

QUANTUM GRAVITY AT THE PLANCK LENGTH

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ABSTRACT

I describe our understanding of physics near the Planck length, in particular the great progress of the last four years in string theory.

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1 Introduction

For obvious reasons, the SLAC Summer Institute is usually concerned with the three particle interactions. It is very appropriate, though, that the subject of the 1998 SSI is gravity, because the next step in understanding the weak, strong, and electromagnetic interactions will probably require the inclusion of gravity as well. There are many reasons for making this statement, but I will focus on two, one based on supersymmetry and one based on the unification of the couplings.

What is supersymmetry? I will answer this in more detail later, but for now let me give two short answers:

- A. A lot of new particles.
- B. A new *spacetime* symmetry.

Answer A is the pragmatic one for a particle experimentalist or phenomenologist. In answer B, I am distinguishing internal symmetries like flavor and electric charge, which act on the fields at each point of spacetime, from symmetries like Lorentz invariance that move the fields from one point to another. Supersymmetry is of the second type. If the widely anticipated discovery of supersymmetry actually takes place in the next few years, it not only means a lot more particles to discover. It also will be the first new spacetime symmetry since the discovery of relativity, bringing the structure of the particle interactions closer to that of gravity; in a sense, supersymmetry is a partial unification of particle physics and gravity.

The unification of the couplings is depicted in figure 1. This is usually drawn with a rather different vertical scale. Here the scale is compressed so that the three gauge couplings can hardly be distinguished, but this makes room for the fourth coupling, the gravitational coupling. Newton's constant is dimensionful, so what is actually drawn is the dimensionless coupling $G_N E^2$ with E the energy scale and $\hbar = c = 1$. This dimensionless gravitational coupling depends strongly on energy, in contrast to the slow running of the gauge couplings.

It is well-known that the three gauge couplings unify to good accuracy (in supersymmetric theories) at an energy around 2×10^{16} GeV. Note however that the fourth coupling does not miss by much, a factor of 20 or 30 in energy scale. This is another way of saying that the grand unification scale is near the Planck scale. In fact, the Planck scale $M_P = 2 \times 10^{19}$ GeV is deceptively high because of various factors like 4π that must be included. Figure 1 suggests that the grand

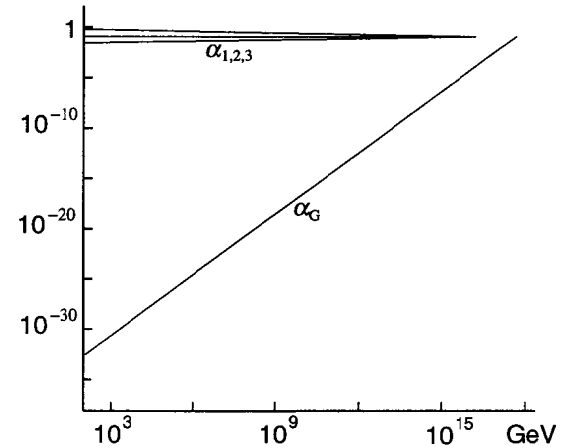


Fig. 1. The three gauge couplings and the dimensionless gravitational coupling as functions of the energy. Here $\alpha_G = G_N E^2 / 8\pi$.

unification of the three gauge interactions will actually be a very grand unification including gravity as well. The failure of the four couplings to meet exactly could be due to any of several small effects, which I will discuss briefly later.*

Figure 1 also shows why the phenomenologies of the gauge interactions and gravity are so different: at accessible energies the coupling strengths are very different. For the same reason, the energy scale where the couplings meet is far removed from experiment. Nevertheless, we believe that we can deduce much of what happens at this scale, and this is the subject of my lectures. At the end I will briefly discuss experimental signatures, and Michael Peskin and Nima Arkani-Hamed will discuss some of these in more detail.

In section 2 I discuss the idea that spacetime has more than four dimensions: first why this is not such a radical idea, and then why it is actually a good idea. In section 3 I review string theory as it stood a few years ago: the motivations from the short distance problem of gravity, from earlier unifying ideas, and from the search for new mathematical ideas, as well as the main problem, vacuum

*I will also discuss briefly the idea of low energy string theory, in which figure 1 is drastically changed.

selection. In section 4 I introduce the idea of duality, including weak–strong and electric–magnetic. I explain how supersymmetry gives information about strongly coupled systems. I then describe the consequences for string theory, including string duality, the eleventh dimension, D-branes, and M-theory. In section 5 I develop an alternative theory of quantum gravity, only to find that “all roads lead to string theory.” In section 6 I explain how the new methods have solved some of the puzzles of black hole quantum mechanics. This in turn leads to the Maldacena dualities, which give detailed new information about supersymmetric gauge field theories. In section 7 I discuss some of the ways that the new ideas might affect particle physics, through the unification of the couplings and the possibility of low energy string theory and large new dimensions. In section 8 I summarize and present the outlook.

