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FIT70 — A KINEMATIC FITTING ROUTINE

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I. INTRODUCTION

This report describes a kinematic fitting program, FIT70, written as a closed subroutine in FORTRAN IV (on IBM System 360/91). This was used to perform the kinematic analysis of a large scale photoproduction experiment[1] with the streamer chamber at the Stanford Linear Accelerator Center. The purpose of the program is to adjust by minimizing a certain χ^2 the input data consisting of mass and momentum for each track (and the full error-matrix) associated with a vertex so that the fitted data satisfy the energy momentum conservation. The resulting χ^2 is also used as a test for the assumed mass hypothesis.

The general formulation of kinematic fitting has been known for many years.[2] The methods used in FIT70 are originally due to G. Ascoli and have been adapted from ILLFIT,[3] the first version of which dates back more than ten years. We describe some features:

1. 3-momentum fit.

The fitting is formulated in such a way that it is possible to ask only for 3-momentum conservation. To test for the visible momentum balance we first do this 3-momentum fit for each constrained fit. A large χ^2 at this stage implies the presence of missing neutrals. Furthermore remaining mass hypotheses of the same class may be skipped as a time-saving device, since the 3-momentum fit is insensitive to masses (apart from small energy-loss corrections). For example, in the reaction,

$$\gamma p \rightarrow \pi^+ \pi^- p \quad ,$$

with unknown photon energy this preliminary fit is a 2C fit requiring the transverse momentum balance. This fit serves as a finer filter than, for instance, testing m_γ^2 of measured values.

On the other hand if the 3-momentum fit is good, then the solutions are such that the subsequent fit including the energy constraint usually takes one step to converge to the final values.

2. Alternate method of iteration.

The numerical difficulties in the conventional method of minimizing, such as outright divergence or too-slow convergence, are all due to constraints. If the constraints were linear, one step would lead to the final answers regardless of the starting values. When these difficulties arise, we take a step in a different method of iteration, which is designed specifically to force the constraints to vanish.

3. Negative discriminant in 4-missing variable case.

This problem arises in the case including one missing neutral and one track with unmeasured momentum, as in the decay,

$$K^o \rightarrow \pi^+ \pi^- \pi^o \quad .$$

One has to solve a quadratic equation for the unmeasured momentum (if the particle has nonvanishing mass). Due to the measurement errors the associated discriminant may be negative. In this case a solution is forced by means of a formal 1C fit with the vanishing of the discriminant as a constraint.

In Section II we give the mathematical formulation, which also serves to introduce notations. In Section III the programs are described. And finally, in Section IV we discuss how this program was used in the photoproduction experiment and summarize results.

II. DESCRIPTION OF METHOD

A. Usual Mode of Iteration

We want to minimize a χ^2 with respect to variables, x_i ($i=1,\ldots,NV$), subject to constraints, $F_\lambda(x)=0$ ($\lambda=1,\ldots,LC$). Using Lagrange multipliers, $2\alpha_\lambda$, we have to find a minimum of the χ^2 defined as,

$$\chi^2 = \sum_{i,j}^{NV} (x_i - x_i^m)\, G_{ij} (x_j - x_j^m) + 2 \sum_{\lambda}^{LC} \alpha_\lambda F_\lambda(x) \quad , \tag{1}$$

where x_i^m are the measured values with the covariance matrix, G_{ij}. (We need to know only the error matrix,

$$G_{ij}^{-1} = \langle \delta x_i^m \delta x_j^m \rangle$$

as seen below.) We are thus led to the following system of NV+LC equations to be solved for NV variables x_i and LC multipliers α_λ,

$$\frac{1}{2} \frac{\partial \chi^2}{\partial x_i} = \sum_j G_{ij}(x_j - x_j^m) + \sum_\mu \alpha_\mu \frac{\partial F_\mu}{\partial x_i} = 0 \quad , \tag{2a}$$

$$\frac{1}{2} \frac{\partial \chi^2}{\partial \alpha_\lambda} = F_\lambda(x) = 0 \quad . \tag{2b}$$

Because of the functions F_λ the problem is not linear. We linearize F_λ and iterate as follows. Put

$$x_i = x_i^{old} + \Delta x_i \tag{3}$$

and take

$$F_\lambda(x) = F_\lambda(x^{old}) + \frac{\partial F_\lambda}{\partial x_i}(x^{old})\, \Delta x_i + 0(\Delta x)^2 \tag{4}$$

Ignoring terms like $0(\Delta x)^2$, Eqs. (2) become a linear system in Δx_i,

$$NV \qquad G_{ij}\, \Delta x_j + G_{ij}(x_j^{old} - x_j^m) + \alpha_\mu \frac{\partial F_\mu}{\partial x_i}(x^{old}) = 0 \quad , \qquad i = 1, NV \quad \mu = 1, LC \tag{5a}$$

$$+ \; LC \qquad \frac{\partial F_\lambda}{\partial x_i}(x^{old})\, \Delta x_i + F_\lambda(x^{old}) = 0 \quad . \tag{5b}$$

The solutions are,

$$\alpha_\lambda = H^{-1}_{\lambda\mu} B_\mu \quad , \tag{6}$$

$$\Delta x_i = x_i^m - x_i^{old} - \alpha_\lambda E_{\lambda_i} \quad , \tag{7}$$

where

$$E_{\lambda_i} \equiv \frac{\partial F_\lambda}{\partial x_j} G^{-1}_{ij} \quad , \tag{8}$$

$$H_{\lambda\mu} \equiv E_{\lambda_i} \frac{\partial F_\mu}{\partial x_i} \quad , \tag{9}$$

and

$$B_\lambda = F_\lambda(x^{old}) + (x_i^m - x_i^{old}) \frac{\partial F_\lambda}{\partial x_i} \quad . \tag{10}$$

So that the new values of variables are, from Eqs. (3) and (7)

$$x_i^{new} = x_i^m - \alpha_\lambda E_{\lambda_i} \quad . \tag{11}$$

And the new value of χ^2 is from Eq. (1)

$$\begin{aligned} \chi^{2^{new}} &= \alpha_\lambda E_{\lambda_i} G^2_{ij} \alpha_\mu E_{\mu j} \\ &= \alpha_\lambda H_{\lambda\mu} \alpha_\mu \\ &= \alpha_\lambda B_\lambda \end{aligned} \tag{12}$$

not including the contribution, $2\alpha_\lambda F_\lambda(x) = 0(\Delta x)^2$, which should finally vanish. The amount χ^2 will decrease because of this step is

$$\delta\chi^2 = 2\alpha_\lambda F_\lambda(x^{old}) + \chi^{2^{old}} - \chi^{2^{new}} \quad . \tag{13}$$

B. Speed of Convergence

Since the remaining part of χ^2 is linear to begin with, it is sufficient to make sure that the constraints are rapidly decreasing to zero in order to prevent non- or too-slow convergence of the iteration scheme described previously. For this purpose we introduce a testing function, which is like a χ^2 for vanishing constraints,

$$<F^2(x)> \equiv F_\lambda(x) <\delta F_\lambda \delta F_\mu>^{-1} F_\mu(x) \tag{14}$$

Since

$$<\delta F_\lambda \delta F_\mu> = \frac{\partial F_\lambda}{\partial x_i} <\delta x_i^m \delta x_j^m> \frac{\partial F_\mu}{\partial x_j} = H_{\lambda\mu} \ , \tag{15}$$

the testing function becomes

$$<F^2(x)> = F_\lambda H^{-1}_{\lambda\mu} F_\mu \tag{16}$$

Let

$$<F^2>^{new} = F_\lambda(x^{new}) H^{-1}_{\lambda\mu} F_\mu(x^{new}) \ , \tag{17a}$$

$$<F^2>^{old} = F_\lambda(x^{old}) H^{-1}_{\lambda\mu} F_\mu(x^{old}) \ . \tag{17b}$$

At the end of each iteration step we test if

$$<F^2>^{new} < <F^2>^{old} \ . \tag{18}$$

Otherwise we take a cut-step (described later) to avoid this bad iteration-step. If the constraints are found to be decreasing, then we test the speed by demanding

$$<F^2>^{new} < \epsilon <F^2>^{old} \ , \tag{19}$$

with $\epsilon = 0.7$ say.

