SEARCHING FOR NEW PHYSICS WITH CP VIOLATING B DECAYS

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ABSTRACT

We explore the possibility of using CP violation in B decays to detect the presence of physics beyond the Standard Model. We first study the possibility of new physics in the B–$\overline{B}$ mixing amplitude. We discuss a construction to extract information about the phase and magnitude of the new physics contribution, as well as the CKM parameters in a model-independent way. We point out the difficulty of carrying through this program induced by hadronic uncertainties and discrete ambiguities, and suggest additional measurements to overcome these problems. We then study the possibility of new physics contributions to the B meson decay amplitudes. We emphasize the sensitivity of the $B \to \phi K_S$ decay to these new contributions and explain how this sensitivity can be quantified using experimental data on $SU(3)$ related decays. Finally, we analyze a number of models where the B decay amplitudes are modified.

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1 Introduction

CP violation has so far only been observed in the decays of neutral K mesons. It is one of the goals of the proposed B factories to find and study CP violation in the decays of B mesons, and thus elucidate the mechanisms by which CP violation manifests itself in the low-energy world. There is a commonly accepted Standard Model of CP violation, namely, that it is a result of the one physical phase in the 3×3 Cabibbo-Kobayashi-Maskawa (CKM) matrix.[1] This scenario has specific predictions for the magnitude as well as patterns of CP violation that will be observed in the B meson decays.[2] However, since there currently exists only one experimental measurement of CP violation, it is possible that the Standard Model explanation for it is incorrect, or more likely that in addition to the one CKM phase, there are additional CP violating phases introduced by whatever new physics lies beyond the Standard Model.

In this lecture we study the possibility of detecting the presence of physics beyond the Standard Model, using the CP violating asymmetries measured in the decays of neutral B_d mesons to CP eigenstates, in a largely model-independent way. (For recent reviews concerning possible outcomes in specific models, see Refs. 3 and 4.) We first introduce the necessary formalism and, in Sec. 2, briefly review the situation concerning these CP asymmetries in the Standard Model. Section 3 deals with the possibility of new physics in the B–$\overline{B}$ mixing amplitude, while in Sec. 4 we study the possibility of new physics in the B decay amplitudes. We present our conclusions in Sec. 5.

1.1 Formalism

In this subsection, we display the well-known formulae for the decays of neutral B mesons into CP eigenstates, and highlight the relevant features that are important when more than one decay amplitude contributes to a particular process.

The time-dependent CP asymmetry for the decays of states that were tagged as pure B^0 or $\overline{B}^0$ at production into CP eigenstates is defined as

$$a_{f_{CP}}(t) \equiv \frac{\Gamma[B^0(t) \to f_{CP}] - \Gamma[\overline{B}^0(t) \to f_{CP}]}{\Gamma[B^0(t) \to f_{CP}] + \Gamma[\overline{B}^0(t) \to f_{CP}]}, \tag{1}$$

and given by

$$a_{f_{CP}}(t) = a_{f_{CP}}^{cos} \cos(\Delta M t) + a_{f_{CP}}^{sin} \sin(\Delta M t), \tag{2}$$

where

$$a_{f_{CP}}^{cos} = \frac{1-|\lambda|^2}{1+|\lambda|^2}; \qquad a_{f_{CP}}^{sin} = -\frac{2\mathrm{Im}\lambda}{1+|\lambda|^2}. \tag{3}$$

Here, ΔM is the mass difference between the two physical states, and

$$\lambda = \left(\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \right) \frac{\langle f_{CP}|\mathcal{H}|\overline{B}^0\rangle}{\langle f_{CP}|\mathcal{H}|B^0\rangle} = e^{-2i\phi_M} \frac{\bar{A}}{A}, \tag{4}$$

where we have used the fact that $M_{12} \gg \Gamma_{12}$ to replace the first fraction in Eq. (4) by $e^{-2i\phi_M}$, the phase of B–$\overline{B}$ mixing.

If the decay amplitude A has only one dominant contribution, $A = |A|e^{i\phi_D}$, then one has $\bar{A} = A^*$ and consequently $|\lambda| = 1$. Thus, in this case, $a_{f_{CP}}^{cos} = 0$, and $a_{f_{CP}}^{sin} = \sin 2(\phi_M + \phi_D)$ is a clean measure of the CP violation due to interference between the mixing and decay amplitudes. In addition, if there is no new physics contribution to the mixing matrix (or if it is in phase with the Standard Model contribution), $a_{f_{CP}}^{sin}$ cleanly measures CP violating phases in the CKM matrix since both ϕ_M and ϕ_D are simply sums of these.[5]

Consider now the case where the decay amplitude A contains contributions from two terms with magnitudes A_i, CP violating phases ϕ_i, and CP conserving phases δ_i. (In what follows, it will be convenient to think of A_1 giving the dominant Standard Model contribution, and A_2 giving the subleading Standard Model contribution or the new physics contribution.)

$$A = A_1 e^{i\phi_1} e^{i\delta_1} + A_2 e^{i\phi_2} e^{i\delta_2}, \qquad \bar{A} = A_1 e^{-i\phi_1} e^{i\delta_1} + A_2 e^{-i\phi_2} e^{i\delta_2}. \tag{5}$$

To first order in $r \equiv A_2/A_1$, Eq. (3) reduces to[6]

$$a_{f_{CP}}^{cos} = -[2r \sin(\phi_{12}) \sin(\delta_{12})] \tag{6}$$

and

$$a_{f_{CP}}^{sin} = -[\sin 2(\phi_M + \phi_1) + 2r \cos 2(\phi_M + \phi_1) \sin(\phi_{12}) \cos(\delta_{12})], \tag{7}$$

where we have defined $\phi_{12} = \phi_1 - \phi_2$ and $\delta_{12} = \delta_1 - \delta_2$.

In the case of $r = 0$ or $\phi_{12} = 0$, one recovers the case studied above, where $a_{f_{CP}}^{sin}$ cleanly measures the CP violating quantity $\sin 2(\phi_M + \phi_1)$. If $r \neq 0$ and $\phi_{12} \neq 0$, we can consider two distinct scenarios:

(a) Direct CP violation ($a_{f_{CP}}^{cos} \neq 0$). This occurs when $\delta_{12} \neq 0$ and can be measured by a careful study of the time dependence, since it gives rise to a

$\cos \Delta Mt$ term in addition to the $\sin \Delta Mt$ term. Such a scenario would also give rise to CP asymmetries in charged B decays.

(b) Different quark level decay channels that measure the same phase when only one amplitude contributes can measure different phases if more than one amplitude contributes, *i.e.*, two different processes with the same ϕ_1, but with different r or ϕ_2.

For the rest of this lecture, we concentrate on the information we can get from $a_{f_{CP}}^{sin}$. To this end we write

$$a_{f_{CP}}^{sin} \equiv a_{f_{CP}} = -\sin 2(\phi_0 + \delta\phi), \tag{8}$$

where $\phi_0 = \phi_M + \phi_1$, and $\delta\phi$ is the correction to it. For small r, $\delta\phi \leq r$. However, for $r > 1$, $\delta\phi$ can take any value. Thus, when we catalog values of $\delta\phi$ for various models, we use $\delta\phi \simeq 1$ to indicate an arbitrary value.

2 The Standard Model

All the information about flavor and CP violation in the Standard Model is encoded in the CKM matrix. Although the CKM matrix could have up to five large phases (only one of which is independent), we know experimentally that only two of these are large. Thus, we can write the CKM matrix as:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & |V_{ub}|e^{-i\gamma} \\ V_{cd} & V_{cs} & V_{cb} \\ |V_{td}|e^{-i\beta} & V_{ts} & V_{tb} \end{pmatrix}. \tag{9}$$

The phase structure and the magnitudes of the elements are most transparent in the Wolfenstein parametrization,[7] where the CKM matrix is given by

$$V_{CKM} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}. \tag{10}$$

Here, $\lambda = 0.22$ is the Cabbibo angle. Unitarity of the CKM matrix implies the relation

$$V_{cd}V_{cb}^* + V_{ud}V_{ub}^* + V_{td}V_{tb}^* = 0, \tag{11}$$

which is usually graphically represented as the "unitarity triangle" in the $\rho - \eta$ plane (see Fig. 1). In principle, one can determine β and γ (or alternatively ρ

and η) from the available data on K and B decays. However, given the large theoretical uncertainties in the input parameters (*e.g.*, B_K, f_B), the size of these phases remains uncertain (for recent reviews, see Refs. 8 and 9).

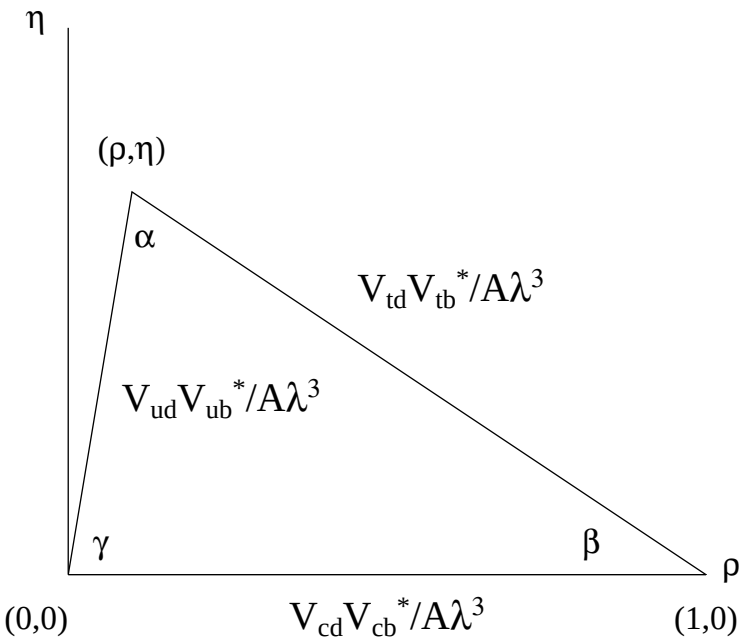

Figure 1. The unitarity triangle.

This is where the CP violating experiments at the B factories come into their own. In the Standard Model, the B–$\overline{B}$ mixing amplitude is dominated by the box diagram with top quarks in the loop. Thus, the phase of the mixing amplitude is given by the phase of $(V_{tb}V_{td}^*)^2$, and in the convention for the CKM matrix above, we get $\phi_M = 2\beta$. In order to extract the CKM phases, we then need decay modes of the B's that are dominated by one decay amplitude, depend on independent CKM phases, and are experimentally feasible. Some examples are:

(*i*) $B \to \psi K_S$ (Ref. 10): The decay is driven by the quark level process $b \to c\bar{c}s$. Moreover, the dominant contribution to K–$\overline{K}$ mixing is proportional to $V_{cs}V_{cd}^*$ (the box diagram with charm quarks). Thus, the CKM elements in the decay amplitude are $(V_{cb}^*V_{cs})(V_{cs}^*V_{cd})$ leading to $\phi_1 = 0$ and subsequently $a_{\psi K_S} = \sin 2\beta$. This decay has a high rate, $BR[B \to \psi K_S] = 4 \times 10^{-4}$ with the ψ tagged by its decay into two leptons, $BR[\psi \to l^+l^-] = 0.12$. Moreover, there is negligible pollution from subleading decay amplitudes.

