TOP QUARK TOPICS

G.L. KANE

Randall Laboratory of Physics
University of Michigan
Ann Arbor, Michigan 48109-1120

ABSTRACT

These lectures survey why the top quark may be unusually interesting if its mass is large. Perhaps the large mass is related to a fundamental role of the top quark. Perhaps it will have non-Standard Model decays, or be sensitive to some new physics.

INTRODUCTION

Lectures like these always begin (appropriately) by praising the Standard Model (SM), and then go on to say how it is hoped that new physics will appear. We will not bury the SM but extend it. To complete our knowledge of the SM the most basic need is to understand the physics of the Higgs sector. We need to know whether Higgs bosons exist, and at what masses. We also need to know whether neutrino masses are zero, and their values if they are not zero. CP violation can be described by the SM, but we are not very sure that is the main mechanism. And finally, we need to know the value of M_t, the top quark mass.

The current experimental limit is $M_t > 89$ GeV[1] , if the t-quark behaves like a SM particle. Once the limit is this high, whatever the value of M_t is, it may be very interesting. M_t has been predicted long ago to be in the 120-150 GeV region by several interesting ideas (fixed point behavior or renormalization group equations, driving the Higgs mechanism in a supersymmetric world, etc., as described below). [Indeed, contrary to what a number of people have remarked, it was not surprising to most interested theorists that $M_t > M_W$; we will examine some of these arguments below.] At even larger M_t it has been speculated that non-perturbative effects cause non-zero expectation values for $\bar{t}t$ and cause spontaneous symmetry breaking. And in another sense, the large phase space available in the decay of a heavy t may allow a non-SM mode to appear. Alternatively, it could happen that in fact 50 GeV $\leq M_t \leq$ 90 GeV, but t has not been observed because it decays by some mode(s) to which experiments were so far not sensitive.

There is no consensus concerning how to think about M_t. In a naive SM approach probably all masses ought to be within a factor of two or so of M_W, and the question should be why are the other fermion masses so small, and so different. From the point of view of composite and dynamical approaches, however, it is difficult to give fermions mass at all, so the problem is why is M_t so large. The quark masses or mass ratios for each family show no instructive pattern. Undoubtedly the quark masses are telling us something profound, but unfortunately no one knows what.

It is amusing, and perhaps important, to note that the top width $\Gamma_t \sim M_t^3$ (for large M_t), so as M_t increases the lifetime gets very short and t-quarks decay before mesons form so that they behave as free quarks. One can write approximately for the lifetime

$$\tau_t \simeq 1.2 \text{ Fermi} \left(\frac{M_W}{M_t}\right)^3 ,$$

and once $\tau_t \leq 1$ Fermi there is not time for binding to occur. In particular, we will see that this may make it possible to test some interesting QCD polarization predictions.

We will cover the following topics below:

- — The top quark exists
- — Upper limit on M_t
- — Constraints on M_t
- — SM top decays, signatures
- — Production of t
  - e^+e^- Collider
  - Hadron Colliders
- — Detecting SM tops — signatures
- — Model-independent lower limit on M_t
- — Determining the charge of a new heavy quark
- — Top decays as a window to new physics
  - Supersymmetric decays
  - Decay to charged Higgs bosons
  - Decays to heavy quarks
  - Flavor-changing neutral current decays
  - Other possibilities
- — New information once top is observed
- — Studying the top decay couplings
- — Top quark at (NLC)
- — Testing QCD polarization predictions
- — Correlation of top spin direction with final b, ℓ^+ directions
- — Measuring M_t from loops
- — Prediction for M_t
- — Is $t \to Wb$ a background for studying TeV WW interactions?
- — Final comments

THE TOP QUARK EXISTS

Nothing can be proved without assumptions, of course. So the theorem here is that (1) given the validity of the $SU(2)$ structure of the SM, and (2) assuming the $SU(2)$ symmetry of the SM is not broken by the explicit absence of states, then the top quark exists. The general validity of the

SM is not in question; it has been too broadly tested by now. Rather, we hope someday to discover extensions of the SM that clarify its foundations, much as the SM is an extension of QED. Such extensions will leave intact the $SU(2)$ structure of the SM.

Assumption (2) is naively more ad hoc, and is based on the total absence so far of any hint of missing $SU(2)$ states. However, I suspect that using a fairly sophisticated analysis would allow the removal of assumption (2), because most likely the absence of any $SU(2)$ state in any multiplet would cause effects that are inconsistent with some other measurement. This is basically the result discussed below that sets an upper on M_t, though the analysis has not to my knowledge been approached in detail from the direction of asking what the consequences of a missing t-quark are.

With this discussion it is easy to state the basic result:

$$\text{EXPERIMENT} \longrightarrow T^b_{3L} = -\frac{1}{2} \longrightarrow \begin{pmatrix} t_L \\ b_L \end{pmatrix}.$$

In other words, data (to be quoted) shows that the left-handed b-quark has weak isospin eigenvalue $T^b_{3L} = -\frac{1}{2}$. Therefore it must be in a multiplet with one or more other states, one of them having $T^b_{3L} = +\frac{1}{2}$. That is by definition the top quark.

One could imagine that some other set of particles could accompany b_L. No attempt to construct such a set has succeeded. We will not spend time on the failed attempts; literature can be traced from ref. 2. Probably a general proof that all other models for particle content will fail is very difficult, so perhaps an exhaustive treatment of classes of models is the best that can be done.

The bottom line (top line?) is that given our assumptions the top quark must exist. The arguments for its existence could be tightened by further analysis. Similar arguments demonstrate that the τ neutrino must exist, so anyone who believes in the τ neutrino must believe in the top quark. In the case of ν_τ, since $m_{\nu_\tau} < m_\tau$ and τ decays into ν_τ, data provides even stronger constraints and models to evade ν_τ existing are easier to destroy. However, I think this is a practical difference; the logical status of t and ν_τ is the same.

There are also some theoretical arguments that independently suggest t must exist.

(a) Without t the minimal SM would have anomalies. However, while t is the simplest way to eliminate anomalies it may not be the only way. Further, it is not clear that there would be any measurable experimental impact of having anomalies, e.g., if the SM were an effective theory at the TeV scale. Thus, this argument is not compelling.

(b) t is needed in loops to generate the observed $B^\circ - \overline{B}^\circ$ mixing, which goes as $|V_{td}|^2 M_t^2$ in the minimal SM. However, other contributions can mimic the top contribution, so this is not unique in general. It is a constraint on model building rather than on experiment. If top is light or very heavy, some other states must (quantitatively) explain the $B^\circ - \overline{B}^\circ$ mixing.

Finally, we summarize the status of the measurements of T^b_{3L}. For this discussion we allow general T_3 eigenvalues for b_L and for b_R, both to be determined by measurement. Because of the interference of the γ and Z contribution in $e^+e^- \to b\bar{b}$, there is a forward-backward symmetry.

$$A_{F-B} \sim \left(T^b_{3L} - T^b_{3R}\right)\left(T^b_{3L} + T^b_{3R} + \frac{2}{3}\sin^2\theta_W\right).$$

Also, from Z decay,

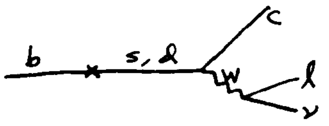
$$\Gamma\left(Z \to b\bar{b}\right) = 664 \text{ MeV}\left\{\left(T^b_{3L} + \frac{1}{3}\sin^2\theta_W\right)^2 + \left(T^b_{3R} + \frac{1}{3}\sin^2\theta_W\right)^2\right\}$$

From these two equations T^b_{3L} and T^b_{3R} can be measured. (Note that the sign of A_{FB} distinguishes T_{3L} from T_{3R}.) I am not aware of any analysis doing that yet. In the past only the $e^+e^- \to b\bar{b}$ data was available. It was used at PETRA, assuming $T^b_{3R} = 0$, to give[3]

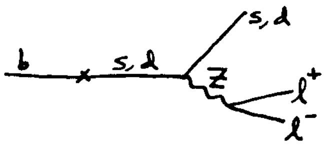
$$\begin{aligned} 2T^b_{3L} &= -(1.15 \pm 0.41), && \text{CELLO} \\ &= -(0.70 \pm 0.22), && \text{JADE} \\ &= -(1.2 \pm 0.5), && \text{TASSO} \\ &= -(1.44 \pm 0.56 \pm 0.13). && \text{AMY} \end{aligned}$$

These are the most recent measurements of which I am aware. Except for AMY, they have been corrected for $B^\circ - \overline{B}^\circ$ mixing, which itself causes a significant asymmetry. They are presently the most accurate measurements of $T^b_{3L} = -1/2$, and have been done assuming $T^b_{3R} = 0$.