C. Special Mode of Iteration

If the speed of convergence in the usual mode of iteration is too slow (Eq. (19) is not satisfied), or cut-down procedures fail to cure the divergence condition (Eq. (18) is not satisfied), then we take a step, which is designed to force $<F^2> = 0$ as described below. Demand

$$\frac{1}{2} \frac{\partial}{\partial x_i} <F^2> = \frac{\partial F_\lambda}{\partial x_i} H^{-1}_{\lambda\mu} F_\mu = 0 \tag{20}$$

Linearize as in Eq. (3) and (4),

$$\frac{\partial F_\lambda}{\partial x_i} H^{-1}_{\lambda\mu} \left(F_\mu + \frac{\partial F_\mu}{\partial x_j} \Delta x_j \right) = 0 \quad . \tag{21}$$

Since

$$\frac{\partial F_\lambda}{\partial x_i} H^{-1}_{\lambda\mu} \frac{\partial F_\mu}{\partial x_j} = G_{ij} \quad , \tag{22}$$

we obtain

$$\Delta x_i = - E_{\lambda_i} H^{-1}_{\lambda\mu} F_\mu \quad , \tag{23}$$

so that

$$x_i^{new} = x_i^{old} + \Delta x_i \quad . \tag{24}$$

The amount χ^2 will decrease because of this step is, by design,

$$\delta\chi^2 = \langle F^2 \rangle^{old} \quad . \tag{25}$$

and we have

$$\chi^{2^{new}} = 2\,\alpha_\lambda F_\lambda (x^{old}) + \chi^{2^{old}} - \delta\chi^2 \ . \tag{26}$$

D. <u>Cut-Down Step</u>

If an iteration step leads to an apparent divergence ($\langle F^2 \rangle$ increasing), or to unphysical values of variables (negative momenta), then instead of the full step Δx_i we take a smaller step (for all variables)

$$x_i^{new} = x_i^{old} + \lambda\, \Delta x_i \tag{27}$$

with $\lambda < 1$.

The new χ^2 due to this cut-down step is

$$\chi^{2^{new}} = \chi^{2^{old}} + 2\lambda\, \Delta x_i G_{ij} (x_j^{old} - x_j^m) + \lambda^2 \Delta x_i G_{ij} \Delta x_j \ . \tag{28}$$

From Eq. (5)

$$\Delta x_i G_{ij} (x_j^{old} - x_j^m) = -\Delta x_i G_{ij} \Delta x_j + \alpha_\lambda F_\lambda (x^{old}) \quad . \tag{29}$$

And using Eqs. (10) and (12)

$$\Delta x_i G_{ij} \Delta x_j = \chi^{2^{\text{old}}} + 2\alpha_\lambda F_\lambda(x^{\text{old}}) - \alpha_\lambda B_\lambda \tag{30}$$

Thus Eq. (28) can be written

$$\begin{aligned} \chi^{2^{\text{new}}} = \chi^{2^{\text{old}}} &- 2\lambda\left(\chi^{2^{\text{old}}} + \alpha_\lambda F_\lambda - \alpha_\lambda B_\lambda\right) \\ &+ \lambda^2\left(\chi^{2^{\text{old}}} + 2\alpha_\lambda F_\lambda - \alpha_\lambda B_\lambda\right) \end{aligned} \tag{31}$$

E. Case of Fewer than 4 Missing Variables

We describe how the unmeasured variables, if any, are solved for and the remaining constraints are obtained in the case involving 3 or fewer unknown variables. The following cases considered are found to be sufficient in practice.

1. One missing momentum.

Let $\vec{F}$ be the total measured momentum,

$$\begin{aligned} F_1 &= \sum s_i p_i \cos\lambda_i \cos\phi_i \ , \\ F_2 &= \sum s_i p_i \cos\lambda_i \sin\phi_i \ , \\ F_3 &= \sum s_i p_i \sin\lambda_i \ , \end{aligned} \tag{32}$$

where $s_i = +1$ (-1) for outgoing (incoming) particle. Consider three orthonormal vectors, $\hat{e}_i$ such that $\hat{e}_3$ is antiparallel to the direction of the particle with unknown momentum p_ℓ (whose angles ϕ_ℓ and λ_ℓ are measured). Explicitly we have

$$\begin{aligned} \hat{e}_3 &= -(\cos\lambda_\ell \cos\phi_\ell, \ \cos\lambda_\ell \sin\phi_\ell, \ \sin\lambda_\ell) \ , \\ \hat{e}_2 &= (\sin\phi_\ell, \ -\cos\phi_\ell, \ 0) \ , \\ \hat{e}_1 &= \hat{e}_2 \times \hat{e}_3 \ . \end{aligned} \tag{33}$$

The momentum conservation reads

$$P_1\hat{e}_1 + P_2\hat{e}_2 + P_3\hat{e} = \vec{F} \tag{34}$$

where P_i is magnitude of the momentum in each direction $\hat{e}_i$. Or in matrix notation

$$U_{ij}P_j = F_i \ , \tag{35}$$

where we have defined

$$U_{ij} \equiv (e_j)_i \tag{36}$$

So that the missing momentum is

$$p_\ell = P_3 = (U^{-1}F)_3 \quad , \tag{37}$$

and the remaining momentum constraints are

$$\begin{aligned} P_1 &= (U^{-1}F)_1 = 0 \quad , \\ P_2 &= (U^{-1}F)_2 = 0 \quad . \end{aligned} \tag{38}$$

2. Two missing momenta.

Instead of Eq. (33) take $\hat{e}_2$ and $\hat{e}_3$ to be unit vectors antiparallel to the directions of two missing momenta respectively and $\hat{e}_1 = \hat{e}_2 \times \hat{e}_3 / |\hat{e}_2 \times \hat{e}_3|$. Then we have for the missing momenta

$$\begin{aligned} p_{\ell_1} &= (U^{-1}F)_2 \quad , \\ p_{\ell_2} &= (U^{-1}F)_3 \quad . \end{aligned} \tag{39}$$

and for the constraint

$$P_1 = (U^{-1}F)_1 = 0 \tag{40}$$

3. Three missing momenta.

Instead of Eq. (33) take the unit vectors $\hat{e}_i$ to be antiparallel to the directions of missing momenta respectively. There is no momentum constraint.

4. One missing neutral.

The momentum of the neutral is

$$\vec{P}_n = - S_n \vec{F} \tag{41}$$

so that the three unknown variables are

$$\begin{aligned} P_n &= \sqrt{P_x^2 + P_y^2 + P_z^2} \quad ; \\ \lambda_n &= \tan^{-1} \frac{P_z}{\sqrt{P_x^2 + P_y^2}} \quad , \\ \phi_n &= \tan^{-1} P_y/P_x \quad . \end{aligned} \tag{42}$$

F. Case of Four Missing Variables

We treat the problem including one neutral (p_n, λ_n and ϕ_n are unknown) and a particle with unknown momentum p_ℓ (the angles λ_ℓ and ϕ_ℓ are measured). We describe below how p_ℓ is obtained. Once p_ℓ is known, the missing neutral $\vec{p}_n$ can be obtained as in Section II. E. 4.

