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Z Decays and Tests of the Standard Model *

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Abstract

Fundamental aspects of Z decays are reviewed. The effects of radiative corrections, both from ordinary QED and from the electroweak interactions are considered from an elementary point of view, but in some detail. The possibility of mixing with an extra Z boson and the effect that technicolor might have are discussed. Additional information that will be obtained from measurements of the W in collider experiments is considered.

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1 Introduction

The successful operations of the SLC and LEP have provided us direct, clear access to the evidence for electroweak unification. Their earliest results gave a precise determination of the mass and width of the Z and showed that there are only three light neutrinos, and by implication that there are only three generations of quarks and leptons. The data available by June, 1990 dramatically confirmed the basic predictions of the standard model for the width and branching ratios.[1, 2] This success is too much of a good thing. To make further progress we need to find new particles or effects that are not predicted in the simplest version of the standard model. Such effects have not yet been found.[3]

How then can we hope to learn something new from LEP or SLC? We must look to high statistics experiments, with either polarized or unpolarized beams. It seems unlikely that there will be direct evidence for production of new particles, although the search for the Higgs boson continues, but there could be signs of higher-energy phenomena that would appear in Z decays through radiative corrections or through mixing.

These lectures explore these themes. However, they can be considered no more than an introduction to a subject that has been investigated in great detail. Among the myriad important references, the CERN publications *Physics at LEP*,[4], *Polarization at LEP*,[5], and *Z Physics at LEP I*,[6] are worthy of special note for their exhaustive treatment of many of the topics considered here. My lectures inevitably overlap the excellent presentation of Michael Peskin at last year's Summer School.[7] Some of the material may be found in my Beijing lectures of 1988.[8]

2 Standard Model Basics

The familiar litany of the standard model is that electroweak interactions arise from a Yang-Mills theory based on the group $SU(2)\times U(1)$. Each factor has its own coupling constant, g for $SU(2)$ and g' for $U(1)$. In its simplest version the symmetry is broken spontaneously when a complex doublet of scalars takes on a vacuum expectation value, $v/\sqrt{2}$, in its neutral component. This generates a term in the Lagrangian that is quadratic in the vector fields:

$$\frac{v^2}{8}\left[g^2(W_1^2+W_2^2)+(gW_3-g'B)^2\right]. \tag{2.1}$$

This is then recognized as the mass term for the W and Z. The properly normalized Z field is

$$Z=\frac{gW_3-g'B}{\sqrt{g^2+g'^2}}, \tag{2.2}$$

and the implied masses are

$$m_W^2=\frac{g^2v^2}{4}, \tag{2.3}$$

$$m_Z^2=\frac{(g^2+g'^2)}{4}v^2. \tag{2.4}$$

The combination of neutral fields orthogonal to the Z is the massless photon:

$$A=\frac{g'W_3+gB}{\sqrt{g^2+g'^2}}. \tag{2.5}$$

The mixing angle between the W_3 and B is called θ_W and

$$\tan\theta_W\equiv g'/g, \tag{2.6}$$

so

$$A = \sin\theta_W\, W_3+\cos\theta_W\, B, \tag{2.7}$$

$$Z = \cos\theta_W\, W_3-\sin\theta_W\, B. \tag{2.8}$$

The couplings of the theory are determined by the covariant derivative, which when written in terms of the physical fields is

$$D_\mu=\partial_\mu+\frac{ig}{\sqrt{2}}(T_+W_\mu^+ + T_-W_\mu^-)+\frac{ig}{\cos\theta_W}(T_3-Q\,\sin^2\theta_W)Z_\mu+ieQA_\mu, \tag{2.9}$$

where the coupling e is defined by

$$\frac{1}{e^2}=\frac{1}{g^2}+\frac{1}{g'^2}. \tag{2.10}$$

The value of v is determined by comparing the amplitude for the decay $\mu^-\to e^-\nu_\mu\bar{\nu}_e$ calculated from the effective Lagrangian derived from Eq. (2.9)

$$\mathcal{L}_{eff}=\left(\frac{g}{\sqrt{2}}\right)^2\frac{1}{m_W^2}\bar{\nu}_\mu\gamma^\lambda\frac{1}{2}(1-\gamma_5)\mu\;\bar{e}\gamma_\lambda\frac{1}{2}(1-\gamma_5)\nu_e, \tag{2.11}$$

with the result obtained from the usual $V - A$ theory. This yields

$$\frac{G_F}{\sqrt{2}} = \frac{v^2}{2} = \frac{g^2}{8m_W^2}. \tag{2.12}$$

All of these relations are calculated in lowest order perturbation theory, at the Born level. There are three fundamental parameters: g, g', and v, but four relations among them, Eqs. (2.3), (2.4), (2.10), and (2.12). Radiative corrections will modify the results by finite amounts. We are free to maintain three of the four relations, as described more fully below, and the fourth then will be changed in a way dictated by the theory. We anticipate the results of the following sections by writing

$$\begin{aligned} \frac{G_F}{\sqrt{2}} &= \frac{g^2}{8m_W^2}(1+\Delta r) \\ &= \frac{v^2}{2}(1+\Delta r). \end{aligned} \tag{2.13}$$

The quantity Δr reflects effects from known and possibly from unknown sources. We shall later calculate many of the contributions, but for now let us note that $\Delta r \approx 0.06$. Using Eqs. (2.3), (2.4), (2.10), and (2.13) we find the relation

$$m_W^2 = \frac{1}{2}\left(1+\sqrt{1-\frac{4\pi\alpha(1+\Delta r)}{\sqrt{2}m_Z^2 G_F}}\right) m_Z^2. \tag{2.14}$$

Often the relation between m_Z and m_W is written in terms of $\sin^2\theta_W$. There is nothing wrong with this, but it can cause confusion because there are various definitions of $\sin^2\theta_W$ that agree at the lowest level (tree level) of perturbation theory but which differ once radiative corrections are included. We shall use the definition advocated by Sirlin [9]:

$$\sin^2\theta_W = 1 - m_W^2/m_Z^2, \tag{2.15}$$

from which it follows that

$$\sin^2\theta = \frac{1}{2}\left(1-\sqrt{1-\frac{4\pi\alpha(1+\Delta r)}{\sqrt{2}m_Z^2 G_F}}\right). \tag{2.16}$$

With $m_Z = 91.17$ GeV, $\Delta r = 0.060$ we find $m_W = 79.99$ GeV and $\sin^2\theta_W = 0.230$.

The partial widths for Z decay may be calculated from the couplings

$$g_{f_L} Z_\mu \bar{f}_L \gamma^\mu f_L; \quad g_{f_R} Z_\mu \bar{f}_R \gamma^\mu f_R \tag{2.17}$$

where g_f is determined by referring to the coupling dictated by the covariant derivative. Of course a real calculation must involve radiative corrections, but heuristically we can write a form that contains most of the radiative correction by replacing

$$g^2 \to 8m_W^2 G_F/\sqrt{2} \tag{2.18}$$

in the lowest order expression for the partial width. In this way the general form,

$$\Gamma(Z \to f_L \bar{f}_R) = \frac{g_f^2 m_Z}{24\pi}, \tag{2.19}$$

gives us the specific predictions

$$\Gamma(Z \to \nu_L \bar{\nu}_R) = \frac{\sqrt{2}G_F m_Z^3}{24\pi} = 165.7 \text{ MeV}, \tag{2.20}$$

$$\Gamma(Z \to f_L \bar{f}_R) = \frac{\sqrt{2}G_F m_Z^3}{6\pi}(T_3 - Q\sin^2\theta_W)^2, \tag{2.21}$$

$$\Gamma(Z \to f_R \bar{f}_L) = \frac{\sqrt{2}G_F m_Z^3}{6\pi}(Q\sin^2\theta_W)^2, \tag{2.22}$$

$$\Gamma(W \to e\bar{\nu}_e) = \frac{\sqrt{2}G_F m_W^3}{12\pi} = 224\text{MeV}. \tag{2.23}$$

We summarize this in Table 2.1.

These predictions may be compared to recent results from LEP in Tables 2.2 and 2.3: It is apparent that the agreement between theory and experiment is remarkably good.

	Γ	$\times QCD$	Γ/gen	gen	Γ
e_L	48.2	1	48.2	3	144.6
e_R	35.1	1	35.1	3	105.3
ν_L	165.7	1	165.7	3	497.1
u_L	79.6	1.04×3	248.4	2	496.7
u_R	15.6	1.04×3	48.7	2	97.3
d_L	118.7	1.04×3	370.3	3	1111.0
d_R	3.9	1.04×3	12.2	3	36.5

Table 2.1: Partial widths in MeV for Z decays predicted by the simple formulas in Eqs. (2.19) – (2.23) with $\sin^2\theta_W$=0.230, m_Z = 91.17 GeV. The total width is predicted to be 2488.5 MeV and the hadronic width 1741.5 MeV. The hadronic widths have a correction factor of $1+\alpha_s(m_Z^2) \approx 1.04$. The final column shows the partial width summed over generations.

	M_Z (GeV)	Γ_Z (GeV)	$\sigma_{had}^{0\ peak}$(nb)	N_ν
ALEPH	91.194 ± 0.016	2.498 ± 0.031	41.86 ± 0.66	2.90 ± 0.12
DELPHI	91.182 ± 0.021	2.495 ± 0.040	41.7 ± 1.2	2.93 ± 0.21
L3	91.148 ± 0.017	2.502 ± 0.033	41.24 ± 0.76	3.04 ± 0.14
OPAL	91.163 ± 0.017	2.497 ± 0.034	41.6 ± 0.8	3.09 ± 0.19

Table 2.2: Results from LEP experiments for the Z mass, Z width, peak cross section, and number of light neutrino species as reported by Fernandez.[1]

	Γ_{lepton} (MeV)	Γ_{hadron} (MeV)	$\Gamma_{invisible}$ (MeV)	$R \equiv \Gamma_{had}/\Gamma_{lep}$
ALEPH	84.4 ± 1.3	1753 ± 27	492 ± 25	20.77 ± 0.39
DELPHI	83.2 ± 2.0	1731 ± 52	514 ± 46	20.8 ± 0.6
L3	84.3 ± 1.3	1733 ± 44	516 ± 42	20.57 ± 0.60
OPAL	83.1 ± 1.9	1804 ± 44	474 ± 43	$21.72\ ^{+0.71}_{-0.65}$

Table 2.3: Results from LEP experiments for partial decay widths of the Z as reported by Fernandez.[1]

3 Decays to the Higgs Boson

If we are to find new physics at the Z it might well come from rare decays. Of such possibilities the one pursued most vigorously is the search for the decay $Z \to Z^*H$, where H is the Higgs boson and Z^* is a virtual Z that decays into some "observable" state. The most obvious choice for this state is a charged lepton pair, but in fact it has been possible to look effectively for the decay into neutrino pairs. From Table 2.1 we see that the sum of the neutrino channels has a branching ratio six times a big as any single charged lepton channel.

This manner of searching for the Higgs boson was first proposed by Ioffe and Khoze [10] and the decay distribution was calculated by Bjorken. [11] In terms of $x = 2E_H/m_Z$, where E_H is the energy of the Higgs boson,

$$E_H = \frac{m_Z^2 + m_H^2 - m_{Z^*}^2}{2m_Z}. \tag{3.1}$$

We have for $Z \to Z^*H \to (\mu^+\mu^-)H$

$$\begin{aligned} \frac{1}{\Gamma(Z\to\mu\mu)}\frac{d\Gamma}{dx} &= \frac{\alpha}{4\pi\sin^2\theta_W\cos^2\theta_W}\left[1 - x + \frac{x^2}{12} + \frac{2}{3}\frac{m_H^2}{m_Z^2}\right] \\ &\times \left(x^2 - \frac{4m_H^2}{m_Z^2}\right)^{1/2}\left(x - \frac{m_H^2}{m_Z^2}\right)^{-2}. \end{aligned} \tag{3.2}$$

The branching ratio for $Z \to Z^*H \to (\text{all}) + H$ is obtained by integrating over x:

$$BR = \frac{\alpha}{4\pi\sin^2\theta_W\cos^2\theta_W} F\left(\frac{m_H^2}{m_Z^2}\right), \tag{3.3}$$

where

$$\begin{aligned} F(y) &= (1-y)\left(-\frac{47}{24} + \frac{13y}{24} - \frac{y^2}{12}\right) \\ &+ \frac{(5y - 2y^2 + y^3/4)}{\sqrt{4y - y^2}}\cos^{-1}\frac{\sqrt{y}(3-y)}{2} \\ &+ \left(1 - \frac{3y}{2} + \frac{y^2}{4}\right)\ln\frac{1}{\sqrt{y}}. \end{aligned} \tag{3.4}$$

Lower bounds on the mass of the Higgs boson, assuming the simple one-doublet model, have been obtained by the various LEP experiments. The best limit so far comes from the ALEPH Collaboration.[12] Data from their experiment are shown in Fig. 3.1. The histogram shows the visible mass. The first plot, (a), shows a large sample of events dominated by ordinary hadronic decays of the Z. The shaded histogram shows the expected distribution from hadronic Z decays alone while the points indicate the data. After a series of cuts is applied that should greatly exclude most normal events, while leaving the true $Z \to H\nu\bar{\nu}$ events, the remaining distribution is as shown in (b). The shaded distribution in (c) shows the expectation for actual $Z \to H\nu\bar{\nu}$ events from a 40 GeV Higgs boson. When the $Z^* \to \ell^+\ell^-$ channels are added the ALEPH limit is $M_H > 41.6$ GeV at 95% C.L. When the $Z^* \to \ell^+\ell^-$ channels are added to $Z \to H\nu\bar{\nu}$ the ALEPH limit is $M_H > 41.6$ GeV at 95% C.L.

An alternative decay of interest is $Z \to H\gamma$. This decay does not occur at lowest order because the photon couples to charge and the Higgs boson and the Z are neutral. There are contributions, however, from one-loop diagrams.[13] Results of such calculations are shown in Fig. 3.2.

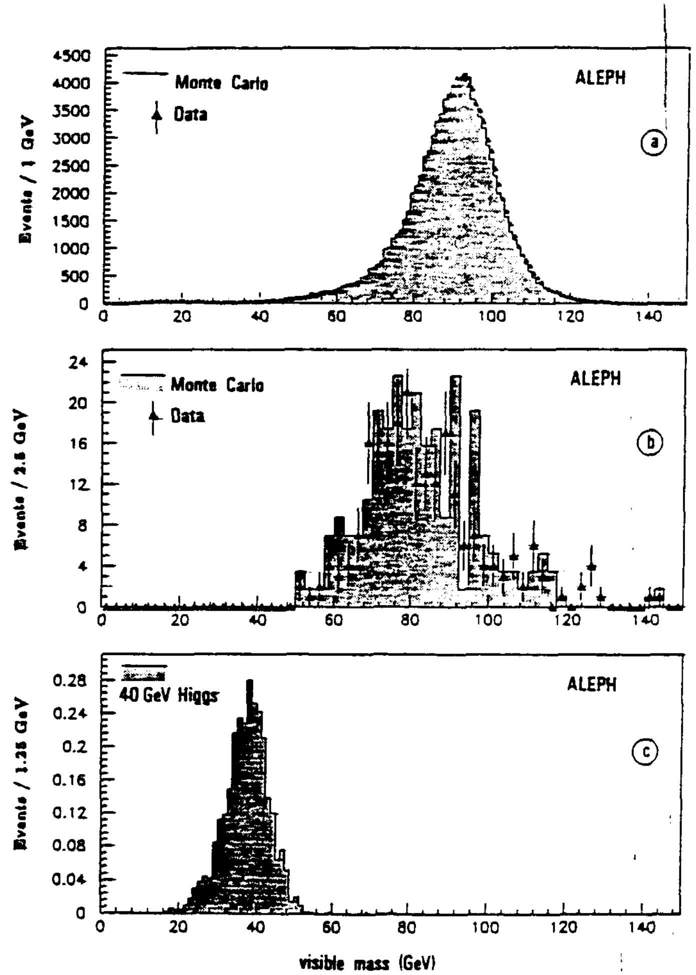


Figure 3.1: Data from the ALEPH Collaboration excluding a conventional Higgs boson of about 40 GeV by looking for the decay $Z \to H\nu\bar{\nu}$.[12]

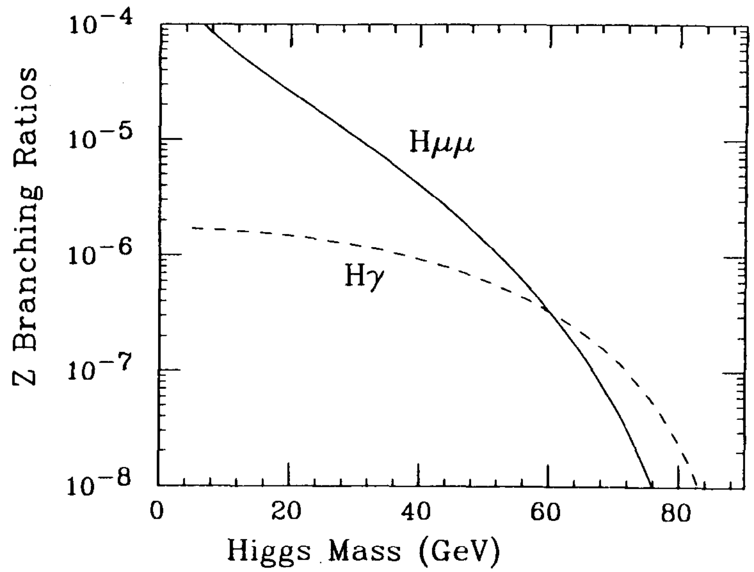


Figure 3.2: The branching ratios for $Z \to Z^{*}H$ and $Z \to H\gamma$ as a function of the mass of the Higgs boson.

