WEAK MATRIX ELEMENTS AND THE DETERMINATION OF THE WEAK MIXING ANGLES*

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In the limit of QCD where heavy quark masses go to infinity new symmetries appear which enable one to predict the hadronic form factors that occur in the semileptonic decays $\bar{B} \to De\bar{\nu}_e, \bar{B} \to D^*e\bar{\nu}_e$ and $\Lambda_b \to \Lambda_c e\bar{\nu}_e$. These results may eventually improve our ability to extract the weak mixing angles from experimental data on $\bar{B}$ and Λ_b decay.

1. INTRODUCTION

The main goal of this talk is to review a new method[1] for predicting, from QCD, the weak matrix elements that are needed to describe the semileptonic decays; $\bar{B} \to De\bar{\nu}_e, \bar{B} \to D^*e\bar{\nu}_e$ and $\Lambda_b \to \Lambda_c e\bar{\nu}_e$. The method also relates $\bar{B} \to \rho e\bar{\nu}_e$ semileptonic decay to $D \to \rho\bar{e}\nu_e$ semileptonic decay. Since these are exclusive decays involving hadronic states nonperturbative strong interaction physics plays a role. The basic idea is to examine QCD in the limit where the heavy charm and bottom quark masses to go infinity. In this limit, QCD has new symmetries that allow us to get a handle on the nonperturbative strong interaction physics that occurs in the semileptonic decays of hadrons containing a single heavy quark.

I will discuss briefly how the results on semileptonic decay matrix elements impact the determination of the weak Cabibbo–Kobayashi–Maskawa mixing angles. These angles parametrize the coupling of the W–bosons to the quarks. In the standard six–quark model

$$\mathcal{L} = -\frac{g_2}{2\sqrt{2}}(\bar{u}, \bar{c}, \bar{t})\gamma_\mu(1-\gamma_5)V \begin{pmatrix} d \\ s \\ b \end{pmatrix} W^\mu + \text{h.c} . \tag{1}$$

In eq. (1) W^μ is the W–boson field and g_2 is the weak SU(2) gauge coupling. The quantity V is a 3×3 unitary matrix that arises from diagonalizing the charge 2/3

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and charge $-1/3$ quark mass matrices. It can be written in terms of four angles $\theta_1, \theta_2, \theta_3$ and δ. A standard parametrization for V is[2]

$$V = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_1s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix} \tag{2}$$

where $c_i \equiv \cos\theta_i$ and $s_i \equiv \sin\theta_i$ for $i = 1,2,3$. Without loss of generality the angles θ_1, θ_2 and θ_3 can be chosen to lie on the first quadrant where their sines and cosines are positive. Then the quadrant of δ has physical significance and cannot be fixed by convention. In the minimal standard model it is the angle δ which occurs in the phase that is responsible for the CP violation observed in kaon decays. Experimental information on $\bar{B}$-meson decays, hyperon and kaon decays and nuclear β decay imply that the angles θ_1, θ_2 and θ_3 are small. From eqs. (1) and (2) it is evident that θ_1 is essentially the Cabibbo angle, the linear combination $(s_3 + s_2e^{i\delta})$ determines the strength of the $b \to c$ coupling and the product s_1s_3 determines the strength of the $b \to u$ weak coupling.

In the minimal standard model the quarks get mass through Yukawa couplings to a Higgs doublet. The Yukawa coupling constants and hence the values of the quark masses and mixing angles are fundamental quantities whose values must be determined by experiment. While there is a feeling in the particle physics community that one should be able to predict the quark mixing angles and masses, this lies beyond the scope of the minimal standard model.

2. EFFECTIVE THEORY FOR HEAVY QUARKS

We are interested in the interaction of a heavy quark Q with light degrees of freedom (quarks and gluons) when the light degrees of freedom carry a four-momentum much less than m_Q. For such a situation it is appropriate to take the limit of QCD where $m_Q \to \infty$ with the four-velocity of the heavy quark $v^\mu \equiv p_Q^\mu/m_Q$ fixed. The four-momentum of the light degrees of freedom are neglected compared with m_Q. In this limit creation of heavy quark–antiquark pairs is suppressed. The heavy quark simply travels in a straight worldline undeflected by its interactions with gluons.