Let $\vec{\pi}$ be the total measured momentum as in Eq. (32) and ϵ be the total measured energy,

$$\epsilon = \sum s_i E_i - m_T \quad , \tag{43}$$

where $s_i = +1$ (-1) for outgoing (incoming) particle and $m_T=0$ if there is no target. The energy-momentum balance reads,

$$\begin{aligned} \vec{\pi} + s_\ell \vec{p}_\ell + s_n \vec{p}_n &= 0 \quad , \\ \epsilon + s_\ell E_\ell + s_n E_n &= 0 \quad . \end{aligned} \tag{44}$$

Eliminating p_n and E_n from the above equations, we obtain an equation for p_ℓ as

$$-\epsilon E_\ell + b p_\ell + s_\ell a = 0 \tag{45}$$

where

$$a \equiv \frac{1}{2} \left(\vec{\pi}^2 - \epsilon^2 - m_\ell^2 + m_n^2 \right) , \tag{46}$$

and b is the projection of $\vec{\pi}$ in the direction of $\vec{p}_\ell$,

$$b \equiv \vec{\pi} \cdot u_\ell \quad . \tag{47}$$

If $m_\ell = 0$, then from Eq. (45) we get

$$p_\ell = \frac{s_\ell a}{\epsilon - b} \quad . \tag{48}$$

In general $m_\ell \neq 0$, and Eq. (45) leads to a quadratic equation for p_ℓ,

$$(\epsilon^2 - b^2) \, p_\ell^2 - 2 s_\ell ab \, p_\ell + (\epsilon^2 m_\ell^2 - a^2) = 0 \quad . \tag{49}$$

Unless the discriminant,

$$D = a^2 - m_\ell^2 (\epsilon^2 - b^2) \; , \tag{50}$$

is negative, we have two roots

$$p_\ell^{(1)} = B\,(1 + \sqrt{1 - C/B^2})\ ,$$
$$p_\ell^{(2)} = C/p_\ell^{(1)}\ , \tag{51}$$

where

$$B \equiv \frac{s_\ell ab}{\epsilon^2 - b^2}$$
$$C \equiv \frac{\epsilon^2 m_\ell^2 - a^2}{\epsilon^2 - b^2}\ . \tag{52}$$

Because of measurement errors events with small values of D will sometimes show negative values of D. The value for p_ℓ obtained by simply taking D=0, will violate energy momentum conservation. Hence we have to readjust the measurements to satisfy the condition, D=0. We achieve this by formally doing a 1C problem of minimizing χ^2 of Eq. (1) subject to the constraint of vanishing D.

In this process we need the following partial derivatives: For measured variables x_i except λ_i and ϕ_ℓ

$$\frac{\partial D}{\partial x_i} = \frac{\partial D}{\partial F_\mu}\,\frac{\partial F_\mu}{\partial x_i} \tag{53}$$

where $\partial F_\mu/\partial x_i$ are the usual partial derivatives given in Section II. G and $\partial D/\partial F_\mu$ are,

$$\frac{\partial D}{\partial \epsilon} = -\,2\,(a + m_\ell^2)\,\epsilon\ ,$$
$$\frac{\partial D}{\partial \pi_x} = 2a\pi_x + 2b\,m_\ell^2 \cos\lambda_\ell \cos\phi_\ell\ ,$$
$$\frac{\partial D}{\partial \pi_y} = 2a\pi_y + 2b\,m_\ell^2 \cos\lambda_\ell \sin\phi_\ell\ , \tag{54}$$
$$\frac{\partial D}{\partial \pi_z} = 2a\pi_z + 2b\,m_\ell^2 \sin\lambda_\ell\ .$$

All partial derivatives with respect to unmeasured variables vanish. And we have

$$\frac{\partial D}{\partial x_\ell} = 2m_\ell^2 b\ (\pi_x \sin\lambda_\ell \cos\phi_\ell + \pi_y \sin\lambda_\ell \sin\phi_\ell - \pi_z \cos\lambda_\ell)\ ,$$

$$\frac{\partial D}{\partial \phi_\ell} = 2m_\ell^2 b\ (\pi_x \cos\lambda_\ell \sin\phi_\ell - \pi_y \cos\lambda_\ell \cos\phi_\ell)\ . \tag{55}$$

G. Derivatives of Constraints

We need derivatives of constraints with respect to measured variables x_k, namely those with nonvanishing diagonal element in the error matrix G_{ij}^{-1}. We assume that unmeasured variables y_ℓ have been solved for as described previously so that we do not have to distinguish. In particular, for this purpose the error matrix is taken to be the full NV-by-NV matrix with elements referring to y_ℓ set to 0.

Our choice of variables are for each particle,

$$x_1 = 1/p\ , \quad x_2 = \lambda \ \text{ and } \ x_3 = \phi \tag{56}$$

Let $(F_\mu) = (\epsilon, \vec{F})$ be the total 4-momentum as in Eqs. (32) and (43) including contributions from particles which had unmeasured variables. Also let P_μ be the 4-vector of a particle, multiplied by $s_k = -1$ if it is incoming. The partial derivatives of energy and momentum are of the form

$$\begin{array}{lll}
\dfrac{\partial F_1}{\partial x_1} = -\dfrac{1}{E}\dfrac{\vec{p}^{\,2}}{x_1}\ , & \dfrac{\partial F_1}{\partial x_2} = 0\ , & \dfrac{\partial F_1}{\partial x_3} = 0\ , \\[2ex]
\dfrac{\partial F_2}{\partial x_1} = -\dfrac{P_x}{x_1}\ , & \dfrac{\partial F_2}{\partial x_2} = -\dfrac{P_x \cdot P_z}{\sqrt{P_x^2 + P_y^2}}\ , & \dfrac{\partial F_2}{\partial x_3} = -P_y\ , \\[2ex]
\dfrac{\partial F_3}{\partial x_1} = -\dfrac{P_y}{x_1}\ , & \dfrac{\partial F_3}{\partial x_2} = -\dfrac{P_y \cdot P_z}{\sqrt{P_x^2 + P_y^2}}\ , & \dfrac{\partial F_3}{\partial x_3} = P_x\ , \\[2ex]
\dfrac{\partial F_4}{\partial x_1} = -\dfrac{P_z}{x_1}\ , & \dfrac{\partial F_4}{\partial x_2} = s_k \cdot \sqrt{P_x^2 + P_y^2}\ , & \dfrac{\partial F_4}{\partial x_3} = 0\ .
\end{array} \tag{57}$$

The above is the final result if there are no missing variables. Otherwise we have to take into account the constraint transformation and the implicit dependence as described below.

Let

$$\frac{\partial F^1_\mu}{\partial x_i} = C_{\mu\nu} \frac{\partial F_\nu}{\partial x_i} \tag{58}$$

where C is the inverse of the 3-by-3 matrix U of Eq. (35), trivially extended to 4-by-4 to accommodate the energy component if necessary (MMD=4). As described earlier, because of the way C is constructed, Eq. (58) with μ=1, . . . , LC, gives the explicit part, $\partial F^1_\lambda/\partial x_i$, of the derivative of the unused constraints. (Strictly speaking this is not correct since the transformation C is taken to be constant, which is allowed because we are iterating.) There is an additional contribution through the implicit dependence, $\partial y_\ell/\partial x_i$.

The remaining part of the derivatives calculated above, i.e., Eq. (58) with μ=LC+1, . . . , MMD, satisfy

$$\frac{\partial F^1_\ell}{\partial x_i} + \frac{\partial F^1_\ell}{\partial y^1_\ell} \frac{\partial y^1_\ell}{\partial y_i} = 0 \tag{59}$$

So that

$$\frac{\partial y_\ell}{\partial x_i} = - \left[\frac{\partial F_{\ell'}}{\partial y_\ell}\right]^{-1} \frac{\partial F_{\ell'}}{\partial x_i} \tag{60}$$

We have as a final result

$$\frac{\partial F_y}{\partial x_i} = \frac{\partial F^1_\lambda}{\partial x_i} + \frac{\partial F^1_\lambda}{\partial y_\ell} \frac{\partial y_\ell}{\partial x_i} \tag{61}$$

where λ=1, . . . , LC.

