$\phi_i(y,\theta) = \widetilde{\psi}_i(y) + \sqrt{2}\theta\psi_i(y) + \theta\theta F_i(y)$$

$$\phi_i\phi_j = \widetilde{\psi}_i(y)\widetilde{\psi}_i(y) + \sqrt{2}\theta[\psi_i\widetilde{\psi}_j + \widetilde{\psi}_i\psi_j] + \theta\theta[\widetilde{\psi}_i F_j + \widetilde{\psi}_j F_i - \psi_i\psi_j]$$

$$\begin{aligned} \phi_i\phi_j\phi_k = {} & \widetilde{\psi}_i\widetilde{\psi}_j\widetilde{\psi}_k + \sqrt{2}\theta[\psi_i\widetilde{\psi}_j\widetilde{\psi}_k + \widetilde{\psi}_i\psi_j\widetilde{\psi}_k + \widetilde{\psi}_i\widetilde{\psi}_j\psi_k] \\ & + \theta\theta[F_i\widetilde{\psi}_j\widetilde{\psi}_k + \widetilde{\psi}_i F_j\widetilde{\psi}_k + \widetilde{\psi}_i\widetilde{\psi}_j F_k - \psi_i\psi_j\widetilde{\psi}_k - \psi_j\psi_k\widetilde{\psi}_i - \psi_k\psi_i\widetilde{\psi}_j] \end{aligned} \tag{I.67}$$

obtained by simple multiplication ignoring θ^3 or higher terms. When we take $\int d^2\theta$ of these we project out the $\theta\theta$ ("F") term. The $\sqrt{2}$ is in keeping with Wess and Bagger notation. For

$$W[\phi_i] = \lambda_i\phi_i + m_{ij}\phi_i\phi_j + g_{ijk}\phi_i\phi_j\phi_k$$

$$\begin{aligned}\mathcal{L}_{POT} &= \int W[\phi_i]d^2\theta = \lambda_i\phi_i|_{\theta\theta\ term} + m_{ij}\phi_i\phi_j|_{\theta\theta\ term} + \phi_i\phi_j\phi_k|_{\theta\theta\ term} \\ &= \lambda_i F_i + m_{ij}[\widetilde{\psi}_i F_j + \widetilde{\psi}_j F_i - \psi_i\psi_j] \\ &+ g_{ijk}[F_i\widetilde{\psi}_j\widetilde{\psi}_k + \widetilde{\psi}_i F_j\widetilde{\psi}_k + \widetilde{\psi}_i\widetilde{\psi}_j F_k - \psi_i\psi_j\widetilde{\psi}_k - \psi_j\psi_k\widetilde{\psi}_i - \psi_k\psi_i\widetilde{\psi}_j] \,.\end{aligned} \tag{I.68}$$

If m_{ij} and g_{ijk} are symmetric then we can write this as

$$\mathcal{L} = \lambda_i F_i + \tfrac{1}{2}m_{ij}\{\widetilde{\psi}_i F_j - \psi_i\psi_j\} + \frac{1}{3}\, g_{ijk}\{F_i\widetilde{\psi}_j\widetilde{\psi}_k - \psi_i\psi_j\widetilde{\psi}_k\} \,. \tag{I.69}$$

Note that, for the term $W[\phi] = \phi_i\phi_j\ldots\phi_N$, that the θ^2 (i.e. "F") coefficient will be

$$\begin{aligned}&\widetilde{\psi}_1\widetilde{\psi}_2\cdots\widetilde{\psi}_{N-1}F_N + \widetilde{\psi}_1\widetilde{\psi}_2\cdots\widetilde{\psi}_{N-2}F_{N-1}\widetilde{\psi}_N + \cdots + F_1\widetilde{\psi}_2\widetilde{\psi}_3\cdots\widetilde{\psi}_N \\ &\qquad +\psi_1\psi_2\widetilde{\psi}_3\widetilde{\psi}_4\widetilde{\psi}_5\cdots\widetilde{\psi}_N + \psi_1\widetilde{\psi}_2\psi_3\widetilde{\psi}_4\widetilde{\psi}_5\cdots\widetilde{\psi}_N \\ &\qquad + \cdots + \widetilde{\psi}_1\widetilde{\psi}_2\widetilde{\psi}_3\cdots\widetilde{\psi}_{N-2}\psi_{N-1}\psi_N\end{aligned} \tag{I.70}$$

which can be written as

$$\left.\frac{\partial W[\phi]}{\partial\phi_i}\right|_{\phi_k=\widetilde{\psi}_k} F_i + \left.\frac{\partial^2 W[\phi]}{\partial\phi_i\partial\phi_j}\right|_{\phi_k=\widetilde{\psi}_k} \psi_i\psi_j \,. \tag{I.71}$$

In particular for $\phi_i = \phi_j\ \forall i,j$, i.e. $W[\phi] = \phi^N$ we have θ^2 coefficient

$$\begin{aligned}&N\widetilde{\psi}^{N-1}F + N(N-1)\,\widetilde{\psi}^{N-1}\psi^2 \\ &= W'F + W''\psi\psi\end{aligned} \tag{I.72}$$

evaluated at $\phi = \widetilde{\psi}$.

A more convenient notation might be

$$\mathcal{L}_{POT}(W) = W_{,i}(\widetilde{\psi})F_i + W_{,ij}(\widetilde{\psi})\psi_i\psi_j \tag{I.73}$$

where $W_{,i}(\widetilde{\psi})$ means $\partial W[\phi_1, \ldots, \phi_N]/\partial\phi_i$ evaluated at $\phi_1 = \widetilde{\psi}_1, \ldots, \phi_N = \widetilde{\psi}_k$.

Solving for F

We may expand the kinetic term in (I.40) by using (I.54) and its conjugate. We find that the θ^4 coefficient of $\overline{\phi}_i(x,\theta,\overline{\theta})\phi_j(x,\theta,\overline{\theta})$ is

$$\begin{aligned}\overline{\phi}_i(x,\theta,\overline{\theta})\phi_j(x,\theta,\overline{\theta})\Big|_{\theta\theta\overline{\theta}\overline{\theta}} &= F_i^* F_j + \frac{1}{4}\left[\widetilde{\psi}_i^*\Box\widetilde{\psi}_j + \widetilde{\psi}_j\Box\widetilde{\psi}_i^*\right] \\ &- \tfrac{1}{2}(\partial_\mu\widetilde{\psi}_i^*)(\partial^\mu\widetilde{\psi}_j) + \frac{i}{2}\left[(\partial_\mu\overline{\psi})\overline{\sigma}^\mu\psi_j - \overline{\psi}_i\overline{\sigma}^\mu\partial_\mu\psi_j\right]\end{aligned} \tag{I.74}$$

writing (thus far)

$$\mathcal{L} = \mathcal{L}_{KIN} + \mathcal{L}_{POT} = \int \overline{\phi}_i\phi_i d^4\theta + \int W[\phi_i]d^2\theta + \int W^*[\overline{\phi}_i]d^2\overline{\theta} \tag{I.75}$$

and integrating $L = \int d^3x\mathcal{L}$ by parts to convert the $(\partial_\mu\widetilde{\psi}_i^*)(\partial^\mu\widetilde{\psi}_i)$ term in (I.74) to the form $\widetilde{\psi}_i^*\Box\,\widetilde{\psi}_i$, we obtain (using (I.69))

$$\begin{aligned}\mathcal{L} &= i(\partial_\mu\overline{\psi}_i)\overline{\sigma}^\mu\psi_i + \widetilde{\psi}_i^*\Box\,\widetilde{\psi}_i + F_i^* F_i \\ &+ \left[m_{ij}(\widetilde{\psi}_i F_j - \tfrac{1}{2}\psi_i\psi_j) + g_{ijk}(\widetilde{\psi}_i\widetilde{\psi}_j F_k - \psi_i\psi_j\widetilde{\psi}_k) + \lambda_i F_i + h.c.\right].\end{aligned} \tag{I.76}$$

By using (I.64) (or its conjugate) for the equation of motion of F

$$\frac{\partial\mathcal{L}}{\partial F_k} = F_k^* + \lambda_k + m_{ik}\widetilde{\psi}_i + g_{ijk}\widetilde{\psi}_i\widetilde{\psi}_j = 0 \tag{I.77}$$

or

$$F_k^* = -(\lambda_k + m_{ik}\widetilde{\psi}_i + g_{ijk}\widetilde{\psi}_i\widetilde{\psi}_j) \ . \tag{I.78}$$

Now examining (I.63) (with $C = 0$) we see that

$$F_k^* = - \left. \frac{\partial W[\phi]}{\partial \phi_k} \right|_{\phi_k = \widetilde{\psi}_k} \ . \tag{I.79}$$

Consequently from (I.71)-(I.73)

$$\begin{aligned} \mathcal{L}_{POT} &= -\sum_i \left| W_{,i}(\widetilde{\psi}) \right|^2 + \sum_{ij} W_{,ij}(\widetilde{\psi}) + h.c. \\ &= -\sum_i |F_i|^2 + \sum_{ij} W_{,ij}(\widetilde{\psi}) + h.c. \end{aligned} \tag{I.80}$$

By using (I.78) in (I.76) we can eliminate F and F^* and place $\mathcal{L}$ in the form

$$\begin{aligned} \mathcal{L} &= i\partial_\mu \psi_i^* \overline{\sigma}^\mu \psi_i + \widetilde{\psi}_i^* \Box \, \widetilde{\psi}_i \\ &\quad - (\tfrac{1}{2} m_{ik}\psi_i\psi_k + g_{ijk}\psi_i\psi_j\widetilde{\psi}_k + h.c.) - \mathcal{V}(\widetilde{\psi}, \widetilde{\psi}^*) \end{aligned} \tag{I.81}$$

where the potential $\mathcal{V}$ is

$$\mathcal{V} = F_i^* F_i \ . \tag{I.82}$$

From (I.78) we see that $\mathcal{V}$ will provide a mass term for $\widetilde{\psi}_i$ (a scalar) of

$$\begin{aligned} m_{\widetilde{\psi}} &= \sum_k m_{ik} m_{ik}^* \widetilde{\psi}_i^* \widetilde{\psi}_i + \left(\sum_{jk} g_{ijk} \lambda_k^* \widetilde{\psi}_i \widetilde{\psi}_j + h.c. \right) \\ m_{\widetilde{\psi}} &= \widetilde{\psi}^\dagger m m^\dagger \widetilde{\psi} + \left(\widetilde{\psi}^T [g_k \lambda_k^*] \widetilde{\psi} + h.c. \right) \end{aligned} \tag{I.83}$$

where the second line was written in matrix form with $(\widetilde{\psi}_i)$ a column vector, $\widetilde{\psi}^T = (\widetilde{\psi}_1 \ \widetilde{\psi}_2 \ldots \widetilde{\psi}_n)$ and m, g_k matrices.

Note that since $Da = \overline{D}a = 0$, where a is a constant, that $\overline{D}(\phi + a) = 0$ is still a scalar superfield. Thus shifting $\phi \to \phi + a$, $\overline{\phi} \to \overline{\phi} + a^*$ in a Lagrangian yields another perfectly valid Lagrangian.[1] For this reason it is often possible to eliminate the $\lambda_i \phi_i$ term from $W[\phi_i]$. If that is done then (I.83) becomes

$$m_{\widetilde{\psi}} = \widetilde{\psi}^\dagger |m|^2 \widetilde{\psi} \qquad (\lambda_i = 0) \tag{I.84}$$

and from (I.81)

$$m_\psi = \tfrac{1}{2}\psi^T m \psi + h.c. \tag{I.85}$$

and we see an explicit symmetry between the fermion (ψ, ψ^*) and boson $(\widetilde{\psi}, \widetilde{\psi}^*)$ masses. In particular when there is only one scalar field, ϕ, present with components $\widetilde{\psi}$ and ψ

$$-\mathcal{L}_{mass} = m^2 \widetilde{\psi}^* \widetilde{\psi} + \tfrac{1}{2} m\psi\psi + \tfrac{1}{2} m\psi^*\psi^* \ . \tag{I.86}$$

When there are multiple ϕ_i, (I.84) and (I.85) lead to non-diagonal mass terms in $\mathcal{L}$. Both may be simultaneously diagonalized by an orthogonal rotation $\psi_i \to O_{ij}\psi_j$; $\widetilde{\psi}_i \to O_{ij}\widetilde{\psi}_j$, *i.e.* the superfields may be mixed $\phi_i \to O_{ij}\phi_j$ with

$M_D = O^T m O$. Note that the fermion mass terms are Majorana. Additional Dirac mass terms can be obtained via the Higgs mechanism in the usual way or via supersymmetry-breaking .

<u>R-Parity</u>

R-parity is an additive quantum number that we frequently impose to further restrict our class of theories. It may formerly be written as a global $U(1)$

symmetry of $\mathcal{L}$ (see references 1, 3 and 39.) It may also be written as (in lesser generality)[14]

$$R = (-1)^{2J+3B+L} \tag{I.87}$$

where J, B, and L are the spin, baryon number and lepton number of a particle. It is also true that

$$\begin{aligned} R &= -1 \quad \text{for superpartners} \\ R &= +1 \quad \text{for ordinary matter.} \end{aligned} \tag{I.88}$$

In a renormalizable Lagrangian, invariance under R may be broken by the terms[40]

$$\phi_i \quad \text{or} \quad \phi_i \phi_j \quad (i \neq j) \; . \tag{I.89}$$

This is not surprising as $\mathcal{L} = \int \lambda \phi_i d^2\theta$ contains a term which would require that a single supersymmetric state be created. Conservation of R-parity implies that supersymmetric states must be created in pairs and consequently the lightest supersymmetric particle is stable. For most scalar superfields the spinor term (ψ) is mundane matter and the scalar ($\widetilde{\psi}$) is the superpartner. The sole exception is the Higgs superfield where scalar component field is the normal Higgs boson H (i.e. $\widetilde{\psi} \equiv H$) and whose fermionic partner has odd R-parity ($\psi \equiv \widetilde{H}$). This explains how a term such as $g_{ijk}\psi_i\psi_j\widetilde{\psi}_k$ can be invariant under R-parity. If $\phi_k = \hat{\mathbf{H}}_k$, so that $\widetilde{\psi}_k = H_k$, and $\psi_i = Q_i = \binom{u}{d}_i$ while $\psi_j = \overline{u}_j$ or $\overline{d}_j$ we have a normal Yukawa-type Higgs term where each field has $R = +1$. If only a single scalar field of the ϕ_i is present in the theory then a term like $g\phi_{\overline{u}}^3$ *would* violate R-parity (as well as lepton number and charge).

Comments on the Auxiliary Fields

We have seen that the component field of highest dimensionality (F for scalar superfields; D for vector superfields) is an *auxiliary* field, that is appears with no derivative couplings in $\mathcal{L}$. Such fields are non-dynamic (i.e. do not propagate) and, via their equations of motion ($\partial\mathcal{L}/\partial F = 0$), provide a constraint on the other fields. The question arises as to who we bother including such fields at all. (This is somewhat akin to asking why we bother including a sigma field in the linear sigma model when we are going to solve for it in producing the non-linear sigma model).

We have seen that F is extremely convenient in characterizing the potential energy term ($\mathcal{V}$) when a scalar superpotential is added. A more fundamental reason lies in the fact that an unbroken supersymmetric theory must have an equal number of fermionic and bosonic degrees of freedom. Indeed the supersymmetry algebra can be derived from this statement[18] (plus a few assumptions and requirements such as closure). Looking back at Eqs. (I.32), (I.34) and (I.35) we may ask why, given the superfield components A (a scalar, called $\widetilde{\psi}$ elsewhere) and ψ (a left-handed Weyl spinor), we could not simply close the algebra with $Q\psi = 0$? (i.e. make θ and $\overline{\theta}$ one-component objects so that $\phi(y,\theta) = A(y) + \sqrt{2}\theta\psi(y)$ and $Q, \overline{Q}$ would also be one-component anti-commuting operators with $Q^2 = \overline{Q}^2 = 0$.)

Indeed in supersymmetric ($N = 1$) quantum mechanics this can be done (corresponding to an "$N = \frac{1}{2}$" field theory) but not in a complete field theory. Consider a free field theory of massless particles. On shell, with A and ψ obeying their equations of motion ($\Box A = 0$; $\sigma^\mu\partial_\mu\psi = 0$), ψ has one degree of freedom (one helicity state), as does A, and all is right with the universe. In a field theory A and ψ must be able to propagate off-shell. Then ψ would have *two* degrees of freedom (just as if it were massive) but A still only has one. The solution is to introduce an additional bosonic degree of freedom, F, off-shell which has no on-shell degree of freedom. Such a state (remember there is no superpotential term W in this free theory) must have the equation of motion $F = 0$. To obtain this from $\delta\mathcal{L}/\delta F = (\partial/\partial_\mu)\ (\delta\mathcal{L}/\delta(\partial^\mu F))$ requires that $\partial_\mu F$ not appear and that F^n appear with $n > 1$. Assuming only terms of quadratic order or less in the field for our free Lagrangian we see that F appears only in the term $\frac{1}{2}F^2$ (and so cannot propagate *or* interact.) By dimensional counting we see that $[F] = m^2$. The invariant action now assumes the form

$$S = \int d^4x\ \left\{ -\tfrac{1}{2}(\partial_\mu A)^2 - \tfrac{1}{2}\psi^\dagger\sigma^\mu\partial_\mu\psi + \tfrac{1}{2}F^2 \right\} .$$

Another way of looking at this is that without F (I.37) would become (dropping the i and $\sqrt{2}$):

$$\delta_\epsilon A = \epsilon\psi \qquad \delta_\epsilon\psi = \sigma^\mu\bar{\epsilon}\partial_\mu A$$

then

$$\begin{aligned} [\delta_{\epsilon_1}, \delta_{\epsilon_2}]\ A &= 2\epsilon_2\sigma^\mu\epsilon_1\partial_\mu A \\ [\delta_{\epsilon_1}, \delta_{\epsilon_2}]\ \psi &= 2\epsilon_2\sigma^\mu\epsilon_1\partial_\mu\psi - \epsilon_2\sigma^\nu\bar{\epsilon}_1\bar{\sigma}_\nu\sigma^\mu\partial_\mu\psi \end{aligned} \tag{I.90}$$

and, *on-shell*, $\sigma^\mu\partial_\mu\psi = 0$ so that we have a closed complete super-algebra. Off shell we need F with

$$\delta_\epsilon\psi = \sigma^\mu\bar{\epsilon}\partial_\mu A + F \qquad \delta_\epsilon F = \epsilon\sigma^\mu\partial_\mu\psi$$

which will cancel the $\sigma^\mu\partial_\mu\psi$ term in (I.90).

The same arguments may be applied to vector superfields. In a free, massless theory we have a gauge boson with two degrees of freedom (transverse polarizations) and a Majorana gauge fermion with two degrees of freedom. Off shell the vector boson gains a longitudinal polarization and the fermion has four degrees of freedom, so, again, another bosonic degree of freedom is required. Thus the D-field term $\frac{1}{2}D^2$ is born. D, as F, transforms into a total derivative of the fermion field λ.

$$\delta_\epsilon D = \bar{\epsilon}\,\bar{\sigma}^\mu \partial_\mu \lambda \qquad\qquad \delta_{\bar{\epsilon}} \overline{D} = \epsilon \sigma^\mu \partial_\mu \overline{\lambda}\ .$$

Super Field Strength

Moving on to the gauge sector we desire to find an analogue of the field strength tensor, $F_{\mu\nu}$, of a standard field theory. We shall again start with Abelian theories.

To motivate our construction we consider that, in a standard field theory

$$F_{\mu\nu} = -i[\mathcal{D}_\mu, \mathcal{D}_\nu] = \partial_\mu A_\nu - \partial_\mu A_\mu + i[A_\mu, A_\nu] \tag{I.91}$$

where the final term is absent in the Abelian case under discussion. In a supersymmetric theory we might try to construct a field strength out of the real vector gauge superfield, V, and covariant (super-)derivatives D and $\overline{D}$. Under a gauge transformation $F_{\mu\nu} \to F_{\mu\nu}$ because it is constructed from covariant (gauge) derivatives. This new object must be invariant under a supergauge transformation as found in (I.49): $V \to V + i(\overline{\Lambda} - \Lambda)$. Writing the field strength as $\widehat{O}V$, a differential operator $\widehat{O}$ acting on V, we demand

$$\widehat{O}V \to \widehat{O}V \quad \text{or} \quad \widehat{O}(\overline{\Lambda} - \Lambda) = 0 \tag{I.92}$$

under the $U(1)$ gauge transformation where we recall that Λ and $\overline{\Lambda}$ are scalar superfields ($D\overline{\Lambda} = 0$; $\overline{D}\Lambda = 0$). Choosing $\widehat{O} = D$ or $\widehat{O} = \overline{D}$ will not work because $D\Lambda \neq 0$ and $\overline{D\Lambda} \neq 0$ (and $D\Lambda \neq \overline{D\Lambda}$). Trying $\widehat{O} = D\overline{D}$ we note that $D\overline{D\Lambda} = -\overline{D}D\overline{\Lambda} + \{D, \overline{D}\}\overline{\Lambda} = \{D, \overline{D}\}\overline{\Lambda}$. Now

$$\{D_\alpha, \overline{D}_{\dot\beta}\} = -2i\sigma^\mu_{\alpha\dot\beta}\partial_\mu \tag{I.93}$$

and

$$[\partial_\mu, D_\alpha] = [\partial_\mu, \overline{D}_{\dot\beta}] = 0 \ . \tag{I.94}$$

Therefore $D\overline{D\Lambda} = -2i\sigma^\mu\partial_\mu\overline{\Lambda} \neq 0$. However, trying the next order

$$\widehat{O} = D^2\overline{D} \tag{I.95}$$

does work

$$\widehat{O}\Lambda = D^2\overline{D}\Lambda = 0 \quad \text{since} \quad \overline{D}\Lambda = 0$$

$$\begin{aligned}
\widehat{O}\overline{\Lambda} = D^2\overline{D\Lambda} &= -D\overline{D}D\overline{\Lambda} + D\{D, \overline{D}\}\overline{\Lambda} \\
&= -2iD\sigma^\mu\partial_\mu\overline{\Lambda} \quad \text{by (I.93)} \\
&= -2i\sigma^\mu\partial_\mu D\overline{\Lambda} \quad \text{by (I.94)} \\
&= 0 \ .
\end{aligned}$$

In the literature $\widehat{O}V$ is called $\overline{W}_{\dot\alpha}$ which is unfortunate since the superpotential is also called W. Its conjugate, W_α, also satisfies the above criteria and is a field strength

$$\overline{W}_{\dot\alpha} = -\frac{1}{4}\, DD\overline{D}_{\dot\alpha}V \qquad W_\alpha = \frac{1}{4}\, \overline{DD}D_\alpha V \ . \tag{I.96}$$

We note some immediately obvious properties of W_α.

1. Since D and $\overline{D}$ are anti-commuting, W_α is an anti-commuting object like θ_α and $\overline{W}_{\dot{\alpha}}$ is like $\overline{\theta}_{\dot{\alpha}}$.
2. Since $\overline{D}^3 = 0$, $\overline{D}W_\alpha = 0$ and W_α is a chiral superfield.
3. From part 2 we can write $W_\alpha = W_\alpha(y,\theta)$ (just as for any chiral field) and $\overline{W}_{\dot{\alpha}} = \overline{W}_{\dot{\alpha}}(y^\dagger,\overline{\theta})$. We can then use the explicit forms of D, $\overline{D}$ and V (in Wess-Zumino gauge, of course!) to expand W_α into its component fields

$$W_\alpha = -i\lambda_\alpha(x) + [D(y)\theta_\alpha - (\sigma^{\mu\nu})_\alpha{}^\beta\theta_\beta F_{\mu\nu}] + \theta^2(\sigma^\mu)_{\alpha\dot{\alpha}}\partial_\mu\overline{\lambda}^{\dot{\alpha}}(y) \ . \quad \text{(I.97)}$$

Note that (on-shell) the final term would vanish by the Dirac equation leaving a gauge fermion, auxiliary field (D) and $\sigma^{\mu\nu}F_{\mu\nu}$ term. Since W_α is fermion it cannot appear alone in $\mathcal{L}$ but must appear in a combination such as $\theta^\alpha W_\alpha$ or $W^\alpha W_\alpha$. Also note that the further relation $D^\alpha W_\alpha = \overline{D}_{\dot{\alpha}}\overline{W}^{\dot{\alpha}}$ is satisfied. Taking $\mathcal{L} = \int \theta^\alpha W_\alpha d^2\theta$ would yield (since this projects out the θ^2 term) a convective $\sigma^{\mu\nu}F_{\mu\nu}$ term in $\mathcal{L}$ which we don't want is a QED-like theory. We desire a term akin to $F_{\mu\nu}F^{\mu\nu}$ so we will try $W^\alpha W_\alpha = W_\alpha\epsilon^{\alpha\beta}W_\beta$.

$$\int W^\alpha W_\alpha d^2\theta = W^\alpha W_\alpha|_{\theta\theta} = -2i\lambda\sigma^\mu\partial_\mu\overline{\lambda} - \tfrac{1}{2}\,F^{\mu\nu}F_{\mu\nu} + D^2 + \frac{i}{4}\,F^{\mu\nu}\widetilde{F}_{\mu\nu} \quad \text{(I.98)}$$

where $\widetilde{F}_{\mu\nu} = F^{\alpha\beta}\epsilon_{\alpha\beta\mu\nu}$. This final term is known to be the source of much pain in anomalous theories. In this paper we will always be concerned with topologically trivial solutions. Then, after some integration by parts, (and discarding the, assumed trivial, surface terms) we have the real supersymmetric version of the Lagrangian for a free vector field

$$\mathcal{L}_{Gauge} = \int W^\alpha W_\alpha d^2\theta + \int \overline{W}_{\dot{\alpha}}\overline{W}^{\dot{\alpha}}d^2\overline{\theta} = \tfrac{1}{2}\,D^2 - \frac{1}{4}\,F^{\mu\nu}F_{\mu\nu} - i\lambda\sigma^\mu\partial_\mu\overline{\lambda} \ . \quad \text{(I.99)}$$

(The $F^{\mu\nu}F_{\mu\nu}$ is like $E^2 - B^2$ and $F^{\mu\nu}\widetilde{F}_{\mu\nu}$ is like $E \cdot B$). In the non-Abelian case, just as we had the gauge transformation change from $V \to V - i(\Lambda - \Lambda)$ to $e^V \to e^{-i\overline{\Lambda}} e^V e^{i\Lambda}$, the superfield strength becomes

$$W_\alpha = -\frac{1}{4}\,\overline{D}^2 e^{-V} D_\alpha e^V \ . \tag{I.100}$$

Constructing Supersymmetric Lagrangia

We now have all of the ingredients necessary for constructing a realistic Lagrangian which is supersymmetrically invariant:

(1) The "Kinetic Term" $\int \overline{\phi} e^{gV} \phi d^4\theta$

This contains kinetic terms for the component fields of ϕ and the matter-gauge coupling terms along with all of the corresponding supersymmetric vertices. In the W-Z gauge these terms conserve R-parity automatically and all vertices may (qualitatively) be obtained by replacing fields from standard vertices in pairs by their supersymmetry counterparts, except when this would result in a new vertex of dimension greater than four. For example, using (the $\sqrt{2}$ is Wess and Bagger notation)

$$\begin{aligned}
\phi(y) &= \widetilde{\psi}(y) + \sqrt{2}\,\theta\psi(y) + \theta^2 F(y) \\
y^\mu &= x^\mu + i\theta\sigma^\mu\overline{\theta} \\
\phi(x) &= [\widetilde{\psi}(x) + i\theta\sigma^\mu\overline{\theta}\partial_\mu A(x) + \tfrac{1}{2}\,\theta^2\overline{\theta}^2\Box A(x)] \\
&\quad + [\sqrt{2}\theta\psi(x) - \frac{i}{\sqrt{2}}\,\theta^2\partial_\mu\psi(x)\sigma^\mu\overline{\theta}] + \theta^2 F(x) \qquad \text{(I.101)} \\
V &= -\theta\sigma^\mu A_\mu - i\overline{\theta}^2\theta\lambda + h.c. + \tfrac{1}{2}\,\theta^2\overline{\theta}^2 D \\
e^{gV} &= 1 + gV + \tfrac{1}{2}\,g^2V^2
\end{aligned}$$

yields (remembering that $d^4\theta$ project the $\theta^2\overline{\theta}^2$ term out of the expression)

$$\begin{aligned}\mathcal{L}_{KIN} = & F^*F + \widetilde{\psi}\Box\widetilde{\psi}^* + i\psi^\dagger\overline{\sigma}^\mu\partial_\mu\psi \\ & + \frac{g}{2}\left(\psi^\dagger\overline{\sigma}^\mu A_\mu\psi + i\widetilde{\psi}^* A_\mu\partial^\mu\widetilde{\psi} - i\widetilde{\psi}A_\mu\partial^\mu\widetilde{\psi}^* - i\sqrt{2}[\widetilde{\psi}\lambda^\dagger\psi^\dagger - \widetilde{\psi}^*\lambda\psi]\right) \\ & + \tfrac{1}{2}\,\widetilde{\psi}^*\widetilde{\psi}(gD - \tfrac{1}{2}\,g^2A_\mu A^\mu)\end{aligned} \tag{I.102}$$

where this is for a simple scalar matter superfield and a single Abelian vector superfield. This contains the $\frac{ig}{\sqrt{2}}\widetilde{\psi}^*\psi\lambda$ "electron-selectron-photino" type of term, the $\frac{ig}{2}\widetilde{\psi}^*\not{A}\widetilde{\psi}$ derivative scalar electrodynamics term (for selections) and the final term, which is a "seagull" term for the selections. There is no order five $\psi^*A_\mu A^\mu\psi$ term unless it is buried in $(g/2)\widetilde{\psi}^*\widetilde{\psi}D$. Normally the only other term with a "D" in it will be $\frac{1}{2}D^2$ coming from the field-strength term below. Thus, solving for the auxiliary field D from its equations of motion, will lead to a $|\widetilde{\psi}^*\widetilde{\psi}|^2$ term (i.e. a four-selection vertex) whose coefficient is proportional to g^2. (In general this will appear as $|\widetilde{\psi}^*T^a\widetilde{\psi}|^2$ where T^a is the group generator.) This term is instrumental in initiating spontaneous supersymmetry breaking (SSB) (see point four below). Note that if multiple fields are involved in $\mathcal{L}$ that mixing terms will naturally result.

(2) The Superpotential term or "F-sector"

$$\int W[\phi]d^2\theta + h.c. = \int(\lambda_i\phi_i + m_{ij}\phi_i\phi_j + g_{ij}\phi_i\phi_j\phi_k)\delta^2\theta + h.c.$$

This term encompasses the mass terms, Yukawa couplings and potential energy term for the matter fields,♮1 plus the various supersymmetric variations.

♮1 These arise in part from FF^* in the "Kinetic Term" but not until a superpotential is added.

R-parity must be imposed. The Higgs structure of a theory is found in this sector. Recall from (I.67) and (I.69) that the individual terms are

$$\int \lambda_i \phi_i d^2\theta = \lambda_i F_i(y) \text{ (often suppressed by } R\text{-parity)}$$

$$\int m_{ij}\phi_i\phi_j d^2\theta = 2m_{ij}\widetilde{\psi}_i F_j - m_{ij}(-)^p\psi_i\psi_j \tag{I.103}$$

$$\int g_{ijk}\phi_i\phi_j\phi_k d^2\theta = 3g_{ijk}\left[F_i\widetilde{\psi}_j\widetilde{\psi}_k - (-)^p\psi_i\psi_j\widetilde{\psi}_k\right]$$

where the $(-)^p$ is to remind us that ψ is anti-commuting and so $\psi_1\psi_2$ and $\psi_2\psi_1$ enter with opposite signs (positive for $\psi_1\psi_2$) and we have assumed that m_{ij} and g_{ijk} are symmetric ($\psi_i\psi_j\widetilde{\psi}_k$ enter with positive sign for ijk cyclic permutations of 123)

We found that this could be written as

$$\mathcal{L} = W'F + W''\psi\psi \text{ (evaluated at } \phi = \widetilde{\psi}) + h.c.$$

$$= \left.\frac{\partial W[\phi]}{\partial \phi_i}\right|_{\phi_k=\widetilde{\psi}_k} F_i + \left.\frac{\partial^2 W[\phi]}{\partial\phi_i\partial\phi_j}\right|_{\phi_k=\widetilde{\psi}_k}\psi_i\psi_j + h.c.$$

and when the kinetic term was added yields

$$\mathcal{L} = \text{Kinetic terms} + \text{gauge-matter couplings} - \left[\tfrac{1}{2}m_{ij}\psi_i\psi_j + g_{ijk}\psi_i\psi_j\widetilde{\psi}_k + h.c.\right]$$
$$- \mathcal{V}_{P.E.}(\widetilde{\psi}, \widetilde{\psi}^*) \tag{I.104}$$

where $\mathcal{V} = F^*F$ is a potential energy term and, since the equation of motion for F is $F = -W^*$, $\mathcal{V} = |W'|^2$.

(3) The Super Field Strength term or "D-sector" $\frac{1}{4}\int W^\alpha W_\alpha d^2\theta + h.c.$

This term includes the free gauge field sector and its supersymmetric counterparts. We find the vector and gaugino kinetic terms and the analogue of the field strength $F^a_{\mu\nu}F^{\mu\nu}_a$ which, in the non-Abelian case, includes all of the trilinear and quadrilinear gauge self-interactions (and gaugino in the trilinear case). Finally we have an auxiliary D_i field for each vector super-field V_i present

$$\mathcal{L} = -i\lambda_i\sigma^\mu\partial_\mu\overline{\lambda}_i - \frac{1}{4}\,F^i_{\mu\nu}F^{\mu\nu}_i + \tfrac{1}{2}\,D^2_i\ . \tag{I.105}$$

(4) Fayet-Iliopoulos Progenitor $\int V d^4\theta$

The only term which has not jet been discussed is a function of the vector superfields, V_i, just as $W[\phi_i]$ is a function of the scalar superfields (there is also a gravity superfield but we will not concern ourselves with it in this paper).

We know that, in the W-Z gauge, the terms V and V^2 arise. The term $m\int V^2 d^4\theta$ would introduce vector and gaugino mass terms. Just as such terms are not gauge invariant in ordinary gauge theories, they are not supergauge-invariant here. Under (the Abelian case) $V \to V - i(\overline{\Lambda} - \Lambda)$ the term $\int V^2 d^4\theta$ changes by

$$\int V^2 d^4\theta \to \int V^2 d^4\theta + 2\int(\overline{\Lambda}\Lambda)d^4\theta \tag{I.106}$$

and may be excluded.

The linear term, $c\int V d^4\theta$, *is* gauge invariant, however. The reason is that $\int d^4\theta$ is, like $d^4/d\theta^4$, a projection operator for the $\theta^2\overline{\theta}^2$ term. Therefore

$$\begin{aligned}\int V(x,\theta,\overline{\theta})\,d^2\theta d^2\overline{\theta} &\to \int V(x,\theta,\overline{\theta})d^2\theta d^2\overline{\theta} + i[\overline{\Lambda}(y,\overline{\theta}) - \Lambda(y,\theta)]\,d^2\theta\,d^2\overline{\theta}\\ &= \int V(x,\theta,\overline{\theta})d^2\theta\,d^2\overline{\theta}\ .\end{aligned} \tag{I.107}$$

So this term cannot be so easily excluded. If V is not a $U(1)$ gauge superfield (i.e. mediates a non-Abelian force), $V \to V_{ij} = T^a_{ij}V^a$, then V carries quantum numbers and $\int V d^4\theta$ would explicitly break the ordinary gauge symmetry and would also be excluded. We are left with the sole possible term

$$\mathcal{L}_{F.I.} = \int c_i V_i d^4\theta = c_i D_i \tag{I.108}$$

where V_i are $U(1)$ vector superfields and the c_i are constants. The $c_i D_i$ are called "Fayet-Iliopoulos" terms. Their sole function in life is to alter the auxiliary D field constraint by a constant and thereby initiate spontaneous supersymmetry-breaking. How this is accomplished will be outlined in the following subsection. Since the theory we will be dealing with will be broken by explicit terms we will assume that no F-I type terms arise. This term (for obvious reasons) is frequently included in the "D-sector" along with the superfield strength term.

Spontaneous Supersymmetry Breaking

I intend to gloss over this very briefly since it is not used elsewhere in this paper. Supersymmetry may be spontaneously broken both in the "F" sector and the "D" sector. Supersymmetry may also be dynamically broken (e.g. "supercolor").

Let us first examine the "F" sector. We recall from (I.13) that

$$H \sim \sum_i |Q_i|^2 \geq 0.$$

Thus, if the vacuum state has non-zero energy it must be true that $Q_i |0\rangle \neq 0$ for at least one supersymmetry generator Q_i. The vacuum state is therefore not annihilated by such a generator and supersymmetry is spontaneously broken. For supersymmetry to be unbroken we require $H = 0$ in the vacuum. H achieves its minimum when the potential energy, $\mathcal{V} = F_i^* F_i = \sum_i \left|\frac{\partial W}{\partial \phi_i}\right|^2$, vanishes. Therefore we require $F_i = 0$; $\left.\frac{\partial W}{\partial \phi_i}\right|_{\phi_i = \widetilde{\psi}_i} = 0 \ \forall_i$ in the vacuum. That $F_i = 0$ for unbroken supersymmetry may be understood from the transformations in (I.37). In the vacuum, since there can be no derivative term, we have ($A \equiv \widetilde{\psi}$)

$$\delta_\epsilon A = \sqrt{2}\,\epsilon\,\langle\psi\rangle \qquad \delta_\epsilon \psi = \sqrt{2}\,\epsilon\,\langle F\rangle \qquad \delta_\epsilon F = 0\ . \tag{I.109}$$

Hence when we vary the scalar field we obtain zero unless $\langle\psi\rangle \neq 0$. This it would be a disaster since it would imply that the vacuum was not Lorentz invariant! (Actually the vacuum can consist of a *single*, massless, momentum-zero fermion.) Ignoring this possibility we see that if $F_i = 0$ then the variation of all of the component fields under a supersymmetric transformation vanishes.

The other formulation of the condition for unbroken supersymmetry, $\left.\frac{\partial W}{\partial \phi_i}\right|_{\phi_i = \widetilde{\psi}_i} = 0 \ \forall\ \phi_i$ for some set of $\widetilde{\psi}_i$ values (i.e. the vacuum), means that supersymmetry is spontaneously broken if the set of equations, $\frac{\partial W}{\partial \phi_i} = 0$ has no consistent solution.

Examples:

1) $W[\phi] = \phi^3$

$$\left.\frac{\partial W}{\partial \phi}\right|_{\phi=\widetilde{\psi}} = 0 \;\Rightarrow\; 3\widetilde{\psi}^2 = 0 \;\Rightarrow\; \widetilde{\psi} = 0 \text{ is a solution.}$$

2) $W[\phi] = g\phi^3 + m\phi^2$

$$\left.\frac{\partial W}{\partial \phi}\right|_{\phi=\widetilde{\psi}} = 0 \;\Rightarrow\; 3g\widetilde{\psi}^2 + 2m\widetilde{\psi} = 0 \;\Rightarrow\; \widetilde{\psi} = 0 \text{ is a solution.} \tag{I.110}$$

3) $W[\phi_1, \phi_2, \phi_3] = \lambda\phi_1 + m\phi_2\phi_3 + g\phi_1\phi_2\phi_2$

$$\frac{\partial W}{\partial \phi_1} = 0 \;\Rightarrow\; \lambda + g{\phi_2}^2 = 0$$

$$\frac{\partial W}{\partial \phi_2} = 0 \;\Rightarrow\; m\phi_3 + 2g\phi_1\phi_2 = 0$$

$$\frac{\partial W}{\partial \phi_3} = 0 \;\Rightarrow\; m\phi_2 = 0\ .$$

The first and third equations are clearly inconsistent when λ and m are non-zero and we have spontaneous supersymmetry-breaking. This is called the O'Raifeartaigh Potential.[41] Unlike internal symmetry breaking, in which a scalar field acquires a vacuum expectation value, spontaneous supersymmetry-breaking occurs when a constraint (auxiliary) field acquires a VEV. (Note how the "dangerous" single superfield term $\lambda\phi$ is again involved. $W = \lambda\phi$ would break supersymmetry by itself but could violate R-parity.)

In the "D" sector the Fayet-Iliopoulos term, $\mathcal{L}_{FI} = c\int V d^4\theta = cD$, provides the breaking. From (I.102), (I.105) and (I.108) we see that (for one Abelian gauge superfield and one scalar superfield), when all of the parts of $\mathcal{L}$ are included, the terms with "D" in them are

$$\mathcal{L}_D = \frac{g}{2}\,\widetilde{\psi}^*\widetilde{\psi}D + \tfrac{1}{2}\,D^2 + cD \tag{I.111}$$

so $\frac{\partial \mathcal{L}}{\partial D} = 0 = \frac{1}{2} g\widetilde{\psi}^* \widetilde{\psi} + c + D$ or

$$D = -\left(c + \frac{g}{2}\, \widetilde{\psi}^* \widetilde{\psi}\right) \qquad \mathcal{L}_D = -\tfrac{1}{2}\left(c + \frac{g}{2}\, \widetilde{\psi}^* \widetilde{\psi}\right)^2 . \tag{I.112}$$

This is a potential term; $\mathcal{V}_D$ is to be added to $\mathcal{V}_F$, which replaces the $|\phi|^4$ term discussed earlier. ($\frac{1}{2} \sum_a g_a |\widetilde{\psi}^* T^a \widetilde{\psi}|^2$ for a non-Abelian gauge group, introducing Clebsch-Gordon coefficients into the relative coupling strengths.) When $c < 0$ the minimum is achieved when $\left\langle \widetilde{\psi} \right\rangle = \sqrt{\frac{2|c|}{g}}$ and the *gauge symmetry* is spontaneously broken. Supersymmetry is unbroken because, in the vacuum, when $\left\langle \widetilde{\psi} \right\rangle = \sqrt{\frac{2|c|}{g}}$ we have a vanishing potential and $H = 0$ is yet possible (unless broken in the F-sector). When $c > 0$, $\mathcal{V}_D > 0$ and we have spontaneous supersymmetry-breaking. In this case the gauge symmetry is unbroken. Note that

$$\begin{aligned} \mathcal{L}_D &= -\tfrac{1}{2}\, D^2 = -\mathcal{V}_D \\ \mathcal{V}_D &= \tfrac{1}{2}\, D^2 \end{aligned} \tag{I.113}$$

so we may write the total potential.

$$\mathcal{V} = \mathcal{V}_F + \mathcal{V}_D = F_i^* F_i + \tfrac{1}{2}\, D_i D_i \; . \tag{I.114}$$

Again we have spontaneous supersymmetry-breaking when the auxiliary field (D) gets a VEV and gauge breaking when the scalar field does. In general gauge and supersymmetries may be independently conserved or spontaneously broken. In Fig. I.1 we summarize this breaking picture by showing potential terms of each of the four possibilities (gauge, supersymmetry either preserved or spontaneously broken). Finally spontaneous supersymmetry-breaking results in a Goldstone fermion ("Goldstino") just as spontaneous gauge symmetry breaking results in a Goldstone boson.

The approach actually utilized is simply to add "soft" breaking terms directly into $\mathcal{L}$. Such terms *could* arise from spontaneous supersymmetry-breaking as discussed (at least *a priori*) at some greater energy scale (specifically from local supersymmetry breaking) but we simply do not worry about such specific origins. In point of fact global spontaneous supersymmetry-breaking has proven singularly unsuccessful in providing realistic models. Supergravity-inspired softly-broken theories appear much more promising.[42]

One of the nicest aspects of supersymmetry-breaking is that it cannot be induced radiatively (due to improved divergence cancellations; there are still non-perturbative instanton-like effects which can break supersymmetry). There is a "set it and forget it" attitude. If supersymmetry is unbroken at the tree level it will remain so to any number of loops. Masses and Yukawa couplings are not renormalized at all in unbroken supersymmetry (and typically receive only *finite* corrections in spontaneously or softly broken theories). Only the wavefunctions need be renormalized. The advantage of using only "soft" breaking terms is that these impressive renormalization properties are not destroyed.

How does this supersymmetry breaking manifest itself? Note that both "D" and "F" spontaneous supersymmetry-breaking give masses to the bosons, but not the fermions, thus lifting the mass degeneracy. The fermions gain mass via the traditional Higgs mechanism. Prior to breaking supersymmetry forces the bosonic superpartners to attain identical masses.