It is easy to derive the Feynman rules for this effective theory by taking the limit of the usual Feynman rules for QCD. In QCD the propagator for the heavy quark is

$$\frac{i(\not{p}_Q + m_Q)}{p_Q^2 - m_Q^2} . \tag{3}$$

For the heavy quark four-momentum we write

$$p_Q = m_Q v + k \tag{4}$$

where k is a residual four-momentum that we shall neglect compared with m_Q. Putting eq. (4) into eq. (3) and using $v^2 = 1$ the propagator becomes

$$\frac{i(\not{v} + 1)}{2v \cdot k} \tag{5}$$

where terms of order (k/m_Q) have been neglected. Eventually the $\not{v}$ will stand next to the heavy quark field Q. Since $\not{v}Q = Q$ the propagator (5) finally becomes

$$\frac{i}{v \cdot k} . \tag{6}$$

In QCD the vertex for a gluon with four-momentum k interacting with a heavy quark is

$$ig\,T^a\,\gamma_\mu \tag{7}$$

with g is the strong coupling constant and T^a a color SU(3) generator. Since the heavy quark remains approximately on-shell as it propagates the γ_μ in eq. (7) is essentially sandwiched between on-shell spinors. Applying the Gordon decomposition and neglecting terms of order k/m_Q the gluon vertex becomes

$$ig\,T^a\,v_\mu . \tag{8}$$

These Feynman rules are a consequence of the Lagrangian density[3,4]

$$\mathcal{L} = i\bar{Q}\,v^\mu D_\mu Q \tag{9}$$

where D_μ is a covariant derivative and Q is the field for a heavy quark with four-velocity v. In eq. (9) the heavy quark field has been rescaled by the factor $e^{imv.x}$

so that the derivative produces a factor of the residual momentum. In the effective theory, the coupling of the heavy quark to gluons is independent of its mass and spin. It does, however, depend on the heavy quark's four-velocity. Note that the effective theory is not a nonrelativistic description of the interactions of the heavy quark. The four-velocity v can have spatial components of order unity.

This effective theory has symmetries not manifest in QCD[1]. Since the interactions of heavy quarks are independent of their masses if there are N heavy flavors Q_j $j = 1, \ldots N$ then the theory has a SU(N) flavor symmetry (actually SU(N)×U(1)). At this point it is worth reviewing the flavor symmetries that occur in nature. The six-quarks u, d, s, c, b, t naturally break into two groups of three. The light u, d and s quarks have masses that are less than the QCD scale ($m_u \simeq 0.005$ GeV, $m_d \simeq 0.01$ GeV, $m_s \simeq 0.15$ GeV). This gives rise to an approximate SU(3) flavor symmetry of QCD. The flavor symmetry arises not because the light quark masses are almost equal, but rather because the quark masses are small compared with the QCD scale, Λ_{QCD}. It is important to understand the origin of this symmetry. For example, even though the pion and kaon are in the same multiplet of this symmetry group it is a mistake to use it to deduce that the pion and kaon masses are almost equal (recall $m_\pi \simeq 0.14$ GeV, $m_K \simeq 0.49$ GeV). The pion and kaon are pseudoGoldstone bosons; their masses go to zero as the light quark masses go to zero. One should apply the SU(3) (light quark) flavor symmetry to quantities which go to a constant as the light quark masses go to zero. For example, a legitimate application would be to deduce that the proton and cascade are almost degenerate. The remaining three quarks c, b, t have masses large compared with the QCD scale ($m_c \simeq 1.8$ GeV, $m_b \simeq 5.2$ GeV, $m_t \geq 90$ GeV). We have just seen that this implies a second SU(3) flavor symmetry of nature. The heavy quark flavor symmetry arises not because the heavy quarks are almost degenerate but rather because their masses are all large compared with the QCD scale. Again, it is important to understand the physical origin of the symmetry. For example, it would be a mistake to use the heavy flavor SU(3) symmetry to deduce that the $\bar{B}$ and D mesons should have the same mass (experimentally $m_{\bar{B}} \simeq 5.3$ GeV, $m_D \simeq 1.9$ GeV). These are quantities which go to infinity as the heavy quark masses go to infinity. The heavy quark flavor symmetry should be applied to relate quantities which go to a constant as the heavy quark masses go to infinity.