H. Fitted Error Matrix

Let x_i be fitted, y_ℓ computed, and x^m_i measured variables. Then as discussed in Ref. 2 it is straightforward to obtain the following result for the error matrix associated with the fitted variables,

$$<\delta x_i \delta x_j> = G^{-1}_{ij} - G_{\lambda i} H^{-1}_{\lambda\mu} E_{\mu j} \tag{62}$$

where as before G_{ij}^{-1} is the error matrix for the measured variables and the matrices $E_{\lambda i}$ and $H_{\lambda\mu}$ have been defined by Eq. (8) and (9) respectively. Also we have

$$<\delta x_i \delta y_\ell> = <\delta x_i \delta x_j> V_{\ell j} \quad , \tag{63}$$

$$<\delta y_\ell \delta y_{\ell'}> = V_{\ell i} <\delta x_i \delta x_j> V'_{\ell j} \tag{64}$$

where $V_{\ell i}$ is the matrix $\partial y_\ell / \partial x_i$ of Eq. (60).

III. DESCRIPTION OF PROGRAM

A. A Bird's Eye View

For each mass hypothesis at a vertex the main program, FIT, is to be called to obtain a fit. As shown in Fig. 1 FIT uses six subroutines, BLOCK1,...,BLOCK6, each carrying out distinct tasks (as summarized in Table I), performs various tests based on the results, and controls the flow of computations. An attempt was made to separate out, from the program, "arithmetic" such as those involving vectors and matrices, so that the problem of optimizing (in speed or space) can be attacked independently of the program itself. Thus there are CALL's for external routines as listed in Table II, some of which may easily be MACRO-CALL's to cause in-line expansions.

A general purpose MACRO processor[4] is utilized in the original source. This processor is designed to easily equip with a MACRO facility, a language (such as FORTRAN, or SUMX Control Cards) lacking one. In particular the declared dimensions are symbolic in this source, which after a pass through this processor turns into an ordinary FORTRAN source.

B. BLOCKs

1. BLOCK1 (& Error return)

This is an input routine to be called once per fit. At entry the variables in the COMMON block /CBLK1/ must be set as follows:

MMD: either 3 to demand 3-momentum conservation only; or 4 to include energy conservation.

NOT4: zero to do 4-missing variable problem including one neutral and a particle with unknown momentum; or nonzero to disallow 0-constraint situation in any other case.

NCALL: 1 to call for first (larger) root in the 4MV case; 2 to call for second (smaller) root.

NP: total number of particles including neutral.

XMEAS(3NP): measured variables x_i^m such that for kth track, XMEAS(3k-2)=1/p, XMEAS(3k-1)=λ, and XMEAS(3k)=ϕ. If a momentum is unmeasured, then XMEAS(3k-2)=0.

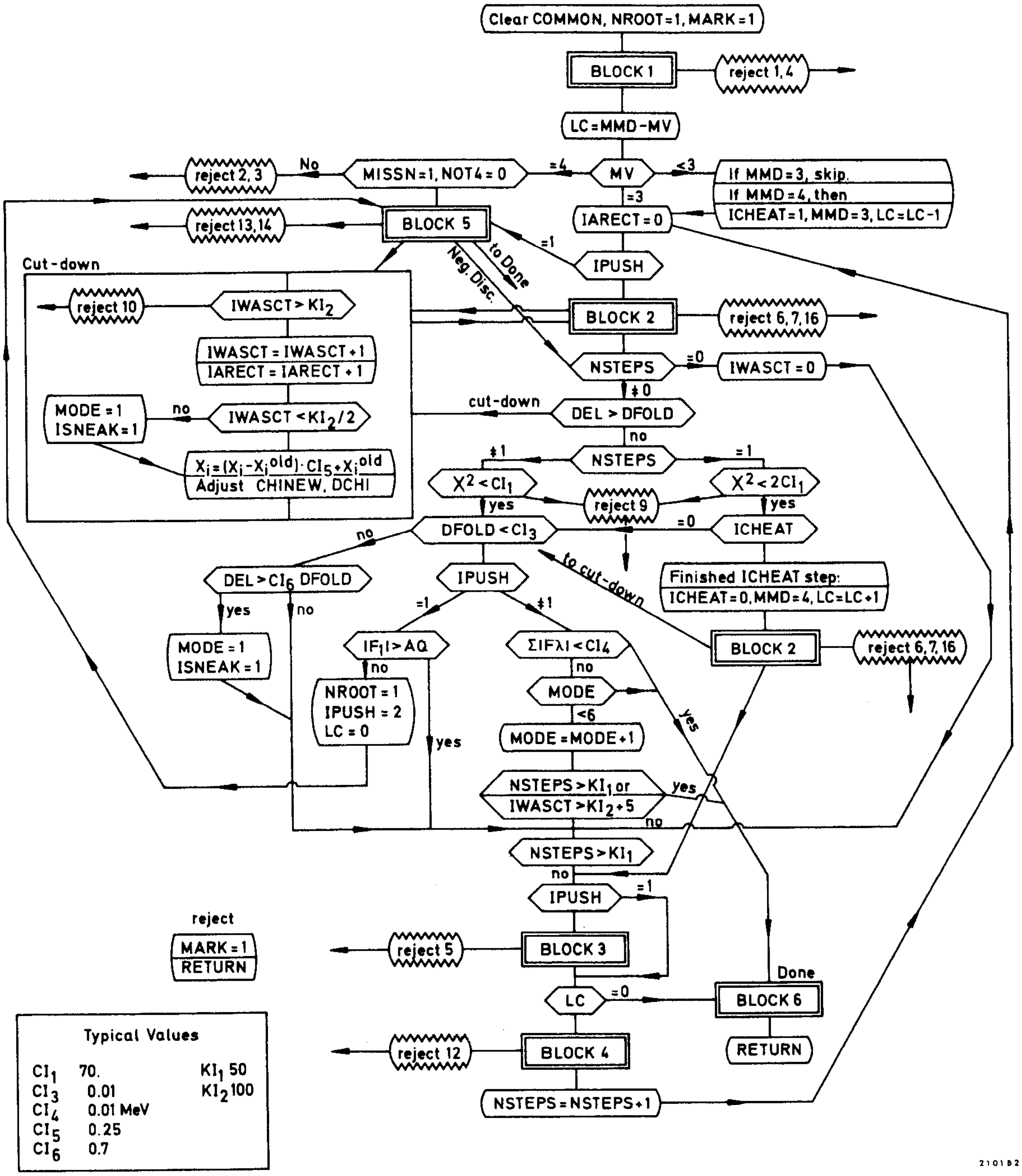

FIG. 1--Subroutine fit. Values of parameters shown are from Exp. 13.