Assembling the Lagrangian: SQED

We are now ready to put all the ingredients we have discussed together to produce a realistic theory. Before going to the supersymmetric standard model we present the much simpler case of unbroken supersymmetric QED. Since it is unbroken we will not require a Fayet-Iliopoulos term (linear in V) or a superpotential term linear in either of the scalar superfields ϕ_L and ϕ_R. Since there are also no Yukawa terms we may do away with trilinear terms in these superfields. V is the photon superfield, ϕ_L contains e_L and its superpartner $\widetilde{e}_L$ (the "left-selection") while ϕ_R contains e_R and $\widetilde{e}_R$. The subscripts on $\widetilde{e}_L$ and $\widetilde{e}_R$ convey no angular momentum information (they are scalars after all!) but indicate which of the spinor states the boson is associated with. Elsewhere ϕ_L is called L and ϕ_R is called R. This is convenient when we consider the extension to multiple generations (L_i, R_i) and nontrivial (doublet) structure under other groups $(SU(2))$. Note that since R is a chiral (left-handed) superfield (as is L) then $\overline{R}$ must contain e_R^+ while R contains e_L^+. This accounts for the differences in charge signs in the kinetic terms.

$$\text{Kinetic Term}: \qquad \overline{L}e^{-eV}L + \overline{R}\,e^{eV}\,R \tag{I.115}$$

Since we desire a Dirac mass, not a Majorana mass, for the electron, we require a term such as $m(\overline{e}_L e_R + \overline{e}_R e_L)$ in the Lagrangian.

$$\text{Superpotential ("}F\text{"-term)}: \qquad m_e(RL + \overline{LR})\ . \tag{I.116}$$

As usual the (Abelian) superfield strength is given by $W^\alpha = -\frac{1}{4}\overline{D}^2 D^\alpha V$.

$$\text{Gauge Field ("}D\text{"-term)}: \qquad \frac{1}{4}\,WW + \frac{1}{4}\overline{WW}\ . \tag{I.117}$$

Together we have

$$\mathcal{L}_{SQED} = \int \left\{ \overline{L}e^{-eV}L + \overline{R}e^{eV}R \right\} d^4\theta + \int \frac{1}{4}\, W^2 d^2\theta + \int \frac{1}{4}\, \overline{W}^2 d^2\overline{\theta}$$
$$+ m_e \left\{ \int RL\, d^2\theta + \int \overline{L}\overline{R} d^2\theta \right\} \ . \tag{I.118}$$

This was presented in Appendix E (Eq. E3). There we used the fact that $\delta(\theta_i) = \theta_i$ for any Grassman variable θ_i

$$\left(\int \theta_i d\theta_i = \int \delta(\theta_i) d\theta_i = 1\,; \quad \int \theta_1^2 d\theta_i = \int \theta_i \delta(\theta_i) d\theta_i = 0 \right)$$

to bring the final terms into the form $d^4\theta = d^2\theta d^2\overline{\theta}$. The expansion into component fields, using the rules assiduously developed in this chapter, were given in Eq. (E4).

I.3 THE SUPERSYMMETRIC STANDARD LAGRANGIAN

We now use the results of the previous sections to construct the supersymmetric version of the "standard model" (as of 1987) consisting of 3 generations and an $SU(3) \times SU(2) \times U(1)$ symmetry group which breaks spontaneously, via the Higgs mechanism, to $SU(3)_{\text{color}} \times U(1)_{EM}$.

Superfield Content

Using the notation we have established, the superfields required are given in table I.1.

The Higgs singlet, $\hat{\mathbf{N}}$, may be included to implement the standard Higgs mechanism while retaining an unbroken supersymmetry . Note that $\hat{\mathbf{Q}}$, $\hat{\mathbf{U}}$ and $\hat{\mathbf{D}}$ are color triplets while $\hat{\mathbf{G}}$ is a color octet.

Kinetic Term

The general form for this term was found to be

$$\mathcal{L}_{KIN} = \int \sum_{\phi} \overline{\phi}_j e^{g_i T^a_{jk(i)} V_a} \ \phi_k d^4\theta \tag{I.119}$$

where "i" sums over all of the gauge groups that ϕ transforms non-trivially under. For instance, if $\phi = \widehat{Q}$, which experiences $SU(3)$, $SU(2)$, and $U(1)$ forces, then

$$(g_i T^a_i V_a)_{jk} = g_{\text{strong}} \lambda^a_{jk} \hat{\mathbf{G}}_a + g_{\text{weak}} \tau^a_{jk} \hat{\mathbf{V}}_a + g_y y \delta_{jk} \hat{\mathbf{V}}' \ . \tag{I.120}$$

Since $\hat{\mathbf{R}}$ is a singlet under $SU(3)$ and $SU(2)$ only the final term would be required when $\phi = \hat{\mathbf{R}}$. Here λ^a are the $SU(3)$ generators.

We shall not be concerned with the action of gluons or gluinos in this paper. Ignoring all but electroweak contributions we can expand $\mathcal{L}_{KIN}$ into its component field terms. These consist of standard kinetic terms for all of the leptons, quarks and Higgses plus the gauge-coupling terms and their supersymmetric counterparts. Letting[12]

$$\hat{\mathbf{V}} \text{ coupling be } g \quad (SU(2)) \qquad \hat{\mathbf{V}}' \text{ coupling be } g' \quad (U(1)_y) \tag{I.121}$$

$$\begin{aligned}\mathcal{L}_{qqV,V'} = &-\tfrac{1}{2}\,(g\tau^a_{ij}V^\mu_a + Y_Q g' V'^\mu \delta_{ij}) q^{\dagger i}\overline{\sigma}_\mu q^j \\ &- \tfrac{1}{2}\, g' Y_U V'^\mu (u_R)^\dagger \overline{\sigma}_\mu u_R - \tfrac{1}{2}\, g' Y_D V'^\mu d_R^\dagger \overline{\sigma}_\mu d_R\end{aligned} \tag{I.122}$$

where $q^i = (u_L \;\; d_L)$ is a weak doublet and $\psi_Q^{i\dagger}\tau_{ij}\psi_Q^j$ is an $SU(2)$ singlet (Lorentz spinor indices have been suppressed). (I.122) must also be summed over generations. Of course we may write $\tau^\pm = \frac{1}{\sqrt{2}}\,(\tau^1 \pm i\tau^2)$ and $W^\pm = \frac{1}{\sqrt{2}}\,(V^1 \pm iV^2)$ in order to put (I.122) in a more conventional form. For the leptons we have

$$\begin{aligned}\mathcal{L}_{\ell\ell V,V'} = &-\tfrac{1}{2}\,(g\tau^a_{ij}V^\mu_a + Y_\ell g' V'^\mu \delta_{ij}) \ell^{\dagger i}\overline{\sigma}_\mu \ell^j \\ &- \tfrac{1}{2}\, g' Y_{\ell^+} V'^\mu (\ell_R^-)^\dagger \overline{\sigma}_\mu (\ell_R^-)\ .\end{aligned} \tag{I.123}$$

since there is no ν_R term. Other terms which arise are (see (I.102))

$$\begin{aligned}\mathcal{L}_{\widetilde{\ell}\widetilde{\ell}V,V'} = &-\frac{i}{2}\,\widetilde{\ell}^{iT}(g\tau^a_{ij}V^\mu_a + Y_\ell g' V'^\mu \delta_{ij})\partial_\mu \widetilde{\ell}^{j*} \\ &- \frac{i}{2}\, g' Y_{\ell^+} \widetilde{\ell}^*_R V'^\mu \partial_\mu \widetilde{\ell}_R\end{aligned} \tag{I.124}$$

with a similar expression for quarks with $\widetilde{\ell}^i \to \widetilde{q}^i$, $\widetilde{\ell}^+_R \to \widetilde{u}_R$ and $\widetilde{d}_R$. Once again generations are summed over. A related term is

$$\begin{aligned}\mathcal{L}_{\widetilde{q}q\lambda} = &\frac{i}{\sqrt{2}}\,\widetilde{q}^{i\dagger}(g\tau^a_{ij}\lambda_a q^j + g' Y_Q \lambda' q^j) \\ &- \frac{i}{\sqrt{2}}\,\widetilde{u}^*_R g' Y_u \lambda' u_R - \frac{i}{\sqrt{2}}\,\widetilde{d}^*_R g' Y_D \lambda' d_R\end{aligned} \tag{I.125}$$

and similarly for leptons. In addition to these there are seagull terms, quartic slepton and squark terms and gauge-Higgs terms. For these terms, re-expressed in terms of more familiar forms ($W^{\pm}$ instead of $W^{1,2}$, etc.), Refs. 12 and 13 are recommended. We already see that the kinetic term alone is enormously complicated. This is only the beginning, however, since many of these states mix (especially when $\left\langle H^0_{1,2} \right\rangle$, $\langle N \rangle \neq 0$). Furthermore we have auxiliary terms in $\mathcal{L}_{KIN}$ which can only be disentangled once the remainder of the Lagrangian is added. Into this we plan to add soft supersymmetry-breaking terms! The result is a prodigiously tangled Lagrangian. For this reason we will not attempt to simplify each term as it is presented. We might as well wait until the entire imbroglio has been accounted for.

Superpotential

The superpotential term, $W[\phi_i]$, may be divided into the pure Higgs potential and the mass-generation terms (all trilinear)

$$W[\phi_i] = g_L^{ij} H_1 L_i R_j + g_D^{ij} H_1 Q_i D_j + g_U^{ij} H_2 Q_i U_j + W_H[H_1, H_2, N] \ . \qquad \text{(I.126)}$$

In (I.126) i and j are generation indices. The weak isospin indices have been suppressed, however it is assumed that each term is contracted into an isosinglet (e.g. $g_L^{ij} \epsilon_{\alpha\beta} H_1^{\alpha} L_i^{\beta} R_j$). It is clear that these terms include the normal Higgs-induced lepton and quark masses in $\mathcal{L}_{POT} = \int W d^2\theta + \int \overline{W} d^2\overline{\theta}$ once the neutral Higgs bosons gain a VEV. It is also clear that quadrilinear terms such as $H_0 H_0 \widetilde{e}_L^2$ will induce identical masses in these sleptonic partners. As discussed previously, two Higgs doublets are required for such a scheme. Note that all of the other interactions which involve Higgsinos will couple with the same Yukawa strengths (g_U, g_D, g_L) which will not require radiative renormalizations. Note also that no bilinear potential terms, which would constitute explicit mass terms, have been included.

In order to have unbroken supersymmetry spontaneously break the gauge symmetry $SU(2)_W \times U(1)_Y \to U(1)_{EM}$ in the Higgs sector it is usual to take

$$W_H[\hat{\mathbf{H}}_1, \hat{\mathbf{H}}_2, \hat{\mathbf{N}}] = g_N[\hat{\mathbf{H}}_1\hat{\mathbf{H}}_2 - \tfrac{1}{2}v^2]\hat{\mathbf{N}} \,. \tag{I.127}$$

The equation of motion for the F-term of N is

$$F_N = g_N[\epsilon_{ij}H_1^i, H_2^j - \tfrac{1}{2}v^2] = g_N[H_1^0 H_2^0 + H_1^- H_2^+ - \tfrac{1}{2}v^2] \tag{I.128}$$

where H_1^i etc. are scalar field components.

Now for supersymmetry to be unbroken we required (previous section) that $\langle F_i \rangle = 0\ \forall_i$. Since $\langle H_1^- \rangle_1 = \langle H_2^+ \rangle = 0$ from charge conservation we require $\langle H_1^0 \rangle\ \langle H_2^0 \rangle \neq 0$. To determine the condition accurately we need to find the potential term, $\mathcal{V} = \sum_F FF^* + \frac{1}{2}\sum_D D^2$, from the entire Lagrangian, including the field-strength sector. This has been done in Ref. 12. The above term (I.128) is the only inhomogeneous term in $\mathcal{V}$ so we may set the VEV of all other terms to zero. Doing so the remaining terms are

$$\begin{aligned}\langle \mathcal{V} \rangle &= \Bigg\langle g_N^2|\epsilon_{ij}H_1^i H_2^j - \tfrac{1}{2}v^2|^2 + \tfrac{1}{2}g\left[|H_1^{i*}H_2^i|^2 - \tfrac{1}{2}(H_1^{i*}H_1^i)(H_2^{i*}H_2^i)\right] \\ &\quad + \frac{1}{8}\,g'^2\left[H_2^{i*}H_2^i - H_1^{j*}H_1^j\right]^2 \Bigg\rangle \\ &= g_N^2|v_1 v_2 - \tfrac{1}{2}v^2|^2 + \frac{1}{8}\,g'^2[v_1^2 - v_2^2]^2\end{aligned} \tag{I.129}$$

where

$$v_1 = \langle H_1^0 \rangle \quad v_2 = \langle H_2^0 \rangle \,.$$

We see that the solution to $\langle \mathcal{V} \rangle = 0$ is

$$v_1 = v_2 = \frac{1}{\sqrt{2}}\, v \ . \tag{I.130}$$

We further observe that if several singlet superfields $N_1, \ldots, N_K$ are employed that (I.130) need not be true. In general it is merely required that

$$v_1^2 + v_2^2 = v^2 \qquad v_1 = \langle H_1^0 \rangle \qquad v_2 = \langle H_2^0 \rangle \ . \tag{I.131}$$

We will, in fact, permit this more general VEV assignment. We will not include any scalar Higgs superfields since our breaking will be accomplished via soft explicit terms. The most general Higgs structure with two weak doublets, one singlet and soft (bilinear and certain trilinear) breaking terms has been analyzed.[13] The presence of such soft terms relieves us of the necessity of employing a Higgs singlet. (See, in particular, Chapter 3, case 2 of the quoted reference.) Knowledge of the precise details underlying the final Higgs structure, which will assume the form $g_H|H_1^0 H_1^{0*} + H_2^0 H_2^{0*} - v^2|^2$, need not concern us since they will affect the Higgs sector which we will not be dealing with directly. It is sufficient for our purposes to know that such a soft-breaking scheme exists.

Super Field Strength

Recall that this contribution was of the form

$$\mathcal{L}_{\text{gauge}} = \sum_i \left\{ \int W_i^{\alpha} W_{\alpha i} d^2\theta + \int \overline{W}_i^{\dot{\alpha}} W_{\dot{\alpha} i} d^2\overline{\theta} \right\} \tag{I.132}$$

where "i" represents each gauge group in the theory and W^{α} was given by (I.95) and (I.100). Using the notation of Table I.1 we have

$$
\begin{aligned}
W_\alpha^{\hat{\mathbf{V}}\,'} &= -\frac{1}{4}\,\overline{D}^2 D_\alpha \hat{\mathbf{V}}\,' \\
W_\alpha^{\hat{\mathbf{V}}} &= -\frac{1}{4}\,\overline{D}^2 e^{-\tau^a \hat{\mathbf{V}}_a} D_\alpha e^{\tau^a \hat{\mathbf{V}}_a} \\
W_\alpha^{\hat{\mathbf{G}}} &= -\frac{1}{4}\,\overline{D}^2 e^{-\lambda^a \hat{\mathbf{G}}_a} D_\alpha e^{\lambda^a \hat{\mathbf{G}}_a}\ .
\end{aligned}
\tag{I.133}
$$

These contain all of the gauge and gaugino kinetic terms and self-interactions. When spontaneous gauge symmetry breaking occurs, resulting in $SU(2)\times U(1) \rightarrow U(1)_{em}$, the $\hat{\mathbf{V}}$ superfield components becomes (equally) massive. Since gaugino mass terms added to $\mathcal{L}$ will break supersymmetry only softly (in fact they are one of the only spin-$\frac{1}{2}$ fields in an $N = 1$ supersymmetry for which this is true)[19] we will do so. Since this, and the fermion mixing to be discussed shortly, will alter what we would expect from (I.133) alone we will refrain from expanding (I.133) and into its myriad terms.

I.4 THE FULL LAGRANGIAN

Into the unsullied, fully supersymmetric Lagrangian of the previous section we now add certain terms which break this symmetry softly.

Soft Breaking Terms

"Soft" terms are those, which when included as explicit terms in the Lagrangian at three level, do not introduce *new* divergences. This is a desirable attribute since it means that the attractive non-renomalization theorems remain valid. An exhaustive tabulation of soft terms has been made.[19] These include all dimension-two operators which would normally arise in $\mathcal{L}$ (once it had been expanded into its component fields) and a few dimension three operators. Specifically included in the latter are gaugino mass terms but not normal fermion masses. Trilinear scalar combinations which arise from the unbroken superpotential are also included. Note that vector mass terms would break ordinary gauge invariance and are not included. Higgsino bilinears of the form $\epsilon^{ij}\widetilde{H}_i\widetilde{H}_j$ are also acceptable.

The soft terms which we will add to $\mathcal{L}_{SGWS}$, the leptonic sector of "standard" supersymmetric Lagrangian, are

$$\begin{aligned}\mathcal{L}_{Br} = &-\tfrac{1}{2}M\tilde{W}^0\tilde{W}^0 - \tfrac{1}{2}M'\tilde{B}^0\tilde{B}^0 - \mu\tilde{\psi}^0_{H_1}\tilde{\psi}^0_{H_2} - M\tilde{W}^+\tilde{W}^- + \mu\tilde{\psi}^-_{H_1}\tilde{\psi}^+_{H_2} \\ &+ m_{ij}\tilde{\ell}^*_i\tilde{\ell}_j + \hat{m}_{ij}\tilde{\nu}_i\tilde{\nu}_j + \text{ Higgs boson terms } + h.c.\end{aligned} \tag{I.134}$$

The Higgs boson terms will be responsible for electroweak spontaneous symmetry breaking as discussed previously. There is no $\hat{\mathrm{N}}$ superfield and only H_1^0

and H_2^0 incur non-zero vacuum expectation values

$$\langle H_1^0 \rangle = v_1 \qquad \langle H_2^0 \rangle = v_2$$
$$v_1^2 + v_2^2 = v^2 = \left(\frac{2m_w}{g}\right)^2 \simeq (250\ GeV)^2 \ . \tag{I.135}$$

It is convenient to define

$$\tan\theta_v = v_1/v_2 \ . \tag{I.136}$$

(This is called $\cot\beta$ in Ref. 13.)

Slepton Mass Lagrangian

As discussed in Chapter 3 we may generally ignore the contribution to the scalar lepton (slepton) masses which arise via the unbroken supersymmetric Higgs mechanism, from the lepton masses, and instead consider the most general mass matrices induced by soft terms:

$$\begin{aligned}\mathcal{L} = &\tfrac{1}{2}\,(\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{e}_L\ \widetilde{e}_R)^*(M_+)(\widetilde{\tau}_L\ \widetilde{\tau}_R\ \widetilde{\mu}_L\ \widetilde{\mu}_R\ \widetilde{e}_L\ \widetilde{e}_R)^T \\ &+ \tfrac{1}{2}\,(\widetilde{\nu}_L^\tau\ \widetilde{\nu}_L^\mu\ \widetilde{\nu}_L^e\ )^*(M_0)(\widetilde{\nu}_L^\tau\ \widetilde{\nu}_L^\mu\ \widetilde{\nu}_L^e\ )^T \ .\end{aligned} \tag{I.137}$$

Here we have ignored right-handed sneutrinos. The complexities of this term in general are alluded to in Chapter 3. For our purposes it is sufficient to simplify the mixing to

$$\begin{aligned}&\tilde{\mu}_L \text{ and } \tilde{\tau}_L \text{ with angle } \theta_L \\ &\tilde{\mu}_R \text{ and } \tilde{\tau}_R \text{ with angle } \theta_R \\ &\tilde{\nu}_\mu \text{ and } \tilde{\nu}_\tau \text{ with angle } \theta_\nu\end{aligned} \tag{I.138}$$

in which case we may write

$$
\begin{aligned}
\tilde{\ell}_{L_1} &= \tilde{\mu}_L \cos\theta_L + \tilde{\tau}_L \sin\theta_L \\
\tilde{\ell}_{L_2} &= -\tilde{\mu}_L \sin\theta_L + \tilde{\tau}_L \cos\theta_L \\
\tilde{\ell}_{R_1} &= \tilde{\mu}_R \cos\theta_R + \tilde{\tau}_R \sin\theta_R \\
\tilde{\ell}_{R_2} &= -\tilde{\mu}_R \sin\theta_R + \tilde{\tau}_R \cos\theta_R \\
\tilde{\nu}_1 &= \tilde{\nu}_\mu \cos\theta_\nu + \tilde{\nu}_\tau \sin\theta_\nu \\
\tilde{\nu}_2 &= -\tilde{\nu}_\mu \sin\theta_\nu + \tilde{\nu}_\tau \cos\theta_\nu \ .
\end{aligned}
\tag{I.139}
$$

Note that $\tilde{\ell}_{L,R_1}$, $\tilde{\ell}_{L,R_2}$ and $\tilde{\nu}_{1,2}$ are the physical mass eigenstates. Conserved global family number is the only artifact which would prevent such a construction.

The absence of $\widetilde{\ell}_L - \widetilde{\ell}_R$ mixing is a great simplification. The importance of such mixing is clear from Chapter 2.

Charginos

The supersymmetric partners of the charged Higgs and vector bosons (Higgsinos and gauginos) are collectively referred to as *charginos.*

The supersymmetric counterparts of the $W^+H_i^-H_j^0$, and $W^-H_i^+H_j^0$ terms in the unbroken standard $\mathcal{L}_{SGWS}$ include

$$\mathcal{L} = -\frac{ig}{\sqrt{2}} \left[H_1^0 \widetilde{W}^+ \tilde{\psi}_{H_1}^- + H_2^0 \widetilde{W}^- \tilde{\psi}_{H_2}^+ \right] + h.c.$$

once $SU(2) \times U(1)$ is broken down to $U(1)_{em}$ and $\left\langle H_{1,2}^0 \right\rangle = v_{1,2}$

$$\mathcal{L} = -\frac{ig}{\sqrt{2}} \left[v_1 \widetilde{W}^+ \tilde{\psi}_{H_1}^- + v_2 \widetilde{W}^- \tilde{\psi}_{H_2}^+ \right] + h.c. \tag{I.140}$$

Note that these are Weyl spinors and we have switched to the basis $\widetilde{W}^\pm = \frac{1}{\sqrt{2}} \left(\widetilde{W}^1 \pm i\widetilde{W}^2 \right)$. These terms will induce mixing between the charged Higgsinos and gauginos. A similar miscibility occurs in the neutral sector. In this sense the Higgs and gauge superfields have become "united". Fayet[43] has taken this idea one step further. It is known that gauge and Higgs bosons can be unified in a single superfield in an $N = 2$ supersymmetry . Fayet showed that this is also possible in our $N = 1$ supersymmetry if one is prepared to abandon the Wess-Zumino gauge. If a different set of component fields is eliminated from a vector superfield via the supergauge mechanism described in section one, is possible to be left with a gauge boson, a Weyl gaugino, a Higgs boson and a Weyl Higgsino and no auxiliary fields.

To these fermion mass terms are added the soft breaking terms of (I.134) to yield the chargino mass Lagrangian

$$\mathcal{L}_{Ch} = M\widetilde{W}^+\widetilde{W}^- - \mu\tilde{\psi}^-_{H_1}\tilde{\psi}^+_{H_2} + \sqrt{2}ig(v_1\widetilde{W}^+\tilde{\psi}^-_{H_1} + v_2\widetilde{W}^-\tilde{\psi}^+_{H_2}) + h.c. \qquad \text{(I.141)}$$

Following Haber and Kane[12] we note that if we gather these Weyl fields into four-component fermions of the form

$$\psi^+ = \begin{pmatrix} -i\widetilde{W}^+ \\ \tilde{\psi}^+_{H_2} \end{pmatrix} \qquad \psi^- = \begin{pmatrix} -i\widetilde{W}^- \\ \tilde{\psi}^-_{H_1} \end{pmatrix} \qquad \text{(I.142)}$$

(where we note that $\overline{\psi}^+ \neq \psi^-$) and write

$$\mathcal{L}_{Ch} = -\tfrac{1}{2}\left[(\psi^+)^T X^T \psi^- + (\psi^-)^T X \psi^+\right] + h.c. \qquad \text{(I.143)}$$

(note the use of ordinary transposes) then we find that

$$X = \begin{pmatrix} M & \sqrt{2}m_W \cos\theta_v \\ \sqrt{2}m_W \sin\theta_v & \mu \end{pmatrix} \qquad \text{(I.144)}$$

will reproduce (I.141) to wit

$$\mathcal{L} = -\tfrac{1}{2}\Bigg\{(-i\widetilde{W}^{+}\ \tilde{\psi}^{+}_{H_2}) \begin{pmatrix} M & \sqrt{2}m_W s_v \\ \sqrt{2}m_W c_v & \mu \end{pmatrix} \begin{pmatrix} -i\widetilde{W}^{-} \\ \tilde{\psi}^{-}_{H_1} \end{pmatrix}$$

$$+ (-i\widetilde{W}^{-}\ \tilde{\psi}^{-}_{H_1}) \begin{pmatrix} M & \sqrt{2}m_W c_v \\ \sqrt{2}m_W s_v & \mu \end{pmatrix} \begin{pmatrix} -i\widetilde{W}^{+} \\ \tilde{\psi}^{+}_{H_2} \end{pmatrix}\Bigg\} + h.c.$$

$$= -\tfrac{1}{2}\Bigg\{ - M\widetilde{W}^{+}\widetilde{W}^{-} - i\sqrt{2}m_W(s_v\widetilde{W}^{+}\tilde{\psi}^{-}_{H_1} + c_v\tilde{\psi}^{+}_{H_2}\widetilde{W}^{-}) + \mu\tilde{\psi}^{+}_{H_2}\tilde{\psi}^{-}_{H_1}$$

$$- M\widetilde{W}^{-}\widetilde{W}^{+} - i\sqrt{2}m_W(c_v\tilde{\psi}^{-}_{H_1}\widetilde{W}^{+} + s_v\widetilde{W}^{-}\tilde{\psi}^{+}_{H_2}) + \mu\tilde{\psi}^{-}_{H_1}\tilde{\psi}^{+}_{H_2}\Bigg\}$$

$$+ h.c.$$

Now, since

$$s_v = \sin\theta_v = \frac{v_1}{v} \qquad c_v = \cos\theta_v = \frac{v_2}{v} \qquad v^2 = v_1^2 + v_2^2 \qquad g = \frac{2m_W}{v} \tag{I.145}$$

$$\mathcal{L}_{Ch} = M\widetilde{W}^{0}\widetilde{W}^{-} + i\sqrt{2}\ \frac{m_W v_1}{v}\widetilde{W}^{+}\tilde{\psi}^{-}_{H_1} + i\sqrt{2}\ \frac{m_W v_2}{v}\widetilde{W}^{-}\tilde{\psi}^{+}_{H_2} - \mu\tilde{\psi}^{-}_{H_1}\tilde{\psi}^{+}_{H_2} + h.c.$$

$$= M\widetilde{W}^{+}\widetilde{W}^{-} - \mu\tilde{\psi}^{-}_{H_1}\tilde{\psi}^{+}_{H_2} + \sqrt{2}ig(v_1\widetilde{W}^{+}\tilde{\psi}^{-}_{H_1} + v_2\widetilde{W}^{-}\tilde{\psi}^{+}_{H_2}) + h.c.$$

once we have substituted for "g". In the next appendix we will diagonalize this to obtain the true chargino mass eigenstates which we denote $\widetilde{\chi}^{+}_{i}$ $(i = 1, 2)$ and $\widetilde{\chi}^{-}_{i}$. When this is diagonalized via

$$\widetilde{\chi}^{+}_{i} = V_{ij}\psi^{+}_{i} \qquad \widetilde{\chi}^{-}_{i} = U_{ij}\psi^{-}_{j} \tag{I.146}$$

we will have

$$\mathcal{L}_{Ch} = -\tfrac{1}{2}\left[m_{\widetilde{\chi}^{+}_{1}}\ \widetilde{\chi}^{-}_{1}\widetilde{\chi}^{+}_{1} + m_{\widetilde{\chi}^{+}_{2}}\ \widetilde{\chi}^{-}_{2}\widetilde{\chi}^{+}_{2}\right] + h.c. \tag{I.147}$$

The matrices U and V (and combinations thereof) will be proliferated through the Feynman rules. It is also possible to express $\widetilde{\chi}^{\pm}$ in terms of the Dirac spinors

$$\widetilde{W} = \begin{pmatrix} -i\widetilde{W}^+ \\ i\overline{\widetilde{W}^-} \end{pmatrix} \qquad \widetilde{H}^+ = \begin{pmatrix} \tilde{\psi}^+_{H_2} \\ \bar{\tilde{\psi}}^-_{H_1} \end{pmatrix} . \tag{I.148}$$

Neutralinos

This situation is similar here but we have more states. Both Higgs doublets have a neutral state, H_1^0 and H_2^0, with corresponding Higgsinos $\tilde{\psi}^0_{H_1}$ and $\tilde{\psi}^0_{H_2}$. The two neutral gauginos are $\widetilde{B}^0$ and $\widetilde{W}^0$, related to the basis $\widetilde{\gamma}$ and $\widetilde{Z}^0$ by the same ratios as their bosonic counterparts

$$\begin{pmatrix} \widetilde{B}^0 \\ \widetilde{W}^0 \end{pmatrix} = \begin{pmatrix} c_w & s_w \\ -s_w & c_w \end{pmatrix} \begin{pmatrix} \widetilde{\gamma} \\ \widetilde{Z}^0 \end{pmatrix} \qquad \begin{pmatrix} \widetilde{\gamma} \\ \widetilde{Z}^0 \end{pmatrix} = \begin{pmatrix} c_w & -s_w \\ s_w & c_w \end{pmatrix} \begin{pmatrix} \widetilde{B}^0 \\ \widetilde{W}^0 \end{pmatrix} \tag{I.149}$$

where we henceforth establish that

$$\begin{aligned} c_w &= \cos\theta_w & s_w &= \sin\theta_w \\ c_v &= \cos\theta_v & s_v &= \sin\theta_v \\ c_{2v} &= \cos 2\theta_v & s_{2v} &= \sin 2\theta_v \\ c_\phi &= \cos\phi & s_\phi &= \sin\phi \; . \end{aligned} \tag{I.150}$$

From this point forth we use the traditional notation

$$\widetilde{\gamma} \equiv \lambda \; . \tag{I.151}$$

The terms derived from the supersymmetrized $W^0 H_i^0 H_j^0$ and $B^0 H_1^0 H_j^0$ vertices yields the mass terms

$$\frac{i}{2}\left(g'\widetilde{B}^0 - g\widetilde{W}^0\right)\left(v_1\tilde{\psi}^0_{H_1} - v_2\tilde{\psi}^0_{H_2}\right) + h.c.$$

which again mixes the neutral Higgsinos and gauge bosons. To this we add the breaking terms

$$-\tfrac{1}{2}\,M\widetilde{W}^0\widetilde{W}^0 - \tfrac{1}{2}\,M'\widetilde{B}^0\widetilde{B}^0 - \mu\tilde{\psi}^0_{H_1}\tilde{\psi}^0_{H_2} + h.c.$$

to yield the general "neutralino" Lagrangian

$$\begin{aligned}\mathcal{L}_0 &= -\tfrac{1}{2}\left[M\widetilde{W}^0\widetilde{W}^0 + M'\widetilde{B}^0\widetilde{B}^0\right] - \mu\tilde{\psi}^0_{H_1}\tilde{\psi}^0_{H_2} \\ &\quad + \tfrac{1}{2}\,(g'\widetilde{B}^0 - g\widetilde{W}^0)\;(v_1\tilde{\psi}^0_{H_1} - v_2\tilde{\psi}^0_{H_2}) + h.c.\end{aligned} \tag{I.152}$$

In terms of $\widetilde{Z}^0$ and λ this is

$$\begin{aligned}\mathcal{L}_0 &= \tfrac{1}{2}\,(M'\cos^2\theta_w + M\sin^2\theta_w)\lambda\lambda + (M - M')\sin\theta_w\cos\theta_w\lambda\widetilde{Z}^0 \\ &\quad + \tfrac{1}{2}\,(M\cos^2\theta_w + M'\sin^2\theta_w)\widetilde{Z}^0\widetilde{Z}^0 + \frac{i}{2}\,\frac{g}{\cos\theta_w}\,\widetilde{Z}^0(v_1\tilde{\psi}^0_{H_1} - v_2\tilde{\psi}^0_{H_1}) \\ &\quad + \mu\tilde{\psi}^0_{H_1}\tilde{\psi}^0_{H_2} + h.c.\end{aligned} \tag{I.153}$$

Haber and Kane[12] establish the notation

$$\begin{aligned}\mathcal{L}_0 &= -\tfrac{1}{2}\,\psi^{0T}Y\psi^0 \\ &= -\tfrac{1}{2}\,\psi^{0'T}Y'\psi^{0'}\end{aligned} \tag{I.154}$$

where two possible bases we work with are

$$\psi^0 = \begin{pmatrix} -i\widetilde{B}^0 \\ -i\widetilde{W}^0 \\ \tilde{\psi}^0_{H_1} \\ \tilde{\psi}^0_{H_2} \end{pmatrix} \qquad \psi^{0'} = \begin{pmatrix} -i\lambda \\ -i\widetilde{Z}^0 \\ s_v\tilde{\psi}^0_{H_1} - c_v\tilde{\psi}^0_{H_2} \\ c_v\tilde{\psi}^0_{H_1} + s_v\tilde{\psi}^0_{H_1} \end{pmatrix} \tag{I.155}$$

These consist of four Weyl spinors each. Explicit calculation shows that

$$Y = \begin{pmatrix} M' & 0 & -m_Z \sin\theta_v \sin\theta_w & m_Z \cos\theta_v \sin\theta_w \\ 0 & M & m_Z \sin\theta_v \cos\theta_w & -m_Z \cos\theta_v \cos\theta_w \\ -m_Z \sin\theta_v \sin\theta_w & m_Z \sin\theta_v \cos\theta_w & 0 & -\mu \\ m_Z \cos\theta_v \sin\theta_w & -m_Z \cos\theta_v \cos\theta_w & -\mu & 0 \end{pmatrix} \tag{I.156}$$

and

$$Y' = \begin{pmatrix} M' c_w^2 + M s_w^2 & (M - M') s_w c_w & 0 & 0 \\ (M - M') s_w c_w & M c_w^2 + M' s_w^2 & m_Z & 0 \\ 0 & m_Z & \mu s_{2v} & \mu c_{2v} \\ 0 & 0 & \mu c_{2v} & -\mu s_{2v} \end{pmatrix} \tag{I.157}$$

These are related via

$$\psi^{0'}{}_{j'} = B_{j'j} \psi^0{}_j \qquad B B^\dagger = 1 \tag{I.158}$$

$$B_{jj'} = \begin{pmatrix} \cos\theta_w & \sin\theta_w & 0 & 0 \\ -\sin\theta_w & \cos\theta_w & 0 & 0 \\ 0 & 0 & \sin\theta_v & -\cos\theta_v \\ 0 & 0 & \cos\theta_v & \sin\theta_v \end{pmatrix} \tag{I.159}$$

The physical mass eigenstates will be

$$\begin{aligned} \widetilde{\chi}_i^0 &= N_{ij} \psi_j^0 \\ \widetilde{\chi}_i^{0'} &= N'_{ij} \psi_j^{0'} \end{aligned} \tag{I.160}$$

where

$$N_{ij} = N'_{ik} B_{kj} \ . \tag{I.161}$$

In correspondence to (I.148) we will frequently express the unmixed neutralinos in the form of four-component Majorana spinors

$$\lambda = \begin{pmatrix} -i\lambda \\ i\overline{\lambda} \end{pmatrix} \quad \widetilde{Z}^0 = \begin{pmatrix} -i\widetilde{Z}^0 \\ i\overline{\widetilde{Z}}^0 \end{pmatrix} \quad \widetilde{H}_1 = \begin{pmatrix} \tilde{\psi}^0_{H_1} \\ \bar{\tilde{\psi}}^0_{H_1} \end{pmatrix} \quad \widetilde{H}_2 = \begin{pmatrix} \tilde{\psi}^0_{H_2} \\ \bar{\tilde{\psi}}^0_{H_2} \end{pmatrix} \tag{I.162}$$

and the eigenstates as

$$\widetilde{\chi}^0_i = \begin{pmatrix} \widetilde{\chi}^0_i \\ \overline{\widetilde{\chi}}^0_2 \end{pmatrix} \qquad i = 1, 2, 3, 4 \ . \tag{I.163}$$

Gauge Couplings to Leptons

These are the same as in the standard electroweak Lagrangian (at least at the tree level) and will not be discussed in detail. (Reference 12 expresses this part of the Lagrangian in notation consistent with the supersymmetric terms.) The general form for $\mathcal{L}_{\psi_L \psi_L V}$ (ψ_L = left-handed spin-$\frac{1}{2}$ Weyl fermions; group generators are T^a) is

$$\mathcal{L}_{\psi\psi V} = -\tfrac{1}{2}\, g_V T^a_{ij} V^a_\mu \psi^{i\dagger}_L \overline{\sigma}^\mu \psi^j_L \ . \tag{I.164}$$

In our case (for one generation)

$$\begin{aligned} \mathcal{L}_{\ell\ell V} &= -\tfrac{1}{2}\,(g\tau^a_{ij} W^a_\mu + y_\ell g' B^0_\mu \delta_{ij})\ell^{\dagger i} \overline{\sigma}^\mu \ell^j \\ &\quad - \tfrac{1}{2}\, g' y_{e_R} B^0_\mu e^\dagger_R \overline{\sigma}^\mu e_R \qquad (a = 1, 2, 3\ ; i = 1, 2) \end{aligned} \tag{I.165}$$

where $\ell^T = (e_L\ \nu_L)$, τ^a are Pauli matrices and y is hypercharge ($y_\ell = -1$; $y_{e_R} = +2$). In terms of $A_\mu, Z_\mu, W^\pm, \tan\theta_w = g'/g$ and $e = g\sin\theta_w = g'\cos\theta_w$

$$\begin{aligned} \mathcal{L}_{\ell\nu W} &= -\frac{g}{\sqrt{2}}\,[W^+_\mu \nu^\dagger \overline{\sigma}^\mu \ell^- + W^-_\mu \ell^+ \overline{\sigma}^\mu \nu] \\ \mathcal{L}_{\ell^+\ell^+\gamma} &= +e\ell^+ \overline{\sigma}^\mu A_\mu \ell^- \end{aligned} \tag{I.166}$$

and so forth. In terms of more conventional Dirac spinors, with

$$\gamma_\pm = \tfrac{1}{2}(1 \pm \gamma_5) \tag{I.167}$$

$$\begin{aligned}\mathcal{L}_{eeV} &= \frac{-g}{\sqrt{2}}\left[W_\mu^+ \overline{\nu}\gamma^\mu\gamma_- e + W_\mu^- \overline{e}\gamma^\mu\gamma_-\nu\right] \\ &\quad - \frac{g}{2\cos\theta_v} Z_\mu \left[\overline{\nu}\gamma^\mu\gamma_-\nu - \cos 2\theta_W \overline{e}\gamma^\mu\gamma_- e - \sin^2\theta_W \overline{e}\gamma^\mu\gamma_+ e\right] \\ &\quad + eA_\mu \overline{e}\gamma^\mu e \ .\end{aligned} \tag{I.168}$$

<u>Gauge Couplings to Sleptons</u>

Following the procedure used when we derived the SQED lagrangian we find that the analogue of (I.164) is

$$\mathcal{L}_{\widetilde{\psi}\widetilde{\psi}V} = -ig_V T^a_{ij} V^a_\mu \widetilde{\psi}^{i*}_L \overleftrightarrow{\partial}^\mu \widetilde{\psi}^j_L \tag{I.169}$$

from which $\mathcal{L}_{\tilde{l}\tilde{l}Z^0}$, etc. follows just as in the previous section.

<u>Gaugino Coupling to Leptons</u>

The corresponding general term for $\mathcal{L}_{\widetilde{\psi}\psi\widetilde{V}}$ is

$$\mathcal{L}_{\widetilde{\psi}\psi\widetilde{V}} = \sqrt{2}\, ig[\widetilde{\psi}^i_L T^{a*}_{ij} \psi^{j\dagger}_L \widetilde{V}^{a\dagger} - \widetilde{\psi}^{i*}_L T^a_{ij} \psi^j_L \widetilde{V}^a] \tag{I.170}$$

where $\psi^{j\dagger}_L \widetilde{V}^{a\dagger}$ has the (spinoral) structure $\overline{\psi}_{\dot{\alpha}} \overline{\widetilde{V}}_{\dot{\beta}} \epsilon^{\dot{\alpha}\dot{\beta}}$, etc. We may again expand this out for $\widetilde{W}^a$ and $\widetilde{B}^0$ and then switch to the $\widetilde{W}^\pm, \widetilde{Z}^0, \lambda$ basis. For instance (in two-component form)

$$\mathcal{L}_{\widetilde{e}_L \nu_e \widetilde{W}} = ig[\widetilde{W}^- \nu_e \widetilde{e}^*_L - \overline{\widetilde{W}}^- \overline{\nu}_e \widetilde{e}_L + \widetilde{W}^+ e_L \widetilde{\nu}^* - \overline{\widetilde{W}}^+ e_L \widetilde{\nu}] \ . \tag{I.171}$$

To write this in terms of chargino mass eigenstates we would have to use (I.142) and (I.146) (or the equivalent expression for (I.146) in terms of (I.148)) and obtain $\mathcal{L}_{\widetilde{e}_L \nu_e \widetilde{\chi}^+}$ which would have U and V matrices in the coefficients. An example would be

$$\mathcal{L}_{\widetilde{e}_L \nu_e \widetilde{\chi}_i^+} = ig[U_{i1} \widetilde{\chi}_i^+ \nu_e \widetilde{e}_L^* - U_{i1}^* \overline{\widetilde{\chi}}^+ \overline{\nu}_e \widetilde{e}_L] \ . \tag{I.172}$$

If we were considering $\mathcal{L}_{\widetilde{\nu}_1 \tau_L \widetilde{\chi}_i^+}$ where we have allowed $\widetilde{\nu}_\mu$ and $\widetilde{\nu}_\tau$ to mix, as in (I.139), then we would find

$$\mathcal{L}_{\widetilde{\nu}_1 \tau_L \widetilde{\chi}_i^+} = -ig[V_{j1} \widetilde{\chi}_i^+ \tau_L \widetilde{\nu}_1^* - V_{j1}^* \overline{\widetilde{\chi}}_1^+ \overline{\tau}_L \widetilde{\nu}_1] \sin\theta_\nu \ . \tag{I.173}$$

There are obviously many terms and it would be exceedingly tedious to list them all. We will restrict our attention to deriving Feynman rules for those vertices which will be of special importance to the calculations in this work (Appendix K). The same is true of the neutralino vertices.

I.5 Gaugino Coupling to Gauge Bosons

These are the analogues of the non-Abelian trilinear gauge couplings. Once the Higgsinos and Gauginos mix to the physical eigenstates $\widetilde{\chi}_i$ the vertices $\widetilde{\chi}_i\widetilde{\chi}_j G_k$ and $\widetilde{\chi}_i\widetilde{\chi}_j H_k$ will involve two mixing matrices.

Just as the $G_i G_j G_k$ gauge term is

$$\mathcal{L}_{GGG} = igf^{ijk}\, G_i^\nu\, G_j^\mu\, \partial_\mu\, G_{\nu k} \tag{I.174}$$

the $\widetilde{G}_i\widetilde{G}_j G_k$ term simplifies to (Weyl notation)

$$\mathcal{L}_{\widetilde{G}\widetilde{G}G} = igf^{ijk}\, \widetilde{G}_i\, \sigma^\mu\, \overline{\widetilde{G}}_j\, G_{k\mu} \tag{I.175}$$

where f^{ijk} are the group structure constants. We will be interested in (I.175) for the cases $G = \gamma, Z^0$.

We will also need the Higgsino-Gauge-Gaugino terms. The form of the HHG, HGG and $HHGG$ terms arises in the usual manner from GGG and $GGGG$ once we implement spontaneous SU(2) $\times$ U(1) breaking. Since $\hat{\mathbf{H}}_1$ and $\hat{\mathbf{H}}_2$ are scalar superfields the Lagrangian for $\widetilde{G}\widetilde{H}H$ and $G\widetilde{H}\widetilde{H}$ arises in the kinetic terms in the same way as $\widetilde{G}\widetilde{e}e$ and $G\widetilde{e}\widetilde{e}$ did. A $G\widetilde{G}\widetilde{H}$ term arises from the $GGHH$ scalar-vector interaction from the kinetic term when one of the Higgs gains a VEV.

Chargino Coupling to Z^0 and γ I: Weak Eigenbasis

We will use the notation of Haber and Kane[12]

$$\mathcal{L}_{Z^0\widetilde{\chi}_i^+\widetilde{\chi}_j^-} \equiv \frac{g}{\cos\theta_W}\, Z_\mu \left[\frac{1}{2}\, \overline{\widetilde{\chi}}_i^+\, \gamma^\mu \left(O_{ij}^{\prime L}\gamma_- + O_{ij}^{\prime R}\gamma_+\right) \widetilde{\chi}_j^+\right] + \text{h.c.} \tag{I.176}$$

to define the mixing matrices O'_{ij} from the matrices U and V defined in (I.146). We will use the 4-component spinors $\widetilde{W}$ and $\widetilde{H}^+$ in (I.148)and the Majorana spinors from (I.8) and (I.9).

The total $\gamma\widetilde{\chi}\widetilde{\chi} + Z^0\widetilde{\chi}\widetilde{\chi}$ term is, in two-component form (using (I.175)):

$$
\begin{aligned}
\mathcal{L}_{\widetilde{\chi}\widetilde{\chi}Z^0} = & -gW_\mu^0\left(\overline{\widetilde{W}^-}\overline{\sigma}^\mu\widetilde{W}^+ - \overline{\widetilde{W}^+}\overline{\sigma}^\mu\widetilde{W}^-\right) \\
& -\frac{1}{2}\left(g\tau_{ij}^3W_\mu^0 + g'y_k\delta_{ij}B_\mu^0\right)\left(\overline{\widetilde{\psi}}_{H_k}\right)^i\overline{\sigma}^\mu\left(\widetilde{\psi}_{H_k}\right)^j + \text{h.c.}
\end{aligned}
\tag{I.177}
$$

where $y_k = \pm 1$ can be read off of Table I.1. The $\tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ terms include both chargino and neutralino parts.