(*ii*) $B \to \pi^+\pi^-$: This decay gets a tree-level contribution from the quark process $b \to u\bar{u}d$. Thus, the CKM elements in the decay amplitude are $V_{ub}^*V_{ud}$ leading to $\phi_1 = \gamma$ and subsequently $a_{\pi\pi} = -\sin 2(\beta + \gamma) = \sin 2\alpha$. The expected rate is $BR[B \to \pi^+\pi^-] \sim 1 \times 10^{-5}$. There is expected to be a substantial pollution to this prediction coming from the penguin-induced $b \to d\bar{u}u$ decay. However, it may be possible to still obtain a measure of α by measuring other isospin-related $B \to \pi\pi$ rates.[11]

(*iii*) $B \to \phi K_S$ (Ref. 12): This decay is driven by the quark level process $b \to s\bar{s}s$. The leading contribution to this decay is a penguin diagram with top quarks in the loop. Thus, the CKM elements in the decay amplitude (after including K–$\overline{K}$ mixing) are $(V_{tb}^*V_{ts})(V_{cs}^*V_{cd})$ leading to $\phi_1 = 0$ and subsequently $a_{\phi K_S} = \sin 2\beta$. The expected rate is $BR[B \to \phi K_S] \sim 1 \times 10^{-5}$, with the ϕ tagged by its decay into two K's: $BR[\phi \to K^+K^-] = 0.5$. As we will discuss later, the CP asymmetry in this mode is particularly sensitive to new physics contributions;[13] moreover, the Standard Model pollution to this mode is small and quantifiable.[14] Thus, this mode provides an interesting consistency check.

3 New Physics in the B–$\overline{B}$ Mixing Amplitude

In this section, we study the possibility of detecting new contributions to the B–$\overline{B}$ mixing amplitudes.[15] We discuss a construction that allows us to extract information about the CKM matrix elements, as well as the phase and magnitude of the new physics contribution. We highlight potential difficulties in carrying out this construction and suggest ways to overcome them.

3.1 The Basic Assumptions and Results

The first two CP asymmetries to be measured in a B factory are likely to be

$$\frac{\Gamma(B^0_{\rm phys}(t)\to\psi K_S)-\Gamma(\overline{B}^0_{\rm phys}(t)\to\psi K_S)}{\Gamma(B^0_{\rm phys}(t)\to\psi K_S)+\Gamma(\overline{B}^0_{\rm phys}(t)\to\psi K_S)}=a_{\psi K_S}\sin(\Delta m_B t), \tag{12}$$

$$\frac{\Gamma(B^0_{\rm phys}(t)\to\pi\pi)-\Gamma(\overline{B}^0_{\rm phys}(t)\to\pi\pi)}{\Gamma(B^0_{\rm phys}(t)\to\pi\pi)+\Gamma(\overline{B}^0_{\rm phys}(t)\to\pi\pi)}=a_{\pi\pi}\sin(\Delta m_B t). \tag{13}$$

In addition, the B factory will improve our knowledge of the B–$\overline{B}$ mixing parameter, $x_d\equiv\frac{\Delta m_B}{\Gamma_B}$, and of the charmless semileptonic branching ratio of the B mesons.

Within the Standard Model, these four measurements are useful in constraining the unitarity triangle. The asymmetries in Eq. (12) and Eq. (13) measure angles of the unitarity triangle:

$$a_{\psi K_S} = \sin 2\beta, \tag{14}$$

$$a_{\pi\pi} = \sin 2\alpha, \tag{15}$$

where

$$\alpha\equiv\ \arg\left[-\frac{V_{td}V^*_{tb}}{V_{ud}V^*_{ub}}\right],\quad \beta\equiv\ \arg\left[-\frac{V_{cd}V^*_{cb}}{V_{td}V^*_{tb}}\right]. \tag{16}$$

In Eq. (14), we have taken into account the fact that the final state is CP-odd. In Eq. (15), we have ignored possible penguin contamination which can, in principle, be eliminated by isospin analysis.[11] The measurement of x_d determines one side of the unitarity triangle (R_t) up to the unknown constant $\sqrt{B_B}f_B$:

$$x_d = C_t R_t^2, \tag{17}$$

where

$$R_t\equiv\left|\frac{V^*_{tb}V_{td}}{V^*_{cb}V_{cd}}\right|, \tag{18}$$

and $C_t=\tau_b\frac{G_F^2}{6\pi^2}\eta_B m_B(B_B f_B^2)m_t^2 f_2(m_t^2/m_W^2)|V^*_{cb}V_{cd}|^2$ (for definitions and notations, see Ref. 2). The present values are $x_d=0.73\pm0.05$ and $C_t\sim 0.4$–0.8 for $\sqrt{B_B}f_B=$ 140–200 MeV (Ref. 16). Measurements of various inclusive and exclusive $b\to u\ell\nu$ processes will determine (up to uncertainties arising from various hadronic models) the length of the other side of the unitarity triangle (R_u):

$$\frac{\Gamma(b\to u\ell\nu)}{\Gamma(b\to c\ell\nu)}=\frac{1}{F_{\rm ps}}\left|\frac{V_{cd}}{V_{ud}}\right|^2 R_u^2, \tag{19}$$

where

$$R_u \equiv \left| \frac{V_{ub}^* V_{ud}}{V_{cb}^* V_{cd}} \right|, \tag{20}$$

and $F_{\rm ps} \approx 0.5$ is a phase space factor. The present value for R_u ranges from 0.27 to 0.45, depending on the hadronic model used to relate the measurement at the endpoint region, or of some exclusive mode, to the total $b \to u$ inclusive rate.[16]

In the presence of new physics, it is quite possible that the Standard Model predictions in Eqs. (14), (15), and (17) are violated. The most likely reason is a new, significant contribution to B–$\overline{B}$ mixing that carries a CP violating phase different from the Standard Model one. Other factors that could affect the construction of the unitarity triangle from these four measurements are unlikely to be significant.[17,18]

(a) The $\bar{b} \to \bar{c}c\bar{s}$ and $\bar{b} \to \bar{u}u\bar{d}$ decays for $a_{\psi K_S}$ and $a_{\pi\pi}$ respectively, as well as the semileptonic B decays for R_u, are mediated by Standard Model tree-level diagrams. In most extensions of the Standard Model, there is no decay mechanism that could significantly compete with these contributions. (For exceptions which could affect the $\bar{b} \to \bar{u}u\bar{d}$ decay, see Ref. 13.)

(b) New physics could contribute significantly to K–$\overline{K}$ mixing. However, the small value of ϵ_K forbids large deviations from the Standard Model phase of the mixing amplitude.

(c) Unitarity of the three-generation CKM matrix is maintained if there are no quarks beyond the three generations of the Standard Model. Even in models with an extended quark sector, the effect on B–$\overline{B}$ mixing is always larger than the violation of CKM unitarity.

Our analysis below applies to models where the above three conditions are not significantly violated. Under these circumstances, the relevant new physics effects can be described by two new parameters, r_d and θ_d (Refs. 19–22), defined by

$$\left(r_d e^{i\theta_d}\right)^2 \equiv \frac{\langle B^0 | \mathcal{H}_{\rm eff}^{\rm full} | \overline{B}^0 \rangle}{\langle B^0 | \mathcal{H}_{\rm eff}^{\rm SM} | \overline{B}^0 \rangle}, \tag{21}$$

where $\mathcal{H}_{\rm eff}^{\rm full}$ is the effective Hamiltonian including both Standard Model and new physics contributions, and $\mathcal{H}_{\rm eff}^{\rm SM}$ only includes the Standard Model box diagrams. In particular, with this definition, the modification of the two CP asymmetries in Eqs. (14) and (15) depends on a *single* new parameter, the phase θ_d:

$$a_{\psi K_S} = \sin(2\beta + 2\theta_d), \tag{22}$$

$$a_{\pi\pi} = \sin(2\alpha - 2\theta_d), \tag{23}$$

while the modification of the B–$\overline{B}$ mixing parameter x_d in Eq. (17) is given by the magnitude rescaling parameter, r_d:

$$x_d = C_t R_t^2 r_d^2. \tag{24}$$

Furthermore, since the determination of R_u from the semileptonic B decays is not affected by the new physics, and since the unitarity triangle remains valid, we have the following relations between the length of its sides and its angles:

$$R_u = \frac{\sin\beta}{\sin\alpha}, \tag{25}$$

$$R_t = \frac{\sin\gamma}{\sin\alpha}, \tag{26}$$

where

$$\gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]. \tag{27}$$

When α, β, and γ are defined to lie in the $\{0, 2\pi\}$ range, they satisfy

$$\alpha + \beta + \gamma = \pi \text{ or } 5\pi. \tag{28}$$

The four measured quantities $a_{\psi K_S}$, $a_{\pi\pi}$, x_d, and R_u can be used to achieve the following:[19]

(i) fully reconstruct the unitarity triangle and, in particular, find α, β and R_t;

(ii) find the magnitude and phase of the new physics contribution to B–$\overline{B}$ mixing, namely, determine r_d and θ_d.

It is straightforward to show that the above tasks are possible. Equations (22), (23), and (25) give three equations for three unknowns, α, β, and θ_d. Once α and β are known, γ can be extracted from Eq. (28), R_t can then be deduced from Eq. (26), and finally r_d is found from Eq. (24).

In the next two subsections, we describe how to determine the parameters, both in the ρ–η plane and in the $\sin 2\alpha$–$\sin 2\beta$ plane. In practice, however, it is quite likely that the combination of experimental and theoretical uncertainties (particularly in the x_d and R_u constraints) and discrete ambiguities will limit the usefulness of the above method rather significantly. We discuss the source of the hadronic uncertainties in Sec. 3.4 and the discrete ambiguities that arise in this calculation in Sec. 3.5. We mention ways to resolve some of the ambiguities in Sec. 3.6.

3.2 The ρ–η Plane

The key point in the extraction of the CKM parameters is that the angle θ_d cancels in the following sum:

$$2(\alpha+\beta) = \arcsin(a_{\psi K_S}) + \arcsin(a_{\pi\pi}). \tag{29}$$

In other words, the angle γ can be determined (up to the discrete ambiguities to be discussed in Sec. 3.5). In the ρ–η plane, a value for γ gives a ray from the origin, while a value for R_u gives a circle that is centered in the origin. The intersection point of the line and the circle gives (ρ,η) of the unitarity triangle and determines it completely.