TABLE I

BLOCKS

Block No.	Purpose
1	Prepares input data
2	Calculates unknown variables and constraints
3	Computes derivatives of constraints and of unknown variables
4	Takes one iteration step
5	Treats four missing variable problem
6	Calculates output error matrix and stretches

TABLE II

ODDS AND ENDS

	Vector Manipulations
DADDJ(X, Y, Z, N)	Z(N)=X(N)+Y(N)
DEQUJ(X, Y, N)	Y(N)=X(N)
DEQUS(A, X, N)	X(N)=A
DNEQUJ(X, Y, N)	Y(N)=-X(N)
DSCAL(X, Y, N)	DSCAL=X(N)·Y(N)
DSMPYJ(A, X, Y, N)	Y(N)=A·X(N)
DSUBJ(X, Y, Z, N)	Z(N)=X(N)-Y(N)
DVECN(X, Y, Z)	$\vec{Z}=\vec{X}\times\vec{Y}/\lvert\vec{X}\times\vec{Y}\rvert$
	Matrix Manipulations
DMMPY(A, B, C, N, M, L)	C(N, L)=A(N, M)·B(M, L)
DMMPY1(A, B, C, N, M, L)	C(N, L)=A(M, N)·B(M, L)=A^TB
DMMPY2(A, B, C, N, M, L)	C(N, L)=A(N, M)·B(L, M)=AB^T
DMMPY3(A, B, C, N, M, L)	C(N, L)=A(M, N)·B(L, M)=A^TB^T
DMTINV(A, N)	A(N, N)=A^{-1}(N, N) and
	DMTINV=Det(A)
DMTRN(A, B, N, M)	B(M, N)=A(N, M)=A^T
	Peculiar to FIT
DGETX(X, P)	Get X=$(1/p, \lambda, \phi)$ from P=(p_x, p_y, p_z)
DGETP(X, P)	Vice versa

GINA(3NP, 3NP): bottom half ($i \geq j$) of error matrix G_{ij}^{-1}. For example, GINA(3k-2, 3k-2)=$<\delta\, 1/p\, \delta\, 1/p>$, GINA(3k-1, 3k-1)=$<\delta\lambda\delta\lambda>$, and GINA(3k, 3k)=$<\delta\phi\delta\phi>$. In particular $<\delta\phi\delta\phi>=0$ for neutral.

SN: -1 if first particle is incident; +1 otherwise.

TMASS: target mass; zero if none.

RMASS(NP): mass list.

BLOCK1 counts unmeasured variables (MV) by testing GINA(3k, 3k)=0 for a neutral particle (3 missing variables) and XMEAS(3k-2), 1/p=0 for unknown momentum (1 missing). It zeroes elements of GINA corresponding to missing variables. It also initializes the variables X (in /CBLK2/) to XMEAS; missing variables are taken as zeroes except momentum, if it is unknown, is set to 1.

Error-returns:

NRJCT=1; number of missing variables (MV) exceed MMD.

4; GINA is not positive-definite.

2. BLOCK2 (& Cut-down, & Error-return)

BLOCK2 first examines X to make sure that every momentum is positive, otherwise does cut-down return. Then it solves for unknown variables (in the case MV $\leq$ 3) using the 3-momentum conservation as described in Section II. E. The total measured momenta are also calculated: If MMD=3, then $F=(\Sigma P_x, \Sigma P_y, \Sigma P_z)$ and C is the 3 by 3 matrix U^{-1} of Section II. E; if MMD=4, then $F \equiv (E, \vec{F})$ and C is a 4 by 4 matrix of the form

$$C = \left(\begin{array}{c|c} 1 & 0 \\ \hline 0 & U^{-1} \end{array}\right).$$

In any case constraints are obtained by requiring first LC=MMD-MV components of (C)(F) to vanish.

Error-returns:

NRJCT=6; two or more parallel missing momenta (U is singular).

7; negative momentum in first iteration step.

16; negative momentum in LC=0 case.

3. <u>BLOCK3 (& Error-return)</u>

BLOCK3 computes partial derivatives of constraints F_λ and of missing variables y_ℓ as described in Section II.G. On return we have

$$FL1 = \begin{array}{cc} & \longleftarrow NV \longrightarrow \\ \updownarrow LC & \left(\dfrac{\partial F_\lambda}{\partial x_i} + \dfrac{\partial F_\lambda}{\partial y_\ell}\dfrac{\partial y_\ell}{\partial x_i}\right) \end{array} ,$$

and

$$V = \begin{array}{cc} & \leftarrow NV \rightarrow \\ \updownarrow MV & \left(\dfrac{\partial y_\ell}{\partial x_i}\right) \end{array} .$$

Error-return:

NRJCT=5; MV by MV matrix $\partial F_{\ell'}/\partial y_\ell$ is singular (see Eq. (60)).

4. <u>BLOCK4 (& Error-return)</u>

This routine does one iteration step. Calculate LC-by-NV matrix ELI defined by Eq. (8), HIN, inverse of LC-by-LC matrix H of Eq. (9), and LC component vector B of Eq. (10). Then in the usual mode of iteration (MODE=0) we have

$$ALFA = (HIN)(B) ,$$

and

$$X = XMEAS - (ALFA)(ELI) ,$$

as in Eqs. (6) and (11). Also take CHIOLD=CHINEW and calculate CHINEW as in Eq. (12). The old value DFOLD of the testing function Eq. (17b) is also computed for later application in FIT.

In the SNEAK mode of iteration (MODE≠0), which is designed to force $<F^2>=0$ as described in Section II.C, we have

$$X = XOLD - (ELI)^T (HIN)(F)$$

as in Eq. (23) and the new value of χ^2 is given by Eq. (26).

Error-return:

NRJCT=12; H is singular.

5. <u>BLOCK5 (& Neg-disc, & Cut-down, & Error-return)</u>

This routine treats the 4-missing variable problem as described in Section II.F. There are in general two roots for the missing momentum p_ℓ (we assume $m_\ell \neq 0$). BLOCK5 is to be called twice with NCALL=1 and 2 respectively to get the first (larger) and second root for p_ℓ. IPUSH is initially zero, is set to 1 in the negative-discriminant case, and to 2 (in FIT) when a solution is found. Both calls, NCALL=1 and 2, will give the identical answer in the latter case. More detail is shown in Fig. 2.

Error-return:

NRJCT=13; No good root.

14; No energy balance. This and test (V) in Section III.C.2 are the only instances in the entire FIT package in which the energy scale is needed. We assume it is MeV and the error is due to an energy imbalance of more than 1 MeV.

6. BLOCK6

The fitted error matrix GINB and stretches STR are calculated in this routine as described in Section II.H.

C. FIT

This is the main control program to be called once (twice) per constrained (OC) hypothesis to obtain a fit. The input requirements have been given in the description of BLOCK1. We sketch below what is done in FIT. More details are shown in Fig. 1.

1. ICHEAT step.

If MMD=4 and the number of unknown variables (MV) is less than 3, then we first do one successful step (during which ICHEAT=1) of the MMD=3 fit, which requires only the visible momentum balance. Large χ^2 leading to reject 9 (see Test (ii) below) at this stage implies, of course, large missing momentum (carried off by neutrals).

2. Iteration Control

The following tests are made, at the end of each iteration step in the order listed:

(i) Make sure of nondivergence by demanding DEL < DFOLD, where DEL $= \langle F^2 \rangle^{new}$, DFOLD $= \langle F^2 \rangle^{old}$ of Eqs. (17a and b). Otherwise take a cut-down step.

(ii) Test χ^2 to see if it is not too large: $\chi^2 < 2$ CI(1) for the first step, in particular, ICHEAT step, and $\chi^2 <$ CI(1) for subsequent steps. Otherwise reject 9.

(iii) Test for termination, DFOLD < CI(3). If not, either convergence is too slow or we have to take another iteration step.