Using (A.1) and (A.3) we may assemble this into Majorana spinors

$$
\begin{aligned}
\mathcal{L} = & -gW_\mu^0(\overline{\widetilde{W}^+})\gamma^\mu\widetilde{W}^+ - \frac{1}{2}\left(gW_\mu^0 + g'B_\mu^0\right)(\overline{\widetilde{H}^+})\gamma^\mu\widetilde{H}^+ \\
& + \frac{1}{4}\left(g\widetilde{W}_\mu^0 - g'\widetilde{B}_\mu^0\right)\left(\overline{\widetilde{H}}_1\gamma^\mu\gamma_5\widetilde{H}_1 - \overline{\widetilde{H}}_2\gamma^\mu\gamma_5\widetilde{H}_2\right) + \text{h.c.}
\end{aligned}
\tag{I.178}
$$

The first two terms constitute the chargino contribution. Using

$$
\begin{aligned}
Z_\mu &= \cos\theta_W\, W_\mu^0 - \sin\theta_W\, B_\mu^0 \\
A_\mu &= \sin\theta_W\, W_\mu^0 + \cos\theta_W\, B_\mu^0
\end{aligned}
\tag{I.179}
$$

results in the chargino part:

$$
\begin{aligned}
\mathcal{L}_{G^0-\text{chargino}} = & -\frac{g}{\cos\theta_W}\,Z_\mu\left[\cos^2\theta_W(\overline{\widetilde{W}^+})\gamma^\mu\widetilde{W} + \frac{1}{2}\cos 2\theta_W(\overline{\widetilde{H}^+})\gamma^\mu\widetilde{H}\right] \\
& - eA_\mu\left[(\overline{\widetilde{W}^+})\gamma^\mu\widetilde{W} + (\overline{\widetilde{H}^+})\gamma^\mu\widetilde{H}\right] + \text{h.c.}
\end{aligned}
\tag{I.180}
$$

Relating Bases

To rewrite this in terms of the physical states $\widetilde{\chi}_i^+$ we need to relate the bases of states discussed. In summary:

$$
\begin{aligned}
&\psi_j^+ = \begin{pmatrix} -i\widetilde{W}^+ \\ \widetilde{\psi}_{H_2}^+ \end{pmatrix} &&\psi_j^- = \begin{pmatrix} -i\widetilde{W}^- \\ \widetilde{\psi}_{H_1}^- \end{pmatrix} &&\text{from (I.142)}\\
&\widetilde{W}_j^+ = \begin{pmatrix} -i\widetilde{W}^+ \\ i(\overline{\widetilde{W}^-}) \end{pmatrix} &&\widetilde{H}_j^+ = \begin{pmatrix} \widetilde{\psi}_{H_2}^+ \\ (\overline{\widetilde{\psi}_{H_1}^-}) \end{pmatrix} &&\text{from (I.148)}\\
&\widetilde{\chi}_{1j}^+ = \begin{pmatrix} \widetilde{\chi}_1^+ \\ \overline{\widetilde{\chi}_1^-} \end{pmatrix} &&\widetilde{\chi}_{2j}^+ = \begin{pmatrix} \widetilde{\chi}_2^+ \\ \overline{\widetilde{\chi}_2^-} \end{pmatrix} &&\text{from (I.148)}
\end{aligned}
\tag{I.181}
$$

where $j = 1, 2$. Recall that

$$
\widetilde{\chi}_i^+ = V_{ij}\,\psi_j^+ \qquad \widetilde{\chi}_i^- = U_{ij}\,\psi_j^-
$$

$(i, j = 1, 2)$ so

$$
\psi_j^+ = (V^\dagger)_{ji}\,\widetilde{\chi}_i^+ \qquad \psi_j^- = (U^\dagger)_{ji}\,\widetilde{\chi}_i^- \tag{I.182}
$$

Now $\gamma_\pm = \frac{1}{2}(1 \pm \gamma_5)$

$$
\gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \gamma_- = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad \gamma_+ = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{I.183}
$$

Hence

$$
\begin{aligned}
&\gamma_-\,\widetilde{W}^+ = \begin{pmatrix} -i\widetilde{W}^+ \\ 0 \end{pmatrix} &&\gamma_+\,\widetilde{W}^+ = \begin{pmatrix} 0 \\ i\overline{\widetilde{W}^-} \end{pmatrix}\\
&\gamma_-\,\widetilde{H}^+ = \begin{pmatrix} \widetilde{\psi}_{H_2}^+ \\ 0 \end{pmatrix} &&\gamma_+\,\widetilde{H}^+ = \begin{pmatrix} 0 \\ \overline{\widetilde{\psi}_{H_1}^-} \end{pmatrix}\\
&\gamma_-\,\widetilde{\chi}_i^+ = \begin{pmatrix} \widetilde{\chi}_i^+ \\ 0 \end{pmatrix} &&\gamma_+\,\widetilde{\chi}_i^+ = \begin{pmatrix} 0 \\ \overline{\widetilde{\chi}_i^-} \end{pmatrix}
\end{aligned}
\tag{I.184}
$$

Now, in anticipation of future needs we introduce the conjugate spinors $\widetilde{W}^c$

and $\widetilde{\chi}_i^c$ (see also sections A.4 and M.1). In general if

$$\psi = \begin{pmatrix} \xi_\alpha \\ \overline{\eta}^{\dot{\alpha}} \end{pmatrix}$$

then

$$\overline{\psi} = (\eta^\alpha \quad \xi_{\dot{\alpha}}) \qquad \psi^c = \begin{pmatrix} \eta_\alpha \\ \xi^{\dot{\alpha}} \end{pmatrix} \tag{I.185}$$

so

$$\widetilde{W}^c = \begin{pmatrix} -i\widetilde{W}^- \\ i\overline{\widetilde{W}}^+ \end{pmatrix} \quad \widetilde{\chi}_1^c = \begin{pmatrix} \widetilde{\chi}_1^- \\ \overline{\widetilde{\chi}}_1^+ \end{pmatrix} \quad \widetilde{\chi}_2^c = \begin{pmatrix} \widetilde{\chi}_2^- \\ \overline{\widetilde{\chi}}_2^+ \end{pmatrix} \tag{I.186}$$

and

$$\gamma_- \widetilde{W}^c = \begin{pmatrix} -i\widetilde{W}^- \\ 0 \end{pmatrix} \qquad \gamma_+ \widetilde{W}^c = \begin{pmatrix} 0 \\ i\overline{\widetilde{W}}^+ \end{pmatrix} \tag{I.187}$$

From (I.182) and (I.184)

$$\begin{aligned} \gamma_- \widetilde{W}_j^+ \ (4-\text{comp.}) &= -i\widetilde{W}^+ \ (2-\text{comp.}) \ = \psi_1^+ = (V^\dagger)_{11}\widetilde{\chi}_1^+ + (V^\dagger)_{12}\widetilde{\chi}_2^+ \\ &= V_{11}^*\widetilde{\chi}_1^+ + V_{21}^*\widetilde{\chi}_2^+ \ (2-\text{comp.}) \\ &= \gamma_-(V_{11}^*\widetilde{\chi}_1^+ + V_{21}^*\widetilde{\chi}_2^+) \ (4-\text{comp.}) \end{aligned} \tag{I.188}$$

Using (I.184) again, so

$$\gamma_- \widetilde{W}^+ = \gamma_-(V_{11}^*\widetilde{\chi}_1^+ + V_{21}^*\widetilde{\chi}_2^+) \ .$$

Similarly, from (I.181) and (I.182)

$$\begin{aligned} -i\widetilde{W}^- = \psi_1^- &= U_{11}^\dagger\widetilde{\chi}_1^- + U_{12}^\dagger\widetilde{\chi}_2^- \\ &= U_{11}^*\widetilde{\chi}_1^- + U_{21}^*\widetilde{\chi}_2^- \end{aligned}$$

thus

$$i\overline{\widetilde{W}}^- = U_{11}\overline{\widetilde{\chi}}_1^- + U_{21}\overline{\widetilde{\chi}}_2^- \ (2-\text{comp.})$$

$$= U_{11}\gamma_+\widetilde{\chi}_1^- + U_{21}\gamma_+\widetilde{\chi}_2^- \ (4-\text{comp.})$$

and since

$$\gamma_+ \widetilde{W}^+ \ (4-\text{comp.}) \ = i\overline{\widetilde{W}}^- \ (2-\text{comp.}) \ \text{from(I.181)}$$

$$\gamma_+ \widetilde{W}^+ = \gamma_+(U_{11}\ \widetilde{\chi}_1^+ + U_{21}\ \widetilde{\chi}_2^+) \tag{I.189}$$

For the Higgsinos

$$\widetilde{\psi}_{H_2}^+ = \psi_2^+ = (V^\dagger)_{21}\widetilde{\chi}_1^+ + (V^\dagger)_{22}\widetilde{\chi}_2^+$$

$$= \gamma_-(V_{12}^*\widetilde{\chi}_1^+ + V_{22}^*\widetilde{\chi}_2^+)$$

and

$$\gamma_-\ \widetilde{H}^+ = \gamma_-(V_{12}^*\widetilde{\chi}_1 + V_{22}^*\widetilde{\chi}_2)$$

$$\widetilde{\psi}_{H_1}^- = \psi_2^- = (U^\dagger)_{21}\widetilde{\chi}_1^- + (U^\dagger)_{22}\widetilde{\chi}_2^-$$

$$\Longrightarrow \qquad \overline{\widetilde{\psi}}_{H_1}^+ = (U^T)_{12}\widetilde{\chi}_1^+ + (U^T)_{22}\widetilde{\chi}_2^+$$

$$= \gamma_+(U_{12}\widetilde{\chi}_1^+ + U_{22}\widetilde{\chi}_2^+)$$

so

$$\gamma_+\ \widetilde{H}^+ = \gamma_+(U_{12}\widetilde{\chi}_1^+ + U_{22}\widetilde{\chi}_2^+)$$

as in (I.188)

$$-i\widetilde{W}^+ = \psi_1^+ = (V^\dagger)_{11}\widetilde{\chi}_1^+ + (V^\dagger)_{12}\widetilde{\chi}_2^+$$

$$\Longrightarrow \qquad i\overline{\widetilde{W}}^+ = (V^T)_{11}\overline{\widetilde{\chi}}_1^+ + (V^T)_{12}\overline{\widetilde{\chi}}_2^+$$

$$\Longrightarrow \qquad \gamma_+\widetilde{W}^c = \gamma_+(V_{11}\widetilde{\chi}_1^c + V_{12}\widetilde{\chi}_2^c)$$

Summarizing these results

$$
\begin{aligned}
\gamma_+\widetilde{W}^+ &= \gamma_+(U_{11}\widetilde{\chi}_1 + U_{21}\widetilde{\chi}_2) \\
\gamma_-\widetilde{W}^+ &= \gamma_-(V_{11}^*\widetilde{\chi}_1^+ + V_{21}^*\widetilde{\chi}_2^+) \\
\gamma_+\widetilde{H}^+ &= \gamma_+(U_{12}\widetilde{\chi}_1^+ + U_{22}\widetilde{\chi}_2^+) \\
\gamma_-\widetilde{H}^+ &= \gamma_-(V_{12}^*\widetilde{\chi}_1^+ + V_{22}^*\widetilde{\chi}_2^+) \\
\gamma_+\widetilde{W}^c &= \gamma_+(V_{11}\widetilde{\chi}_1^c + V_{21}\widetilde{\chi}_2^c)
\end{aligned}
\tag{I.190}
$$

Taking the normal Hermitian conjugate in (I.185) ($\overline{\gamma}_+ = \gamma_-$)

$$
\begin{aligned}
\overline{\widetilde{W}}\,\gamma_+ &= (V_{11}\overline{\widetilde{\chi}}_1 + V_{21}\overline{\widetilde{\chi}}_2)\gamma_+ \\
\overline{\widetilde{W}}\,\gamma_- &= (U_{11}^*\overline{\widetilde{\chi}}_1 + U_{21}^*\overline{\widetilde{\chi}}_2)\gamma_- \\
\overline{\widetilde{H}}\,\gamma_+ &= (V_{12}\overline{\widetilde{\chi}}_1 + V_{22}\overline{\widetilde{\chi}}_2)\gamma_+ \\
\overline{\widetilde{H}}\,\gamma_- &= (U_{12}^*\overline{\widetilde{\chi}}_1 + U_{22}^*\overline{\widetilde{\chi}}_2)\gamma_-
\end{aligned}
\tag{I.191}
$$

Chargino Coupling to Z^0 and γ II: Mass Eigenbasis

We now substitute (I.190) and (I.191) into (I.180). We use

$$
\{\gamma_\pm, \gamma^\mu\} = 0 \qquad \gamma_+ + \gamma_- = 1 \qquad \gamma_\pm^2 = 1 \tag{I.192}
$$

to note that

$$
\begin{aligned}
\gamma^\mu &= (\gamma_+ + \gamma_-)\,\gamma^\mu(\gamma_+ + \gamma_-) \\
&= \gamma_+\,\gamma^\mu\,\gamma_- + \gamma_-\,\gamma^\mu\,\gamma_+ \,.
\end{aligned}
\tag{I.193}
$$

Then

$$\mathcal{L}_{Z^\circ \widetilde{\chi}^+ \widetilde{\chi}^-} = \frac{g}{\cos\theta_W}\, Z_\mu \Big[-\cos^2\theta_W \overline{\widetilde{W}} (\gamma_-\gamma^\mu\gamma_+ + \gamma_+\gamma^\mu\gamma_-)\widetilde{W}^+$$

$$-\cos 2\theta_W \;\overline{\widetilde{H}}(\gamma_-\gamma^\mu\gamma_+ + \gamma_+\gamma^\mu\gamma_-)\widetilde{W}^- \Big] + \text{h.c.}$$

$$= \frac{g}{\cos\theta_W}\, Z \Big\{ -\cos^2\theta_W \; \Big[(U^*_{11}\overline{\widetilde{\chi}}_1 + U^*_{21}\overline{\widetilde{\chi}}_2)\gamma_-\gamma^\mu\gamma_+ (U_{11}\widetilde{\chi}^+_1 + U_{21}\widetilde{\chi}^+_2)$$

$$+ (V_{11}\overline{\widetilde{\chi}}_1 + V_{21}\overline{\widetilde{\chi}}_2)\gamma_+\gamma^\mu\gamma_- (V^*_{11}\widetilde{\chi}^+_1 + V^*_{21}\widetilde{\chi}^+_2)\Big]$$

$$-\frac{1}{2}\cos 2\theta_W \; \Big[(U^*_{12}\overline{\widetilde{\chi}}_1 + U^*_{22}\overline{\widetilde{\chi}}_2)\gamma_-\gamma^\mu\gamma_+ (U_{12}\widetilde{\chi}^+_1 + U_{22}\widetilde{\chi}^+_2)$$

$$+ (V_{12}\overline{\widetilde{\chi}}_1 + V_{22}\overline{\widetilde{\chi}}_2)\gamma_+\gamma^\mu\gamma_- (V^*_{12}\widetilde{\chi}^+_1 + V^*_{22}\widetilde{\chi}^+_2)\Big]\Big\} + \text{h.c.}$$

Expanding out and collecting terms into the form of (I.176) results in

$$O'^L_{ij} = -\cos^2\theta_W \, V_{i1}\, V^*_{j1} - \frac{1}{2}\cos 2\theta_W \, V_{i2}\, V^*_{j2}$$

$$O'^R_{ij} = -\cos^2\theta_W \, U^*_{i1}\, U^*_{j1} - \frac{1}{2}\cos 2\theta_W \, U^*_{i2}\, U_{j2} \tag{I.194}$$

Therefore

$$O'^L_{ij} = (\sin^2\theta_W - 1) V_{i1}\, V^*_{j1} - \frac{1}{2}(1 - 2\sin^2\theta_W) V_{i2}\, V^*_{j2}$$

$$= \sin^2\theta_W (V_{i1}\, V^*_{j1} + V_{i2}\, V^*_{j2}) - V_{i1}\, V^*_{j1} - \frac{1}{2} V_{i2}\, V^*_{j2}$$

The term in brackets is $(VV^+)_{ij} = \delta_{ij}$ since U and V are unitary. So finally

$$O'^L_{ij} = \delta_{ij}\sin^2\theta_W - V_{i1}\, V^*_{j1} - \frac{1}{2} V_{i2}\, V^*_{j2}$$

$$O'^R_{ij} = \delta_{ij}\sin^2\theta_W - U^*_{i1}\, V_{j1} - \frac{1}{2} U^*_{i2}\, V_{j2} \tag{I.195}$$

$$\mathcal{L}_{Z^\circ \widetilde{\chi}^+_i \widetilde{\chi}^-_j} = \frac{g}{\cos\theta_W}\, Z_\mu \left[\frac{1}{2}\, \overline{\widetilde{\chi}}^+_i \gamma^\mu \left(O'^L_{ij}\gamma_- + O'^R_{ij}\gamma_+ \right) \widetilde{\chi}^+_j \right] + \text{h.c.}$$

The photon term comes out directly

$$\mathcal{L}_{\gamma \widetilde{\chi}^+ \widetilde{\chi}^-} = -ie\gamma^\mu \tag{I.196}$$

as one might expect.

Neutralino Coupling to Z^0 and γ^μ

From (I.178) we see that

$$\begin{aligned}\mathcal{L}_{G^0\text{–neutralino}} &= \frac{1}{4}(gW^0_\mu - g'\,B^0_\mu)(\overline{\widetilde{H}}_1\,\gamma^\mu\,\gamma_5\,\widetilde{H}_1 - \overline{\widetilde{H}}_2\,\gamma^\mu\,\gamma_5\,\widetilde{H}_2) + \text{h.c.} \\ &= \frac{g}{\cos\theta_W}\;Z_\mu \cdot \frac{1}{4}(\overline{\widetilde{H}}_1\,\gamma^\mu\,\gamma_5\,\widetilde{H}_1 - \overline{\widetilde{H}}_2\,\gamma^\mu\,\gamma_5\,\widetilde{H}_2) + \text{h.c.}\end{aligned} \tag{I.197}$$

where the second line follows from (I.179). Since there is normally no Z^3 or $\gamma H^0 H^0$ vertex it is not surprising that the only vertex is the supersymmetric analogue of ZH^0H^0. The terms like $\overline{\widetilde{H}}_i\gamma^\mu\widetilde{H}_i$ vanish because the neutralinos are Majorana fermions ($\widetilde{\chi}^{0c} = \widetilde{\chi}^0$). By analogy with the chargino case we define

$$\mathcal{L}_{Z^0\widetilde{\chi}^0_i\widetilde{\chi}^0_j} = \frac{g}{\cos\theta_W}\;Z_\mu\left[\frac{1}{2}\,\overline{\widetilde{\chi}}{}^0_i\,\gamma^\mu\left(O''^L_{ij}\,\gamma_- + O''^R_{ij}\,\gamma_+\right)\widetilde{\chi}^0_j\right] + \text{h.c.} \tag{I.198}$$

Recalling that $\widetilde{\chi}^0_i = N_{ij}\psi^0_j$ we may derive expressions similar to (I.190) and

(I.191):

$$
\begin{aligned}
\gamma_- \widetilde{B}^0 &= \gamma_- N_{j1}^* \,\widetilde{\chi}_j^0 \\
\gamma_+ \widetilde{B}^0 &= \gamma_+ N_{j1} \,\widetilde{\chi}_j^0 \\
\gamma_- \widetilde{W}^0 &= \gamma_- N_{j2}^* \,\widetilde{\chi}_j^0 \\
\gamma_+ \widetilde{W}^0 &= \gamma_+ N_{j2} \,\widetilde{\chi}_j^0 \\
\gamma_- \widetilde{H}_1 &= \gamma_- N_{j3}^* \,\widetilde{\chi}_j^0 \\
\gamma_+ \widetilde{H}_1 &= \gamma_+ N_{j3} \,\widetilde{\chi}_j^0 \\
\gamma_- \widetilde{H}_2 &= \gamma_- N_{j4}^* \,\widetilde{\chi}_j^0 \\
\gamma_+ \widetilde{H}_2 &= \gamma_+ N_{j4} \,\widetilde{\chi}_j^0
\end{aligned}
\tag{I.199}
$$

where j is summed over $(j = 1,2,3,4)$. The remarkable symmetry is due to the fact that $\widetilde{\chi}_i^0$ is a Majorana fermion. Substituting (I.199) into (I.197) and using

$$
\gamma_5 = \gamma_+ - \gamma_- \tag{I.200}
$$

with (I.192) and (I.193), and then placing the result in the form of (I.198) gives:

$$
\begin{aligned}
O_{ij}^{\prime\prime L} &= -\frac{1}{2}\left[N_{i3}\,N_{j3}^* - N_{i4}\,N_{j4}^*\right] \\
O_{ij}^{\prime\prime R} &= \frac{1}{2}\left[N_{i3}^*\,N_{j3} - N_{i4}^*\,N_{j4}\right]
\end{aligned}
\tag{I.201}
$$

Note that

$$
O_{ij}^{\prime\prime R} = -O_{ij}^{\prime\prime L*} \tag{I.202}
$$

and that photons do not couple to the neutralinos.

Table I.1. Fields Appearing in the Minimal Model

Superfield	Ordinary Matter	Superpartners	Weak Isospin	y
		Vector (Gauge) Multiplets		
$\hat{\mathbf{G}}$	$g^i,\ i=1,...,8$	$\tilde{g}^i,\ i=1,...,8$	Singlet	0
$\hat{\mathbf{V}}$	$W^{\pm,0}$	$\tilde{W}^{\pm,0}$	Triplet	0
$\hat{\mathbf{V}}'$	B^0	$\tilde{B}^0$	Singlet	0
		Scalar Multiplets		
$\hat{\mathbf{Q}}_i$	u_{L_i}, d_{L_i}	$\tilde{u}_{L_i}, \tilde{d}_{L_i}$	Doublet	1/3
$\hat{\mathbf{U}}_i$	$(\bar{u}_i)_L = (u_{R_i})^*$	$(\tilde{u}_{R_i})^*$	Singlet	-4/3
$\hat{\mathbf{D}}_i$	$(\bar{d}_i)_L = (d_{R_i})^*$	$(\tilde{d}_{R_i})^*$	Singlet	2/3
$\hat{\mathbf{L}}_i$	$\nu_i, \ell_{L_i}{}^-$	$\tilde{\nu}_i, \tilde{\ell}_{L_i}$	Doublet	-1
$\hat{\mathbf{R}}_i$	$(\ell_i^+)_L = (\ell_{R_i}^-)^*$	$(\tilde{\ell}_{R_i}^-)^*$	Singlet	2
$\hat{\mathbf{H}}_1$	$H_1{}^0, H_1{}^-$	$\tilde{\psi}_{H_1}^0, \tilde{\psi}_{H_1}^-$	Doublet	-1
$\hat{\mathbf{H}}_2$	$H_2{}^+, H_2{}^0$	$\tilde{\psi}_{H_2}^+, \tilde{\psi}_{H_2}^0$	Doublet	1
		Optional Higgs Singlet		
$\hat{\mathbf{N}}$	N	$\tilde{N}_L$	Singlet	0

FIGURE CAPTIONS

1. Patterns of Gauge and Supersymmetry Breaking.

    (a) Supersymmetry Unbroken and Gauge Symmetry Unbroken.

    (b) Supersymmetry Broken and Gauge Symmetry Unbroken.

    (c) Supersymmetry Unbroken and Gauge Symmetry Broken.

    (d) Supersymmetry Broken and Gauge Symmetry Broken.

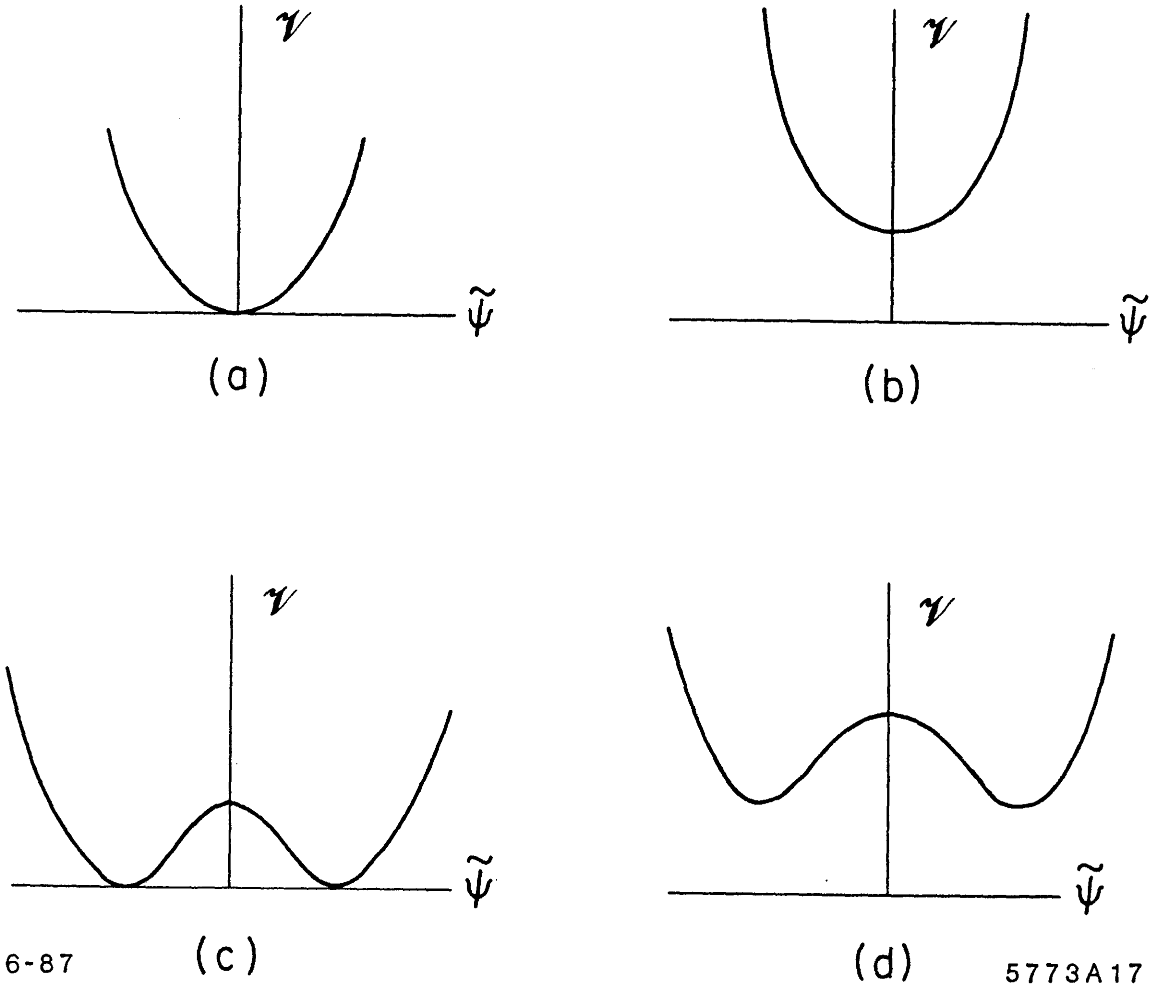


Fig. I.1

APPENDIX J

Mixing Matrices and Mass Eigenvalues

J.1 CHARGINOS

As we saw in the previous appendix (eqns I.142-I.148) the chargino mass terms may be written as

$$\mathcal{L}_{Ch} = -\tfrac{1}{2}\left[(\psi^+)^T X^T \psi^- + (\psi^-)^T X \psi^+\right] + h.c. \tag{J.1}$$

where $\psi^\pm$ are four-component fermions of the form

$$\psi^+ = \begin{pmatrix} -i\widetilde{W}^+ \\ \tilde{\psi}^+_{H_2} \end{pmatrix} \qquad \psi^- = \begin{pmatrix} -i\widetilde{W}^- \\ \tilde{\psi}^-_{H_1} \end{pmatrix} \tag{J.2}$$

and

$$X = \begin{pmatrix} M & \sqrt{2}m_W \cos\theta_v \\ \sqrt{2}m_W \sin\theta_v & \mu \end{pmatrix} . \tag{J.3}$$

We may diagonalize this to obtain the true chargino mass eigenstates which we denote $\widetilde{\chi}^+_i$ $(i = 1,2)$ and $\widetilde{\chi}^-_i$. When this is diagonalized via

$$\widetilde{\chi}^+_i = V_{ij}\psi^+_i \qquad \widetilde{\chi}^-_i = U_{ij}\psi^-_j \tag{J.4}$$

we will have

$$\mathcal{L}_{Ch} = -\tfrac{1}{2}\left[m_{\widetilde{\chi}^+_1}\, \widetilde{\chi}^-_1 \widetilde{\chi}^+_1 + m_{\widetilde{\chi}^+_2}\, \widetilde{\chi}^-_2 \widetilde{\chi}^+_2\right] + h.c. \tag{J.5}$$

The matrices U and V are unitary 2×2 matrices which diagonalize the mass matrix via

$$M_{\text{Diag}} = U^* X V^{-1} . \tag{J.6}$$

Diagonalization of General 2×2 Matrix

In general to diagonalize a regular 2×2 matrix

$$\begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}$$

by $M_D = U^* M V^\dagger$, U and V unitary, to

$$M_D = \begin{pmatrix} M_+ & 0 \\ 0 & M_- \end{pmatrix}$$

we use

$$U = \begin{pmatrix} \cos\phi_- & \sin\phi_- \\ -\sin\phi_- & \cos\phi_- \end{pmatrix} \qquad V = \begin{pmatrix} \cos\phi_+ & \sin\phi_+ \\ -\sin\phi_+ & \cos\phi_+ \end{pmatrix} \qquad D > 0$$

$$U = \begin{pmatrix} \cos\phi_- & \sin\phi_- \\ -i\sin\phi_- & i\cos\phi_- \end{pmatrix} \qquad V = \begin{pmatrix} \cos\phi_+ & \sin\phi_+ \\ -i\sin\phi_+ & i\cos\phi_+ \end{pmatrix} \qquad D < 0$$

where

$$D = \det M = M_{11}M_{22} - M_{12}M_{21}$$

yielding the positive definite mass eigenvalues

$$\begin{aligned} M_\pm &= \tfrac{1}{2}\{\mathrm{M}_+ \pm \mathrm{M}_-\} \qquad && D > 0 \\ M_\pm &= \tfrac{1}{2}\{\mathrm{M}_- \pm \mathrm{M}_+\} \qquad && D < 0\,. \end{aligned} \tag{J.7}$$

Here

$$\mathrm{M}_\pm = \sqrt{(M_{11} \mp M_{22})^2 + (M_{12} \pm M_{21})^2}\,. \tag{J.8}$$

This is an appropriate time to interject a word concerning the positivity of the eigenmasses. Normally, when diagonalizing a matrix one cannot be so cavalier about altering the sign of an eigenvalue. When the determinant of a real 2×2 matrix is negative it indicates one positive and one negative eigenvalue. It has been noted (and can be easily demonstrated) that negating the eigenmass of a spinor is physically equivalent to retaining the original sign and performing the substitution $\psi \rightarrow \gamma_5 \psi$ (or equivalently $\psi \rightarrow -\gamma_5 \psi$). Here we have accomplished the same end by multiplying a given row of both U and V by a factor of i. This negates the corresponding eigenvalue because we have used (J.6) instead of the purely unitary $M_D = UMV^\dagger$. In particular we note that

$$UU^\dagger = VV^\dagger = \mathbf{1} \qquad D > 0$$

$$UU^\dagger = VV^\dagger = -\gamma_5 \qquad D > 0 \, .$$

We may similarly negate neutralino masses. The angular entries satisfy

$$\begin{aligned}
\tan(\phi_+ \pm \phi_-) &= \frac{M_{12} \pm M_{21}}{M_{11} \mp M_{22}} \\
\cos(\phi_+ \pm \phi_-) &= \frac{M_{11} \mp M_{22}}{\mathrm{M}_\pm} \\
\sin(\phi_+ \pm \phi_-) &= \frac{M_{12} \pm M_{21}}{\mathrm{M}_\pm} \\
\tan 2\phi_\pm &= \frac{B_\pm}{A_\pm} \\
& \qquad A_\pm = (M_{11}^2 - M_{22}^2) \pm (M_{21}^2 - M_{12}^2) \\
& \qquad B_+ = 2[M_{11}M_{12} + M_{21}M_{22}] \\
& \qquad B_- = 2[M_{11}M_{21} + M_{12}M_{22}] \\
\tan \phi_\pm &= \frac{\sqrt{A_\pm^2 + B_\pm^2} - A_\pm}{B_\pm} \, .
\end{aligned} \tag{J.9}$$

Charginos: General Case

We proceed to use the above results for the general chargino pattern given by (J.3). The masses of the charginos ($\tilde{\chi}^+$) are given in terms of the supersymmetry breaking terms of M, M' and μ of eqn (3.4) and the Higgs VEV angle, θ_v, of eqn (3.6) (see section I.4 also) by

$$m_{\tilde{\chi}^+_{1,2}} = \tfrac{1}{2}\left\{\mathrm{M}_+ \pm \mathrm{M}_-\right\}\eta_\pm \qquad \mathrm{M}_\pm = \sqrt{(M \pm \mu)^2 + 2M_W^2(1 \mp \sin 2\theta_v)^2}$$
$$\text{with } \eta_+ = 1 \text{ and } \eta_- = \mathrm{sign}[M\mu - M_W^2 \sin 2\theta_v] \tag{J.10}$$

The masses $m_{\tilde{\chi}^+_i}$ have been plotted, for typical parameter values, in Fig. J.1a.

The chargino vertex matrices are given by[44]

$$U_{ij} = \begin{pmatrix} \cos\phi_- & \sin\phi_- \\ -\sin\phi_- & \cos\phi_- \end{pmatrix} \qquad V_{ij} = \begin{pmatrix} \cos\phi_+ & \sin\phi_+ \\ -\sin\phi_+ & \cos\phi_+ \end{pmatrix} \qquad D > 0$$

$$U_{ij} = \begin{pmatrix} \cos\phi_- & \sin\phi_- \\ -i\sin\phi_- & i\cos\phi_- \end{pmatrix} \qquad V_{ij} = \begin{pmatrix} \cos\phi_+ & \sin\phi_+ \\ -i\sin\phi_+ & i\cos\phi_+ \end{pmatrix} \qquad D < 0 \tag{J.11}$$

where

$$D = M\mu - M_W^2 \sin 2\theta_v \tag{J.12}$$

$$\tan\phi_\pm = \frac{\sqrt{{A_\pm}^2 + {B_\pm}^2} - A_\pm}{B_\pm} \tag{J.13}$$

$$\begin{aligned} A_\pm &= (M^2 - \mu^2) \mp 2M_W^2 \cos 2\theta_v \\ B_+ &= 2\sqrt{2}M_W(M\cos\theta_v + \mu\sin\theta_v) \\ B_- &= 2\sqrt{2}M_W(M\sin\theta_v + \mu\cos\theta_v)\ . \end{aligned} \tag{J.14}$$

Recall from Appendix I that the general form of the non-Abelian supersymmetric vertex was given by

$$
\begin{aligned}
O^{L}_{ij}{}' &= -V_{i1}V_{j1}{}^* - \tfrac{1}{2}V_{i2}V_{j2}{}^* + \delta_{ij}\sin^2\theta_w \\
O^{R}_{ij}{}' &= -U_{i1}{}^*U_{j1} - \tfrac{1}{2}U_{i2}{}^*U_{j2} + \delta_{ij}\sin^2\theta_w
\end{aligned}
\tag{J.15}
$$

resulting in the general expressions

$$
\begin{aligned}
O^{L}_{ij}{}' &= \begin{pmatrix} \frac{1}{2}\sin^2\phi_+ - \cos^2\theta_w & \frac{1}{2}\sin\phi_+\cos\phi_+ \\ \frac{1}{2}\sin\phi_+\cos\phi_+ & \frac{1}{2}\cos^2\phi_+ - \cos^2\theta_w \end{pmatrix} \\
O^{R}_{ij}{}' &= \begin{pmatrix} \frac{1}{2}\sin^2\phi_- - \cos^2\theta_w & \frac{1}{2}\sin\phi_-\cos\phi_- \\ \frac{1}{2}\sin\phi_-\cos\phi_- & \frac{1}{2}\cos^2\phi_- - \cos^2\theta_w \end{pmatrix} \\
&\text{for } D > 0
\end{aligned}
\tag{J.16}
$$

$$
\begin{aligned}
O^{L}_{ij}{}' &= \begin{pmatrix} \frac{1}{2}\sin^2\phi_+ - \cos^2\theta_w & -{}^{i}\!/_{2}\sin\phi_+\cos\phi_+ \\ {}^{i}\!/_{2}\sin\phi_+\cos\phi_+ & \frac{1}{2}\cos^2\phi_+ - \cos^2\theta_w \end{pmatrix} \\
O^{R}_{ij}{}' &= \begin{pmatrix} \frac{1}{2}\sin^2\phi_- - \cos^2\theta_w & {}^{i}\!/_{2}\sin\phi_-\cos\phi_- \\ -{}^{i}\!/_{2}\sin\phi_-\cos\phi_- & \frac{1}{2}\cos^2\phi_- - \cos^2\theta_w \end{pmatrix} \\
&\text{for } D < 0 \,.
\end{aligned}
\tag{J.17}
$$

Charginos: Special Case

Here we examine a few specific subcases of interest.

1) $M_{12} = M_{21}$ $(v_1 = v_2)$

When diagonalizing the general 2×2 symmetric matrix $(M_{12} = M_{21})$ we find

$$U = V = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \qquad D > 0$$

$$U = V = \begin{pmatrix} \cos\phi & \sin\phi \\ -i\sin\phi & i\cos\phi \end{pmatrix} \qquad D < 0$$

where

$$D = \det M = M_{11}M_{22} - M_{12}^2$$

and in this instance

$$\phi_+ = \phi_- = \phi$$

which is given by

$$\tan\phi = \frac{\sqrt{A^2 + B^2} - A}{B}$$

$$A = M_{11}^2 - M_{22}^2$$

$$B = 2M_{12}(M_{11} + M_{22}) \; .$$

The positive eigenvalues are

$$M_{\pm} = \tfrac{1}{2}\left\{\sqrt{(M_{11} - M_{22})^2 + 4M_{12}^2} \pm (M_{11} + M_{22})\right\} \; . \tag{J.18}$$

For charginos the symmetric condition implies that $\tan\theta_v = 1$. Restricting ourselves to $-\pi \leq \theta_v \leq \pi$ we conclude that $v_1 = v_2$. Then

$$\tan\phi = \frac{\sqrt{A^2+B^2}-A}{B} \tag{J.19}$$

$$\begin{aligned} A &= M^2 - \mu^2 \\ B &= 2M_W(M+\mu) \end{aligned} \tag{J.20}$$

and

$$M_\pm = \tfrac{1}{2}\left\{\sqrt{(M-\mu)^2 + M_W^2} \pm (M+\mu)\right\} \tag{J.21}$$

are the masses.

2) $M \to 0,\ \mu \to 0\ v_1 \neq v_2$

This corresponds to the instance of the general 2×2 matrix in whi ch it is completely off-diagonal, *i.e.* $M_{11} = M_{22} = 0$ but $M_{12} \neq M_{21}$. In this case some care is required when using the general expressions. There are two equally valid solutions:

$$\begin{aligned} \phi_- = 0,\ \phi_+ = \frac{\pi}{2} \text{ leading to } M_D &= \begin{pmatrix} M_{12} & 0 \\ 0 & M_{21} \end{pmatrix} \\ \phi_- = \frac{\pi}{2},\ \phi_- = 0 \text{ leading to } M_D &= \begin{pmatrix} M_{21} & 0 \\ 0 & M_{12} \end{pmatrix}. \end{aligned} \tag{J.22}$$

For charginos $M_{12} = \sqrt{2}m_w \cos\theta_v$ and $M_{21} = \sqrt{2}m_w \sin\theta_v$. The corresponding

U, V and O' matrices are, for the first case,

$$U = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \qquad \begin{pmatrix} 0 & 1 \\ -i & 0 \end{pmatrix} \tag{J.23}$$

$$O^{L}_{ij}{}' = \begin{pmatrix} \frac{1}{2} - \cos^2\theta_w & 0 \\ 0 & -\cos^2\theta_w \end{pmatrix} \qquad O^{R}_{ij}{}' = \begin{pmatrix} -\cos^2\theta_w & 0 \\ 0 & \frac{1}{2} - \cos^2\theta_w \end{pmatrix} . \tag{J.24}$$

3) "SUSY" Limit: $M = \mu = 0$, $v_1 = v_2$

As above, the mass matrix is completely anti-diagonal however, in this case, the off-diagonal terms are equal as well. The solutions $\phi_\pm = 0, \frac{\pi}{2}$ apply as before but now there are additional possibilities. Any ϕ_-, ϕ_+ subject to $\phi_- + \phi_+ = \frac{\pi}{2}$ is a valid solution. For æstetic reasons we will chose to use the symmetric result

$$\phi_- = \phi_+ = \frac{\pi}{4} .$$

The resulting mixing matrices are

$$\begin{aligned} U = V &= \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -i & i \end{pmatrix} \\ O^{L}_{ij}{}' &= \frac{1}{4} \begin{pmatrix} 4\sin^2\theta_w - 3 & -i \\ i & 4\sin^2\theta_w - 3 \end{pmatrix} \\ O^{R}_{ij}{}' &= \frac{1}{4} \begin{pmatrix} 4\sin^2\theta_w - 3 & i \\ -i & 4\sin^2\theta_w - 3 \end{pmatrix} \end{aligned} \tag{J.25}$$

4) "Unmixed Limit" $M \to \infty,\ \mu = 0$

Assuming that we set $\mu = 0$ prior to taking the limit $M \to \infty$, we see that $D = \det X$ is negative. We find

$$
\begin{aligned}
&M_+ = M \to \infty \\
&M_- \to 0 \text{ as } M_W^2/M
\end{aligned}
\tag{J.26}
$$

ergo we have one very massive and one massless particle. We also determine that

$$
\tan\phi_\pm \to 0 \text{ as } M_W/M
$$

so that

$$
\begin{aligned}
U = V &= \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \\
O_{ij}^{L\,\prime} = O_{ij}^{R\,\prime} &= \begin{pmatrix} -\cos^2\theta_w & 0 \\ 0 & \frac{1}{2} - \cos^2\theta_w \end{pmatrix}
\end{aligned}
\tag{J.27}
$$

J.2 NEUTRALINOS

From $\mathcal{L}_{Br} + \mathcal{L}_{Mix}$ in eqns (3.4) and (3.6) we can write the neutralino mass terms in the form:[13]

$$
\mathcal{L}_{mass} = -\tfrac{1}{2}\,(\psi^0)^T Y \psi^0 + h.c. \tag{J.28}
$$

where Y was given by eqn (3.8) or eqn (I.156) and

$$
\left(\psi_j^0\right)^T = (-i\tilde{B}^0, -i\tilde{W}^0, \tilde{\psi}_{H_1}^0, \tilde{\psi}_{H_2}^0)\ . \tag{J.29}
$$

Y is diagonalized by the unitary matrix N via $M_D = N^* Y N^\dagger$ (we can choose N^* instead of N because we use $(\psi^0)^T$ instead of $(\psi^0)^\dagger$.) The *mass eigenstates* then satisfy $\tilde{\chi}_i^0 = N_{ij}\psi_j^0$ $(i = 1,2,3,4)$. The masses $m_{\tilde{\chi}_i^+}$ and $m_{\tilde{\chi}_i^0}$ have been plotted, for typical parameter values, in Fig. J.1b. The neutralino vertex matrices are given by N and by[12]

$$O^{L''}_{ij} = \tfrac{1}{2}\left(N_{i4}N_{j4}^* - N_{i3}N_{j3}^*\right) \tag{J.30}$$

$$O^{R''}_{ij} = -O^{L''*}_{ij} \tag{J.31}$$

The General Case

The general 4×4 mass matrix may be diagonalized much as in the chargino case. We now must solve a quartic equation. The general roots are sufficiently involved that it is almost always more expedient to diagonalize specific cases numerically. Furthermore if a fifth neutralino, such as N, is added the resulting quintic is typically unsovable in closed form. There are a number of special cases which arise which are tractable analytically.