While the above is the best measurement today of T^b_{3L}, historically[4,5] a different argument was first used to imply the top quark exists. If b were an $SU(2)$ singlet then top would not have to exist, so assume b is an $SU(2)$ singlet. Then b has no charged current interactions so it cannot decay by W emission. But it does decay, so it must do so by some kind of mixing with lighter quarks s, d as in fig. 2 (the result will not depend on the mixing mechanism).



But if that happens then so must the process



In the ratio of rates for these, the unknown mixing cancels. One can make this quantitative[5] and show that

$$BR\,(b \longrightarrow \ell^+\ell^- X) > 0.013.$$

Experimentally this BR is less than 0.0012 according to the CLEO group.[6] This establishes that b_L is not an $SU(2)$ singlet, so other particles must accompany b. As remarked above, no alternative except a simple $\binom{t}{b}_L$ doublet has been found that is not excluded by other data.

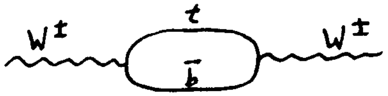
Finally, since an $SU(2)$ rotation from $b \rightarrow t$ must commute with space time properties, and we believe the $SU(2)$ breaking only affects the masses, all space-time properties such as $V - A$ couplings measured for the b should apply to the t.

THE UPPER LIMIT ON M_t

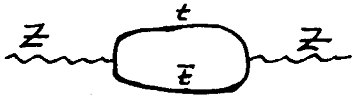
Having found that the top quark exists, the next problem is to determine its mass. Until it is directly detected (which cannot happen before a year from now — see below) only indirect arguments can be help. Using an argument basically due to Veltman,[7] applied recently by a number of people,[8,9] one can show that

IN ANY THEORY ONE CAN SET AN UPPER LIMIT ON M_t.

The upper limit always exists and we will denote it with $\overline{M}_t$. The value of $\overline{M}_t$ depends on the theory, and particularly on whether other particles beyond those of the minimal SM occur. Basically the argument comes from the observation that the W mass gets a contribution from the diagram



and the value of the contribution depends on M_t and M_b. Similarly, the Z mass gets contributions from



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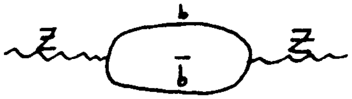

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It is customary to show the results for the quantity $\rho = M_W/M_Z \cos\theta_W$ and to neglect M_b^2/M_t^2, in which case for the minimal SM,[7]

$$\rho = 1 + \frac{3G_F}{8\sqrt{2}\pi^2}\, M_t^2 \simeq 1 + 0.003 \left(\frac{M_t}{100\ \mathrm{GeV}}\right)^2 .$$

If one chooses a prescription for radiative corrections where $\rho \equiv 1$, then a similar effect shows up for some other observable. The basic physics is the same, namely as $SU(2)$ is broken by separating the masses of the particles in the multiplet, observable effects of the symmetry breaking arise and became large enough to be detected.[8,9] Since ρ is measured to better than a per cent to be 1, M_t cannot get too large.

Unfortunately, as was emphasized above, the limit holds in any theory — but since we do not yet know the full theory of everything we cannot draw any firm conclusions about the actual value of M_t. Loop diagrams such as those above could have contributions from supersymmetric partners or other new particles, of either sign relative to these diagrams; the value of $\overline{M}_t$ could be larger or smaller than for the minimal SM. Presumably such effects are likely since most of us believe some physics beyond the minimal SM will occur.

Further, tree level effects can also occur. For example, if a Higgs boson triplet representation were part of the complete theory, with a triplet vacuum expectation value v_T for the neutral triplet state with $T_3 = -1$, and a doublet VEV v_D, then it turns out[10] that

$$\rho \simeq \frac{1 + 2v_T^2/v_D^2}{1 + 4v_T^2/v_D^2} \left[1 + \frac{3G_F}{8\sqrt{2}\pi^2} M_t^2 \right]$$

and since the first factor is necessarily less than unity it is easy for the effects of a larger M_t to be hidden. We are not recommending such an alternative,[11] only emphasizing that the value of $\overline{M}_t$ is not unique.[12]

Having said that, we note that some very recent calculations based on the precise LEP data have given the result[13]

$$M_t = 135 + 27 - 31 \text{ GeV}$$

in the minimal SM. In another calculation, these authors[13] have looked at the effect of including supersymmetric partners in the loops. With some assumptions, they conclude that

$$M_t = 131 + 24 - 28 \text{ GeV}.$$

These analyses give a range rather than only an upper limit since the data is now so precise that it is somewhat sensitive to the actual value of M_t once $M_t \geq M_W$. The errors are "one-standard deviation," defined in a complicated way, so presumably they should not be interpreted too rigidly. Complexities in the data analysis giving the input numbers can affect the final results too. For example, Pumplin has argued[14] that "higher twist" effects in ν scattering could shift the value of $\sin^2\theta_W$ deduced there by as much as 0.01.

If M_t does lie in those ranges it implies either that there is little other physics at the weak scale that affects such questions, or that the new physics comes in pairs whose effects cancel. It puts a strong constraint on any ideas. If M_t lies outside those ranges it is more interesting, since then new physics must exist at the weak scale. Of course, these analyses rely heavily on experimental measurements, so the conclusions only are valid if the data does not change.

CONSTRAINTS ON M_t

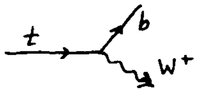
In any given theory other constraints can exist on M_t in order to have the theory be consistent, or perturbatively unifiable, or have an appropriate minimum for the Higgs potential. None of these constraints applies independent of the assumptions of the particular theory in question. If the constraints were violated in some theory when M_t is known, it would exclude that theory. One of the earliest of these is Cabibbo et al., ref. 15. A useful review is that of Sher, ref. 16; ref. 17 is a recent paper on cosmological constraints, from which the literature can be traced. Since these approaches are more relevant to constructing theories than to M_t itself, we will not pursue them further here.

SM TOP DECAYS, SIGNATURES

Now we turn to top quark decays and signatures. First, we examine the SM behavior of top,[18] and then we can consider some possible non-SM behaviors. If top decays are (or are dominantly) SM, then the experimental lower limit of 89 GeV is valid so we assume here that the dominant decay is the two-body process $t \to W^+ b$.

The SM top decays are

(a) $\Gamma_b \sim | V_{tb} |^2 \sim 1$



(b) $\Gamma_s \sim | V_{ts} |^2 \sim | V_{bc} |^2 \sim 0.0025$

(c) $\Gamma_d \sim | V_{td} |^2 \sim 10^{-4}$

The approximate sizes are given for comparison purposes. Γ_s has too small a branching ratio for a measurement at FNAL (either in top decays or by production from an s-quark) but may be measurable at future high luminosity hadron colliders or at NLC. Γ_d is probably too small and has too difficult a signature to ever directly measure it, but once M_t is known and other parameters are better fixed it is fairly well determined by $B^\circ - \bar{B}^\circ$ mixing; the 10^{-4} estimate comes from the $B^\circ - \bar{B}^\circ$ mixing approximate result $M_t \mid V_{td} \mid \simeq 1$ GeV with M_t set at 100 GeV (see below).

Consider the W decay. In the W rest frame longitudinally polarized W's, W_L, have decay distribution

$$W_L \sim \sin^2\theta,$$

and transversely polarized W's, $W_\pm$, have decay distribution

$$W_\pm \sim (1 \mp \cos\theta)^2,$$

where θ is the angle from one of the fermions from W decay to the direction the W was traveling.

Suppose we set $M_b = 0$ so the b-quark must be left-handed for a SM t decay. Consider the decay in the t rest frame, and quantize along the direction of $b - W$ motion.

If the t helicity initially pointed in the direction of W motion, and the b helicity does the same because it is left-handed, then the W must have helicity zero, *i.e.*, be longitudinally polarized, W_L. If the t helicity initially pointed in the direction of b motion, then the W must have its helicity in the same direction since the b helicity is still left-handed. Then the W is left-handed, which we denote W_-. There is a unique W polarization for each t polarization. Neither t helicity gives W_+, so that is a SM prediction. There are only two independent helicity amplitudes, one for W_L and one for W_-.

Define in general

$$\begin{aligned} \Gamma_L &= \text{ decay width to } W_L \\ \Gamma_+ &= \text{ decay width to } W_+ \\ \Gamma_- &= \text{ decay width to } W_- \end{aligned}$$

so

$$\begin{aligned} \Gamma_T &= \text{ decay width to transverse } W\text{'s} = \Gamma_+ + \Gamma_- \\ \Gamma &= \text{ total decay width} = \Gamma_T + \Gamma_L . \end{aligned}$$

Then the angular distribution of W decay for the case $M_b = 0$ will be the longitudinal width times the longitudinal angular distribution plus the transverse width times the transverse distribution,

$$\begin{aligned} \frac{d\Gamma}{d\cos\theta} &\sim \Gamma_L \sin^2\theta + \Gamma_-\,(1+\cos\theta)^2 \\ &= \Gamma\left\{1 + \frac{2\Gamma_-}{\Gamma}\cos\theta + \frac{\Gamma_- - \Gamma_L}{\Gamma}\cos^2\theta\right\} \end{aligned}$$

ignoring an overall numerical factor. In the general case where either $M_b \neq 0$ or non-SM interaction terms are allowed, in the second terms make the replacement $\Gamma_- \longrightarrow \Gamma_- - \Gamma_+$, and in the third term $\Gamma_- \longrightarrow \Gamma_- + \Gamma_+$.