4 Other Rare Decays

The topic of rare decays of the Z has been summarized by Glover and van der Bij.[14] Among the possibilities are

$Z \to HHf\bar{f}$. This is essentially excluded given the present limits on the mass of the Higgs boson.

$Z \to q\bar{q}\ell^+\ell^-$. The rate is not too small since the partial width summed over leptons is about 1 MeV, but neither is the process very interesting since the lighter of the quark pair and lepton pair is simply produced by a virtual photon emitted by the other pair.

$Z \to Wf\bar{f}$. This is interesting in principle, but the rate is extraordinarily small because all diagrams have at least one very virtual particle. Glover and van der Bij conclude that more than 10^8 events are needed to see this mode.

$Z \to ggg$. The decay proceeds through quark loops, but with a rate that is not observable (BR$\approx 2 \cdot 10^{-6}$) given the $q\bar{q}g$ background.

$Z \to \gamma gg$. The rate is even smaller than for ggg, and the $q\bar{q}\gamma$ background is formidable.

$Z \to \gamma\gamma\gamma$. Ridiculously small in the standard model, BR$< 10^{-9}$.

$Z \to \gamma\psi$. The rate to decay into a photon and a heavy quark-antiquark bound state can be calculated. The largest is to $\gamma\psi$, but its branching ratio is less than 10^{-7}.

$Z \to b\bar{s}$. In the standard model this and other flavor-changing neutral current decays are very small, typically 10^{-7} and therefore out of range.

$Z \to \gamma\pi_{TC}, \eta_{TC}$. Here π_{TC} is a neutral technipion and η_{TC} is a technieta. This decay was considered as early as 1981. [15] The most recent calculation [16] for a one family model of technifermions evaluated for the η_{TC} with N technicolors gives a partial decay rate of 0.7 keV $(N/4)^2$, which is only observable for a large value of N, say 7 or more.

$Z \to W\pi$. An interesting, calculable [17], and unobservable decay with a partial width of less than $4 \cdot 10^{-10}$ GeV.

$Z \to \gamma\pi^0$. Despite arguments to the contrary [18], this also has a very tiny partial width[17], about 10^{-10} GeV.

5 Initial State Radiation

The most important radiation correction in $e^+e^- \to Z \to X$ is initial state radiation, a primarily classical phenomenon leading to emission of real photons along the directions of the incident beams. While there are numerous extensive calculations and Monte Carlo routines to evaluate initial state radiation, for many practical purposes it suffices to use some simple analytic formulas.[19]

Were it not for radiative corrections, the e^+e^- annihilation cross section near the Z would be given, to a very good approximation, by the standard Breit-Wigner formula

$$\begin{aligned} \sigma &= \left[\frac{4\pi}{k^2}\frac{(2J+1)B(Z\to e^+e^-)}{(2S_1+1)(2S_2+1)}\right] \\ &\quad \times \frac{\Gamma^2/4}{(W-M)^2+\Gamma^2/4}, \end{aligned} \tag{5.1}$$

where $k = \sqrt{s}/2$ is the cm momentum in the initial state, $J = 1$ is the spin of the produced resonance, $2S_1 + 1 = 2S_2 + 1 = 2$ is the spin multiplicity of the initial particles, $B(Z \to e^+e^-) = 0.033$ is the branching ratio in the incident channel, $W = \sqrt{s}$ is the cm energy, and Γ is the total width. With $m_Z = 91.17$ GeV the peak cross section is 58 nb. The radiative corrections necessarily involve $\alpha = 1/137$ but they are not small because they are accompanied by a large logarithm, explicitly in the combination

$$t = \frac{2\alpha}{\pi}\left[\ln\frac{m_Z^2}{m_e^2} - 1\right] \approx 0.108. \tag{5.2}$$

The effect of the radiative corrections is obtained as a convolution, using the results of Kuraev and Fadin [20]:

$$\sigma(W) = \left(1+\frac{3t}{4}\right)t\int_0^E \frac{dk}{k}\left(\frac{k}{E}\right)^t \sigma_0(W-k). \tag{5.3}$$

Here $E = W/2$ is the beam energy and σ_0 represents the cross section before radiative corrections. Introducing $x = 2k/\Gamma$ and $\lambda = 2(W-M)/\Gamma$ we have

$$\frac{\sigma(W)}{\sigma_{\max}} \cong \left(1+\frac{3t}{4}\right)t\left(\frac{\Gamma}{W}\right)^t \int_0^{2E/\Gamma} dx x^{t-1}\left[1+(x-\lambda)^2\right]^{-1}. \tag{5.4}$$

Since $E >> \Gamma$, the upper limit may be set equal to infinity. If we now define

$$\begin{aligned} \Phi(\lambda) &\equiv t\int_0^\infty dx\, x^{t-1}[1+(x-\lambda)^2]^{-1} \\ &= \frac{\pi t}{\sin \pi t}(1+\lambda^2)^{(t-1)/2} \\ &\quad \times \sin\left[(1-t)\cos^{-1}\left(\frac{-\lambda}{\sqrt{1+\lambda^2}}\right)\right], \end{aligned} \tag{5.5}$$

we can write the radiatively corrected cross section as

$$\frac{\sigma(W)}{\sigma_{\max}} = \left(1+\frac{3t}{4}\right)\left(\frac{\Gamma}{W}\right)^t \Phi(\lambda). \tag{5.6}$$

In particular we see that

$$\Phi(0) = \frac{\pi t/2}{\sin(\pi t/2)}, \tag{5.7}$$

and thus at the old peak ($\lambda = 0$) there is a reduction of the cross section

$$\begin{aligned} \frac{\sigma(m_Z)}{\sigma_{\max}} &= \left(1+\frac{3t}{4}\right)\left(\frac{\Gamma}{W}\right)^t \frac{\pi t/2}{\sin \pi t/2} \\ &= 1.081 \times 0.678 \times 1.005 \\ &= 0.737, \end{aligned} \tag{5.8}$$

that is, a 26% reduction. In Fig. 5.1 the effect of the initial state radiative correction is shown.

These results can be extended to include interference of the Z with the virtual photon intermediate state and to include an energy dependent width for the Z.[19] With a full knowledge of the initial state radiation the line shape for the Z is determined and it is possible to show convincingly that there is no room for a fourth species of light neutrino. This is seen in the data from ALEPH [21] in Fig. 5.2.

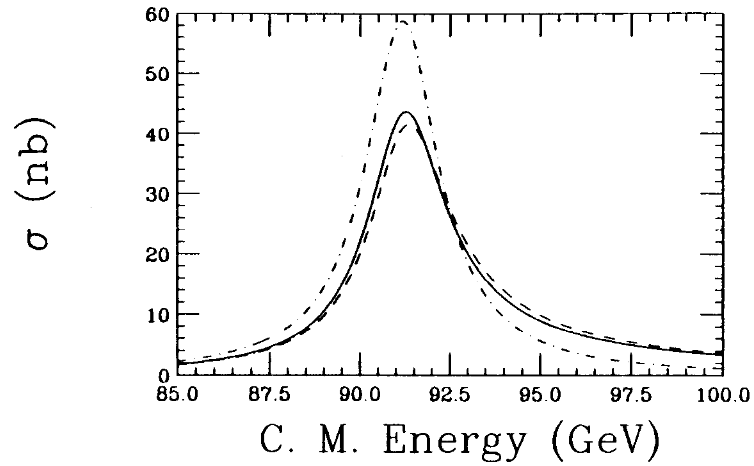


Figure 5.1: A comparison of the uncorrected Breit-Wigner shape for the Z resonance (dot-dash) with the fully corrected form of Eq. (5.6) (solid) and the result using just first order corrections (dashed). The cross section shown is for all final states, including neutrinos.

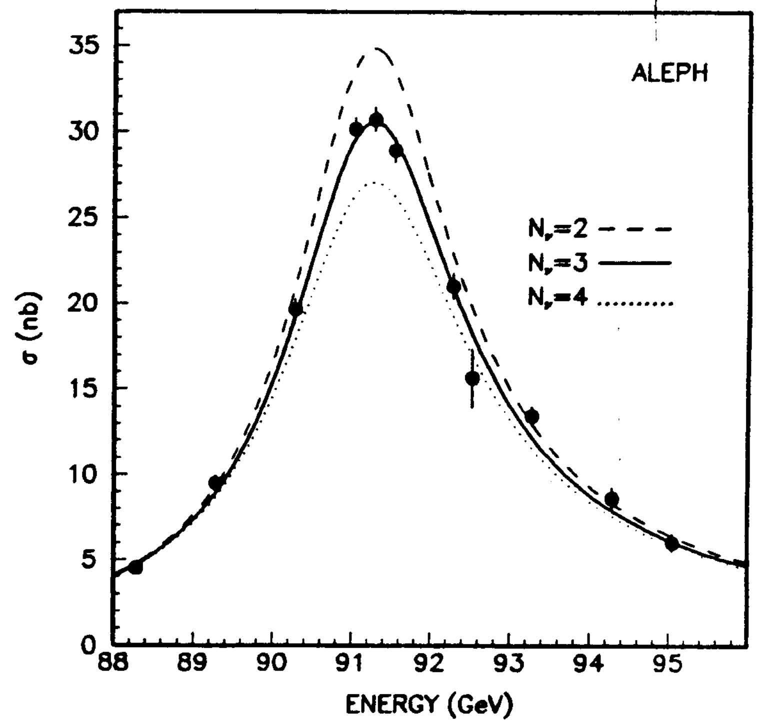


Figure 5.2: Data from ALEPH [21] for hadronic decays of the Z showing fits with 2, 3, and 4 neutrino species.

6 Review of Radiative Corrections

Because we believe the Standard Model contains important truths, it is of great importance to test it in as great detail as possible, not so much to verify it as to find its limitations. The Standard Model is really a *theory* in that it is completely calculable: it is renormalizable. This means we can go beyond leading order in calculating the physical observables. Just as quantum electrodynamics was established by the agreement between the measured and predicted values of $g-2$ and the Lamb shift, so we would like to look for measurable electroweak radiative corrections.

The essence of renormalizability is that all physical quantities, like cross sections and decay rates, can be expressed in terms of a finite number of initial physical parameters, like the masses and coupling constants of the particles in the theory. In the Standard Model, as outlined in Section 2, the basic parameters that appear in the theory are g, g', and v. To fix these we need three physical quantities. One possible choice is α_{em}, m_Z, and m_W. Since m_W is not well measured, it is better to choose α_{em}, m_Z, and G_F. Before m_Z was well measured, it was convenient to use α_{em}, G_F, and some quantity measured in neutral-current neutrino scattering. Of course there are other physical quantities that must be specified – the quark masses, Kobayashi-Maskawa mixing angles, and the mass of the Higgs boson. These are incorporated in a straightforward fashion.

The weak mixing angle is not to be regarded as a fundamental parameter. It may be convenient to introduce $\sin^2\theta_W$ for the purpose of doing calculations. When this is done, it is essential to provide a precise definition. In some a basic sense $\sin^2\theta_W$ is irrelevant. We can always express any physical prediction as *Observable* $= f(\alpha, G_F, m_Z, m_H, m_t, \ldots)$ without the appearance of $\sin^2\theta_W$.

We shall sketch the evaluation of radiative corrections following the procedure of Sirlin.[9] The basic idea is to say that the parameters that appear in the Lagrangian, which we called g, g', and v^2, should be indicated instead by g_0, g_0', and v_0^2, and regarded as *bare* parameters. We write them as

$$\begin{aligned} g_0 &= g - \delta g, && (6.1)\\ g_0' &= g' - \delta g', && (6.2)\\ v_0^2 &= v^2 - \delta v^2. && (6.3) \end{aligned}$$

Now suppose we calculate a physical quantity like the mass of the W. In lowest order we will find

$$m_W^2 = g_0^2 v_0^2/4. \qquad (6.4)$$

In the next order we will find a correction of order g_0^2 relative to this one. Suppose we insist that the actual relation be the one we first obtained in Section 2:

$$m_W^2 = g^2 v^2/4. \qquad (6.5)$$

To do this we can adjust δg and δv^2 so that they just cancel the higher order correction. Then, to the next order in perturbation theory, Eq. (6.5) is restored. Now we only have three δs to adjust, so we can restore only three of the four basic relations we started with in Section 2:

$$\begin{aligned} m_Z^2 &= \frac{1}{4}(g^2 + g'^2)v^2, && (6.6)\\ m_W^2 &= \frac{1}{4}g^2 v^2, && (6.7)\\ \frac{G_F}{\sqrt{2}} &= \frac{g^2}{8m_W^2}, && (6.8)\\ \frac{1}{e^2} &= \frac{1}{4\pi\alpha} = \frac{1}{g^2} + \frac{1}{g'^2}. && (6.9) \end{aligned}$$

We choose to maintain Eqs. (6.6), (6.7), and (6.9). This is the most convenient choice although, ultimately, we will specify the physical input in terms of α, G_F, and m_Z.

Now let us write in the mass term of the Lagrangian in terms of the bare parameters (c.f. Eq. (2.1)):

$$\mathcal{L} = \frac{v_0^2}{2}\left[\frac{g_0^2}{2}W_\mu^+ W^{-\mu} + \frac{1}{4}\left(g_0 W_{3\mu} - g_0' B_\mu\right)^2\right], \qquad (6.10)$$

and then substitute the expressions for the bare parameters in terms of the renormalized parameters, g, g', and v^2 and the δs. If we keep only terms first order in δ, we have

$$\begin{aligned} \mathcal{L} &= \frac{m_Z^2}{2}Z_\mu Z^\mu + m_W^2 W_\mu^+ W^{-\mu}\\ &\quad - \left(\frac{\delta m_Z^2}{2}Z_\mu Z^\mu + \delta m_W^2 W_\mu^+ W^{-\mu} - \delta m_{ZA}^2 Z_\mu A^\mu\right), \qquad (6.11) \end{aligned}$$

with

$$m_W^2 = \frac{1}{4}\left(g^2v^2\right), \tag{6.12}$$

$$m_Z^2 = \frac{1}{4}\left(g^2+g'^2\right)v^2, \tag{6.13}$$

$$\delta m_W^2 = \frac{1}{4}\delta\left(g^2v^2\right) = \frac{1}{4}\left(v^2\delta g^2 + g^2\delta v^2\right), \tag{6.14}$$

$$\delta m_Z^2 = \frac{1}{4}\delta\left[\left(g^2+g'^2\right)v^2\right], \tag{6.15}$$

$$\delta m_{ZA}^2 = \frac{m_Z^2}{\sqrt{g^2+g'^2}}\left(c\delta g' - s\delta g\right), \tag{6.16}$$

$$= \frac{v^2}{4}(-g'\delta g + g\delta g'), \tag{6.17}$$

where δg^2 means $2g\delta g$, etc. We have made the definitions

$$\tan\theta_W = g'/g, \tag{6.18}$$

$$c = \cos\theta_W, \qquad s = \sin\theta_W, \tag{6.19}$$

so that always

$$m_W^2/m_Z^2 = \cos^2\theta_W. \tag{6.20}$$

Now our next step is to calculate the one loop corrections to m_W^2, m_Z^2, and e^2 so that we can adjust the three δs so as to restore our fundamental relations, Eqs. (6.6), (6.7), and (6.9). When we are done, we will find that there are finite corrections to (6.8).