A Special Case

An example of an instance in which the 4×4 Y matrix may be readily diagonalized occurs when $v_1 = v_2$ and $M = M'$. The various general forms following eqn (I156) then become:

$$N' = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\omega & \sin\omega & 0 \\ 0 & -\sin\omega & \cos\omega & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\tan\omega = \frac{\mathrm{M} + \mu - M}{2M_Z} \qquad \tan 2\omega = \frac{2M_Z}{M - \mu}\,. \tag{J.32}$$

Here

$$\mathrm{M} = \sqrt{(M-\mu)^2 + 4M_Z^2}\ . \tag{J.33}$$

Since

$$B = \begin{pmatrix} \cos\theta_w & \sin\theta_w & 0 & 0 \\ -\sin\theta_w & \cos\theta_w & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

we obtain

$$N = \begin{pmatrix} \cos\theta_w & \sin\theta_w & 0 & 0 \\ -\cos\omega\sin\theta_w & \cos\omega\cos\theta_w & \frac{1}{\sqrt{2}}\sin\omega & -\frac{1}{\sqrt{2}}\sin\omega \\ \sin\omega\sin\theta_w & -\sin\omega\cos\theta_w & \frac{1}{\sqrt{2}}\cos\omega & -\frac{1}{\sqrt{2}}\cos\omega \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}\ . \tag{J.34}$$

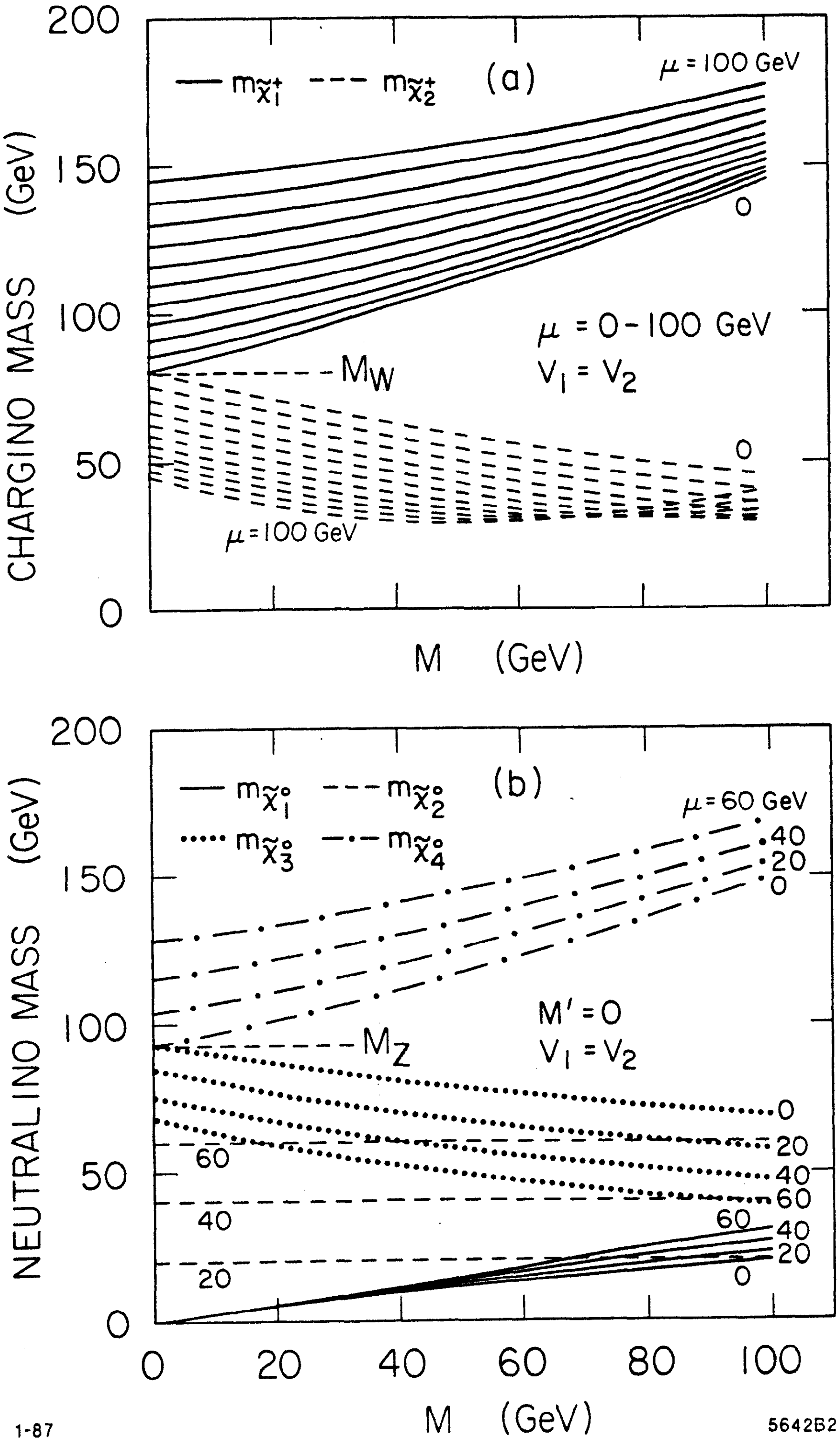
The mass eigenvalues are

$$\begin{aligned} m_{\tilde{\chi}_1^0} &= M \\ m_{\tilde{\chi}_2^0} &= \tfrac{1}{2}\left\{M + \mu + \mathrm{M}\right\} \\ m_{\tilde{\chi}_3^0} &= \tfrac{1}{2}\left\{M + \mu - \mathrm{M}\right\} \\ m_{\tilde{\chi}_4^0} &= -\mu\ . \end{aligned} \tag{J.35}$$

To negate $m_{\tilde{\chi}_i^0}$ multiply the i^{th} row of N by $\pm i$ as discussed in the section on charginos.

FIGURE CAPTIONS

1. Masses for the indicated supersymmetry-breaking parameters.
   a) Charginos
   b) Neutralinos



200
150
100
50
0
CHARGINO MASS (GeV)
$m_{\tilde{\chi}_1^+}$
$m_{\tilde{\chi}_2^+}$
(a)
μ = 100 GeV
0
μ = 0–100 GeV
$V_1 = V_2$
M_W
0
μ=100 GeV
M (GeV)
200
150
100
50
0
NEUTRALINO MASS (GeV)
$m_{\tilde{\chi}_1^0}$
$m_{\tilde{\chi}_2^0}$
$m_{\tilde{\chi}_3^0}$
$m_{\tilde{\chi}_4^0}$
(b)
μ=60 GeV
40
20
0
M′ = 0
$V_1 = V_2$
M_Z
0
20
40
60
60
40
20
60
40
20
0
0
20
40
60
80
100
M (GeV)
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Fig. J.1

APPENDIX K

Feynman Rules

K.1 INTRODUCTION

Given the Lagrangian $\mathcal{L}$ it is a straightforward process, in principle, to generate the Feynman rules for a theory. The generating functional is

$$W[J] = \int [d\theta]\, e^{i\left[S+\int J(x)\phi(x,\theta,\overline{\theta})d^4\theta\right]} \tag{K.1}$$

where the action is given by

$$S = \int \mathcal{L}(x,\theta,\overline{\theta})\, d^4x\, d^4\theta \tag{K.2}$$

for a supersymmetric theory. We then generate connected Green's functions via

$$G^{(n)}(x_1,\ldots,x_N) = \langle 0|T(\phi(x_1)\ldots\phi(x_n))|0\rangle = i^n\, \frac{\delta^n \ell n\, W[J]}{\delta J(x_1)\ldots\delta J(x_n)}\ . \tag{K.3}$$

This can be done with the superfields directly leading to the so-called "supergraph" approach. Such a procedure has proved useful in proving non-renormalization theorems, deriving super-Slavnov-Taylor identities and so forth but has limited utility in performing practical calculations when the supersymmetry has been explicitly broken. In practice it is far easier to read the Feynman rules directly from $\mathcal{L}$ once it has been fully expanded (remembering the factor of "i" from (K.1) and any symmetry factors). There is yet another complication in the gaugino sector. Because we have *Majorana* fermions, which carry no conserved internal quantum number, the direction which a spinor line points is somewhat arbitrary. Using the procedure and construction of refs. 12 and 13 we assign the direction conventionally and establish the corresponding rules.

K.2 Majorana Fermions

Charge Conjugation

The positive and negative-energy solutions to the Dirac equation

$$(i \not{\partial} - e \not{A} - m)\psi = 0$$

are $u(p,s)$ and $v(p,s)$ respectively. It is convenient to define the anti-fermion, ψ^c, which is a positive-energy solution to $(i \not{\partial} + e \not{A} - m)\psi = 0$.

Given (since $\overline{\psi} = \psi^\dagger \gamma^0$)

$$\psi = \begin{pmatrix} \xi_\alpha \\ \overline{\eta}^{\dot{\alpha}} \end{pmatrix} \qquad \overline{\psi} = (\eta^\alpha \; \overline{\xi}_{\dot{\alpha}}) \qquad \psi^c = \begin{pmatrix} \eta_\beta \\ \overline{\xi}^{\dot{\beta}} \end{pmatrix} \tag{K.4}$$

$$\xi^\alpha = \epsilon^{\alpha\beta}\xi_\beta \qquad C = i\gamma^2\gamma^0 = \begin{pmatrix} \epsilon_{\alpha\beta} & 0 \\ 0 & \epsilon^{\dot{\alpha}\dot{\beta}} \end{pmatrix} \tag{K.5}$$

we see that

$$\psi^C = C\overline{\psi}^T \; . \tag{K.6}$$

Note the following useful relations:

$$C^\dagger = C^{-1} \qquad C = \text{charge conjugation matrix} \tag{K.7}$$

$$C^T = -C \tag{K.8}$$

$$C^{-1}\gamma_5 C = (\gamma_5)^T \qquad C^{-1}\gamma^\mu C = -(\gamma^\mu)^T \tag{K.9}$$

where in our notation

$$\gamma_{\pm} = \tfrac{1}{2}\left(1 \pm \gamma_5\right) \qquad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{K.10}$$

For any matrix M, $(M^T)^{-1} = (M^{-1})^T$, in particular

$$(C^T)^{-1} = (C^{-1})^T \tag{K.11}$$

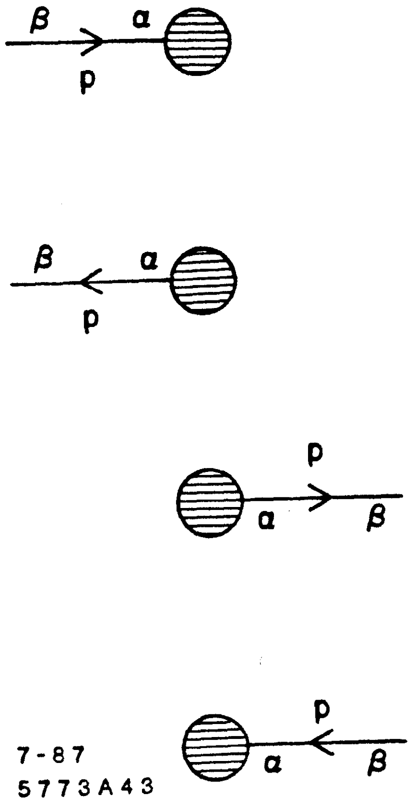
$$u^T(k,s) = \overline{v}(k,s)C^T \qquad v^T(k,s) = \overline{u}(k,s)C^T$$

$$\overline{u}^T(k,s) = C^{-1}v(k,s) \qquad \overline{v}^T(k,s) = C^{-1}u(k,s) \tag{K.12}$$

$$\gamma^\mu \gamma_{\pm} = \gamma_{\mp}\gamma^\mu \qquad (\gamma_{\pm})^2 = \gamma_{\pm} \qquad \gamma_{\pm}\gamma_{\mp} = 0$$

In any process we attach spinor legs according to

Initial State Final State



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$u(p)_{\alpha\beta}$

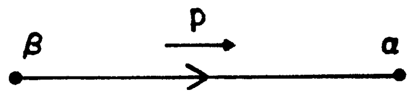
$\overline{v}(p)_{\beta\alpha}$

$\overline{u}(p)_{\beta\alpha}$

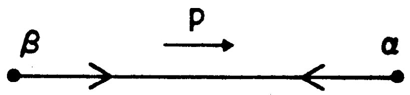
$v(p)_{\alpha\beta}$

(K.13)

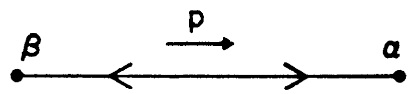
Majorana Propagators:



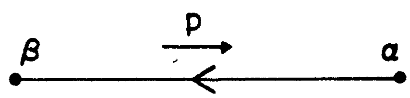
$$\left(\frac{i(\not{p}+m)}{p^2-m^2+i\epsilon}\right)_{\alpha\beta}$$



$$\left(\frac{iC^{-1}(\not{p}+m)}{p^2-m^2+i\epsilon}\right)_{\alpha\beta}$$



$$\left(\frac{-i(\not{p}+m)C}{p^2-m^2+i\epsilon}\right)_{\alpha\beta}$$



$$\left(\frac{i(-\not{p}+m)}{p^2-m^2+i\epsilon}\right)_{\alpha\beta}$$

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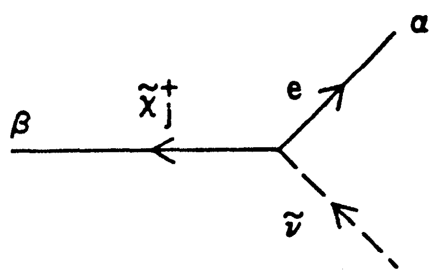
Vertex Manipulation

Let λ, ψ, c be a general Majorana fermion, Dirac fermion and boson field respectively, converging at a single vertex. Then[12] the various permutations of the direction arrows and corresponding rules are illustrated in Fig. K.1. The ambiguities arise because Majorana fermions carry no conserved quantum number.

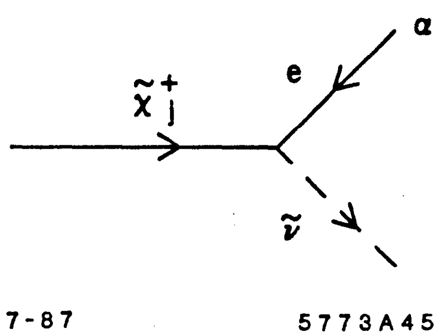
K.3 Chargino Vertices

Let e, ν indicate a representative lepton doublet with no family mixing (arrows indicate the flow of positive charge).[13] We ignore m_e since in the matrix element it is negligible relative to m_Z. The vertex is



$$[-igV_{j1}\gamma_+C]_{\alpha\beta}$$

(K.14)



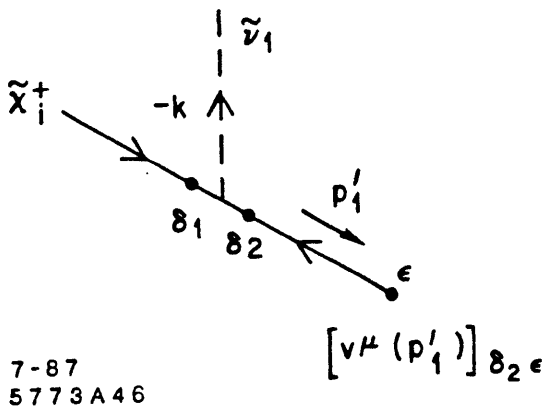
$$[igC^{-1}V_{j1}^*\gamma_-]_{\beta\alpha}$$

Where V_{ij} was defined in Appendices I and J.

When we go to the case where $\tilde{\nu}$ is $\tilde{\nu}_1$ or $\tilde{\nu}_2$ instead of $\tilde{\nu}_e$ (*i.e.* a mixture in flavor space) we will in addition have mixing angles $(\sin\theta_\nu, \cos\theta_\nu)$ in the vertex (in general when all 3 generators mix, we will have a mixing matrix element).

To see how some of the above rules work we will derive two new effective rules for the previous vertices when the Dirac lepton (e) is an external particle and the corresponding spinor has *not* been truncated. We will go directly to the case of $\mu - \tau$ mixing,

$$\widetilde{\nu}_1 = \widetilde{\nu}_\mu \cos\theta_\nu + \widetilde{\nu}_\tau \sin\theta_\nu \qquad \widetilde{\nu}_2 = -\widetilde{\nu}_\mu \sin\theta_\nu + \widetilde{\nu}_\tau \cos\theta_\nu \tag{K.15}$$

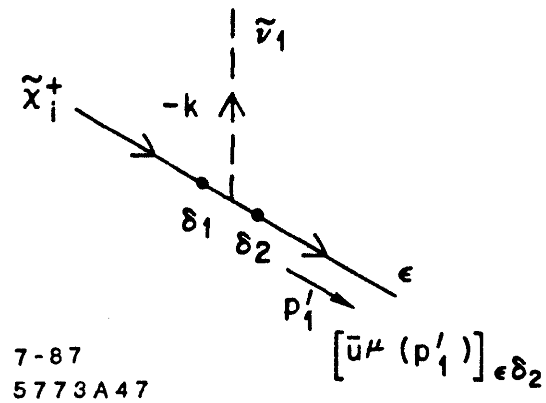


$$[igC^{-1}V_{i1}^*\gamma_-]_{\delta_1\delta_2}\cos\theta_\nu \tag{K.16}$$

This part of the matrix element is

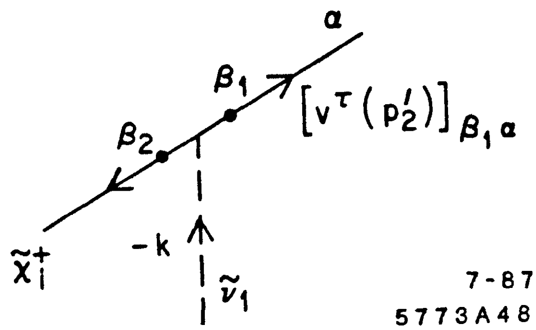
$$\begin{aligned}
&[v^\mu(p_1')]_{\delta_2\epsilon}[igC^{-1}V_{i1}^*\gamma_-]_{\delta_1\delta_2}\cos\theta_\nu \\
&= ig\cos\theta_\nu \left([v^\mu(p_1')]^T\right)_{\epsilon\delta_2} V_{i1}^* \left([C^{-1}\gamma_-]^T\right)_{\delta_2\delta_1} \\
&= igV_{i1}^*\cos\theta_\nu \left([v^\mu(p_1')]^T[C^{-1}\gamma_-]^T\right)_{\epsilon\delta_1} \\
&= igV_{i1}^*\cos\theta_\nu \left(\overline{u}^\mu(p_1')C^T[(\gamma_-)^T C^{-1}]^T\right)_{\epsilon\delta_1} \quad \text{using (K.9) and (K.12)} \qquad \text{(K.17)} \\
&= igV_{i1}^*\cos\theta_\nu \left(\overline{u}^\mu(p_1')C^T(C^{-1})^T\gamma_-\right)_{\epsilon\delta_1} \\
&= igV_{i1}^*\cos\theta_\nu \left(\overline{u}^\mu(p_1')C^T(C^T)^{-1}\gamma_-\right)_{\epsilon\delta_1} \quad \text{using (K.11)} \\
&= igV_{i1}^*\cos\theta_\nu \left(\overline{u}^\mu(p_1')\gamma_-\right)_{\epsilon\delta_1}
\end{aligned}$$

Now regarding Fig. K.1 we can graphically depict this as



$$[\overline{u}^\mu(p_1')]_{\epsilon\delta_2}[igV_{i1}^*\gamma_-]_{\delta_2\delta_1}\cos\theta_\nu \tag{K.18}$$

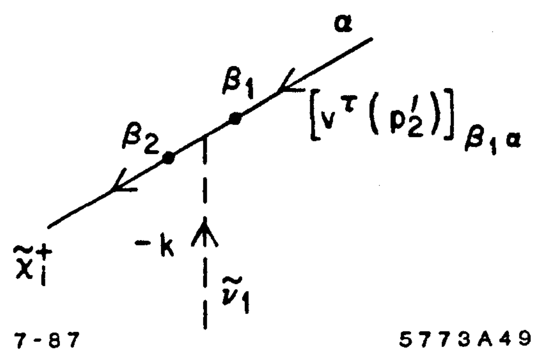
Thus (K.18) is a new effective non-truncated vertex (which will prove useful). Similarly, from (K.14) the conjugate of the previous example gives us



$$[-igV_{i1}\gamma_+C]_{\beta_1\beta_2}\sin\theta_\nu \tag{K.19}$$

$$\begin{aligned}
&= [-igV_{i1}\gamma_+C]_{\beta_1\beta_2}\sin\theta_\nu[\overline{u}^\tau(p_2')]_{\alpha\beta_1} \\
&= -igV_{i1}\sin\theta_\nu[\gamma_+C]^T_{\beta_2\beta_1}[\overline{u}^\tau(p_2')]^T_{\beta_1\alpha} \\
&= -igV_{i1}\sin\theta_\nu\left([C\gamma_+^T]^T[C^{-1}v^\tau(p_2')]\right)_{\beta_2\alpha} \\
&= -igV_{i1}\sin\theta_\nu\left(\gamma_+C^TC^{-1}v^\tau(p_2')\right)_{\beta_2\alpha} \\
&= -igV_{i1}\sin\theta_\nu\left(\gamma_+(-C)C^{-1}v^\tau(p_2')\right)_{\beta_2\alpha} \\
&= [igV_{i1}\sin\theta_\nu\gamma_+v^\tau(p_2')]_{\beta_2\alpha}
\end{aligned} \tag{K.20}$$

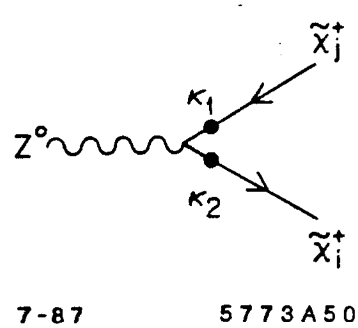
Depicted graphically as



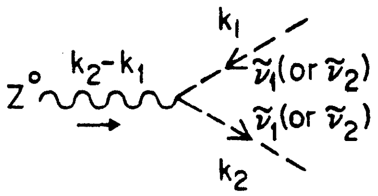
$$[igV_{i1}\gamma_+]_{\beta_2\beta_1}\sin\theta_\nu[v^\tau(p_2')]_{\beta_1\alpha} \tag{K.21}$$

It must be emphasized that (K.18) and (K.21) are valid *only* for the specific case where the lepton (μ or τ) is (1) external and (2) in the final state (as in our problem). In general (K.16) and (K.19) must be used. We will refer to (K.18) and (K.21) as "derived" Feynman rules and use them to great effect in the next appendix.

Other "chargino" vertices

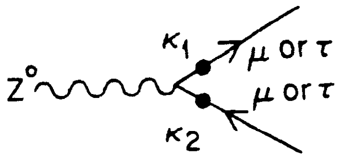


$$\frac{ig}{\cos\theta_w}\,[\gamma^\omega(O'^L_{ij}\gamma_- + O'^R_{ij}\gamma_+)]_{\kappa_2\kappa_1} \tag{K.22}$$



$$\frac{-ig}{2\cos\theta_w}\,[k_1+k_2]^\omega \tag{K.23}$$

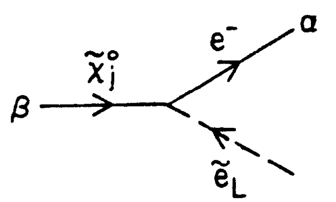
And the normal Z^0-lepton vertex



$$\frac{ig}{2\cos\theta_w}\,[\gamma^\omega(\cos 2\theta_w\gamma_- - 2\sin^2\theta_w\gamma_+)]_{\kappa_1\kappa_2}$$

$$= ie[\gamma^\omega(\cot 2\theta_w\gamma_- - \tan\theta_w\gamma_+)]_{\kappa_1\kappa_2} \tag{K.24}$$

K.4 Neutralino Vertices



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$$\frac{ig}{\sqrt{2}\cos\theta_w}\left[\sin 2\theta_w N'_{j1} + \cos 2\theta_w N'_{j2}\right]\gamma_+ \tag{K.25}$$

With no family mixing.

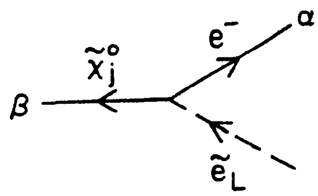
We wish to express our results in terms of N_{ij} (N and N' were defined in Appendix I).

$$N'_{j1} = N_{j1}\cos\theta_w + N_{j2}\sin\theta_w \qquad N'_{j2} = -N_{j1}\sin\theta_w + N_{j2}\cos\theta_w \tag{K.26}$$

The above vertex becomes

$$\begin{aligned} V_{\tilde{\chi}^0 e\tilde{e}} = \frac{ig}{\sqrt{2}\cos\theta_w} &\Big[2\sin\theta_w\cos\theta_w(N_{j1}\cos\theta_w + N_{j2}\sin\theta_w) \\ &+ (\cos^2\theta_w - \sin^2\theta_w)(-N_{j1}\sin\theta_w + N_{j2}\cos\theta_w)\Big]\gamma_+ \\ = &\left\{\frac{ig}{\sqrt{2}}\left[\tan\theta_w N_{j1} + N_{j2}\right]\gamma_+\right\}_{\alpha\beta} \end{aligned} \tag{K.27}$$

Therefore using Fig. K.1



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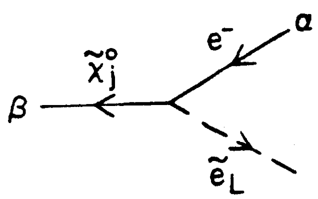
$$\left\{\frac{ig}{\sqrt{2}}\left[\tan\theta_w N_{j1} + N_{j2}\right]\gamma_+ C\right\}_{\alpha\beta} \tag{K.28}$$

Reversing all arrows in (K.28) yields



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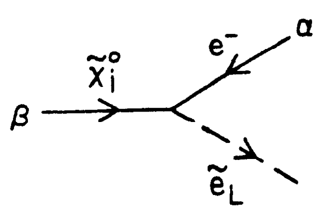
$$\overline{\left\{\frac{ig}{\sqrt{2}}\left[\tan\theta_w N_{j1} + N_{j2}\right]\gamma_+ C\right\}_{\alpha\beta}} \tag{K.29}$$

$$= \left\{\frac{-ig}{\sqrt{2}}\left[\tan\theta_w N^*_{j1} + N^*_{j2}\right]C^{-1}\gamma_-\right\}_{\beta\alpha}$$

Again from Fig. K.1



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$$= \left\{\frac{-ig}{\sqrt{2}}\left[\tan\theta_w N^*_{j1} + N^*_{j2}\right]C^{-1}\gamma_-\right\}_{\beta\alpha} \tag{K.30}$$

Similarly for the $\widetilde{\chi}_j^0 e \widetilde{e}_R$ vertex

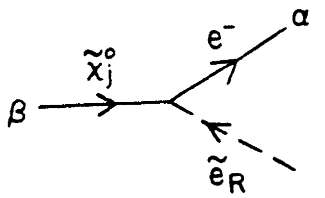
$$\frac{i}{\sqrt{2}}\left[g \sin\theta_w N'^*_{j1} - \frac{g \sin^2\theta_w}{\cos\theta_w} N'^*_{j2}\right](1-\gamma_5)$$

$$= \frac{2}{\cos\theta_w}\left(\frac{-ig}{\sqrt{2}}\right)[\sin\theta_w\cos\theta_w(N^*_{j1}\cos\theta_w + N^*_{j2}\sin\theta_w)$$

$$-\sin^2\theta_w(-N^*_{j1}\sin\theta_w + N^*_{j2}\cos\theta_w]\gamma_-$$

$$= \left\{-\frac{ie\sqrt{2}}{\cos\theta_w} N^*_{j1}\gamma_-\right\}_{\alpha\beta}$$

so the vertex is



$$\left\{-\frac{ie\sqrt{2}}{\cos\theta_w} N^*_{j1}\gamma_-\right\}_{\alpha\beta} \qquad \text{(K.31)}$$

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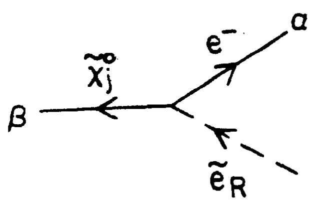
Using Fig. K.1 we obtain these equivalent vertices:



$$\left\{-ig\sqrt{2}\tan\theta_w N^*_{j1}\gamma_- C\right\}_{\alpha\beta} \qquad \text{(K.32)}$$

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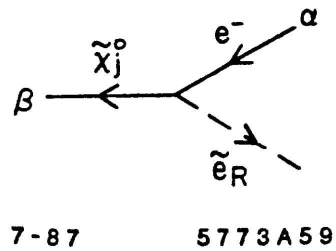
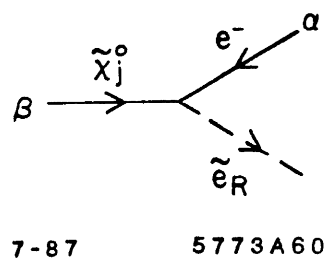


$$\left\{ig\sqrt{2}\tan\theta_w N_{j1}\gamma_+\right\}_{\beta\alpha} \tag{K.33}$$



$$\left\{ig\sqrt{2}\tan\theta_w N_{j1}C^{-1}\gamma_+\right\}_{\beta\alpha} \tag{K.34}$$

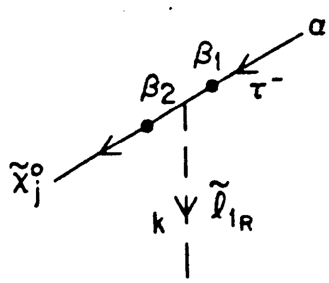
In all cases above the rules have been demonstrated for *no* flavor mixing. (*i.e.* for $\widetilde{e}_L, \widetilde{e}_R$ only.) By analogy with (K.15) we have sleptons mixing:

$$\begin{aligned}
\widetilde{\ell}_{1L} &= \widetilde{\mu}_L\cos\theta_L + \widetilde{\tau}_L\sin\theta_L \\
\widetilde{\ell}_{2L} &= -\widetilde{\mu}_L\sin\theta_L + \widetilde{\tau}_L\cos\theta_L \\
\widetilde{\ell}_{1R} &= \widetilde{\mu}_R\cos\theta_R + \widetilde{\tau}_R\sin\theta_R \\
\widetilde{\ell}_{2R} &= -\widetilde{\mu}_R\sin\theta_R + \widetilde{\tau}_R\cos\theta_R
\end{aligned} \tag{K.35}$$

In the simplest case

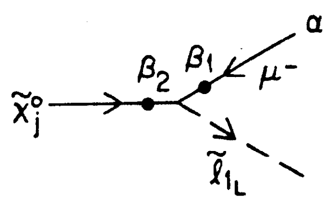
$$\theta_\nu = \theta_L = \theta_R \equiv \theta \tag{K.36}$$

In the most general case $\tilde{e}_L, \tilde{\mu}_L, \tilde{\tau}_L, \tilde{e}_R, \tilde{\mu}_R$ and $\tilde{\tau}_R$ all mix together as do $\tilde{\nu}_e, \tilde{\nu}_\mu$ and $\tilde{\nu}_\tau$. This leads to rules like:



$$[ig\sqrt{2}\tan\theta_w N_{j1}\gamma_+]_{\beta_2\beta_1}\sin\theta_R$$

(K.37)



$$\left\{\frac{-ig}{\sqrt{2}}\,[\tan\theta_w N^*_{j1} + N^*_{j2}]C^{-1}\gamma_-\right\}_{\beta_1\beta_2}\cos\theta_L$$

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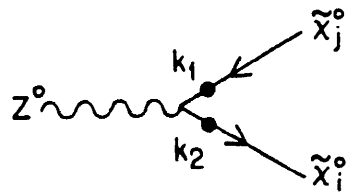
Other "Neutralino" Vertices



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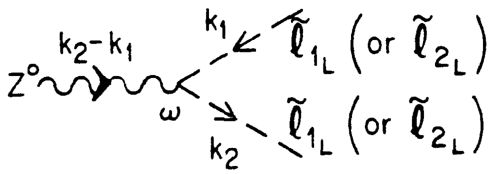
$$\frac{ig}{\cos\theta_w}\,[\gamma^\omega(O''^{L}_{ij}\gamma_- + O''^{R}_{ij}\gamma_-)]_{\kappa_2\kappa_1} \tag{K.38}$$



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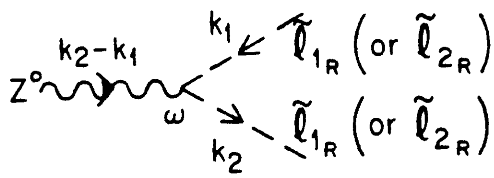
$$\frac{ig}{2\cos\theta_w}\;\cos 2\theta_w(k_1+k_2)^\omega \tag{K.39}$$



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$$\frac{-ig}{2\cos\theta_w}\cdot 2\sin^2\theta_w(k_1+k_2)^\omega \tag{K.40}$$

These are all of the Feynman rules required for the process $Z^0 \to \mu^-\tau^+$.

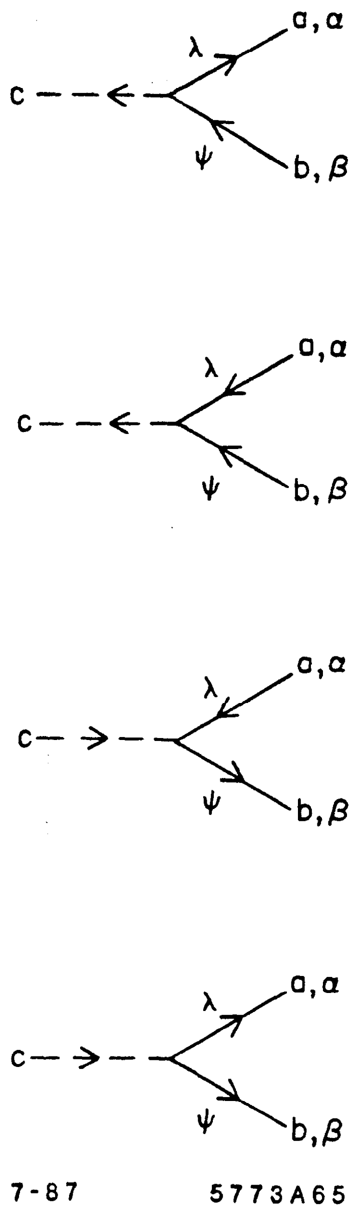
FIGURE CAPTIONS

1. The various permutations of the direction arrows and corresponding rules for a vertex consisting of λ, ψ, c which are a general Majorana fermion, Dirac fermion and boson field respectively.

   Here

$$\Gamma_i = 1, i\gamma_5, \gamma_\mu\gamma_5, \gamma_\mu,\ \sigma_{\mu\nu}\ .$$

Fig. K.1



$$iK^{i}_{abc}\Gamma_{i\alpha\beta}$$

$$-iK^{i}_{abc}(C^{-1}\Gamma_i)_{\alpha\beta}$$

$$iK^{i*}_{abc}\Gamma_{i\beta\alpha}$$

$$iK^{i*}_{abc}(\Gamma_i C)_{\beta\alpha}$$

APPENDIX L

Computation of $Z^0 \to \tau\mu$ Matrix Elements

L.1 Chargino Diagrams

Diagram 1

We demonstrate the use of the "manipulated" Feynman rules (K.18) and (K.21) here. Diagram one is illustrated in Fig. 1. From the Feynman rules of Appendix K we may read off the matrix element:

$$\begin{aligned}
\mathcal{M}_1{}^{\omega} &= \int \frac{d^4k}{(2\pi)^4} \quad \left[\overline{u}^{\mu}(p_1')\right]_{\epsilon\delta_2} \left[igV_{i1}{}^{*}\gamma_-\right]_{\delta_2\delta_1} \cos\theta_\nu \\
&\otimes \quad \left\{ \frac{i\left[(\not p_1' - \not k) + m_{\tilde{\chi}_i^+}\right]}{(p_1'-k)^2 - m_{\tilde{\chi}_i^+}{}^2} \right\}_{\delta_1\kappa_2} \quad \frac{ig}{\cos\theta_w}\left[\gamma^{\omega}(O_{ij}^{L}{}'\,\gamma_- + O_{ij}^{R}{}'\,\gamma_+)\right]_{\kappa_2\kappa_1} \\
&\otimes \quad \left\{ \frac{i\left[-(\not p_2' + \not k) + m_{\tilde{\chi}_j^+}\right]}{(p_2'+k)^2 - m_{\tilde{\chi}_j^+}{}^2} \right\}_{\kappa_1\beta_2} \quad \left[igV_{j1}\gamma_+\right]_{\beta_2\beta_1} \\
&\otimes \quad \sin\theta_\nu \frac{i}{k^2 - m_{\tilde{\nu}_1}{}^2} \left[v^{\tau}(p_2')\right]_{\beta_1\alpha} \\
&= \frac{-g^3 \sin\theta_\nu \cos\theta_\nu V_{j1} V_{i1}{}^{*}}{\cos\theta_w} \int \frac{d^4k}{(2\pi)^4} \\
&\times \quad \overline{u}^{\mu}(p_1') \frac{\gamma_-}{k^2 - m_{\tilde{\nu}_1}{}^2} \frac{\left[(\not p_1' - \not k) + m_{\tilde{\chi}_i^+}\right]}{(p_1'-k)^2 - m_{\tilde{\chi}_i^+}{}^2} \\
&\times \quad \gamma^{\omega}\left[O_{ij}^{L}{}'\,\gamma_- + O_{ij}^{R}{}'\,\gamma_+\right] \frac{\left[-(\not p_2' + \not k) + m_{\tilde{\chi}_j^+}\right]}{(p_2'+k)^2 - m_{\tilde{\chi}_j^+}{}^2} \gamma_+ v^{\tau}(p_2')
\end{aligned} \tag{L.1}$$

Notice how simple the matrix index structure is. This is a result of using the manipulated rules (K.18) and (K.21). If these had not been utilized we would essentially have derived them when simplifying the expression which would have resulted.

$$\mathcal{M}_1{}^{\omega} = \frac{g^3 \sin\theta_\nu \cos\theta_\nu V_{j1} V_{i1}{}^*}{\cos\theta_w} \int \frac{d^4k}{(2\pi)^4}$$

$$\times \quad \overline{u}^{\mu}(p_1') \frac{\gamma_- \left[(\not{p}_1' - \not{k}) \gamma^{\omega} O_{ij}^{L\,\prime} (\not{p}_2' + \not{k}) - m_{\tilde{\chi}_i^+} m_{\tilde{\chi}_j^+} O_{ij}^{R\,\prime} \right] \gamma_+}{\left[(p_1' - k)^2 - m_{\tilde{\chi}_i^+}{}^2 \right] \left[(p_2' + k)^2 - m_{\tilde{\chi}_j^+}{}^2 \right] \left[k^2 - m_{\tilde{\nu}_1}{}^2 \right]} v^{\tau}(p_2') \tag{L.2}$$

where we have used

$$\gamma^{\mu}\gamma_{\pm} = \gamma_{\mp}\gamma^{\mu} \qquad (\gamma_{\pm})^2 = \gamma_{\pm} \qquad \gamma_{\pm}\gamma_{\mp} = 0$$

several times. Using

$$\frac{1}{k^2 - m_{\tilde{\nu}_1}{}^2} - \frac{1}{k^2 - m_{\tilde{\nu}_2}{}^2} = - \int_{m_{\tilde{\nu}_1}{}^2}^{m_{\tilde{\nu}_2}{}^2} \frac{dt}{(k^2 - t)^2}$$

we obtain

$$\mathcal{M}_1{}^{\omega} = - \frac{2e^3 \sin\theta_\nu \cos\theta_\nu V_{j1} V_{i1}{}^*}{\sin^2\theta_w \sin 2\theta_w} \int \frac{d^4k}{(2\pi)^4} \int_{m_{\tilde{\nu}_1}{}^2}^{m_{\tilde{\nu}_2}{}^2} dt$$

$$\times \quad \frac{\overline{u}^{\mu}(p_1') \gamma_- \left[(\not{p}_1' - \not{k}) \gamma^{\omega} O_{ij}^{L\,\prime} (\not{p}_2' + \not{k}) - m_{\tilde{\chi}_i^+} m_{\tilde{\chi}_j^+} O_{ij}^{R\,\prime} \right] \gamma_+ v^{\tau}(p_2')}{\left[k^2 - t \right]^2 \left[k^2 - 2p_1' \cdot k + m_\mu^2 - m_{\tilde{\chi}_i^+}{}^2 \right] \left[k^2 + 2p_2' \cdot k + m_\tau^2 - m_{\tilde{\chi}_j^+}{}^2 \right]} \tag{L.3}$$

We proceed to ignore all m_μ^2, m_τ^2 terms (relative to $q^2, m_{\tilde{\nu}_1}^2, m_{\tilde{\chi}_i^+}{}^2$ etc. terms).