Since the strange quark mass is not very small compared with Λ_{QCD} there are sizable corrections (typically ~ 30 %) to the predictions of light quark flavor symmetry. Similarly, the charm quark mass is not extremely large compared with the QCD scale so we can expect sizable corrections to the predictions of heavy quark flavor symmetry.

The effective theory for the interaction of a heavy quark Q with light quarks and gluons has an SU(2) symmetry generated by the spin $\vec{S}_Q$ of the heavy quark.[1] This spin symmetry has important experimental consequences. Consider the low-lying mesons with $Q_j\bar{q}$ flavor quantum numbers (Q_j is a heavy quark of flavor j.) Both the spin of the heavy quark $\vec{S}_{Q_j}$ and the spin of the light degrees of freedom $\vec{S}_\ell = \vec{S} - \vec{S}_{Q_j}$ are good quantum numbers. States of total spin S are made from combining $S_{Q_j} = 1/2$ with S_ℓ. Since the dynamics are completely independent of S_{Q_j} the spectrum will consist of degenerate multiplets related by the spin of the heavy quark. The lowest lying mesons with $Q_j\bar{q}$ flavor quantum numbers have $S_\ell = 1/2$ (this is a dynamical assumption about QCD that is consistent with the observed spectrum) and so we get degenerate $S = 0$ and $S = 1$ states denoted by P_j and P_j^*. The P_j and P_j^* states (at rest with $S^3 = 0$) are

$$|P_j\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\} \tag{10a}$$

$$|P_j^*\rangle = \frac{1}{\sqrt{2}}\{|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle\} \tag{10b}$$

in a notation where the first arrow refers to $S^3_{Q_j}$ and the second arrow refers to S^3_ℓ. The spin of the heavy quark relates these mesons

$$S^3_{Q_j}|P_j\rangle = \frac{1}{2}|P_j^*\rangle \ . \tag{11}$$

Experimentally $m_{D^*} - m_D \simeq 0.14$ GeV. The fact that this mass splitting (which arises from a $1/m_c$ correction to the Lagrangian density in eq. (9)[5]) is small compared with the QCD scale lends support to the point of view that the charm quark mass is large enough to apply the heavy quark spin and flavor symmetries discussed above.

The heavy quark spin and flavor symmetries of the effective theory endow us with considerable predictive power. Consider, for example, the matrix element

$$\langle P_j(\vec{v}')|\bar{\mathbf{Q}}_j\gamma_\mu\mathbf{Q}_i|P_i(\vec{v})\rangle / \sqrt{m_{P_i}m_{P_j}} = \tilde{f}_+(v+v')_\mu + \tilde{f}_-(v-v')_\mu \,. \tag{12}$$

In eq. (12) we have used the conventional normalization of meson states which contains a factor of the mass of the state. Because of this normalization it is the matrix element divided by $\sqrt{m_{P_i}m_{P_j}}$ (as appears on the l.h.s. of eq. (12)) that is independent of the heavy quark masses. We have chosen to label the meson states by their velocity rather than by their momentum. This is convenient because the heavy quark flavor symmetry relates states of the same velocity but different mass and hence different momentum. On the right- hand side of eq. (12) $\tilde{f}_+$ and $\tilde{f}_-$ are Lorentz invariant form factors which are functions of $v\cdot v'$. We have put a tilde over them since eq. (12) is not the usual definition for the form factors. Conventionally, one writes ($p_\mu = m_{P_i}v_\mu, p'_\mu = m_{P_j}v'_\mu$)

$$\langle P_j(\vec{v}')|\bar{\mathbf{Q}}_j\gamma_\mu\mathbf{Q}_i|P_i(\vec{v})\rangle = f_+(p+p')_\mu + f_-(p-p')_\mu \,. \tag{13}$$