In order to calculate the one-loop effects, we must reconsider the interactions of the gauge bosons with the particles that might go around the loop. The interactions are governed by the covariant derivative, which enters the Lagrangian as $\bar{\psi} i \not{D}\psi$ so we examine

$$\mathcal{L} = -\bar{\psi}\left(g_0 \not{W}\cdot\frac{\tau}{2} + g_0'\frac{Y}{2}\not{B}\right)\psi, \tag{6.21}$$

and substitute again for the bare parameters. The result is

$$\begin{aligned}\mathcal{L} = & -\bar{\psi}\left[\frac{g}{\sqrt{2}}\left(T_+\not{W}^+ + T_-\not{W}^-\right) + \frac{g}{c}\left(T_{3L}-Qs^2\right)\not{Z} + eQ\not{A}\right]\psi \\ & +\bar{\psi}\left[\frac{\delta g}{\sqrt{2}}\left(T_+\not{W}^+ + T_-\not{W}^-\right) + \left(\delta\sqrt{g^2+g'^2}\right)\left(T_{3L}-Qs^2\right)\not{Z}\right. \\ & +\delta eQ\not{A} + esc\left(\frac{\delta g}{g}-\frac{\delta g'}{g'}\right)Q\not{Z} \\ & \left. +e\left(\frac{\delta g}{g}-\frac{\delta g'}{g'}\right)\left(T_{3L}-Qs^2\right)\not{A}\right]\psi.\end{aligned} \tag{6.22}$$

Here, of course, we mean by δe

$$\delta\frac{1}{e^2} = -2\frac{\delta e}{e^3} = \delta\left(\frac{1}{g^2}+\frac{1}{g'^2}\right). \tag{6.23}$$

We recognize the electromagnetic and weak neutral currents:

$$\begin{aligned} J_\gamma^\mu &= \bar{\psi}\gamma^\mu Q\psi, \\ J_Z^\mu &= \bar{\psi}\gamma^\mu\left(T_{3L}-s^2Q\right)\psi. \end{aligned} \tag{6.24}$$

In terms of these the Lagrangian is

$$\begin{aligned}\mathcal{L} = & -\frac{g}{\sqrt{2}}\bar{\psi}(T_+\not{W}^+ + T_-\not{W}^-)\psi - \frac{g}{c}J_Z^\mu Z_\mu - eJ_\gamma^\mu A_\mu \\ & +\frac{\delta g}{\sqrt{2}}\bar{\psi}(T_+\not{W}^+ + T_-\not{W}^-)\psi + A_\mu J_\gamma^\mu\delta e + Z_\mu J_Z^\mu\delta\sqrt{g^2+g'^2} \\ & +J_\gamma^\mu Z_\mu g s^2 c\left(\frac{\delta g}{g}-\frac{\delta g'}{g'}\right) + J_Z^\mu A_\mu e\left(\frac{\delta g}{g}-\frac{\delta g'}{g'}\right).\end{aligned} \tag{6.25}$$

Now let us return to the question of the mass of the Z. Ordinarily we would write the Z propagator as

$$\frac{-i}{q^2-m_Z^2}\left(g_{\mu\nu}-q_\mu q_\nu/m_Z^2\right). \tag{6.26}$$

Here we have chosen the unitary gauge. In another gauge the piece proportional to $q_\mu q_\nu$ would be different, but the piece proportional to $g_{\mu\nu}$ would be the same. It suffices for our purposes to consider just the $g_{\mu\nu}$ part. Now consider the effect of the extra interaction implied by Eq. (6.11) as shown in Fig. 6.1.

There are also contributions from the one loop diagrams as shown in the same Figure. The value of the $g_{\mu\nu}$ piece of this contribution, $i\Pi_{ZZ}(q^2)$, depends

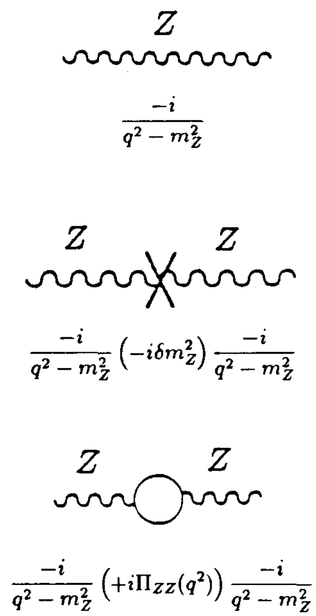


Figure 6.1: Diagrams contributing to the Z mass. The first contribution comes from the renormalized mass term. The second comes from the insertion of δm_Z^2. The third is the one loop contribution.

on the invariant mass squared, q^2, carried by the incoming line. There are additional diagrams that contain repetitions of these basic (one-particle irreducible) pieces. If we add the repetitions, we see we get just a geometric series (the tensor factor, $g_{\mu\nu}$ just reproduces itself). The full sum is

$$\frac{-i}{q^2 - m_Z^2} \frac{1}{1 + \frac{\delta m_Z^2 - \Pi_{ZZ}(q^2)}{q^2 - m_Z^2}} = \frac{-i}{q^2 - m_Z^2 + \delta m_Z^2 - \Pi_{ZZ}(q^2)}. \tag{6.27}$$

Now if we want the mass of the Z to be m_Z we must have δm_Z^2 cancel the real part ofΠ_{ZZ} exactly at $q^2 = m_Z^2$:

$$\delta m_Z^2 = \delta \left[\frac{v^2}{4}(g^2 + g'^2)\right] = \mathrm{Re}\,\Pi_{ZZ}(m_Z^2). \tag{6.28}$$

The imaginary part of Π_{ZZ} determines the width of the Z. Similarly we must choose

$$\delta m_W^2 = \delta \left[\frac{v^2}{4}g^2\right] = \mathrm{Re}\,\Pi_{WW}(m_W^2). \tag{6.29}$$

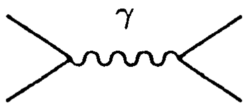
Of course these cancellations occur only at the specified points, so Π_{ZZ} and Π_{WW} will give rise to important corrections when evaluated for other values of q^2.

Finally we must arrange it so that the charge of the electron is actually $-e$. To do this, we calculate Coulomb scattering at some low momentum transfer. As shown in Fig. 6.2, there are two contributions. One comes from exchange of a single photon and the other from a photon interrupted by a vacuum polarization bubble. There are other diagrams we can draw, but these do not contribute in the limit $q^2 \to 0$. For example, a simple Z exchange is unimportant because instead of varying as $1/q^2$, it goes to $1/m_Z^2$. The diagram with a bubble connecting one photon and one Z is unimportant because as $q^2 \to 0$, $\Pi_{\gamma Z}(q^2) \to 0$. As a result we have just the sum

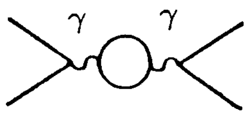
$$\frac{-i}{q^2} J_\gamma^1 (e - \delta e)^2 J_\gamma^2 + e^2 \frac{-i}{q^2} J_\gamma^1 \frac{-i}{q^2} \left(i\Pi_{\gamma\gamma}(q^2)\right) J_\gamma^2. \tag{6.30}$$

To get the required form we must demand that

$$-2e\delta e + e^2 \frac{\Pi_{\gamma\gamma}(q^2)}{q^2}\Big|_{q^2=0} = 0. \tag{6.31}$$



$$\frac{-i}{q^2} J^1_\gamma (e - \delta e)^2 J^2_\gamma$$



$$\frac{-i}{q^2} J^1_\gamma \frac{-i}{q^2} \left(i\Pi_{\gamma\gamma}(q^2) \right) J^2_\gamma$$

Figure 6.2: Diagrams contributing to Coulomb scattering at low q^2.

Now we have determined all three of the δs:

$$\delta\left[\frac{1}{4}v^2(g^2 + g'^2)\right] = \mathrm{Re}\,\Pi_{ZZ}(m_Z^2), \tag{6.32}$$

$$\delta\left[\frac{1}{4}v^2 g^2\right] = \mathrm{Re}\,\Pi_{WW}(m_W^2), \tag{6.33}$$

$$\frac{\delta e^2}{e^2} = \Pi'_{\gamma\gamma}(0). \tag{6.34}$$

We can solve explicitly for δg and $\delta g'$:

$$\delta g = \frac{e}{2s}\Pi'_{\gamma\gamma}(0) - \frac{ec^2}{2s^3}\left[\frac{\mathrm{Re}\,\Pi_{ZZ}(m_Z^2)}{m_Z^2} - \frac{\mathrm{Re}\,\Pi_{WW}(m_W^2)}{m_W^2}\right], \tag{6.35}$$

$$\delta g' = \frac{e}{2c}\Pi'_{\gamma\gamma}(0) + \frac{e}{2c}\left[\frac{\mathrm{Re}\,\Pi_{ZZ}(m_Z^2)}{m_Z^2} - \frac{\mathrm{Re}\,\Pi_{WW}(m_W^2)}{m_W^2}\right]. \tag{6.36}$$

We are now in a position to calculate physical processes.

7 Muon Decay

There are three diagrams to be considered for the decay $\mu \to e\nu\overline{\nu}$, as shown in Fig. 7.1. Each is proportional to the result, $\mathcal{M}$, that would have been obtained in lowest order. Summing the contributions shown in the Figure we obtain

$$\mathcal{M}\left(1 - \frac{2\delta g}{g} + \frac{\mathrm{Re}\Pi_{WW}(m_W^2) - \Pi_{WW}(0)}{m_W^2}\right) \equiv \mathcal{M}(1 + \Delta r) \ . \tag{7.1}$$

This is the result promised in Section 2 and defines Δr. Using our expression for $\delta g/g$

$$\begin{aligned}\Delta r &= -\Pi'_{\gamma\gamma}(0) + \frac{c^2}{s^2}\left(\frac{\mathrm{Re}\Pi_{ZZ}(m_Z^2) - \Pi_{ZZ}(0)}{m_Z^2}\right)\\ &+ \frac{s^2 - c^2}{s^2}\left(\frac{\mathrm{Re}\Pi_{WW}(m_W^2) - \Pi_{WW}(0)}{m_W^2}\right)\\ &+ \frac{c^2}{s^2}\left(\frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2}\right).\end{aligned} \tag{7.2}$$

It is a little easier to understand this result if we cast it in a slightly different way. Instead of looking at the currents that couple to Z, W, and the photon,

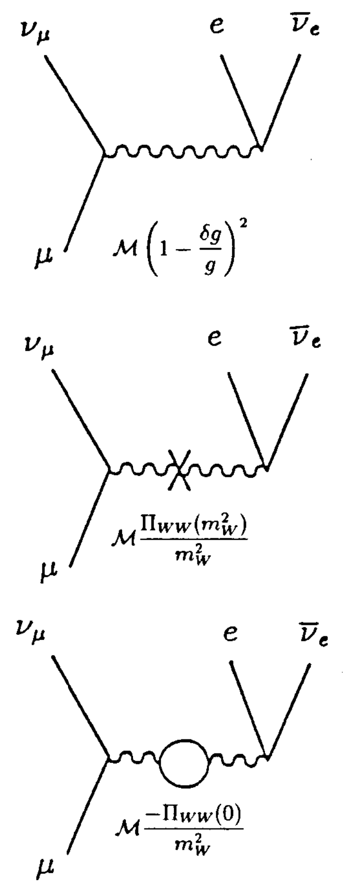


Figure 7.1: Diagrams contributing to muon decay.

let us use those that couple to W_3, W_1, and the photon and extract the coupling constants. Explicitly, let

$$\Pi_{ZZ} = \frac{g^2}{c^2}\left(\Pi_{33} - 2s^2\Pi_{3Q} + s^4\Pi_{QQ}\right), \tag{7.3}$$

$$\Pi_{WW} = g^2\Pi_{11}, \tag{7.4}$$

$$\Pi_{\gamma\gamma} = e^2\Pi_{QQ}, \tag{7.5}$$

and generically write

$$\Pi'(q^2) = \frac{\Pi(q^2) - \Pi(0)}{q^2}. \tag{7.6}$$

Then we can rewrite

$$\begin{aligned}\Delta r &= -e^2\Pi'_{QQ}(0) + \frac{g^2}{s^2}\left[\Pi'_{33}(m_Z^2) - 2s^2\Pi'_{3Q}(m_Z^2) + s^4\Pi'_{QQ}(m_Z^2)\right] \\ &\quad + g^2\frac{s^2-c^2}{s^2}\Pi'_{11}(m_W^2) + \frac{c^2}{s^2}\frac{g^2}{m_W^2}\left[\Pi_{33}(0) - \Pi_{11}(0)\right].\end{aligned} \tag{7.7}$$

From our charge renormalization calculation we see that Coulomb scattering at a momentum transfer q^2 comes with a factor

$$J^1_\gamma J^2_\gamma e^2\left(1 - e^2[\Pi'_{QQ}(0) + \Pi'_{QQ}(q^2)]\right). \tag{7.8}$$

It follows that

$$\frac{\alpha(m_Z^2) - \alpha(0)}{\alpha(0)} = e^2\left(\Pi'_{QQ}(m_Z^2) - \Pi'_{QQ}(0)\right) \equiv \Delta\alpha. \tag{7.9}$$

We see that the two Π'_{QQ} terms in the expression for Δr can be rewritten in terms of $\Delta\alpha$.

If weak isospin were a good symmetry then we would have $\Pi_{11}(q^2) = \Pi_{33}(q^2)$. Of course weak isospin is broken and this is reflected in the quantity

$$\Delta\rho = \frac{g^2}{m_W^2}\left[\Pi_{11}(0) - \Pi_{33}(0)\right]. \tag{7.10}$$

We can re-express Δr using $\Delta\alpha$ and $\Delta\rho$ as

$$\Delta r = \Delta\alpha - \frac{c^2}{s^2}\Delta\rho + \frac{g^2}{s^2}\left[\Pi'_{33}(m_Z^2) - 2s^2\Pi'_{3Q}(m_Z^2)\right] + g^2\frac{s^2-c^2}{s^2}\Pi'_{11}(m_W^2). \tag{7.11}$$

The introduction of "new" physics – a heavy t-quark, technicolor, a heavy Higgs boson – will change the values of the vacuum polarization bubbles and thus change Δr. This induces a change in the value of m_W since m_Z, G_F and α are fixed. From Eq. (2.14),

$$m_W^2 = \frac{1}{2}\left(1+\sqrt{1-\frac{4\pi\alpha(1+\Delta r)}{\sqrt{2}m_Z^2 G_F}}\right)m_Z^2, \tag{7.12}$$

we find

$$\delta m_W^2/m_W^2 = -\frac{s^2}{1-2s^2}\delta\Delta r. \tag{7.13}$$

8 Dispersion Relations

How are we to evaluate the various vacuum polarization functions, $\Pi(q^2)$? The standard way is to write down the corresponding Feynman diagram. Here we explore an alternative: dispersion relations. The functions $\Pi(q^2)$ turn out to be analytic functions of their argument. Moreover, they have the special property that they are *real analytic* functions. An analytic function $f(z)$ is said to be real analytic if there is some interval on the real axis on which $f(z)$ is real. In this region $f(z)$ has a power series expansion with real coefficients. It is not hard to prove and easy to believe that for a real analytic function $f(z^*) = f^*(z)$.

Consider a real analytic function with a branch point at $z = a$ and a branch cut running from a to ∞. An example might be $f(z) = (a-z)^\nu$ with a real and f defined to be real for z real and $z < a$. Now Cauchy's theorem tells us we can always write

$$f(z) = \frac{1}{2\pi i}\oint \frac{dz'}{z'-z}f(z'), \tag{8.1}$$

where the contour encircles (counterclock-wise) the point z. Now suppose we take a contour as shown in Fig. 8.1. Now on crossing from one side of the cut to the other the real part of f is unchanged and the imaginary part changes sign (because $f^*(z) = f(z^*)$). On the other hand, dz' goes one way below the cut and the other way above the cut. As a result, only the imaginary part contributes and

$$f(z) = \frac{1}{\pi}\int_{cut}\frac{dz'}{z'-z}\mathrm{Im}f(z') + \frac{1}{2\pi i}\int_{circle}\frac{dz'}{z'-z}f(z'), \tag{8.2}$$

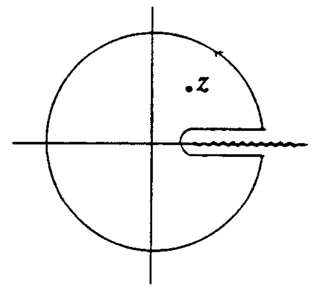


Figure 8.1: A contour useful for evaluating Cauchy's formula for a real analytic function.

where $\mathrm{Im}f(z')$ means the imaginary part *above* the real axis. If f is small enough for large $|z|$ we will be able to drop the contribution from the large circle. Thus if f is cut from a to ∞

$$f(z) = \frac{1}{\pi}\int_a^\infty \frac{dz'}{z'-z}\mathrm{Im}f(z'). \tag{8.3}$$

If the point z is on the real axis and $z < a$ the integral is purely real, as it must be. If the point z approaches the cut from above the left hand side will have to have an imaginary part that is just $\mathrm{Im}f(z)$. The real part will be given by the principal part of the integral.