From equation (H.5)

$$\frac{1}{a^2bc} = 6\int_0^1 dx \int_0^{1-x} dy\ \frac{1-x-y}{[a+(b-a)x+(c-a)y]^4}$$

Therefore

$$\mathcal{M}_1{}^{\omega} = -\frac{2e^3\sin\theta_\nu\cos\theta_\nu V_{j1}V_{i1}{}^*}{\sin^2\theta_w\sin 2\theta_w}\int\frac{d^4k}{(2\pi)^4}\int_{m_{\tilde{\nu}_1}{}^2}^{m_{\tilde{\nu}_2}{}^2} dt\quad 6\int_0^1 dx\int_0^{1-x} dy\ (1-x-y)$$

$$\times\quad \frac{\overline{u}^{\mu}(p_1')\gamma_-\left[(\not{p}_1'-\not{k})\gamma^{\omega}O_{ij}^{L}{}'\,(\not{p}_2'+\not{k})-m_{\tilde{\chi}_i^+}\,m_{\tilde{\chi}_j^+}\,O_{ij}^{R}{}'\right]\gamma_+v^{\tau}(p_2')}{\left[k^2-t+\left(t-2p_1'\cdot k-m_{\tilde{\chi}_i^+}{}^2\right)x+\left(t+2p_2'\cdot k-m_{\tilde{\chi}_j^+}{}^2\right)y\right]^4} \tag{L.4}$$

From (H.10) we can switch variables of integration.

$$z\equiv x+y \qquad\qquad \tilde{z}\equiv x-y$$

so that

$$xy=\frac{1}{4}\left(z^2-\tilde{z}^2\right)$$

$$\int_0^1 dx\int_0^{1-x} dy = \tfrac{1}{2}\int_0^1 dz\int_{-z}^{z} d\tilde{z}$$

We now employ the formulae of (H.7)

$$\int\frac{d^4k}{(2\pi)^4}\frac{1;k^{\mu}}{(k^2-2p\cdot k+s-i\epsilon)^4} \quad=\quad \frac{i[1;p^{\mu}]}{96\pi^2(s-p^2)^2}$$

$$\int\frac{d^4k}{(2\pi)^4}\frac{k_{\mu}k^{\nu}}{(k^2-2p\cdot k+s-i\epsilon)^4} \quad=\quad \frac{i[p_{\mu}p^{\nu}+\frac{1}{2}(s-p^2)\delta_{\mu}{}^{\nu}]}{96\pi^2(s-p^2)^2}$$

to re-cast our integral into the form

$$\mathcal{M}_1{}^{\omega} = -\frac{2e^3 \sin\theta_\nu \cos\theta_\nu V_{j1} V_{i1}{}^*}{\sin^2\theta_w \sin 2\theta_w} \int\limits_{m_{\tilde{\nu}_1}{}^2}^{m_{\tilde{\nu}_2}{}^2} dt \quad \frac{i}{32\pi^2} \int\limits_0^1 dz \int\limits_{-z}^{z} d\tilde{z}\ (1-z)$$

$$\times \quad \overline{u}^{\mu}(p_1')\gamma_- \left\{ \frac{\left[(\not{p}_1' - \not{P})\gamma^{\omega} O_{ij}^{L}{}' (\not{p}_2' + \not{P}) - m_{\tilde{\chi}_i^+}\, m_{\tilde{\chi}_j^+}\, O_{ij}^{R}{}'\right]}{\left(s - P^2\right)^2} \right. \tag{L.5}$$

$$\left. - \quad \frac{\frac{1}{2}\gamma^{\mu}\gamma^{\omega}\gamma_{\nu}\delta_{\mu}{}^{\nu} O_{ij}^{L}{}'}{s - P^2} \right\} \gamma_+ v^{\tau}(p_2')$$

with

$$P = p_1' x - p_2' y$$

$$s = -t(1-x-y) - \left[m_{\tilde{\chi}_i^+}{}^2 x + m_{\tilde{\chi}_j^+}{}^2 y\right] \tag{L.6}$$

$$= -\left\{t(1-z) + \tfrac{1}{2}\left[\left(m_{\tilde{\chi}_i^+}{}^2 + m_{\tilde{\chi}_j^+}{}^2\right) z + \left(m_{\tilde{\chi}_i^+}{}^2 - m_{\tilde{\chi}_j^+}{}^2\right)\tilde{z}\right]\right\}.$$

As in (H.17 a) we see that

$$P^2 \doteq \frac{q^2}{4}\left(\tilde{z}^2 - z^2\right)$$

and so

$$s - P^2 \equiv -D(t)$$

$$= -\left\{t(1-z) + \tfrac{1}{2}\left[\left(m_{\tilde{\chi}_i^+}{}^2 + m_{\tilde{\chi}_j^+}{}^2\right) z + \left(m_{\tilde{\chi}_i^+}{}^2 - m_{\tilde{\chi}_j^+}{}^2\right)\tilde{z}\right]\right.$$

$$\left. + \ \frac{q^2}{4}\left(\tilde{z}^2 - z^2\right)\right\}. \tag{L.7}$$

From (L.5)

$$\mathcal{M}_1{}^{\omega} = -\frac{2e^3 \sin\theta_\nu \cos\theta_\nu V_{j1} V_{i1}{}^*}{\sin^2\theta_w \sin 2\theta_w} \cdot \frac{i}{32\pi^2} \int_{m_{\tilde{\nu}_1}{}^2}^{m_{\tilde{\nu}_2}{}^2} dt \int_0^1 dz \int_{-z}^{z} d\tilde{z}\,(1-z)$$
$$\times \quad \overline{u}^{\mu}(p_1')\gamma_- \left\{ \frac{T - m_{\tilde{\chi}_i^+} m_{\tilde{\chi}_j^+} O_{ij}^{R\,\prime}}{[D(t)]^2} + \frac{\gamma^{\omega} O_{ij}^{L\,\prime}}{-D(t)} \right\} \gamma_+ v^{\tau}(p_2') \tag{L.8}$$

since $\gamma^{\mu}\gamma^{\omega}\gamma_{\mu} = -2\gamma^{\omega}$. In (L.8)

$$\overline{u}^{\mu}(p_1')Tv^{\tau}(p_2') = \overline{u}^{\mu}(p_1')\gamma_- \left[(\not{p}_1'[1-x] + \not{p}_2' y)\gamma^{\omega} O_{ij}^{L\,\prime}\,(\not{p}_2'[1-y] + \not{p}_1' x) \right] \gamma_+ v^{\tau}(p_2') \tag{L.9}$$

Using the Dirac equation

$$\overline{u}^{\mu}(p_1')\ \not{p}_1' \approx 0 \qquad \not{p}_2' v^{\tau}(p_2') \approx 0$$

this becomes

$$\begin{aligned} \overline{u}^{\mu}(p_1')Tv^{\tau}(p_2') &\approx \overline{u}^{\mu}(p_1')\gamma_-\ \not{p}_2' y \gamma^{\omega}\ \not{p}_1' x \gamma_+ v^{\tau}(p_2') \\ &\approx -2xy\ p_1' \cdot p_2' \overline{u}^{\mu}(p_1')\gamma_- \gamma^{\omega}\gamma_+ v^{\tau}(p_2') \\ &\approx \frac{1}{4} q^2 \left(\tilde{z}^2 - z^2\right) \xi^{\omega}{}_L \end{aligned}$$

where we recall from (3.9 a) that

$$\xi^{\omega}{}_{L,R} = \bar{u}^{\mu}(p_1{}')\gamma_{\mp}\gamma^{\omega}\gamma_{\pm} v^{\tau}(p_2{}') \tag{L.10}$$

Also recall (from (3.9 d) that

$$\mathcal{K}_\nu = \frac{i}{16\pi^2} e^3 \sin\theta_\nu \cos\theta_\nu \tag{L.11}$$

and

$$\frac{2e^3 \sin\theta_\nu \cos\theta_\nu V_{j1} V_{i1}{}^*}{\sin^2\theta_w \sin 2\theta_w} \cdot \frac{i}{32\pi^2} = -\frac{K_\nu V_{j1} V_{i1}{}^*}{\sin^2\theta_w \sin 2\theta_w} \ .$$

Performing the t integration and substituting for T in (L.8)

$$\begin{aligned} \mathcal{M}_1{}^\omega = \sum_{ij} \xi^\omega{}_L \frac{K_\nu V_{j1} V_{i1}{}^*}{\sin^2\theta_w \sin 2\theta_w} \int_0^1 dz \int_{-z}^{z} d\tilde{z} \\ \times \left\{ \frac{m_{\tilde{\chi}_i^+} m_{\tilde{\chi}_j^+} O_{ij}^{R\,\prime} - \frac{1}{4} q^2 (\tilde{z}^2 - z^2) O_{ij}^{L\,\prime}}{D(m_{\tilde{\nu}_1}{}^2)} - m_{\tilde{\nu}_1} \to m_{\tilde{\nu}_2} \right. \\ \left. + \ O_{ij}^{L\,\prime} \ln\left[\frac{D(m_{\tilde{\nu}_2}{}^2)}{D(m_{\tilde{\nu}_1}{}^2)}\right] \right\} \end{aligned} \qquad \text{(L.12)}$$

where

$$\begin{aligned} D(m_{\tilde{\nu}_1}{}^2) &= m_{\tilde{\nu}_1}{}^2 (1 - z) \\ &+ \tfrac{1}{2} \left[\left(m_{\tilde{\chi}_i^+}{}^2 + m_{\tilde{\chi}_j^+}{}^2 \right) z + \left(m_{\tilde{\chi}_i^+}{}^2 - m_{\tilde{\chi}_j^+}{}^2 \right) \tilde{z} \right] + \frac{1}{4} q^2 (\tilde{z}^2 - z^2) \end{aligned}$$

If we had employed a traditional Pauli-Villars regularization scheme we would have obtained

$$\begin{aligned} \mathcal{M}_1{}^\omega = \sum_{ij} \xi^\omega{}_L \frac{K_\nu V_{j1} V_{i1}{}^*}{\sin^2\theta_w \sin 2\theta_w} \int_0^1 dz \int_{-z}^{z} d\tilde{z} \\ \times \left\{ \frac{m_{\tilde{\chi}_i^+} m_{\tilde{\chi}_j^+} O_{ij}^{R\,\prime} - \frac{1}{4} q^2 (\tilde{z}^2 - z^2) O_{ij}^{L\,\prime}}{D(m_{\tilde{\nu}_1}{}^2)} \right. \\ \left. + \ O_{ij}^{L\,\prime} \ln\left[\frac{\Lambda^2 (1 - z) + m_{\tilde{\chi}_i^+} m_{\tilde{\chi}_j^+} z + \frac{1}{4} q^2 (\tilde{z}^2 - z^2)}{D(m_{\tilde{\nu}_1}{}^2)}\right] \right\} \\ - \ m_{\tilde{\nu}_1} \to m_{\tilde{\nu}_2} \end{aligned}$$

Here $\Lambda^2 \to \infty$ is the Pauli-Villars regulator which is superfluous in this instance since we have equally well written the log term in (L.12) as a ratio of "D's" where the numerator "D" is equal to the denominator with with $\tilde{\nu}_1$ replaced by $\tilde{\nu}_2$. Dividing (L.12) through by $m_{\tilde{\chi}_i^+}{}^2$ and using the definitions of the ratios R, S and T from eqn. (3.10) and the integral functions in the table at the conclusion of the appendix (Table L.1) gives us the final form

$$
\begin{aligned}
\mathcal{M}_1{}^\omega &= \sum_{ij} \xi^\omega{}_L \frac{2\mathcal{K}_\nu V_{j1} V_{i1}{}^*}{\sin^2\theta_w \sin 2\theta_w} \\
&\times \quad \Big\{ \sqrt{S_{ij}^+}\, O_{ij}^{R\,\prime}\, \Delta\check{I}_{[1]}(T_i^+, S_{ij}^+, R_i^+) \\
&- \quad R_i^+ O_{ij}^{L\,\prime}\, \Delta\check{I}_{[\widetilde{\mathbf{Z}}^2 - \mathbf{Z}^2]}(T_i^+, S_{ij}^+, R_i^+) + O_{ij}^{L\,\prime}\, \Delta\check{G}(T_i^+, S_{ij}^+, R_i^+) \Big\}
\end{aligned}
\tag{L.13}
$$

<u>Diagram 2</u>

Diagram two is illustrated in Fig. 2. From the Feynman rules of Appendix K we may read off the matrix element:

$$
\begin{aligned}
\mathcal{M}_2{}^\omega &= \int \frac{d^4k}{(2\pi)^4} \quad [\overline{u}^\mu(p_1')]_{\epsilon\delta_2} [igV_{i1}{}^*\gamma_-]_{\delta_2\delta_1} \cos\theta_\nu \\
&\otimes \quad \left\{ \frac{i\left[(\not k + m_{\tilde{\chi}_i^+}) \right]}{k^2 - m_{\tilde{\chi}_i^+}{}^2} \right\}_{\delta_1\beta_2} [igV_{i1}\gamma_+]_{\beta_2\beta_1} \sin\theta_\nu \left[v^\tau(p_2') \right]_{\beta_1\alpha} \\
&\otimes \quad \frac{i}{(p_1'-k)^2 - m_{\tilde{\nu}_1}{}^2} \cdot \frac{-ig}{2\cos\theta_w} \left[p_1' - p_2' - 2k \right]^\omega \cdot \frac{i}{(p_2'+k)^2 - m_{\tilde{\nu}_1}{}^2} \\
&= \frac{g^3 \sin\theta_\nu \cos\theta_\nu V_{i1} V_{i1}{}^*}{2\cos\theta_w} \int \frac{d^4k}{(2\pi)^4} \\
&\times \quad \overline{u}^\mu(p_1') \frac{\gamma_-(\not k + m_{\tilde{\chi}_i^+})\gamma_+(2k + p_2' - p_1')^\omega}{\left[k^2 - m_{\tilde{\chi}_i^+}{}^2\right]\left[(p_1'-k)^2 - m_{\tilde{\nu}_1}{}^2\right]\left[(p_2'+k)^2 - m_{\tilde{\nu}_1}{}^2\right]} v^\tau(p_2')
\end{aligned}
\tag{L.14}
$$

We see that use of (K.18) and (K.21) have again simplified the algebra. In this case we use a Pauli-Villars cut-off. Introducing

$$\frac{1}{k^2 - m_{\tilde{\chi}_i^+}{}^2} = - \int\limits_{m_{\tilde{\chi}_i^+}{}^2}^{\Lambda^2} \frac{dt}{(k^2 - t)^2}$$

Following the techniques used above we obtain

$$\mathcal{M}_2{}^\omega = \frac{e^3 \sin\theta_\nu \cos\theta_\nu V_{i1} V_{i1}{}^*}{\sin^2\theta_w \sin 2\theta_w} \int \frac{d^4k}{(2\pi)^4} \int\limits_{m_{\tilde{\chi}_i^+}{}^2}^{\Lambda^2} 6 \int\limits_0^1 dx \int\limits_0^{1-x} dy\ (1 - x - y)$$

$$\times \quad \frac{\overline{u}^\mu(p_1')\gamma_+ \not{k}\gamma_-(2k + \tilde{q})^\omega v^\tau(p_2')}{\left[k^2 - t + \left(t - 2p_1' \cdot k - m_{\tilde{\nu}_1}{}^2\right) x + \left(t + 2p_2' \cdot k - m_{\tilde{\nu}_1}{}^2\right) y\right]^4}$$

$$-\tilde{\nu}_1 \to \tilde{\nu}_2$$

where

$$q^\omega = (p_1' + p_2')^\omega \qquad \tilde{q}^\omega = (p_2' - p_1')^\omega\ . \tag{L.15}$$

This leads to

$$\mathcal{M}_2{}^\omega = \frac{K_\nu\,|V_{i1}|^2}{\sin^2\theta_w \sin 2\theta_w}\ \frac{1}{2} \int\limits_0^1 dx \int\limits_0^{1-x} dy \int\limits_{m_{\tilde{\chi}_i^+}{}^2}^{\Lambda^2} (1 - x - y)$$

$$\times \quad \frac{\overline{u}^\mu(p_1')\gamma_+ \left\{(\not{p}_1' x - \not{p}_2' y)(p_1'^\omega[2x - 1] + p_2'^\omega[1 - 2y]) + \gamma^\omega(s - P^2)\right\} v^\tau(p_2')}{(s - P^2)^2}$$

$$-\tilde{\nu}_1 \to \tilde{\nu}_2$$

where

$$s - P^2 \approx -t(1 - z) - m_{\tilde{\nu}_1}{}^2 z - \frac{q^2}{4}(\tilde{z}^2 - z^2)\ .$$

The $\not p$ terms will introduce factors of m_μ, m_τ and so will be down by

$$O(m_\tau/m_{SUSY})$$

relative to the γ^ω term. Thus

$$\begin{aligned}
\mathcal{M}_2{}^\omega &= \sum_{ij} \frac{\xi^\omega{}_L \, K_\nu \, |V_{i1}|^2}{\sin^2\theta_w \sin 2\theta_w} \int_0^1 dz \int_{-z}^{z} d\tilde{z} \\
&\times \quad O_{ij}^{L}{}' \ln\left[\frac{\Lambda^2(1-z) + m_{\tilde{\nu}_1}{}^2 z + \frac{1}{4}q^2\left(\tilde{z}^2 - z^2\right)}{m_{\tilde{\chi}_i^+}{}^2(1-z) + m_{\tilde{\nu}_1}{}^2 z + \frac{1}{4}q^2\left(\tilde{z}^2 - z^2\right)}\right] \\
&- \quad m_{\tilde{\nu}_1} \to m_{\tilde{\nu}_2} \; + O\left(\frac{m_\tau}{M_{SUSY}}\right)
\end{aligned} \tag{L.16}$$

and because, as we shall seen the total matrix element is finite as $\Lambda^2 \to \infty$ we may write this sans regularization as

$$\begin{aligned}
\mathcal{M}_2{}^\omega &= \sum_{i} \frac{\xi^\omega{}_L \, K_\nu \, |V_{i1}|^2}{\sin^2\theta_w \sin 2\theta_w} \int_0^1 dz \int_{-z}^{z} d\tilde{z} \\
&\times \quad O_{ij}^{L}{}' \ln\left[\frac{m_{\tilde{\chi}_i^+}{}^2(1-z) + m_{\tilde{\nu}_2}{}^2 z + \frac{1}{4}q^2\left(\tilde{z}^2 - z^2\right)}{m_{\tilde{\chi}_i^+}{}^2(1-z) + m_{\tilde{\nu}_1}{}^2 z + \frac{1}{4}q^2\left(\tilde{z}^2 - z^2\right)}\right] \\
&+ \quad O\left(\frac{m_\tau}{M_{SUSY}}\right)
\end{aligned} \tag{L.17}$$

Where K_ν is given by (L.11) and $\xi^\omega{}_L$ by (L.10). Dividing through by $m_{\tilde{\chi}_i^+}{}^2$ and using the definition of the integral functions in the table at the conclusion of this appendix yields

$$\mathcal{M}_2{}^\omega = \sum_{i} \frac{\xi^\omega{}_L \, K_\nu \, |V_{i1}|^2}{\sin^2\theta_w \sin 2\theta_w} \Delta G(T_i^+, R_i^+) \tag{L.18}$$

Diagram Three

Diagrams three and four are leg corrections. For these two sets of diagrams we will not use (K.18) and (K.21) in an effort to demonstate some of the C-matrix manipulations which occur. From Fig. 3 we see that

$$\begin{aligned}
\mathcal{M}_3{}^{\omega} = \int \frac{d^4k}{(2\pi)^4} \quad & \left[v^{\mu}(p_1')\right]_{\epsilon\delta_2} \left[igC^{-1}V_{i1}{}^{*}\gamma_-\right]_{\delta_1\delta_2} \cos\theta_\nu \\
\otimes \quad & \left\{ \frac{i\left[(\not{p}_1' - \not{k}) + m_{\tilde{\chi}_i^+}\right]}{(p_1'-k)^2 - m_{\tilde{\chi}_i^+}{}^2} \right\}_{\delta_1\beta_2} \left[-igV_{i1}\gamma_+ C\right]_{\beta_1\beta_2} \\
\otimes \quad & \sin\theta_\nu \frac{i}{k^2 - m_{\tilde{\nu}_1}{}^2} \left\{ \frac{i(-\not{p}_1' + m_\tau)}{(p_1')^2 - m_\tau^2} \right\}_{\kappa_2\beta_1} \\
\otimes \quad & \frac{ig}{2\cos\theta_w} \left[\gamma^{\omega}(\cos 2\theta_w\gamma_- - \sin^2\theta_w\gamma_+)\right]_{\kappa_1\kappa_2} \left[\overline{u}^{\tau}(p_2')\right]_{\alpha\kappa_1}
\end{aligned} \tag{L.19}$$

Now we take a few transposes in order to place this in the form of a matrix product in the spin indicies.

$$\begin{aligned}
\mathcal{M}_3{}^{\omega} = \; & \frac{g^3 \sin\theta_\nu \cos\theta_\nu V_{i1} V_{i1}{}^{*}}{2\cos\theta_w (m_\mu^2 - m_\tau^2)} \int \frac{d^4k}{(2\pi)^4} \\
\times \quad & \left[v^{\mu}(p_1')\right]^T \left[C^{-1}\gamma_-\right]^T \frac{\left[(\not{p}_1' - \not{k}) + m_{\tilde{\chi}_i^+}\right]}{(p_1'-k)^2 - m_{\tilde{\chi}_i^+}{}^2} \\
\times \quad & \left[\gamma_+ C\right]^T \left[-\not{p}_1' + m_\tau\right]^T \frac{1}{k^2 - m_{\tilde{\nu}_1}{}^2} \\
\times \quad & \left[\gamma^{\omega}(\cos 2\theta_w\gamma_- - \sin^2\theta_w\gamma_+)\right]^T \left[\overline{u}^{\tau}(p_2')\right]^T
\end{aligned} \tag{L.20}$$

We now proceed to use the various identities given at the beginning of Appendix K.

Since $C^{-1}\gamma_- = \gamma_-^T C^{-1}$ and $\gamma_+ C = C\gamma_+^T$ using (K.12)

$$\mathcal{M}_3{}^{\omega} = \frac{g^3 \sin\theta_\nu \cos\theta_\nu V_{i1} V_{i1}{}^*}{2\cos\theta_w (m_\mu^2 - m_\tau^2)} \int \frac{d^4k}{(2\pi)^4}$$

$$\times \quad \left[\overline{u}^\mu(p_1')C^T\right] \left[\gamma_-^T C^{-1}\right]^T \frac{\left[(\not{p}_1' - \not{k}) + m_{\tilde{\chi}_i^+}\right]}{(p_1' - k)^2 - m_{\tilde{\chi}_i^+}{}^2} \tag{L.21}$$

$$\times \quad \left[C\gamma_+^T\right]^T \left[-\not{p}_1' + m_\tau\right]^T \frac{1}{k^2 - m_{\tilde{\nu}_1}{}^2}$$

$$\times \quad \left[(\cos 2\theta_w \gamma_-^T - \sin^2\theta_w \gamma_+^T)(\gamma^\omega)^T\right] \left[C^{-1} v^\tau(p_2')\right]$$

Now using (K.11)

$$(\gamma_-^T C^{-1})^T = (C^T)^{-1}\gamma_-$$

and using (K.8)

$$(C\gamma_+^T)^T = -\gamma_+ C$$

so

$$\mathcal{M}_3{}^{\omega} = \frac{g^3 \sin\theta_\nu \cos\theta_\nu V_{i1} V_{i1}{}^*}{2\cos\theta_w (m_\mu^2 - m_\tau^2)} \int \frac{d^4k}{(2\pi)^4}$$

$$\times \quad \overline{u}^\mu(p_1')C^T \left(C^T\right)^{-1} \gamma_- \frac{\left[(\not{p}_1' - \not{k}) + m_{\tilde{\chi}_i^+}\right]}{(p_1' - k)^2 - m_{\tilde{\chi}_i^+}{}^2} \tag{L.22}$$

$$\times \quad \left[-\gamma_+ C\right] \left[-\not{p}_1' + m_\tau\right]^T \frac{1}{k^2 - m_{\tilde{\nu}_1}{}^2}$$

$$\times \quad (\cos 2\theta_w \gamma_-^T - \sin^2\theta_w \gamma_+^T)(\gamma^\omega)^T C^{-1} v^\tau(p_2')$$

We now "slide" the C^{-1} near the end of (L.22) until it combines with a C via

$CC^{-1} = 1$ and repeatedly use (K.11) to obtain

$$\mathcal{M}_3{}^{\omega} = \frac{g^3 \sin\theta_\nu \cos\theta_\nu V_{i1} V_{i1}{}^*}{2\cos\theta_w (m_\mu^2 - m_\tau^2)} \int \frac{d^4k}{(2\pi)^4}$$

$$\times \quad \overline{u}^\mu(p_1')\gamma_- \frac{\left[(\not{p}_1' - \not{k}) + m_{\tilde{\chi}_i^+}\right]}{(p_1' - k)^2 - m_{\tilde{\chi}_i^+}{}^2} \qquad \text{(L.23)}$$

$$\times \quad (-\gamma_+)\left[\not{p}_1' + m_\tau\right] \frac{1}{k^2 - m_{\tilde{\nu}_1}{}^2}$$

$$\times \quad (\cos 2\theta_w \gamma_- - 2\sin^2\theta_w \gamma_+)(-\gamma^\omega) v^\tau(p_2')$$

and since $\gamma_\pm\gamma_\mp = 0$ and $\gamma_\pm\gamma_\pm = \gamma_\pm$

$$\mathcal{M}_3{}^{\omega} = \frac{g^3 \sin\theta_\nu \cos\theta_\nu V_{i1} V_{i1}{}^*}{2\cos\theta_w (m_\mu^2 - m_\tau^2)} \int \frac{d^4k}{(2\pi)^4}$$

$$\times \quad \overline{u}^\mu(p_1') \frac{\gamma_-(\not{p}_1' - \not{k})\gamma_+}{(p_1' - k)^2 - m_{\tilde{\chi}_i^+}{}^2} \quad \frac{(\not{p}_1' \cos 2\theta_w \gamma_- - 2m_\tau \sin^2\theta_w \gamma_+)(\gamma^\omega)}{k^2 - m_{\tilde{\nu}_1}{}^2} v^\tau(p_2') \qquad \text{(L.24)}$$

We proceed as we did for the previous diagram, introducing a Pauli-Villars regulator via $\int dt$, but use (H.3) instead of (H.5). We find (after eliminating the regulator)

$$\mathcal{M}_3{}^{\omega} = -\sum_i \frac{K_\nu \, |V_{i1}|^2}{\sin^2\theta_w \sin 2\theta_w} \int_{m_{\tilde{\nu}_1}{}^2}^{m_{\tilde{\nu}_2}{}^2} dt \int_0^1 \frac{x^2 \, dx}{tx + m_{\tilde{\chi}_i^+}{}^2(1-x)} \qquad \text{(L.25)}$$

$$\times \quad \frac{\cos 2\theta_w m_\mu^2 \xi^\omega{}_L - 2\sin^2\theta_w m_\mu m_\tau \xi^\omega{}_R}{m_\mu^2 - m_\tau^2} \, .$$

From Table L.1 we see that this integral is like $G(T, \mathcal{R})$ when $\mathcal{R} = 0$ (*i.e.* $q^2 = 0$).

Therefore we can write this as

$$\mathcal{M}_3{}^\omega = \sum_i \frac{K_\nu\ |V_{i1}|^2}{\sin^2\theta_w \sin 2\theta_w} \frac{\cos 2\theta_w m_\mu^2 \xi^\omega{}_L - 2\sin^2\theta_w m_\mu m_\tau \xi^\omega{}_R}{m_\mu^2 - m_\tau^2}\ \Delta G(T, \mathcal{R} = 0) \tag{L.26}$$

<u>Diagram Four</u>

Diagram four is computed in precisely the same manner as the previous contribution From Fig. 4 we may write

$$\begin{aligned}
\mathcal{M}_4{}^\omega = \int \frac{d^4k}{(2\pi)^4} \quad & \left[v^\mu(p_1')\right]_{\kappa_2\epsilon} \frac{ig}{2\cos\theta_w} \\
\otimes \quad & \left[\gamma^\omega(\cos 2\theta_w\gamma_- - \sin^2\theta_w\gamma_+)\right]_{\kappa_1\kappa_2} \left\{\frac{i\,(\not{p}_2' + m_\mu)}{(p_2')^2 - m_\mu^2}\right\}_{\delta_2\kappa_1} \\
\otimes \quad & \left[igC^{-1}V_{i1}{}^*\gamma_-\right]_{\delta_1\delta_2} \cos\theta_\nu \left\{\frac{i\left[-(\not{p}_2' - \not{k}) + m_{\tilde{\chi}_i^+}\right]}{(p_2' - k)^2 - m_{\tilde{\chi}_i^+}{}^2}\right\}_{\delta_1\beta_2} \\
\otimes \quad & \left[-igV_{i1}\gamma_+C\right]_{\beta_1\beta_2} \sin\theta_\nu \frac{i}{k^2 - m_{\tilde{\nu}_1}{}^2} \left[\overline{u}^\tau(p_2')\right]_{\alpha\beta_1}
\end{aligned} \tag{L.27}$$

Following the same steps leads us to

$$\mathcal{M}_4{}^\omega = -\sum_i \frac{K_\nu\ |V_{i1}|^2}{\sin^2\theta_w \sin 2\theta_w} \frac{\cos 2\theta_w m_\tau^2 \xi^\omega{}_L - 2\sin^2\theta_w m_\mu m_\tau \xi^\omega{}_R}{m_\mu^2 - m_\tau^2}\ \Delta G(T, \mathcal{R} = 0) \tag{L.28}$$

From (L.26) and (L.28) we conclude

$$(\mathcal{M}_3 + \mathcal{M}_4)^\omega = K_\nu \sum_{i=1}^{2} \frac{\xi^\omega{}_L\ |V_{i1}|^2}{\sin^2\theta_w \sin 2\theta_w} \cos 2\theta_w\ \Delta G(T, \mathcal{R} = 0)\ . \tag{L.29}$$

Note that the $\xi^\omega{}_R$ term has completely cancelled between the two leg correction diagrams.

Total Chargino Matrix Element

We may now gather together the individual contributions and present the complete chargino matrix element. From (L.13), (L.18) and (L.29)

$$\begin{aligned}
\mathcal{M}_{\pm}{}^{\omega} &= 2\mathcal{K}_\nu \xi^{\omega}{}_L \sum_{ij=1}^{2} \frac{V_{j1}V_{i1}{}^*}{\sin^2\theta_w \sin 2\theta_w} \\
&\times \quad \Big\{ \sqrt{S_{ij}^+}\, O_{ij}^{R}{}' \, \Delta\check{I}_{[1]}(T_i^+, S_{ij}^+, R_i^+) \\
&- \quad R_i^+ O_{ij}^{L}{}' \, \Delta\check{I}_{[\widetilde{Z}^2 - Z^2]}(T_i^+, S_{ij}^+, R_i^+) + O_{ij}^{L}{}' \, \Delta\check{G}(T_i^+, S_{ij}^+, R_i^+) \Big\} \\
&+ \quad \mathcal{K}_\nu \xi^{\omega}{}_L \sum_{i=1}^{2} \frac{|V_{i1}|^2}{\sin^2\theta_w \sin 2\theta_w} \left[\Delta G(T_i^+, R_i^+) + \cos 2\theta_w \, \Delta G(T, \mathcal{R}=0) \right] \ .
\end{aligned} \tag{L.30}$$

Large Mass Limits

Various limiting forms of the integral functions are presented in section M.7 of the following appendix. We assert that (L.30) is well-behaved in the limit that any of the masses involved or q^2 or any of the defined ratios thereof (T,S and R) becomes large. In this limit the decoupling theorem enters with full force and contributions from very heavy states become negligible. This is readily evident for most of the limits. When we take T large (*i.e.* $m_{\tilde{\ell}}$ large) this requires a bit of work. As a reminder:

$$T_i = \frac{m_{\tilde{\ell}}^2}{m_{\tilde{\chi}_i}^2} \qquad S_{ij} = \frac{m_{\tilde{\chi}_j}^2}{m_{\tilde{\chi}_i}^2} \qquad R_i = \frac{q^2}{4m_{\tilde{\chi}_i}^2}$$

$$
\begin{aligned}
&\lim_{T\to\infty} \check{I}_{[\mathrm{N(Z,\tilde{Z})}]}(T,S,R) \to 0 \\
&\lim_{T\to\infty} \tilde{G}(T,S,R) \to 0 \\
&\lim_{T\to\infty} G(T,S,R) \to -\tfrac{1}{2}\ln T \\
&\lim_{T\to\infty} \check{G}(T,S,R) \to -\tfrac{1}{2}\ln T
\end{aligned}
\tag{L.31}
$$

Suppose that we consider what happens if $m_{\tilde{\nu}_1}, m_{\tilde{\nu}_2} \to \infty$. We find that

$$
\Delta G, \Delta\check{G} \to -\tfrac{1}{2}\ln \frac{{m_{\tilde{\nu}_1}}^2}{{m_{\tilde{\nu}_2}}^2} \tag{L.32}
$$

and the chargino matrix element becomes

$$
\begin{aligned}
\mathcal{M}_{\pm}{}^{\omega} \to\ & \frac{\mathcal{K}_\nu \xi^\omega{}_L}{\sin^2\theta_w \sin 2\theta_w}\left(-\tfrac{1}{2}\ln\frac{{m_{\tilde{\nu}_1}}^2}{{m_{\tilde{\nu}_2}}^2}\right) \\
\times\ & \left\{\sum_{i=1}^{2} |V_{i1}|^2 (1+\cos 2\theta_w) + 2\sum_{ij=1}^{2} V_{j1}V_{i1}{}^{*} O^{L}_{ij}{}' \right\}
\end{aligned}
\tag{L.33}
$$

and we recall that

$$
O^{L}_{ij}{}' = \delta_{ij}\sin^2\theta_w - V_{i1}V^{*}_{j1} - \tfrac{1}{2}V_{i2}V^{*}_{j2}
$$

and that V is unitary so that $(V^{\dagger}V)_{ij} = \delta_{ij}$. Using these it is simple to show that

$$
\sum_{i=1}^{2} |V_{i1}|^2 (1+\cos 2\theta_w) = 1 + \cos 2\theta_w
$$

$$
\sum_{ij=1}^{2} V_{j1}V_{i1}{}^{*} O^{L}_{ij}{}' = \sin^2\theta_w - 1
$$

and so

$$\mathcal{M}_{\pm}{}^{\omega} \rightarrow \frac{\mathcal{K}_{\nu}\xi^{\omega}{}_{L}}{\sin^2\theta_w \sin 2\theta_w}\left(-\tfrac{1}{2}\ln\frac{m_{\tilde{\nu}_1}{}^2}{m_{\tilde{\nu}_2}{}^2}\right)$$

$$\times \quad \left\{(1+\cos 2\theta_w) + 2(1-\sin^2\theta_w)\right\} \tag{L.34}$$

$$= 0$$

Therefore $\mathcal{M}_{\pm} \rightarrow 0$ as $m_{\tilde{\nu}_{1,2}} \rightarrow \infty$.

L.2 NEUTRALINO DIAGRAMS

The neutralino contributions may be obtained from those of the charginos by making appropriate substitutions. There are two sectors to consider. The left and right sleptons behave somewhat differently. Rather than present all of the crossing relations, which may be obtained by examining the Feynman rules of the previous appendix, we will simply present the final result.

$$\mathcal{M}^{\omega} = \mathcal{M}_L\xi^{\omega}{}_L + \mathcal{M}_R\xi^{\omega}{}_R$$

$$\xi^{\omega}{}_{L,R} = \bar{u}^{\mu}(p_1{}')\gamma_{\mp}\gamma^{\omega}\gamma_{\pm}v^{\tau}(p_2{}')$$

$$\mathcal{M}_L^0 = -\frac{\mathcal{K}_L}{2\sin^2\theta_w \sin 2\theta_w} \sum_{i=1}^{4} \Big\{ |\tan\theta_w N_{i1} + N_{i2}|^2 \cos 2\theta_w \Delta\tilde{G}({T_L}^0{}_i, R_i^0)$$

$$-2\sum_{j=1}^{4} (\tan\theta_w N^*{}_{i1} + N^*{}_{i2})(\tan\theta_w N_{j1} + N_{j2})$$

$$\Big[\sqrt{S_{ij}^0}\, {O^R}''_{ij}\, \Delta\check{I}_{[1]}({T_L}^0{}_i, S_{ij}^0, R_i^0)$$

$$- R_i^0 {O^L}''_{ij}\, \Delta\check{I}_{[\widetilde{Z}^2 - Z^2]}({T_L}^0{}_i, S_{ij}^0, R_i^0)$$

$$+ {O^L}''_{ij}\, \Delta\check{G}({T_L}^0{}_i, S_{ij}^0, R_i^0) \Big] \Big\}$$

$$\mathcal{M}_R^0 = \frac{2\mathcal{K}_R}{\sin^2\theta_w \sin 2\theta_w} \sum_{i=1}^{4} \Big\{ |\tan\theta_w N^*{}_{i1}|^2 2\sin^2\theta_w \Delta\tilde{G}({T_R}^0{}_i, R_i^0)$$

$$+2\sum_{j=1}^{4} (\tan\theta_w N_{i1})(\tan\theta_w N^*{}_{j1})$$

$$\Big[\sqrt{S_{ij}^0}\, {O^L}''_{ij}\, \Delta\check{I}_{[1]}({T_R}^0{}_i, S_{ij}^0, R_i^0)$$

$$- R_i^0 {O^R}''_{ij}\, \Delta\check{I}_{[\widetilde{Z}^2 - Z^2]}({T_R}^0{}_i, S_{ij}^0, R_i^0)$$

$$+ {O^R}''_{ij}\, \Delta\check{G}({T_R}^0{}_i, S_{ij}^0, R_i^0) \Big] \Big\}$$

$$\mathcal{K}_\nu = \frac{i}{16\pi^2} e^3 \sin\theta_\nu \cos\theta_\nu$$

$$\mathcal{K}_L = \frac{i}{16\pi^2} e^3 \sin\theta_L \cos\theta_L$$

$$\mathcal{K}_R = \frac{i}{16\pi^2} e^3 \sin\theta_R \cos\theta_R$$

R, S, and T are ratios of masses:

$$T_{L,R}{}^0{}_i = \frac{{m_{\tilde{\ell}_{L,R_i}}}^2}{{m_{\tilde{\chi}_i^0}}^2}$$

$$S_{ij}^0 = \frac{m_{\tilde{\chi}_j^0}{}^2}{m_{\tilde{\chi}_i^0}{}^2}$$

$$R_i^0 = \frac{q^2}{4m_{\tilde{\chi}_i^0}{}^2}$$

where $m_{\tilde{\ell}_{L_1}}$ and $m_{\tilde{\ell}_{R_1}}$ are the slepton masses.

A typical substition would involve going from the $\tau^-\tilde{\nu}_1\tilde{\chi}_j^+$ to the $\tau^-\tilde{\ell}_{R_1}\tilde{\chi}_j^0$ vertex. We would make the substitutions

$$-igV_{j1} \rightarrow -i\sqrt{2}g\tan\theta_w N_{j1}^* \qquad \gamma_+ \rightarrow \gamma_- \qquad \theta_\nu \rightarrow \theta_R\ .$$

L.3 Complete Matrix Element

Summarizing, the complete matrix element is the total of the chargino and neutralino contributions. It is given by

$$\mathcal{M}^\omega = \mathcal{M}_L \xi^\omega{}_L + \mathcal{M}_R \xi^\omega{}_R \tag{L.35}$$

$$\xi^\omega{}_{L,R} = \bar{u}^\mu(p_1{}')\gamma_\mp\gamma^\omega\gamma_\pm v^\tau(p_2{}') \qquad\qquad \gamma_\pm = \tfrac{1}{2}\left(1 \pm \gamma_5\right)$$

$$\mathcal{K}_\nu = \frac{i}{16\pi^2}e^3\sin\theta_\nu\cos\theta_\nu$$

$$\mathcal{K}_L = \frac{i}{16\pi^2}e^3\sin\theta_L\cos\theta_L$$

$$\mathcal{K}_R = \frac{i}{16\pi^2}e^3\sin\theta_R\cos\theta_R$$

$$
\begin{aligned}
\mathcal{M}_L = \frac{\mathcal{K}_\nu}{\sin^2\theta_w \sin 2\theta_w} \sum_{i=1}^{2} \Big\{ & |V_{i1}|^2 \left[\Delta G(T_i^+, R_i^+) + \cos 2\theta_w \Delta G(T_i^+, 0)\right] \\
& + 2\sum_{j=1}^{2} V_{i1}^* V_{j1} \Big[\sqrt{S_{ij}^+}\, O_{ij}^{R\,\prime}\, \Delta\check{I}_{[1]}(T_i^+, S_{ij}^+, R_i^+) \\
& \qquad - R_i^+ O_{ij}^{L\,\prime}\, \Delta\check{I}_{[\tilde{Z}^2 - Z^2]}(T_i^+, S_{ij}^+, R_i^+) \\
& \qquad + O_{ij}^{L\,\prime}\, \Delta\check{G}(T_i^+, S_{ij}^+, R_i^+) \Big] \Big\} \\
- \frac{\mathcal{K}_L}{2\sin^2\theta_w \sin 2\theta_w} \sum_{i=1}^{4} \Big\{ & |\tan\theta_w N_{i1} + N_{i2}|^2 \cos 2\theta_w \Delta\tilde{G}({T_L}^0{}_i, R_i^0) \\
& - 2\sum_{j=1}^{4} (\tan\theta_w N^*{}_{i1} + N^*{}_{i2})(\tan\theta_w N_{j1} + N_{j2}) \\
& \Big[\sqrt{S_{ij}^0}\, {O^{R\,\prime\prime}}_{ij}\, \Delta\check{I}_{[1]}({T_L}^0{}_i, S_{ij}^0, R_i^0) \\
& \quad - R_i^0 {O^{L\,\prime\prime}}_{ij}\, \Delta\check{I}_{[\tilde{Z}^2 - Z^2]}({T_L}^0{}_i, S_{ij}^0, R_i^0) \\
& \quad + {O^{L\,\prime\prime}}_{ij}\, \Delta\check{G}({T_L}^0{}_i, S_{ij}^0, R_i^0) \Big] \Big\}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_R = \frac{2\mathcal{K}_R}{\sin^2\theta_w \sin 2\theta_w} \sum_{i=1}^{4} \Big\{ & |\tan\theta_w N^*{}_{i1}|^2 2\sin^2\theta_w \Delta\tilde{G}({T_R}^0{}_i, R_i^0) \\
& + 2\sum_{j=1}^{4} (\tan\theta_w N_{i1})(\tan\theta_w N^*{}_{j1}) \\
& \Big[\sqrt{S_{ij}^0}\, {O^{L\,\prime\prime}}_{ij}\, \Delta\check{I}_{[1]}({T_R}^0{}_i, S_{ij}^0, R_i^0) \\
& \quad - R_i^0 {O^{R\,\prime\prime}}_{ij}\, \Delta\check{I}_{[\tilde{Z}^2 - Z^2]}({T_R}^0{}_i, S_{ij}^0, R_i^0) \\
& \quad + {O^{R\,\prime\prime}}_{ij}\, \Delta\check{G}({T_R}^0{}_i, S_{ij}^0, R_i^0) \Big] \Big\}
\end{aligned}
$$

R, S, and T are ratios of masses:

$$T_i^+ = \frac{m_{\tilde{\nu}_i}{}^2}{m_{\tilde{\chi}_i^+}{}^2} \qquad T_{L,R}{}^0{}_i = \frac{m_{\tilde{\ell}_{L,R_i}}{}^2}{m_{\tilde{\chi}_i^0}{}^2}$$

$$S_{ij}^+ = \frac{m_{\tilde{\chi}_j^+}{}^2}{m_{\tilde{\chi}_i^+}{}^2} \qquad S_{ij}^0 = \frac{m_{\tilde{\chi}_j^0}{}^2}{m_{\tilde{\chi}_i^0}{}^2} \tag{L.36}$$

$$R_i^+ = \frac{q^2}{4m_{\tilde{\chi}_i^+}{}^2} \qquad R_i^0 = \frac{q^2}{4m_{\tilde{\chi}_i^0}{}^2}$$

where $m_{\widetilde{\ell}_{L_1}}$ and $m_{\widetilde{\ell}_{R_1}}$ are the masses of the charged slepton of the first generation mixture (see eqn. 3a) and $m_{\widetilde{\nu}_1}$ is the mass of the first sneutrino ($\widetilde{\nu}_1 = \widetilde{\nu}_\mu \cos\theta_\nu + \widetilde{\nu}_\tau \sin\theta_\nu$). The neutralino masses are $m_{\tilde{\chi}_i^0} \quad (i = 1,2,3,4)$ and chargino masses are $m_{\tilde{\chi}_i^+} \quad (i = 1,2)$.