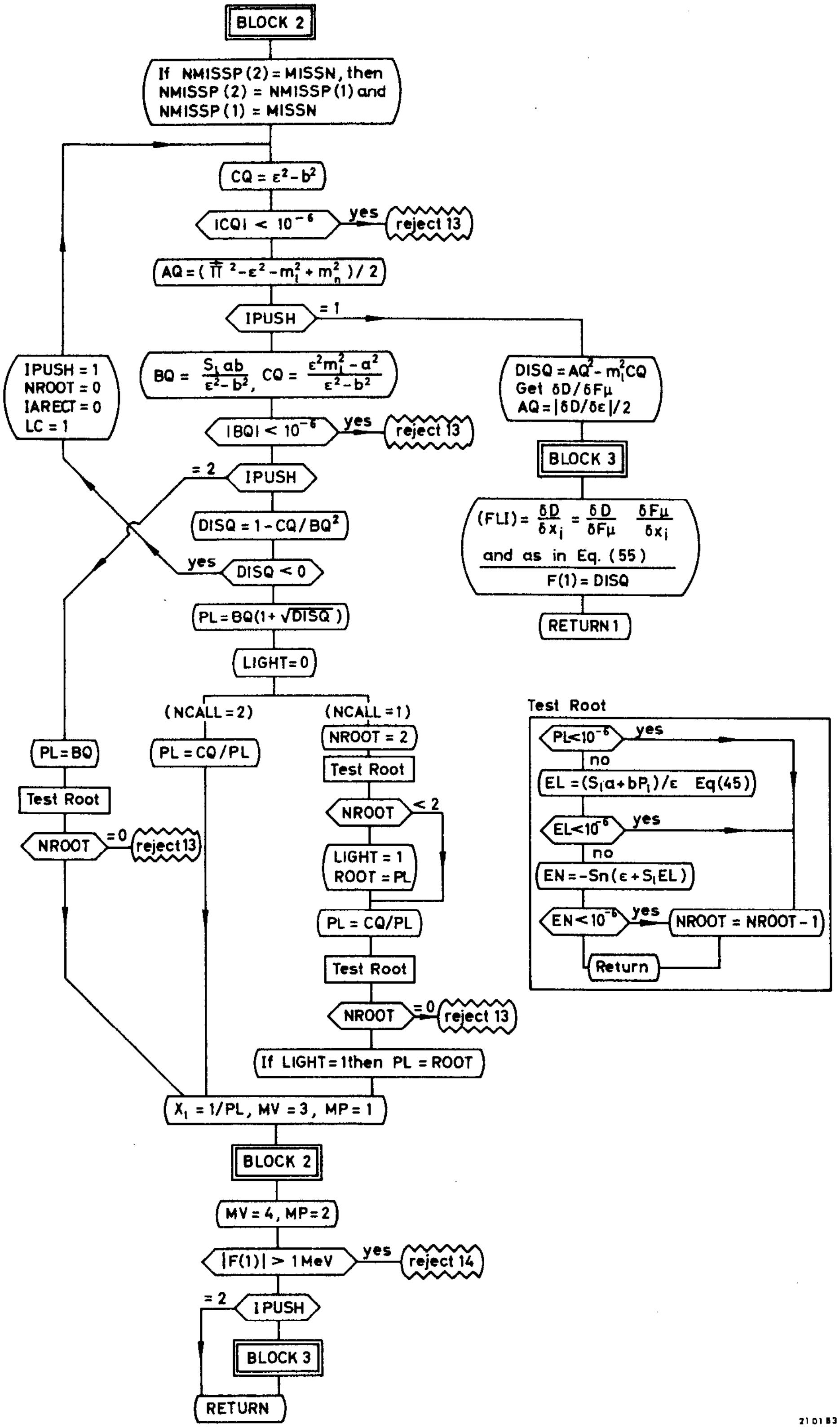

FIG. 2--Subroutine BLOCK5.

(iv) Test speed of convergence, DEL < CI(6) DFOLD. If not, take a step in SNEAK mode.

(v) Check the final solution to see constraints are actually zero, $\Sigma |F_\lambda| <$ CI(4). Otherwise increment MODE and take a SNEAK step, unless MODE > 5 or NSTEPS > KI(1) or IWASCT > KI(2)+5, in which case terminate.

3. <u>SNEAK Mode of Iteration.</u>

This is the special procedure designed to force constraints to vanish ($\langle F^2 \rangle = 0$) as described in Section II.C, which is invoked when

(i) tests (iii) and (iv) are not satisfied, meaning too-slow convergence, or

(ii) too many cut steps are called for (because the iteration is diverging), or

(iii) the final solution does not yet satisfy test (v) above.

4. <u>Cut-down.</u>

If BLOCK2 finds that the current iteration step will lead to negative momenta, or if FIT detects the divergence condition (test (i) above), then the cut-down steps as described in Section II.D are taken until the undesired condition vanishes. If too many such steps are called for, i.e., IWASCT > KI(2)/2, (presumably because of nondecreasing $\langle F^2 \rangle$), then SNEAK mode is invoked. If this fails then eventually IWASCT > KI(2) leading to reject 10. IARECT counts consecutive cut-steps taken during one iteration step.

5. When everything fails so that the iteration counter NSTEP exceeds KI(1) the fit is terminated via reject 11.

IV. APPLICATION AND PERFORMANCE

The present version FIT 70 was used to perform the kinematic analysis in the second photoproduction experiment[1] of Group D (Experiment 13) at the Stanford Linear Accelerator Center (SLAC). In this experiment the 2-meter streamer chamber with a pressurized hydrogen gas target was exposed to an 18 GeV bremsstrahlung beam. 640,000 pictures (in 3 views) were taken by triggering the chamber for hadronic events.

Events were measured using conventional hand-guided machines (SPVB and NRI at SLAC) and were then processed through the geometry program SYBIL.[5] As explained in Ref. 5 the geometric reconstruction of events was approached in a manner unlike those of the usual bubble chamber programs (like THRESH or TVGP) because of the following aspects of the experimental setup:

(i) The target tube (made of Mylar, 4 mil in thickness, 1/2" in diameter and extending the whole visible length along the beam) made the vertex region and some path length downstream invisible. Tracks associated with a given vertex were fitted simultaneously together with the vertex coordinates from the beginning.

(ii) The top set of magnet coils had no iron core (to let the cameras see through) giving rise to a comparatively more complicated field in the chamber. Rather than relying on a (piecewise) analytic expression for tracks in space, the equation of motion for a charged particle,

$$\frac{d\hat{u}}{ds} = \frac{K}{p}\hat{u} \times \vec{B}$$

with momentum $p\,\hat{u}(\vec{x})$ through a magnetic field $\vec{B}(\vec{x})$, was integrated numerically (via a Runge-Kutta method). In each view reprojecting and comparing with measurements the residuals were computed, the sum of squares of which defined the χ^2 to be minimized with respect to the track parameters and vertex coordinates.

The resulting track parameters, $(1/p, \lambda, \phi)$ at the vertex for each track, and the full error matrix including elements correlating different tracks were then input to FIT.

From the programming point of view the kinematic analysis in this experiment was particularly simple, because (i) the reactions analyzed were all one-vertex type, and (ii) Coulomb scatterings (in gas) were negligible so that the error matrix of the least-square fit in SYBIL (accounting only for measurement errors) was used directly without modifications.

Table IV gives a brief description of additional routines (except those of the standard IBM support) used in this analysis. The input/output tape formats were kept identical to those of the SYBIL-TEUTA system used in the preceding experiment[7] as described in Refs. 5 and 6, in order to minimize effort in interfacing to then existing programs. Thus several routines were taken intact from TEUTA disregarding much duplicity in COMMON space. Also, to get results quickly little optimizing was done in the utility routines such as those in Table II. In this application most of them were in FORTRAN generating much overhead and furthermore the advantage of dealing often with symmetric matrices were ignored.

A total of 39,313 hadronic events were reconstructed by SYBIL at the rate of 2.3 CPU sec/event. (The total CPU time amounted to 9 hours and 26 minutes.) The composition of different prong numbers is shown in Table V. After applying small energy loss corrections due to the mylar tube, which is negligible for particles with laboratory momenta (scaled to proton) higher than 300 MeV, these events were input to FIT to test the mass hypotheses as listed in Table VI. A total of 193,145 3C fits were made, not counting those hypotheses which were skipped because of large missing momenta, at the rate of 22 msec/fit (total CPU time of 73 minutes). As summarized in Table VII there were almost no cases (one out of 2000) in which FIT failed to converge to a solution. In this run no other possible rejects (see Table III) were found.