Now what is the nature of $\mathrm{Im}\Pi(s)$? There is an imaginary part to $\Pi_{ZZ}(s)$, say, only if there is an actual physical state with cm energy squared s into which a virtual Z can decay. Thus, technically, $\Pi_{ZZ}(s)$ has a cut beginning at (two times the mass of the lightest neutrino)$^2 \approx 0$, while the cut for $\Pi_{WW}(s)$ begins at m_e^2. Let us indicate the threshold generally by s_{th}. Then, if we ignore the contribution on the periphery,

$$\Pi(s_0) = \frac{1}{\pi}\int_{s_{th}} \frac{ds}{(s-s_0)}\mathrm{Im}\Pi(s). \tag{8.4}$$

Now we know that $\mathrm{Im}\Pi$ is related to real physical processes, but how? We have already seen that propagators look like

$$\frac{i}{s-m^2-\Pi(s)}. \tag{8.5}$$

On the other hand, we are used to the form

$$\frac{i}{s-m^2+i\Gamma m}, \tag{8.6}$$

that appears in the Breit-Wigner formula. This suggests the identification

$$\mathrm{Im}\Pi(s) = -\sqrt{s}\Gamma(s). \tag{8.7}$$

We must write $\sqrt{s}$ instead of m because the bubble doesn't know the mass of the real particle, it knows only about the value of $q^2 = s$. Moreover, we must remember that the width of the virtual particle in question depends on s. Thus $\Gamma(s)$ is to be thought of as the width a Z would have if it had a mass squared s. This could be measured, in principle, in neutrino antineutrino annihilation! More practically it could be studied in e^+e^- annihilation, where it would interfere with the electromagnetic annihilation.

For the case of the virtual photon we can prove that

$$\mathrm{Im}\Pi_{\gamma\gamma}(s) = -\frac{s^2\sigma(e^+e^-)}{4\pi\alpha}. \tag{8.8}$$

The optical theorem relates the forward scattering amplitude to the total cross section:

$$\begin{aligned} \mathrm{Im}\mathcal{M}(p_1,p_2;p_1,p_2) &= -2\sqrt{s}p_{cm}\sigma_{tot} \qquad (8.9)\\ &\approx -s\sigma_{tot} \qquad (8.10)\end{aligned}$$

where $\mathcal{M}$ is evaluated for the same initial and final spins. If we write $\mathcal{M}$ for the diagram that includes a vacuum polarization bubble (without it, $\mathcal{M}$ is real) we have

$$-i\mathcal{M} = \overline{u}(p_1)ie\gamma_\mu v(p_2)\frac{-i}{q^2}i\Pi_{\gamma\gamma}(q^2)\overline{v}(p_2)ie\gamma^\mu u(p_1)\frac{-i}{q^2}. \tag{8.11}$$

Averaging over the spins, remembering that initial and final spins are the same, and dropping the electron mass,

$$\begin{aligned} \mathcal{M} &= -\frac{1}{4}\mathrm{Tr}\,\not{p}_1\gamma_\mu\not{p}_2\gamma^\mu\frac{e^2}{q^4}\Pi_{\gamma\gamma}(q^2) \qquad (8.12)\\ &= \frac{4\pi\alpha}{s}\Pi_{\gamma\gamma}(q^2). \qquad (8.13)\end{aligned}$$

Inserting this into the optical theorem gives the stated relation. Analogous relations exist for Π_{WW} and Π_{ZZ}.

If we wish to use the relation between Γ and $\mathrm{Im}\Pi$, we need to calculate the width a vector meson would have as a function of its mass. The amplitude for the decay of a vector into a fermion – antifermion pair is

$$\mathcal{M} = \overline{u}(p)\gamma_\mu(g_V+g_A\gamma_5)v(p')\epsilon^\mu, \tag{8.14}$$

where ϵ is the polarization of the vector meson. In terms of these

$$\begin{aligned} \Gamma(s) &= \frac{p}{4\pi s}\left\{g_V^2\left[s-(m-m')^2-4p^2/3\right]\right.\\ &\quad\left.+\,g_A^2\left[s-(m+m')^2-4p^2/3\right]\right\}. \qquad (8.15)\end{aligned}$$

For the special case where $g_V = \pm g_A$:

$$\Gamma = \frac{pg^2}{2\pi s}\left(s - m^2 - m'^2 - 4p^2/3\right). \tag{8.16}$$

For the equal mass case, $m = m'$:

$$\Gamma = \frac{p}{6\pi}\left[g_V^2(1+2m^2/s) + g_A^2(1-4m^2/s)\right]. \tag{8.17}$$

Let us use these relations to calculate $\Delta\alpha$:

$$\begin{aligned}\Delta\alpha &= e^2\left[\Pi'_{QQ}(m_Z^2) - \Pi'_{QQ}(0)\right] \\ &= \left[\Pi'_{\gamma\gamma}(m_Z^2) - \Pi'_{\gamma\gamma}(0)\right].\end{aligned} \tag{8.18}$$

We write formally (that means we don't worry about convergence)

$$\Pi(s_0) = \frac{1}{\pi}\int_{s_{th}}^{\infty}\frac{ds}{(s-s_0)}\mathrm{Im}\Pi(s), \tag{8.19}$$

and

$$\Pi'(s_0) = \frac{\Pi(s_0)-\Pi(0)}{s_0} = \frac{1}{\pi}\int_{s_{th}}^{\infty}\frac{ds}{s(s-s_0)}\mathrm{Im}\Pi(s), \tag{8.20}$$

and further

$$\Pi'(s_0) - \Pi'(0) = \frac{1}{\pi}\int_{s_{th}}^{\infty} ds\frac{s_0}{s^2(s-s_0)}\mathrm{Im}\Pi(s). \tag{8.21}$$

Now $\mathrm{Im}\Pi(s)$ is proportional to s for large s, so the integral is convergent and we assume we can drop the contour around the large circle. Using Eq. (8.17) we find that a Dirac particle of mass m and charge e contributes to $\mathrm{Im}\Pi$ as

$$\mathrm{Im}\Pi_{\gamma\gamma}(s) = -\frac{2\alpha}{3}p_{cm}\sqrt{s}\left(1+\frac{2m^2}{s}\right). \tag{8.22}$$

Thus the contribution to $\Delta\alpha$ is

$$\begin{aligned}\mathrm{Re}\Pi'_{\gamma\gamma}(m_Z^2) - \mathrm{Re}\Pi'_{\gamma\gamma}(0) &= -\frac{\alpha}{3\pi}\int_{s_{th}}^{\infty}\frac{ds}{s^2}\frac{m_Z^2}{s-m_Z^2}\sqrt{s(s-4m^2)}\left(1+\frac{2m^2}{s}\right) \\ &= -\frac{\alpha}{3\pi}\int_0^1 dz\frac{z^2(3-z^2)}{\mu-1+z^2},\end{aligned} \tag{8.23}$$

where $\mu = 4m^2/m_Z^2$. If the fermion is light so that $\mu < 1$ the integral is complex, but we actually need just the real part of the integral – the principal part. It is a straightforward exercise to show that the result is

$$\Delta\alpha = -\frac{\alpha}{3\pi}\left\{-\frac{1}{3} + (2+\mu)\left[1 - \frac{1}{2}\sqrt{1-\mu}\ln\frac{1+\sqrt{1-\mu}}{1-\sqrt{1-\mu}}\right]\right\}, \tag{8.24}$$

	e	μ	τ	u	d	s	c	b	t
mass (GeV)	0.0005	0.106	1.8	0.15	0.15	0.35	1.5	5.0	100
$\Delta r \times 10^2$	1.74	0.91	0.48	1.15	0.29	0.24	0.67	0.11	−0.02

Table 8.1: Table 2.1. Contributions to Δr from Eqs. (8.24) and (8.26) using the fermion masses shown. The total here is about 0.056.

which for light fermions gives

$$\Delta\alpha = \frac{\alpha}{3\pi}\left[-\frac{5}{3} + \ln\frac{m_Z^2}{m^2}\right]. \tag{8.25}$$

For the t quark, which is too heavy to be produced in Z decay, the corresponding formula is, from Eq. (8.23),

$$\Delta\alpha = -\frac{\alpha}{3\pi}\left\{-\frac{1}{3} + (2+\mu)\left[1 - \sqrt{\mu-1}\tan^{-1}\frac{1}{\sqrt{\mu-1}}\right]\right\}. \tag{8.26}$$

In Table 8.1 we show the contributions from the charged leptons and the quarks. Clearly the treatment of the hadronic piece is unconvincing. How should we specify the quark masses? The correct procedure is to use Eqs. (8.8) and (8.21) to obtain

$$\begin{aligned}\Pi'_{\gamma\gamma}(m_Z^2) - \Pi'_{\gamma\gamma}(0) &= -\frac{1}{\pi}\int ds\frac{m_Z^2}{s-m_Z^2}\frac{\sigma(e^+e^-\to \text{hadrons})}{4\pi\alpha} &(8.27)\\ &= -\frac{1}{\pi}\int ds\frac{m_Z^2}{s-m_Z^2}\frac{\alpha}{3s}\mathcal{R}(s) &(8.28)\end{aligned}$$

where $\mathcal{R}$ is the ratio of the e^+e^- hadronic annihilation cross section to the cross section to muons,

$$\sigma_{\text{point}} = \frac{4\pi\alpha^2}{3s}. \tag{8.29}$$

A careful evaluation [22] gives 0.0288 ± 0.0009, rather than the 0.0244 given in Table 8.1, resulting in $\Delta\alpha = 0.060$.

9 Heavy t quark

Using the results of the earlier sections we can now determine the contribution a heavy t quark to $\Delta\rho$ and thus to the $W-Z$ mass splitting. The value of $\Delta\rho$ depends on $\Pi_{11}(0)-\Pi_{33}(0)$. We consider contributions from loops involving t and b quarks, beginning with Π_{33}. From Eq. (8.16), with $g_V=-g_A=gT_3/2$, the decay width of a W_3 of mass squared s into a quark–antiquark pair with quark mass m is

$$\Gamma_3=\frac{g^2p}{48\pi}\left(1-m^2/s\right), \tag{9.1}$$

where the cm momentum, p, is given by

$$p^2=(s-4m^2)/4. \tag{9.2}$$

Thus, formally (and remembering to drop the g^2 in accordance with our definition of Π_{33})

$$\Pi_{33}(0)=-\frac{1}{96\pi^2}\int_{4m^2}^{\Lambda^2}\frac{ds}{s}\sqrt{s(s-4m^2)}\left(1-m^2/s\right). \tag{9.3}$$

A cutoff, Λ, must be introduced since the integral diverges. This is not a real problem since $\Delta\rho$ will turn out to be finite. It is simply more convenient to calculate Π_{33} and Π_{11} separately. A tedious but elementary calculation shows that for large Λ,

$$\Pi_{33}(0)=-\frac{1}{96\pi^2}\left(\Lambda^2-3m^2\ln\frac{\Lambda^2}{m^2}\right). \tag{9.4}$$

Adding the t and b contributions,

$$\Pi_{33}(0)=-\frac{1}{48\pi^2}\left(\Lambda^2-\frac{3}{2}m_t^2\ln\frac{\Lambda^2}{m_t^2}-\frac{3}{2}m_b^2\ln\frac{\Lambda^2}{m_b^2}\right). \tag{9.5}$$

For Π_{11} we consider the decay of a virtual charged W into $t\bar{b}$. For the width we have

$$\Gamma_1=\frac{pg^2}{2\pi s}\left(s-m_t^2-m_b^2-4p^2/3\right), \tag{9.6}$$

where the cm momentum is given by

$$p^2=\frac{s^2-2s(m_t^2+m_b^2)+(m_t^2-m_b^2)^2}{4s}. \tag{9.7}$$

In this way we find

$$\begin{aligned}\Pi_{11}(0) &= -\frac{1}{48\pi^2}\int_{(m_t+m_b)^2}^{\Lambda^2}\frac{ds}{s}\sqrt{s^2-2s(m_t^2+m_b^2)+(m_t^2-m_b^2)^2}\\ &\quad\times\left(1-\frac{m_t^2+m_b^2}{2s}-\frac{(m_t^2-m_b^2)^2}{2s^2}\right).\end{aligned} \tag{9.8}$$

Another tedious, elementary integration gives

$$\begin{aligned}\Pi_{11}(0) &= -\frac{1}{48\pi^2}\left[\Lambda^2-\frac{3}{2}(m_t^2+m_b^2)\ln\frac{\Lambda^2}{m_tm_b}\right.\\ &\quad\left.+\frac{3}{2}\frac{m_t^4+m_b^4}{(m_t^2-m_b^2)}\ln\frac{m_t^2}{m_b^2}-\frac{3}{4}(m_t^2+m_b^2)\right].\end{aligned} \tag{9.9}$$

Combining these results we find [23, 24]

$$\begin{aligned}\Delta\rho &= \frac{g^2}{m_W^2}\left[\Pi_{11}(0)-\Pi_{33}(0)\right]\\ &= \frac{3g^2}{64\pi^2m_W^2}\left(m_t^2+m_b^2-\frac{2m_t^2m_b^2}{m_t^2-m_b^2}\ln\frac{m_t^2}{m_b^2}\right).\end{aligned} \tag{9.10}$$

The factor of 3 comes from summing over the quark colors.

For $m_t >> m_b$ we have

$$\Delta\rho=\frac{3G_Fm_t^2}{8\pi^2\sqrt{2}}. \tag{9.11}$$

This gives a contribution to Δr of

$$\Delta r=-\frac{c^2}{s^2}\frac{3G_Fm_t^2}{8\pi^2\sqrt{2}}=-0.0105(m_t/100\text{ GeV})^2. \tag{9.12}$$

The corresponding shift in the $W-Z$ mass splitting is

$$\Delta m_W=-\frac{s^2}{1-2s^2}\frac{m_W}{2}\Delta r=-180\text{ MeV}(m_t/100\text{ GeV})^2. \tag{9.13}$$

The additional terms in the expression for Δr are of the form $\Pi'_{33}(m_Z^2)$, $\Pi'_{11}(m_W^2)$, etc. Their formal expressions separately diverge logarithmically, though their sum is finite. However, taking the limit $m_Z\to 0$ or $m_W\to 0$ in each piece introduces no extra singularities. Thus $\Pi'_{33}(m_Z^2)$ contains terms like $\ln(\Lambda^2/m_t^2)$ and m_Z^2/m_t^2, but not m_t^2/m_Z^2. It follows that the contribution of all these terms, which is finite, can be at most logarithmic in m_t^2.

10 The Effective Lagrangian

It is straightforward to calculate fermion-fermion scattering including the one-loop corrections to the various vector propagators. Of course, this is not a complete calculation of all effects at this order since it ignores vertex corrections and box diagrams. Nonetheless, it does contain most of the physics. Moreover, the results can be written in an especially clear fashion, namely they are just like the lowest level results except that some of the parameters are shifted slightly. This approach has been developed and advocated by Lynn and by Peskin and their co-workers.[25, 26, 7, 33]

Our approach is similar, but not identical, to that of the originators. We shall simply calculate fermion-fermion neutral current scattering. We need not specify whether the exchange is in the t channel or s channel. When we are done we shall see that the result can be written [7]

$$\begin{aligned}\mathcal{M}^{NC}_{eff} &= -e_*^2 Q \frac{1}{q^2} Q' \\ &\quad - \frac{e_*^2}{s_*^2 c_*^2}\left(T_3 - s_*^2 Q\right)\frac{Z_*}{q^2 - m_*^2}\left(T'_3 - s_*^2 Q'\right) \qquad (10.1)\end{aligned}$$

where the third components of the isospin of the external fermions are T_3 and T_3' and their charges are Q and Q'. At Born level the same formula would hold, with $e_*^2 = e^2$, $s_*^2 = \sin^2\theta_W$, $c_*^2 = \cos^2\theta_W$, $m_*^2 = m_Z^2$, and $Z_* = 1$. It is appropriate to recall at this time that we have *always*

$$\cos^2\theta_W = \frac{m_W^2}{m_Z^2} = \frac{g^2}{g^2 + g'^2}, \qquad (10.2)$$

$$\sin^2\theta_W = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4\pi\alpha(1+\Delta r)}{\sqrt{2} m_Z^2 G_F}}\right). \qquad (10.3)$$

The vertices we need for the calculation follow from Eqs. (6.11)-(6.17) and (6.25) and are shown in Fig. 10.1.

The diagram with a single Z exchange contributes

$$\begin{aligned}\mathcal{M} &= -\frac{(T_3 - s^2 Q)(T_3' - s^2 Q')}{q^2 - m_Z^2}\left[g^2 + g'^2 - \delta(g^2 + g'^2)\right] \\ &\quad + \frac{Q(T_3' - s^2 Q') + Q'(T_3 - s^2 Q)}{q^2 - m_Z^2} e^2 \left(\frac{\delta g}{g} - \frac{\delta g'}{g'}\right). \qquad (10.4)\end{aligned}$$

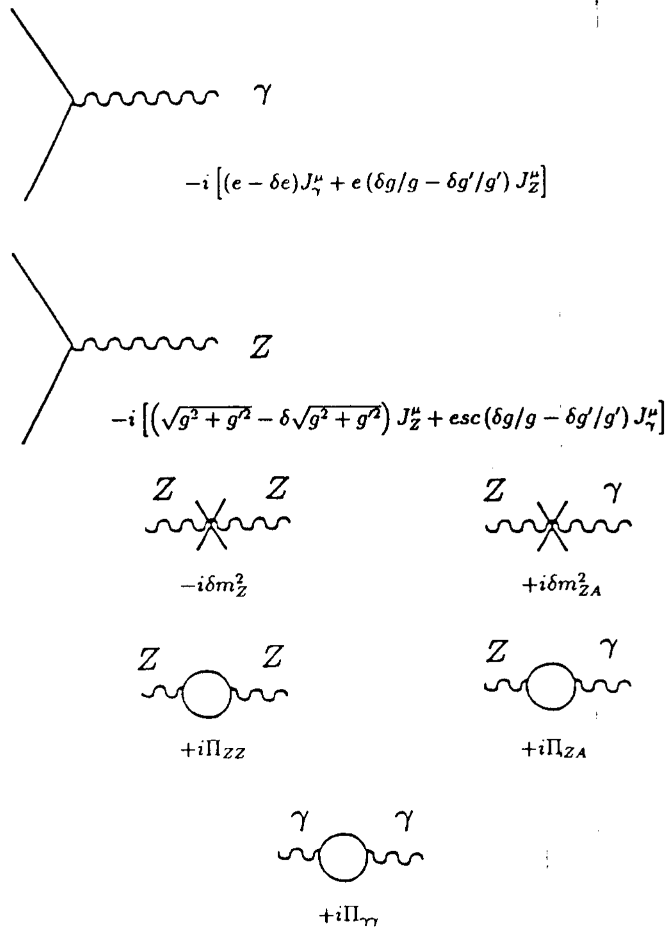


Figure 10.1: Factors associated with vertices and insertions needed for the radiative corrections to fermion-fermion scattering.