Table L.1 Integral Functions

$$I_{[\mathrm{N}]}(T,R) = \tfrac{1}{2}\int\limits_0^1 dz \int\limits_{-z}^{z} d\tilde{z}\, \frac{N(z,\tilde{z})}{z(1-T)+1+\mathcal{R}(\tilde{z}^2-z^2)}$$

$$\bar{I}_{[\mathrm{N}]}(T,R) = \tfrac{1}{2}\int\limits_0^1 dz \int\limits_{-z}^{z} d\tilde{z}\, \frac{N(z,\tilde{z})}{(1-z)T+z+\mathcal{R}(\tilde{z}^2-z^2)}$$

$$\check{I}_{[\mathrm{N}]}(T,S,R) = \tfrac{1}{2}\int\limits_0^1 dz \int\limits_{-z}^{z} d\tilde{z}\, \frac{N(z,\tilde{z})}{(1-z)T+\frac{1}{2}\left[(1+S)z+(1-S)\tilde{z}\right]+\mathcal{R}(\tilde{z}^2-z^2)}$$

$$\check{G}(T,S,R) = -\tfrac{1}{2}\int\limits_0^1 dz \int\limits_{-z}^{z} d\tilde{z}\, \ln\left[(1-z)T+\tfrac{1}{2}\left[(1+S)z+(1-S)\tilde{z}\right]+\mathcal{R}(\tilde{z}^2-z^2)\right] - 3/4$$

$$\bar{G}(T,R) = -\tfrac{1}{2}\int\limits_0^1 dz \int\limits_{-z}^{z} d\tilde{z}\, \ln\left[(1-z)T+z+\mathcal{R}(\tilde{z}^2-z^2)\right] - 3/4$$

$$G(T,R) = -\tfrac{1}{2}\int\limits_0^1 dz \int\limits_{-z}^{z} d\tilde{z}\, \ln\left[z(1-T)+1+\mathcal{R}(\tilde{z}^2-z^2)\right] - 1/4$$

$$\tilde{G}(T,R) \equiv G(T,R) - G(T,R\equiv 0)$$

Table L.1 Integral Functions (Continued)

Note that

$$\bar{G}(T,R) = \check{G}(T,S=1,R)$$

$$\bar{I}_{[\mathbf{N}]}(T,R) = \check{I}_{[\mathbf{N}]}(T,S=1,R)$$

$$\bar{I}_{[\mathbf{N}]}(T,R) = \frac{1}{T} I_{[\mathbf{N}]}\left(\frac{1}{T},\frac{R}{T}\right)$$

$$I_{[\mathbf{N}]}(T,R) = \frac{1}{T} \bar{I}_{[\mathbf{N}]}\left(\frac{1}{T},\frac{R}{T}\right)$$

$$\bar{G}(T,R) = G\left(\frac{1}{T},\frac{R}{T}\right) + \tfrac{1}{2}\ln T$$

$$I_{[\tilde{\mathbf{z}}^2]}(T,R) = \frac{1}{2\mathcal{R}} \tilde{G}(T,R)$$

For any function, $\mathbf{F}$, of ${T_i^{0,+}}_{L,R}$

$$\Delta F \equiv F|_{\tilde{\ell}_1,\tilde{\nu}_1} - F|_{\tilde{\ell}_2,\tilde{\nu}_2}$$

Table L.1 Integral Functions (Continued)

$$I'_{[\mathrm{N}]}(T,R) = \frac{T}{2}\int_{0}^{1} dz \int_{-z}^{z} d\tilde{z} \frac{zN(z,\tilde{z})}{\left[z(1-T)+1+\mathcal{R}(\tilde{z}^2-z^2)\right]^2}$$

$$\bar{I}'_{[\mathrm{N}]}(T,R) = \frac{T}{2}\int_{0}^{1} dz \int_{-z}^{z} d\tilde{z} \frac{N(z,\tilde{z})}{\left[(1-z)T+z+\mathcal{R}(\tilde{z}^2-z^2)\right]^2}$$

$$\check{I}'_{[\mathrm{N}]}(T,S,R) = \frac{T}{2}\int_{0}^{1} dz \int_{-z}^{z} d\tilde{z} \frac{N(z,\tilde{z})}{\left[(1-z)T+\frac{1}{2}\left[(1+S)z+(1-S)\tilde{z}\right]+\mathcal{R}(\tilde{z}^2-z^2)\right]^2}$$

$$\check{G}'(T,S,R) = -T\check{I}_{[\mathbf{1-z}]}(T,S,R)$$

$$\bar{G}'(T,R) = -T\bar{I}_{[\mathbf{1-z}]}(T,R)$$

$$G'(T,R) = -TI_{[\mathbf{1-z}]}(T,R)$$

$$\tilde{G}'(T,R) = -T\left[I_{[\mathbf{z}]}(T,R) - I_{[\mathbf{z}]}(T,R\equiv 0)\right]$$

$$\breve{G}'(T,R) = \tfrac{1}{2} - G'\left(\frac{1}{T},\frac{R}{T}\right)$$

$$I'_{[\mathbf{z}]}(T,R=0) = \frac{2T\ln T + 1 - T^2}{(T-1)^3}$$

FIGURE CAPTIONS

1. Diagram One.
2. Diagram Two.
3. Diagram Three.
4. Diagram Four.

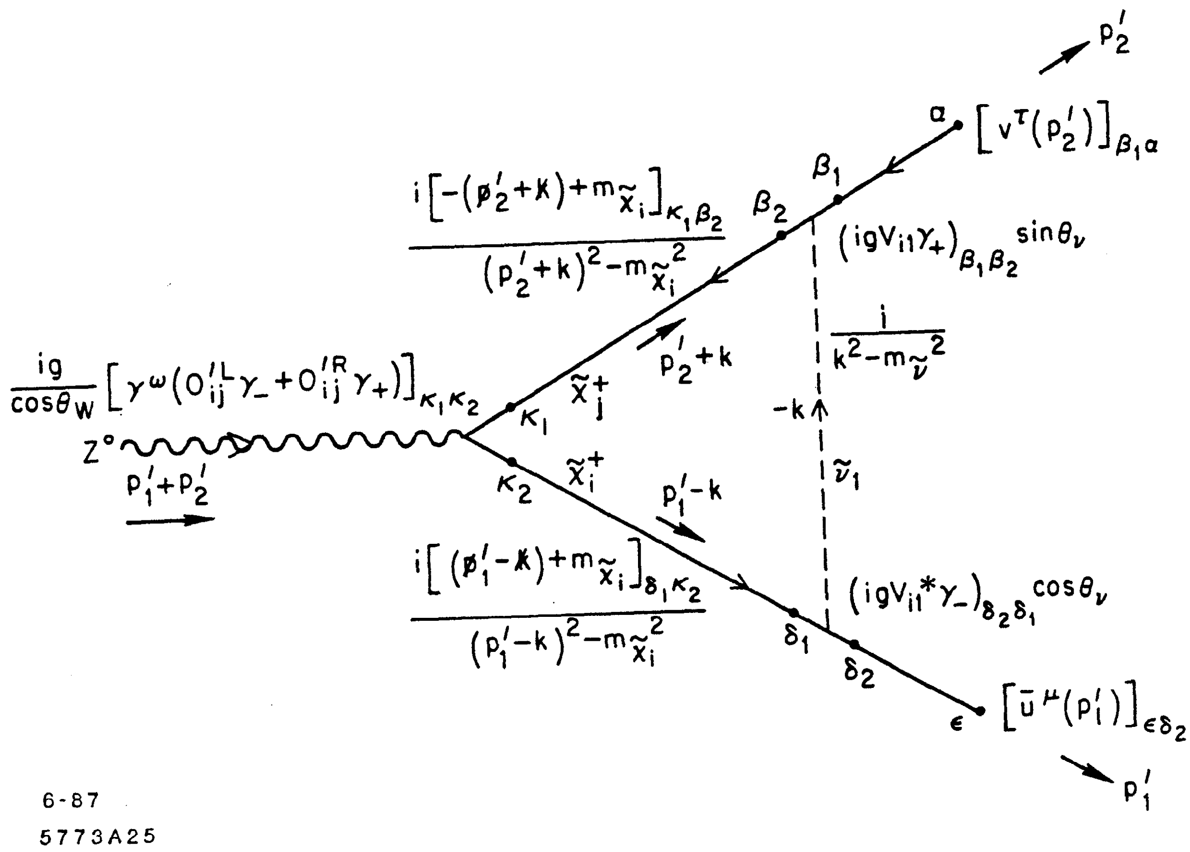
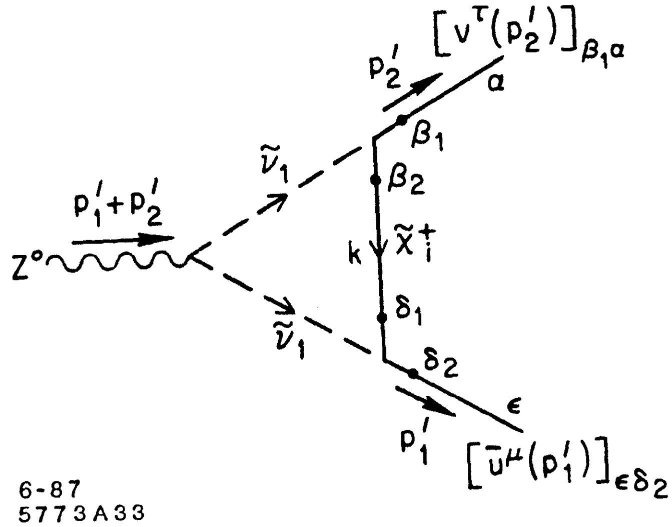

p_2'
α
$[v^T(p_2')]_{\beta_1\alpha}$
β_1
β_2
$\dfrac{i[-(\not p_2'+\not k)+m_{\tilde\chi_i}]_{\kappa_1\beta_2}}{(p_2'+k)^2-m_{\tilde\chi_i}^2}$
$(igV_{i1}\gamma_+)_{\beta_1\beta_2}\sin\theta_\nu$
$\dfrac{i}{k^2-m_{\tilde\nu}^2}$
$\dfrac{ig}{\cos\theta_W}[\gamma^\omega(O_{ij}'^L\gamma_-+O_{ij}'^R\gamma_+)]_{\kappa_1\kappa_2}$
κ_1
$\tilde\chi_j^+$
$p_2'+k$
$-k$
Z°
$p_1'+p_2'$
κ_2
$\tilde\chi_i^+$
$p_1'-k$
$\tilde\nu_1$
$\dfrac{i[(\not p_1'-\not k)+m_{\tilde\chi_i}]_{\delta_1\kappa_2}}{(p_1'-k)^2-m_{\tilde\chi_i}^2}$
$(igV_{i1}^*\gamma_-)_{\delta_2\delta_1}\cos\theta_\nu$
δ_1
δ_2
ϵ
$[\bar u^\mu(p_1')]_{\epsilon\delta_2}$
p_1'
6-87
5773A25

Fig. L.1



$[v^T(p_2')]_{\beta_1 \alpha}$
p_2'
α
β_1
$\tilde{\nu}_1$
β_2
$p_1' + p_2'$
k
$\tilde{\chi}_i^+$
Z^0
δ_1
$\tilde{\bar{\nu}}_1$
δ_2
ϵ
p_1'
$[\bar{u}^\mu(p_1')]_{\epsilon\delta_2}$
6-87
5773A33

Fig. L.2

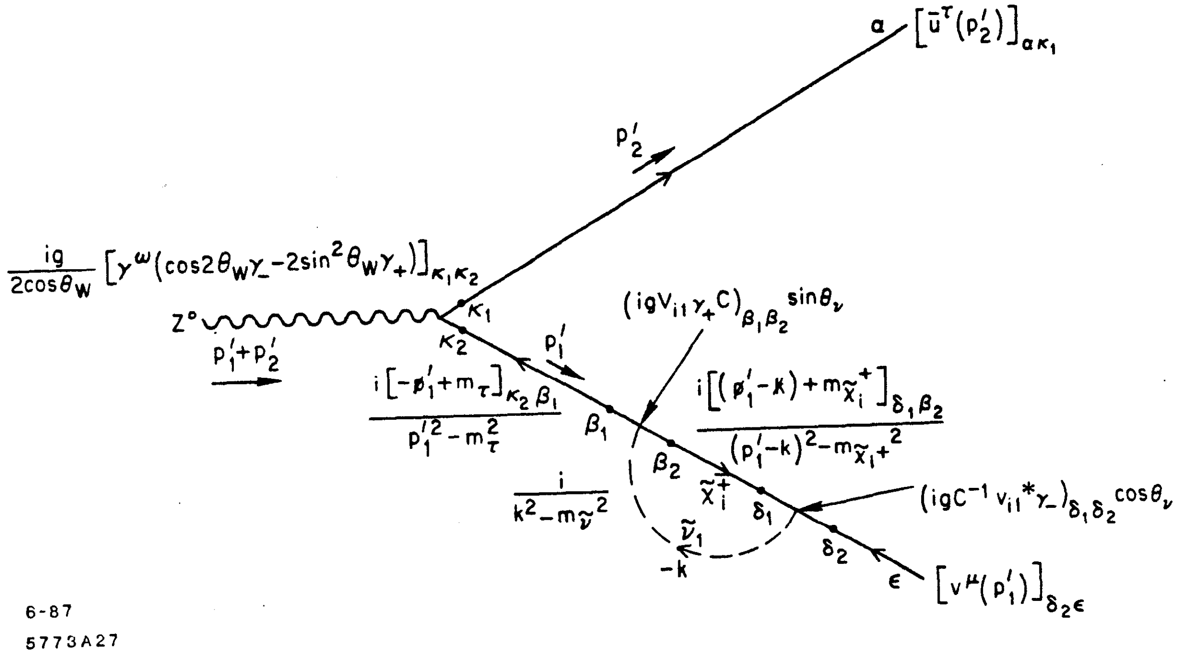
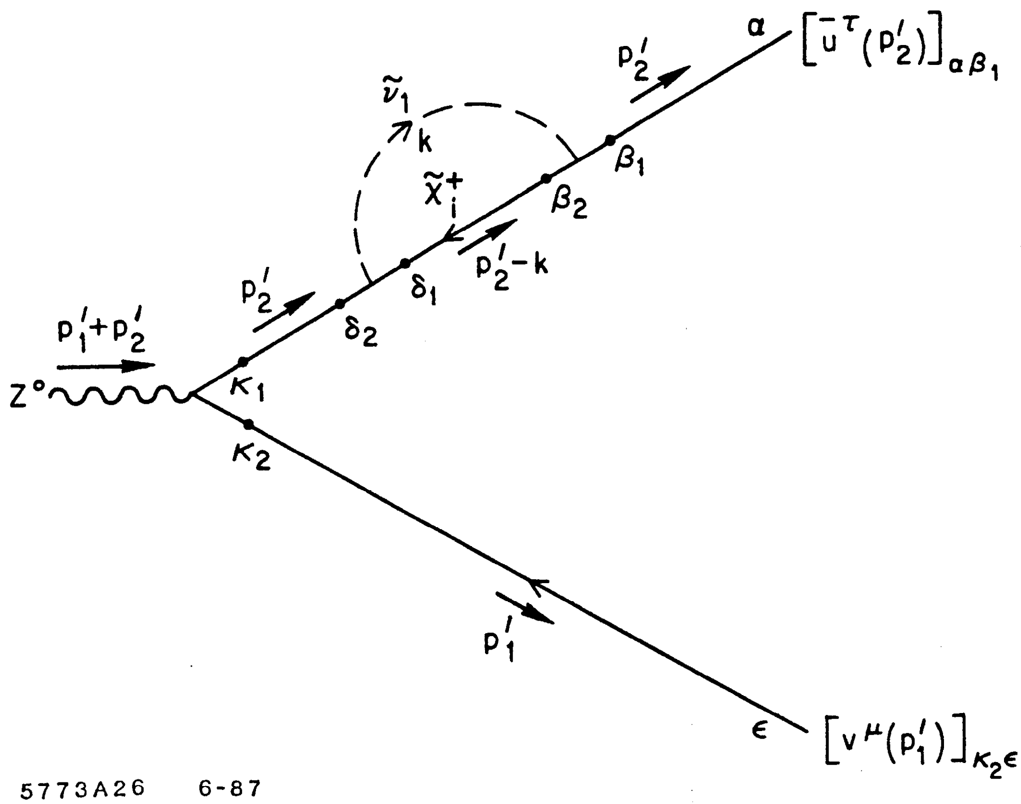

α
$[\bar{u}^\tau(p_2')]_{\alpha\kappa_1}$
p_2'
$\frac{ig}{2\cos\theta_W}[\gamma^\omega(\cos2\theta_W\gamma_- - 2\sin^2\theta_W\gamma_+)]_{\kappa_1\kappa_2}$
Z°
κ_1
κ_2
$p_1'+p_2'$
p_1'
$(igV_{i1}\gamma_+C)_{\beta_1\beta_2}\sin\theta_\nu$
$\frac{i[-\not p_1'+m_\tau]_{\kappa_2\beta_1}}{p_1'^2-m_\tau^2}$
$\frac{i[(\not p_1'-\not k)+m_{\tilde{\chi}_i^+}]_{\delta_1\beta_2}}{(p_1'-k)^2-m_{\tilde{\chi}_1^+}^2}$
β_1
β_2
$\tilde{\chi}_i^+$
δ_1
$\frac{i}{k^2-m_{\tilde{\nu}}^2}$
$(igC^{-1}v_{i1}^*\gamma_-)_{\delta_1\delta_2}\cos\theta_\nu$
$\tilde{\nu}_1$
$-k$
δ_2
ϵ
$[v^\mu(p_1')]_{\delta_2\epsilon}$
6-87
5773A27

Fig. L.3



Z°
$p_1'+p_2'$
p_2'
κ_1
κ_2
δ_2
δ_1
$\tilde{\nu}_1$
k
$\tilde{\chi}_i^+$
$p_2'-k$
β_2
β_1
p_2'
α
$[\bar{u}^\tau(p_2')]_{\alpha\beta_1}$
p_1'
ϵ
$[v^\mu(p_1')]_{\kappa_2\epsilon}$
5773A26 6-87

Fig. L.4

APPENDIX M

Analysis of Integral Functions

M.1 Invariant Form of Ǐ and Ǧ Integrals: S_{ij} versus S_{ji}

We know from physical intuition that when we interchange $\widetilde{\chi}_i$ and $\widetilde{\chi}_j$ in the vertex shown in Fig. 1 that this should not affect our answer.

Here $\widetilde{\chi}_i$ is $\widetilde{\chi}_i^{\pm,0}$, $m_{\widetilde{\ell}} = m_{\widetilde{\ell}_{L,R}}$, etc. Using these abreviated forms we may also shorten the mass ratios originally defined in eqn. 3.10 to

$$T_{i,j} = \frac{m_{\widetilde{\ell}}^2}{(m_{\widetilde{\chi}_{i,j}})^2} \quad S_{ij} = \frac{m_{\widetilde{\chi}_j}^2}{m_{\widetilde{\chi}_i}^2} \quad S_{ji} = \frac{m_{\widetilde{\chi}_i}^2}{m_{\widetilde{\chi}_j}^2} \quad R_{i,j} = \frac{q^2}{4(m_{\widetilde{\chi}_{i,j}})^2} \tag{M.1}$$

$$S_{ij} = \frac{1}{S_{ji}} \tag{M.2}$$

Then

$$\check{I}_{[\mathrm{N}]}(T_i, S_{ij}, R_i) = \tfrac{1}{2} \int_0^1 dz \int_{-z}^{z} d\tilde{z}$$
$$\times \frac{N(z,\tilde{z})}{(1-z)T_i + \frac{1}{2}\left[(1+S_{ij})z + (1-S_{ij})\tilde{z}\right] + R_i(\tilde{z}^2 - z^2)} \, . \tag{M.3}$$

Now multiply by S_{ji}/S_{ji} to use (M.2)

$$\check{I}_{[\mathrm{N}]}(T_i, S_{ij}, R_i)$$
$$= \tfrac{1}{2} \int_0^1 dz \int_{-z}^{z} d\tilde{z} \, \frac{S_{ji} N(z,\tilde{z})}{(1-z)T_i S_{ji} + \frac{1}{2}\left[(1+S_{ji})z - (1-S_{ji})\tilde{z}\right] + R_i S_{ji}(\tilde{z}^2 - z^2)} \, . \tag{M.4}$$

Now

$$T_iS_{ji} = T_j \quad \text{and} \quad R_iS_{ji} = R_j \ . \tag{M.5}$$

If $N(z,\tilde{z}) = N(z,-\tilde{z})$ we let $\tilde{z} \to -\tilde{z}$ in (M.4) and using (M.5) get

$$\begin{aligned} \check{I}_{[\mathrm{N}]}(T_i, S_{ij}, R_i) &= (S_{ji})\tfrac{1}{2} \int\limits_0^1 dz \int\limits_{-z}^{z} d\tilde{z} \\ &\quad \times \frac{N(z,\tilde{z})}{(1-z)T_j + \frac{1}{2}\left[(1+S_{ji})z + (1-S_{ji})\tilde{z}\right] + R_i(\tilde{z}^2 - z^2)} \\ &= S_{ji}\check{I}_{[\mathrm{N}]}(T_i, S_{ji}, R_j) \end{aligned} \tag{M.6}$$

or

$$\sqrt{S_{ij}}\ \check{I}_{[\mathrm{N}]}(T_i, S_{ij}, R_i) = \sqrt{S_{ji}}\ \check{I}_{[\mathrm{N}]}(T_j, S_{ji}, R_j) \ . \tag{M.7}$$

Similarly from (M.5) and (M.6)

$$T_i\check{I}_{[\mathrm{N}]}(T_i, S_{ij}, R_i) = T_j\check{I}_{[\mathrm{N}]}(T_i, S_{ji}, R_i) \tag{M.8}$$

$$R_i\check{I}_{[\mathrm{N}]}(T_i, S_{ij}, R_i) = R_j\check{I}_{[\mathrm{N}]}(T_j, S_{ji}, R_j) \ . \tag{M.9}$$

In particular, of the functions which appear in (3.9), $\sqrt{S}\ \check{\mathrm{I}}_{[1]}$ and $R\check{\mathrm{I}}_{[\tilde{z}^2-z^2]}$,

$$\sum_{ij} \sqrt{S}\,\check{\mathrm{I}}_{[1]} = 2 \sum_{\substack{ij \\ m_{\tilde{\chi}_j} > m_{\tilde{\chi}_i}}} \sqrt{S}\,\check{\mathrm{I}}_{[1]} + \sum_i \bar{I}_{[1]} \tag{M.10}$$

The latter term being the $i = j$ term since $\bar{I} = \check{I}$ with $S \equiv 1$. Likewise

$$\begin{aligned} &\sum_{ij} R_i\check{\mathrm{I}}_{[\tilde{z}^2-z^2]}(T_i, S_{ij}, R_i) \\ &\quad = 2 \sum_{\substack{ij \\ m_{\tilde{\chi}_j} > m_{\tilde{\chi}_i}}} R_i\check{\mathrm{I}}_{[\tilde{z}^2-z^2]}(T_i, S_{ij}, R_i) + \sum_i R_i\bar{I}_{[\tilde{z}^2-z^2]}(T_i, R_i) \ . \end{aligned} \tag{M.11}$$

The sums

$$\sum_{\substack{i,j \\ m_{\tilde{\chi}_j} > m_{\tilde{\chi}_i}}}$$

can also be written

$$\sum_{\substack{i,j \\ S>1}} \tag{M.12}$$

A similar result holds for the $\check{\mathrm{G}}$ functions. From Table L.1, letting $\tilde{z} \to -\tilde{z}$ if $N(z, \tilde{z})$ is even in $\tilde{z}$, yields

$$\check{\mathrm{G}}(T_i, S_{ij}, R_i) = \check{\mathrm{G}}(T_j, S_{ji}, R_j) + \tfrac{1}{2}\, \ell n\, S_{ji} \tag{M.13}$$

and the "invariant" form is

$$\check{\mathrm{G}}(T_i, S_{ij}, R_i) + \frac{1}{4}\, \ell n\, S_{ij} = \check{\mathrm{G}}(T_j, S_{ji}, R_j) + \frac{1}{4}\, \ell n\, S_{ji} \tag{M.14}$$

and similarly

$$\check{\mathrm{G}}(T_i, S_{ij}, R_i) + \tfrac{1}{2}\, \ell n\, R_i = \check{\mathrm{G}}(T_j, S_{ji}, R_j) + \tfrac{1}{2}\, \ell n\, R_j \tag{M.15}$$

$$\check{\mathrm{G}}(T_i, S_{ij}, R_i) + \tfrac{1}{2}\, \ell n\, T_i = \check{\mathrm{G}}(T_j, S_{ji}, R_j) + \tfrac{1}{2}\, \ell n\, T_j\ . \tag{M.16}$$

From Table L.1

$$\check{\mathrm{G}}'(T_i, S_{ij}, R_i) = -T_i \check{\mathrm{I}}_{[1-z]}(T_i, S_{ij}, R_i) \tag{M.17}$$

so that

$$\check{\mathrm{G}}'(T_i, S_{ij}, R_i) = \check{\mathrm{G}}'(T_j, S_{ji}, R_j)\ . \tag{M.18}$$

Thus in all of the terms in $\mathcal{M}^{\omega}$, both large and small mass-splitting cases considered, we may restrict our attention to the $S_{ji} \geq 1$ contributions. This will prove convenient for numerical evaluation.

M.2 $\check{I}_{[N(z,\tilde{z})]}$ Integrals: Imaginary Part

Some fun with complex analysis

The propagator for the i^{th} boson [fermion] is

$$\Delta_{i\,\mathrm{propagator}} = \frac{i[(\not{p}_i + m_i)]}{p_i^2 - m_i^2 + i\epsilon_i} \tag{M.19}$$

so we just absorb the $i\epsilon$ into the m^2 term

$$m_i^2 \to m_i^2 - i\epsilon_i \ . \tag{M.20}$$

The $\check{I}_{[N]}$ terms came from diagrams like those of Fig. 2 involving loops of three propagators, each with their own, arbitrary $i\epsilon$. From eqn. (L.3) we are lead to integrals like

$$I \sim \int_0^1 dz \int_{-z}^{z} d\tilde{z}$$

$$\times \frac{m_{\tilde{\chi}_i}^2}{m_{\tilde{\ell}}^2(1-z) + \frac{1}{2}\left[(m_{\tilde{\chi}_i}^2 + m_{\tilde{\chi}_j}^2)z + (m_{\tilde{\chi}_i}^2 - m_{\tilde{\chi}_j}^2)\tilde{z}\right] + \frac{1}{4}\, q^2(\tilde{z}^2 - z^2) - i\bar{\epsilon}} \tag{M.21}$$

where

$$\bar{\epsilon} = \bar{\epsilon}_{\tilde{\ell}}(1-z) + \tfrac{1}{2}\left[(\bar{\epsilon}_{\chi_i} + \bar{\epsilon}_{\chi_j})z + (\bar{\epsilon}_{\chi_i} - \bar{\epsilon}_{\chi_j})\tilde{z}\right] . \tag{M.22}$$

Since each ϵ_i is of arbitrary relative magnitude the overall sign of $\bar{\epsilon}$ might appear to be parameter-dependent in a way which could render the *relative* signs of different contributions ambigous. Our task is to prove that this is not the case and to derive the correct sign.

Divide through by $m^2_{\tilde{\chi}_i}$

$$I \sim \int_0^1 dx \int_{-z}^{z} d\tilde{z} \, \frac{1}{T_i(1-z) + \frac{1}{2}\left(1 + S_{ij})z + (1 - S_{ij})\tilde{z}\right] + R_i(\tilde{z}^2 - z^2) - i\epsilon} \tag{M.23}$$

where

$$T_i \equiv \frac{m^2_{\tilde{\ell}}}{m^2_{\tilde{\chi}_i}} \quad S_{ij} \equiv \frac{m^2_{\tilde{\chi}_j}}{m^2_{\tilde{\chi}_i}} \quad R_i \equiv \frac{q^2}{4m_{\tilde{\chi}^2_i}} \equiv \frac{m^2_Z}{4\tilde{\chi}^2_i} \tag{M.24}$$

here (as in (M.1)). We re-define

$$\epsilon \equiv \frac{\bar{\epsilon}}{m^2_{\tilde{\chi}_i}}$$

and, in general,

$$\epsilon_j \equiv \frac{\bar{\epsilon}_j}{m^2_{\tilde{\chi}_i}} \tag{M.25}$$

(i subscript understood).

The slepton masses, $m_{\tilde{\ell}}$, are $(m_{\tilde{\ell}_{L_i}}, m_{\tilde{\ell}_{R_i}}, m_{\tilde{\nu}_i})$ and $m_{\tilde{\chi}_i}$ is i^{th} gaugino/higgsino mass. We can choose $S_{ij} \geq 1$ as discussed above and so we have

$$q^2 = m^2_Z > 0 \quad T_i \geq 0 \quad R_i \geq 0 \quad S_{ij} \geq 1 \quad 0 \leq z \leq 1 \quad -z \leq \tilde{z} \leq z$$
$$\text{and all} \quad \epsilon > 0\,. \tag{M.26}$$

Now look at the most general such integral of the type in (M.23). Pulling "R_i" out of (M.23) we may write (making the expression exact)

$$\check{\mathrm{I}}_{[N(z,\tilde{z})]} = \frac{1}{2R} \int_0^1 dz \int_{-z}^{z} d\tilde{z} \, \frac{N(z,\tilde{z})}{\tilde{z}^2 + (b + i\delta_b)\tilde{z} + (c + i\delta_c)}\,. \tag{M.27}$$

Comparing with (M.23), (M.24), and (M.25)

$$
\begin{aligned}
b &= \frac{1-S}{2\mathcal{R}} < 0 \\
\delta_b &= -\frac{2}{q^2}\,(\epsilon_{\chi_i} - \epsilon_{\chi_j})
\end{aligned}
\tag{M.28}
$$

which is of no fixed sign since ϵ_i, ϵ_j are arbitrary and independent

$$
\begin{aligned}
c &= -\left[z^2 - \frac{T(1-z) + \frac{1}{2}(1+S)z}{\mathcal{R}}\right] \\
\delta_c &= -\frac{4}{q^2}\,[\epsilon_{\tilde{l}}(1-z) + \tfrac{1}{2}(\epsilon_{\chi_j} + \epsilon_{\chi_j})z] < 0
\end{aligned}
\tag{M.29}
$$

where the inequality follows from (M.26) since all the bracketed terms are positive. Since $|\epsilon_{\widetilde{\chi}_i} - \epsilon_{\widetilde{\chi}_j}| < \epsilon_{\widetilde{\chi}_i} + \epsilon_{\widetilde{\chi}_j}$ and $0 \leq z \leq 1$ we can say that

$$
0 < z|\delta_b| < -\delta_c\ . \tag{M.30}
$$

For (M.27) write

$$
\mathrm{I}_{[N(z,\tilde{z})]} = \frac{1}{2R}\int_0^1 dz \int_{-z}^{z} d\tilde{z}\,\frac{N(z,\tilde{z})}{(\tilde{z}-\tilde{z}_+)(\tilde{z}-\tilde{z}_-)} \tag{M.31}
$$

where

$$
\tilde{z}_\pm \doteq \tfrac{1}{2}\left[-(b+i\delta_b) \pm \sqrt{b^2 + 2ib\delta_b - 4c - 4\delta_c}\right]\ . \tag{M.32}
$$

Letting

$$
\Delta = b^2 - 4c \equiv 4Q = 4\left[\left(z + \frac{b}{2}\right)^2 - \frac{(1-z)T + z}{R}\right] \tag{M.33}
$$

defines Q as

$$
Q \equiv \left(z + \frac{b}{2}\right)^2 - \frac{(1-z)T + z}{R} \geq 0 \tag{M.34}
$$

since $\Delta \geq 0$ or else there *is* no imaginary part. This follows since Δ is, essentially the discriminant of the denominator of (M.22). If $\Delta < 0$ then the denominator has no real roots and is always positive or negative definite over the range of integration. The result is a real integral. Note that since $(1-z), z, T$ and $R \geq 0$ that $\frac{(1-z)T+z}{R} \geq 0$ and since $Q \geq 0$ for $Im\,\check{\mathrm{I}}$ to exist it follows that that

$$Q < (z + b/2)^2 \; . \tag{M.35}$$

Define

$$\begin{aligned} r_{\pm} &= Re\left[\tilde{z}_{\pm}\right] \\ &= -\frac{b}{2} \pm \sqrt{Q} \end{aligned} \tag{M.36}$$

so $r_{+} > r_{-}$ $(Q \geq 0)$

$$\tilde{z}_{\pm} = r_{\pm} + \tfrac{1}{2}\, i \left[\delta_b \pm i\, \frac{(b\delta_b - 2\delta_c)}{2\sqrt{Q}}\right] \; . \tag{M.37}$$

Writing

$$\tilde{z}_{\pm}(z) = r_{\pm}(z) + i\epsilon_{\pm}(z) \tag{M.38}$$

our analysis will reduce to determining the signs of $\epsilon_{\pm}$.

$$\begin{aligned} \epsilon_{\pm} &= -\left[\delta_b \mp \frac{b\delta_b}{2\sqrt{Q}} \pm \frac{2\delta_c}{2\sqrt{Q}}\right] = -\frac{1}{2\sqrt{Q}}\left[\delta_b\left(\sqrt{Q} \mp \frac{b}{2}\right) \pm \delta_c\right] \\ &= \mp\frac{1}{2\sqrt{Q}}\left[r_{\pm}\delta_b + \delta_c\right] \end{aligned} \tag{M.39}$$

Define

$$\bar{r}_{\pm} = -\frac{b}{2} \pm \sqrt{\frac{(1-z)T + z}{R}} \tag{M.40}$$

and note that

$$\bar{r}_{+} > \bar{r}_{-} \; .$$

Then

$$Q = \left(z + \frac{b}{2}\right)^2 - \frac{(1-z)T + z}{R} = (z - \bar{r}_+)(z - \bar{r}_-) \ . \tag{M.41}$$

Of course from (M.35)

$$(z - \bar{r}_+)(z - \bar{r}_-) = Q > 0 \ . \tag{M.42}$$

So the cases are

$$(i) \quad z - \bar{r}_+ > 0 \quad \text{and} \quad z - \bar{r}_- > 0 \tag{M.43}$$

$$(ii) \quad z - \bar{r}_+ < 0 \quad \text{and} \quad z - \bar{r}_- < 0 \tag{M.44}$$

Likewise

$$(z - r_+)(z - r_-) = \frac{(1-z)T + z}{R} > 0 \tag{M.45}$$

Cases

$$(i) \quad z - r_+ > 0 \quad \text{and} \quad z - r_- > 0 \tag{M.46}$$

$$(ii) \quad z - r_+ < 0 \quad \text{and} \quad z - r_- < 0 \tag{M.47}$$

and in all cases $0 \le z \le 1$. From (M.31) and (M.38)

$$\check{I}_{[N(z,\tilde{z})]} = \frac{1}{2R} \int_0^1 dz \int_{-z}^{z} d\tilde{z} \ \frac{N(z,\tilde{z})}{(\tilde{z} - r_+ - i\epsilon_+)(\tilde{z} - r_- - i\epsilon_-)} \tag{M.48}$$

$$= \frac{1}{2R} \int_0^1 \int_{-z}^{z} d\tilde{z} \ \frac{N(z,\tilde{z})}{(r_+ - r_-)} \left[\frac{1}{\tilde{z} - r_+ - i\epsilon_+} - \frac{1}{\tilde{z} - r_- - i\epsilon_-}\right]$$

with

$$r_\pm = -\frac{b}{2} \pm \sqrt{Q}$$

and therefore

$$\check{\mathrm{I}}_{[N(z,\tilde{z})]} = \frac{1}{4R} \int_0^1 \int_{-z}^{z} d\tilde{z} \, \frac{N(z,\tilde{z})}{\sqrt{Q}} \left[\frac{1}{\tilde{z} - r_+ - i\epsilon_+} - \frac{1}{\tilde{z} - r_- - i\epsilon_-} \right] . \tag{M.49}$$

<u>Principal Value Formula</u>

Under $\int dx$:

$$\frac{1}{x - \alpha \pm i\epsilon} \rightarrow \frac{1}{x - \alpha} \mp i\pi\delta(x - \alpha) \tag{M.50}$$

$$\begin{aligned} Im\, \check{\mathrm{I}}_{[N(z,\tilde{z})]} &= \frac{i\pi}{4R} \int_0^1 \int_{-z}^{z} d\tilde{z} \\ &\times \frac{\theta(Q)}{\sqrt{Q}} N(z,\tilde{z}) \left[\mathrm{sign}(\epsilon_+)\delta(\tilde{z} - r_+) - \mathrm{sign}(\epsilon_-)\delta(\tilde{z} - r_-) \right] \end{aligned} \tag{M.51}$$

since $Q > 0$ for an Im part to exist. The signum function, $\mathrm{sign}(X)$, is defined to be :

$$\mathrm{sign}(x) = \begin{cases} -1 & x < 0 \\ 0 & x = 0 \\ 1 & x > 0 \end{cases} \tag{M.52}$$

$$\begin{aligned} Im\, \check{\mathrm{I}}_{[N(z,\tilde{z})]} = \frac{i\pi}{4R} \int_0^1 dz \, \frac{\theta(Q)}{\sqrt{Q}} \Big[&\mathrm{sign}(\epsilon_+) N(z, r_+) \{\theta(z - r_+) - \theta(-z - r_+)\} \\ &- \mathrm{sign}(\epsilon_-) N(z, r_-) \{\theta(z - r_-) - \theta(-z - r_-)\} \Big] . \end{aligned} \tag{M.53}$$

Now from (M.35)

$$\sqrt{Q} < \left|z + \frac{b}{2}\right| \leq \left|z - \frac{b}{2}\right| \qquad (z \geq 0 \quad b \leq 0) \tag{M.54}$$

and since

$$z - \frac{b}{2} > 0 \qquad \sqrt{Q} < z - \frac{b}{2} \tag{M.55}$$

we find

$$-\left(z - \frac{b}{2}\right) \pm \sqrt{Q} < 0 \tag{M.56}$$

and therefore

$$\theta(-z - r_{\pm}) = \theta(-z + \frac{b}{2} \mp \sqrt{Q}) = \theta\left(-\left[z - \frac{b}{2}\right] \mp \sqrt{Q}\right) = 0 \tag{M.57}$$

with the result that (M.53) reduces to

$$Im\,\mathrm{I}_{[N(z,\bar{z})]} = \frac{i\pi}{4R} \int_0^1 dz$$

$$\frac{\theta(Q)}{\sqrt{Q}} \left[N(z, r_+)\mathrm{sign}(\epsilon_+)\theta(z - r_+) - N(z, r_-)\mathrm{sign}(\epsilon_-)\theta(z - r_-)\right] . \tag{M.58}$$

Since the two terms are proportional to $\theta(z - r_+)$ and $\theta(z - r_-)$ we require $z - r_+$ and/or $z - r_-$ to be positive in order to get a contribution. Looking at (M.46) and (M.47) we see that this rules out case (ii).

We must have

$$z - r_{\pm} > 0 \tag{M.59}$$

This can be understood in the following way:

Since $\dfrac{(1-z)T+z}{R} > 0$ we could write a factor of

$$\theta\left(\frac{(1-z)T+z}{R}\right) = 1 \tag{M.60}$$

in $\check{\mathrm{I}}_{[N]}$ just like $\theta(Q)$

$$\theta\left(\frac{(1-z)T+z}{R}\right) = \theta([z-r_+]\,[z-r_-]) \tag{M.61}$$

so from (M.58)

$$Im\,\check{\mathrm{I}}_{[N(z,\bar{z})]} = \frac{i\pi}{4R}\int\limits_0^1 dz\;\frac{\theta(Q)}{\sqrt{Q}}\;\theta([z-r_+][z-r_-]) \times [\theta(z-r_+)\mathrm{sign}(\epsilon_+)N(z,r_+) - \theta(z-r_-)\mathrm{sign}(\epsilon_-)N(z,r_-)] \tag{M.62}$$

but

$$\theta([z-r_+][z-r_-])\theta(z-r_\pm) = \theta(z-r_+)\theta(z-r_-) = \theta(z-r_+) \tag{M.63}$$

since $r_+ > r_-$, i.e.

$$0 < r_+ < z \leq 1 \quad \text{also} \quad r_- < r_+ < z \leq 1 \tag{M.64}$$

the net result being

$$Im\,\check{\mathrm{I}}_{[N(z,\bar{z})]} = \frac{i\pi}{4R}\int\limits_0^1 dz\;\frac{\theta(Q)\theta(z-r_+)}{\sqrt{Q}}\;[\mathrm{sign}(\epsilon_+)N(z,r_+) - \mathrm{sign}(\epsilon_-)N(z,r_-)]\,. \tag{M.65}$$

From (M.57) $-z - r_- < 0$ and therefore $-z < r_-$. With this and from (M.64)

$$1 \leq -z < r_- < r_+ < z \leq 1 \quad \text{with} \quad 0 < r_+ < z \leq 1 \tag{M.66}$$

From (M.36) and (M.40)

$$\begin{aligned} \bar{r}_+ - r_- &= \sqrt{\frac{(1-z)T + z}{R}} + \sqrt{Q} > 0 \\ r_+ - \bar{r}_- &= \sqrt{\frac{(1-z)T + z}{R}} + \sqrt{Q} > 0 \end{aligned} \tag{M.67}$$

so

$$\bar{r}_- < r_+ < z \leq 1 \tag{M.68}$$

and hence

$$z - \bar{r}_- > 0 \tag{M.69}$$

From (M.43) and (M.44) case (ii) has just been eliminated leading to the inescapable result that

$$z - \bar{r}_\pm > 0\ . \tag{M.70}$$

Thus (because $r_+ > r_-$)

$$\theta(Q) = \theta([z - \bar{r}_+][z - \bar{r}_-]) = \theta(z - \bar{r}_+)\theta(z - \bar{r}_-) = \theta(z - \bar{r}_+)\ . \tag{M.71}$$

Also

$$\bar{r}_+ - z < 0 \Rightarrow -z - \frac{b}{2} + \sqrt{\frac{(1-z)T + z}{R}} < 0\ . \tag{M.72}$$

Now

$$-z - \bar{r}_- = -z + \frac{b}{2} + \sqrt{\frac{(1-z)T + z}{R}} < -z - \frac{b}{2} + \sqrt{\frac{(1-z)T + z}{R}} < 0 \tag{M.73}$$

from (M.72) since $b < 0$.

Therefore

$$\overline{r}_- > -z \tag{M.74}$$

Also note that from (M.72)

$$-1 \le -z < r_-, \overline{r}_- < r, \overline{r}_+ < z \le 1 \qquad r_+ > 0 \tag{M.75}$$

$$0 < \sqrt{\frac{(1-z)T+z}{R}} < z + \frac{b}{2} \tag{M.76}$$

i.e.

$$z + \frac{b}{2} > 0 \tag{M.77}$$

so using (M.71) and (M.65)

$$Im\,\check{\mathrm{I}}_{[N(z,\bar{z})]} = \frac{i\pi}{4R} \int_0^1 dz\, \frac{\theta(z-\overline{r}_+)\theta(z-r_+)}{\sqrt{Q}} \left[\mathrm{sign}(\epsilon_+)N(z,r_+) - \mathrm{sign}(\epsilon_-)N(z,r_-)\right] . \tag{M.78}$$

Note that

$$\theta(z-\overline{r}_+)\theta(z-r_+) = \theta(z - \max[r_+, \overline{r}_+]) . \tag{M.79}$$

From (M.75)

$$|r_\pm| < z . \tag{M.80}$$

Recall from (M.30) and (M.39) that

$$\epsilon_\pm = \mp \frac{1}{2\sqrt{Q}} \left[r_\pm \delta_b + \delta_c\right] \quad \text{and} \quad z|\delta_b| < \delta_c < 0 . \tag{M.81}$$

Since

$$|r_-| < z \Rightarrow |r_\pm \delta_b| < z|\delta_b| < |\delta_c| \tag{M.82}$$

we see that

$$\text{sign}[r_{\pm}\delta_b + \delta_c] = \text{sign}[\delta_c] = -1 \tag{M.83}$$

and since $\frac{1}{2\sqrt{Q}} > 0$

$$\text{sign}(\epsilon_{\pm}) = \pm 1 \tag{M.84}$$

we may finally conclude that

$$Im\, \bar{\text{I}}_{[N(z,\bar{z})]} = \frac{i\pi}{4R} \int_0^1 dz\, \frac{\theta(z - \bar{r}_+)\theta(z - r_+)}{\sqrt{Q}}\, [N(z, r_+) + N(z, r_-)] \tag{M.85}$$

where we recall that

$$\begin{aligned}
\bar{r}_{\pm} &= -\frac{b}{2} \pm \sqrt{\frac{(1-z)T + z}{R}} \\
r_{\pm} &= -\frac{b}{2} \pm \sqrt{Q} \\
Q &= \left(z + \frac{b}{2}\right)^2 - \frac{(1-z)T + z}{R} \\
b &= \frac{(1-S)}{2R}
\end{aligned} \tag{M.86}$$

T,S, and R are as in (M.24).

M.3 $\bar{I}_{[N]}$ AND $I_{[N]}$ INTEGRALS: IMAGINARY PART

We may also obtain $\bar{I}_{[N(z,\bar{z})]}$ directly from (M.85) by letting $s \to 1$. Then $b = 0$ and

$$\bar{r}_{\pm} \to \pm\sqrt{\frac{(1-z)T + z}{R}} \tag{M.87}$$

$$r_{\pm} \to \sqrt{\overline{Q}} \tag{M.88}$$

with

$$\overline{Q} = z^2 - \frac{(1-z)T + z}{R} \tag{M.89}$$

$Im\,\overline{I}_{[N(z,\tilde{z})]}$

$$= \frac{i\pi}{4R} \int_0^1 dz \, \frac{\theta\left(z - \sqrt{\frac{(1-z)T+z}{R}}\right) \theta(z - \sqrt{\overline{Q}})}{\sqrt{\overline{Q}}} \left[N(z, \sqrt{\overline{Q}}) + N(z, -\sqrt{\overline{Q}})\right] . \tag{M.90}$$

Note that if $N(z, \tilde{z})$ is even in $\tilde{z}$, i.e. $N(z, \tilde{z}) = N(z, -\tilde{z})$ then

$$Im\,\overline{I}_{[N(z,-\tilde{z})=N(z,\tilde{z})]} = \frac{i\pi}{2R} \int_0^1 dz \, \frac{\theta\left(z - \sqrt{\frac{(1-z)T+z}{R}}\right) \theta(z - \sqrt{\overline{Q}})}{\sqrt{\overline{Q}}} N(z, \sqrt{\overline{Q}}) . \tag{M.91}$$

If $N(z, \tilde{z})$ is odd in $\tilde{z}$, i.e. $N(z, -\tilde{z}) = -N(z, \tilde{z})$ then

$$Im\,I_{[N(z,\tilde{z}) \text{ odd in } \tilde{z}]} = 0 . \tag{M.92}$$

We compare I and $\overline{I}$ in Table L.1.

We see that the above analysis goes through for $I_{[\mathrm{N}]}$ if we make the changes

$$\overline{r}_{\pm} \to \overline{r}'_{\pm} = \pm\sqrt{\frac{(1-z) + Tz}{R}} \tag{M.93}$$

$$r_{\pm} \to r'_{\pm} = \pm\sqrt{W} \tag{M.94}$$

with

$$W = z^2 - \frac{(1-z) + Tz}{R} \tag{M.95}$$

Therefore

$$Im\, I_{[N]} = \frac{i\pi}{4R} \int_0^1 dz \, \frac{\theta\left(z - \frac{(1-z)+Tz}{R}\right) \theta(z - \sqrt{W})}{\sqrt{W}} \left[N(z, \sqrt{W}) + N(z, -\sqrt{W})\right] . \tag{M.96}$$

If $N(z, \tilde{z}) = N(z, -\tilde{z})$ is even in $\tilde{z}$

$$Im\, I_{[N]} = \frac{i\pi}{2R} \int_0^1 dz \, \frac{\theta\left(z - \frac{(1-z)+Tz}{R}\right) \theta(z - \sqrt{W})}{\sqrt{W}} N(z, \sqrt{W}) . \tag{M.97}$$

If $N(z, \tilde{z}) = N(z, -\tilde{z})$ is odd in $\tilde{z}$

$$Im\, I_{[N]} = 0 . \tag{M.98}$$

M.4 $I_{[N]}$, $\bar{I}_{[N]}$ and $\check{I}_{[N]}$ Integrals: Real Parts

$$Re\, I_{[N]} = \frac{1}{2R} \int_0^1 dz \int_{-z}^{z} d\tilde{z} \, \frac{N(z, \tilde{z})}{\tilde{z}^2 - W(z)} \tag{M.99}$$

where W is given by (M.95). We will assume for the moment that $N = N(z)$. If $N(z, \tilde{z}) = \tilde{z}^2 - z^2$ then we express it in terms of the above (see the next section). Then

$$Re\, I_{[N]} = \frac{1}{R} \int_0^1 dz \, N(z) \int_0^z d\tilde{z}$$

$$\times \left\{ \left[\frac{1}{\tilde{z} - \sqrt{W}} - \frac{1}{\tilde{z} + \sqrt{W}}\right] \frac{1}{2\sqrt{W}} \theta(W) + \frac{1}{\tilde{z}^2 + |W(z)|} \theta(-W) \right\}$$

and from the Principal Value Theorem,

$$=\frac{1}{R}\int_0^1 dz\, N(z)\Big\{\frac{1}{2\sqrt{W}}\ \Big[\ell n(\tilde{z}-\sqrt{W})-\ell n(\tilde{z}+\sqrt{W})\Big]\ \theta(W)$$

$$+\frac{1}{\sqrt{-W}}\tan^{-1}\frac{\tilde{z}}{\sqrt{-W}}\ \theta(-W)\Big\}\Big|_{\tilde{z}=0}^{\tilde{z}=z}$$

ergo

$Re\, I_{[N]}$

$$=\frac{1}{R}\int_0^1 dz\,\frac{N(z)}{\sqrt{|W|}}\left\{\tfrac{1}{2}\,\ell n\left[\frac{(\sqrt{W}-z)}{(\sqrt{W}+z)}\right]\ \theta(W)\ +\ \tan^{-1}\frac{z}{\sqrt{-W}}\ \theta(-W)\right\}\ . \tag{M.100}$$

The procedure for $\check{\mathrm{I}}_{[N]}$ is similar

$$Re\,\check{\mathrm{I}}_{[N(z)]}=\frac{1}{2R}\int_{-z}^{z} d\tilde{z}\ \frac{N(z)}{\tilde{z}^2+b\tilde{z}+c} \tag{M.101}$$

with

$$b=\frac{1-S}{2\mathcal{R}}\qquad c=\frac{1}{\mathcal{R}}\left\{-\mathcal{R}z^2+\frac{1+S}{2}\,z+(1-z)T\right\}\ . \tag{M.102}$$

As in (M.33) and (M.34) let

$$Q=\frac{1}{4}\,(b^2-4c)=\left(z+\frac{b}{2}\right)^2-\frac{z(1-T)+R}{R} \tag{M.103}$$

then

$$Re\,\check{I}_{[\mathrm{N}]}=\frac{1}{4R}\int_0^1\frac{N(z)}{\sqrt{|Q|}}$$

$$\times\left\{2\tan^{-1}\frac{\tilde{z}+b/2}{\sqrt{-Q}}\ \theta(-Q)+\frac{1}{2}\ \ell n\ \frac{\tilde{z}+b/2-\sqrt{Q}}{\tilde{z}+b/2+\sqrt{Q}}\Big|_{-z}^{z}\ \theta(Q)\right\}$$

or

$$Re\,\check{I}_{[\mathrm{N}]} = \frac{1}{2R}\int_0^1 \frac{N(z)}{\sqrt{|Q|}}\left\{\left[\tan^{-1}\left(\frac{z+b/2}{\sqrt{-Q}}\right) - \tan^{-1}\left(\frac{-z+b/2}{\sqrt{-Q}}\right)\right]\theta(-Q)\right.$$

$$\left. +\frac{1}{2}\,\ell n\left[\frac{\left(\frac{b}{2}\right)^2-\left(z-\sqrt{Q}\right)^2}{\left(\frac{b}{2}\right)^2-\left(z+\sqrt{Q}\right)^2}\right]\theta(Q)\right\}. \tag{M.104}$$

For $Re\,\overline{I}_{[N]}$ we let $b \to 0$ in (M.104) above giving us back (M.100) with "W" replaced by $\overline{Q}$ as given in (M.89).