The full SM width for $t \longrightarrow bW$ is

$$\begin{aligned} \Gamma &= \frac{G_F}{8\pi\sqrt{2}}\left(M_t^2 - M_W^2\right)^3 \left\{1 - \frac{3M_W^2}{M_t^2 - M_W^2} + \frac{6M_W^2 M_t^2}{\left(M_t^2 - M_W^2\right)^2}\right\} \\ &\approx \frac{G_f M_t^3}{8\pi\sqrt{2}}\left(1 - \frac{M_W^2}{M_t^2}\right)^2\left(1 + \frac{2M_W^2}{M_t^2}\right) \\ &\xrightarrow[M_t \gg M_W]{} \frac{G_F\,M_t^3}{8\pi\sqrt{2}} \\ &\simeq 170\text{ MeV}\left(\frac{M_t}{M_W}\right)^3 . \end{aligned}$$

The last two forms are not a good numerical approximation for M_t a little larger than M_W, but do indicate the general structure.

Finally, the t-quark could be produced polarized transverse to the production $(x - Z)$ plane,[19,20] with polarization P_T^t in the $+y$ direction, in which case the partial widths become, still for $M_b = 0$,

$$\begin{aligned} \frac{4\pi d\Gamma_L}{\Gamma d\Omega} &= \frac{M_t^2/M_W^2}{2 + M_t^2/M_W^2}\left\{1 - P_T^t \sin\theta_b\ \sin\varphi_b\right\}, \\ \frac{4\pi d\Gamma_-}{\Gamma d\Omega} &= \frac{2}{2 + M_t^2/M_W^2}\left\{1 + P_T^t \sin\theta_b\ \sin\varphi_b\right\}, \\ d\Gamma_+ &= 0. \end{aligned}$$

Here θ_b, φ_b are the angles of the b-quark direction in the t rest frame. P_T^t is the polarization of t in the direction of $\overrightarrow{\text{beam}} \times \vec{t}$ where t is produced by a beam

— antibeam interaction (e.g., $g+\bar{g} \longrightarrow t+\bar{t}$, or $e^+e^- \longrightarrow t+\bar{t}$). Note the ratio Γ_L/Γ_T grows with M_t as $M_t^2/2M_W^2$. The above formulas are calculated by writing the matrix element for each t and W helicity and adding the squares of appropriate ones. QCD and electroweak corrections to top decays have been studied.[21,22] The corrections to the total width including soft gluons are about -8%, a small effect, but the corrections to the tree level decay rate are large, as much as 80% for $M_t \to 200$ GeV; they are sensitive to the cutoff on extra jet energy and to M_t/M_b. Thus, ref. 22 argues that the fraction of top decays that have only $W + b$ and no gluon jets should be small.

PRODUCTION OF t

e^+e^- Colliders

(a) At LEP200 a SM top quark is at best of marginal interest. If $M_t \geq 95$ GeV its cross section will be quite small there, and it will already have been discovered at FNAL in 1991 before LEP can increase its energy. However, if t is hidden because of non-SM decays then LEP could still be a top factory. The region 60 GeV $\leq M_t \leq$ 85 GeV will remain a possibility for some time, until either (i) top is found at FNAL, or (ii) this region is explored at LEP.

(b) The NLC, by which we mean an e^+e^- collider with

$$400 \text{ GeV} \leq \sqrt{s} \leq 500 \text{ GeV}$$

and

$$L \geq 3 \times 10^{32} \text{ cm}^{-2} \text{ sec}^{-1}$$

is a very powerful device for top quark physics. First, if top is not found at FNAL, NLC is guaranteed to detect it however it decays, or to have shown that the limit of about 200 GeV discussed above is exceeded and therefore new physics exists. Second, whether top is first found at FNAL or not, NLC can be a real top factory, studying t in depth, including some kinds of analyses that probably cannot be done at hadron colliders (see below).

As an indication of the rate the cross section for $t\bar{t}$ production when $M_t^2 \gg M_Z^2$ is $\sigma(t\bar{t}) \simeq 2.1\ \sigma_{\text{point}} \simeq 2.1 \times 4\pi\alpha^2/3s \simeq 180\text{fb}/s(\text{TeV}^2)$.

Hadron Colliders

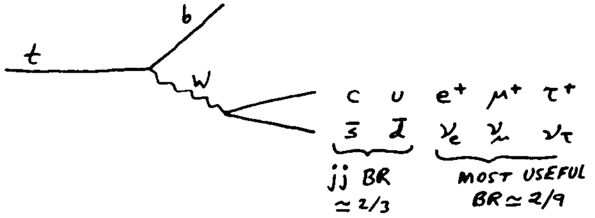
The cross section for $gg \to t\bar{t}$ including some higher order effects has been calculated by Dawson, Ellis, and Nason;[23] it is now rather well known theoretically, and may be the most accurate way to determine M_t. At FNAL it is about 100 pb for $M_t = 100$ GeV, and falls almost as M_t^{-5}.

At FNAL an integrated luminosity of about 30 pb^{-1} should[24] allow detecting t if M_t is less than about 145 GeV and decays as a SM particle; the next run, starting about August 1991, should accumulate that much data. A search to 200 GeV will take about 130 pb^{-1}. A SM top quark can be detected at FNAL in the next few years up to about 220 GeV in mass if sufficient funds are input to upgrade the intensity (and energy to 2 TeV). As discussed above, if none is found in that range we know new physics exists on the weak scale. But it is by no means settled that the necessary funds will be available to carry out the needed luminosity upgrades.

By 2000 we should have collisions at both LHC at CERN, and SSC. Both will produce over 10^8 t-quarks in a year of 10^7 sec at $L = 10^{33}$ cm^{-2} sec^{-1} if $M_t \leq 200$ GeV, with SSC producing about three times more than LHC. Detecting a SM top will not be difficult, and because of the large event rate some non-SM top decays could be detected; careful simulations will be needed because the EW/QCD backgrounds to non-SM decays are never negligible. In general the non-SM modes will be studied by tagging one top with $t \to Wb$, $W \to \ell\nu$, and examining the decay of the associated $\bar{t}$.

DETECTING SM TOPS — SIGNATURES

If SM decays dominate, almost all tops will decay as shown,



The best signature comes from semileptonic decays, so

$$t + \bar{t} \longrightarrow \ell^+ + \ell^- + b + \bar{b} + \text{missing momentum.}$$

When $\ell^+\ell^- = \mu^+e^-$ or μ^-e^+, there is essentially no background if leptons are isolated, so even two events could indicate a signal. Unfortunately, the branching fraction suppression is about a factor of 40, so large cross sections are needed. To keep enthusiasm up, the CDF group has reported one such event, with only a fraction of an event expected from background.

If a signal is claimed in these modes, it must be accompanied by e^+e^- and $\mu^+\mu^-$ events, as well as by the signatures discussed below, so it will not be easy to fake.

Another signature that is useful and has a larger rate triggers by having t or $\bar{t}$ decay to $\ell^{\pm} + \nu +$ jet(s), and the other to jets. This is 32% of all decays. There is more background, but the background is calculable and independently measurable. When a cut is made insisting that three or more of the jets are hard, the background is not expected to be a problem.

Both methods will work better with vertex detection to tag b's, since most background has u, d, s, g jets. The CDF detector will have such vertex detection in the next run at FNAL. On the basis of searches for a top quark using the above techniques, the CDF group has published[1] the result that for a SM top quark,

$$M_t \ > \ 89 \text{ GeV}$$

at 90% CL. This result can be marginally improved over the next year as further decay modes are incorporated.

The signatures discussed so far are rather straightforward. Other interesting distributions can be used as well, both for searching and as confirmation of a signal. For example, as Barger & Phillips[25] have emphasized, the main background source of hard leptons is $b\bar{b}$ events, but at FNAL the leptons from $b\bar{b}$ tend to be back-to-back with each other, while the leptons from $t\bar{t}$ are essentially isotropic because the tops are heavy and their decays are essentially uncorrelated. If $M_t \simeq M_W$ special effects could occur; see for example ref 26.