The diagram with a single photon exchange gives

$$\mathcal{M} = -\frac{QQ'}{q^2}\left[e^2 - \delta e^2\right] + \frac{Q(T_3' - s^2Q') + Q'(T_3 - s^2Q)}{q^2} e^2 \left(\frac{\delta g}{g} - \frac{\delta g'}{g'}\right). \tag{10.5}$$

The remaining diagrams have either a δm type insertion or a vacuum polarization bubble. The diagrams with Z leading in and out of the insertion or bubble give

$$\mathcal{M} = -\frac{(T_3 - s^2Q)(T_3' - s^2Q')}{(q^2 - m_Z^2)^2}(g^2 + g'^2)\left[\Pi_{ZZ}(q^2) - \mathrm{Re}\Pi_{ZZ}(m_Z^2)\right] \tag{10.6}$$

Combining this with the single Z exchange gives

$$\mathcal{M} = -\frac{(T_3 - s^2Q)(T_3' - s^2Q')\left[g^2 + g'^2 - \delta(g^2 + g'^2)\right]}{q^2 - m_Z^2 - [\Pi_{ZZ}(q^2) - \Pi_{ZZ}(m_Z^2)]} + \frac{Q(T_3' - s^2Q') + Q'(T_3 - s^2Q)}{q^2 - m_Z^2} e^2 \left(\frac{\delta g}{g} - \frac{\delta g'}{g'}\right). \tag{10.7}$$

Similarly, combining the single photon exchange with the diagram with the photon vacuum polarization gives

$$\mathcal{M} = -\frac{QQ'}{q^2\left[1 - \Pi_{\gamma\gamma}(q^2)/q^2\right]}\left[e^2 - \delta e^2\right] + \frac{Q(T_3' - s^2Q') + Q'(T_3 - s^2Q)}{q^2} e^2 \left(\frac{\delta g}{g} - \frac{\delta g'}{g'}\right). \tag{10.8}$$

There remain only the diagrams that mix the photon and the Z. These give

$$\mathcal{M} = -\frac{Q(T_3' - s^2Q') + Q'(T_3 - s^2Q)}{q^2(q^2 - m_Z^2)}\frac{e^2}{sc}\left[\Pi_{ZA}(q^2) - scm_Z^2\left(\frac{\delta g}{g} - \frac{\delta g'}{g'}\right)\right]. \tag{10.9}$$

If we combine the terms with the coefficient $Q(T_3' - s^2Q') + Q'(T_3 - s^2Q)$ from Eqs. (10.7) - (10.9) we find their sum is

$$\left[Q(T_3' - s^2Q') + Q'(T_3 - s^2Q)\right]\frac{e^2}{q^2 - m_Z^2}\left[2\left(\frac{\delta g}{g} - \frac{\delta g'}{g'}\right) - \frac{1}{sc}\Pi'_{ZA}(q^2)\right]. \tag{10.10}$$

Now we combine this with the first term in Eq. (10.7) to obtain

$$\begin{aligned}\mathcal{M} = &-\frac{g^2 + g'^2}{q^2 - m_Z^2}\left[1 - \frac{\delta(g^2 + g'^2)}{g^2 + g'^2} + \frac{\Pi_{ZZ}(q^2) - \Pi_{ZZ}(m_Z^2)}{q^2 - m_Z^2}\right] \\ &\times\left\{(T_3 - s^2Q) + \left[sc\Pi'_{ZA}(q^2) - 2s^2c^2\left(\frac{\delta g}{g} - \frac{\delta g'}{g'}\right)\right]Q\right\} \\ &\times\left\{(T_3' - s^2Q') + \left[sc\Pi'_{ZA}(q^2) - 2s^2c^2\left(\frac{\delta g}{g} - \frac{\delta g'}{g'}\right)\right]Q'\right\}.\end{aligned} \tag{10.11}$$

Using the relation

$$\left(\frac{\delta g}{g} - \frac{\delta g'}{g'}\right) = -\frac{1}{2s^2}\left[\frac{\mathrm{Re}\,\Pi_{ZZ}(m_Z^2)}{m_Z^2} - \frac{\mathrm{Re}\,\Pi_{WW}(m_W^2)}{m_W^2}\right], \tag{10.12}$$

which follows from Eqs. (6.35) and (6.36), we see that

$$s_*^2 = s^2 - sc\Pi'_{ZA}(q^2) - c^2\left[\frac{\mathrm{Re}\,\Pi_{ZZ}(m_Z^2)}{m_Z^2} - \frac{\mathrm{Re}\,\Pi_{WW}(m_W^2)}{m_W^2}\right]. \tag{10.13}$$

From Eq. (6.34) $\delta e^2 = e^2\Pi'_{\gamma\gamma}(0)$. Combining this with Eq. (10.5) we see that

$$e_*^2 = e^2\left[1 + \Pi'_{\gamma\gamma}(q^2) - \Pi'_{\gamma\gamma}(0)\right], \tag{10.14}$$

in agreement with the discussion surrounding Eq. (7.9).

We still have to identify Z_* and m_*^2. As it stands there is ambiguity because effects can be transferred between Z_* and m_*^2. To resolve the ambiguity we insist [7] not only that

$$m_*^2(m_Z^2) = m_Z^2, \tag{10.15}$$

but also that

$$\frac{d}{dq^2}m_*^2|_{m_Z^2} = 0. \tag{10.16}$$

This can be done by defining m_*^2 through

$$(q^2 - m_*^2) \equiv (q^2 - m_Z^2)\left[1 - \frac{\Pi_{ZZ}(q^2) - \Pi_{ZZ}(m_Z^2)}{q^2 - m_Z^2} + \frac{d}{dq^2}\Pi_{ZZ}(q^2)|_{m_Z^2}\right]. \tag{10.17}$$

Now Z_* is determined by

$$\frac{e_*^2}{s_*^2c_*^2}Z_* = (g^2 + g'^2)\left[1 - \frac{\delta(g^2 + g'^2)}{g^2 + g'^2} + \frac{d}{dq^2}\Pi_{ZZ}(q^2)|_{m_Z^2}\right]. \tag{10.18}$$

Substituting in the values of e_*^2, s_*^2, and c_*^2 and using

$$\frac{\delta(g^2+g'^2)}{g^2+g'^2} = \Pi'_{\gamma\gamma}(0) + \frac{s^2-c^2}{s^2}\left[\frac{\mathrm{Re}\,\Pi_{ZZ}(m_Z^2)}{m_Z^2} - \frac{\mathrm{Re}\,\Pi_{WW}(m_W^2)}{m_W^2}\right], \qquad (10.19)$$

we find

$$Z_* = 1 + \frac{s^2-c^2}{sc}\Pi'_{ZA}(q^2) + \frac{d}{dq^2}\Pi_{ZZ}(q^2)|_{m_Z^2} - \Pi'_{\gamma\gamma}(q^2). \qquad (10.20)$$

All of the starred quantities of the effective Lagrangian are now defined, so we can read off the consequences for a physical quantity like A_{LR}. The formula for A_{LR} at the peak of the Z is obtained just by replacing s^2 by s_*^2. In this sense the effective Lagrangian is just a convenient way of summarizing the one-loop corrections to the single vector-exchange graphs. But there is another advantage to the formulation: it isolates the large, trivial radiative correction, $\Delta\alpha$.

To see this, consider the formula for s_*^2, Eq. (10.13). The corrections to be subtracted from s^2 are evaluated at q^2 or at m_Z^2 and if q^2 is of the order of m_Z^2 these cannot lead to a large logarithm of the sort occurring in $\Delta\alpha = \Pi'_{\gamma\gamma}(m_Z^2) - \Pi'_{\gamma\gamma}(0)$. There is a large logarithm that does occur in s^2. We see this in Eq. (10.3) where Δr appears. Now $\Delta r = \Delta\alpha$ plus some other pieces (see Eq. (7.11)). Since we understand the contributions to $\Delta\alpha$ from the known quarks and leptons, it makes sense to isolate these. If there is new physics it will make its appearance in the remaining parts of the expression. To this end, following the lead of Lynn and Peskin [25, 7], we define

$$s^2_{\text{standard}} = \frac{1}{2}\left(1-\sqrt{1-\frac{4\pi\alpha_0(m_Z^2)}{\sqrt{2}m_Z^2G_F}}\right), \qquad (10.21)$$

where by $\alpha_0(m_Z^2)$ we mean the value of $\alpha(m_Z^2)$ based on the known quarks and leptons. This could differ slightly from the actual value if there are additional contributions. If we calculate the difference between s_*^2 and s^2_{standard} there will be no $\Delta\alpha$ contribution except from new physics. Indeed, explicitly

$$\begin{aligned} s_*^2 &= s^2_{\text{standard}} + \frac{s^2c^2}{c^2-s^2}\Delta\alpha^{\text{new}} \\ &+ \frac{e^2}{c^2-s^2}\left[\frac{\Pi_{33}(m_Z^2) - 2s^2\Pi_{3Q}(m_Z^2) - \Pi_{11}(0)}{m_Z^2}\right] - e^2\Pi'_{3Q}(q^2) \\ &+ e^2s^2\left[\Pi'_{QQ}(q^2) - \Pi'_{QQ}(m_Z^2)\right]. \qquad (10.22)\end{aligned}$$

This is similar to Eq. (5.36) of Ref. [7], but apparently not identical to it. If we assume that the old physics makes an insignificant contribution then all the Π can be regarded as coming from new physics. If the same assumption is made in Peskin's formula, then the two agree. In any event, we can rewrite the expression make its finiteness manifest:

$$\begin{aligned} s_*^2 &= s^2_{\text{standard}} + \frac{s^2c^2}{c^2-s^2}(\Delta\alpha^{\text{new}} - \Delta\rho) \\ &+ e^2\left[\Pi'_{3Q}(m_Z^2) - \Pi'_{3Q}(q^2)\right] - \frac{e^2}{c^2-s^2}\Pi'_{3,Y/2}(m_Z^2). \qquad (10.23)\end{aligned}$$

The finiteness is now easy to demonstrate. The contribution of a single quark to Π_{33} behaves as $a\Lambda^2 + bm^2\ln(\Lambda^2/m^2)$ as seen in Eq. (9.4). Taking a derivative makes Π'_{33} more convergent and reduces its dimension by two, so its behavior is $\Pi'_{33} \propto \ln(\Lambda^2/m^2)$. Taking still another derivative makes Π''_{33} convergent and it must behave like $1/m^2$. The term involving Π'_{3Q} in Eq. (10.23) is effectively $(m_Z^2 - q^2)\Pi''_{3Q}$ and is thus convergent since Π_{3Q} and Π_{33} have similar behavior. The final term, which involves $\Pi'_{3,Y/2}$ receives opposing contributions from the $T_3 = 1/2$ and $T_3 = -1/2$ quarks. These cancel the $\ln\Lambda^2$ and leave a convergent result.

11 Technicolor

The orthodox version of the Standard Model with a single, elementary Higgs doublet is by no means the only possibility. Indeed it is both ad hoc and theoretically suspect. In principle, it would be much more desirable to have a model in which the spontaneous symmetry breaking was the result of dynamics – so-called dynamical symmetry breaking.[27, 28, 29] The basic idea of technicolor is to introduce a new set of fermions – technifermions – with very strong (technicolor) interactions between them. Just as in QCD where the theory of (almost) massless quarks develops a vacuum expectation value for $\overline{u}u + \overline{d}d$, the corresponding combination of techniquarks, $\overline{U}U + \overline{D}D$, will develop a vacuum expectation value. This breaks various symmetries.

We saw that mass terms like $\overline{u}u$ break $SU(2)\times U(1)$ because they are actually the sum of two terms with $T = 1/2$:

$$\overline{u}u = \overline{u}_Lu_R + \overline{u}_Ru_L. \qquad (11.1)$$

Thus, just like the conventional $<\phi>$, the technicolor condensate breaks the gauge symmetry.

In QCD with n flavors of massless quarks, there is a large symmetry, $SU(n) \times SU(n)$, that arises because there is a separate flavor symmetry for the left-handed and right-handed quarks. These are separate because the gauge interactions preserve the handedness. A mass term couples left-handed to right-handed and destroys the (chiral) $SU(n) \times SU(n)$ symmetry. If all the quarks were given an identical mass, there would still be a flavor $SU(n)$ symmetry, analogous to isospin. This would be a vector (as opposed to axial) symmetry. The condensate, like a mass term, breaks the chiral symmetry.

Now the Goldstone theorem tells us that spontaneously broken global symmetries produce massless particles, so we know that the technicolor theory will generate massless particles. For example, suppose there are just two techniflavors, U and D. Then there is initially an $SU(2) \times SU(2)$ symmetry, which is broken to the vector $SU(2)$. The axial $SU(2)$ is lost and its three generators must give three massless scalars. But these are just what we need for the W and Z to become massive. In this way we can obtain all the good features of the standard model without introducing elementary scalars.

How can we describe all this? Well, it is actually completely analogous to QCD and so we can rely on techniques developed long ago. If we were dealing with a theory with two massless quarks, U and D, we would have two separate isospins that acted on the left-handed and right-handed quarks. We know that ultimately we will get three massless technipions so we want to write a theory in terms of them, but still displaying the chiral $SU(2) \times SU(2)$ symmetry. The best way to do this is with a chiral Lagrangian. Let π^a, $a = 1, 2, 3$, be the three (techni)pion fields and indicate the 2×2 generators of $SU(2)$ by

$$T^a = \tau^a/2, \tag{11.2}$$

so that

$$\mathrm{Tr}\, T^a T^b = \frac{1}{2}\delta_{ab}. \tag{11.3}$$

Now define

$$\Sigma = \exp(i2\pi \cdot T/f_\pi) \tag{11.4}$$

where f_π is a constant, which for QCD is actually the pion-decay constant and which for our technicolor model will be determined below. A general element of $SU(2) \times SU(2)$ is specified by a left-handed rotation, $L = \exp(-i\alpha_L \cdot T)$ and a right-handed rotation, $R = \exp(-i\alpha_R \cdot T)$. These are the matrices that would act on the left-handed and right-handed quarks. How shall we have them act on the pions? The solution is [30]

$$\Sigma \to L\Sigma R^\dagger = \exp(-i\alpha_L \cdot T)\Sigma \exp(+i\alpha_R \cdot T). \tag{11.5}$$

What is nice about this is that if we have a rotation that is just an ordinary $SU(2)$ transformation, that is, an isospin rotation, then $L = R$ and Σ transforms in a sensible way,

$$\Sigma \to U\Sigma U^{-1}, \tag{11.6}$$

where $U = L = R$. We see that Σ does carry the full $SU(2) \times SU(2)$ symmetry in that if we perform two successive transformations we get the same result as we would have if we found the product transformation and applied it directly to Σ.

Let us write a Lagrangian that incorporates the chiral symmetry. The Σs are matrices, so we will need to take traces of combinations of Σs. Consider, for example, $\mathrm{Tr}\,\Sigma\Sigma$. Under a chiral transformation this becomes $\mathrm{Tr}\, L\Sigma R^\dagger L\Sigma R^\dagger$, which is not the same. On the other hand $\mathrm{Tr}\,\Sigma\Sigma^\dagger$ transforms properly, $\mathrm{Tr}\,\Sigma\Sigma^\dagger \to Tr(L\Sigma R^\dagger)(L\Sigma R^\dagger)^\dagger = \mathrm{Tr}(L\Sigma R^\dagger R\Sigma^\dagger L^\dagger) = \mathrm{Tr}\,\Sigma\Sigma^\dagger$. Unfortunately this is useless since $\Sigma\Sigma^\dagger = I$. It does point the way, however. We take

$$\mathcal{L} = \frac{f_\pi^2}{4}\,\mathrm{Tr}\,\partial_\mu\Sigma\partial^\mu\Sigma^\dagger. \tag{11.7}$$

If we expand in powers of the π field we find

$$\partial_\mu\Sigma = i2\partial_\mu\pi \cdot T/f_\pi + \ldots \tag{11.8}$$

and thus the quadratic part of our Lagrangian is

$$\mathcal{L} = \mathrm{Tr}\,\partial_\mu\pi \cdot T\partial^\mu\pi \cdot T = \frac{1}{2}\partial_\mu\pi^a\partial^\mu\pi^a, \tag{11.9}$$

which is exactly correct for a scalar field theory. On the other hand there is no mass term. This is just what we expected: Goldstone bosons. There are interactions, but every interaction contains some derivatives. Now this Lagrangian contains terms with many powers of the π field. It is not renormalizable. We use it only at the Born level.