A question arises as to the behavior of $Re\,\check{\mathrm{I}}$ at $Q=0$. Using the asymptotic form $(x\to\infty)$ $\tan^{-1}x = \pi/2 - 1/x + 1/3x^3 - \ldots$ valid in the arctan sheet used

$$Re\,\check{\mathrm{I}}(Q\to 0) \to\sim \frac{1}{2R}\int_0^1 dz\,\frac{N(z)}{\sqrt{Q}}$$

$$\times\left\{\left(-\frac{\sqrt{Q}}{z+b/2}+\frac{\sqrt{Q}}{-z+b/2}\right)\theta(-Q)+\tfrac{1}{2}\,\ell n\left[\frac{1+\frac{2z\sqrt{Q}}{(b/2)^2-z^2}}{1-\frac{2z\sqrt{Q}}{(b/2)^2-z^2}}\right]\theta(Q)\right\}. \tag{M.105}$$

Accordingly

$$Re\,\check{\mathrm{I}}(Q\to 0)\approx\frac{1}{2R}\int_0^1 dz\,N(z)\left\{\frac{2z}{(b/2)^2-z^2}\,\theta(Q)+\frac{2z}{(b/2)^2-z^2}\,\theta(-Q)\right\} \tag{M.106}$$

so the integrand is continuous at $Q=0$. In fact at $Q=0$ since, $\theta(Q)+\theta(-Q)=1$ we have

$$Re\,\check{\mathrm{I}}_{[N(z)]}(Q=0)\to -\frac{1}{R}\int_0^1 dz\,\frac{zN(z)}{z^2-(b/2)^2}\,. \tag{M.107}$$

Note that when $b = 0$ in the $\overline{I}$ case

$$Re\,\overline{I}_{[N(z)]}(\overline{Q}(z) \to 0) \to -\frac{1}{R}\int\limits_0^1 dz\ \frac{N(z)}{z}\ . \tag{M.108}$$

This causes no problems unless $\overline{Q}(z = 0)$ happens to be zero, i.e. unless the spot where $\overline{Q}$ vanishes just *happens* to be $z = 0$. But

$$\overline{Q} = z^2 - \frac{(1-z)T + z}{R} \tag{M.109}$$

from (M.89) so

$$\overline{Q}(z = 0) = \frac{T}{R} > 0 \tag{M.110}$$

since we have taken $T, R > 0$. So (M.108) is good everywhere ($T/R = 4m_{\tilde{\ell}}/q^2$ so there is a possible problem when $m_{\tilde{\ell}}$, the slepton mass, vanishes.)

M.5 $\check{I}_{[\tilde{z}^2 - z^2]}$

The formula for $Im\,\check{I}_{[N]}$ works well for $N = N(z, \tilde{z})$, but (M.104) was restricted to $N = N(z)$. Recall from (M.27) that we may write

$$\check{I}_{[\mathrm{N}]}(T, S, R) = \frac{1}{2R}\int\limits_0^1 dz \int\limits_{-z}^{z} d\tilde{z}\ \frac{N(z, \tilde{z})}{\tilde{z}^2 + b\tilde{z} + c + i\epsilon \text{ terms}} \tag{M.111}$$

$$b = \frac{1-S}{2R} \qquad c = \left\{-z^2 + \left(\frac{1}{R} - b\right)z + \frac{(1-z)T}{R}\right\}\ . \tag{M.112}$$

From

$$\int d\tilde{z}\ \frac{\tilde{z}^2}{\tilde{z}^2 + b\tilde{z} + c} = \tilde{z} - \frac{b}{2}\,\ell n(\tilde{z}^2 + b\tilde{z} + c) + \frac{b^2 - 2c}{2}\int \frac{d\tilde{z}}{\tilde{z}^2 + b\tilde{z} + c} \tag{M.113}$$

we have

$$
\check{\mathrm{I}}_{[\tilde{z}^2-z^2]}(T,S,R) = \frac{1}{2R}\int\limits_0^1 dz
$$
$$
\times\left\{2z - \frac{b}{2}\,\ell n\;\frac{z^2+bz+c}{z^2-bz+c} + \frac{b^2-2c-2z^2}{2}\int\limits_{-z}^{z}\frac{d\tilde{z}}{\tilde{z}^2+b\tilde{z}+c}\right\}\,. \tag{M.114}
$$

Therefore

$$
Re\,\check{\mathrm{I}}_{[\tilde{z}^2-z^2]}(T,S,R) = \frac{1}{2R}\left\{1-\frac{b}{2}\int\limits_0^1 dz\;\ell n\left[\frac{z^2+bz+c}{z^2-bz+c}\right]\right\}
$$
$$
+\,Re\,\check{\mathrm{I}}_{[\frac{1}{2}b^2-c-z^2]}(T,S,R) \tag{M.115}
$$

and

$$
\tfrac{1}{2}\,b^2-c-z^2 = \tfrac{1}{2}\,b^2+\left(b-\frac{1}{R}\right)z-(1-z)\,\frac{T}{R}
$$
$$
=\frac{1}{R}\;\left[z(T-1+bR)+(\tfrac{1}{2}\,b^2R-T)\right] \tag{M.116}
$$

as well as

$$
z^2+bz+c=\frac{1}{R}\,\left\{z(1-T)+T\right\} \qquad z^2-bz+c=\frac{1}{R}\,\left\{z(1-T-2bR)+T\right\} \tag{M.117}
$$

so

$$
\frac{z^2+bz+c}{z^2-bz+c}=\frac{z(1-T)+T}{z(1-T-2bR)+T}\,. \tag{M.118}
$$

Let

$$
N'(z)\equiv 2\left[z(T-1+bR)+(\tfrac{1}{2}\,b^2R-T)\right] \tag{M.119}
$$

then from (M.115), (M.116), and (M.117)

$$Re\,\check{I}_{[\bar{z}^2-z^2]}(T,S,R) = \frac{1}{2R}\left\{1 - \frac{b}{2}\int_0^1 dz\;\ell n\left|\frac{z(1-T)+T}{z(1-T-2bR)+T}\right| \right. \tag{M.120}$$

$$\left. + Re\,\check{I}_{[N'(z)]}(T,S,R)\right\} .$$

Now both the numerator and denominator of $\frac{z(1-T)+T}{z(1-T-2bR)+T}$ are always positive ($b < 0$) so we can easily integrate out this term. Since

$$\int_0^1 dz\;\ell n(\alpha z+\beta) = \ell n(\alpha+\beta) - 1 + \frac{\beta}{\alpha}\,\ell n\left(\frac{\alpha}{\beta}+1\right) \tag{M.121}$$

in general we have

$$\int_0^1 dz\,\ell n\left(\frac{\alpha z+\beta}{\alpha' z+\beta'}\right) = \ell n\left(\frac{\alpha+\beta}{\alpha'+\beta'}\right) + \frac{\beta}{\alpha}\,\ell n\left(\frac{\alpha}{\beta}+1\right) - \frac{\beta'}{\alpha'}\,\ell n\left(\frac{\alpha'}{\beta'}+1\right) . \tag{M.122}$$

with

$$\alpha = 1-T \qquad \alpha' = 1-T-2bR \qquad \beta = \beta' = T \tag{M.123}$$

for the integral under consideration. Since

$$\alpha+\beta = 1 \quad \text{and} \quad \alpha'+\beta' = 1-2bR \tag{M.124}$$

we may write

$$\int_0^1 dz\ \ell n\ \left(\frac{z(1-T)+T}{z(1-T-2bR)+T}\right)$$

$$= \frac{T}{1-T}\ \ell n\ \left(\frac{1}{T}\right) - \ell n\ (1-2bR) - \frac{T}{1-T-2bR}\ \ell n\ \left(\frac{1-2bR}{T}\right)$$

$$= \frac{2bRT\ell nT}{(1-T)(1-T-2bR)} - \frac{1-2bR}{1-T-2bR}\ \ell n(1-2bR) \tag{M.125}$$

or letting

$$B \equiv bR = \frac{1-S}{2} \tag{M.126}$$

$$\int_0^1 dz\ \ell n\left(\frac{z(1-T)+T}{z(1-T-2B)+T}\right) \tag{M.127}$$

$$= \frac{1}{1-T-2B}\ \left[\frac{2BT\ell nT}{1-T} - (1-2B)\ell n(1-2B)\right]$$

and (M.120) becomes

$$Re\,\check{\mathrm{I}}_{[\bar{z}^2-z^2]}(T,S,R)$$

$$= \frac{1}{2R}\left\{1 - \frac{1}{2R}\cdot\frac{B}{1-T-2B}\ \left[\frac{2BT\ell nT}{1-T} - (1-2B)\ell n(1-2B)\right]\right. \tag{M.128}$$

$$\left. + Re\,\check{\mathrm{I}}_{[N'(z)]}(T,S,R)\right\}$$

or

$$Re\,\check{\mathrm{I}}_{[\bar{z}^2-z^2]}(T,S,R)$$

$$= \frac{1}{2R}\left\{1 - \frac{1}{2R}\cdot\frac{B}{S-T}\ \left[\frac{1-S}{1-T}T\ell nT - S\ell nS\right]\right. \tag{M.129}$$

$$\left. + Re\,\check{\mathrm{I}}_{[N'(z)]}(T,S,R)\right\}$$

since

$$B = \frac{1-S}{2} \qquad b = \frac{B}{R} \qquad 1-2B = S \tag{M.130}$$

$$N'(z) = 2\left[z(T-1+B) + \left(\tfrac{1}{2}\,b^2R - T\right)\right] \ . \tag{M.131}$$

When $S = 1$ $(B = b = 0)$ this becomes

$$Re\,\overline{I}_{[\tilde{z}^2 - z^2]}(T, S=1, R) = \frac{1}{2R}\Big\{1 - 2Re\,\overline{I}_{[z(1-T)+T]}(T, S=1, R)\Big\} \ . \tag{M.132}$$

M.6 G AND Ğ

G comes from, for instance, the γ^w term of $\mathcal{M}_2^\omega$ in Appendix L.

$$\mathcal{M}_2^\omega \sim \gamma^w \int_0^1 dz\, \tfrac{1}{2} \int_{-z}^{z} d\tilde{z} \int_{m_{\tilde{\chi}_i}^2}^{\Lambda^2} dt$$
$$\times \frac{1-z}{(t - i\epsilon_{\chi_i})(1-z)^+ (m_\nu^2 - i\epsilon_\nu)z + \frac{1}{4}q^2(\tilde{z}^2 - z^2)} \ . \tag{M.133}$$

Comparing the definition of G from Table L.1 or the next section we see that

$$G(T_i, R) = \tfrac{1}{2} \int_0^1 dz \int_{-z}^{z} d\tilde{z} \int_{m_{\tilde{\chi}_i}^2}^{\Lambda^2} dt$$
$$\times \frac{1-z}{(t - i\epsilon_{\chi_i})(1-z) + (m_{\tilde{\ell}}^2 - i\epsilon_{\tilde{\ell}})z + \frac{1}{4}q^2(\tilde{z}^2 - z^2)} \tag{M.134}$$

and since $(1-z), z > 0$, when we extract $(1-z)$ we obtain

$$G = \tfrac{1}{2} \int_0^1 dz \int_{-z}^{z} d\tilde{z} \int_{m_{\tilde{\chi}_i}^2}^{\Lambda^2} dt$$
$$\times \frac{1}{t + \frac{1}{1-z}[m_{\tilde{\ell}}^2 z + \frac{1}{4}q^2(\tilde{z}^2 - z^2)] - i\epsilon} \tag{M.135}$$

where $\epsilon > 0$. Now use the Principal Value Theorem (M.50)

$$\frac{1}{x-\alpha\pm i\epsilon} \rightarrow \frac{1}{x-\alpha} \mp i\pi\ \delta(x-\alpha)$$

under the integral sign to give

$$Im\, G = \tfrac{1}{2} \int\limits_0^1 dz \int\limits_{-z}^{z} d\tilde{z} \int\limits_{m^2_{\tilde{\chi}_i}}^{\Lambda^2} dt\ i\pi\, \delta\left(t + \frac{1}{1-z}\left[m^2_{\tilde{\ell}} z + \frac{q^2}{4}\ (\tilde{z}^2 - z^2)\right]\right)$$

$$= i\pi \int\limits_0^1 dz\ \tfrac{1}{2} \int\limits_{-z}^{z} d\tilde{z}\ \theta\left(\Lambda^2 + \frac{1}{1-z}\left[m^2_{\tilde{\ell}} z + \frac{q^2}{4}\ (\tilde{z}^2 - z^2)\right]\right) - (\Lambda^2 \rightarrow m^2_{\tilde{\chi}_i})\ .$$

Since $\Lambda^2 \rightarrow \infty$ $\quad\theta(\Lambda^2 + \ldots) = 1$ always and

$$Im\, G = i\pi \int\limits_0^1 dz\ \tfrac{1}{2} \int\limits_{-z}^{z} d\tilde{z}\ \left[1 - \theta(m^2_{\tilde{\chi}_i}[1-z] + m^2_{\tilde{\ell}} z + \frac{q^2}{4}\ [\tilde{z}^2 - z^2]\right]\ . \qquad \text{(M.136)}$$

Now

$$1 - \theta(x-a) = \theta(a-x) \qquad \text{(M.137)}$$

hence

$$Im\, G = i\pi \int\limits_0^1 dz\ \tfrac{1}{2} \int\limits_{-z}^{z} d\tilde{z}\ \theta\left[-\left(R[\tilde{z}^2 - z^2] + (1-z) + Tz\right)\right] \qquad \text{(M.138)}$$

where we have divided through by $m^2_{\tilde{\chi}_i} > 0$ (i.e. $\theta(f(x)) = \theta(af(x))$ if $a > 0$).

From (M.95)

$$W(z) = z^2 - \frac{(1-z) + Tz}{R} > 0\ . \tag{M.139}$$

Dividing through by R ($R > 0$ if $q^2 > 0$), (M.139) becomes

$$Im\, G = i\pi \int_0^1 dz\, \tfrac{1}{2} \int_{-z}^{z} d\tilde{z}\ \theta\Big(W(z) - \tilde{z}^2\Big)\ . \tag{M.140}$$

This could be solved analytically (by cases). From (M.135) and (M.50)

$$ReG = -\tfrac{1}{2} \int_0^1 dz \int_{-z}^{z} d\tilde{z}\ \ell n \left| \mathcal{R} \left(\tilde{z}^2 - W(z)\right)\right| \tag{M.141}$$

$$T = \frac{m_\ell^2}{m_{\tilde{\chi}_i}^2} \qquad R = \frac{q^2}{4m_{\tilde{\chi}_i}^2} \tag{M.142}$$

Ğ is evaluated in a similar manner.

$$Im\, \check{G} = i\pi \int_0^1 dz\, \tfrac{1}{2} \int_{-z}^{z} d\tilde{z}\ \theta\left(-\left[\tilde{z}^2 + b\tilde{z} - z^2 + \frac{1+S}{2\mathcal{R}} z + (1-z)\frac{T}{\mathcal{R}}\right]\right) \tag{M.143}$$

$$Re\check{G} = -\tfrac{1}{2} \int_0^1 dz \int_{-z}^{z} d\tilde{z}\ \ell n \left| \mathcal{R}\left(\tilde{z}^2 - z^2\right) + \tfrac{1}{2}\left\{(1+S)z + (1-S)\tilde{z}\right\} + T(1-z)\right| \tag{M.144}$$

$$S = S_{ij} = m_{\tilde{\chi}_i}^2 / m_{\tilde{\chi}_i}^2 \qquad T, R \text{ as in(M.142)} \qquad b \equiv \frac{1-S}{2R}\ . \tag{M.145}$$

Note that in $Re\, G$ and $Re\,$Ğ that as $m_\ell^2 \to \infty$ (i.e. $T \to \infty$) that

$$Re\, \check{G}, Re\, G \to\sim \ell n\, m_\ell^2$$

which would seem unnatural for a term in a cross section.

M.7 G AND Ǧ "RE-DEFINED". LIMITING FORMS.

All portions of the matrix elements are finite as $m_{\tilde{\chi}_i^+}$, $m_{\tilde{\chi}_i^0} \to 0$ or $q^2 \to \infty$ *i.e.* as $R \to \infty$ or $S_{ij} \to \infty$. There is a troublesome point in the chargino sector as $m_{\tilde{\nu}} \to \infty$, i.e. as $T \to \infty$. We desire the infinite $-\frac{1}{2}\,\ell n\, T$ contributions from G and Ǧ to cancel, at least in the SUSY limit.

If we define

$$G(T,R) = -\tfrac{1}{2} \int_0^1 dz \int_{-z}^{z} d\tilde{z}\, \ell n[z(T-1) + 1 + R(\tilde{z}^2 - z^2)] \tag{M.146}$$

$$\check{G}(T,S,R) = -\tfrac{1}{2} \int_0^1 dz \int_{-z}^{z} d\tilde{z}\, \ell n[T(1-z) + \tfrac{1}{2}\{(1+S)z + (1-S)\tilde{z}\} + R(\tilde{z}^2 - z^2)] \tag{M.147}$$

then

$$G(T \to \infty, R) \to -\tfrac{1}{2} \int_0^1 dz \int_{-z}^{z} d\tilde{z}\, \ell n[zT] = -\tfrac{1}{2}\,\ell n T + \frac{1}{4} \tag{M.148}$$

$$\check{G}(T \to \infty, S, R) \to -\tfrac{1}{2} \int_0^1 dz \int_{-z}^{z} d\tilde{z}\, \ell n[T(1-z)] = -\tfrac{1}{2}\,\ell n T + \frac{3}{4} \tag{M.149}$$

Since all "real" processes will involve ΔG, $\Delta \check{G}$ or $\Delta \widetilde{G}$ where

$$\Delta F \equiv F\big|_{\tilde{\ell}_1,\tilde{\nu}_1} - F\big|_{\tilde{\ell}_2,\tilde{\nu}_2} \qquad \widetilde{F}(T,R) \equiv F(T,R) - F(T,R=0) \tag{M.150}$$

for any function F, the constants will always cancel. It is only in numerical calculations where, perhaps, the $\tilde{\ell}_1$ (or $\tilde{\ell}_2$) sector is being artificially examined by itself, that a re-definition of these integral functions becomes desirable. The

$I_{[\mathrm{N}]}, \check{I}_{[\mathrm{N}]}$ functions need not be re-defined, nor the G', $\check{G}'$ functions (since the derivative of a constant vanishes). Thus we choose the redefinition

$$G(T,R) = -\tfrac{1}{2}\int_0^1 dz \int_{-z}^{z} d\tilde{z}\, \ell n[z(T-1)+1+R(\tilde{z}^2 - z^2)] - \frac{1}{4} \tag{M.151}$$

$$\begin{aligned}\check{G}(T,S,R) = -\tfrac{1}{2}\int_0^1 dz \int_{-z}^{z} d\tilde{z} & \\ \times \quad \ell n\left[T(1-z) + \tfrac{1}{2}\{(1+S)z + (1-S)\tilde{z}\} + R(\tilde{z}^2 - z^2)\right] - \frac{3}{4}\,. &\end{aligned} \tag{M.152}$$

For numerical investigations, if we use the definitions in (M.146)

$$\begin{aligned} G(T,R\to\infty) &\to -\tfrac{1}{2}\,\ell n\, R + C_{GR} \\ \overline{G}(T,R\to\infty) &\to -\tfrac{1}{2}\,\ell n\, R + C_{GR} \\ \check{G}(T,S,R\to\infty) &\to -\tfrac{1}{2}\,\ell n\, R + C_{GR} \end{aligned} \tag{M.153}$$

where

$$C_{GR} = 0.80684 - 1.57079570257\, i \tag{M.154}$$

and

$$\check{G}(T,S\to\infty,R) \to -\ell n\, S + C_{GS} \tag{M.155}$$

where

$$C_{GS} = 0.75\ . \tag{M.156}$$

Note that when we use the definition (M.152) that

$$\check{G}(T,S,R\to\infty) \to -\tfrac{1}{2}\,\ell n\, R + 0.05684 - 1.57079\, i \tag{M.157}$$

$$\check{G}(T, S \to \infty, R) \to -\ell n\, S \;. \tag{M.158}$$

In all of the $R \to \infty$, $S \to \infty$ limits $\Delta G \to 0$ and $\Delta \widetilde{G} \to 0$ of course. Note that as $m_{\tilde{\chi}_i} \to \infty$ that $T_i, R_i, S_{ij} \to 0$. In this limit the quondam definitions of (M.146) and (M.147) would lead to

$$\begin{aligned}
G(R \to 0, T \to 0) &\to -\tfrac{1}{2} \int\limits_0^1 dz \int\limits_{-z}^{z} \ell n\,(1-z) = \frac{3}{4} \\
\overline{G}(R \to 0, T \to 0) &\to -\tfrac{1}{2} \int\limits_0^1 dz \int\limits_{-z}^{z} d\tilde{z} \ell n\, z = \frac{1}{4} \\
\check{G}(R \to 0, S > 1, T \to 0) &\to \frac{1}{4}\, \frac{2S\,\ell n S + 3(1-S)}{1-S}
\end{aligned} \tag{M.159}$$

$\check{G}$ need not be considered for $S < 1$ since this is not used.

Note that

$$\lim_{S \to 1} \check{G}(R \to 0, S, T \to 0) = \overline{G}(T \to 0, R \to) = \frac{1}{4} \tag{M.160}$$

as desired.

If the $m_{\tilde{\chi}_i^+}, m_{\tilde{\chi}_i^0} \to \infty$ limits were thought to be of greater import than the $m_{\tilde{\ell}} \to \infty$ then a different re-definition could have been made in which the constants -1/4, -3/4 in (M.151) and (M.152) could be interchanged to bring the limiting forms to zero. Using our revised definitions in (M.152)

$$\begin{aligned}
G(T \to 0, R \to 0) &= \tfrac{1}{2} \\
\overline{G}(T \to 0, R \to 0) &= -\tfrac{1}{2} \\
\check{G}(T \to 0, S > 0, R \to 0) &= \tfrac{1}{2}\, \frac{S \ell n\, S}{1-S} \;.
\end{aligned} \tag{M.161}$$

Which are much more symmetric than was the case with the functions as they were previously defined.

In conclusion we present certain $q^2 = 0$ limiting cases of interest:

$$G(T,0) = -\tfrac{1}{2}\left[\ell n\, T - \frac{\ell n\, T^2 + (3-T)(1-T)}{2(1-T)^2}\right] - \frac{1}{4} \qquad G(T \to 1,0) \to -\frac{1}{4}$$

$$\overline{G}(T,0) = -\frac{T^2 \ell n\, T^2 + (3T-1)(T-1)}{4(T-1)^2} - \frac{3}{4} \qquad \overline{G}(T \to 1,0) \to -\frac{7}{4}$$

$$\bar{I}_{[1]}(T,0) = \frac{T \ell n\, T + 1 - T}{(1-T)^2} \qquad \bar{I}_{[1]}(T \to 1,0) \to \tfrac{1}{2} \ . \tag{M.162}$$

The formulae developed in this chapter have been used to plot the more important of the integral functions over a range of parameter regimes. It should be mentioned that the Feynman diagrams may be equivalently expressed in terms of what may be becoming a standardized set of integral functions originally put forth by Passarino and Veltman.[45]

FIGURE CAPTIONS

1. Tree Level Vertex with more than one Neutralino.

2. Typical diagram contributing to $\check{I}_{[N]}$ terms.

3. $G(T,R)$ vs. $m_{\tilde{\ell}}$ for $m_{\tilde{\chi}} = 0.1, 20\ GeV$.

$$T = \frac{m_{\tilde{\ell}}^2}{m_{\tilde{\chi}}^2} \qquad R = \frac{q^2}{4m_{\tilde{\chi}}^2} \qquad q^2 = m_Z^2 = (90\ GeV)^2$$

4. $G'(T,R)$ vs. $m_{\tilde{\ell}}$ for $m_{\tilde{\chi}} = 0.1, 20, 45, 50, 100\ GeV$.

$$T = \frac{m_{\tilde{\ell}}^2}{m_{\tilde{\chi}}^2} \qquad R = \frac{q^2}{4m_{\tilde{\chi}}^2} \qquad q^2 = m_Z^2 = (90\ GeV)^2$$

5. $\check{G}(T,S,R)$ vs. $m_{\tilde{\ell}}$ for $m_{\tilde{\chi}_i} = 0.1\ GeV$ and $S = 1, 10000$.

$$T = \frac{m_{\tilde{\ell}}^2}{m_{\tilde{\chi}_i}^2} \qquad R = \frac{q^2}{4m_{\tilde{\chi}_i}^2} \qquad S = \frac{m_{\tilde{\chi}_j}^2}{m_{\tilde{\chi}_i}^2} \qquad q^2 = m_Z^2 = (90\ GeV)^2$$

6. $\check{G}'(T,S,R)$ vs. $m_{\tilde{\ell}}$ for $m_{\tilde{\chi}_i} = 0.1\ GeV$ and $S = 1, 2$

(Indistinguishable).

$$T = \frac{m_{\tilde{\ell}}^2}{m_{\tilde{\chi}_i}^2} \qquad R = \frac{q^2}{4m_{\tilde{\chi}_i}^2} \qquad S = \frac{m_{\tilde{\chi}_j}^2}{m_{\tilde{\chi}_i}^2} \qquad q^2 = m_Z^2 = (90\ GeV)^2$$

7. $\tilde{G}(T,R)$ vs. $m_{\tilde{\ell}}$ for $m_{\tilde{\chi}} = 1.0\ MeV$.

$$T = \frac{m_{\tilde{\ell}}^2}{m_{\tilde{\chi}}^2} \qquad R = \frac{q^2}{4m_{\tilde{\chi}}^2} \qquad q^2 = m_Z^2 = (92.9\ GeV)^2$$

8. $R \times \bar{I}_{[\tilde{Z}^2 - Z^2]}(T,R)$ vs. $m_{\tilde{\chi}}$ for $m_{\tilde{\ell}} = 20, 40, 60, 80, 100\ GeV$.

$$T = \frac{m_{\tilde{\ell}}^2}{m_{\tilde{\chi}}^2} \qquad R = \frac{q^2}{4m_{\tilde{\chi}}^2} \qquad q^2 = m_Z^2 = (92.9\ GeV)^2$$

9. $\check{I}_{[1]}(T,S,R)$ vs. $m_{\tilde{\ell}}$ for $m_{\tilde{\chi}_i} = 0.1\ GeV$ and $S = 1, 6 \times 10^5, 8 \times 10^5,\ 10^6$.

$$T = \frac{m_{\tilde{\ell}}^2}{m_{\tilde{\chi}_i}^2} \qquad R = \frac{q^2}{4m_{\tilde{\chi}_i}^2} \qquad S = \frac{m_{\tilde{\chi}_j}^2}{m_{\tilde{\chi}_i}^2} \qquad q^2 = m_Z^2 = (90\ GeV)^2$$

10. $\check{I}'_{[1]}(T,S,R)$ vs. $m_{\tilde{\ell}}$ for $m_{\tilde{\chi}_i} = 0.1\ GeV$ and $S = 1.4, 1.6$.

$$T = \frac{m_{\tilde{\ell}}^2}{m_{\tilde{\chi}_i}^2} \qquad R = \frac{q^2}{4m_{\tilde{\chi}_i}^2} \qquad S = \frac{m_{\tilde{\chi}_j}^2}{m_{\tilde{\chi}_i}^2} \qquad q^2 = m_Z^2 = (90\ GeV)^2$$

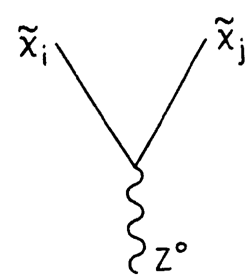


Fig. M.1

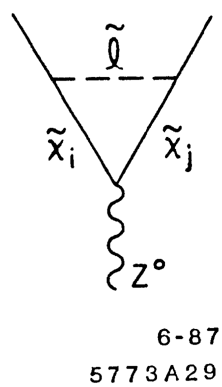


Fig. M.2

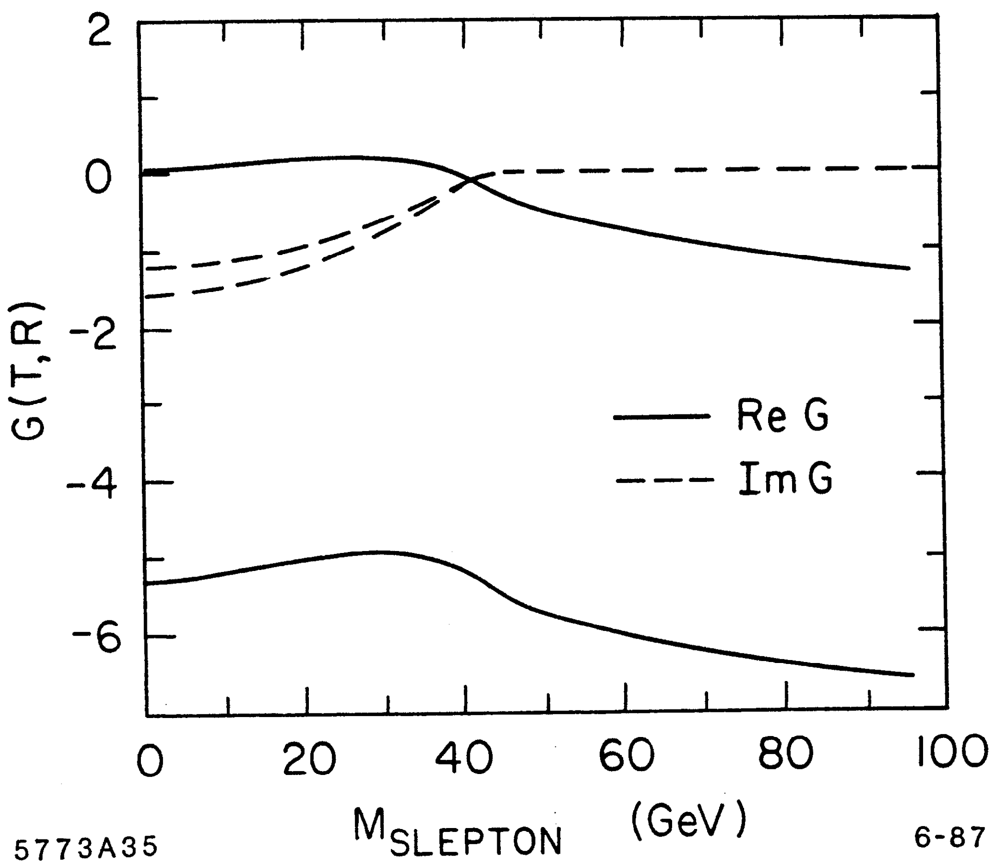


Fig. M.3

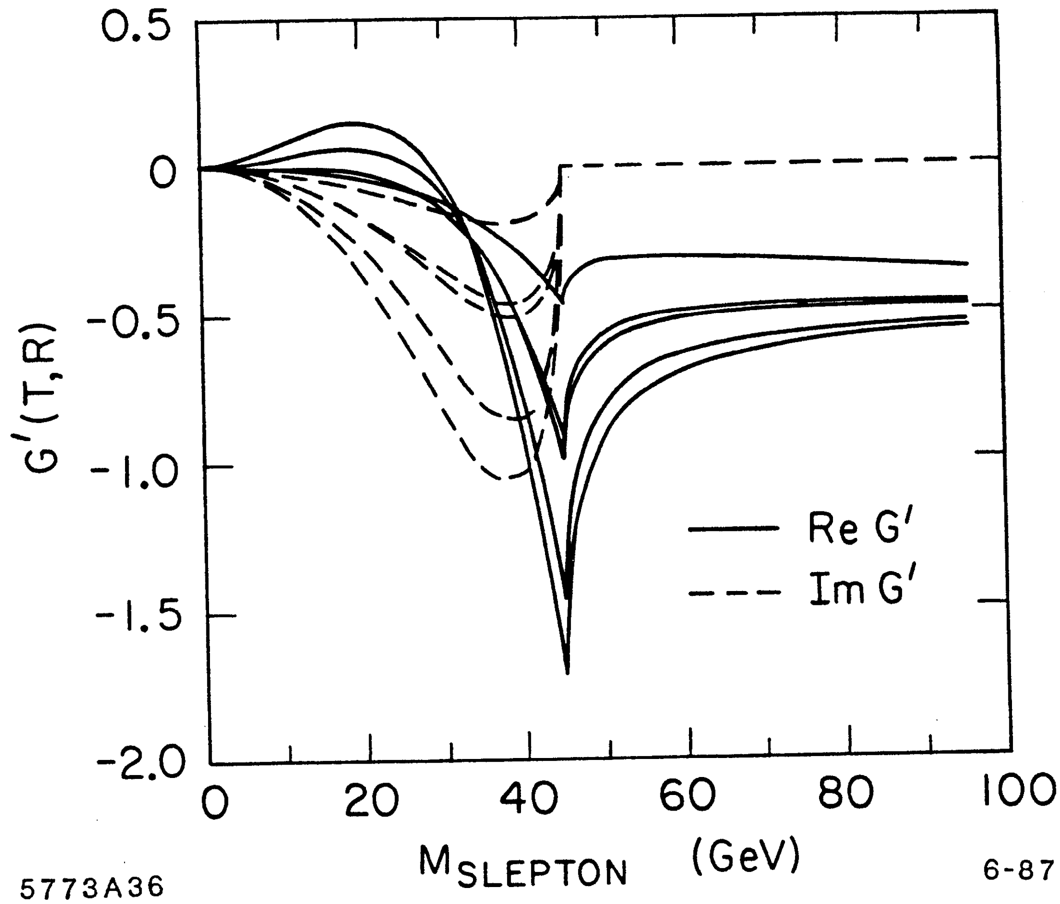

0.5
0
-0.5
-1.0
-1.5
-2.0
G'(T,R)
0
20
40
60
80
100
M_SLEPTON (GeV)
Re G'
Im G'
5773A36
6-87

Fig. M.4

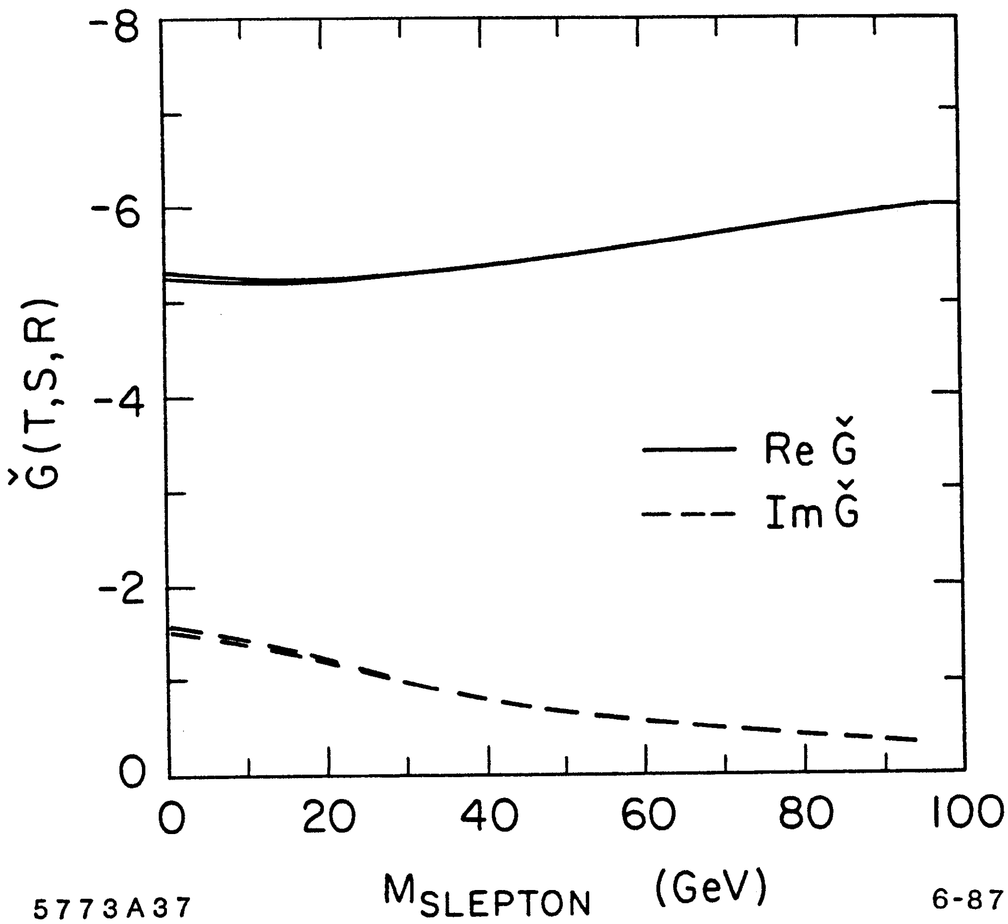
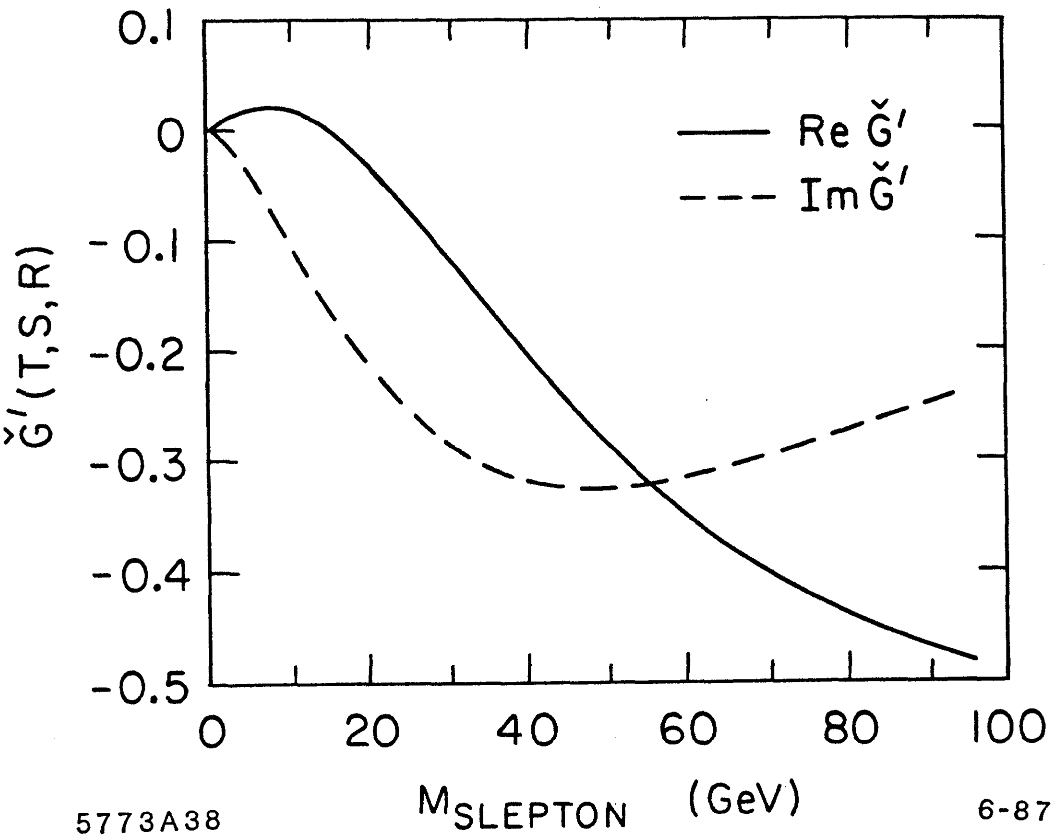

-8
-6
-4
-2
0
$\check{G}(T,S,R)$
Re $\check{G}$
Im $\check{G}$
0
20
40
60
80
100
$M_{SLEPTON}$ (GeV)
5773A37
6-87

Fig. M.5



0.1
0
-0.1
-0.2
-0.3
-0.4
-0.5
0
20
40
60
80
100
$\check{G}'(T,S,R)$
Re $\check{G}'$
Im $\check{G}'$
$M_{SLEPTON}$ (GeV)
5773A38
6-87

Fig. M.6

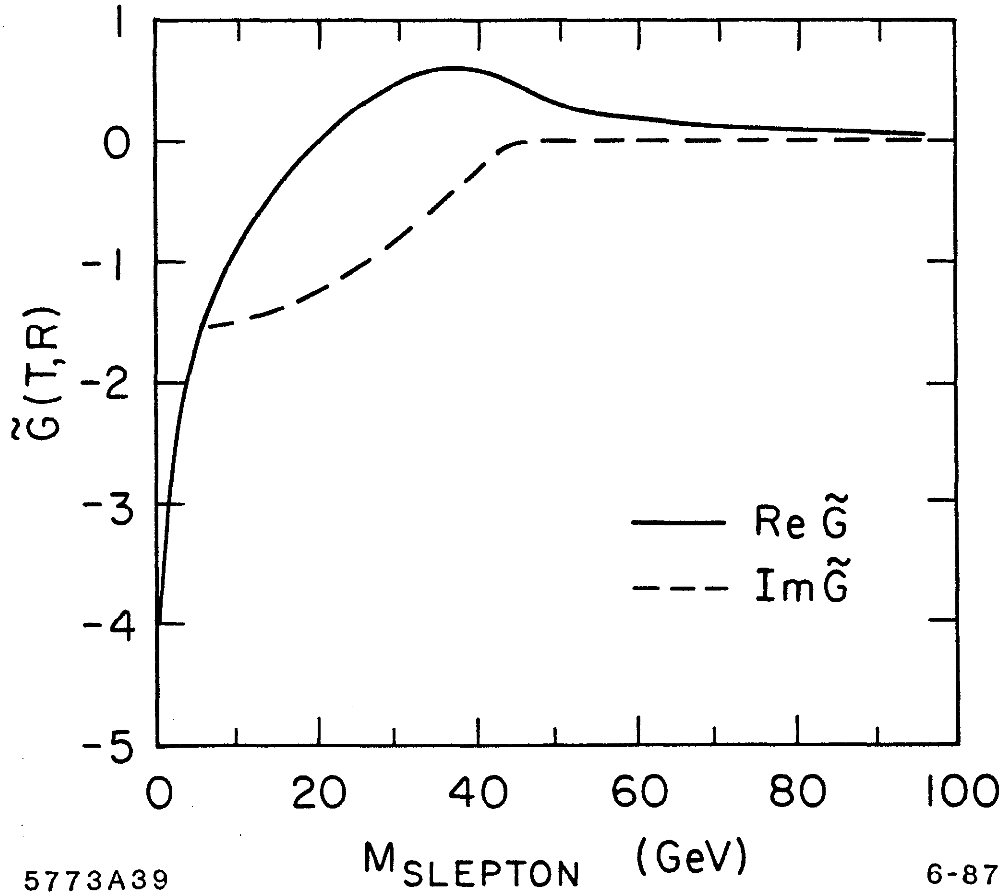

1
0
-1
-2
-3
-4
-5
$\tilde{G}(T,R)$
Re $\tilde{G}$
Im $\tilde{G}$
0
20
40
60
80
100
$M_{SLEPTON}$ (GeV)
5773A39
6-87