Comparing eqs. (12) and (13) gives the following relationship between $f_\pm$ and $\tilde{f}_\pm$

$$f_+ = \frac{1}{2}\left(\sqrt{\frac{m_{P_j}}{m_{P_i}}} + \sqrt{\frac{m_{P_i}}{m_{P_j}}}\right)\tilde{f}_+ + \frac{1}{2}\left(\sqrt{\frac{m_{P_j}}{m_{P_i}}} - \sqrt{\frac{m_{P_i}}{m_{P_j}}}\right)\tilde{f}_- \,, \tag{14a}$$

$$f_- = \frac{1}{2}\left(\sqrt{\frac{m_{P_j}}{m_{P_i}}} - \sqrt{\frac{m_{P_i}}{m_{P_j}}}\right)\tilde{f}_+ + \frac{1}{2}\left(\sqrt{\frac{m_{P_j}}{m_{P_i}}} + \sqrt{\frac{m_{P_i}}{m_{P_j}}}\right)\tilde{f}_- \,. \tag{14b}$$

In a transition where Q_i with four-velocity v is changed by the action of the current into Q_j with four-velocity v' the heavy quark fields satisfy

$$\not{v}Q_i = Q_i \quad ; \quad \bar{Q}_j\not{v}' = \bar{Q}_j \,. \tag{15}$$

This implies that

$$\tilde{f}_- = 0 \,. \tag{16a}$$

When $v = v'$ the operator $\bar{\mathbf{Q}}_j\gamma_0\mathbf{Q}_i$ is related to a generator of the heavy quark flavor symmetry. Since the action of the generators on the states is fixed its matrix element is known. This yields

$$\tilde{f}_+(1) = 1 \,. \tag{16b}$$

Note that eqs. (16a) and (16b) are not the same as what would be obtained from an "ordinary" manifest flavor symmetry which predicts[6] $f_- = 0$ and $f_+(1) = 1$ (eqs. (16) do reduce to this when $m_{Q_i} \simeq m_{Q_j}$).

It is appropriate to use the effective theory to calculate the matrix element (12). Initially, the light antiquark and gluons carry a four-momentum of order $\Lambda_{\rm QCD}v$ and finally they carry a four-momentum of order $\Lambda_{\rm QCD}v'$. So even though the total momentum transfer squared

$$t = (p'-p)^2 = m^2_{P_j} + m^2_{P_i} - 2m_{P_i}m_{P_j}v\cdot v' \tag{17}$$

is large the momentum transfer squared felt by the light degrees of freedom is only of order

$$\Lambda^2_{\rm QCD}v\cdot v' \,. \tag{18}$$

The heavy quark spin symmetry can be used to determine other matrix elements which involve $P_i \to P^*_j$ transitions. Consider

$$\langle P^*_j(\vec{v}',\epsilon)|\bar{\mathbf{Q}}_j\gamma_\mu\gamma_5\mathbf{Q}_i|P_i(\vec{v})\rangle / \sqrt{m_{P_i}m_{P^*_j}} = \tilde{f}\epsilon^*_\mu + \tilde{a}_+(\epsilon^*\cdot v)(v+v')_\mu + \tilde{a}_-(\epsilon^*\cdot v)(v-v')_\mu \,, \tag{19}$$

$$\langle P^*_j(\vec{v}',\epsilon)|\bar{\mathbf{Q}}_j\gamma_\mu\mathbf{Q}_i|P_i(\vec{v})\rangle / \sqrt{m_{P_i}m_{P^*_j}} = i\tilde{g}\epsilon_{\mu\nu\alpha\beta}\,\epsilon^{*\nu}v'^{\alpha}v^{\beta} \,. \tag{20}$$