MODEL-INDEPENDENT LOWER LIMIT ON M_t

Many people have noted that W decays can be used to observe t or to put a lower limit on M_t. If t is lighter than $M_W - M_b$, then $W \to t + \bar{b}$. Since

$$BR(W \to e\nu) = \frac{\Gamma(W \to e\nu)}{\Gamma(W \to e\nu) + \Gamma(W \to \mu\nu) + \ldots + \Gamma(W \to t\bar{b})},$$

a measurement of $BR(W \to e\nu)$ combined with the use of SM values for the other partial widths, allows a limit to be set on $\Gamma(W \to t + b)$, which in turn sets a lower limit on M_t. Numerically, Γ_W decreases by 20% as M_t increases from 45 GeV to $M_W - M_b$. The theory value can be calculated to better than a percent accuracy.

In practice what is measured is $\sigma(W) \cdot BR(W \to e\nu)$ so the ability to calculate $\sigma(W)$ enters.[27] So far UA2 and CDF have proceeded by taking ratios of $\sigma(W)BR(W \to e\nu)$ and $\sigma(Z)BR(Z \to ee)$, which eliminates many systematic and calculational errors. The best result is presently $M_t > 41$ GeV (90% CL) from CDF, based solely on the electron channels. It turns out that the size of the errors is determined by the limited number of Z°'s. As the $\mu\nu$ and $\mu^+\mu^-$ channels get included the limit will rise to nearly 50 GeV if a positive signal is not seen. With the integrated luminosity of the 1991 data the limit could get to 60 GeV. In ref. 28 it is argued that combining all relevant data from different experiments pushes the CDF 41 GeV limit up to 52 GeV already.

These limits do not depend at all on how t decays, so they could see an effect that was very difficult to detect directly if t had dominant non-SM decays, as we discuss shortly. Pursuing such limits (or seeing an effect) will be very important until a top quark is found.

DETERMINING THE CHARGE OF A NEW HEAVY QUARK[29]

Suppose a new heavy quark is discovered. Can we decide if it has $q = 2/3$ or $q = 1/3$ (or —)? In principle, they have different ΔR values, but the situation is not so simple as at low energies, because (i) $\Delta R/R \ll 1$, and (ii) $\Delta R_{2/3} \simeq \Delta R_{-1/3}$. It turns out that a good way to decide is available, the analysis of the forward/backward asymmetry in the semileptonic decay. The differential cross section is of the form

$$\frac{d\sigma}{d\cos\theta}\left(f\bar{f} \xrightarrow[\gamma,Z]{} Q\bar{Q}\right) = A + B\cos^2\theta + C\cos\theta$$

where $\cos\theta = \hat{p}_f \cdot \hat{p}_Q$. The coefficient C determines the asymmetry and it is (V, A are the vector and axial vector couplings of f, Q)

$$\begin{aligned} C &\sim 2V_f A_f V_Q A_Q \mid X \mid^2 + e_Q e_f A_Q A_f ReX \\ X &= s/\left(s - M_Z^2 + iM_Z\Gamma_Z\right). \end{aligned}$$

If we interchange a t and a b' quark, for example, all of V_Q, A_Q, e_q change sign so C does not change sign, and t and b' have the same forward/backward asymmetry. The size of the asymmetry changes a little, but it is hard to measure accurately. However, when they decay

$$\begin{aligned} t &\longrightarrow \ell^+\nu X \\ b' &\longrightarrow \ell^-\bar{\nu} X. \end{aligned}$$

So the lepton forward/backward symmetry is opposite and allows us to easily distinguish the two cases, at hadron or electron colliders.

TOP DECAYS — A WINDOW TO NEW PHYSICS?

Since the top quark is heavy, it could have decays to new objects that have not been detected other ways, or major non-SM decays to conventional particles. If these new decays dominate they could have signatures very different from the SM ones discussed above, so that the reported limit $M_t >$ 89 GeV would not apply. The 41 GeV limit still applies. Or the new decays could be rare ones that would not affect the limits, but would be exciting new physics if detected.

If in fact $M_t < M_W + M_b$ and t is decaying in a non-SM way, then the non-SM decay could dominate if it were a two-body decay with large phase space compared to the SM decay via a virtual W. But if $M_t > M_W + M_b$, then the decay $t \to W + b$ is also two-body and is full strength electroweak, so it will never be negligible, and the signature from $t \to W + b$ can always be used to put strong lower limits on M_t. Thus at present the region

$$41 \text{ GeV} < M_t < 87 \text{ GeV}$$

is NOT excluded. After the full analysis of the next FNAL run, if no top quark is found the approximate model independent allowed values for M_t will be

$$60 \text{ GeV} \leq M_t \leq 87 \text{ GeV}$$

and

$$M_t \geq 140 \text{ GeV};$$

the first of these will be excluded for a SM top but not for a general top.

Let us examine some possible non-SM top decays. Our approach is not that these will or should occur, but that they do occur in interesting, reasonable extensions of the SM and therefore they might occur in reality. They should be taken seriously. These possibilities are not new; most of them have been discussed for some number of years.

Supersymmetric decays

If nature is supersymmetric the left-handed and right-handed top quarks have supersymmetric partners $\tilde{t}_L$ and $\tilde{t}_R$. These electroweak eigenstates give rise to mass eigenstates $\tilde{t}_1$ and $\tilde{t}_2$ after a rotation. Models exist in which one of the mass eigenstates is lighter than the t either because of typical mixing level repulsion effects, or in some cases because the masses of the partners are determined for all fermions by a result such as

$$\widetilde{M}_f^{\,2} = \widetilde{M}^2 - M_f^2,$$

where $\widetilde{M}$ is a general mass parameter. Then since M_t is large, if $M_t \sim \widetilde{M}$ it can happen that $\widetilde{M}_t < M_t$ while all other $\widetilde{M}_q \sim \widetilde{M}$ are large.

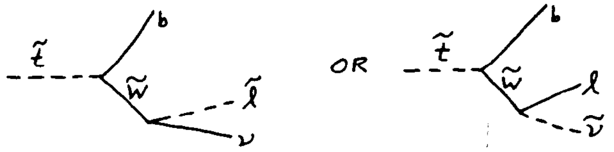
In that case the decay

$$t \longrightarrow \tilde{t}_1 + \text{LSP}$$

is allowed, where LSP is the lightest supersymmetric partner. The LSP could be a photino, but it can more generally be a combination of gauginos. These decays have been discussed as a possibility since 1983. See ref. 31 for more recent analysis and references.

For example, if $M_t = 75$ GeV and $\widetilde{M}_{t_1} = 60$ GeV and $M_{\text{LSP}} = 10$ GeV this decay is not excluded by any data. Limits on squark masses have been published that are larger than 60 GeV, but they assume[32] all twelve mass eigenstates are degenerate. Up-type squarks with $\widetilde{M} < 42$ GeV and down-type squarks with $\widetilde{M} < 43$ GeV have been excluded.[33] The limits would be significantly smaller for stops of one chirality. Eventually LEP200 could detect or exclude stops up to the needed 82 GeV, and possibly it could be done at FNAL. The expected branching ratio is very model dependent. Since it is effectively a neutral current decay most calculations give $\Gamma(t \to \tilde{t} + \text{LSP})/\Gamma(t \to Wb)$ proportional to $\sin^2\theta_W$ times a mass dependent factor that can be of order unity once $M_t > M_W + M_b$.

If $t \to \tilde{t} +$ LSP occurs the next question is how $\tilde{t}$ decays. That is model dependent.[34,30] If $\tilde{\nu}$ and $\tilde{\ell}^{\pm}$ are light, then

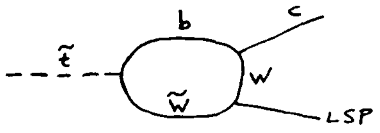


could dominate; $\tilde{\ell} \to \ell + \tilde{\gamma}$. The final state in both cases is $b + \ell^{\pm} +$ missing momentum. The final state from $t + \bar{t}$ is then $b + \bar{b} + \ell^{+} + \ell^{-} +$ several escaping LSP's and ν's. While this can be analyzed, and is a possibility, it is not favored in some models.

If $\tilde{\nu}$ and $\tilde{\ell}$ are both heavier than $\tilde{t}$, this becomes a 4-body decay and it is severely suppressed by 4-body phase space. Then it is likely that the 2-body decay

$$\tilde{t} \longrightarrow c + \text{LSP}$$

dominates, by one loop,



Then $t+\bar{t}$ gives $c+\bar{c}+$ two LSP's, also a signature that can be examined. See ref. 34 for study of $\tilde{t} \to c+$ LSP.

The CDF and D0 detector groups could search for $t \to \tilde{t}+$ LSP in the range 41 GeV $< M_t <$ 87 GeV and detect it or exclude it over much of the parameter range. If t is not discovered as a SM particle in the 1991 data, this will be an important analysis.