2 Beyond Four Dimensions

Gravity is the dynamics of spacetime. It is very likely that at lengths near the Planck scale ($L_P = 10^{-33}$ cm) it becomes evident that spacetime has more than the four dimensions that are visible to us. That is, spacetime is as shown in figure 2a, with four large dimensions (including time) and some additional number of small and highly curved spatial dimensions. A physicist who probes this spacetime with wavelengths long compared to the size of the small dimensions sees only the large ones, as in figure 2b. I will first give two reasons why this is a natural possibility to consider, and then explain why it is a good idea.

The first argument is cosmological. The universe is expanding, so the dimensions that we see were once smaller and highly curved. It may have been that initially there were more than four small dimensions, and that only the four that are evident to us began to expand. That is, we know of no reason that the initial expansion had to be isotropic.

The second argument is based on symmetry breaking. Most of the symmetry in nature is spontaneously broken or otherwise hidden from us. For example, of the $SU(3) \times SU(2) \times U(1)$ gauge symmetries, only a $U(1)$ is visible. Similarly the flavor symmetry is partly broken, as are the symmetries in many condensed matter systems. This symmetry breaking is part of what makes physics so rich: if all of the symmetry of the underlying theory were unbroken, it would be much easier to figure out what that theory is!

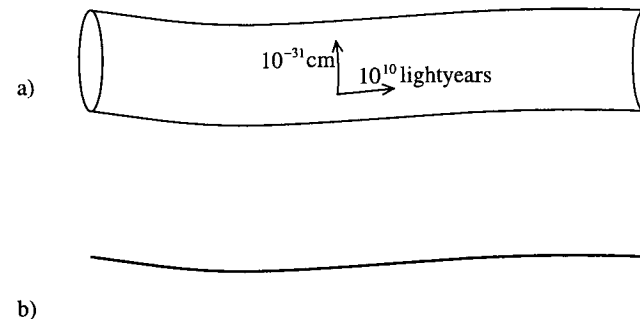


Fig. 2. a) A spacetime with one large dimension and one small one. We assume here that the small dimensions are nearly Planck sized; the possibility of larger dimensions will be considered later. b) The same spacetime as seen by a low energy observer.

Suppose that this same symmetry breaking principle holds for the spacetime symmetries. The visible spacetime symmetry is $SO(3, 1)$, the Lorentz invariance of special relativity consisting of the boosts and rotations. A larger symmetry would be $SO(d, 1)$ for $d > 3$, the Lorentz invariance of $d + 1$ spacetime dimensions. Figure 2 shows how this symmetry would be broken by the geometry of spacetime.

So extra dimensions are cosmologically plausible, and are a natural extension of the familiar phenomenon of spontaneous symmetry breaking. In addition, they may be responsible for some of the physics that we see in nature. To see why this is so, consider first the following cartoon version of grand unification. The traceless 3×3 and 2×2 matrices for the strong and weak gauge interactions fit into a 5×5 matrix, with room for an extra $U(1)$ down the diagonal:

$$\left[\begin{array}{c|c} 3 \times 3 & X, Y \\ \hline X, Y & 2 \times 2 \end{array} \right]. \quad (1)$$

Now let us try to do something similar, but for gravity and electromagnetism. Gravity is described by a metric $g_{\mu\nu}$, which is a 4×4 matrix, and electromagnetism

by a 4-vector A_μ . These fit into a 5×5 matrix:

$$\left[\begin{array}{c|c} g_{\mu\nu} & A_\mu \\ \hline A_\nu & \phi \end{array} \right]. \quad (2)$$

In fact, if one takes Einstein's equations in five dimensions, and writes them out in terms of the components (2), they become Einstein's equations for the four-dimensional metric $g_{\mu\nu}$ plus Maxwell's equation for the vector potential A_μ . This elegant unification of gravity and electromagnetism is known as *Kaluza-Klein theory*.

If one looks at the Dirac equation in the higher-dimensional space, one finds a possible explanation for another of the striking patterns in nature, the existence of quark and lepton generations. That is, a single spinor field in the higher-dimensional space generally reduces to several four-dimensional spinor fields, with repeated copies of the same gauge quantum numbers.

Unification is accompanied by new physics. In the case of grand unification this includes the X and Y bosons, which mediate proton decay. In Kaluza-Klein theory it includes the dilaton ϕ , which is the last element in the matrix (2). I will discuss the dilaton further later, but for now let me note that it is likely not to have observable effects. Of course, in Kaluza-Klein theory there is more new physics: the extra dimension(s)!

Finally, let me consider the threshold behavior as one passes from figure 2b to figure 2a. At energies greater than the inverse size of the small dimensions, one can excite particles moving in those directions. The states are quantized because of the finite size, and each state of motion looks, from the lower-dimensional point of view, like a different kind of particle. Thus the signature of passing such a threshold is a whole tower of new particles, with a spectrum characteristic of the shape of the extra dimensions.