Fig. M.7

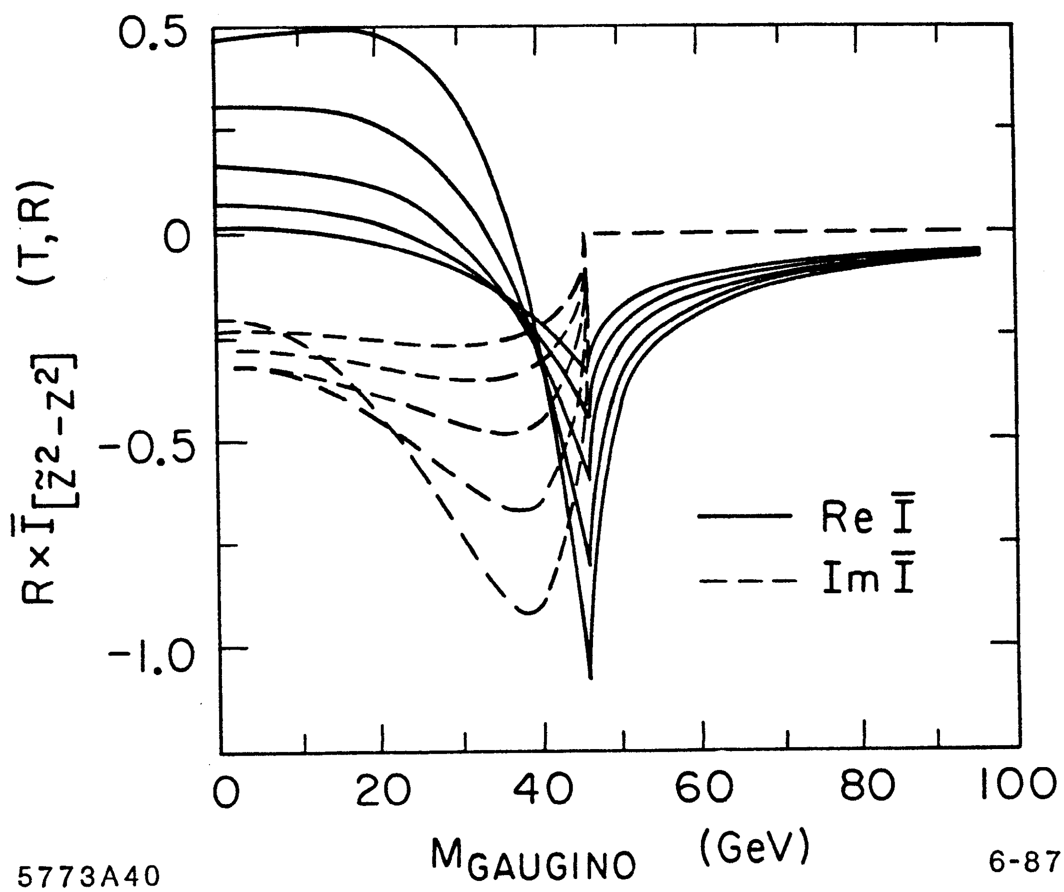
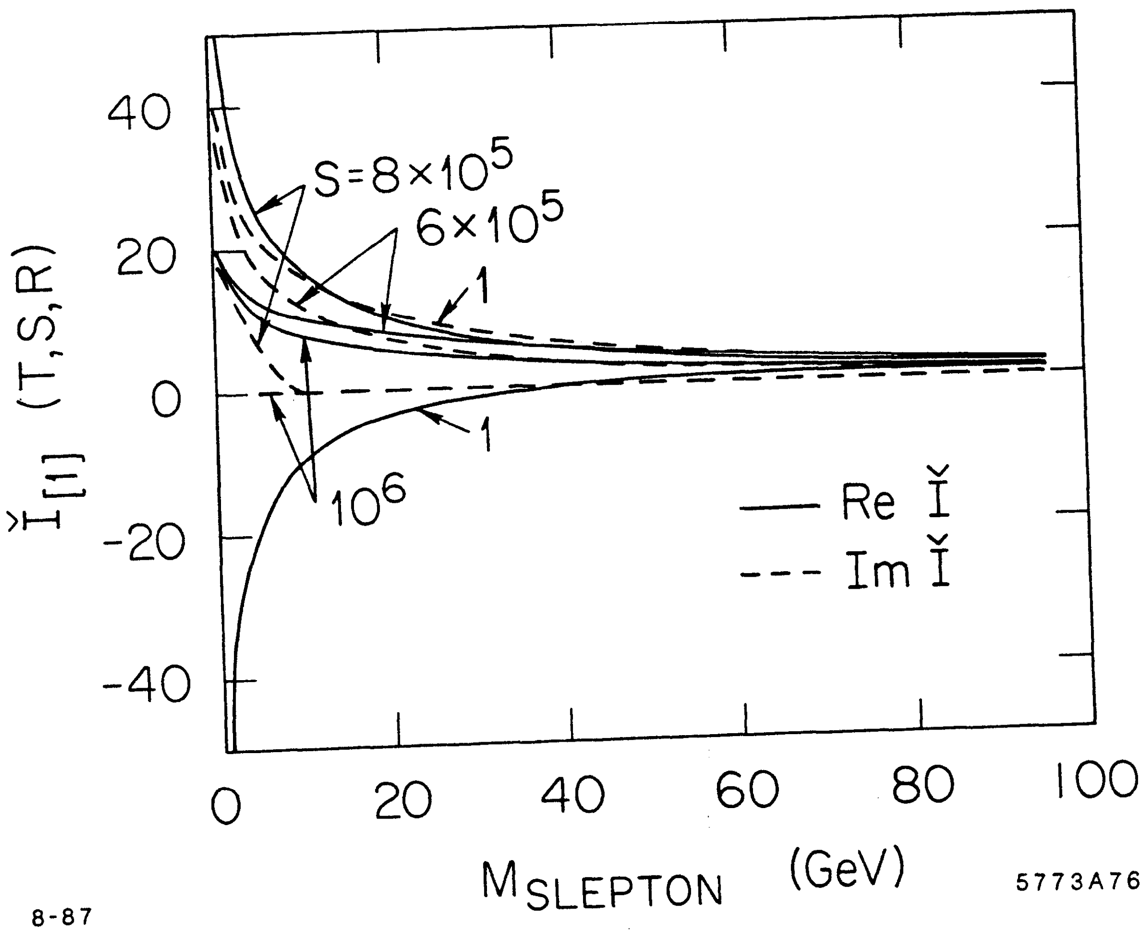

0.5
0
-0.5
-1.0
$R\times\bar{I}_{[\tilde{Z}^2-Z^2]}$ (T,R)
0
20
40
60
80
100
$M_{GAUGINO}$ (GeV)
Re $\bar{I}$
Im $\bar{I}$
5773A40
6-87

Fig. M.8



40
20
0
-20
-40
$\check{I}_{[1]}$ (T,S,R)
S=8×10^5
6×10^5
1
1
10^6
Re Ǐ
Im Ǐ
0
20
40
60
80
100
M_SLEPTON (GeV)
8-87
5773A76

Fig. M.9

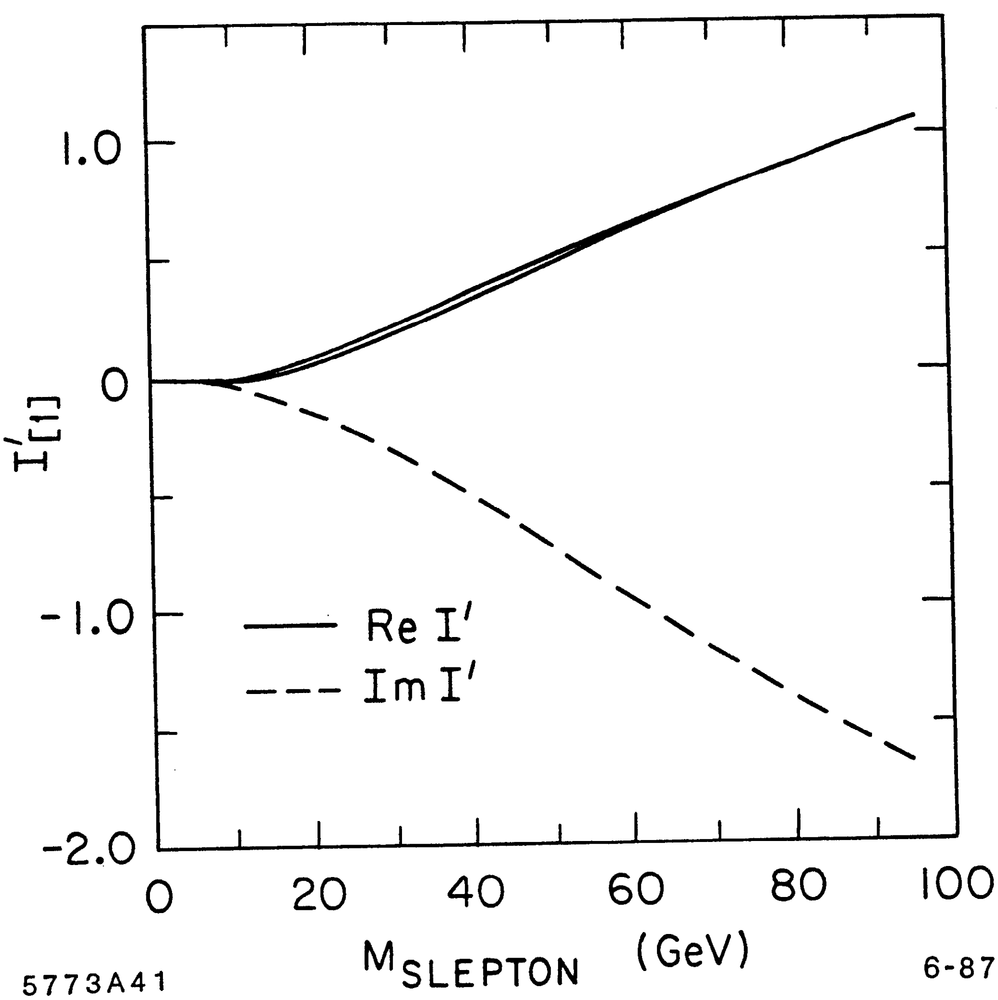


Fig. M.10

APPENDIX N

Background

We wish to consider the difficulties involved in observing $e^+e^- \to \mu\tau$ at SLC (or LEP) while operating at a Z^0-resonance whereat $(p_{e^-}+p_{e^+})^2 = m_Z^2$ which, for the purposes of numerical estimates, we will take as (90 GeV)^2 in this appendix. The detector signal is a $\mu^\pm$ and a $\tau^\mp$ exiting back-to-back, each with essentially half of the total energy and with no missing mass (from ν's, γ's, $\tilde{\gamma}$'s, etc.)

This signal may be mimicked by the far more common decay mode $Z^0 \to \tau\tau$ with the subsequent decay of one or both of the τ's into a muon and neutrinos. Since the τ will travel $\sim$ 0.25 cm prior to decay, and such a kink would be unobservable if the decay products of the τ do not come out at a large angle with respect to the τ's direction, the event will resemble a muon and the decay products of a τ. In either process the τ's must emerge back-to-back, both with $E = 45$ GeV (to within a few MeV), but since all decay modes of the tau involve at least one undetectable particle (the tau neutrino), and many ($\tau \to \mu\nu\nu, e\nu\nu$, etc.) give rise to several such states, accurate energy and momentum reconstruction is difficult. There is no problem with the modes $\tau \to$ HADRONS or $\tau^+ \to e^+\nu_e\overline{\nu}_\tau$, but $\tau^+ \to \mu^+\nu_\mu\overline{\nu}_\tau$ provides a substantial background. Some of the μ^+ will have virtually all of the τ^+ momentum (soft neutrinos) and so the event will appear as $e^+e^- \to \tau^+\mu^-$ instead of $\tau^+\tau^-$.

One method of reducing the background would be to eliminate all signals which have an opening angle which is discernibly different than 180 degrees. The difficulty in distinguishing the two decay modes $Z^0 \to \tau\mu$ and $Z^0 \to \tau\tau \to \tau\mu +$ soft neutrinos is that both involve at least one tau reconstruction. Since such a reconstruction will necessarily introduce an error in the inferred tau momentum angle (which varies over the range of the detector) there will be some $\Delta\theta$ (for the MARK II detector this is actually $\Delta\phi$, the error in the 'acoplanarity') below which it will be impossible to distinguish a back-to-back event from one whose opening angle is $\pi - \Delta\theta$. For a background event the opening angle will range up to approximately twice what it was for a genuine $\mu\tau$ event. Thus we will be able to experimentally eliminate those events with the largest opening angles. But these are precisely the events which will be most easily differentiated on the basis of calorimetric information (since events with larger opening angles will tend to be those with stiffer neutrinos). Thus it appears that angular information will not be useful in discriminating the background from the actual signal.[46] An analogous argument applies to using the impact parameter.

It appears that we must rely upon energy and momentum considerations to exclude the background contribution. The difficulty of eliminating the spurious component depends crucially upon the energy resolution of the detector and what the tail end of the energy distribution of $\tau(45\ GeV) \to \mu\nu\nu$ $(p_\mu = 45GeV)$ looks like. When $p_\mu \doteq p_\tau$ the angular kink is negligible (the $\tau \to \mu$ decay point is 2.5 mm from the interaction region) and will be indiscernible. The time of flight (delay because $m_\tau > m_\mu$) difference is ~ 0.25 cm$/(1-\beta_\tau)c$ with $\beta_\tau = \sqrt{1-(1/\gamma_\tau^2)}$ where

$$\gamma_\tau \simeq 4.5\ GeV/(m_\tau \approx 1.784\ GeV) \approx 25.2\ . \tag{N.1}$$

Therefore

$$\beta_\tau \approx 1 - \frac{1}{2\gamma_\tau^2} \tag{N.2}$$

and

$$1 - \beta_\tau \approx \frac{1}{2\gamma_\tau^2} \tag{N.3}$$

yielding

$$\Delta t = \frac{2.5 \times 10^{-3} m}{3 \times 10^{+8} m/s} \cdot \frac{1}{2(25.2)^2} \approx 6 \cdot 6 \times 10^{-15} sec. \tag{N.4}$$

So the delay time is about 6.6 femtoseconds and experimentally unobservable.

We need to calculate the energy spectrum of the $\mu^\pm$ produced from the decay of a $\tau^\pm$. We will perform an idealized background calculation. We will consider only events which have passed cuts imposed by the detector topology and ignore

1. Beam width (*i.e.* assume that the beam energy is precisely known).
2. Bremsstrahlung. The μ and τ will bremsstrahlung in the strong magnetic field of the detector (even over the 2.5 mm decay distance).
3. Chance of particle interaction (imperfect vacuum) in detector (secondary scattering).
4. Imperfect particle identification.

At SLC the e^- beam will be 100% polarized (say in the $\widehat{Z}$ direction). Thus to make a (spin = 1) Z^0 only the e^+ component with spin in the $+\widehat{Z}$ direction also (almost all of it) will be used. As a result nearly all of the Z^0 will be created polarized in the $+\widehat{Z}$ direction and at rest. Examining the helicity states of μ^-, τ^+ in $e^+e^- \to Z^0 \to \mu^-\tau^+$, as illustrated in Fig. N.1, we see that polarization effects are expected to be important.

Let us consider the τ decaying at rest and then "boost" this to the lab ($p_\tau = 45$ GeV/c) frame as in Fig. N.2. We know that the τ will likely be polarized. The normalized number distribution of the muons in the tau rest frame[47] (using the 4-Fermi Lagrangian, which is an adequate approximation here, and $m_\mu \ll m_\tau$) is

$$d^2N_\mu = x^2[3 - 2x \pm \alpha(1-2x)\cos\theta_{\rm Boost}]dx\,d\cos\theta \tag{N.5}$$

$$+ \text{ for } \tau^- \to \mu^-$$

$$- \text{ for } \tau^+ \to \mu^+$$

$$\theta = \theta_{\rm Boost} \text{ here}$$

$$\begin{aligned} \alpha &= \text{degree of polarization} \\ (\alpha &= 0 \text{ unpolarized} \\ \alpha &= 1 \text{ 100\% polarized}) \end{aligned} \tag{N.6}$$

$$x = \frac{E_i}{\frac{1}{2}m_\tau}\,. \tag{N.7}$$

$$E_i = E_\mu \text{ in } \tau \text{ rest frame, prior to boosting by } \tfrac{1}{2}m_Z.$$

$$\tfrac{1}{2}m_\tau = \max[E_i] \text{ in rest frame so } 0 \le x \le 1.$$

Therefore

$$\frac{dN_\mu}{dE_i} = \frac{dN_\mu}{dx}\left(\frac{x}{E_i}\right) = \frac{2}{m_\tau}\left[\frac{2E_i}{m_\tau}\right]^2$$

$$\times\left[3 - 2\left(\frac{2E_i}{m_\tau}\right) \pm \alpha\begin{pmatrix}\cos\theta \\ \cos[\theta+\pi]\end{pmatrix}\left(1 - 2\left[\frac{2E_i}{m_\tau}\right]\right)\right]d\cos\theta \tag{N.8}$$

$$= \frac{16E_i^2}{m_\tau^2}\;[3m_\tau - 4E_i - \alpha(m_\tau - 4E_i)\cos\theta]\;\;d\cos\theta\,.$$

Note that the "-" sign holds for both μ^+ and μ^- because for μ^+ the polarization is in the momentum direction (same as boost) whereas for μ^- it is in the opposite direction. This just compensates for the sign difference in (N.5).

The (normalized) number of μ's in the range $[\theta, \theta + d\theta]$ (*i.e.* τ rest frame) with energy E_i is

$$dN_\mu(E_i, \theta) = \frac{16E_i^2}{m_\tau^4}\ [3m_\tau - 4E_i - \alpha(m_\tau - 4E_i)\cos\theta]\ \frac{2\pi d\cos\theta}{4\pi}\ dE_i \qquad \text{(N.9)}$$

(since there is no ϕ dependence and $\int d\Omega = 4\pi$).

We now boost (in $\widehat{Z}$ direction say) up to $E_\tau = m_Z/2$. In general[48]

$$\gamma_f = \gamma_{\mu[\tau-rest]}\gamma_{\tau[z-rest]}\ \left(1 + \frac{\vec{V}_\mu \cdot \vec{V}_\tau}{c^2}\right) \qquad \text{(N.10)}$$

and since $\beta_\mu \approx \beta_\tau \approx 1$

$$\gamma_f = \frac{E_i}{m_\mu} \cdot \frac{\frac{1}{2}m_Z}{m_\tau}\ (1 + \beta_\mu\beta_\tau\cos\theta) \approx \frac{E_i E_Z}{m_\mu m_\tau}\ \frac{(1 + cos\theta)}{2}\ . \qquad \text{(N.11)}$$

Let E_f be the final energy of the μ in the lab (Z^0 at rest) frame

$$E_f = m_\mu\gamma_f = \tfrac{1}{2}\,E_i\ \frac{m_Z}{m_\tau}\ (1 + \cos\theta) \qquad \text{(N.12)}$$

(remembering that θ is in the τ rest frame). The probability of the final state having energy E_f in the lab frame, given energy E_i and angle θ in the τ rest frame, is

$$E_f = \int\ (\text{Prob. of } E_i)(\text{Prob. of } \theta) \qquad \text{(N.13)}$$

up to a normalization, where $E_f = \frac{1}{2}\,E_i\ \frac{m_Z}{m_\tau}\ (1 + \cos\theta)$ from (N.12). Letting

$$y \equiv \frac{2E_f m_\tau}{m_Z(1 + \cos\theta)} \qquad \text{(N.14)}$$

this condition is $E_i = y$.

The probability of E_f being the final μ energy is, therefore, (N is a normalization factor)

$$N \int_{E_{i\,\min}}^{E_{i\,\max}} dE_i \int_{-1}^{1} d\cos\theta \, \frac{16E_i^2}{m_\tau^4} \, [m_\tau(3-\alpha\cos\theta) - 4E_i(1-\alpha\cos\theta)] \, \delta[E_1 - y] \quad \text{(N.15)}$$

with $E_{i\,\min} = 0$ and $E_{i\,\max} = m_\tau/2$. Letting $z = \cos\theta$ The energy constraint equation becomes

$$y = \frac{2E_f m_\tau}{m_Z(1+z)}$$

and the number of muons expected at a given lab energy is

$$\frac{dN_\mu}{dE_f} = N \int_0^{m_\tau/2} dE_i \int_{-1}^{1} dz \, \frac{16E_i^2}{m_\tau^4} \, [m_\tau(3-\alpha z) - 4E_i(1-\alpha z] \, \delta[E_i - y]$$

$$= N \int_{-y}^{m_\tau/2-y} d(E_i - y) \int_{-1}^{1} dz \, \frac{16E_i^2}{m_\tau^4} \, [m_\tau(3-\alpha z) - 4E_i(1-\alpha z] \, \delta[E_i - y]$$

$$= N \int_{-1}^{1} \tfrac{1}{2}\, dz \, \frac{16}{m_\tau^4} \left[\frac{2E_f m_\tau}{m_Z(1+z)}\right]^2$$

$$\times \left[m_\tau(3-\alpha z) - 4\cdot\frac{2E_f m_\tau}{m_Z(1+z)}\,(1-\alpha z)\right] \theta\left(\frac{m_\tau}{2} - \frac{2E_f m_\tau}{m_Z(1+z)}\right) . \quad \text{(N.16)}$$

The argument of the theta-function must exceed zero requiring that

$$\frac{m_\tau}{2} - \frac{2E_f m_\tau}{m_Z(1+z)} > 0 \Rightarrow z > \frac{4E_f}{m_Z} - 1 .$$

Therefore

$$\frac{dN_\mu}{dE_f} = N \int_{(4E_f/m_Z)-1}^{1} dz \, \frac{E_f^2}{m_\tau^2 m_Z^2} \left[\frac{m_\tau(3-\alpha z)}{(1+z)^2} - \frac{8E_f m_\tau(1-\alpha z)}{m_Z(1+z)^3}\right] \quad \text{(N.17)}$$

and letting $\omega = 1 + z$

$$\frac{dN_\mu}{dE_f} = N\ \frac{E_f^2}{m_\tau m_Z^2} \int\limits_{4E_f/m_Z}^{2} d\omega \left[\frac{3-\alpha(\omega-1)}{\omega^2} - \frac{8E_f}{m_Z}\ \frac{1-\alpha(\omega-1)}{\omega^3}\right]$$

$$= N\frac{E_f^2}{m_\tau m_Z^3} \int\limits_{4E_f/m_Z}^{2} d\omega \left[-\frac{m_Z\alpha}{\omega} + \frac{(3+\alpha)m_Z + 8\alpha E_f}{\omega^2} - \frac{8(1+\alpha)E_f}{\omega^3}\right]$$

$$= N\ \frac{E_f}{4m_\tau m_Z^4}\ \Big[4\alpha m_Z E_f \ell n\ \frac{2E_f}{m_Z} + 6(\alpha-1)m_Z E_f$$

$$+ 4(1-3\alpha)E_f^3 + 2m_Z^2\Big] \tag{N.18}$$

Let

$$\eta = \frac{N}{4m_\tau m_Z^4}\ .$$

The normalization is established by requiring that

$$\int\limits_0^{m_Z/2} \left(\frac{dN_\mu}{dE_f}\right)\ dE_f = 1$$

and so

$$\eta \int\limits_0^{m_Z/2} dE_f\ E_f \tag{N.19}$$

$$\times\ \left\{4\alpha m_Z E_f \ell n\ \frac{2E_f}{m_Z} + 6(\alpha-1)m_Z E_f + 4(1-3\alpha)E_f^2 + 2m_Z^2\right\} = 1\ .$$

Performing the integration yields the result

$$1 = \eta\left\{\frac{m_Z^4}{144}\ (9+\alpha)\right\}$$

or

$$\eta = \frac{144}{m_Z^4(9+\alpha)}$$

which, when used in (N.18), gives

$$\frac{dN_\mu}{dE_f} = \frac{288E_f}{(9+\alpha)m_Z^4} \left\{ m_Z^2 + 3(\alpha-1)m_Z E_f + 2(1-3\alpha)E_f^2 + 2\alpha m_Z E_f \ell n \, \frac{2E_f}{m_Z} \right\} \tag{N.20}$$

Where $\alpha = 0$ for the unpolarized case and $\alpha = 1$ in the completely polarized case.

For $\alpha = 0$:

$$\frac{dN_\mu}{dE_f} = \frac{32E_f(E_f - m_Z)(2E_f - m_Z)}{m_Z^4} \quad \text{(unpolarized case).} \tag{N.21}$$

For $\alpha = 1$:

$$\frac{dN_\mu}{dE_f} = \frac{144}{5}\,\frac{E_f}{m_Z^4} \left\{ (m_Z^2 - 4E_f^2) + 2m_Z E_f \ell n \, \frac{2E_f}{m_Z} \right\} \quad \text{(polarized case).} \tag{N.22}$$

These distributions are depicted in Fig. N.3.

We are interested in those μ's with $E_\mu \doteq E_\tau = m_Z/2$. The number, $N(\Delta\varepsilon)$, of μ's within energy $\Delta\varepsilon$ of $m_Z/2$ is

$$N(\Delta\varepsilon) = \int_{(m_Z/2)-\Delta\varepsilon}^{m_Z/2} dE_f \, \frac{dN_\mu}{dE_f} \,. \tag{N.23}$$

Therefore

$$N(\Delta\varepsilon) = 1 - \frac{288}{9+\alpha} \int_0^{(m_z/2)-\Delta\varepsilon} \frac{dE_f E_f}{m_Z^4}$$

$$\times \left\{ m_Z^2 + 3(\alpha-1)m_Z E_f + 2(1-3\alpha)E_f^2 + 2\alpha m_Z E_f \ell n\, \frac{2E_f}{m_Z} \right\}$$

$$= 1 - \frac{288}{9+\alpha} \frac{1}{m_Z^4}$$

$$\times \left\{ \tfrac{1}{2} m_Z^2 E_f^2 + (\alpha-1) m_Z E_f^3 + \tfrac{1}{2}(1-3\alpha)E_f^4 \right.$$

$$\left. + (\frac{2}{3}\alpha m_Z \ell n\, \frac{2}{m_Z})E_f^3 + 2\alpha m_Z E_f^3 (\frac{1}{3}\ell n\, E_f - \frac{1}{9}) \right\} \Bigg|_0^{(m_Z/2)-\Delta\varepsilon}$$

and expanding all terms to order $(\Delta\varepsilon)^2$ yields (eventually)

$$N(\Delta\varepsilon) = \frac{3-\alpha}{9+\alpha} \cdot 24\, \frac{\Delta\varepsilon^2}{m_Z^2} \tag{N.24}$$

$$\alpha = 0 \quad \text{gives} \quad N(\Delta\varepsilon) = \ 8\, \frac{(\Delta\varepsilon)^2}{m_Z^2} \tag{N.25}$$

$$\alpha = 1 \quad \text{gives} \quad N(\Delta\varepsilon) = \frac{24}{5}\, \frac{(\Delta\varepsilon)^2}{m_Z^2}\ . \tag{N.26}$$

Thus polarization effects decrease the background by a factor of 5/3.

The energy-momentum resolution for stiff muons for the Mark II detector at the SLC will be, at best, (once the close vertex detector has been installed)[28]

$$\frac{\Delta p}{p} \gtrsim 0.1\% \text{ per } GeV. \tag{N.27}$$

For muons in the 45 GeV range this implies that

$$\frac{\Delta p}{p} \gtrsim 5\%. \tag{N.28}$$

(There is an additional 1.4% error anticipated from multiple scatterings, within the drift chamber alone, which must be added in quadrature.) Taking the "super-optimal" momentum resolution, $\Delta\varepsilon_{\rm OPT}$, at SLC to be 5% then

$$\Delta\varepsilon_{OPT} = 5\% \left(\frac{m_Z}{2}\right) = 0.05 \left(\frac{m_Z}{2}\right) \tag{N.29}$$

and we observe from (N.24) that

$$N(\Delta\varepsilon_{OPT}) = \frac{3-\alpha}{9+\alpha} \cdot 24 \left(\frac{0.05 \cdot \frac{1}{2} m_Z}{m_Z}\right)^2 = \left(\frac{3-\alpha}{9+\alpha}\right) 1.5 \times 10^{-2} \,. \tag{N.30}$$

$$\text{For} \quad \alpha = 0 \quad N(\Delta\varepsilon_{OPT}) = 5 \times 10^{-3} \tag{N.31}$$

$$\text{For} \quad \alpha = 1 \quad N(\Delta\varepsilon_{OPT}) = 3 \times 10^{-3} \,. \tag{N.32}$$

Note that the difference between E_μ and $m_Z/2$ (due to $m_\tau \neq m_\mu$) is only about 17 MeV, far below the energy resolution.

Now

$$BR(Z \to \tau\tau) \simeq 3\% \qquad BR(Z \to \mu\mu) \simeq 3\% \tag{N.33}$$

$$BR(\tau \to \mu\bar{\nu}\nu) \simeq 17.5\% \qquad BR(\tau \to e\bar{\nu}\nu) \simeq 17.5\% \tag{N.34}$$

and therefore an expected sample (ultimate fiducial yearly yield at SLC) of

10^6 Z^0's yields 30,000 $\tau\tau$ events. We expect roughly

$$10^6 \times 0.03 \times 2 \times 0.175 \times (3 \times 10^{-3}) \sim 30 \qquad \text{(N.35)}$$

misidentified stiff μ's from $e^+e^- \to \tau\tau$ events (many of which will go undetected due to their decay configuration). We also expect $\sim$ 30,000 $\mu\mu$ events. From chapter three recall that

$$\frac{\sigma_{\mu\tau}}{\sigma_{\mu\mu}} \lesssim 3 \times 10^{-6} \qquad \text{(N.36)}$$

and so we would expect fewer than

$$10^6 \times 0.03 \times 2 \times 3 \times 10^{-6} \approx 2 \times 10^{-1} \text{ events per year.} \qquad \text{(N.37)}$$

We may simultaneously search for both $\tau^+\mu^-$ and $\tau^-\mu^+$ events, accounting for the additional factor of two. Again some of this signal will be lost.

The question arises as to whether we could improve these results if the principal mixing arose in the $\widetilde{e} - \widetilde{\mu}$ sector rather than the $\widetilde{\mu} - \widetilde{\tau}$ sector. Since factors of the lepton masses have cancelled from (N.24), the same expression is valid for $\tau \to e\overline{\nu}\nu$. Similarly $e^+e^- \to Z^0 \to \mu^+e^-$ will have the same rate as $e^+e^- \to Z^0 \to \tau^+\mu^-$ however the primary background will now arise from

$$\begin{aligned} &e^+e^- \to Z^0 \to \tau^+\tau^- \\ &\qquad \tau^+ \to \mu^+\overline{\nu}\nu \\ &\qquad \tau^- \to e^-\overline{\nu}\nu \ . \end{aligned} \qquad \text{(N.38)}$$

We expect that ($\mathcal{P}$ means 'probability of')

$$\mathcal{P}\{\tau \to \mu(\Delta\varepsilon)\overline{\nu}\nu;\ \tau \to e(\Delta\varepsilon)\overline{\nu}\nu\} = F\mathcal{P}\{\tau \to \mu(\Delta\varepsilon)\overline{\nu}\nu\} \cdot \mathcal{P}\{\tau \to e(\Delta\varepsilon)\overline{\nu}\nu\} \tag{N.39}$$

where F is a factor of order unity. This describes events in which the taus emerge back-to-back and decay into an electron and muon, both of which retain almost all of the momentum of their parent taus, and appear to continue diametrically back-to-back with no apparent energy loss or deviation. From (N.24) we know that

$$N(\Delta\varepsilon)_{(\mu\tau)} = \frac{3-\alpha}{9+\alpha} \cdot 6\ \frac{(\Delta p)^2}{p^2} \tag{N.40}$$

therefore

$$N(\Delta\varepsilon)_{(e\mu)} \doteq \left(\frac{3-\alpha}{9+\alpha}\right)^2 \cdot 36\ \left(\frac{\Delta p}{p}\right)^2 . \tag{N.41}$$

For $\alpha = 1$ this becomes

$$N(\Delta\varepsilon)_{(e\mu)} \simeq 1.4 \left(\frac{\Delta p}{p}\right)^4 . \tag{N.42}$$

which, for a 5% error gives

$$N(\Delta p) \simeq 9 \times 10^{-6} .$$

From the branching ratios in (N.34) we expect about

$$10^6 \times 0.03 \times 2 \times 0.175 \times 0.175 \times (9 \times 10^{-6}) \sim 2 \times 10^{-2} \tag{N.43}$$

misidentified $e^+e^- \to e^\pm\mu^\mp$ events per sample of 10^6 Z^0's. Comparing with (N.37) we see that this could, in theory, be as much as an order of magnitude less that the signal rate. This conclusion is, of course, based upon somewhat idealized ("super-optimal") background estimates and production rates. It should be noted that, in partial compensation, it will be substantially easier to eliminate background on the basis of angular information in this system since no τ-track momentum reconstruction will be required.

FIGURE CAPTIONS

1. The decay $Z^0 \to \tau\mu$ in the laboratory rest frame illustrating the configurations which are and are not helicity-suppressed.

2. The decay $\tau \to \mu\overline{\nu}\nu$ in τ rest frame.

   $\theta =$ the angle of the μ momentum with respect to τ polarization factor.

   θ is also the boost direction angle for the μ^+ case.

   $\theta + \pi$ is boost direction angle for the $\tau^- \to \mu^-$ case.

3. The distribution of emergent muons, $\frac{dN_\mu}{dE_f}$, from $\tau \to \mu\overline{\nu}\nu$ versus E_f, the muon energy in the laboratory frame.

   The degree of polarization of the $\tau^\pm$ is α.

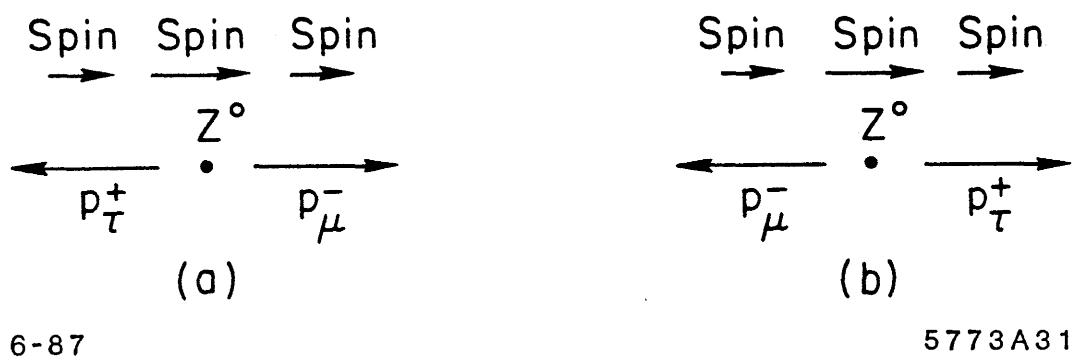

Spin
Spin
Spin
Z°
p_τ^+
p_μ^-
(a)
6-87
Spin
Spin
Spin
Z°
p_μ^-
p_τ^+
(b)
5773A31

Fig. N.1

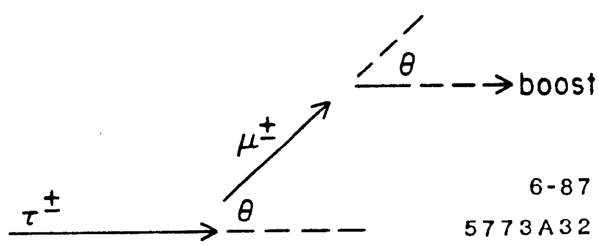


Fig. N.2

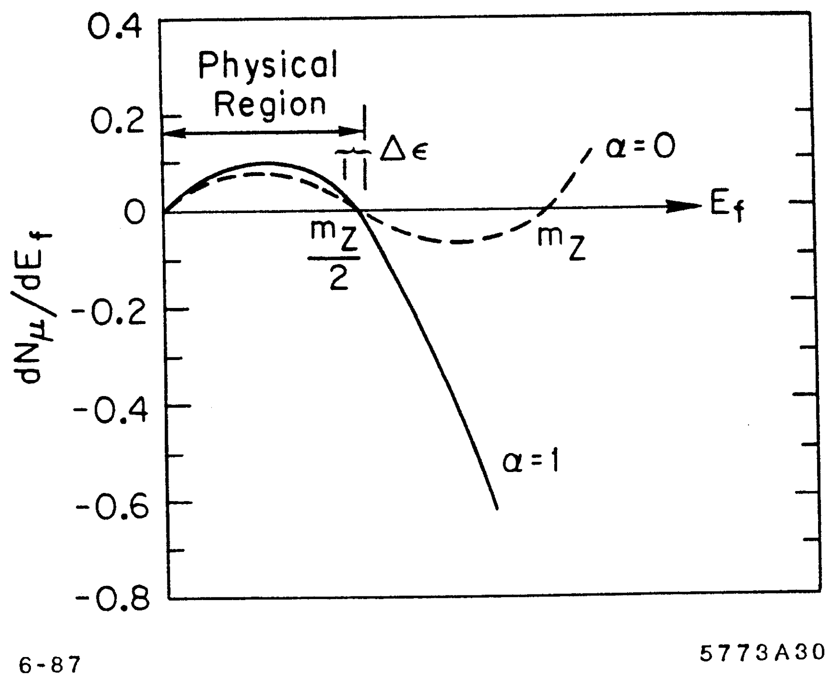

0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
Physical
Region
$\Delta\epsilon$
$\alpha=0$
E_f
$\frac{m_Z}{2}$
m_Z
$\alpha=1$
dN_μ/dE_f
6-87
5773A30

Fig. N.3

APPENDIX O

Matrix Element for $\mu \to e\gamma$

O.1 INTRODUCTION

A one loop calculation of the $\mu e\gamma$ vertex has been performed. Unlike the pretermission in the $Z \to \mu\tau$ computations, here we shall not neglect the masses of the external fermions and assume the photon to be on-shell. The result may be added to the effective Lagrangian of the theory and is valid whenever the leptons are on-shell (when all three states are off-shell the result is somewhat more involved since some portion of the m_i^2 terms will be replaced by p_i^2 and gauge invariance will be more difficult to employ as a check on the final form of the result). The calculation was performed with the two-component formalism for a purely SQED theory. We know from considerations of Lorentz and gauge invariance[29] that the results may be placed in the form:

$$\begin{aligned}\mathcal{M}_{\mu e\gamma}(q^2) = K\overline{\psi}^{e}(p',s')\Big\{ &\left[F_1(q^2) + F_1^5(q^2)\gamma_5\right] \frac{\not{q}\sigma^{\mu\nu}q_\nu}{m_\lambda^2} \\ &+ \left[F_2(q^2) + F_2^5(q^2)\gamma_5\right] \sigma^{\mu\nu}q_\nu \Big\}\psi^{\mu}(p,s)\end{aligned} \tag{O.1}$$

$$K = -\frac{\alpha^{3/2}\sin\theta\cos\theta}{144\sqrt{\pi}} \qquad q_\omega = p_\omega - p'_\omega \tag{O.2}$$

where θ is the $\tilde{e} - \tilde{\mu}$ mixing angle.

There are twenty Weyl-spinor diagrams in all, falling into the catagories of triangle and leg-correction contributions. Each contributes a piece to both the F_1 and F_2 terms as well as other pieces which would be disallowed by our general considerations. These can be shown explicitly (and rather laboriously) to cancel among themselves. To facilitate this it was found useful develop a projection technique, similar to that employed by Brodsky and Sullivan,[49] to project out of each two-component expression that portion which contributed to each of the terms in (O.1). The tangle of algebra which followed was made tractable by judicious (*i.e.* extensive) use of the symbolic manipulation routines REDUCE and MACSYMA.

O.2 SQED Form Factors

In the SQED case we use the integral functions of Table L.1 but the mass ratios of eqn. (3.10) become

$$\tau_{L,R} = \frac{m^2_{\tilde{\ell}_1}}{m^2_{\lambda}} \qquad\qquad \mathcal{R} = \frac{q^2}{4m^2_{\lambda}} \ . \tag{O.3}$$

We also define

$$m_+ \equiv \tfrac{1}{2}\left(m_\mu + m_e\right) \qquad\qquad m_- \equiv \tfrac{1}{2}\left(m_\mu - m_e\right) \ . \tag{O.4}$$

It is then found that

$$
\begin{aligned}
F_1(q^2) &= -9\frac{\Delta\tilde{G}(T_R,R)}{\mathcal{R}} \\
&\quad - \frac{36[3m_+^2+m_-^2][q^2-2(2m_+^2-3m_+m_-+3m_-^2)]}{[q^2-4m_+^2][q^2-4m_-^2]} \\
&\quad \times \left[\Delta I_{[\mathbf{Z(1-Z)}]}(T_R,R)-\Delta I_{[\mathbf{Z(1-Z)}]}(T_R,\mathcal{R}=0)\right] \;+\; R\to L \\
F_1^5(q^2) &= -9\frac{\Delta\tilde{G}(T_R,R)}{\mathcal{R}} \\
&\quad - \frac{36[3m_-^2+m_+^2][q^2-2(2m_-^2-3m_-m_++3m_+^2)]}{[q^2-4m_+^2][q^2-4m_-^2]} \\
&\quad \times \left[\Delta I_{[\mathbf{Z(1-Z)}]}(T_R,R)-\Delta I_{[\mathbf{Z(1-Z)}]}(T_R,\mathcal{R}=0)\right] \;-\; R\to L \\
F_2(q^2) &= -\frac{36}{m_\lambda^2[q^2-4m_+^2][q^2-4m_-^2]}\Bigg\{N_2\quad \Delta I_{[\mathbf{Z(1-Z)}]}(T_R,R) \\
&\quad -2m_-[3m_+^2+m_-^2][q^2+2m_+(m_+-3m_-)]\quad \Delta I_{[\mathbf{Z(1-Z)}]}(T_R,\mathcal{R}=0)\Bigg\} \\
&\quad + \; R\to L \\
N_2 &= m_+q^4-2(m_+-m_-)(2m_+^2-m_+m_-+m_-^2)q^2 \\
&\quad +4m_+m_-(3m_+^3-5m_+^2m_-+m_+m_-^2-3m_-^3) \\
F_2^5(q^2) &= -\frac{36}{m_\lambda^2[q^2-4m_+^2][q^2-4m_-^2]}\Bigg\{N_2^5\quad \Delta I_{[\mathbf{Z(1-Z)}]}(T_R,R) \\
&\quad -2m_+[3m_-^2+m_+^2][q^2+2m_-(m_--3m_+)]\quad \Delta I_{[\mathbf{Z(1-Z)}]}(T_R,\mathcal{R}=0)\Bigg\} \\
&\quad - \; R\to L \\
N_2^5 &= m_-q^4+2(m_+-m_-)(2m_-^2-m_-m_++m_+^2)q^2 \\
&\quad +4m_+m_-(3m_-^3-5m_-^2m_++m_-m_+^2-3m_+^3)
\end{aligned}
\tag{O.5}
$$

These expressions are valid for all q^2. Note that F_1^5 and F_2^5 are the same as F_1 and F_2 respectively except for $m_+ \leftrightarrow m_-$ and the relative signs of the left and right sectors. In deriving these form factors algebraic simplification was facilitated by use of the equation

$$I_{[\bar{z}^2]}(T,R) = \frac{1}{2\mathcal{R}}\tilde{G}(T,R) \tag{O.6}$$

as found in Table L.1 of Appendix L. Since $\mathcal{R} = \frac{q^2}{4m_\lambda^2}$, as $q^2 \to 0$ (photon on shell) we have $\mathcal{R} \to 0$ and we see that the $\Delta I_{[Z(1-Z)]}$ terms in F_1 and F_1^5 explicitly vanish. It is also true that $\tilde{G}(T,R)$ vanishes as $\mathcal{R} \to 0$ but

$$\lim_{\mathcal{R}\to 0} \frac{\tilde{G}(T,R)}{\mathcal{R}} \to \frac{(1-T)(2T^2-7T+11)}{(1-T)^4}\,. \tag{O.7}$$

This expression is regular at T=1:

$$\lim_{T\to 1}\lim_{\mathcal{R}\to 0} \frac{\tilde{G}(T,R)}{\mathcal{R}} \to -\frac{3}{2}\,. \tag{O.8}$$

Similar considerations enable us to evaluate (O.5) in various regimes of momentum space. Relatively simple closed-form expressions have been derived for the form factors in the large and small $|q|^2$ limits. For example

$$F_1(q^2 \to 0) \to \Delta f_1(T_R) - \frac{\mathcal{R}}{5}\Delta f_1^0(T_R) \quad + \quad (R \to L)$$

where

$$f_1(x) \equiv \frac{(1-x)(2x^2-7x+11)+6\ell n x}{(1-x)^4} \qquad f_1(1) = -\frac{3}{2}$$

$$f_1^0(x) \equiv \frac{(x-1)(3x^4-17x^3+43x^2-77x-12)+60\; x\ell n x}{x(1-x)^6} \qquad f_1^0(1) = 2$$

O.3 EXACT ANALYTIC EXPRESSIONS

The various integral functions may be expressed in closed form although, in practice, little is gained thereby. As an example

$$\tilde{G}(T,R) = 1 - \Big\{G_+(U_+) - G_+(L) + G_-(U_-) - G_-(L)\Big\}$$

with the following tedious definitions

$$G_\pm(z) \equiv \frac{1}{4}\left(\frac{A_\pm}{B_\pm} - C_\pm - D_\pm\right)$$

$$\begin{aligned} A_\pm = {} & 2\ln z[z^4\rho^2 \pm 4z^3\rho^3 + z^2(\chi^4 + 5\rho^4) \mp 2z\rho(\chi^4 + \rho^4)] \\ + {} & [-z^4\rho^2 \pm 6z^3\rho^3 - z^2\rho^4 \pm 2z\rho(2\rho^4 - \chi^4) + 2\rho^2\chi^4] \end{aligned}$$

$$B_\pm = 4\rho^2[z \pm \rho]^2$$

$$C_\pm = \frac{\chi^4}{2\rho^2}\ln(\mp z - \rho)$$

$$D_\pm = \chi^2[2\ln z \cdot \ln(1 \pm z/\rho) - \mathrm{li}_2(\pm z/\rho)]$$

with li_2 is the dilogarithmic function defined by

$$\mathrm{li}_2(x) \equiv -\int_0^x \frac{\ln(1-x)}{x}\ dx\ . \tag{O.9}$$

Note that $\mathrm{li}_2(x) = -\int\ \ln(1-x)\ d(\ln x)$ and that

$$\int \frac{\ln x}{x+1}\ dx = \ln x\ \ \ln(1+x) + \mathrm{li}_2(-x)\ .$$

Continuing with the definitions:

$$\rho \equiv \frac{1-T}{2\mathcal{R}}$$

$$\chi \equiv \sqrt{\rho^2 - 1/\mathcal{R}}$$

$$L \equiv \sqrt{\rho^2 - \chi^2}$$

$$= \frac{1}{\sqrt{\mathcal{R}}}$$

$$U_{\pm} \equiv \sqrt{(1+\rho)^2 - \chi^2} \pm 1$$

Similar expressions exist for the other integral functions.

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