A graphical way to carry out these calculations in the ρ–η plane is the following (see Fig. 2 and Ref. 22). One draws the four curves that correspond to Eqs. (14), (15), (17), and (19) [even though only Eq. (19) is valid in the presence of new physics]. The next step is to draw the ray from the origin that passes through the intersection point of the β-ray and the α-circle: this is the *correct* γ-ray (see the dashed line in Fig. 2). The intersection point of the γ-ray and the R_u-circle gives the *correct* vertex of the unitarity triangle, (ρ,η), namely,

$$\begin{aligned} \tan\beta &= \frac{\eta}{1-\rho}, \\ R_t^2 &= \eta^2 + (1-\rho)^2. \end{aligned} \tag{30}$$

The information about the new physics contribution to B–$\overline{B}$ mixing is found from the intersection point of the β-ray and the x_d-circle, (ρ',η'), namely,

$$\begin{aligned} \theta_d &= \arctan\frac{\eta'}{1-\rho'} - \arctan\frac{\eta}{1-\rho}, \\ r_d^2 &= \frac{\eta'^2 + (1-\rho')^2}{\eta^2 + (1-\rho)^2}. \end{aligned} \tag{31}$$

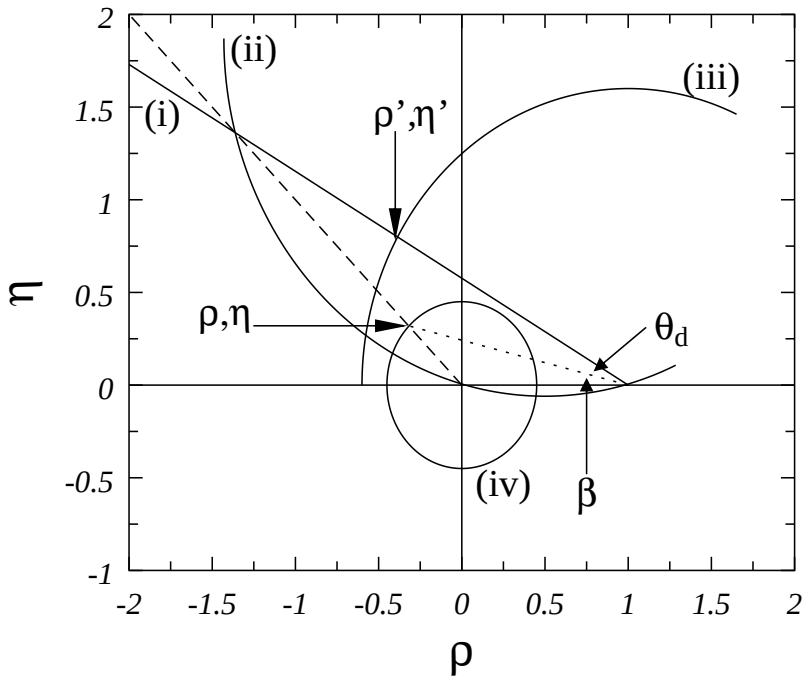

Figure 2. The model-independent analysis in the ρ–η plane: (i) the $a_{\psi K_S}$ ray, (ii) the $a_{\pi\pi}$ circle, (iii) the x_d circle, and (iv) the R_u circle. The γ ray is given by the dashed line. The true β ray is given by the dotted line. Also shown are the true vertex of the unitarity triangle (ρ, η) and the (ρ', η') point that serves to find θ_d and r_d.

3.3 The $\sin 2\alpha$–$\sin 2\beta$ Plane

A presentation of the various constraints in the $\sin 2\alpha$–$\sin 2\beta$ plane[19,23,24] is useful because the two angles are usually correlated.[25] The model-independent analysis is demonstrated in Fig. 3. The R_u constraint gives an eight-shaped curve on which the physical values have to lie. The various solutions for Eq. (29) fall on two ellipses, the intersections of which with the R_u curve determine the allowed values of $\sin 2\alpha$ and $\sin 2\beta$. Note that these ellipses cross the eight-shaped curve in 16 points, but only eight of these points are true solutions. The inconsistent intersection points can be found by noting that the slopes of the ellipse at the consistent points should be $(\cos 2\alpha, -\cos 2\beta)$. The eight correct solutions are denoted by the filled circles in Fig. 3.

In the above, we showed how to use measured values of the CP asymmetries $a_{\psi K_S}$ and $a_{\pi\pi}$ to find the allowed values for α and β. The presentation in the $\sin 2\alpha$–$\sin 2\beta$ plane is also useful for the opposite situation. Some models predict specific values for α and β. (Such predictions can arise naturally from horizontal symmetries.) On the other hand, the models often allow new contributions to B–$\overline{B}$ mixing of unknown magnitude and phase. In this case, the predicted value of $(\sin 2\alpha, \sin 2\beta)$ is just a point in the plane, and the ellipse in Eq. (29) actually gives the allowed (and correlated) values of $(a_{\pi\pi}, a_{\psi K_S})$. (Such an analysis was carried out in Ref. 26.)

More generally, even in models that make no specific predictions for CKM parameters, we usually have some constraints on the allowed range for α and β. For example, in this work we assume the validity of the limits on R_u from charmless semileptonic B decays, which constrains the ratio $\sin\beta/\sin\alpha$ through Eq. (25). Note, however, that this constraint by itself cannot exclude any region in the $a_{\pi\pi}$–$a_{\psi K_S}$ plane. The reason is the following: For any value of R_u, neither α nor θ_d are constrained. (The angle β is constrained for any $R_u < 1$ and certainly by the present range, $0.27 < R_u < 0.45$.) Then, any value of $a_{\psi K_S}$ can be accommodated by an appropriate choice of θ_d and any value of $a_{\pi\pi}$ can be fitted by further choosing an appropriate α. Obviously, to get predictions for the CP asymmetries beyond the Standard Model, one has to make some assumptions that go beyond our generic analysis.

For example, consider models where ϵ_K is dominated by the Standard Model box diagrams (while B–$\overline{B}$ mixing is not). Then, we know that $0 < \gamma < \pi$. This

already excludes part of the allowed range. In particular, $(a_{\pi\pi}, a_{\psi K_S}) = (1, -1)$ or $(-1, 1)$ requires $\gamma = 0$ or π, and is therefore excluded in this class of models. More generally, in any class of models where $\sin^2\gamma$ cannot assume any value between zero and one, some regions in the $a_{\pi\pi}$–$a_{\psi K_S}$ plane are excluded.

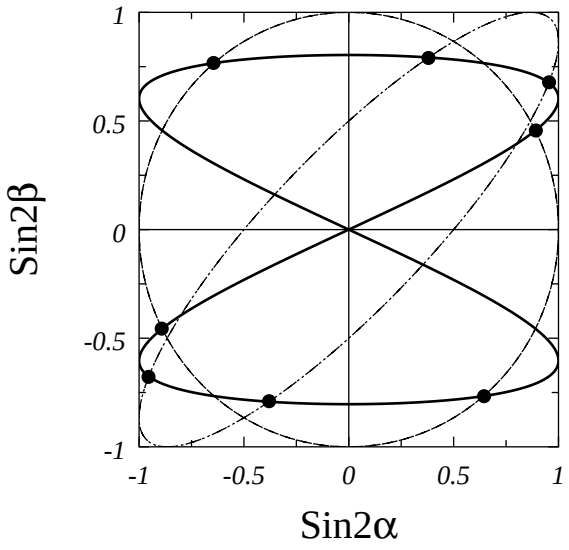

Figure 3. The $\alpha + \beta$ constraint in Eq. (29) and the R_u constraint in Eq. (25) in the $\sin 2\alpha$–$\sin 2\beta$ plane. The eight possible solutions for the unitarity triangle are given by the filled circles.

3.4 Hadronic Uncertainties

In Sec. 2, we argued that the CP asymmetries in B decays that are a result of interference between mixing and decay give us a clean measurement of CP violating quantities that are free of hadronic uncertainties if only one decay amplitude contributes. Yet in the presence of new contributions to the B–$\overline{B}$ mixing amplitude, we find we are once again limited by our theoretical understanding of hadronic physics. To understand the source of the hadronic uncertainty, it is instructive to compare the CP violation in neutral B decays to CP eigenstates with that in neutral K decays to CP eigenstates.

The CP violation in the decay $K_L \to \pi\pi$ is also a result of interference between mixing and decay. The quark level decay is given by the process $s \to u\bar{u}d$ with CKM matrix elements $V_{us}^* V_{ud}$ which are real in the convention we have chosen. Thus, as argued above, the CP asymmetry in this mode cleanly measures $\sin 2\phi_{M_K}$, the phase of the K–$\overline{K}$ mixing amplitude. The problem arises in trying to relate ϕ_{M_K} to phases of CKM matrix elements. This is because although the decay was dominated by one contribution, in this case the mixing amplitude has more than one contribution with unknown relative magnitudes and different (but known) dependence on CKM matrix elements. In particular, there is a large, unknown, long-distance contribution to the K–$\overline{K}$ mixing amplitude, making the interpretation in terms of CKM parameters dependent on poorly known hadronic quantities like B_K.

Similarly, in the presence of new, unknown contributions to the B–$\overline{B}$ mixing amplitude, although $a_{\psi K_S}$ cleanly measures $\sin 2\phi_{M_B}$, it is not possible to relate this to fundamental parameters like the CKM matrix elements without a knowledge of the relative magnitudes of the different contributions. The clean information we had before is lost, and the extraction of CKM parameters is once again dependent on hadronic parameters like $B_B f_B^2$ that are not well-determined at present.

3.5 Discrete Ambiguities

A major obstacle in carrying out the above program will be the discrete ambiguities in determining γ. We now describe these ambiguities.

A physically meaningful range for an angle is 2π. We choose this range to be $\{0, 2\pi\}$. Measurement of any single asymmetry, $\sin 2\phi$, determines the corre-

sponding angle only up to a fourfold ambiguity: ϕ, $\pi/2-\phi$, $\pi+\phi$, and $3\pi/2-\phi$ (mod 2π). Specifically, let us denote by $\bar{\alpha}$ and $\bar{\beta}$ some solution of the equations

$$a_{\psi K_S} = \sin 2\bar{\beta}, \qquad a_{\pi\pi} = \sin 2\bar{\alpha}. \tag{32}$$

Thus, measurements of the two asymmetries leads to a 16-fold ambiguity in the values of the $\{\bar{\alpha}, \bar{\beta}\}$ pair. However, since $\bar{\alpha} = \alpha - \theta_d$ and $\bar{\beta} = \beta + \theta_d$, and unitarity is not violated, γ still satisfies the condition

$$\bar{\alpha} + \bar{\beta} + \gamma = \pi \text{ (mod } 2\pi). \tag{33}$$

Then, the 16 possibilities for γ are divided into two groups of eight that are related by the combined operation $\bar{\alpha} \to \bar{\alpha}+\pi$ and $\bar{\beta} \to \bar{\beta}+\pi$. This in turn shifts the value of γ by 2π. However, since γ is only defined modulo 2π, the ambiguity in γ is reduced to eightfold. We emphasize that this reduction of the ambiguity depends only on the definition of γ. Defining

$$\phi_\pm = \bar{\alpha} \pm \bar{\beta}, \tag{34}$$

the eight possible solutions for γ are

$$\gamma = \pm\phi_+,\ \pi \pm \phi_+,\ \pi/2 \pm \phi_-,\ 3\pi/2 \pm \phi_- \text{ (mod } 2\pi). \tag{35}$$

Note that the eight solutions come in pairs of $\pm\gamma$. This in turn implies that the ambiguity on R_t is only fourfold.

In any model where the three angles $\bar{\alpha}$, $\bar{\beta}$, and γ form a triangle, the ambiguity is further reduced:[2] the requirement that the angles are either all in the range $\{0, \pi\}$ or all in the range $\{\pi, 2\pi\}$ reduces the ambiguity in γ to fourfold. It is enough to know the signs of $a_{\psi K_S}$ and $a_{\pi\pi}$ to carry out this step. Finally, within the Standard Model, the bound $0 < \beta < \pi/4$ (obtained from the sign of ϵ_K and from $R_u < 1/\sqrt{2}$) reduces the ambiguity in γ to twofold.

When we allow for the possibility of new physics effects in the mixing, knowing the signs of $a_{\psi K_S}$ and $a_{\pi\pi}$ does not lead to further reduction in the ambiguity, which remains eightfold. The three angles $\bar{\alpha}$, $\bar{\beta}$, and γ are not angles that define a triangle and therefore further constraints cannot be imposed. It is possible, for example, that both γ and $\bar{\beta}$ lie in the range $\{\pi/2, \pi\}$. Further, the sign of ϵ_K may not be related to the sign of η.