The Lorentz invariant form factors $\tilde{f}, \tilde{a}_+, \tilde{a}_-$ and $\tilde{g}$ are functions of $v\cdot v'$. These form factors can be related to $\tilde{f}_+$ using the spin symmetry. For example,

$$\begin{aligned}
&\left\langle P_j(\vec{0})|\bar{\mathbf{Q}}_j\gamma_3\mathbf{Q}_i|P_i(\vec{v})\right\rangle \,, \\
&= 2\left\langle S^3_{Q_j}P^*_j(\vec{0},0)|\bar{\mathbf{Q}}_j\gamma_3\mathbf{Q}_i|P_i(\vec{v})\right\rangle \,, \\
&= 2\left\langle P^*_j(\vec{0},0)|[S^3_{Q_j},\bar{\mathbf{Q}}_j\gamma_3\mathbf{Q}_i]|P_i(\vec{v})\right\rangle \,, \\
&= -\left\langle P^*_j(\vec{0},0)|\bar{\mathbf{Q}}_j\gamma_0\gamma_5\mathbf{Q}_i|P_i(\vec{v})\right\rangle \,.
\end{aligned} \tag{21}$$

Other relations of this type, eq. (21), eqs. (15) and eqs. (16) allow us to express all the form factors $\tilde{f}_\pm, \tilde{a}_\pm, \tilde{f}, \tilde{g}$ in terms of a single function $\xi(v\cdot v')$ normalized at threshold ($v = v'$) to unity. Explicitly[1]

$$\tilde{f}_+ = \xi(v \cdot v') \quad , \quad \tilde{f}_- = 0 \tag{22a}$$

$$\tilde{f} = [1 + v \cdot v']\xi(v \cdot v') \tag{22b}$$

$$\tilde{g} = \xi(v \cdot v') \tag{22c}$$

$$(\tilde{a}_+ + \tilde{a}_-) = 0 \quad , \quad (\tilde{a}_+ - \tilde{a}_-) = -\xi(v \cdot v') \,. \tag{22d}$$

(See refs. 7 and 8 for a simpler method of deriving these relations.) In the case $Q_i = b\ Q_j = c$ these are the form factors relevant for the semileptonic decays $\bar{B} \to De\bar{\nu}_e$ and $\bar{B} \to D^* e\bar{\nu}_e$. We have seen that in the effective theory all the form factors for these decays are determined at threshold (where in the rest frame of the decaying $\bar{B}$ meson the final D or D^* meson is also at rest). Away from threshold their shape is determined in terms of a single function $\xi(v \cdot v')$. However, these results are true in the effective theory; what precisely is the relationship between the full theory of QCD and the effective theory?

3. RELATIONSHIP BETWEEN QCD AND THE EFFECTIVE THEORY

Consider the vector current $\bar{Q}_j \gamma_\mu Q_i$. In QCD this operator does not require renormalization since it is a partially conserved current. However, at one loop its matrix elements (e.g., those considered in the previous section) do involve logarithms of the heavy quark masses (divided by a typical momentum of the light degrees of freedom). In the effective theory these finite logarithms become ultraviolet divergences, since the heavy quark masses are taken to infinity. In the effective theory the vector current $\bar{\mathbf{Q}}_j \gamma_\mu \mathbf{Q}_i$ requires renormalization and it has dependence on the subtraction point μ. The relationship between the vector current in QCD and the vector current in the effective theory is (no sum on i and j)

$$\bar{Q}_j \gamma_\mu Q_i = C_{ji} \bar{\mathbf{Q}}_j \gamma_\mu \mathbf{Q}_i + \ldots \tag{23}$$