To take a closer look we have reprocessed a sample consisting of 300 5-prong events (each having at least one acceptable 3C fit) outputting (via the subroutine LOOK, which otherwise is a dummy) a short summary record for each fit to be SUMXed. In this run a total of 4500 fits were made, 3683 of which were rejected because of too large χ^2 (greater than 70). The iteration counter, NSTEP, the accumulative cut-step counter, IWASCT, and MODE are displayed in Fig. 3. MODE, when ISNEAK=0, counts the number of steps taken in the special mode of iteration designed to force the constraints to vanish. This is invoked when $\Sigma|F_\lambda| > 0.01$ MeV. Otherwise

TABLE III

ERROR CODES

Code	Why	Where
1	Too many missing variables	BLOCK1
2	NOT4≠0 in 4MV case	FIT
3	No missing neutral in 4MV case	FIT
4	Input error matrix not positive definite	BLOCK1
5	Singular $\partial F_\lambda / \partial y_\ell$	BLOCK3
6	Parallel missing momenta	BLOCK2
7	Unmeasured momentum negative	BLOCK2
9	χ^2 too large	FIT
10	Too many cut-steps	FIT
11	Too many iterations	FIT
12	Singular $H_{\mu\nu}$	BLOCK4
13	No root in 4MV case	BLOCK5
14	No energy balance in 4MV case	BLOCK5
16	Negative momentum in OC case	BLOCK2

TABLE IV

ROUTINES USED IN EXP. 13 KINEMATICS

Name	Type	Additional Entries and Description
BLOCKA	XF	SELECT, ISTORE, IPRINT: Experiment dependent Main Control; Prepares input such as nominal beam parameters, energy loss corrections and in conjunction with IDRIVE drives FIT and the rest.
DCLEV	GF	Calculates confidence level for given χ^2 and NDF.
DMASS	GA	Mass codes, characters, and values.
DPFR	GF	DRFP: Momentum from range and vice versa for proton in mylar.
DTQ3	GF	Quadratic interpolation.
DUMP	GA	DUMPF: Used for debugging purpose; F-entry NOPes BALR in Caller.
ELOS	XF	Energy loss through mylar. (Contains experiment dependent constants.)
FAREAD	GA	FAWRIT: FORTRAN array I/O.
ICHOPD	GF	Binary chops a table.
IDRIVE	XA	Event types and mass hypotheses for Exp. 13.
READ1	XF	READ2, . . . , READ5, WRITE1: Selected reading (writing) for TEUTA output records.
SCRIB	XF	Printout.
BREAD	TF	Reading for TEUTA output tape.
BWRITE	TF	Same for writing.
ENDREP	TF	Generates run-report from summary-records.
ORDER	TF	Orders fit-record in mass and charge.
OUTPUT	TF	BTALLY: Prepares output record.
TOUT	TF	Writes fit-record.
WSOM	TF	Writes summary-record.

Type characterizes each routine as either in FORTRAN(F) or in ASSEMBLER(A) and G for general purpose; X for Exp. 13 use; T meaning it is from TEUTA.

TABLE V

EVENTS IN EXPERIMENT 13

Prong Number	Events [a]	3C-Events
3	21,055	7,304
5	15,305	4,162
7	1,918	368
9	180	31
Other [b]	855	

a. Correspond to different film samples; 5 prongs from all, whereas 3, 7 and 9 from 28/64 of all.

b. Mostly V-events from a sample of film.

TABLE VI

MASS HYPOTHESES IN EXPERIMENT 13

```
*               TYPE  TABLE
TYPBG           TP            500,T500
                TP            300,T300
                TP            700,T700
                TP            200,T200
                TP            900,T900
TYPND           EQU           *
*               FIT TABLE     FOR  TYPE  500
&NMEZ           SETA          5
&CNTRL          SETA          &MMD4+&PROD
T500            FW            501,(GAM,P,P,PIM,PIP,PIM,PIP)
                FW            502,(GAM,P,PIP,PIM,P,PIM,PIP)
                FW            503,(GAM,P,PIP,PIM,PIP,PIM,P)
                FW            504,(GAM,P,P,KM,KP,PIM,PIP)
                FW            505,(GAM,P,P,PIM,KP,KM,PIP)
                FW            506,(GAM,P,P,KM,PIP,PIM,KP)
                FW            507,(GAM,P,P,PIM,PIP,KM,KP)
                FW            508,(GAM,P,KP,KM,P,PIM,PIP)
                FW            509,(GAM,P,KP,PIM,P,KM,PIP)
                FW            510,(GAM,P,PIP,KM,P,PIM,KP)
                FW            511,(GAM,P,PIP,PIM,P,KM,KP)
                FW            512,(GAM,P,KP,KM,PIP,PIM,P)
                FW            513,(GAM,P,KP,PIM,PIP,KM,P)
                FW            514,(GAM,P,PIP,KM,KP,PIM,P)
                FW            515,(GAM,P,PIP,PIM,KP,KM,P)
                FW             ,
*               FIT TABLE     FOR  TYPE  300
&NMEZ           SETA          3
T300            FW            301,(GAM,P,P,PIM,PIP)
                FW            302,(GAM,P,PIP,PIM,P)
                FW            303,(GAM,P,P,KM,KP)
                FW            304,(GAM,P,KP,KM,P)
                FW            305,(GAM,P,P,P,P)
                FW             ,
*               FIT TABLE     FOR  TYPE  700
&NMEZ           SETA          7
T700            FW            701,(GAM,P,P,PIM,PIP,PIM,PIP,PIM,PIP)
                FW            702,(GAM,P,PIP,PIM,P,PIM,PIP,PIM,PIP)
                FW            703,(GAM,P,PIP,PIM,PIP,PIM,P,PIM,PIP)
                FW            704,(GAM,P,PIP,PIM,PIP,PIM,PIP,PIM,P)
                FW             ,
*               FIT TABLE     FOR  TYPE  900
&NMEZ           SETA          9
T900            FW            901,(GAM,P,P,PIM,PIP,PIM,PIP,PIM,PIP,PIM,PIP)
                FW            902,(GAM,P,PIP,PIM,P,PIM,PIP,PIM,PIP,PIM,PIP)
                FW            903,(GAM,P,PIP,PIM,PIP,PIM,P,PIM,PIP,PIM,PIP)
                FW            904,(GAM,P,PIP,PIM,PIP,PIM,PIP,PIM,P,PIM,PIP)
                FW            905,(GAM,P,PIP,PIM,PIP,PIM,PIP,PIM,PIP,PIM,P)
                FW             ,
*               FIT TABLE     FOR  TYPE  200
&CNTRL          SETA          &MMD4+&DECAY
&NMEZ           SETA          2
T200            FW            201,(KO,PIP,PIM)
                FW            202,(LDA,P,PIM)
                FW            203,(GAM,EP,EM)
                FW             ,
```

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TABLE VII

FIT70 RUN

Reject Code	Fits	Remarks
0	21,220	$\chi^2 < 70$ [a]
9	171,828	$\chi^2 > 70$
10	93	too many (> 100) cut-steps
11	4	too many (> 50) iterations

a. Much larger than the final cutoff used in post-kinematic analysis.