So far we have a theory only of the massless pseudoscalars. We must couple it to the gauge theory to learn something interesting. Of the $SU(2) \times SU(2)$ currents, only some of them are gauged. In particular, the gauge theory uses all the $SU(2)_L$, but not all the $SU(2)_R$. If we wanted to find the covariant derivative acting on left-handed quarks we would write

$$D_\mu = \partial_\mu + igW_\mu \cdot T + ig'B_\mu(Y/2), \tag{11.10}$$

where $Y/2$ would be a 2×2 matrix equal to $(1/6)I$. On right-handed quarks we would have $T = 0$ and the weak hypercharge would be arranged so that it accounted for the entirety of the electric charge Q. In terms of the matrices that act on the left-handed quarks, we would have on the right-handed quarks,

$$D_\mu = \partial_\mu + ig'(T_3 + Y/2)B_\mu. \tag{11.11}$$

This tells us how to make the covariant derivative that acts on Σ. Remembering that it is $R^\dagger$ that occurs in the transformation law, we take

$$D_\mu\Sigma = \partial_\mu\Sigma + iL_\mu\Sigma - i\Sigma R_\mu, \tag{11.12}$$

with

$$L_\mu = gW_\mu \cdot T + g'B_\mu Y/2, \tag{11.13}$$

$$R_\mu = g'B_\mu(T_3 + Y/2). \tag{11.14}$$

Now we can write a Lagrangian with gauge interactions:

$$\mathcal{L} = \frac{f_\pi^2}{4} \operatorname{Tr} D_\mu\Sigma(D^\mu\Sigma)^\dagger. \tag{11.15}$$

This isn't actually invariant under $SU(2) \times SU(2)$. The reason is that in R_μ we have T_3. This singles out a particular direction. Of course if g' were zero, the symmetry would be restored. What does Eq. (11.15) actually contain physically? To find out we must expand through quadratic order in the fields. Writing $\tilde{\pi} = \pi \cdot T$

$$\begin{aligned}\operatorname{Tr} D_\mu\Sigma(D^\mu\Sigma)^\dagger \rightarrow \operatorname{Tr} \Big(&+2i\partial_\mu\tilde{\pi}/f_\pi - 2\partial_\mu\tilde{\pi}\tilde{\pi}/f_\pi^2 - 2\tilde{\pi}\partial_\mu\tilde{\pi}/f_\pi^2 \\ &+iL_\mu - 2L_\mu\tilde{\pi}/f_\pi - 2iL_\mu\tilde{\pi}\tilde{\pi}/f_\pi^2 \\ &-iR_\mu + 2\tilde{\pi}R_\mu/f_\pi + 2i\tilde{\pi}\tilde{\pi}R_\mu/f_\pi^2\Big) \\ \times \Big(&-2i\partial^\mu\tilde{\pi}/f_\pi - 2\partial^\mu\tilde{\pi}\tilde{\pi}/f_\pi^2 - 2\tilde{\pi}\partial^\mu\tilde{\pi}/f_\pi^2 \\ &-iL^\mu - 2\tilde{\pi}L^\mu/f_\pi + 2i\tilde{\pi}\tilde{\pi}L^\mu/f_\pi^2 \\ &+iR^\mu + 2R^\mu\tilde{\pi}/f_\pi - 2iR^\mu\tilde{\pi}\tilde{\pi}/f_\pi^2\Big).\end{aligned} \tag{11.16}$$

Keeping terms through $\tilde{\pi}^2$ and restoring the $f_\pi^2/4$ to obtain the Lagrangian,

$$\begin{aligned}\mathcal{L} = \operatorname{Tr}\Big\{&\partial_\mu\tilde{\pi}\partial^\mu\tilde{\pi} + \frac{f_\pi^2}{4}(L_\mu - R_\mu)(L^\mu - R^\mu) + f_\pi\partial_\mu\tilde{\pi}(L^\mu - R^\mu) \\ &-if_\pi^2[R_\mu, L^\mu]\tilde{\pi} - i\partial_\mu\tilde{\pi}[L^\mu + R^\mu, \tilde{\pi}] - [R_\mu, \tilde{\pi}][L^\mu, \tilde{\pi}]\Big\}.\end{aligned} \tag{11.17}$$

The first term of Eq. (11.17) is just the kinetic energy of the technipions. The second term gives mass to the vectors as we see from

$$\begin{aligned}L_\mu - R_\mu &= gW_\mu \cdot T + g'B_\mu Y/2 - g'B_\mu(Y/2 + T_3) \\ &= \frac{g}{\sqrt{2}}(T^+W_\mu^+ + T^-W_\mu^-) + (gW_{3\mu} - g'B)T_3,\end{aligned} \tag{11.18}$$

and

$$\begin{aligned}\frac{f_\pi^2}{4}\frac{g^2}{2}\operatorname{Tr}(T^+W^+ + T^-W^-)^2 &= \frac{f_\pi^2 g^2}{8}\operatorname{Tr} 2T^+T^-W^+W^- \\ &= \frac{f_\pi^2 g^2}{4}W^+W^- = \frac{f_\pi^2 g^2}{8}(W_1^2 + W_2^2).\end{aligned} \tag{11.19}$$

Comparing with Eq. (2.1) we see that for our technicolor model with just one technifermion doublet to give the correct mass for the W we need $f_\pi = v$.

Moreover, the calculation of the Z mass

$$\begin{aligned}\frac{f_\pi^2}{4}\operatorname{Tr}(gW_3 - g'B)^2 T_3^2 &= \frac{f_\pi^2}{8}(gW_3 - g'B)^2 \\ &= \frac{f_\pi^2 g^2}{8}\frac{g^2+g'^2}{g^2}Z^2, \qquad (11.20)\end{aligned}$$

yields the standard relation: $m_W^2/m_Z^2 = g^2/(g^2+g'^2) = \cos^2\theta_W$.

The term with $\partial_\mu\tilde{\pi}(L^\mu - R^\mu)$ displays how the W and Z eat the appropriate Goldstone bosons. From Eq. (11.18) we see that the charged Ws eat the charged technipions and the Z eats the neutral one. The term involving $\partial_\mu\tilde{\pi}[L^\mu + R^\mu, \tilde{\pi}]$ would seem to give the couplings of the W, Z, and photon to technipion pairs. The only problem is that in our simple model there are only three technipions and all of them are eaten. This coupling then is simply part of the trilinear gauge boson coupling after the spontaneous symmetry breaking. If we want a model with real technipions, we need to start with more technifermions.

A logical choice is to take a full generation of technifermions.[31] We have a U and D, each in three ordinary colors, and an E and N, each lacking ordinary color. In addition, all couple to the superstrong technicolor interactions. The original $SU(8)\times SU(8)$ global symmetry is broken by the technicolor condensate to the vector $SU(8)$.

Each of the 63 generators of the broken $SU(8)$ presents us with a Goldstone boson. Three of these are eaten by the W and Z. We can enumerate the Goldstone bosons as follows. There are technipions, which are isotriplets, e.g., $U\overline{D}$, $E\overline{N}$. Since there are three colors, there are nine $Q\overline{Q}$ technipions, of which one is colorless and eight are color-octet. There are three colored $Q\overline{L}$ triplets and three colored $\overline{Q}L$ triplets. In addition there is one $L\overline{L}$ triplet. Altogether there are 16 isotriplets, of which one must be eaten. Similarly, there are 16 isosinglets, except that the combination that is an $SU(8)$ singlet is not a Goldstone boson. In sum there are $16\times 3 + 15 = 63$ Goldstone bosons, of which 3 are eaten.

If we return to our formulas for the W and Z masses, Eqs. (11.19) and (11.20), we see that the trace extends over four isodoublets, the three colored techniquarks (U,D) and the colorless technileptons, (N,E). As a result, f_π needs to be only half as large, $v/2$.

Our goal is to calculate the technicolor contribution to Δr and thus to the $W - Z$ mass splitting. Our general expression for Δr, from Eq. (7.2), is

$$\begin{aligned}\Delta r &= -\Pi'_{\gamma\gamma}(0) + \frac{c^2}{s^2}\Pi'_{ZZ}(m_Z^2) + \frac{s^2-c^2}{s^2}\Pi'_{WW}(m_W^2) \\ &\quad + \frac{c^2}{s^2}\left(\frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2}\right) \qquad . \qquad (11.21)\end{aligned}$$

At this point we have to confess to a swindle. We have contended we can calculate the Πs just with dispersion relations. However, this cannot be completely true since, for example we must have $\Pi_{\gamma\gamma}(0) = 0$ because of conservation of the electromagnetic current. Similarly $\Pi_{3Q}(0) = 0$. These relations do not follow from our dispersion relations, which are of course positive definite and divergent for $\Pi_{\gamma\gamma}(0)$ and $\Pi_{3Q}(0)$. If we enforce these conditions, then $\Pi_{ZZ}(0) = (g/c)^2\Pi_{33}(0)$. The final term in Eq. (11.21) is then proportional to $\Pi_{33}(0) - \Pi_{11}(0)$, that is, proportional to $\Delta\rho$. In our model with degenerate technimultiplets this vanishes. Thus, writing $x = \sin^2\theta_W$, we are left with

$$\begin{aligned}\Delta r &= -\frac{1}{\pi}\int\frac{ds}{s^2}\operatorname{Im}\Pi_{\gamma\gamma}(s) + \frac{1}{\pi}\int ds\frac{\operatorname{Im}\Pi_{WW}(s)}{s(s-m_W^2)} \\ &\quad + \frac{1-x}{x}\frac{1}{\pi}\int ds\left[\frac{\operatorname{Im}\Pi_{ZZ}(s)}{s(s-m_Z^2)} - \frac{\operatorname{Im}\Pi_{WW}(s)}{s(s-m_W^2)}\right] \quad . \qquad (11.22)\end{aligned}$$

To evaluate this expression we need the partial widths of the virtual photon, W, and Z into technipions. Let us begin by calculating the couplings of the gauge bosons to the technipions from the term

$$\operatorname{Tr}\partial_\mu\tilde{\pi}[L^\mu + R^\mu, \tilde{\pi}] = \operatorname{Tr}(L^\mu + R^\mu)[\tilde{\pi}, \partial_\mu\tilde{\pi}] \quad . \qquad (11.23)$$

First we calculate

$$\begin{aligned}L_\mu + R_\mu &= gW_\mu\cdot T + g'B_\mu(T_3 + Y) \\ &= \frac{g}{\sqrt{2}}(T^+W_\mu^+ + T^-W_\mu^-) + \frac{g}{c}(T_3 - 2xQ)Z_\mu + 2eQA_\mu. \qquad (11.24)\end{aligned}$$

For simplicity, look at just the coupling of the photon.

$$-2ieA_\mu\operatorname{Tr}Q[\tilde{\pi}, \partial^\mu\tilde{\pi}] = -2ieA_\mu\operatorname{Tr}\partial^\mu\tilde{\pi}[Q,\tilde{\pi}] \quad . \qquad (11.25)$$

Now commuting with Q just multiplies each component of the field by its charge. For some particular technipion, π_x and its antiparticle $\overline{\pi_x}$, taking into account the normalization $\mathrm{Tr}\, T^a T^b = \delta_{ab}$ we get

$$-ieQ_x A_\mu(\pi_x \partial^\mu \overline{\pi_x} - \overline{\pi_x}\partial^\mu \pi_x) \tag{11.26}$$

where Q_x is the charge of π_x in units of e. This is exactly what we expect from ordinary electrodynamics. Thus we infer that the coupling is just one-half of what we read from Eq. (11.24), so Z couples as

$$\frac{g}{c}\left(\frac{1}{2}T_3 - xQ\right) \tag{11.27}$$

and W^+ as

$$\frac{g}{2\sqrt{2}}T^+ \tag{11.28}$$

and γ as

$$eQ. \tag{11.29}$$

A coupling to a vector of the form

$$aV_\mu(\pi_i\partial^\mu\pi_j - \pi_j\partial^\mu\pi_i) \tag{11.30}$$

gives a decay matrix element

$$\mathcal{M} = a\epsilon\cdot(p-p') \tag{11.31}$$

where ϵ is the polarization vector of the decaying particle. The angular average of the square of the matrix element is

$$|\mathcal{M}|^2 = \frac{4}{3}a^2 p_{\rm cm}^2 \tag{11.32}$$

and thus

$$\Gamma(V \to \pi\pi') = \frac{a^2 p_{\rm cm}^3}{6\pi s} \tag{11.33}$$

where the mass squared of the decaying vector particle is s and

$$p_{\rm cm}^2 = (s-4m^2)/4. \tag{11.34}$$

We assume that all the technipions in a single multiplet are degenerate, with mass m. Combining the above results we can find the partial widths for photon, Z, and W decay. Following [32] we consider technipion multiplets of isospin t and identify each member by its third component of isospin, t_3 and the multiplet's hypercharge y. Then for Z decay we have, writing $\sin^2\theta_W = x$

$$\Gamma(Z \to \Pi(t,t_3,y),\overline{\Pi}(t,-t_3,-y)) = \frac{g^2}{1-x}\frac{p_{\rm cm}^3}{6\pi s}\left[(1/2-x)t_3 - xy/2\right]^2. \tag{11.35}$$

Similarly

$$\Gamma(W^+ \to \Pi(t,t_3,y),\overline{\Pi}(t,-t_3+1,-y)) = \frac{g^2}{8}\frac{p_{\rm cm}^3}{6\pi s}[t(t+1) - t_3(t_3+1)] \tag{11.36}$$

and

$$\Gamma(\gamma \to \Pi(t,t_3,y),\overline{\Pi}(t,-t_3,-y)) = g^2 x\frac{p_{\rm cm}^3}{6\pi s}\left[t_3 + y/2\right]^2. \tag{11.37}$$

If we sum these expressions over the elements of an isomultiplet, recognizing that the mean value of t_3^2 is clearly one-third of $T^2 = t(t+1)$, we see that

$$\Gamma(Z \to \Pi(t,y),\overline{\Pi}(t,-y)) = \frac{g^2}{1-x}\frac{p_{\rm cm}^3}{6\pi s}(2t+1)\left[(1/2-x)^2\frac{t(t+1)}{3} + x^2y^2/4\right], \tag{11.38}$$

$$\Gamma(W^+ \to \Pi(t,y),\overline{\Pi}(t,-y)) = \frac{g^2}{8}\frac{p_{\rm cm}^3}{6\pi s}(2t+1)\frac{2t(t+1)}{3}, \tag{11.39}$$

$$\Gamma(\gamma \to \Pi(t,y),\overline{\Pi}(t,-y)) = g^2 x\frac{p_{\rm cm}^3}{6\pi s}(2t+1)\left[\frac{t(t+1)}{3} + \frac{y^2}{4}\right]. \tag{11.40}$$

These expressions are to be inserted into Eq. (11.22). Each integral extends from the threshold for the production of a pair of technipions, $s_{\rm thr} = 4m^2$. For the upper limit we write Λ^2, indicating a value above which our description in terms of technipions ought to be replaced by a description in terms of techniquarks. This is entirely analogous to QCD where at very low energies e^+e^- annihilation must be viewed in terms of the few-pion final states, while at high energies it is viewed in terms of the underlying quarks. For the moment, let us ignore m_Z^2 and m_W^2. Then Eq. (11.22) yields the formula of Golden and Randall:

$$\Delta r = \frac{g^2}{6\pi^2}\frac{t(t+1)(2t+1)}{3}\int_{4m^2}^{\Lambda^2}\frac{ds}{s^3}p_{\rm cm}^3\frac{\sqrt{s}}{2}$$

$$= \frac{g^2}{96\pi^2}\frac{t(t+1)(2t+1)}{3}\ln\frac{\Lambda^2}{4m^2} \tag{11.41}$$

We shall return to an examination of the limitations of this expression, but for the present let's just estimate its value in the one-generation model described above. We follow Ref. [32] by letting $\Lambda = 4\pi v$, $m = m_Z$. Note that Eq. (11.41) assumes that the multiplets are not self-conjugate and must be divided by two for our 16-1=15 multiplets. With $g^2/8 = G_F m_W^2/\sqrt{2} = 0.0528$ we find $\Delta r = 0.0378$ and from Eq. (7.13)

$$\delta m_W = -644 \text{ MeV}. \tag{11.42}$$

This result [32] is in agreement with the value given by Lynn, Peskin, and Stuart. [34] Of course there is arbitrariness in the choice of the values for Λ and m, but nevertheless the shift is quite large and not supported by the existing data.