The following example will make the situation clear. Take

$$a_{\pi\pi} = 1/2, \quad a_{\psi K_S} = \sqrt{3}/2. \tag{36}$$

Then, we could have

$$\bar{\alpha} = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}, \quad \bar{\beta} = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}. \tag{37}$$

The eight solutions for γ are

$$\gamma = \frac{\pi}{4},\ \frac{5\pi}{12},\ \frac{7\pi}{12},\ \frac{3\pi}{4},\ \frac{5\pi}{4},\ \frac{17\pi}{12},\ \frac{19\pi}{12},\ \frac{7\pi}{4}. \tag{38}$$

If $\bar{\alpha}, \bar{\beta}, \gamma$ define a triangle, then only four solutions are allowed:

$$(\bar{\alpha}, \bar{\beta}, \gamma) = \left(\frac{\pi}{12}, \frac{\pi}{6}, \frac{3\pi}{4}\right), \left(\frac{\pi}{12}, \frac{\pi}{3}, \frac{7\pi}{12}\right), \left(\frac{5\pi}{12}, \frac{\pi}{6}, \frac{5\pi}{12}\right), \left(\frac{5\pi}{12}, \frac{\pi}{3}, \frac{\pi}{4}\right). \tag{39}$$

Assuming $0 < \bar{\beta} < \pi/4$ as in the Standard Model leaves only the first two choices.

In various specific cases, the discrete ambiguity is smaller. If the two asymmetries are equal in magnitude, there is only a sixfold ambiguity:

$$\begin{aligned} a_{\pi\pi} = a_{\psi K_S} \implies \quad & \gamma = \pm 2\bar{\beta},\ \pi \pm 2\bar{\beta},\ \pi/2,\ 3\pi/2 \ (\mathrm{mod}\ 2\pi), \\ a_{\pi\pi} = -a_{\psi K_S} \implies \quad & \gamma = 0,\ \pi,\ \pi/2 \pm 2\bar{\beta},\ 3\pi/2 \pm 2\bar{\beta} \ (\mathrm{mod}\ 2\pi). \end{aligned} \tag{40}$$

If one of the asymmetries is maximal, there is a fourfold ambiguity, e.g.,

$$\begin{aligned} a_{\pi\pi} = +1 \implies \quad & \gamma = \pm(\pi/4 + \bar{\beta}),\ \ \pm(3\pi/4 - \bar{\beta}) \ \ (\mathrm{mod}\ 2\pi), \\ a_{\pi\pi} = -1 \implies \quad & \gamma = \pm(\pi/4 - \bar{\beta}),\ \ \pm(3\pi/4 + \bar{\beta}) \ \ (\mathrm{mod}\ 2\pi). \end{aligned} \tag{41}$$

If both asymmetries are maximal, the ambiguity is twofold. If the two asymmetries vanish, there is only a fourfold ambiguity:

$$a_{\pi\pi} = a_{\psi K_S} = 0 \implies \gamma = 0,\ \pi/2,\ \pi,\ 3\pi/2. \tag{42}$$

This is an interesting case, because it is predicted by models with approximate CP symmetry (e.g., in some supersymmetric models).[4] Only two of the solutions $(0, \pi)$ correspond to the CP symmetric case while in the other two $(\pi/2, 3\pi/2)$, the zero asymmetries are accidental.

So far, we have ignored the penguin contamination in $a_{\pi\pi}$. The isospin analysis eliminates the penguin contamination only up to a fourfold ambiguity.[11] Therefore, if the isospin analysis is needed, the ambiguities are increased.

In addition, for each value of γ there are two possibilities for θ_d related by $\theta_d \to \theta_d + \pi$. As long as the new physics is such that the $\Delta b = 2$ operator that contributes to B–$\overline{B}$ mixing can be separated into two $\Delta b = 1$ operators, the $\theta_d \to \theta_d + \pi$ ambiguity is physical. Otherwise, it is not physical.

3.6 Final Comments

We argued that the most likely effect of new physics on CP asymmetries in neutral B decays into CP eigenstates will be a significant contribution to the mixing. This is because we have concentrated on decays that are allowed at tree level in the Standard Model. Thus, the new physics effects on the decay amplitudes and on CKM unitarity can be neglected in a large class of models.* We explained that in this class of models, the unitarity triangle can be constructed model independently and the new physics contribution to the mixing can be disentangled from the Standard Model one.

However, the combination of hadronic uncertainties and discrete ambiguities puts serious obstacles in carrying out this calculation. In particular, there is an eightfold ambiguity in the construction of the triangle. In order to get useful results, it will be necessary to reduce the hadronic uncertainties and discrete ambiguities.

One way to eliminate some of the allowed solutions can be provided by a rough knowledge of $\cos(2\alpha–2\theta_d)$, $\cos(2\beta+2\theta_d)$, or $\cos 2\gamma$ (Ref. 27). For example, $\cos(2\alpha–2\theta_d)$ can be determined from the CP asymmetry in $B \to \rho\pi$ (Ref. 28) and $\cos 2\gamma$ from $B \to DK$ (Ref. 29). While a precise measurement of either of these is not expected in the first stages of a B factory, a knowledge of the sign of the cosine is already useful for our purposes: knowing $\mathrm{sign}[\cos 2(\alpha–\theta_d)]$, $\mathrm{sign}[\cos 2(\beta+\theta_d)]$, or $\mathrm{sign}[\cos 2\gamma]$ reduces the ambiguity in γ to fourfold. Knowing two of them reduces it to twofold. (Knowing the three of them, however, cannot be combined to completely eliminate the ambiguity.)

The ambiguity associated with the isospin analysis can be removed by measuring the time-dependent CP asymmetry in $B \to \pi^0\pi^0$ (Ref. 11). Another way is by studying $B \to \rho\pi$ (Refs. 27 and 28). Here, due to interference between several amplitudes, isospin relations can be used to determine $\sin 2\alpha$ without penguin contamination, and without any discrete ambiguity.

A different approach is to make further assumptions about the new physics that is responsible for the effects discussed above. For example, in the Standard Model, there is a strong correlation between $a_{\psi K_S}$ and $a_{\pi\nu\bar{\nu}} \equiv \Gamma(K_L \to \pi^0\nu\bar{\nu})/\Gamma(K^+ \to \pi^+\nu\bar{\nu})$ (Ref. 30), which we illustrate in Fig. 4. However, in most supersymmet-

*The new physics effects may significantly alter the patterns of CP asymmetries in decays that are dominated by penguins in the Standard Model.[13] See Sec. 4 for a discussion of this point.

ric models, processes involving third-generation quarks, such as B–$\overline{B}$ mixing, are significantly modified by the new physics, but processes with only light quarks, such as $K \to \pi\nu\bar{\nu}$, are not.[31] Thus, finding $(a_{\psi K_S}, a_{\pi\nu\bar{\nu}})$ outside the allowed region in Fig. 4 would most likely be due to new physics in the B–$\overline{B}$ mixing amplitude. Then, measurements of $K^+ \to \pi^+\nu\bar{\nu}$ and $K_L \to \pi^0\nu\bar{\nu}$ will provide the true values of R_t or $|\eta|$, respectively. Although one could construct contrived supersymmetric models with large contributions to the $K \to \pi\nu\bar{\nu}$ decays, this possibility is often signalled by large, observable D–$\overline{D}$ mixing.[31] The unitarity triangle can be determined from these up to a fourfold ambiguity. The additional input of R_u reduces this to a twofold ambiguity. The determination of γ by the methods described above will provide a *test* of this class of models. It will not resolve the twofold ambiguity.

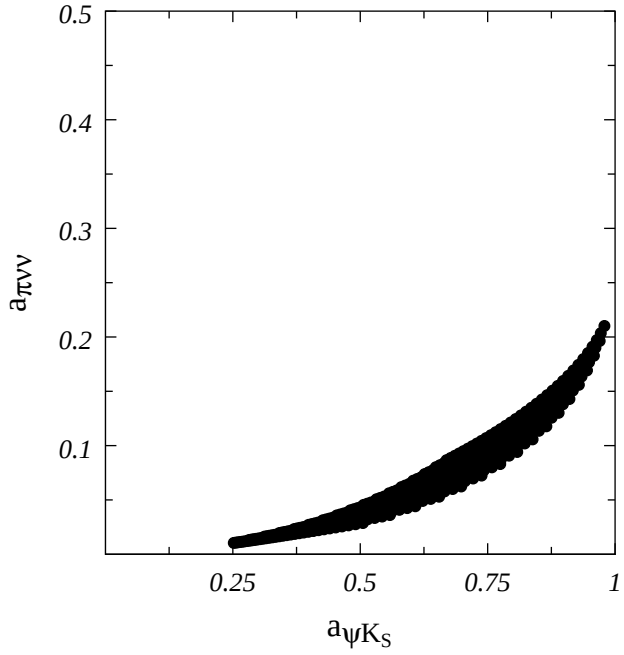

Figure 4. The Standard Model allowed region in the $a_{\psi K_S} - a_{\pi\nu\bar{\nu}}$ plane. We have used $-0.25 \le \rho \le 0.40$, $0.16 \le \eta \le 0.50$.

In some models,[32] there is a significant contribution to both B_d and B_s mixing, but the ratio between the two obeys the Standard Model relation,

$$\frac{\Delta m_{B_d}}{\Delta m_{B_s}} = F_{SU(3)} \sin^2\theta_C R_t^2, \tag{43}$$

where $F_{SU(3)}$ is an $SU(3)$-isospin breaking parameter. Then, a measurement of Δm_{B_s} will provide the correct R_t and, again, the unitarity triangle can be determined, up to a twofold discrete ambiguity, from R_u and R_t. The determination of γ by our analysis is in this case, again, a test and will not resolve the twofold ambiguity. Note, however, that in most models where the *ratio* between B_d and B_s mixing obeys Eq. (43), the *phases* in the B_s, B_d mixing amplitudes are the same as in the Standard Model, namely $\theta_d = 0$. Then, r_d is the only new parameter, and the whole analysis becomes trivial.

In a large class of models, ϵ_K has only small contributions from new physics. If ϵ_K is dominated by the Standard Model, this implies that all angles of the unitarity triangle are in the range $\{0, \pi\}$, and the ambiguity is reduced to fourfold.

Of course, one can combine several of these measurements and assumptions to get a better handle on the true form of the unitarity triangle. It is obvious, however, that the model-independent construction of the triangle, while possible in principle, will pose a serious theoretical and experimental challenge.

4 New Physics in the B Decay Amplitudes

4.1 Introduction

In this section, we make a systematic analysis of the effects of new physics in the B decay amplitudes on the CP asymmetries in neutral B decays.[13] Although these are expected to be smaller than new physics effects on the mixing amplitude, they are easier to probe in some cases. This is based on the fact that given the current uncertainties in the values of the CKM phases, the only precise predictions concerning the CP asymmetries made by the Standard Model are the following:

(i) The CP asymmetries in all B_d decays that do not involve direct $b \to u$ (or $b \to d$) transitions have to be the same.

This prediction holds for the B_s system in an even stronger form:

(ii) The CP asymmetries in all B_s decays that do not involve direct $b \to u$ (or $b \to d$) transition not only have to be the same, but also approximately vanish.

Thus, the best place to look for evidence of new CP violating physics is obviously the B_s system.[33,34] The B factories, however, will initially take data at the $\Upsilon(4s)$ where only the B_d can be studied.