3 String Theory

3.1 The UV Problem

To motivate string theory, I will start with the UV problem of quantum gravity. A very similar problem arose in the early days of the weak interaction. The original

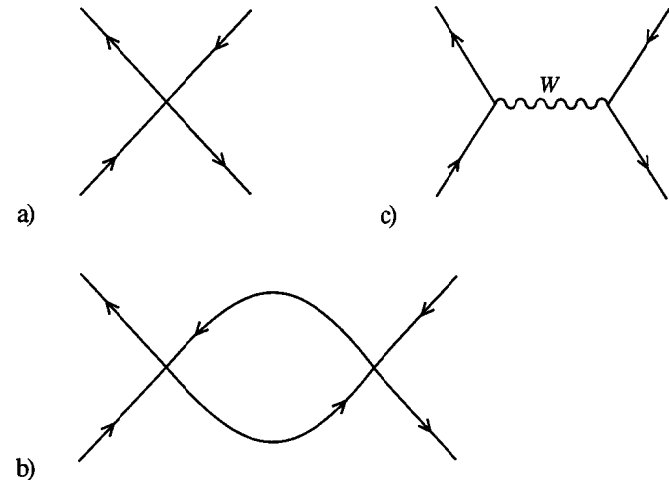


Fig. 3. a) A leptonic weak interaction in four-fermi theory. b) A divergent second order amplitude. c) The weak interaction in Weinberg-Salam theory. At short distance the contact interaction is resolved into the exchange of a W boson.

Fermi theory was based on an interaction of four fermionic fields at a spacetime point as depicted in figure 3a. The Fermi coupling constant G_F has units of length-squared, or inverse energy-squared. In a process with a characteristic energy E the effective dimensionless coupling is then $G_F E^2$. It follows that at sufficiently high energy the coupling becomes arbitrarily strong, and this also implies divergences in the perturbation theory. The second order weak amplitude of figure 3b is dimensionally of the form

$$G_F^2 \int^\infty E' dE', \quad (3)$$

where E' is the energy of the virtual state in the second order process, and this diverges at high energy. In position space the divergence comes when the two weak interactions occur at the same spacetime point (high energy = short distance). The divergences become worse at each higher order of perturbation theory and so cannot be controlled even with renormalization.

Such a divergence suggests that the theory one is working with is only valid

up to some energy scale, beyond which new physics appears. The new physics should have the effect of smearing out the interaction in spacetime and so softening the high energy behavior. One might imagine that this could be done in many ways, but in fact it is quite difficult to do without spoiling Lorentz invariance or causality; this is because Lorentz invariance requires that if the interaction is spread out in space it is also spread out in time. The solution to the short-distance problem of the weak interaction is not quite unique, but combined with two of the broad features of the weak interaction — its $V - A$ structure and its universal coupling to different quarks and leptons — a unique solution emerges. This is depicted in figure 3c, where the four-fermi interaction is resolved into the exchange of a vector boson. Moreover, this vector boson must be of a very specific kind, coming from a spontaneously broken gauge invariance. And indeed, this is the way that nature works.[†]

For gravity the discussion is much the same. The gravitational interaction is depicted in figure 4a. As we have already noted in discussing figure 1, the gravitational coupling G_N has units of length-squared and so the dimensionless coupling is $G_N E^2$. This grows large at high energy and gives again a nonrenormalizable perturbation theory.[‡] Again the natural suspicion is that new short-distance physics smears out the interaction, and again there is only one known way to do this. It involves a bigger step than in the case of the weak interaction: it requires that at the Planck length the graviton and other particles turn out to be not points but one-dimensional objects, loops of “string,” figure 5a. Their spacetime histories are then two-dimensional surfaces as shown in figure 4b.

At first sight this is an odd idea. It is not obvious why it should work and not other possibilities. It may simply be that we have not been imaginative enough, but because UV problems are so hard to solve we should consider carefully this one solution that we have found. And in this case the idea becomes increasingly attractive as we consider it.

[†]It could also have been that the divergences are an artifact of perturbation theory but do not appear in the exact amplitudes. This is a logical possibility, a “nontrivial UV fixed point.” Although possible, it seems unlikely, and it is not what happens in the case of the weak interaction.

[‡]Note that the bad gravitational interaction of figure 4a is the same graph as the smeared-out weak interaction of figure 3c. However, its high energy behavior is worse because gravity couples to energy rather than charge.

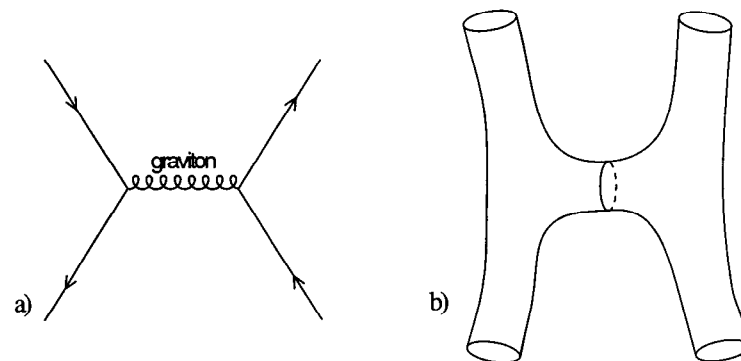


Fig. 4. a) Exchange of a graviton between two elementary particles. b) The same interaction in string theory. The amplitude is given by the sum over histories, over all embeddings of the string world-sheet in spacetime. The world-sheet is smooth: there is no distinguished point at which the interaction occurs (the cross section on the intermediate line is only for illustration).