12 More on Technicolor

How can we make the technicolor result more precise? First notice that the shift in Δr (or m_W) is intrinsically finite, despite the logarithm in Eq. (11.41). At high energies the theory is one of techniquarks and since we have a full generation of them, the theory is finite. In reality, then, the dispersion relations for the various Πs in the combinations we need would actually converge. We have simply used a model of non-interacting technipions whose couplings to the gauge particles are dictated by the underlying symmetry. How can we make a more reliable model?

The approach of Peskin and Takeuchi [33] is to rely on data from QCD, that is low-energy meson physics, and scale it to simulate technicolor. Since the technicolor interactions conserve parity and isospin just like QCD, it is convenient to express the various currents in terms of vector and axial currents. We have the correspondence

$$J_+ = \frac{g}{2\sqrt{2}}(V_+ - A_+), \tag{12.1}$$

$$J_Z = \frac{g}{c}\left[\frac{1}{2}(V_3 - A_3) - xV_Q\right], \tag{12.2}$$

$$J_\gamma = eV_Q \tag{12.3}$$

where V_+ is the vector isospin raising current, V_Q is the electromagnetic current, and so on. If we use these expressions and again treat m_Z and m_W as small we can rewrite Eq. (11.22) as

$$\Delta r = -\frac{g^2}{2\pi}\int\frac{ds}{s^2}\left[\operatorname{Im}\Pi_{VV}(s) - \operatorname{Im}\Pi_{AA}(s)\right] \tag{12.4}$$

where $\Pi_{VV} = \Pi_{V_3V_3}$, etc. In obtaining Eq. (12.4) we assumed the technimultiplets were each degenerate so that the average value of T_3Y vanishes. Restoring m_Z and m_W^2 would induce some small corrections.

In the model of the previous section only the decays of the vectors into a pair of technipions were considered. Since the $V \to PP$ decay goes only through the vector, as opposed to axial, coupling, we treated Π_{AA} as being zero. Inserting the partial width for $V \to \pi\pi$ from Eq. (11.33),

$$\Gamma = \frac{g^2 p_{cm}^3 T_3^2}{6\pi s} \tag{12.5}$$

into Eq. (12.4) recovers Eq. (11.41) since summing over all possible technipion intermediate states introduces a factor $\sum T_3^2 = t(t+1)(2t+1)/3$.

More generally we expect decays of the vector particles into multitechnipion states, analogous to the decays of vector mesons into multipion states. The implications of chiral symmetry for such decays was investigated in the classic work of Weinberg.[35] The situation here is closely related.[33] We can evaluate the contribution of vector (technirho) and axial (techni-A_1) states to Eq. (12.4) in a simple fashion. The amplitude for a gauge-vector V to decay into a technipion pair through a technirho is just

$$\begin{aligned} \mathcal{M} &= \frac{g_V}{s - m_\rho^2 + i\Gamma_\rho m_\rho} g_{\rho\pi\pi}\epsilon\cdot(p - p') \\ &= \frac{g_V}{s - m_\rho^2 + i\Gamma_\rho m_\rho}\mathcal{M}_{\rho\pi\pi} \end{aligned} \tag{12.6}$$

where g_V gives the direct $\rho - V$ coupling. Thus

$$\begin{aligned} \Gamma_V(s) &= \Gamma_\rho\frac{g_V^2}{(s - m_\rho^2)^2 + \Gamma_\rho^2 m_\rho^2} \\ &= \frac{\pi g_V^2}{m_\rho}\delta(s - m_\rho^2) \end{aligned} \tag{12.7}$$

so for $\operatorname{Im}\Pi_{VV}$ the technirho contribution is

$$\operatorname{Im}\Pi_{VV} = -\pi g_\rho^2 \delta(s - m_\rho^2) \ . \tag{12.8}$$

A similar formula applies to the axial-vector A_1:

$$\operatorname{Im}\Pi_{AA} = -\pi g_{a1}^2 \delta(s - m_{a1}^2) \ . \tag{12.9}$$

Substituting in Eq. (12.4) we get a contribution to Δr of

$$\Delta r = \frac{g^2}{2}\left[\frac{g_\rho^2}{m_\rho^4} - \frac{g_{a1}^2}{m_{a1}^4}\right]. \tag{12.10}$$

This leaves us with the problem of determining the masses and couplings of the technirho and techni-$A1$. Such considerations would take us too far afield. An attractive approach [33] uses the Weinberg sum rules and the KSFR relation[36], both of which were developed for ordinary pion physics in the 1960s.

13 Observables at Z

After this long interlude on technicolor, let us return to the world of experimental measurement. Experiments at LEP and SLC should give us data on Γ_{tot}, Γ_μ/Γ_{had}, A_{LR}, A_{FB}, $P(\tau)$, etc. Each is sensitive to m_t, m_H, and new physics. It is worthwhile understanding how well we might measure some of these quantities and to what extent such measurements would actually test the standard model.

Let us consider in particular the left-right asymmetry:

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}. \tag{13.1}$$

The left-right asymmetry is not a parity-violating effect. It receives contributions from the two-photon intermediate state. Setting that aside, we are interested *in the amplitude from the Z and from Z-photon interference.* It is easiest to analyze the process by considering one helicity at a time. For $e_L^- e_R^+ \to f_L \bar{f}_R$ the amplitude is proportional to

$$\frac{Q_L^{\gamma e} Q_L^{\gamma f}}{s} + \frac{Q_L^{Ze} Q_L^{Zf}}{s - m_Z^2 + i\Gamma m_Z}, \tag{13.2}$$

where in the obvious notation Q_L^{Ze} gives the coupling of the left-handed electron to the Z, etc. In every instance we specify the helicity of the fermion (not the antifermion). The relative importance of the Z relative to the photon is given by

$$\xi = \left(1 - \frac{m_Z^2}{s} - i\frac{\Gamma m_Z}{s}\right)^{-1}. \tag{13.3}$$

In terms of this variable we can calculate simply the cross sections for the four relevant processes. Like and opposite helicity combinations have characteristic angular dependences that reflect conservation of angular momentum:

$$\begin{aligned}
|\mathcal{M}(e_L^- e_R^+ \to f_L \bar{f}_R)|^2 &\propto |Q_L^{\gamma e} Q_L^{\gamma f} + \xi Q_L^{Ze} Q_L^{Zf}|^2 \\
&\quad \times (1 + \cos\theta)^2, && (13.4)\\
|\mathcal{M}(e_L^- e_R^+ \to f_R \bar{f}_L)|^2 &\propto |Q_L^{\gamma e} Q_R^{\gamma f} + \xi Q_L^{Ze} Q_R^{Zf}|^2 \\
&\quad \times (1 - \cos\theta)^2, && (13.5)\\
|\mathcal{M}(e_R^- e_L^+ \to f_L \bar{f}_R)|^2 &\propto |Q_R^{\gamma e} Q_L^{\gamma f} + \xi Q_R^{Ze} Q_L^{Zf}|^2 \\
&\quad \times (1 - \cos\theta)^2, && (13.6)\\
|\mathcal{M}(e_R^- e_L^+ \to f_R \bar{f}_L)|^2 &\propto |Q_R^{\gamma e} Q_R^{\gamma f} + \xi Q_R^{Ze} Q_R^{Zf}|^2 \\
&\quad \times (1 + \cos\theta)^2. && (13.7)
\end{aligned}$$

From these we can determine both the forward-backward and left-right asymmetries. In particular, near the Z peak, where the photon's contribution can be ignored, we have

$$A_{FB} = \frac{3}{4}\cdot\frac{Q_L^{Ze^2} - Q_R^{Ze^2}}{Q_L^{Ze^2} + Q_R^{Ze^2}}\cdot\frac{Q_L^{Zf^2} - Q_R^{Zf^2}}{Q_L^{Zf^2} + Q_R^{Zf^2}}, \tag{13.8}$$

$$A_{LR} = \frac{Q_L^{Ze^2} - Q_R^{Ze^2}}{Q_L^{Ze^2} + Q_R^{Ze^2}}. \tag{13.9}$$

A careful evaluation of radiative corrections shows that we should use an effective $x = 0.235$, which gives $A_{LR} = 0.12$, $A_{FB} = 0.011$.

How well can these be measured? Measurement of the left-right asymmetry requires counting the number of observed events with left-handed electrons

	$N = 10^4$	10^5	10^6
$\delta P/P = 0.05$	0.023	0.009	0.006
$\delta P/P = 0.03$	0.023	0.008	0.004
$\delta P/P = 0.01$	0.022	0.007	0.003

Table 13.1: Uncertainty in A_{LR} from statistics and limitations in the accuracy of the determination of the polarization, P. The value of N is the number of observed Zs.

$N =$	10^4	10^5	10^6
$\delta P/P = 0.05$	0.003	0.0014	0.0010
$\delta P/P = 0.03$	0.003	0.0011	0.0005
$\delta P/P = 0.01$	0.003	0.0009	0.0004

Table 13.2: The uncertainty in $\sin^2\theta_W$ obtained from the measurement of A_{LR} using the results of Table 13.1.

incident versus the number with right-handed electrons incident, assuming the integrated luminosity was the same for the two. Then if the degree of polarization is P

$$A_{LR} = \frac{1}{P}\left(\frac{N_L - N_R}{N_L + N_R}\right). \tag{13.10}$$

Uncertainties are introduced in particular by statistical fluctuations and errors in the measurement of the polarization. These introduce an uncertainty in A_{LR}

$$\delta A_{LR} = \sqrt{A_{LR}^2\left(\frac{\delta P}{P}\right)^2 + \frac{1}{NP^2}}. \tag{13.11}$$

Taking as nominal values

$$P^2 \approx 0.20; \quad A_{LR} = 0.12, \tag{13.12}$$

we obtain the results shown in Tables 13.1 and 13.2.

As a point of comparison, consider the capability of LEP in measuring the forward-backward asymmetry.[37]

statistical error	± 0.002
systematic error due to uncertainty in m_Z	± 0.0015
systematic error due to uncertainty in QED correction	± 0.002
systematic error due to apparatus effects	± 0.001
total	± 0.0035

Table 13.3: Anticipated errors in measurement of the forward-backward asymmetry in the muon final state at LEP.[37]

From Eqs. (13.8) and (13.9) we see that for a measurement of the forward-backward asymmetry with uncertainty ΔA_{FB} to be equivalent to a measurement of the left-right asymmetry, we must have

$$\begin{aligned} \Delta A_{FB} &= \frac{3}{4} 2 A_{LR} \Delta A_{LR} \\ &= 0.18 \Delta A_{LR}, \end{aligned} \tag{13.13}$$

so the anticipated LEP result would have the value of a left-right asymmetry measurement with $\Delta A_{LR} = 0.019$, a result that could be achieved at SLC with somewhat more than 10,000 observed Zs. LEP itself might be able to obtain polarized beams and measure A_{LR} with both high statistics and high precision determination of the polarization, as discussed by Treille at this summer school.[38]

14 Mixing with a New Z

We have discussed the possible signatures for new physics through radiative corrections to the standard model, but a more direct manifestation would be the mixing of the Z of the regular with a massive one. It is not simply idle speculation to talk of extra Zs since the $SU(3) \times SU(2) \times U(1)$ model surely appears incomplete. Models with $SO(10)$ or E_6 as symmetries have additional neutral gauge bosons, which could mix with the ordinary Z. Of course, the most direct evidence for such a Z' would be its direct observation. The Fermilab Tevatron is the best place to look right now and CDF would probably have found a Z' with a mass less than about 400 GeV or so, the actual limit depending on the couplings of the new Z to the fermions.

In analyzing the effect of a new Z at the Z peak it is important to keep in mind how the data are actually used. Typically the mass of the Z is interpreted by inferring a value for $\sin^2\theta_W$, but the relation used to do this is true only in the standard model with a single Z. As we shall see, this incorrect value of $\sin^2\theta_W$, as well as the mixing itself, both contribute to the shift of observables from the values predicted by the standard model.

Let us define quite generally a mixing angle λ between the Z_{10} from the standard model and the new Z_{20} that gives the physical states:

$$Z_1 = \cos\lambda\ Z_{10} + \sin\lambda\ Z_{20}, \tag{14.1}$$

$$Z_2 = -\sin\lambda\ Z_{10} + \cos\lambda\ Z_{20}. \tag{14.2}$$

The new Zs couple with new charges that are linear combinations of the charges to which the unmixed Zs coupled:

$$Q_1^f = \cos\lambda\ Q_{10}^f - \sin\lambda\ Q_{20}^f, \tag{14.3}$$

$$Q_2^f = \sin\lambda\ Q_{10}^f + \cos\lambda\ Q_{20}^f. \tag{14.4}$$

Here Q_{10}^f is the standard neutral current charge:

$$Q_{10}^f = \frac{e}{\sin\theta_W\cos\theta_W}\left(T_{3L}^f - \sin^2\theta_W Q_{em}\right). \tag{14.5}$$

Because of the mixing, there is a mass shift,

$$M_1^2 - M_{10}^2 \approx -\lambda^2(M_{20}^2 - M_{10}^2), \tag{14.6}$$

$$\approx -\lambda^2(M_2^2 - M_1^2). \tag{14.7}$$

Now having measured the physical Z mass we would deduce the *wrong* value of $\sin^2\theta_W = \tilde{x}_W$:

$$\tilde{x}_W = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4\pi\alpha(1+\Delta r)}{\sqrt{2}M_1^2 G_F}}\right). \tag{14.8}$$

The correct value could only be deduced from the unmixed mass:

$$x_W = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4\pi\alpha(1+\Delta r)}{\sqrt{2}M_{10}^2 G_F}}\right). \tag{14.9}$$

Thus there is an error in the deduced value of x_W

$$\delta x_W = x_W - \tilde{x}_W \tag{14.10}$$

$$\approx -\lambda^2 \cdot \frac{x_W(1-x_W)}{1-2x_W} \cdot \frac{M_2^2 - M_1^2}{M_1^2}. \tag{14.11}$$

It is conventional to take as the parameters of mixing the mass of the heavier Z and the mixing angle λ. I find that this is really not the most convenient approach. Taking δx_W and λ as the two parameters for the mixing has some advantages over the standard choice of M_2^2 and λ. The deviations in the observable quantities in e^+e^- annihilation at the Z are linear in δx_W and λ, so each measurement yields a linear band as an allowed region in the δx_W - λ plane. In addition, the allowed range will turn out to be a finite portion of the plane. The curves of fixed M_2^2 are parabolas and it is not hard to read the value of M_2 from the plot.

As an example of a model with extra Zs let us consider E_6. [39, 40, 41] One possible scheme involves a two-stage breaking,

$$\begin{aligned} E_6 &\rightarrow SO(10) \times U(1)_\psi, \\ SO(10) &\rightarrow SU(5) \times U(1)_\chi, \end{aligned} \tag{14.12}$$

that provides two potential Zs, Z_ψ and Z_χ, one for each $U(1)$ factor. String theory suggests that a particular linear combination, Z_η is the lighter of the two new physical Zs. The couplings of Z_η are related to those of Z_χ and Z_ψ by

$$Q_\eta = -\sqrt{5/8}Q_\psi + \sqrt{3/8}Q_\chi. \tag{14.13}$$

The $U(1)$s that are associated with the new Zs both commute with $SU(5)$ so the new charges of all the fermions in a single representation of $SU(5)$ are identical. Explicitly

$$Q_\psi(e_L^-, e_L^+, \nu_L, u_L, ..) = \sqrt{5/72} \times (e/\cos\theta_W), \tag{14.14}$$

$$Q_\chi(u_L, d_L, e_L^+, \overline{u}_L) = -\sqrt{1/24} \times (e/\cos\theta_W), \tag{14.15}$$

$$Q_\chi(e_L^-, \nu_L, \overline{d}_L) = 3\sqrt{1/24} \times (e/\cos\theta_W), \tag{14.16}$$

$$Q_\eta(u_L, d_L, e_L^+, \overline{u}_L) = -(1/3) \times (e/\cos\theta_W), \tag{14.17}$$

$$Q_\eta(e_L^-, \nu_L, \overline{d}_L) = (1/6) \times (e/\cos\theta_W). \tag{14.18}$$

If we restrict ourselves to measurements at the peak of the Z and ignore the purely photonic background (the interference term vanishes at the peak of the Z since the Z amplitude is purely imaginary while the photonic amplitude is purely real) all observables can be expressed in terms of partial widths to final states with fermions of specified helicities. Let us then determine the effect of Z mixing on each of these partial widths.