New physics could in principle contribute to both the mixing matrix and to the decay amplitudes. As discussed in the previous section, it is plausible that the new contributions to the mixing could be of the same size as the Standard Model contribution since it is already a one-loop effect. This is why most of the existing studies on the effects of new physics on CP violating B meson decays have concentrated on effects in the mixing matrix, and assume the decay amplitudes are those in the Standard Model.[2,21,35–37] (In Ref. 36 a more general analysis was done where the authors allow for new contributions to the penguin-dominated Standard Model decay amplitudes.) The distinguishing feature of new physics in mixing matrices is that its effect is universal, *i.e.*, although it changes the magnitude of the asymmetries, it does not change the patterns predicted by the Standard Model. Thus, the best way to search for these effects would be to compare the observed CP asymmetry in a particular decay mode with the asymmetry predicted in the Standard Model. This is straightforward for the leading B_s decay modes where the Standard Model predicts vanishing CP asymmetries. However, due to the large uncertainties in the Standard Model predictions for the B_d decays, these new effects would have to be large in order for us to distinguish them from the Standard Model. As discussed in the previous section, one would require additional measurements in order to reduce the hadronic uncertainties and discrete ambiguities that make these effects difficult to detect. In any case, the Standard Model prediction (i) concerning B_d decays still holds.

In contrast, the effects of new physics in decay amplitudes are manifestly non-universal, *i.e.*, they depend on the specific process and decay channel under consideration. Experiments on different decay modes that would measure the same CP violating quantity in the absence of new contributions to decay amplitudes, now actually measure different CP violating quantities. Thus, the Standard Model prediction (i), concerning B_d decays, can be violated. Even though the possibility

of new physics in decay amplitudes is more constrained than that in mixing amplitudes, one could detect these smaller effects by exploiting the fact that now one does not care about the predicted value for some quantity, only that two experiments that should measure the same quantity, in fact, do not. It is this possibility that we wish to study in this section.

We first discuss the possible decay channels, and the uncertainties in the universality predictions introduced within the Standard Model itself by subleading effects. To this end we pay special attention to the decay $B \to \phi K_S$, mediated by the neutral current process $b \to s\bar{s}s$. We explain its usefulness in probing for new physics and discuss the possibility of unexpected long-distance effects polluting this sensitivity. We propose an experimental test to constrain this Standard Model pollution. Finally, we present a brief study of models of new physics that could contain new CP violating decay amplitudes, and their expected size.

4.2 The Different Decay Channels

There are 12 different hadronic decay channels for the b quark: eight of them are charged-current mediated

$$\begin{aligned} &(c1)\ b \to c\bar{c}s\,, \quad (c2)\ b \to c\bar{c}d\,, \quad (c3)\ b \to c\bar{u}d\,, \quad (c4)\ b \to c\bar{u}s\,, \\ &(c5)\ b \to u\bar{c}d\,, \quad (c6)\ b \to u\bar{c}s\,, \quad (c7)\ b \to u\bar{u}d\,, \quad (c8)\ b \to u\bar{u}s, \end{aligned} \tag{44}$$

and four are neutral current

$$(n1)\ b \to s\bar{s}s\,, \quad (n2)\ b \to s\bar{s}d\,, \quad (n3)\ b \to s\bar{d}d\,, \quad (n4)\ b \to d\bar{d}d\,. \tag{45}$$

If only one Standard Model decay amplitude dominates all of these decay channels, *i.e.*, $r = 0$ in Eq. (7), then up to $\mathcal{O}(\lambda^2)$ (where $\lambda \approx 0.22$ is the expansion parameter in the Wolfenstein approximation), the CP asymmetries in B meson decays all measure one of the four phases:

$$\begin{aligned} \alpha &\equiv \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right), \qquad \beta \equiv \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right), \\ \gamma &\equiv \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right), \qquad \beta' \equiv \arg\left(-\frac{V_{cs}V_{cb}^*}{V_{ts}V_{tb}^*}\right) \simeq 0. \end{aligned} \tag{46}$$

This situation is nicely summarized, along with relevant decay modes in Table 1 of Ref. 38. Note that $\beta' < 2.5 \times 10^{-2}$ is very small in the SM,[9] but in principle

measurable. For our purpose, however, this small value is a subleading correction to the clean SM prediction (*ii*). We will study corrections to this idealized limit, as well as to the $r = 0$ limit, in the next subsection. We now discuss the effects that new physics in b quark decay amplitudes could have on the predictions of Eq. (46).

In the Standard Model, the CP asymmetries in the decay modes (*c*1) $b \to c\bar{c}s$ (*e.g.*, $B_d \to \psi K_S$, $B_s \to D_s^+ D_s^-$), (*c*2) $b \to c\bar{c}d$ (*e.g.*, $B_d \to D^+ D^-$, $B_s \to \psi K_S$), and (*c*3) $b \to c\bar{u}d$ (*e.g.*, $B_d \to D^0_{CP}\rho$, $B_s \to D^0_{CP} K_S$) all measure the angle β in B_d decay and β' in B_s decays. [(*c*5) $b \to u\bar{c}d$ acts as a correction to (*c*3) and will be addressed later.] In the presence of new contributions to the B–$\overline{B}$ mixing matrix, the CP asymmetries in these modes would no longer be measuring the CKM angles β and β'. However, they would all still measure the same angles $(\beta + \delta_{m_d}, \beta' + \delta_{m_s})$, where $(\delta_{m_d}, \delta_{m_s})$ are the new contributions to the $B_{(d,s)}$–$\overline{B}_{(d,s)}$ mixing phase. In contrast, new contributions to the b quark decay amplitudes could affect each of these modes differently, and thus they would each be measuring different CP violating quantities.

Several methods[29] have been proposed, based on the fact that the two amplitudes (*c*4) $b \to c\bar{u}s$ and (*c*6) $b \to u\bar{c}s$ (*e.g.*, $B_d \to D_{CP} K_S$, $B_s \to D_{CP}\phi$) are comparable in size, and contribute dominantly to the D^0 or $\bar{D}^0$ parts of D_{CP} respectively, to extract the quantity

$$\arg(b \to c\bar{u}s) + \arg(c \to d\bar{d}u) - \arg(b \to u\bar{c}s) - \arg(\bar{c} \to \bar{d}d\bar{u}) \equiv \gamma. \qquad (47)$$

This measurement of γ is manifestly independent of the B–$\overline{B}$ mixing phase[†].

The mode (*c*7) $b \to u\bar{u}d$ (*e.g.*, $B_d \to \pi\pi$, $B_s \to \rho K_s$) measures the angles $(\beta + \gamma, \beta' + \gamma)$ in the Standard Model. We can combine this measurement with the phase (β, β') measured in the (*c*1) $b \to c\bar{c}s$ mode to get another determination of γ that is independent of the phase in the B–$\bar{B}$ mixing matrix, *e.g.*, comparing $a_{CP}(t)[B_d \to \psi K_S]$ to $a_{CP}(t)[B_d \to \pi\pi]$ allows us to extract

$$\arg(b \to c\bar{c}d) - \arg(b \to u\bar{u}d) \equiv \gamma. \qquad (48)$$

[†] We emphasize that CP asymmetries into final states that contain D_{CP} cannot be affected by possible new contributions to D–$\overline{D}$ mixing. One identifies D_{CP} by looking for CP eigenstate decay products like K^+K^-, $\pi\pi$, or πK_S. As $(\Delta\Gamma/\Gamma)_D$ is known to be tiny, the mass eigenstates cannot be identified. The relevant quantity that enters in the calculation of the CP asymmetry is the D meson decay amplitude and not the D–$\overline{D}$ mixing amplitude. Thus, the only new physics in the D sector that could affect the standard analysis are new contributions to the D decay amplitudes.

Since both of the above evaluations of γ, Eqs. (47) and (48), are manifestly independent of any phases in the neutral meson mixing matrices, the only way they can differ is if there are new contributions to the B or D meson decay amplitudes.

The remaining charged current decay mode $(c8)$ $b \to u\bar{u}s$ suffers from large theoretical uncertainty since the tree and penguin contributions are similar in magnitude, and we will therefore not study it here.

For the neutral current modes, we will first assume that the dominant Standard Model contribution is from a penguin diagram with a top quark in the loop, and discuss corrections to this later. Since these are loop-mediated processes even in the Standard Model, CP asymmetries into final states that can only be produced by flavor-changing neutral current vertices are likely to be fairly sensitive to the possibility of new physics in the B meson decay amplitudes. The modes $(n3)$ $b \to s\bar{d}d$ and $(n4)$ $b \to d\bar{d}d$, however, result in CP eigenstate final states that are the same as for the charged current modes $(c8)$ $b \to u\bar{u}s$ and $(c7)$ $b \to u\bar{u}d$, respectively. Hence, they cannot be used to study CP violation, but rather act as corrections to the charged current modes.

In the Standard Model, the mode $(n1)$ $b \to s\bar{s}s$ (*e.g.*, $B_d \to \phi K_S$, $B_s \to \phi\eta'$) measures the angle β or 0 in B_d and B_s decays. We can once again try and isolate new physics in the decay amplitudes by comparing these measurements with the charged current measurements of β. Finally, $(n2)$ $b \to d\bar{s}s$ (*e.g.*, $B_d \to K_S K_S$, $B_s \to \phi K_S$) measures the angle 0 and β for Standard Model B_d and B_s decays.

4.3 Standard Model Pollution

In all of the preceding discussion, we have considered the idealized case where only one Standard Model amplitude contributes to a particular decay process and we worked to first order in the Wolfenstein approximation. We would now like to estimate the size of the subleading Standard Model corrections to the above processes, which then allows us to quantify how large the new physics effects have to be in order for them to be probed, and what are the most promising modes to study. In this subsection, we concentrate on the charged current modes, and one neutral current mode, $(n2)$ $b \to d\bar{s}s$. We reserve the study of $(n1)$ $b \to s\bar{s}s$ to the next subsection.

There is a Standard Model penguin contribution to $(c1)$ $b \to c\bar{c}s$. However, as is well-known, this contribution has the same phase as the tree-level contribution (up to corrections of order β') and hence $\delta\phi = 0$ in Eq. (8). Thus, in the absence of new contributions to decay amplitudes, the decay $B_d \to \psi K_S$ cleanly measures the phase $\beta + \delta_{m_d}$ (where δ_{m_d} denotes any new contribution to the mixing phase). The mode $(c2)$ $b \to c\bar{c}d$ also has a penguin correction in the Standard Model. However, in this case $\phi_{12} = \mathcal{O}(1)$ and we estimate the correction as[3,39]

$$\delta\phi_{SM}(b \to c\bar{c}d) \simeq \frac{V_{tb}V_{td}^*}{V_{cb}V_{cd}^*}\frac{\alpha_s(m_b)}{12\pi}\log(m_b^2/m_t^2) \lesssim 0.1, \tag{49}$$

where the upper bound is obtained for $|V_{td}| < 0.02$, $m_t = 180$ GeV, and $\alpha_s(m_b) = 0.2$. The mode $(c3)$ $b \to c\bar{u}d$ does not get penguin corrections; however, there is a doubly Cabibbo-suppressed, tree-level correction coming from $(c5)$ $b \to u\bar{c}d$. Thus, $B_d \to D_{CP}\rho$ gets a second contribution with different CKM elements. While in general $\delta\phi$ can be a function of hadronic matrix elements, here we expect this dependence to be very weak.[40] In the factorization approximation, the matrix elements of the leading and subleading amplitude are identical, as are the final state rescattering effects. Moreover, both these cases get contributions from only one electroweak diagram, thus reducing the possibility of complicated interference patterns. We then estimate

$$\delta\phi_{SM}(b \to c\bar{u}d) = \frac{V_{ub}V_{cd}^*}{V_{cb}V_{ud}^*}r_{FA} \leq 0.05, \tag{50}$$

where r_{FA} is the ratio of matrix elements with $r_{FA} = 1$ in the factorization approximation. We have used $|V_{ub}/V_{cb}| < 0.11$, and used what we believe is a reasonable limit for the matrix elements ratio, $r_{FA} < 2$, to obtain the upper bound.