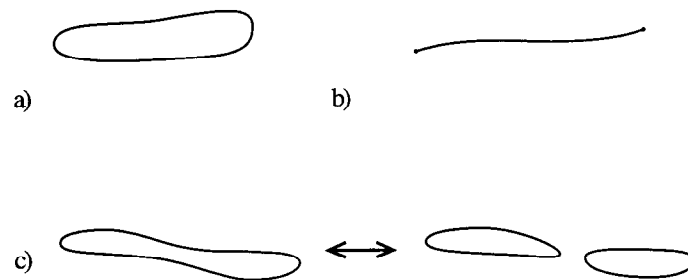


Fig. 5. a) A closed loop of string. b) An open string, which appears in some theories. c) The basic splitting-joining interaction.

3.2 All Roads Lead to String Theory

The basic idea is that the string has different states with the properties of different particles. Its internal vibrations are quantized, and depending on which oscillators are excited it can look like a scalar, a gauge boson, a graviton, or a fermion. Thus the full Standard Model plus gravity can be obtained from this one building block. The basic string interaction is as in figure 5c, one string splitting in two or the reverse. This one interaction, depending on the states of the strings involved, can look like any of the interactions in nature: gauge, gravitational, Yukawa.

A promising fact is that string theory is unique: we have known for some time that there are only a small number of string theories, and now have learned that these are actually all the same. (For now, this does not lead to predictive power because the theory has many vacuum states, with different physics.)

Further, string theory dovetails very nicely with previous ideas for extending the Standard Model. First, string theory automatically incorporates supersymmetry: it turns out that in order for the theory to be consistent the strings must move in a “superspace” which has “fermionic” dimensions in addition to the ordinary ones. Second, the spacetime symmetry of string theory is $SO(9, 1)$, meaning that the strings move in ten dimensions. As I have already explained, this is a likely way to explain some of the features of nature, and it is incorporated in string theory. Third, string theory can incorporate ordinary grand unification: some of the simplest string vacua have the same gauge groups and matter that one finds in unifying the Standard Model.

From another point of view, if one searches for higher symmetries or new mathematical structures that might be useful in physics, one again finds many connections to string theory. It is worthwhile to note that these three kinds of motivation — solving the divergence problem, explaining the broad patterns in the Standard Model, and the connection with mathematics, were also present in the weak interaction. Weinberg emphasized the divergence problem as I have done. Salam was more guided by the idea that non-Abelian gauge theory was a beautiful mathematical structure that should be incorporated in physics. Experiment gave no direct indication that the weak interaction was anything but the pointlike interaction of figure 3a, and no direct clue as to the new physics that smears it out, just as today it gives no direct indication of what lies beyond the Standard Model. But it did show certain broad patterns — universality and the $V - A$

structure — that were telltale signs that the weak interaction is due to exchange of a gauge boson. It appears that nature is kind to us, in providing many trails to a correct theory.

3.3 Vacuum Selection and Dynamics

So how do we go from explaining broad patterns to making precise predictions? The main problem is that string theory has many approximately stable vacua, corresponding to different shapes and sizes for the rolled-up dimensions. The physics that we see depends on which of these vacua we are in. Thus we need to understand the dynamics of the theory in great detail, so as to determine which vacua are truly stable, and how cosmology selects one among the stable vacua.

Until recently our understanding of string theory was based entirely on perturbation theory, the analog of the Feynman graph expansion, describing small numbers of strings interacting weakly. However, we know from quantum field theory that there are many important dynamical effects that arise when we have large numbers of degrees of freedom and/or strong couplings. Some of these effects, such as confinement, the Higgs mechanism, and dynamical symmetry breaking, play an essential role in the Standard Model. If one did not know about them, one could not understand how the Standard Model Hamiltonian actually gives rise to the physics that we see.

String theory is seemingly much more complicated than field theory, and so undoubtedly has new dynamical effects of its own. I am sure that all the experimentalists would like to know, “How do I falsify string theory? How do I make it go away and not come back?” Well, you can’t. Not yet. To understand why, remember that in the ’50s Wolfgang Pauli thought that he had falsified Yang–Mills theory, because it seems to predict long range forces not seen in nature. The field equations for the weak and strong forces are closely parallel to those for electromagnetism, and so apparently of infinite range. It is the dynamical effects, symmetry breaking and confinement, that make these short range forces. Just as one couldn’t falsify Yang–Mills theory in the ’50s, one cannot falsify string theory today. In particular, because we cannot reach the analog of the parton regime where the stringy physics is directly visible, the physics that we see is filtered through a great deal of complicated dynamics.

There is a deeper problem as well. The Feynman graph expansion does not

converge, in field theory or string theory. Thus it does not define the theory at finite nonzero coupling. One needs more, the analog of the path integral and renormalization group of field theory.

Happily, since 1994 we have many new methods for understanding both field theories and string theory at strong coupling. These have led to steady progress on the questions that we need to answer, and to many new results and many surprises. This progress is the subject of the rest of my lectures.