where the ellipsis denotes other possible Lorentz structures (e.g., $\bar{Q}_j Q_i v_\mu$ and $\bar{Q}_j Q_i v'_\mu$) and higher dimension operators. Their effects are suppressed by the QCD fine structure constant evaluated at a heavy quark mass scale or by powers of Λ_{QCD}/m_{Q_i} and Λ_{QCD}/m_{Q_j}. The coefficient C_{ji} has been determined, in the leading logarithmic approximation. For $m_{Q_i} \geq m_{Q_j} >> \mu$[8,9]

$$C_{ji}(\mu) = \left[\frac{\alpha_s(m_{Q_i})}{\alpha_s(m_{Q_j})}\right]^{a_I} \left[\frac{\alpha_s(m_{Q_j})}{\alpha_s(\mu)}\right]^{a_L} \,. \tag{24}$$

Here

$$a_I = -\frac{6}{33 - 2N_f} \quad , \tag{25}$$

where N_f is the number of quark flavors which contribute to the beta function in the momentum interval between m_{Q_i} and m_{Q_j}, and

$$a_L = \frac{8[v \cdot v' r(v \cdot v') - 1]}{33 - 2N'_f} \tag{26}$$

with N'_f is the number of quark flavors appropriate to the momentum interval between m_{Q_j} and μ. In eq. (26)

$$r(v \cdot v') = \frac{1}{\sqrt{(v \cdot v')^2 - 1}} \ell n \left(v \cdot v' + \sqrt{(v \cdot v')^2 - 1}\right) \,. \tag{27}$$

At $v = v'$ the operator $\bar{\mathbf{Q}}_j \gamma_\mu \mathbf{Q}_i$ is related to a generator of heavy quark flavor symmetry and it should not be renormalized. Indeed, we see from eqs. (24), (26) and (27) that, since $r(1) = 1, C_{ji}(\mu)$ is independent of μ for $v = v'$. The relationship between the axial current in QCD and in the effective theory has the same form (this is ensured by the spin symmetries)

$$\bar{Q}_j \gamma_\mu \gamma_5 Q_i = C_{ji} \bar{\mathbf{Q}}_j \gamma_\mu \gamma_5 \mathbf{Q}_i + \ldots \tag{28}$$

with C_{ji}'the same as occurs in eq. (23). Therefore in QCD the form factors $\tilde{f}_\pm, \tilde{a}_\pm, \tilde{f}$ and $\tilde{g}$, are given by making the replacement

$$\xi \to C_{ji} \xi \tag{29}$$

in eqs. (22).

Note that, because the currents require renormalization in the effective theory, the function ξ depends on the subtraction point μ. This dependence cancels that of C_{ji} so that the form factors (which are physical quantities) are subtraction point independent. For μ of order the QCD scale ξ contains no large logarithms and it is expected to have a slope at threshold of order unity. On the other hand, for very large heavy quark masses the dependence of C_{ji} on $v \cdot v'$ is rapid. It follows that for very large heavy quark masses, in the region near threshold, the dependence of ξ on $v \cdot v'$ can be neglected (i.e., ξ can be set to unity) and it is C_{ji} that determines the shape of the form factors. In the physical case $Q_i = b, Q_j = c$ (relevant for semileptonic $\bar{B}$ decay) this is not likely to be a good approximation. For $\bar{B} \to De\bar{\nu}_e$ decay over the whole available phase space, $v \cdot v' = 1$ to $v \cdot v' = 1.6$, the function C_{ji} decreases by only about 10% (for $\alpha_s(\mu) = 1$).