The real partial width is given by

$$\Gamma_f \equiv \Gamma(Z \to f_L \overline{f}_R) = \frac{m_Z}{24\pi} Q_1^{f2}, \tag{14.19}$$

while the width inferred from the measured Z mass and the standard model is

$$\tilde{\Gamma}_f = \frac{m_Z}{24\pi} Q_{10}^{f2}, \tag{14.20}$$

with Q_{10}^f evaluated using $\tilde{x}_W$ in place of x_W. To first order in δx_W and λ,

$$\frac{\delta\Gamma_f}{\Gamma_f} = \frac{\Gamma_f - \tilde{\Gamma}_f}{\Gamma_f} = \lambda \frac{\partial \ln Q_1^{f2}}{\partial\lambda} + \delta x_W \frac{\partial \ln Q_{10}^{f2}}{\partial x_W}, \tag{14.21}$$

$$\equiv -2\lambda q^f + \delta x_W r^f, \tag{14.22}$$

where

$$q^f = \frac{Q_{20}^f}{Q_{10}^f}, \tag{14.23}$$

	Q_{10}^f	B^f	r^f
e^-	-0.19	0.019	−10.4
e^+	−0.17	0.015	5.6
ν	0.37	0.07	−2.9
u	0.25	0.10	−6.8
$\overline{u}$	0.12	0.02	5.6
d	−0.31	0.15	−4.6
$\overline{d}$	−0.06	0.01	5.6

Table 14.1: Values of the weak neutral charge Q_{10}^f, branching ratios in the standard model, and the coefficients r^f for the various fermions, f.

is the ratio of the new Z charge to the old Z charge, and

$$r^f = \frac{-1 + 2x_W}{x_W(1 - x_W)} - \frac{2Q_{em}^f}{T_{3L}^f - x_W Q_{em}^f}. \tag{14.24}$$

This latter quantity gives the importance of the shift in the apparent $\sin^2\theta_W$ and does not depend on the couplings of the new Z. Its values for the various fermions are given in Table 14.1. The effects of the new Z couplings are reflected in the coefficients q^f, shown in Table 14.2.

The total width is given by

$$\Gamma = 3(\Gamma_{e^-} + \Gamma_{e^+} + \Gamma_\nu + \Gamma_d + \Gamma_{\overline{d}}) + 2(\Gamma_u + \Gamma_{\overline{u}}). \tag{14.25}$$

It is simple to see that the deviation in the total width, that is, the difference between the standard model expectation based on the measured value of the Z mass, and the correct value is given by

$$\begin{aligned}\frac{\delta\Gamma}{\Gamma} = \frac{\Gamma - \tilde{\Gamma}}{\Gamma} &= -2\lambda \left[3(q^{e^-}B^{e^-} + q^{e^+}B^{e^+} + q^\nu B^\nu + q^d B^d + q^{\overline{d}} B^{\overline{d}}) \right. \\ &\quad \left. +2(q^u B^u + q^{\overline{u}} B^{\overline{u}})\right] \\ &\quad +\delta x_W \left[3(r^{e^-}B^{e^-} + r^{e^+}B^{e^+} + r^\nu B^\nu + r^d B^d + r^{\overline{d}} B^{\overline{d}})\right.\end{aligned}$$

	q_χ^f	q_ψ^f	q_η^f
e^-	-1.12	-0.48	-0.30
e^+	0.42	-0.54	0.69
ν	0.59	0.26	0.16
u	-0.29	0.37	-0.47
$\overline{u}$	-0.63	0.82	-1.03
d	0.23	-0.30	0.38
$\overline{d}$	-3.8	-1.63	-1.03

Table 14.2: Values of the ratio $q^f = Q_{20}^f/Q_{10}^f$ of the neutral weak charges for Z_χ, Z_ψ, and Z_η for the various fermions, f.

$$+2(r^u B^u + r^{\overline{u}} B^{\overline{u}})\Big] . \tag{14.26}$$

From Tables 14.1 and 14.2 we find for the three cases of Z_χ, Z_ϕ, and Z_η respectively,

$$\frac{\delta\Gamma}{\Gamma} = -2\lambda \left\{ \begin{array}{c} 0.041 \\ -0.053 \\ 0.067 \end{array} \right\} + \delta x_W(-3.97). \tag{14.27}$$

From the data reviewed in Section 2 we see that the total width is known to about 1%. Supposing that we have perfect accord with theory, there is a constraint on λ and δx_W:

$$\left| -2\lambda \left\{ \begin{array}{c} 0.041 \\ -0.053 \\ 0.067 \end{array} \right\} - 3.97\delta x_W \right| < 0.01. \tag{14.28}$$

In Figures 14.1, 14.2, and 14.3 we show the constraints from the total width measurement on the Z_χ, Z_ψ, and Z_η.

As discussed in Section 12, the left-right asymmetry is given by

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}. \tag{14.29}$$

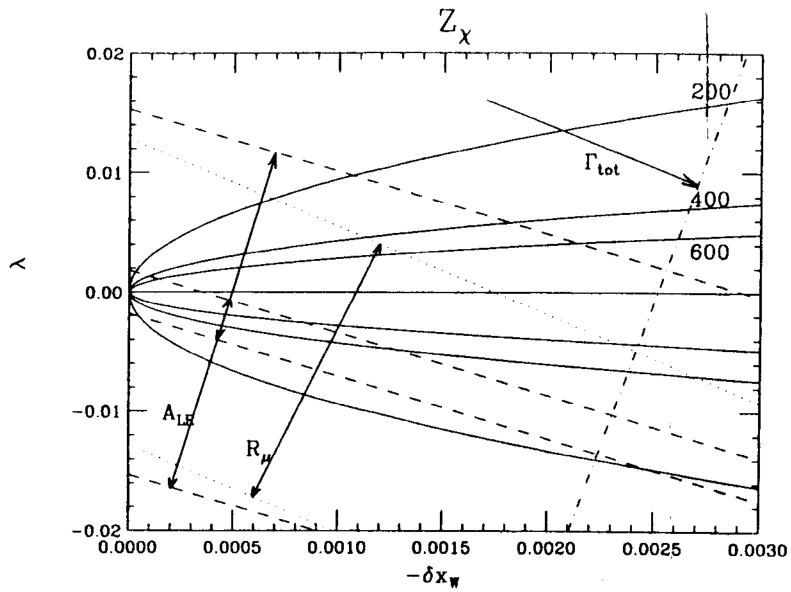


Figure 14.1: The $\delta x_W - \lambda$ plane showing the region that would be allowed by a measurement of R_μ to 1% (between the dotted lines), by a measurement of A_{LR} to ± 0.025 (between the dashed lines) or to 0.003 (between the inner dashed lines), and a measurement of the total width to 1% (to the left of the dot-dash line) for the Z_χ. From outside to inside, the parabolas indicate M_2= 200 GeV, 400 GeV, and 600 GeV.

This asymmetry depends only on the couplings of the Z to the electron and clearly is given by

$$A_{LR} = \frac{\Gamma_{e^-} - \Gamma_{e^+}}{\Gamma_{e^-} + \Gamma_{e^+}} \quad . \tag{14.30}$$

It follows that the deviation of the asymmetry is given by

$$\begin{aligned} \frac{\delta A_{LR}}{A_{LR}} &= \frac{A_{LR} - \tilde{A}_{LR}}{A_{LR}} \\ &= -2\frac{\Gamma_{e^-}\Gamma_{e^+}}{\Gamma_{e^-}^2 - \Gamma_{e^+}^2}\left(\frac{\delta\Gamma_{e^+}}{\Gamma_{e^+}} - \frac{\delta\Gamma_{e^-}}{\Gamma_{e^-}}\right) \end{aligned}$$

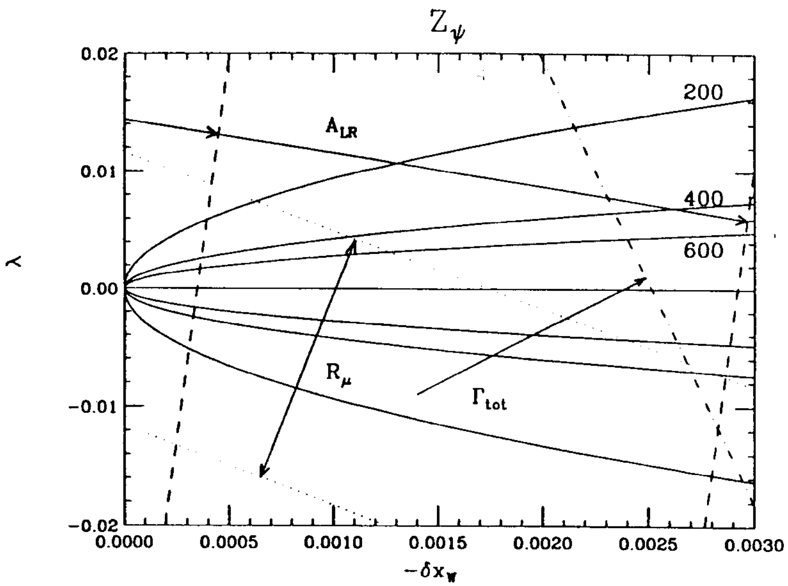


Figure 14.2: The $\delta x_W - \lambda$ plane showing the region that would be allowed by a measurement of R_μ to 1% (between the dotted lines), by a measurement of A_{LR} to ± 0.025 (to the left of the right dashed line) or to 0.003 (to the left of the left dashed lines), and a measurement of the total width to 1% (to the left of the dot-dash line) for the Z_ψ. From outside to inside, the parabolas indicate M_2= 200 GeV, 400 GeV, and 600 GeV.

$$= -2\frac{B^{e^-}B^{e^+}}{B^{e^-2} - B^{e^+2}}\left[-2\lambda(q^{e^+} - q^{e^-}) + \delta x_W(r^{e^+} - r^{e^-})\right]. \quad (14.31)$$

Using Tables 14.1 and 14.2 for Z_χ, Z_ψ, and Z_η,

$$\frac{\delta A_{LR}}{A_{LR}} = -4.122\left[-2\lambda\left\{\begin{array}{c}1.54\\-0.062\\0.99\end{array}\right\} + \delta x_W(16.1)\right] \quad . \qquad (14.32)$$

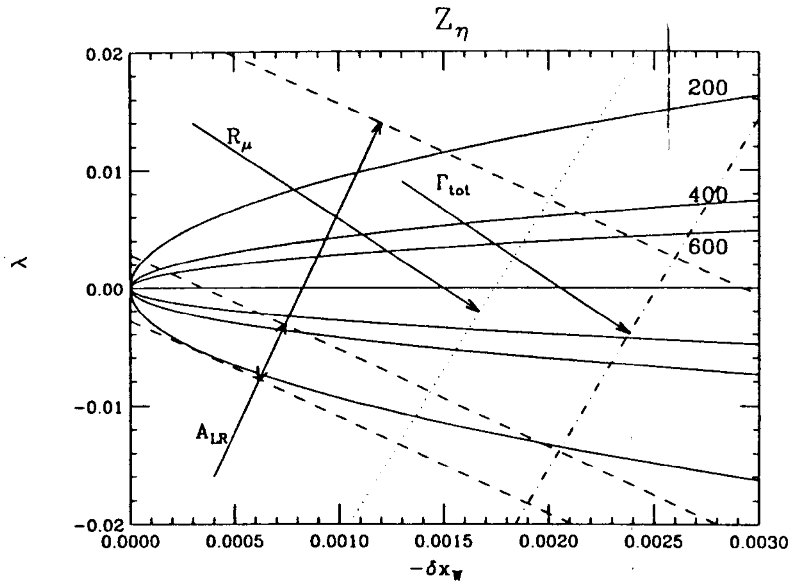


Figure 14.3: The $\delta x_W - \lambda$ plane showing the region that would be allowed by a measurement of R_μ to 1% (to the left of the dotted line), by a measurement of A_{LR} to ± 0.025 (below the upper dashed line) or to 0.003 (between the indicated dashed lines), and a measurement of the total width to 1% (to the left of the dot-dash line) for the Z_η. From outside to inside, the parabolas indicate M_2= 200 GeV, 400 GeV, and 600 GeV.

The constraint from measurement of A_{LR} to $\delta A_{LR} = 0.025$ gives

$$\left|-2\lambda\left\{\begin{array}{c}1.54\\-0.062\\0.99\end{array}\right\} + 16.1\delta x_W\right| < 0.047. \qquad (14.33)$$

The measurement of $R_\mu = \Gamma_\mu/\Gamma_{had} = B_\mu/B_{had}$ provides an independent test of the standard model. The deviation in R_μ is

$$\frac{\delta R_\mu}{R_\mu} = \frac{\delta\Gamma^{\mu^-}}{\Gamma^{\mu^-}}\cdot\frac{\Gamma^{\mu^-}}{\Gamma^{\mu^-}+\Gamma^{\mu^+}} + \frac{\delta\Gamma^{\mu^+}}{\Gamma^{\mu^+}}\cdot\frac{\Gamma^{\mu^+}}{\Gamma^{\mu^-}+\Gamma^{\mu^+}}$$

$$-2\left(\frac{\delta\Gamma^u}{\Gamma^u}\frac{B^u}{B^{had}}+\frac{\delta\Gamma^{\overline{u}}}{\Gamma^{\overline{u}}}\frac{B^{\overline{u}}}{B^{had}}\right)$$

$$-3\left(\frac{\delta\Gamma^d}{\Gamma^d}\frac{B^d}{B^{had}}+\frac{\delta\Gamma^{\overline{d}}}{\Gamma^{\overline{d}}}\frac{B^{\overline{d}}}{B^{had}}\right) \tag{14.34}$$

Here Γ^u represents the partial width for $Z \to u_L\overline{u}_R$. The coefficients 2 and 3 arise from the number of generations that contribute. Using the values in Tables 14.1 and 14.2,

$$\frac{\delta R_\mu}{R_\mu} = -2\lambda\left\{\begin{array}{c}-0.39\\-0.43\\0.10\end{array}\right\} + \delta x_W(-5.74). \tag{14.35}$$

Constraining this by a 1% measurement gives the band

$$\left|-2\lambda\left\{\begin{array}{c}-0.39\\-0.43\\0.10\end{array}\right\} + \delta x_W(-5.74)\right| < 0.01. \tag{14.36}$$

All of these results are displayed in Figures 14.1, 14.2, and 14.3.

In models where there are extra Zs but no extra Ws the W mass is independent of the Z mixing angle. Since the Z mass is always constrained to be its measured value, the Z mixing then has the effect of shifting the mass of the W. Since the W mass is inversely proportional to x_W for fixed G_F

$$\frac{\delta x_W}{x_W} = -\frac{2\delta m_W}{m_W}. \tag{14.37}$$

Recent results from CDF and UA2 were reviewed by Froidevaux who cited [42]

$$\frac{m_W{}^{CDF}}{m_Z} = 0.8775 \pm 0.0047 \pm 0.0021, \tag{14.38}$$

$$\frac{m_W{}^{UA2}}{m_Z} = 0.8831 \pm 0.0048 \pm 0.0026, \tag{14.39}$$

from which he concluded, perhaps a bit optimistically,

$$m_W = 80.07 \pm 0.22 \text{ GeV}. \tag{14.40}$$

This provides a constraint $-\delta x < 0.0013$. In addition CDF will set an explicit constraint of approximately

$$m_{Z'} < 400 \text{ GeV}, \tag{14.41}$$

though careful analysis would set different limit for each type of Z'.

Referring to Figures 14.1, 14.2, and 14.3 we see that these are powerful constraints indeed. For example, consider Z_η. Here the measurements of R_μ and Γ_{tot} provide no new limitations after the results from the hadron colliders are imposed. On the other hand, a high precision measurement of A_{LR} with δA_{LR} would significantly restrict the region in the λ - δx_W plane.

15 Summary

High statistics, high precision measurements at the Z are a primary means of searching for deviations from the standard electroweak model. To date no such deviations have been found, nor have any signs of the the much-awaited Higgs boson been seen. Improved data will lead to more and more restrictions on hypothetical models such as technicolor and multiple Zs. Of particular interest is the measurement of the left-right asymmetry using polarized electron beams. Complementary information from hadron colliders can provide lower limits for extra Zs and measure accurately the mass difference between the W and the Z.

The analysis of these data call for careful work on radiative corrections. The initial state radiative corrections can be understood quite easily and simple analytic expressions exist that are rather accurate. The true electroweak corrections are more complex, but they are dominated by corrections to the propagators of the gauge bosons. These can be pictured easily by considering the imaginary parts, which correspond to physical decays of the gauge bosons (albeit of variable mass), and then obtaining the real parts using dispersion relations. This not only provides a convenient conceptual framework, but allows the consideration of strong interactions, like those of technicolor.

To test the standard model it is necessary to pose an alternative. A particularly interesting alternative is furnished by adding an extra Z that mixes with the Z of the standard model. Aside from the couplings of the new Z, there are two parameters to consider, a mixing angle λ and the mass of the heavier Z. The mixing leads to a shift in the mass of the lighter Z and a consequent shift in the value of $\sin^2\theta_W$. It is easier to analyze data using as the parameters λ and this shift, δx_W. Each experiment at the Z yields an allowed linear band in the λ - δx_W plane. Already the data severly restrict this class of models for Zs that arise from E_6 theories.

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