4 Duality in Field and String Theory

4.1 Dualities

One important idea in the recent developments is *duality*. This refers to the equivalence between seemingly distinct physical systems. One starts with different Hamiltonians, and even with different fields, but after solving the theory one finds that the spectra and the transition amplitudes are identical. Often this occurs because a quantum system has more than one classical limit, so that one gets back to the same quantum theory by “quantizing” either classical theory.

This phenomenon is common in quantum field theories in two spacetime dimensions. The duality of the Sine-Gordon and Thirring models is one example; the high-temperature–low-temperature duality of the Ising model is another. The great surprise of the recent developments is that it is also common in quantum field theories in four dimensions, and in string theory.

A particularly important phenomenon is *weak–strong duality*. I have emphasized that perturbation theory does not converge. It gives the asymptotics as the coupling g goes to zero, but it misses important physics at finite coupling, and at large coupling it becomes more and more useless. In some cases, though, when g becomes very large there is a simple alternate description, a weakly coupled dual theory with $g' = 1/g$. In one sense, as $g \rightarrow \infty$ the quantum fluctuations of the original fields become very large (non-Gaussian), but one can find a dual set of fields which become more and more classical.

Another important idea is *electric–magnetic duality*. A striking feature of Maxwell’s equations is the symmetry of the left-hand side under $\mathbf{E} \rightarrow \mathbf{B}$ and $\mathbf{B} \rightarrow -\mathbf{E}$. This symmetry suggests that there should be magnetic as well as electric charges. This idea became more interesting with Dirac’s discovery of the

quantization condition

$$q_e q_m = 2\pi n \hbar, \quad (4)$$

which relates the quantization of the electric charge (its equal magnitude for protons and electrons) to the existence of magnetic monopoles. A further key step was the discovery by ’t Hooft and Polyakov that grand unified theories predict magnetic monopoles. These monopoles are solitons, smooth classical field configurations. Thus they look rather different from the electric charges, which are the basic quanta: the latter are light, pointlike, and weakly coupled while monopoles are heavy, “fuzzy,” and (as a consequence of the Dirac quantization) strongly coupled.

In 1977 Montonen and Olive proposed that in certain supersymmetric unified theories the situation at strong coupling would be reversed: the electric objects would be big, heavy, and strongly coupled and the magnetic objects small, light, and weakly coupled. The symmetry of the sourceless Maxwell’s equations would then be extended to the interacting theory, with an inversion of the coupling constant. Thus electric–magnetic duality would be a special case of weak–strong duality, with the magnetically charged fields being the dual variables for the strongly coupled theory.

The evidence for this conjecture was circumstantial: no one could actually find the dual magnetic variables. For this reason the reaction to this conjecture was skeptical for many years. In fact the evidence remains circumstantial, but in recent years it has become so much stronger that the existence of this duality is in little doubt.

4.2 Supersymmetry and Strong Coupling

The key that makes it possible to discuss the strongly coupled theory is *supersymmetry*. One way to think about supersymmetry is in terms of extra dimensions — but unlike the dimensions that we see, and unlike the small dimensions discussed earlier, these dimensions are “fermionic.” In other words, the coordinates for ordinary dimensions are real numbers and so commute with each other: they are “bosonic;” the fermionic coordinates instead satisfy

$$\theta_i \theta_j = -\theta_j \theta_i. \quad (5)$$

For $i = j$ this implies that $\theta_i^2 = 0$, so in some sense these dimensions have zero size. This may sound rather mysterious but in practice the effect is the same as having just the bosonic dimensions but with an extra symmetry that relates the masses and couplings of fermions to those of bosons.

To understand how supersymmetry gives new information about strong coupling, let us recall the distinction between *symmetry* and *dynamics*. Symmetry tells us that some quantities (masses or amplitudes) vanish, and others are equal to one another. To actually determine the values of the masses or amplitudes is a dynamical question. In fact, supersymmetry gives some information that one would normally consider dynamical. To see this, let us consider in quantum theory the Hamiltonian operator H , the charge operator G associated with an ordinary symmetry like electric charge or baryon number, and the operator Q associated with a supersymmetry. The statement that G is a symmetry means that it commutes with the Hamiltonian,

$$[H, G] = 0 . \quad (6)$$

For supersymmetry one has the same,

$$[H, Q] = 0 , \quad (7)$$

but there is an additional relation

$$Q^2 = H + G , \quad (8)$$

in which the Hamiltonian and ordinary symmetries appear on the right. There are usually several G s and several Q s, so that there should be additional indices and constants in these equations, but this schematic form is enough to explain the point. It is this second equation that gives the extra information. To see one example of this, consider a state $|\psi\rangle$ having the special property that it is neutral under supersymmetry:

$$Q|\psi\rangle = 0 . \quad (9)$$

To be precise, since we have said that there are usually several Q s, we are interested in states that are neutral under *at least one* Q but usually not all of them. These are known as *BPS (Bogomolnyi-Prasad-Sommerfield) states*. Now take the expectation value of the second relation (8) in this state:

$$\langle\psi|Q^2|\psi\rangle = \langle\psi|H|\psi\rangle + \langle\psi|G|\psi\rangle . \quad (10)$$

The left side vanishes by the BPS property, while the two terms on the right are the energy E of the state $|\psi\rangle$ and its charge q under the operator G . Thus

$$E = -q , \quad (11)$$

and so the energy of the state is determined in terms of its charge. But the energy is a dynamical quantity: even in quantum mechanics we must solve Schrödinger's equation to obtain it. Here, it is determined entirely by symmetry information. (There is a constant of proportionality missing in (11), because we omitted it from (8) for simplicity, but it is determined by the symmetry.)