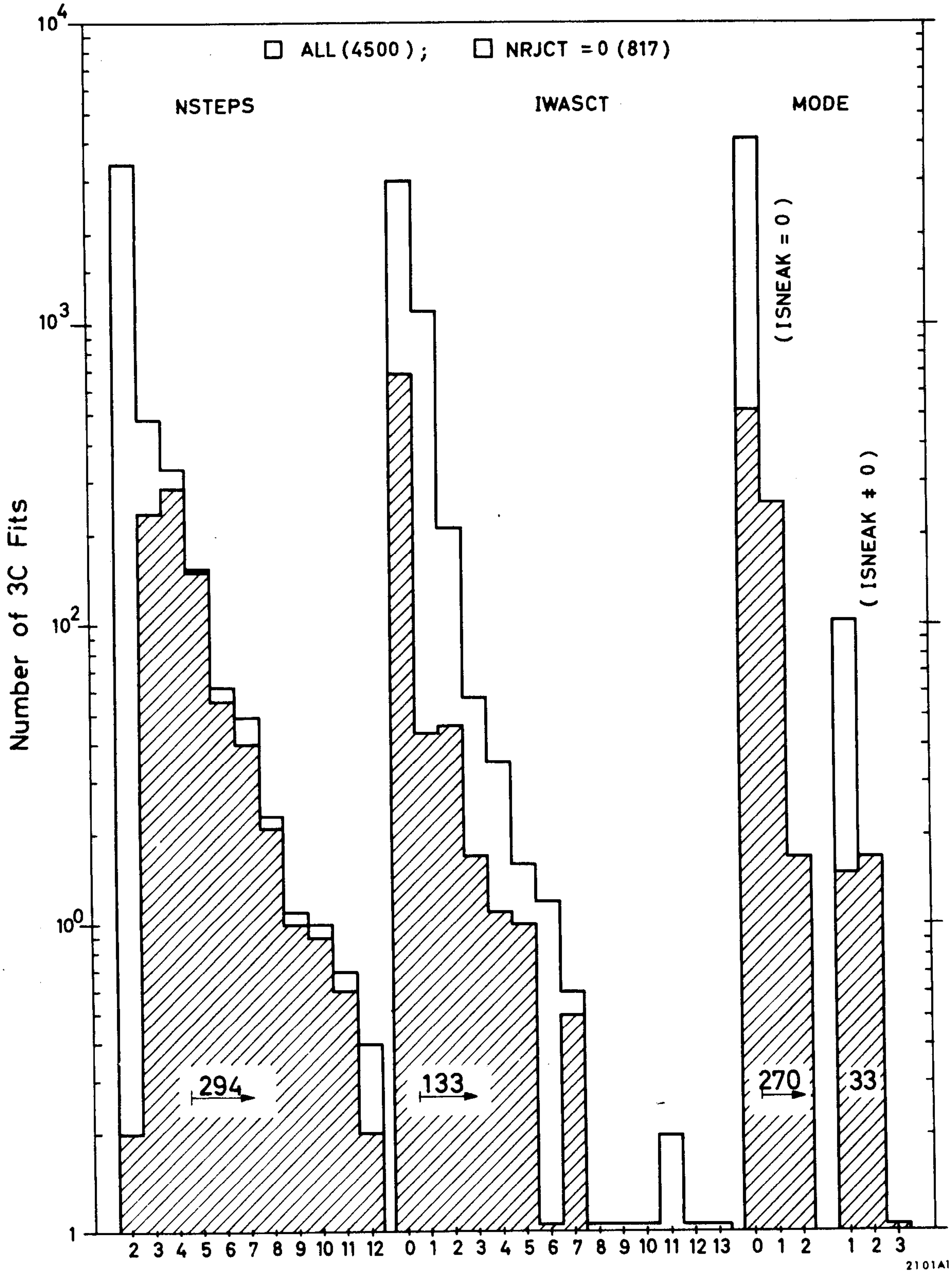

FIG. 3--Number of steps in 5-prong 3C fits.

(ISNEAK$\neq$0) it is set to one to take a step in the special mode in order to speed up the convergence as described in Sections II.C and III.C. It is seen from the figure that in this sample 16% of the successful fits required cut-steps and 37% required the special mode of iteration, without which 4% would not have converged.

Figure 4 shows the fraction (%) of total CPU time spent in each subroutine (only the control section names are given). This was obtained (during the same 5-prong 3C fit run as above) by sampling (total 9982) the Program-Status-Word (PSW) at the rate of one in about 10 msec through an STIMER-EXIT[8] loop. From the figure we find that the vector and matrix manipulations (Table II) account for 60% of the total CPU time. Optimizing here will result in a substantial gain in time.

DIFFERENT CONTROL SECTIONS

% (X10) OF TOTAL PSW

```
 266                        4
 259                        X
 252                        X
 245                        X
 238                        X
 231                        X
 224                        X
 217                        X
 210                        X
 203                        X
 196                        X
 189                        X
 182                        X
 175                        X
 168                        X
 161                        X
 154                        X
 147                        X
 140                        X
 133                        X
 126                        X
 119                        X
 112                        X
 105                        X
  98                        X   2
  91                        X   X
  84                        X   X
  77                        X  3X
  70                        X  XX                        3
  63                    3   X  XX                        X    5
  56                    X   X  XX                        X    X
  49                    X   X  XX               2        X    X
  42                    X   X  XX               X        X    X
  35              2      6X2  X  XX               X        X    X
  28              X      XXX  X 5XX5   4          X        X    X
  21              X4     XXX  X3XXXX4  X          X   3    X  6 X
  14             6XX     XXX1 XXXXXXX  X          X   X    X  X X
   7     1       XXX4  14XXXX4XXXXXXX20X1012003202X5  X    XOOX X  3 4
SIGN
                            2
CON-             131     353  6127921  2          4   1    6  2 6
TENT     1       3084  144908437633682 51 12  32 245  7    6  0 1  3 4
      .0080000044740024703929553286413031737736416000000094620900805000
       0000000025310001615218353753005000000100081003000020040100101000

CHAN            1111111111222222222233333333334444444444555555555566666
NOS.   1234567890123456789012345678901234567890123456789012345678901234

       RCLCCCCCBBBBB TBFDDDDDDDDDDDIDMB DDDEISBBEOOTWDAFSVIIIIIIIIIIPIIII
       ELOBBBBOLLLLLELIGASSVMMMMMMDMALCRTLCCRWNRUOSUDACEHHHHHHHHHHRHHHH
       AOOLLLLNOOOOOSOTEDCMEMMMM TTRAIOLFQOHRERDDTUOMDRACCCCCCCCCCGCCCC
       DCKKKKKSCCCCCTC TDAPCPPPPIRISNCEP3SOIAIREPTMPJEL LLLLLNECSFMEEUU
       1K 2345TKKKKKRK XJLY YYYYNNVS KV   PBDTERU    AA ASEESACOSCCFROA
        1      12345T6    J  123V E  A    D  EP T    DR TCRXQMOMQVHIRPT
CONTENTS ALL CHAN.=   1000.00                            NNFPREMHRTCOMTB
                                                         2   TLH2THKS  L
```

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FIG. 4--5-prong 3C fit run profile.

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1. Most recent publications from this experiment, in which more references can be found, are: J. Park *et al.*, Nucl. Phys. **B36**, 404 (1972); W. P. Swanson *et al.*, Phys. Rev. Letters **21** 1472 (1971); Technical aspects of the experiment are described in D. Fries *et al.*, *Performance of a Large Streamer Chamber Magnet System*, Stanford Linear Accelerator Center, to be published in Nucl. Instr. Methods.
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7. M. Davier *et al.*, Phys. Rev. **D1**, 790 (1970).
8. IBM System/360 OS Macro. See *IBM System/360 Operating System Supervisor and Data Management Services*, Form C28-6646-3 and *Supervisor and Data Management Macro Instructions*, Form C28-6647-3. The idea of sampling PSW was suggested by PROGTIME of T. Y. Johnson and R. H. Johnson, User Note 33, *Program Performance Measurement*, SLAC Facility, Stanford Computation Center (1970). Because it is carried out from a user's point of view (also without much knowledge of how the system functions), our scheme is extremely simple and more efficient. Typically, as in the run of Fig. 3, only 28 out of a total of 10,000 PSW's sampled did not belong to our load-module, although this result depends somewhat on the environment.