4. WEAK MIXING ANGLES

There are several approaches available to determine the magnitude of the Cabibbo-Kobayashi-Maskawa matrix elements V_{bu} and V_{bc}. Since the b-quark is heavy compared to the QCD scale inclusive semileptonic $\bar{B}$ meson decay can be approximated by b-quark decay. Corrections to this picture are presumably suppressed by factors of $\Lambda_{\rm QCD}/m_b$. The b-quark decay either proceeds through the $b \to c$ or $b \to u$ W-boson couplings. So

$$\Gamma(\bar{B} \to X e\bar{\nu}_e) \simeq \Gamma(b \to ce\bar{\nu}_e) + \Gamma(b \to ue\bar{\nu}_e) \tag{30}$$

where

$$\Gamma(b \to ce\bar{\nu}_e) = \frac{|V_{bc}|^2 G_F^2 m_b^5}{192\pi^3} g(m_c/m_b) \ , \tag{31a}$$

$$\Gamma(b \to ue\bar{\nu}_e) = \frac{|V_{bu}|^2 G_F^2 m_b^5}{192\pi^3} \ . \tag{31b}$$

In eq. (31) g is a function that takes into account the effects of the charm quark mass; it is about equal to one-half. Experimentally the $b \to c$ transition dominates the rate. From the measured $\bar{B}$-meson lifetime $\tau_B \simeq 10^{-12}$ sec and semileptonic branching ratio ($\sim 10\%$) eq. (31a) implies (for $m_b = 5.2$ GeV)

$$|V_{bc}|^2 \simeq 0.2 \times 10^{-2} \ . \tag{32}$$

Because eq. (31a) is proportional to m_b^5 it is very sensitive to the value of the b-quark mass. If we had used $m_b = 4.7$ GeV then value of $|V_{bc}|^2$ would increase by 60% over what is given in eq. (32).

Another approach to determining mixing angles is to use predictions for particular final states. Experimentally, the rate for semileptonic $\bar{B}$ decay is dominated by two processes; $\bar{B} \to De\bar{\nu}_e$ and $\bar{B} \to D^* e\bar{\nu}_e$. The rates for these processes are determined by the magnitude of V_{bc} and by the matrix elements

$$\langle D(\vec{v}')|\bar{c}\gamma_\mu b|\bar{B}(\vec{v})\rangle \ , \tag{33a}$$

$$\langle D^*(\vec{v}')|\bar{c}\gamma_\mu b|\bar{B}(\vec{v})\rangle \ , \tag{33b}$$

$$\langle D^*(\vec{v}')|\bar{c}\gamma_\mu\gamma_5 b|\bar{B}(\vec{v})\rangle \ . \tag{33c}$$

The matrix elements of eqs. (33) involve nonperturbative strong interaction physics. Despite this, we have seen in the previous two sections that the form factors that characterize these matrix elements are determined at threshold (where $v = v'$) and that they have a shape determined by a single function of $v \cdot v'$. (In principle the shape can be measured.) These predictions are enough to extract the magnitude of V_{cb} from data on the exclusive $\bar{B}$ semileptonic decays. (At present the data is not good enough for this method to be superior to that based on the inclusive decay rate.) The weakest feature of this approach is treating the charm quark as very heavy. Perturbative corrections of order $\alpha_s(m_c)/\pi$ have been computed[8,10] and they are not large. Recently the Λ_{QCD}/m_c corrections have been characterized.[11] These involve the matrix elements of new operators that are not computable without relying on numerical lattice Monte Carlo methods. However, at threshold there are no Λ_{QCD}/m_c corrections to the form factor $\tilde{f}$. This should eventually allow a very accurate determination of $|V_{cb}|$ from experimental data on $B \to D^* e\bar{\nu}_e$.