Since the calculation of E uses only symmetry information, it does not depend on any coupling being weak: it an exact property of the theory. Thus we know something about the spectrum at strong coupling. Actually, this argument only gives the allowed values of E , not the ones that actually appear in the spectrum. The latter requires an extra step: we first calculate the spectrum of BPS states at weak coupling, and then adiabatically continue the spectrum: the BPS property enables us to follow the spectrum to strong coupling.

The BPS states are only a small part of the spectrum, but by using this and similar types of information from supersymmetry, together with general properties of quantum systems, one can usually recognize a distinctive pattern in the strongly coupled theory and so deduce the dual theory. Actually, this argument was already made by Montonen and Olive in 1977, but only in 1994, after this kind of reasoning was applied in a systematic way in many examples starting with Seiberg, did it become clear that it works and that electric-magnetic duality is a real property of supersymmetric gauge theories.

4.3 String Duality, D-Branes, M-Theory

Thus far the discussion of duality has focused on quantum field theory, but the same ideas apply to string theory. Prior to 1994 there were various conjectures about duality in string theory, but after the developments described above, Hull, Townsend, and Witten considered the issue in a systematic way. They found that for each strongly coupled string theory (with enough supersymmetry) there was a unique candidate for a weakly coupled dual. These conjectures fit together in an intricate and consistent way as dimensions are compactified, and evidence for them rapidly mounted. Thus weak-strong duality seems to be a general property in string theory.

Weak–strong duality in field theory interchanged the pointlike quanta of the original fields with smooth solitons constructed from those fields. In string theory, the duality mixes up various kinds of objects: the basic quanta (which are now strings), smooth solitons, black holes (which are like solitons, but with horizons and singularities), and new stringy objects known as *D-branes*.

The D-branes play a major role, so I will describe them in more detail. In string theory strings usually move freely. However, some string theories also predict localized objects, sort of like defects in a crystal, where strings can break open and their endpoints get stuck. These are known as D-branes, short for Dirichlet (a kind of boundary condition—see Jackson, *Classical Electrodynamics*) membranes. Depicted in figure 6, they can be points (D0-branes), curves (D1-branes), sheets (D2-branes), or higher-dimensional objects. They are dynamical objects—they can move, and bend—and their properties, at weak coupling, can be determined with the same machinery used elsewhere in string theory.

Even before string duality it was found that one could make D-branes starting with just ordinary strings (for string theorists, I am talking about *T*-duality). Now we know that they are needed to fill out the duality multiplets. They have many interesting properties. One is that they are smaller than strings; one cannot really see this pictorially, because it includes the quantum fluctuations, but it follows from calculations of the relevant form factors. Since we are used to thinking that smaller means more fundamental, this is intriguing, and we will return to it.

Returning to string duality, figure 7 gives a schematic picture of what was learned in 1995. Before that time there were five known string theories. These differed primarily in the way that supersymmetry acts on the string, and also the type I theory in that it includes open strings. We now know that starting with any one of these theories and going to strong coupling, we can reach any of the others. Again, the idea is that one follows the BPS states and recognizes distinctive patterns in the limits. The parameter space in the figure can be thought of as two coupling constants, or as the radii of two compact dimensions.

In figure 7 there is a sixth limit, labeled *M*-theory. We have emphasized that the underlying spacetime symmetry of string theory is $SO(9, 1)$. However, the M-theory point in the figure is in fact a point of $SO(10, 1)$ symmetry: the spacetime symmetry of string theory is larger than had been suspected. The extra piece is badly spontaneously broken, at weak coupling, and not visible in the perturbation theory, but it is a property of the exact theory. It is interesting that $SO(10, 1)$ is

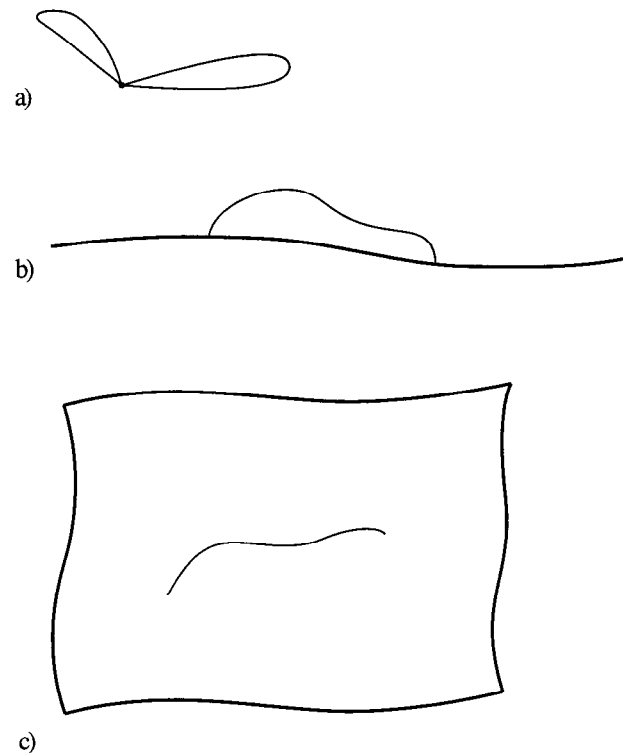


Fig. 6. (a) A D0-brane with two attached strings. (b) A D1-brane (**bold**) with attached string. (c) A D2-brane with attached string.