A decay which is perhaps not so likely but should be kept in mind once top is detected is $t \to \widetilde{W} + \tilde{b}$. If $\widetilde{W}, \tilde{b}$ are light enough this would be a copious source.

Decay to charged Higgs bosons

Another possible non-SM top decay that has been considered seriously at least since 1979[35] is $t \longrightarrow b + H^+$ where H^+ is a charged Higgs boson that occurs when more than one (non-singlet) Higgs representation is included in the theory. H^+ must exist in a supersymmetric world, and in many others.

Again there are two regions. Z° decays[36] tell us that $m_{H^\pm} \geq 35.4$-43 GeV depending on decay modes, and we assume that if $M_H^\pm \leq 45$ GeV it will be detected at LEP, so if $t \to b + H^+$ is dominant then

$$50 \text{ GeV} \leq M_t \leq 87 \text{ GeV}$$

and t was not observed so far because the signature of this decay is unlike the SM signatures. In this region the H^+ decays of significance are

$$H^+ \longrightarrow c\bar{s},\ \tau\nu_\tau,\ c\bar{b}.$$

Because of the Higgs origin of H^+ it will have a fermion mass factor, and a factor of 3 for color for quark channels. A factor V_{cb} from the KM matrix is expected to suppress the $c\bar{b}$ decay. All of these factors are model dependent. There will also be a factor of the ratio of the two vacuum expectation values if H^+ arises in a two doublet world. If $t \to H^+ b$ is the only two-body top decay it will dominate regardless of details, giving almost all of the decays. Then how H^+ decays is relevant for its signature, and one finds approximately in the SUSY-like models,[37]

$$\frac{BR\left(H^+ \to \tau^+\nu\right)}{BR(H^+ \to c\bar{s})} \approx$$
$$\frac{P_\tau}{3P_c}\, \frac{M_\tau^2 \tan^2\beta \left(M_{H^+}^2 - M_\tau^2\right)}{\left(M_s^2 \tan^2\beta + M_c^2 \cot^2\beta\right)\left(M_{H^+}^2 - M_c^2\right) - 4M_c^2 M_s^2 \tan\beta \cot\beta},$$

where $\tan\beta$ is the ratio of vacuum expectation values, and P_τ and P_c are the τ and c momenta. If $M_t/M_b \gg 1$ is associated with the vacuum expectation values then $\tan\beta > 1$. Searches for

$$t \longrightarrow H^+ b,\ H^+ \to c\bar{s} \text{ or } \tau\nu$$

can detect or exclude this as the dominant t decay.

If $M_t > M_W + M_b$, then $t \to W + b$ is also two-body and never negligible. In SUSY-like models,[37]

$$\frac{BR\left(t \to H^+ b\right)}{BR(t \to W^+ b)} \approx$$
$$\frac{P_{H^+}}{P_{W^+}}\, \frac{\left(M_b^2 + M_t^2 - M_H^2\right)\left(M_b^2 \tan^2\beta + M_t^2 \cot^2\beta\right) + 4M_b^2 M_t^2 \tan\beta \cot\beta}{M_W^2\left(M_t^2 + M_b^2 - 2M_W^2\right) + \left(M_t^2 - M_b^2\right)^2}$$
$$\xrightarrow[M_t^2 \gg M_W^2,\, M_H^2]{} \cot^2\beta$$

which is of order 0.1 if M_H is not too different from M_W, unless $\tan\beta$ is very different from unity; for small $\tan\beta$ this ratio gets large compared to unity. Thus once $M_t > M_W + M_b$, top will be detected by its SM mode, and the new physics mode can be seen by a careful study of events when sufficient statistics are in hand.

Once $M_{H^+} > M_W$ additional possible decays arise. In any model the mode

$$H^+ \longrightarrow W^+ + h^\circ$$

will be large if allowed,[37,38] where h° is the lightest of the neutral Higgs bosons. This is a typical situation in supersymmetric worlds, where $M_{H^+} > M_W$ is required in minimal theories and is generally true, and where h° is usually not heavy. Then this is the dominant H^+ decay. Normally $h^\circ \to b\bar{b}$. H^+ can also have large decays to SUSY partners or other new particles. Once top is found, if $M_t > M_W + M_b$ it will be very important to search for new top decays such as these signatures, at the few per cent level.

Decays to heavy quarks

If additional heavy quarks exist, it can happen that (say) a new quark with electric charge -1/3, b', exists. Then

$$t \longrightarrow b'\ell^+\nu$$

could occur, or be important. The heavy quark b' could be from a fourth family, or the $SU(2)$ singlet from an E_6 family, or something else. If this occurs the ℓ^+ and ν are softer, and the published limits on t are not right. The decays of b' are model dependent, but most likely $b' \to c\ell\nu,\ cq'\bar{q}$.

Flavor-changing neutral current decays

A possible set of decays of great importance are

$$\begin{aligned} t &\longrightarrow c+g, \\ &\longrightarrow c+\gamma, \\ &\longrightarrow c+Z, \\ &\longrightarrow c+h^\circ. \end{aligned}$$

In theories with additional particles that are not in $SU(2)$ doublets and that can mix with t, these can arise at tree level and in principle could be large or even dominant. Except for $c+g$ good signatures exist, so these could be detected. The decay of h° is dominantly $b\bar{b}$, so enhanced capability of b detection would probably be necessary for finding this mode.

More likely, perhaps, is finding these decays as rare branching ratios. None of them occur at detectable levels in the SM, so detecting them at all would be a breakthrough into the new physics world. They can occur from mixing of t with a heavier object, or from loop effects.[39] As above, if t is not too heavy their two-body phase space can enhance them relative to the SM decays, while once $t \to W + b$ is fully open the flavor changing neutral decays are likely to be small.[40]

Other possibilities

The decays discussed above are the most likely non-SM top decays from the point of view of today's ideas, but by no means the only possibilities. Decays to other objects from unified models can be written, for example. If the top quark is not found the possibility should be taken very seriously that it is decaying in a non-SM way. If it is found dominantly decaying to $W+b$, the second top in each event should be carefully studied as a window for small non-SM modes.

NEW INFORMATION ONCE TOP IS OBSERVED

If top is seen with only SM decays one can immediately conclude that $M_{H^+} > M_t - M_b, M_{\tilde{t}} + M_{\text{LSP}} > M_t$, etc. Every model will be constrained by several such relations.

STUDYING THE TOP DECAY COUPLINGS[29]

Suppose we produce a new quark. How can we tell if its decay is a normal $V-A$ one or if new interactions are present? For example, many theories require the presence of "mirror fermions" that decay to lighter fermions via a $V+A$ interaction.

Note that this question is applicable to the t-quark, even though the arguments given above that b-quark decays and interactions require the existence of a t-quark imply that the b and t have the same space-time properties, and there is already evidence that the b has $V-A$ decays, because there could be a small $V+A$ (or other) interaction whose effect grows with mass scale, e.g., as M_q^2/Λ^2.

The answer is that one good way to tell is from the semi-leptonic decay distributions looked at on a Dalitz plot. A similar argument would hold for the b-quark. Consider an $e_q = 2/3$ quark, decaying in its rest frame, $Q \to q\ell\nu$, and assume all final state masses can be neglected. Then we can make a table,

	$V-A$	$V+A$
$\mid M\mid^2 \sim$	$(P_Q \cdot P_\ell)(P_q \cdot P_\nu)$	$(P_Q \cdot P_\nu)(P_q \cdot P_\ell)$
$d\Gamma/dE_j dE_\ell \sim$	$E_\ell(m_Q/2 - E_\ell)$	$E_\nu(m_Q/2 - E_\nu)$
		$E_\nu = m_Q - E_j - E_\ell$

where j stands for the quark jet. Then the Dalitz plots look very different. For $V-A$ lines are dense near the center, while for $V+A$ they are dense in a band that does not go through the center. Distinguishing in practice would not be difficult.

For an $e_Q = -1/3$ quark, $V-A \leftrightarrow V+A$ relative to a top quark. These arguments could be applied to b decay too. For a new quark the charge may have to be determined by the forward/backward asymmetry of the lepton in the semileptonic decay, as discussed a few sections above.

TOP QUARK AT NLC

The top quark physics that could be done at NLC is a major justification for constructing such a facility (only one of several such justifications).

If top is not found at FNAL, then NLC would either find it or prove it is heavier than about 250 GeV, in which case new physics must be present on the electroweak scale as discussed above. The advantage NLC would have over FNAL in this situation is the simplicity of e^+e^- collisions. If top is not found at FNAL it could either be that it is too heavy or that it is decaying in a way not examined by the CDF and D0 detectors, while at NLC all decays could be seen. Thus NLC would be guaranteed a major result either way.

If top is found at FNAL it can still be studied in unique ways at NLC.