In Z^0 decays at LEP many Λ_b baryons will be produced. This state has $S_\ell = 0$. The form factors for the semileptonic decay $\Lambda_b \to \Lambda_c e\bar{\nu}_e$ can be predicted using the heavy quark spin and flavor symmetries. Writing

$$\langle\Lambda_c(\vec{v}',s')|\bar{c}\gamma_\mu b|\Lambda_b(\vec{v},s)\rangle = \bar{u}(\vec{v}',s')\Big[F_1\gamma_\mu + F_2 v_\mu + F_3 v'_\mu\Big]u(\vec{v},s) \qquad (34)$$

and,

$$\langle\Lambda_c(\vec{v}',s')|\bar{c}\gamma_\mu\gamma_5 b|\Lambda_b(\vec{v},s)\rangle = \bar{u}(\vec{v}',s')\Big[G_1\gamma_\mu\gamma_5 + G_2 v_\mu\gamma_5 + G_3 v'_\mu\gamma_5\Big]u(\vec{v},s) \quad (35)$$

the heavy quark spin and flavor symmetries imply that the six Lorentz invariant form factors G_1, G_2, G_3, F_1, F_2 and F_3 can be expressed in terms of a single function $\eta(v\cdot v')$ normalized to unity at threshold (i.e., $\eta(1)=1$). Explicitly[12] (baryon states are normalized to E_B/m_B)

$$F_1 = G_1 = \left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{a_I}\left[\frac{\alpha_s(m_c)}{\alpha_s(\mu)}\right]^{a_L}\eta(v\cdot v') \qquad (36a)$$

$$F_2 = F_3 = G_2 = G_3 = 0\,. \qquad (36b)$$

The Λ_{QCD}/m_c corrections to these form factors have also been characterized.[11] Amazingly these introduce no new functions of $v\cdot v'$ but only add pieces to (36) proportional to $\bar{\Lambda}/m_c$, where $\bar{\Lambda} = m_{\Lambda_b} - m_b = m_{\Lambda_c} - m_c$. This decay may eventually also play a role in determining $|V_{bc}|$.

Although the $b\to u$ coupling is responsible for only a small fraction of the semileptonic $\bar{B}$ decays, it is possible to isolate this contribution (without reconstructing hadronic final states) by examining the electron spectrum

$$\frac{d\Gamma}{dE_e}(\bar{B}\to Xe\bar{\nu}_e)\,. \qquad (37)$$

Kinematically, the production of final hadronic states X with mass m_X is forbidden for electron energies greater than

$$(E_e)_{\max} = \left(m_{\bar{B}}^2 - m_X^2\right)/2m_{\bar{B}}\,. \qquad (38)$$

The lowest mass particle containing a charm quark is the D meson with a mass of 1.9 GeV. So by examining the endpoint region $E_e > 2.3$ GeV one is sure to be focussing on the $b\to u$ contribution. What is needed is a method for normalizing the electron spectrum in this region. Unfortunately, one cannot justify the use of the free quark decay picture for the production of low mass hadronic final states[11] (e.g., ρ and π). The heavy flavor symmetries allow another approach to determining $|V_{bu}|$. The heavy flavor symmetry plus ordinary isospin symmetry implies, for example, that[1,13]

$$\left\langle\rho(\vec{k},\epsilon)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}(\vec{v})\right\rangle = \left(\frac{m_B}{m_D}\right)^{1/2}\left[\frac{\alpha_s(m_b)}{\alpha_s(m_c)}\right]^{-6/25} \left\langle\rho(\vec{k},\epsilon)|\bar{d}\gamma_\mu(1-\gamma_5)c|D(\vec{v})\right\rangle\,. \qquad (39)$$

Eq. (39) is valid (in the rest frame of the $\bar{B}$ and D) for light four-momenta k small compared with the heavy quark masses. Since in Cabibbo suppressed semileptonic decay $D\to\rho\bar{e}\nu_e$ the weak mixing angles are known the right hand side of eq. (39) can be determined experimentally. With this information experimental data on $\bar{B}\to\rho e\bar{\nu}_e$ will allow a determination of $|V_{bu}|$. If one uses light quark SU(3) flavor symmetry, instead of isospin symmetry, then the decay $D\to K^*\bar{e}\nu_e$ can be used for the r.h.s of eq. (39). The form factors for this decay have already been measured.[14] Again, the weakest feature of this program is the neglect of Λ_{QCD}/m_c corrections. Considerable theoretical and experimental effort will be required to assess the accuracy of this method.

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