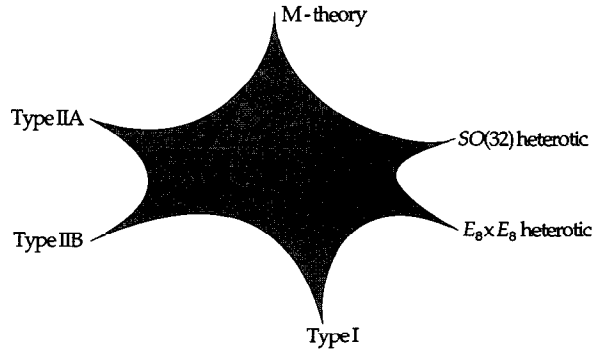


Fig. 7. The five string theories, and M-theory, as limits of a single theory.

known to be the largest spacetime symmetry compatible with supersymmetry.

Another way to describe this is that in the M-theory limit the theory lives in eleven spacetime dimensions: a new dimension has appeared. This is one of the surprising discoveries of the past few years. How does one discover a new dimension? It is worthwhile explaining this in some more detail. The D0-brane mass is related to the characteristic string mass scale m_s and the dimensionless string coupling g_s , by

$$m_{D0} = \frac{m_s}{g_s} . \quad (12)$$

When g_s is small this is heavier than the string scale, but when g_s is large it is lighter. Further, the D0-brane is a BPS state and so this result is exact. If one considers now a state with N D0-branes, the mass is bounded below by Nm_{D0} , and in fact this bound is saturated: there is a BPS bound state with

$$m_{ND0} = N \frac{m_s}{g_s} \quad (13)$$

exactly. Now observe that for g_s large, all of these masses become small. What can the physics be? In fact, this is the spectrum associated with passing a threshold where a new spacetime dimension becomes visible. The radius of this dimension is

$$R = \frac{g_s}{m_s} . \quad (14)$$

That is, small g_s is small R and large g_s is large R . In particular, perturbation theory in g_s is an expansion around $R = 0$: this is why this dimension has always been invisible!

5 An Alternative to String Theory?

On Lance Dixon's tentative outline for my lectures, one of the items was "Alternatives to String Theory." My first reaction was that this was silly, there are no alternatives, but on reflection I realized that there was an interesting alternative to discuss. So let us try to construct a quantum theory of gravity based on a new principle, not string theory. We will fail, of course, but we will fail in an interesting way.

Let us start as follows. In quantum mechanics we have the usual position-momentum uncertainty relation

$$\delta x \delta p \geq \hbar^2 . \quad (15)$$

Quantum gravity seems to imply a breakdown in spacetime at the Planck length, so perhaps there is also a position-position uncertainty relation

$$\delta x \delta x \geq L_P^2 . \quad (16)$$

This has been discussed many times, and there are many ways that one might try to implement it. We will do this as follows. Suppose that we have N nonrelativistic particles. In normal quantum mechanics the state would be defined by N position vectors

$$\mathbf{X}_i , \quad i = 1, \dots, N . \quad (17)$$

Let us instead make these into Hermitian matrices in the particle-number index

$$\mathbf{X}_{ij} , \quad i, j = 1, \dots, N . \quad (18)$$

It is not obvious what this means, but we will see that it leads to an interesting result. For the Hamiltonian we take

$$H = \frac{1}{2M} \sum_{m=1}^{D-1} \sum_{i,j=1}^N (p_{ij}^m)^2 + M^{15} \sum_{m,n=1}^{D-1} \sum_{i,j=1}^N |[X^m, X^n]_{ij}|^2 . \quad (19)$$

The first term is just an ordinary nonrelativistic kinetic term, except that we now have N^2 coordinate vectors rather than N so there is a momentum for each, and

we sum the squares of all of them. The indices m and n run over the $D - 1$ spatial directions, and M and M' are large masses, of order of the Planck scale. The potential term is chosen as follows. We want to recover ordinary quantum mechanics at low energy. The potential is the sum of the squares of all of the components of all of the commutators of the matrices \mathbf{X}_{ij} , with a large coefficient. It is therefore large unless all of these matrices commute. In states with energies below the Planck scale, the matrices will then commute to good approximation, so we do not see the new uncertainty (16) and we recover the usual quantum mechanics. In particular, we can find a basis which diagonalizes all the commuting X_{ij}^m . Thus the effective coordinates are just the N diagonal elements X_{ii}^m of each matrix in this basis, which is the right count for N particles in ordinary quantum mechanics: the X_{ii}^m behave like ordinary coordinates.