1. Peskin and Strassler[41] have recently shown that M_t can be measured to better than one GeV from the threshold behavior of $t\bar{t}$ production, and that the top width can be measured to 25% or better accuracy. One should ask how well we need to know M_t. It will be measured to ± 10 GeV at hadron colliders (some analyses claim ± 7 GeV).But a simple argument suggests ± 1 GeV will eventually be needed. That comes from looking at a graph of the left-right polarization asymmetry at e^+e^- colliders vs. M_t (e.g., fig. 32 of ref. 42). Over the range of interest in M_t, A_{LR} changes by about 0.04 as M_t changes by about 150 GeV, and the change is approximately linear, so $\Delta A_{LR}/\Delta M_t \simeq 0.0003$ GeV^{-1}. SLD experimenters have argued that ultimately they hope to measure A_{LR} to 1% of its value, about ±0.0013. If this measurement is to be a new constraint on the theory, the uncertainty in A_{LR} due to our (lack of) knowledge of M_t must be significantly smaller than 0.0013, so ΔM_t should be less than about 2 GeV, i.e., ±1 GeV. Thus a measurement of M_t to considerably greater accuracy than is possible at hadron colliders will eventually be needed. To put it differently, if $\Delta M_t \simeq 20$ GeV, then one cannot interpret a measurement of A_{LR} to better than about 5% accuracy as a constraint on the theory, though it could be used to extract a better value for M_t.
2. At NLC, depending on details of M_t, energy, and luminosity, rare decays of top can be searched for down to a branching ratio of about 10^{-4}. For some modes it should be possible to do better at SSC/LHC, but a systematic and general search will only be possible at an electron collider.

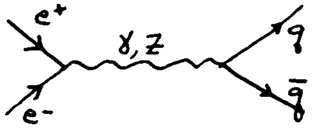
TESTING QCD POLARIZATION PREDICTIONS

As noted in the introduction, heavy tops decay so quickly that they do not form hadrons before decaying. They effectively decay as free quarks. That gives us the possibility of testing some interesting QCD predictions.

Consider the polarization transverse to the production plane,[19,20] of a quark. For example, in

$$e^+e^- \to q\,\bar{q}$$

the dominant, tree level process is

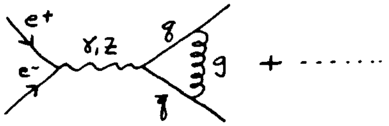


If the amplitude is $M_{\lambda'\lambda}$, with $\lambda', \lambda = \pm$ representing the quark spins, the transverse polarization is

$$P_q^T = 2Im\,(M_{++}M^*_{+-})\,/\,(|\,M_{++}\,|^2 + |\,M_{+-}\,|^2)\,.$$

At tree level this is zero because the amplitudes are relatively real. In addition the spin flip amplitude is proportional to quark mass. Thus at tree level QCD predicts that quarks are produced with zero transverse polarization.

By adding the one-loop gluon contribution



an imaginary part is introduced into the amplitude. Then the polarization is predicted to be

$$P_q^T = \frac{4}{3}\alpha_s\frac{M_q}{\sqrt{s}}\,\frac{\sin\theta\cos\theta}{1+\cos^2\theta}\ +\dots$$

where the first term is the γ contribution,[19] and θ is the production angle of q.

This is obviously numerically negligible for all quarks but top; even for top it is at most about $\alpha_s/2 \simeq 0.06$, which is very hard to see. Testing this prediction tests the loop corrections and helicity structure of QCD in new ways.

Before discussing the experimental tests, we note the calculation is interesting. The loop-amplitude can be written[19]

$$M_{\mathrm{LOOP}} = BI + R$$

where B is the tree diagram, and I is a spin-independent integral that is infrared divergent from the diagram shown, and becomes finite by cancellations from other diagrams. R is infrared finite and spin dependent. Since the polarization arises by interfering the loop contribution with the Born term B, no polarization can arise from the term BI, which therefore need not be calculated. The procedure is considerably simplified by only needing to determine R_{++}, R_{+-}.

The form of the result for P_q^T is clear. The α_s arises from the gluon and the $M_q/\sqrt{s}$ from the helicity flip. They must be multiplied by a number of order unity and by an angular distribution.

In general it is not clear how to measure a quark polarization. There are probably two unambiguous possibilities. First, since QCD says a quark has at most a very small polarization, the only direction that can be associated with a quark jet is its momentum. If any non-zero value can be found for a transverse direction, or an observable such as

$$\sum_{\substack{\text{hadrons } h \\ \text{in jet}}} f(E_h)\vec{P}_h \cdot \left(\vec{P}_{\text{beam}} \times \vec{P}_{\text{jet}}\right)$$

for some weighting function $f(E_h)$, then the QCD prediction is violated. For $f(E_h) = 1$ conservation of momentum guarantees no effect, but choices such as $f(E_h) = E_h$ could be considered. So far no one has tried this analysis.

The second possibility for testing the prediction, which is why it is in these notes, is that the top quark decays as a free quark if it is heavy enough, and once it is heavy enough to decay to $W + b$ it easily analyzes its own polarization in the decay. In a paper[20] to be published we will explain in detail how to measure P_t^T and test the QCD prediction at NLC and at hadron colliders.

The top polarization measurement is of even more general interest. First, although we have presented the result for e^+e^- as is suitable for lectures at SLAC, of course the same tests can be made at hadron colliders such as SSC, LHC. Too few tops will be produced at FNAL for a precise test, though the symmetry tests mentioned below should be carried out there. At a hadron collider the dominant production process are $gg \to t\bar{t}$ and $q\bar{q} \to t\bar{t}$. The former dominates for lighter M_t or at the SSC, while the latter dominates for $M_t > 100$ GeV at FNAL. The quark polarization for $gg \to t\bar{t}$ has been calculated in ref. 43. The predictions for FNAL, LHC, and SSC are presented in ref. 20. The top quark still analyzes its polarization by its decay to $W + b$.

Second, there are no tests of symmetries in high energy collisions. If parity or CP were violated, even maximally, in collisions at FNAL, how would we know? In general it is very hard to find observables that cleanly allow such tests, and the top polarization analyzed by $t \to W + b$ will be a very good one.

CORRELATION OF TOP SPIN DIRECTION WITH FINAL b, ℓ^+ DIRECTIONS[20]

Consider $t(p) \to b(p') + \ell^+(\ell) + \nu(k)$ where momenta are shown in parentheses. The matrix element is

$$M \sim \left(\bar{u}(p')\gamma_\lambda P_L u(p)\right)\left(\bar{u}(k)\gamma^\lambda P_L v(\ell)\right)$$

so the width is proportional to

$$\begin{aligned} d\Gamma &\sim Tr\left[\gamma \cdot p'\gamma_\lambda P_L\,(\gamma \cdot P + m_t)\,(1 + \gamma_5\gamma \cdot s)\,\gamma_\sigma P_L\right] Tr\left[\gamma \cdot k\lambda^\gamma P_L \gamma \cdot \ell\gamma^\sigma P_L\right] \\ &\sim p' \cdot k\,(p \cdot \ell - m_t s \cdot \ell)\,, \end{aligned}$$

where s is the top spin four-vector. In the t rest frame, $s = (0, \vec{s}), p = (m_t, \vec{0})$, and $\ell_0 = |\vec{\ell}|$ so

$$d\Gamma \sim m_t E_\ell \left(1 + \vec{s} \cdot \hat{\ell}\right) \sim 1 + \cos\theta_{s\ell}$$

where $\hat{\ell}$ is a unit vector in the direction of $\vec{\ell}$ so $\theta_{s\ell}$ is the angle between the lepton momentum and the t spin directions. This result assumes 100% polarization of the present quark, but is otherwise general. It is different from the result for muon decay, when the $\vec{s} \cdot \hat{\ell}$ correlation depends on the lepton energy (vanishing when $E_\ell = M_\mu/4$). This result can have important implications, since it implies a strong correlation, with more leptons emitted in the direction of the t spin, the rate going to zero when the lepton is antiparallel to the t spin.

For a $\bar{t}$ the appropriate projection operator is

$$(\gamma \cdot p - m)\ (1 + \gamma_5\gamma \cdot s)$$

so effectively the sign of m changes, and the correlation is

$$1 - \hat{s} \cdot \hat{\ell}^-.$$

In any particular application it is necessary to check whether both t and $\bar{t}$

are present and whether the effects can cancel if charges are not measured or t, $\bar{t}$ cannot be distinguished.

MEASURING M_t FROM LOOPS

As we discussed above, some of the most precisely measured observables depend on M_t, and requiring a consistent set of results fixes M_t to be between about 100 GeV and 180 GeV. As increasingly precise measurements of M_W, M_Z, $\Gamma(Z \to b\bar{b})$, and other quantities are made, M_t will be determined with increasing accuracy, following the original idea of Veltman.