The technique proposed to extract γ using the modes $(c4)$ $b \to c\bar{u}s$ and $(c6)$ $b \to u\bar{c}s$ is manifestly independent of any "Standard Model pollution." Finally, $(c7)$ $b \to u\bar{u}d$ suffers from significant Standard Model penguin pollution, which we estimate as[3,39]

$$\delta\phi_{SM}(b \to u\bar{u}d) \simeq \frac{V_{tb}V_{td}^*}{V_{ub}V_{ud}^*}\frac{\alpha_s(m_b)}{12\pi}\log(m_b^2/m_t^2) \lesssim 0.4, \tag{51}$$

where the upper bound is for $|V_{td}| < 0.02$, $|V_{ub}| > 0.002$, $m_t = 180$ GeV, and $\alpha_s(m_b) = 0.2$. The effects of the Standard Model penguin can be removed by an

isospin analysis.[11] However, this technique would then also rotate away any new physics contributions to the gluonic penguin operator.

Finally, $(n2)$ $b \to d\bar{s}s$ suffers from an $\mathcal{O}(30\%)$ correction due to Standard Model penguins with up and charm quarks.[41]

In summary, the cleanest modes are $b \to c\bar{c}s$ and $b \to c\bar{u}s$ since they are essentially free of any subleading effects. The mode $b \to c\bar{u}d$ suffers only small theoretical uncertainty, less than 0.05. For $b \to c\bar{c}d$ the uncertainty is larger, $\mathcal{O}(0.1)$, and moreover cannot be estimated reliably since it depends on the ratio of tree and penguin matrix elements. Finally, the $b \to u\bar{u}d$ and $b \to d\bar{s}s$ modes suffer from large uncertainties.

4.4 $\mathbf{B \to \phi K_S}$

In this subsection, we would like to carefully analyze the possibility of using the CP asymmetry in $B \to \phi K_S$ as a probe of new physics.[14] To this end we carry out a rigorous analysis of the expected size of the Standard Model pollution. Although we expect a perturbative estimate of the expected size of the pollution along the lines of those carried out above for the other decay modes to be essentially correct, the importance of this mode in searching for new physics warrants a more careful treatment. The sensitivity to new physics of the $B \to \phi K$ decay mode stems from the fact that it is a loop-induced process in the Standard Model, and hence could receive contributions from virtual new physics of comparable size to the Standard Model contribution.

It is well-known that in the Standard Model the time-dependent CP violating asymmetry in $B_d \to \psi K_S$ $[a_{\psi K_S}]$ measures $\sin 2\beta$, where $\beta = \arg(-V_{cd}V_{cb}^*/V_{td}V_{tb}^*)$ and V_{ij} denote the CKM matrix elements. Moreover, being dominated by the tree-level transition $b \to c\bar{c}s$, the decay amplitude of $B_d \to \psi K_S$ is unlikely to receive significant corrections from new physics.[‡] Interestingly, within the Standard Model, the CP asymmetry in $B_d \to \phi K_S$ $[a_{\phi K_S}]$ also measures $\sin 2\beta$ if, as naively expected, the decay amplitude is dominated by the short-distance penguin transition $b \to s\bar{s}s$ (Ref. 12). Since $B_d \to \phi K_S$ is a loop-mediated process within the Standard Model, it is not unlikely that new physics could have a significant effect on it.[13] The expected branching ratio and the high identification

[‡] There is, of course, a possible new contribution to the B^0–$\overline{B}^0$ mixing amplitude. This does not affect the generality of our arguments or the conclusions.[13]

efficiency for this decay suggests that $a_{\phi K_S}$ is experimentally accessible at the early stages of the asymmetric B factories. Thus, the search for a difference between $a_{\psi K_S}$ and $a_{\phi K_S}$ is a promising way to look for physics beyond the Standard Model.[13,42–45]

If, indeed, it turns out that $a_{\psi K_S}$ is not equal to $a_{\phi K_S}$, it would be extremely important to know how precise the Standard Model prediction of them being equal is. In particular, one has to rule out the possibility of unexpected long-distance effects altering the prediction that $a_{\phi K_S}$ measures $\sin 2\beta$ in the Standard Model.

The weak phases of the transition amplitudes are ruled by products of CKM matrix elements. In the $b \to sq\bar{q}$ case, relevant to both $B_d \to \psi K_S$ and $B_d \to \phi K_S$, we denote these by $\lambda_q^{(s)} = V_{qb}V_{qs}^*$. For the purpose of CP violation studies, it is instructive to use CKM unitarity and express any decay amplitude as a sum of two terms. In particular, for $b \to sq\bar{q}$ we eliminate $\lambda_t^{(s)}$ and write

$$A_f = \lambda_c^{(s)} A_f^{cs} + \lambda_u^{(s)} A_f^{us}. \tag{52}$$

The unitarity and the experimental hierarchy of the CKM matrix imply $\lambda_t^{(s)} \simeq \lambda_c^{(s)} \simeq A\lambda^2 + \mathcal{O}(\lambda^4)$ and $\lambda_u^{(s)} = A\lambda^4 e^{i\gamma}$, where $A \approx 0.8$, $\lambda = \sin\theta_c = 0.22$, and γ is a phase of order one. Thus, the first and dominant term is real (we work in the standard parametrization). The correction due to the second term, which is complex and doubly Cabibbo suppressed, is negligibly small unless $A_f^{us} \gg A_f^{cs}$.

The A_f^{qs} amplitudes cannot be calculated exactly since they depend on hadronic matrix elements. However, in some cases we can reliably estimate their relative sizes. For $B \to \psi K_S$, the dominant term includes a tree-level diagram while the CKM-suppressed term contains only one-loop (penguin) and higher order diagrams. This leads to $A_{\psi K_S}^{cs} \gg A_{\psi K_S}^{us}$, and thus ensures that $a_{\psi K_S}$ measures $\sin 2\beta$ in the Standard Model. Since both terms for $B \to \phi K_S$ begin at one-loop order, one naively expects $A_{\phi K_S}^{cs} \sim A_{\phi K_S}^{us}$. In this case $a_{\phi K_S}$ also measures $\sin 2\beta$ in the Standard Model up to corrections of $\mathcal{O}(\lambda^2)$. However, any unexpected enhancement of $A_{\phi K_S}^{us}$ would violate this result. In particular, an enhancement of $\mathcal{O}(\lambda^{-2}) \sim 25$ (analogous to the $\Delta I = 1/2$ rule in K decays) leads to $\mathcal{O}(1)$ violations, and subsequently to $a_{\psi K_S} \neq a_{\phi K_S}$, even in the Standard Model.

In this subsection, we argue against this possibility, presenting different arguments that suggest the pollution of $A_{\phi K_S}^{us}$ in $B_d \to \phi K_S$ is very small. Moreover, we will propose some experimental tests that in the near future could provide quantitative bounds on this pollution.

The natural tool to describe the B decays of interest is by means of an effective $b \to s\bar{q}q$ Hamiltonian. This can be generally written as

$$\mathcal{H}_{eff}^{(s)} = \frac{G_F}{\sqrt{2}} \left\{ \lambda_t^{(s)} \sum_{k=3\ldots10} C_k(\mu) Q_k^s + \lambda_c^{(s)} \sum_{k=1,2} C_k(\mu) Q_k^{cs} + \lambda_u^{(s)} \sum_{k=1,2} C_k(\mu) Q_k^{us} \right\} , \tag{53}$$

where Q_k^i denote the local four-fermion operators and $C_k(\mu)$ the corresponding Wilson coefficients, to be evaluated at a renormalization scale $\mu \sim \mathcal{O}(m_b)$. For our discussion it is useful to emphasize the flavor structure of the operators: $Q_{1,2}^{qs} \sim \bar{b}s\bar{q}q$ and $Q_{3\ldots8}^s \sim \bar{b}s \sum_{q=u,d,s,c} \bar{q}q$, as well as the order of magnitude of their Wilson coefficients: $C_{1,2} \sim \mathcal{O}(1)$ and $C_{3\ldots8} \sim \mathcal{O}(10^{-2})$. The estimates of the $C_k(\mu)$ beyond the leading logarithmic approximation and the definitions of the Q_k^i can be found in Ref. 9. To an accuracy of $O(\lambda^2)$ in the weak phases, $\mathcal{H}_{eff}^{(s)}$ can be rewritten as

$$\mathcal{H}_{eff}^{(s)} = \frac{G_F}{\sqrt{2}} \left\{ \lambda_c^{(s)} \left[\sum_{k=1,2} C_k(\mu) Q_k^{cs} - \sum_{k=3\ldots10} C_k(\mu) Q_k^s \right] + \lambda_u^{(s)} \sum_{k=1,2} C_k(\mu) Q_k^{us} \right\} . \tag{54}$$

It is clear that, when sandwiched between the B_d initial state and the ϕK_S final state, the first term corresponds to $A_{\phi K_S}^{cs}$ and the second to $A_{\phi K_S}^{us}$ [see Eq. (52)]. The pollution is then generated by $Q_{1,2}^{us}$, corresponding to the $b \to s\bar{u}u$ transition.

Since the matrix elements of the Q_k^i have to be evaluated at $\mu \sim \mathcal{O}(m_b)$, a realistic estimate of their relative sizes can be obtained within perturbative QCD. We recall that the $|\phi\rangle$ is an almost pure $|\bar{s}s\rangle$ state. The ω–ϕ mixing angle is estimated to be below 5% (Refs. 16 and 46). We neglect this small mixing in the following. Then, the matrix elements of $Q_{1,2}^{us}$ and $Q_{1,2}^{cs}$ evaluated at the leading order (LO) in the factorization approximation are identically zero. At LO only $Q_{3\ldots8}$, *i.e.*, the short-distance $b \to s\bar{s}s$ penguins, have a nonvanishing matrix element in $B_d \to \phi K_S$. As a consequence, the weak phase of the $B_d \to \phi K_S$ decay amplitude is essentially zero. Nonetheless, given the large Wilson coefficients of $Q_{1,2}^{qs}$, a more accurate estimate of their contribution is required.

At next-to-leading order (NLO), working in a modified factorization approximation, one obtains additional contributions from penguin-like matrix elements of the operators Q_2^{us} and Q_2^{cs} (Ref. 47). These have been reevaluated recently, and shown to be important in explaining the CLEO data on charmless two-body B decays.[48–50] However, even in this case the $b \to s\bar{u}u$ pollution in $B_d \to \phi K_S$ is very small. The reason is that, in the limit where we can neglect both the charm

and the up quark masses with respect to m_b, the matrix elements of $Q^{us}_{1,2}$ and $Q^{cs}_{1,2}$ are identical from the point of view of perturbative QCD [up to corrections of $\mathcal{O}(m_c/m_b) \sim 0.3$]. However, the overall contribution of the charm operators $Q^{cs}_{1,2}$ is enhanced by a factor λ^{-2} with respect to the one of $Q^{us}_{1,2}$. Thus, either if the $B_d \to \phi K_S$ transition is dominated by Q^s_{3-10} (short-distance penguins) or if it is dominated by $Q^{cs}_{1,2}$ (long-distance charming penguins), the weak phase is vanishingly small.