But if M_t is only determined indirectly in this way how sure can we be that some new physics is not sneaking in and biasing the result? With only one measurement it is indeed not difficult to affect the result, as we mentioned above.

However, on the same time scale of a few years a number of independent measurements that depend sensitively on M_t will[44] be made. Although no one has done a systematic analysis to my knowledge, it seems clear that the new physics that enters in one place will enter differently in others, so once several independent observables are available, M_t will effectively be uniquely determined, perhaps as accurately as ±15 GeV. Some of the relevant observables are:

(1) As observed by Flynn and Randall,[45] ϵ'/ϵ depends sensitively on M_t once M_t is large. Their result has been parameterized in ref. 46 as

$$\epsilon'/\epsilon \simeq (0.02 \pm 0.1) \left\{ 1 - \frac{1}{2} \left(\frac{M_t(\text{GeV})}{100} - 0.7 \right)^2 \right\} N$$

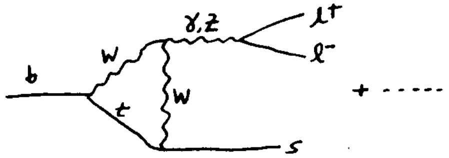
where N is a calculable number. Since this has a zero for $M_t \simeq 210$ GeV, while the bracket is of order unity for $M_t \simeq M_W$, clearly a good measurement will help. The present error in the coefficient will decrease as other parameters are better measured.

(2) $B^\circ - \overline{B}^\circ$ mixing,[47] both for B_s° and B_d°, give results approximately proportional to ${M_t}^2$ in the SM.
For example, in the minimal SM, the mixing parameter x_d, which has an experimental value of 0.7 ± 0.2, is given by

$$x_d \simeq \frac{\tau_b G_F^2 M_t^2 f_B^2 M_B}{6\pi^2} \left(V_{td}^* V_{tb}\right)^2 F(y_t)$$

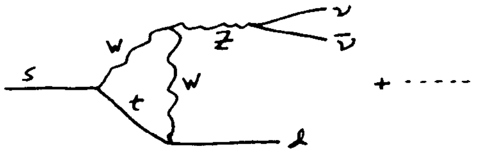
where τ_b is the b lifetime, f_B the B wavefunction at the origin, and $F(y_t)$ a slowly varying function of masses which is $F(y_t) \simeq 1$ if $y_t = M_t^2/M_W^2 \ll 1$, $F(y_t) \simeq 1/4$ if $y_t \gg 1$.

(3) The rare decay $B \to K\ell^+\ell^-$ only arises at one loop and is dominated by the top loop,



It should be detectable at CESR in 2-3 years; the expected BR is above 10^{-6}.

(4) The rare decay $K^+ \to \pi^+\nu\bar{\nu}$ is being looked for in dedicated experiment at Brookhaven. It also arises from a loop diagram,



The top contribution is similar for methods (3) and (4), but other new physics effects such as a fourth generation would not be. The SM prediction for this is $10^{-10} - 10^{-11}$, depending on M_t; it may be detectable in about three years at BNL.

(5) Also at BNL, $K_L \to \mu^+\mu^-$ has been measured to be very nearly given by just the imaginary contribution of the diagram

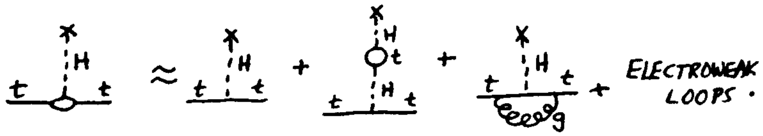
which is calculable by unitarity. The real part must also contribute, and so the loops that have t are very tightly constrained.

PREDICTION FOR M_t

Next we summarize briefly a series of analyses and predictions of the value of M_t. Many speculations exist, or course, and some may be right. Whether any are right for the right reasons is not so clear. Two related ones seem more likely to me to be correct, one corresponding to M_t being a fixed point solution of the renormalization group equations, and the second where M_t must be large enough so that supersymmetry gives us an explanation of the Higgs mechanism. However, at the present time such arguments are mainly a matter of taste, and no one has given compelling arguments as to why any of the following ideas (or any others) should be correct. We present them as a guide for the reader who wishes to study the question further.

One of the important points to note from the following examples is that many theorists have long expected M_t to be rather large, typically $M_t \gtrsim \frac{3}{2}M_W$. Certanly that is the case since the mid-1980's. On the other hand, our fundamental understanding of fermion masses is so weak that no one was sure M_t would be large.

(1) One of the earliest clear arguments for a large M_t came from Pendelton and Ross[48] in 1981, and was studied in more depth by Hill[49] later. The basic idea is that taking into account higher order effects gives for the top quark-Higgs coupling (and therefore for M_t) a result which is graphically

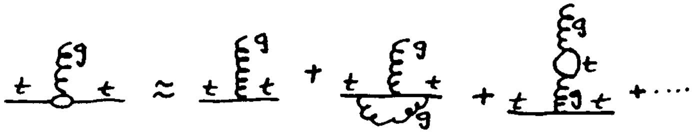


This gives an equation for $g_t = M_t/(v/\sqrt{2})$,

$$16\pi^2 \frac{dg_t}{d\ln Q} \simeq g_t \left\{ \frac{9}{2}g_t^2 - 8g_3^2 - \left(\frac{9}{4}g_2^2 + \frac{17}{12}g_1^2 \right) \right\}.$$

Here g_1, g_2, g_3 are the $U(1), SU(2)$, and $SU(3)$ couplings, v is the Higgs vacuum expectations value, and Q the momentum transfer. An equation such as this arises whenever higher order effects are taken into account, and gives the "running" couplings, *i.e.*, couplings that change with momentum transfer, because the relative size of the loop contributions changes with momentum transfer. The first term on the right comes from the diagram with a top quark loop, which has three Higgs-top couplings and therefore a g_t^3; the next term has one Higgs-top coupling so a factor g_t, and two gluon-top couplings g_3. The last term comes from the electroweak (γ, W, Z) loops.

The gluon coupling satisfies a similar equation,



which becomes

$$16\pi^2 \frac{dg_3}{d\ln Q} \simeq -b_0 g_3^3$$

where $b_0 = 11 - \frac{2}{3}n_f$. This can be used to eliminate the g_3^2 term above, and since the electroweak contributions are numerically small one can see the general behavior without them and then correct for them. This gives the basic equation

$$16\pi^2 \frac{d}{d\ln Q}\left[\ln \frac{g_t}{g_3}\right] \approx \frac{9}{2}g_t^2 - (8 - b_0)\,g_3^2$$

to solve. By inspection, if the right hand side vanishes then g_t and g_3 will simply stay in a fixed ratio as $\ln Q$ changes. Thus whatever value M_t starts with, its value at our scale is basically fixed (fixed-point solution of the first order differential equation) to be given by

$$g_t^2 = 2{M_t}^2/v^2 = \frac{2}{9}(8 - b_0)\,g_3^2$$

where $g_3^2/4\pi = \alpha_s =$ the observed QCD coupling. Putting in electroweak corrections and errors gives

$$M_t \simeq 120 - 150 \text{ GeV},$$

just in the range favored by the data. Hill and others have argued that subtleties of the analysis will increase this number, and that it is not quite so independent of the GUT one begins with as one would hope. However, a value in this range would encourage a belief that the SM will be tied to grand unification physics.[50]

(2) In 1983-1984 it was observed[51,52,53,54] that under certain conditions supersymmetry grand unified theories could explain physically the Higgs mechanism! The essential physics is simple. One writes the renormalization group equations for the change of M^2_{Higgs} with scale, and observes that it has contributions of both signs. Qualitatively,

$$\frac{dM^2_{\text{Higgs}}}{d\ln Q} \simeq -A^2M^2 + B^2g_t^2$$

where A^2 and B^2 are positive, $g_t = \sqrt{2}M_t/v$ as above, and M is a mass parameter related to the electroweak masses. Since spontaneous symmetry breaking occurs when $M^2_{\text{Higgs}} < 0$, it will occur if g_t behaves in a certain way, and that will happen if M_t is sufficiently large, larger than about 125 GeV. The coefficients are determined by the particle content of the theory, and the effect is absent for non-supersymmetric theories. Thus an M_t in the favored region encourages the idea that a real physical explanation for the Higgs mechanism would exist.

Recent studies [55] indicate that considerable care is needed before claiming this works in practice. The one-loop corrections to the Higgs potential can be significant, and there can be major scale dependence.

Of course, this view requires that supersymmetric partners and Higgs bosons be detected as well. But such an interpretation of the Higgs mechanism would not have been possible for a lighter top quark.

(3) A very interesting approach to M_t is that of Barbieri and Hall[56] (B&H). They suppose that M_t is large because of mixing with a fermion from a sector of particle physics at the TeV scale. The Yukawa couplings of the quarks and leptons could than be less different than the masses themselves, and perhaps easier to understand.

However, it is not so simple to find natural-seeming models with fermions having same quantum numbers as the t-quark except for $SU(2)$, so that when $SU(2)$ is broken they will mix, and the mass eigenstate we call top will be heavy. B&H do so by adding a gauge boson that is a color triplet, $SU(2)$ singlet. Then its supersymmetric partner is the same as a top quark except for $SU(2)$. The resulting model turns out to have some problems, but is interesting to study. Simpler models can be written to mix b-quarks with heavier particles and reinterpret M_b and the CKM angles, particularly since E_6 representations have an $SU(2)$ singlet b-type fermion, but that does not help to understand M_t. It would be good if a new kind of model could be found here.

The B&H model should be viewed, as they say, as an existence proof of a theory where the top quark is heavy even though it does not have a large Yukawa coupling. The model does have some problems, but it is clever and provocative. It only makes sense if new physics does exist at the TeV scale, *i.e.*, there is not a desert between the electroweak scale and the GUT scale.

One characteristic prediction of such approaches is flavor-changing neutral currents, since the GIM mechanism is not operating. Decays such as $t \to c + X$, $X = \gamma, g, Z^\circ, H^\circ$ all occur at "tree level" (it is not really tree level since there is a mixing present). These provide an example of the new physics decays discussed in our section above. If $t \to cH^\circ$, $H^\circ \to b\bar{b}$ were the dominant decay, then there would be about 300 six-jet events of $t\bar{t} \to c\bar{c}H^\circ H^\circ \to c\bar{c}b\bar{b}b\bar{b}$ in the existing CDF data for $M_t = 70$ GeV, $M_{H^\circ} = 50$ GeV; the fact two $b\bar{b}$ pairs have the same mass would be helpful in the signature.

(4) In the early 1980's, Veltman[57] suggested that perhaps gauge theories should be constructed to be free of quadratic divergences at the one loop level (which was all that could be examined then for technical reasons). In the SM that leads to the condition

$$M_t{}^2 = \frac{1}{4}M_H^2 + \frac{3}{4}M_W^2 + \frac{1}{4}M_W^2 \tan^2\theta_W.$$

For $M_H = M_Z$, this gives $M_t = 87$ GeV, and obviously for larger M_H, M_t increases. One cannot of course judge the correctness of a value for M_t here unless M_H is known. If this were consistent with experiment it would be an interesting point of view.

Recently, Jack and Jones[58] have examined this view more closely. There is also a two-loop condition. Is it the same as the one-loop one? They examined the situation for classes of theories. For supersymmetry the conditions are different from the SM, being satisfied at each order by cancellations among particles and their superpartners. They could not find any other interesting theory for which the one and two-loop conditions were both satisfied. In particular, for the minimal SM the one-loop and the two-loop equations give two equations for the two masses M_t, M_H; unfortunately the two equations have no solution (and such different trajectories in the M_t, M_H plane that one is clearly not missing a solution in a slightly extended theory).

Apparently this point of view is not turning out to be fruitful. It is being pursued further by the authors of ref. 58, 59 to see if interesting theories can be found that automatically satisfy higher loop conditions once they satisfy the one-loop condition.

(5) Recently several authors[60,61,62,63] have speculated that the large value of M_t may cause, or be a manifestation of, a dynamical breaking of the electroweak symmetry. The mechanism involved would be some as-yet-unknown dynamics that would lead to a non-zero vacuum expectation value for $t\bar{t}$. These speculations are motivated by having M_t be large, so the top Yukawa coupling is larger than the gauge coupling g_2, and of order unity.

One attractive feature of these approaches is that M_t is calculable. No fundamental Higgs boson occurs in these approaches, but a composite scalar arises that behaves like a Higgs boson. Its mass is also calculable. However, the calculated values of M_t, M_H depend significantly on an assumed scale for new physics. In order to get values of M_t small enough to be (barely) consistent with radiative corrections these approaches probably require a desert between the weak scale and the GUT scale, though if there is new physics at an intermediate scale it could conspire to raise M_t and cancel its effects in the radiative corrections. If there is a desert, then studying the top and Higgs properties would be the only source of new physics information accessible at high energy colliders.

In the desert scenarios the value of M_t comes out to be

$$M_t \sim 225 \text{ GeV}.$$

This is essentially the minimum value that can emerge from these approaches in their minimal form, but other new physics on the weak scale such as a fourth family could allow smaller values.

The scalar boson mass that emerges is $M_H = 2M_t$ in the simplest version of the theory, but the renormalization group constraints reduce M_H so that a value of $M_H \sim 250$ GeV corresponds to a value of $M_t \sim 225$ GeV. Basically, the numbers emerge from the infrared fixed points of the renormalization group equations, interpreted form the point of view of ref. 49 rather than ref. 48. These models give a prediction for M_t, M_H because the assumptions force them into the region of the cusp in the M_t, M_H plane of reference 15. In principle the "Higgs"-fermion couplings would be different in this composite case when integrated up to the unification scale from these of a fundamental Higgs boson.

To make progress these approaches probably have to gain some insight into the origins of the hypothetical dynamical mechanisms, and into how other fermions might get mass. If M_t is in the 120-150 GeV region people may lose interest in these approaches, while if $M_t \geq 200$ GeV there will surely be more serious thinking about these approaches.

IS $t \to Wb$ A BACKGROUND FOR STUDYING TEV WW INTERACTIONS?

Longitudinal W bosons arise from the Higgs mechanism or some equivalent physics. It may be essential to study their interactions in the TeV region in order to untangle the physics involved in the Higgs mechanism, and even if other discoveries (such as a Higgs boson) help, we will never be sure we understand the underlying processes until the TeV interactions have been studied. [Since one can Lorentz-transform any single W to rest and rotate longitudinal ↔ transverse, the symmetries of the theory guarantee that one cannot learn about the Higgs mechanism by studying single W's. Further threshold symmetries force any effects large enough to be observable into the TeV region, $\sqrt{s} \gg M_W$.]

Since production of $t\bar{t}$ gives WW + soft b's once $M_t > M_W + M_b$, people have been concerned that this background can obscure a signal from real longitudinal W's. The W's from t decay are increasingly longitudinally polarized, as discussed earlier in these notes, so it is a serious possibility.

Fortunately, the result of analyses is that $t\bar{t}$ is not a problem so long as one is in the TeV WW region. Yuan and collaborators[64,65] have shown that both for $t\bar{t} \to WWb\bar{b}$ and for $Wt\bar{b} \to WWb\bar{b}$ it is possible to isolate a real WW interaction from these backgrounds.

Physically their results are understandable once one thinks about the characteristics of the events. There are basically three large effects, plus a number of smaller ones. The large effects are

(1) For the signal the W's will have a large transverse momentum, $P_T^W > 350$ GeV, in order for $M_{WW} \gtrsim 1$ TeV. But the background $t\bar{t}$ typically have $P_T^t \sim M_t$, so $P_T^W \sim \frac{1}{2}M_t \ll 350$ GeV. Thus if one insists that all W candidates have $P_T^{jj} > 350$ GeV if $W \to jj$, and $P_T^{\ell^\pm} > 100$ GeV if $W \to \ell\nu$, then very few of them will be W's from $t\bar{t}$.

(2) To get $M_{WW} \gtrsim 1$ TeV for W's from top decay, the tops must be very energetic, so the b and the W are nearby. Then the $\ell^\pm$ from the trigger W is not very isolated from the b jet, so it is easy to remove such events with an isolation cut on the trigger lepton.

(3) The background $gg \to t\bar{t}$ has a large hadronic multiplicity, while the signal has a small hadronic multiplicity.[66,67] A simple analysis suggests the average multiplicities for TeV region events should differ by about a factor of three. Even though there is a large multiplicity spread, this should be a powerful way to reject background.

It is important to emphasize that the above procedure can be done in such a way[64] that is does not bias the W polarization analysis, which is very important since ultimately it is the rate and characteristics of longitudinal *WW* events that we need data about.

FINAL COMMENTS

If the t-quark is a SM particle in its properties, then $M_t \sim 140$ GeV (± about 30 GeV) and it should be found at FNAL in the next few years. If not, either because it is too heavy or because it decays very differently, it may not be found for over a decade.

If top is found at FNAL and has the expected decays, one of our best opportunities for experimental hints of where to search for the physics that provides the foundations of the SM will have been thwarted. On the other hand, once M_t is known, and rare decays are not observed, many predictions for other experiments will become much more precise, and many new physics ideas will be constrained. Further SM tests will be possible.

If M_t is above about 120 GeV, opportunities exist to relate it to the origins of the electroweak symmetry breaking and/or grand unified theories. These will be very active areas of theoretical research.

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