Of course, one could not exclude *a priori* a scenario where the contributions of $Q^s_{3\ldots8}$ and $Q^{cs}_{1,2}$ cancel each other to an accuracy of $O(\lambda^2)$. However, this extremely unlikely possibility would result in an unobservably small $BR(B_d \to \phi K_S)$, rendering this entire discussion moot.

As discussed above, any enhancement of $\langle \phi K_S | Q^{us}_{1,2} | B_d \rangle$ that could spoil the prediction that $a_{\phi K_S}$ measures $\sin 2\beta$ in the Standard Model should occur at low energies in order not to be compensated by a corresponding enhancement of $\langle \phi K_S | Q^{cs}_{1,2} | B_d \rangle$. This possibility is not only disfavored by the OZI rule,[§][51] but is also suppressed by the smallness of the energy range where the enhancement should occur with respect to the scale of the process. We are not aware of any dynamical mechanism that could favor this scenario. Inelastic rescattering effects in B decays due to Pomeron exchange have been argued not to be negligible and to violate the factorization limit.[53] However, even within this context, violations of the OZI rule are likely to be suppressed.[54]

There are experimental tests of our arguments that can be achieved in the sector of $b \to d$ transitions. These are described by an effective Hamiltonian $\mathcal{H}^{(d)}_{eff}$ completely similar to the one in Eq. (53) except for the substitution $s \to d$ in the flavor indices of both CKM factors and four-fermion operators. $SU(3)$ flavor symmetry can be used to obtain relation among several matrix elements. In particular

$$\sqrt{2}\,\langle \phi K_S | Q^{us}_{1,2} | B_d \rangle = \langle \phi \pi^+ | Q^{ud}_{1,2} | B^+ \rangle + \langle K^* K^+ | Q^{ud}_{1,2} | B^+ \rangle \,. \tag{55}$$

[$SU(3)$ breaking effects, which are typically at the 30% level, are neglected here.] The coefficients of these matrix elements are, however, proportional to different CKM factors. This is illustrated in Table 1, where we show the relevant B

[§]This nonperturbative prescription has never been fully understood in the framework of perturbative QCD, but can be justified in the framework of the $1/N_c$ expansion, and is known to work well in most cases and particularly in the vector meson sector.[52]

Decay mode	Operators and CKM factors		
	penguins	c–trees	u–trees
$B_d \to \phi K_S$	$Q^s_{3\ldots8}$ $\lambda_t^{(s)} \sim \lambda^2$	$Q^{cs}_{1,2}$ $\lambda_c^{(s)} \sim \lambda^2$	$Q^{us}_{1,2}$ $\lambda_u^{(s)} \sim \lambda^4$
$B^+ \to \phi\pi^+$ and $B^+ \to K^*K^+$	$Q^d_{3\ldots8}$ $\lambda_t^{(d)} \sim \lambda^3$	$Q^{cd}_{1,2}$ $\lambda_c^{(d)} \sim \lambda^3$	$Q^{ud}_{1,2}$ $\lambda_u^{(d)} \sim \lambda^3$

Table 1. $SU(3)$ related B decay modes that allow us to quantify the Standard Model pollution in $a_{\phi K_S}$.

decay modes along with the CKM factors corresponding to the leading and subleading contributions to the decay amplitudes. If our arguments hold, one expects $BR(B_d \to \phi K_S) \sim \mathcal{O}(\lambda^4)$ and $BR(B^+ \to K^*K^+)$, $BR(B^+ \to \phi\pi^+) \sim \mathcal{O}(\lambda^6)$. Notice, however, that the overall contribution of $Q^{ud}_{1,2}$ in $B^+ \to K^*K^+$ and $B^+ \to \phi\pi^+$ is enhanced with respect to the one of $Q^{us}_{1,2}$ in $B_d \to \phi K_S$ by the corresponding CKM factors: $\lambda_u^{(d)}/\lambda_u^{(s)} = \mathcal{O}(\lambda^{-1})$. Thus, if $\langle \phi K_S | Q^{us}_{1,2} | B_d \rangle$ is enhanced by $\mathcal{O}(\lambda^{-2})$ in order to interfere with the dominant $\mathcal{O}(\lambda^2)$ contributions, then $BR(B^+ \to \phi\pi^+)$ and/or $BR(B^+ \to K^*K^+)$ would be dominated by the similarly enhanced matrix elements of $Q^{ud}_{1,2}$. This would result in an enhancement of the naively Cabibbo-suppressed modes, *i.e.*, we should observe $BR(B^+ \to \phi\pi^+) \sim \mathcal{O}(\lambda^2)$ and/or $BR(B^+ \to K^*K^+) \sim \mathcal{O}(\lambda^2)$ [while $BR(B_d \to \phi K_S)$ is still $\sim \mathcal{O}(\lambda^4)$]. Similar arguments hold for the corresponding B_d decay modes; however, in that case the $SU(3)$ relation is not quite as precise.

To get a quantitative bound, we define the ratios

$$R_1 = \frac{BR(B^+ \to \phi\pi^+)}{BR(B_d \to \phi K_S)}, \qquad R_2 = \frac{BR(B^+ \to K^*K^+)}{BR(B_d \to \phi K_S)}, \tag{56}$$

such that in the Standard Model the following inequality holds:

$$|a_{\psi K_S} - a_{\phi K_S}| < \sqrt{2}\lambda \left(\sqrt{R_1} + \sqrt{R_2}\right)[1 + R_{SU(3)}] + \mathcal{O}(\lambda^2), \tag{57}$$

where $R_{SU(3)}$ represents the $SU(3)$ breaking effects. While measuring $a_{\phi K_S}$ it should be possible to set limits at least of order one on R_1 and R_2, and thus to control by means of Eq. (57) the accuracy to which $a_{\phi K_S}$ measures $\sin 2\beta$ in

the Standard Model. The limits $\sqrt{R_1}, \sqrt{R_2} \lesssim 0.25$ would reduce the theoretical uncertainty to the 10% level.

It may be possible to confirm that $BR(B^+ \to \phi\pi^+)$ and $BR(B^+ \to K^*K^+)$ are not drastically enhanced based just on the current CLEO data. The CLEO Collaboration already has reported the bounds $BR(B^+ \to \phi\pi^+) < 0.56 \times 10^{-5}$ (Ref. 55) and $BR(B^+ \to K^*\pi^+) < 4.1 \times 10^{-5}$ (Ref. 56). Given the similarity of energetic K's and π's in the CLEO environment, it is plausible that a bound similar to the latter can also be derived for the mode $B^+ \to K^*K^+$. Bounds on these branching ratios of $\mathcal{O}(10^{-5})$ would clearly imply that the rates are not $\mathcal{O}(\lambda^2)$ as they would be if the matrix elements of $Q_{1,2}^{ud}$ were enhanced by $\mathcal{O}(\lambda^{-2})$.

The above experimental test can only confirm that $a_{\phi K_S}$ measures $\sin 2\beta$ in the Standard Model. If it turns out that R_1 or R_2 is large, this may be either due to the failure of our conjectures or due to new physics. If, however, R_1 and R_2 are small, and $a_{\psi K_S}$–$a_{\phi K_S}$ violates the Standard Model prediction of Eq. (57), this would be an unambiguous sign of new physics.

Another possible check of our conjecture could be achieved through the measurement of the CP asymmetry in $B_d \to \eta' K_S$. Recently, CLEO has measured a large branching ratio for the related decay $B^+ \to \eta' K^+$, suggesting these processes are penguin dominated and thus that $a_{\eta' K_S}$ also should measure $\sin 2\beta$ in the Standard Model.[44] Nonetheless, the $|\eta'\rangle$ has a nonnegligible $|\bar{u}u\rangle$ component that could enhance the $b \to u\bar{u}s$ pollution and the η' mass is one of the few exceptions where the OZI rule is known to be badly broken. Thus, without fine tuning, a sufficient condition to support our claim on $a_{\phi K_S}$ could be obtained by an experimental evidence of $a_{\eta' K_S} = a_{\phi K_S}$. This would imply that the $b \to u\bar{u}s$ pollution is negligible in both cases.

To summarize, we have argued that the deviation from the prediction that $a_{\phi K_S}$ measures $\sin 2\beta$ in the Standard Model is of $\mathcal{O}(\lambda^2) \sim 5\%$. Moreover, we have shown how the accuracy of this prediction can be tested experimentally. While we concentrated on the time-dependent CP asymmetry, it is clear that our arguments hold also for direct CP violation in charged and neutral $B \to \phi K$ decays, namely, that in the Standard Model the direct CP asymmetry is $\mathcal{O}(\lambda^2)$. Experimentally, we can hope to get an accuracy for both the time-dependent and the direct CP violation of about 10%. Therefore, any measurable direct CP violation in $B \to \phi K$ or an indication that $a_{\psi K_S} \neq a_{\phi K_S}$, combined with

experimental evidence that the Standard Model pollution is of $\mathcal{O}(\lambda^2)$, will signal physics beyond the Standard Model.

4.5 Models

In this subsection, we discuss three models that could have experimentally detectable effects on B meson decay amplitudes, and violate the Standard Model predictions (i) and (ii). We also discuss ways to distinguish these models from each other.

(a) **Effective Supersymmetry:** This is a supersymmetric extension of the Standard Model that seeks to retain the naturalness properties of supersymmetric theories, while avoiding the use of family symmetries or ad-hoc supersymmetry breaking boundary conditions that are required to solve the flavor problems generic to these models.[57,58] In this model, the $\tilde{t}_L$, $\tilde{b}_L$, $\tilde{t}_R$, and the gauginos are light (below 1 TeV), while the rest of the superpartners are heavy ($\sim$ 20 TeV). The bounds on the squark mixing angles in this model can be found in Ref. 22. Using the formulae in Ref. 59 we find that for $\tilde{b}_L$ and gluino masses in the 100–300 GeV range, this model generates $b \to sq\bar{q}$ and $b \to dq\bar{q}$ transition amplitudes via gluonic penguins that could be up to twice as large as the Standard Model gluonic penguins, and with an unknown phase. Thus, this model could result in significant deviations from the predicted patterns of CP violation in the Standard Model. We estimate these corrections to be

$$\begin{aligned} &\delta\phi_A(b \to c\bar{c}s) \lesssim 0.1, \qquad \delta\phi_A(b \to c\bar{c}d) \lesssim 0.2, \qquad \delta\phi_A(b \to u\bar{u}d) \lesssim 0.8, \\ &\delta\phi_A(b \to s\bar{s}s) \lesssim 1, \qquad \delta\phi_A(b \to d\bar{s}s) \lesssim 1. \end{aligned} \tag{58}$$

(b) **Models with Enhanced Chromomagnetic Dipole Operators:** These models have been proposed to explain the discrepancies between the B semileptonic branching ratio, the charm multiplicity in B decays, and the Standard Model prediction for these quantities. These enhanced chromomagnetic dipole operators come from gluonic penguins that arise naturally in TeV scale models of flavor physics.[60] In order to explain the above discrepancies with the Standard Model, these models have amplitudes for $b \to sg$ that are about seven times larger than the Standard Model amplitude. The $b \to sq\bar{q}$ transition in this model is dominated by the dipole operator for $b \to sg$ through the chain $b \to sg^* \to sq\bar{q}$. This interferes with the Standard Model $b \to sq\bar{q}$ amplitude. For the $B \to X_s\phi$,

the net result is that the new amplitudes can be up to a factor of two larger than the Standard Model penguins and with arbitrary phase.[61] It is thus plausible that similar enhancements can be present in the exclusive $b \to c\bar{c}s$ transitions as well. In addition, $b \to dg$ can be as large as $b \to sg$. However, in the Standard Model the $b \to d$ penguins are Cabibbo-suppressed compared to the $b \to s$ penguins. Thus, in this model the corrections to the $b \to d\bar{q}q$ modes could be much larger than the corrections to the $b \to s\bar{q}q$ modes. In the explicit models that have been studied, the relative corrections to the $b \to dg$ Standard Model amplitude are up to three times larger than those to the Standard Model $b \to sg$ amplitude.[61] We estimate the following corrections to the dominant Standard Model amplitudes

$$\begin{aligned} &\delta\phi_B(b \to c\bar{c}s) \lesssim 0.1, \qquad &&\delta\phi_B(b \to c\bar{c}d) \lesssim 0.6, \qquad &&\delta\phi_B(b \to u\bar{u}d) \lesssim 1, \\ &\delta\phi_B(b \to s\bar{s}s) \lesssim 1, \qquad &&\delta\phi_B(b \to d\bar{s}s) \lesssim 1. \end{aligned} \tag{59}$$

(c) **Supersymmetry without R-parity:** Supersymmetric extensions of the Standard Model usually assume the existence of a new symmetry called R-parity. However, phenomenologically viable models have been constructed where R-parity is not conserved.[62] In the absence of R-parity, baryon and lepton number violating terms are allowed in the superpotential. Here, we assume that lepton number is conserved in order to avoid bounds from proton decay and study the effects of possible baryon number violating terms. The relevant terms in the superpotential are of the form $\lambda''_{ijk}\bar{u}_i\bar{d}_j\bar{d}_k$, where antisymmetry under $SU(2)$ demands $j \neq k$. The tree-level decay amplitudes induced by these couplings are then given by

$$A(b \to u_i\bar{u}_jd_k) \approx \frac{\lambda''_{i3l}\lambda''_{jkl}}{2m^2_{\tilde{q}}}, \qquad A(b \to d_i\bar{d}_jd_k) \approx \frac{\lambda''_{l3j}\lambda''_{lik}}{2m^2_{\tilde{q}}}. \tag{60}$$

Note that due to the requirement $i \neq k$ in the neutral current mode, the decay $b \to s\bar{s}s$ will not be corrected. If we use $m_{\tilde{q}} \simeq M_W$ for the squark masses, and assume that there are no significant cancellations between the (possibly several) terms that contribute to a single decay, then the bounds for the relevant coupling constants are[63]

$$\lambda''_{ibs}\lambda''_{ids} \lesssim 5\times10^{-3}, \qquad \lambda''_{ibd}\lambda''_{isd} \lesssim 4.1\times10^{-3}, \qquad \lambda''_{ubs}\lambda''_{cds} \lesssim 2\times10^{-2}. \tag{61}$$

[We have imposed the last bound in Eq. (61) by demanding that the new contribution to the B hadronic width be less than the contribution from the Standard

Model $b \to c\bar{u}d$ decay mode]. These lead to the following corrections to the dominant Standard Model amplitudes

$$\begin{aligned} \delta\phi_C(b \to c\bar{c}s) &\lesssim 0.1, & \delta\phi_C(b \to c\bar{c}d) &\lesssim 0.6, \\ \delta\phi_C(b \to c\bar{u}d) &\lesssim 0.5, & \delta\phi_C(b \to d\bar{s}s) &\lesssim 1. \end{aligned} \tag{62}$$

The observed pattern of CP asymmetries can also distinguish between different classes of new contributions to the B decay amplitudes. Here, we list a few examples:
(1) In model (a) the maximum allowable relative corrections to the $b \to s$ and the $b \to d$ Standard Model amplitudes are similar in size, while in model (b) the relative corrections to the $b \to d$ amplitude can be much larger.
(2) In both models (a) and (b), the neutral current decay $b \to s\bar{s}s$ can get significant $[\mathcal{O}(1)]$ corrections. In model (c), however, this mode is essentially unmodified.
(3) The fact that the $b \to c\bar{u}d$ channel can be significantly affected in model (c) is in contrast with the other two models. In those models the new decay amplitudes were penguin induced and required the up-type quarks in the final state to be a flavor singlet ($c\bar{c}$ or $u\bar{u}$), thus giving no correction to the $b \to c\bar{u}d$ decay.

4.6 Discussion

Table 2 summarizes the relevant decay modes with their Standard Model uncertainty, and the expected deviation from the Standard Model prediction in the three models we gave as examples. New physics can be probed by comparing two experiments that measure the same phase ϕ_0 in the Standard Model [see Eq. (8)]. A signal of new physics will be if these two measurements differ by an amount greater than the Standard Model uncertainty (and the experimental sensitivity), *i.e.*,

$$|\phi(B \to f_1) - \phi(B \to f_2)| > \delta\phi_{SM}(B \to f_1) + \delta\phi_{SM}(B \to f_2), \tag{63}$$

where $\phi(B \to f)$ is the angle obtained from the asymmetry measurement in the $B \to f$ decay.

The most promising way to look for new physics effects in decay amplitudes is to compare all the B_d decay modes that measure β in the Standard Model (and the B_s decay modes that measure β' in the Standard Model). The theoretical uncertainties among all the decays considered are at most $\mathcal{O}(10\%)$, and they have relatively large rates. The best mode is $B_d \to \Psi K_S$, which has a sizable rate

Mode	SM angle (ϕ_0)	$\delta\phi_{SM}$	$\delta\phi_A$	$\delta\phi_B$	$\delta\phi_C$	BR
$b \to c\bar{c}s$	β	0	0.1	0.1	0.1	7×10^{-4}
$b \to c\bar{c}d$	β	0.1	0.2	0.6	0.6	4×10^{-4}
$b \to c\bar{u}d$	β	0.05	0	0	0.5	10^{-5}
$b \to s\bar{s}s$	β	0.04	1	1	0	10^{-5}
$b \to u\bar{u}d$	$\beta + \gamma$	0.4	0.4	1	0	10^{-5}
$b \to u\bar{c}s$	γ	0	0	0	0	10^{-6}
$b \to d\bar{s}s$	0	0.3	1	1	1	10^{-6}

Table 2. Summary of the useful modes. The "SM angle" entry corresponds to the angle obtained from B_d decays assuming one decay amplitude and to first order in the Wolfenstein approximation. The angle γ in the mode $b \to u\bar{c}s$ is measured after combining with the mode $b \to c\bar{u}s$. New contributions to the mixing amplitude would shift all the entries by θ_d. $\delta\phi$ [defined in Eq. (8)] corresponds to the (absolute value of the) correction to the universality prediction within each model: $\delta\phi_{SM}$–Standard Model, $\delta\phi_A$–Effective Supersymmetry, $\delta\phi_B$–Models with Enhanced Chromomagnetic Dipole Operators, and $\delta\phi_C$–Supersymmetry without R-parity. One means that the phase can get any value. The BR is taken from Ref. 64 and is an order-of-magnitude estimate for one of the exclusive channels that can be used in each inclusive mode. For the $b \to c\bar{u}d$ mode, the BR stands for the product $BR(B_d \to \overline{D}\rho) \times BR(\overline{D} \to f_{CP})$, where f_{CP} is a CP eigenstate.

and negligible theoretical uncertainty. This mode should be the reference mode to which all other measurements are compared. The $b \to c\bar{u}d$ and $b \to s\bar{s}s$ modes are also theoretically very clean. In addition, with $b \to s\bar{s}s$ being a loop-mediated process in the Standard Model, it is particularly sensitive to new physics effects. In both cases the conservative upper bound on the theoretical uncertainty is less than 0.05, and can be reduced with more experimental data. Moreover, the rates for the relevant hadronic states are $\mathcal{O}(10^{-5})$, which is not extremely small. Thus, the two "gold-plated" relations are

$$|\phi(B_d \to \psi K_S) - \phi(B_d \to \phi K_S)| < 0.05, \tag{64}$$

and

$$|\phi(B_d \to \psi K_S) - \phi(B_d \to D_{CP}\rho)| < 0.05. \tag{65}$$

Any deviation from these two relations will be a clear indication for new physics in decay amplitudes.

Although not as precise as the previous predictions, looking for violations of the relation

$$|\phi(B_d \to \psi K_S) - \phi(B_d \to D^+D^-)| < 0.1 \tag{66}$$

is another important way to search for new physics in the B decay amplitudes. The advantage is that the relevant rates are rather large, $BR(B_d \to D^+D^-) \approx 4 \times 10^{-4}$. However, the theoretical uncertainty is large too, and our estimate of 10% should stand as a central value of it. As long as we do not know how to calculate hadronic matrix elements, it will be hard to place a conservative upper bound.

New physics can also be discovered by comparing the two ways to measure γ in the Standard Model, *i.e.*, from $b \to c\bar{c}d$ combined with $b \to u\bar{u}d$, and $b \to c\bar{u}s$ combined with $b \to u\bar{c}s$. This is not so promising since the rates are relatively small, and the theoretical uncertainties are larger. Thus, one would require larger effects in order for them to be observable. Moreover, the isospin analysis that would substantially reduce the Standard Model uncertainty in the $b \to u\bar{u}d$ would simultaneously remove the isospin invariant new physics effects from this mode, thus requiring effects in the $b \to c\bar{u}s$ mode (which were not found in the three models studied here).

5 Conclusions

In this lecture, we have studied the possibility of using the time-dependent CP asymmetries in B_d decays to CP eigenstates that will be measured at the asymmetric B factories as a probe of physics beyond the Standard Model. The types of new physics that could affect these experiments can be logically divided into two classes: new $\Delta B = 2$ physics, affecting the B–$\overline{B}$ mixing amplitude, and new $\Delta B = 1$ physics, affecting the B decay amplitudes.

We argued that even in the presence of new $\Delta B = 2$ physics, we can use the CP asymmetries in $B \to \psi K_S$ ($a_{\psi K_S}$) and in $B \to \pi\pi$ ($a_{\pi\pi}$) to reconstruct the unitarity triangle in a model-independent way. In practice, however, hadronic uncertainties and discrete ambiguities in the angles of the unitarity triangle make this a difficult program to carry out. In certain classes of models, such as most models of low-energy supersymmetry, the $K \to \pi\nu\bar{\nu}$ decay rates are not affected by new physics. One could then use these rates to accurately constrain the unitarity triangle. Moreover, discrete ambiguities can be removed by a rough measurement of CP asymmetry in modes such as $B_d \to \rho\pi$.

We presented a detailed, model-independent study of the possibility of detecting new $\Delta B = 1$ physics. This possibility affects the precisely known patterns of CP violation predicted in the Standard Model. Thus, the experiments are potentially sensitive to small effects. We pointed out that the CP asymmetry in the rare $B \to \phi K_S$ decay is particularly sensitive to new physics since it is a loop-mediated process in the Standard Model that is theoretically clean and experimentally accessible. We undertook a detailed study of the possible Standard Model contamination to the sensitivity of this mode and proposed a way to bound this contamination experimentally. Finally, we analyzed a number of models of new physics and showed that not only is it possible that the B decay amplitudes are modified in an experimentally discernible way, but that it is possible to discriminate between classes of models of new physics using these CP violating measurements.

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