The Hamiltonian (19) has interesting connections with other parts of physics. First, the commutator-squared term has the exact same structure as the four-gluon interaction in Yang–Mills theory. This is no accident, as we will see later on. Second, there is a close connection to supersymmetry. In supersymmetric quantum mechanics, one has operators satisfying the algebra (7,8). Again in general there are several supersymmetry charges, and the number \mathcal{N} of these Q s is significant. For small values of \mathcal{N} , like 1, 2, or 4, there are many Hamiltonians with the symmetry. As \mathcal{N} increases the symmetry becomes more constraining, and $\mathcal{N} = 16$ is the maximum number. For $\mathcal{N} = 16$ there is only one invariant Hamiltonian, and it is none other than our model (19). To be precise, supersymmetry requires that the particles have spin, that the Hamiltonian also has a spin-dependent piece, and that the spacetime dimension D be 10. In fact, supersymmetry is necessary for this idea to work. The vanishing of the potential for commuting configurations was needed, but we only considered the classical potential, not the quantum corrections. The latter vanish only if the theory is supersymmetric.

So this model has interesting connections, but let us return to the idea that we want a theory of gravity. The interactions among low energy particles come about as follows. We have argued that the potential forces the \mathbf{X}_{ij} to be diagonal: the off-diagonal pieces are very massive. Still, virtual off-diagonal excitations induce interactions among the low-energy states. In fact, the leading effect, from one loop of the massive states, produces precisely the (super)gravity interaction among the low energy particles.

So this simple idea seems to be working quite well, but we said that we were going to fail in our attempt to find an alternative to string theory. In fact we have failed because this is not an alternative: it *is* string theory. It is actually one piece of string theory, namely the Hamiltonian describing the low energy dynamics of N D0-branes. This illustrates the following principle: that all good ideas are part of string theory. That sounds arrogant, but with all the recent progress in string theory, and a fuller understanding of the dualities and dynamical possibilities, string theory has extended its reach into more areas of mathematics and has absorbed previous ideas for unification (including $D = 11$ supergravity).

We have discussed this model not just to introduce this principle, but because the model is important for a number of other reasons. In fact, it is conjectured that it is not just a piece of string theory, but is actually a *complete* description. The idea is that if we view any state in string theory from a very highly boosted frame, it will be described by the Hamiltonian (19) with N large. Particle physicists are familiar with the idea that systems look different as one boosts them: the parton distributions evolve. The idea here is that the D0-branes are the partons for string theory; in effect the string is a necklace of partons. This is the *matrix theory* idea of Banks, Fischler, Shenker, and Susskind (based on earlier ideas of Thorn), and at this point it seems very likely to be correct or at least a step in the correct direction.

To put this in context, let us return to the illustration in figure 7 of the space of string vacua, and to the point made earlier that the perturbation theory does not define the theory for finite g . In fact, every indication is that the string description is useful only near the five cusps of the figure in which the string coupling becomes weak. In the center of the parameter space, not only do we not know the Hamiltonian but we do not know what degrees of freedom are supposed to appear in it. It is likely that they are not the one-dimensional objects that one usually thinks of in string theory; it is more likely that they are the coordinate matrices of the D-branes.

6 Black Hole Quantum Mechanics

6.1 Black Hole Thermodynamics

In the '70s it was found that there is a close analogy between the laws of black hole mechanics and the laws of thermodynamics. In particular, the event horizon area (in Planck units) is like the entropy. It is nondecreasing in classical gravitational processes, and the sum of this *Bekenstein–Hawking entropy* and the entropy of radiation is nondecreasing when Hawking radiation is included. For more than 20 years it has been a goal to find the statistical mechanical picture from which this thermodynamics derives — that is, to count the quantum states of a black hole. There have been suggestive ideas over the years, but no systematic framework for addressing the question.

I have described the new ideas we have for understanding strongly interacting strings. A black hole certainly has strong gravitational interactions, so we might hope that the new tools would be useful here. Pursuing this line of thought, Strominger and Vafa were able in early 1996 to count the quantum states of a black hole for the first time. They did this with the following thought experiment. Start with a black hole and imagine adiabatically reducing the gravitational coupling G_N . At some point the gravitational binding becomes weak enough that the black hole can no longer stay black, but must turn into ordinary matter. A complete theory of quantum gravity must predict what the final state will look like. The answer depends on what kind of black hole we begin with — in other words, what are its electric, magnetic, and other charges (the no-hair theorem says that this is all that identifies a black hole). For the state counting we want to take a supersymmetric black hole, one that corresponds to a BPS state in the quantum theory. For the simplest such black holes, the charges that they carry determine that at weak coupling they will turn into a gas of weakly coupled D-branes, as depicted in figure 8. For these we know the Hamiltonian, so we can count the states and continue back to strong coupling where the system is a black hole. Indeed, the answer is found to agree precisely with the Bekenstein–Hawking entropy. Our initial motivation was one problem of quantum gravity, the UV divergences. Now, many years later, string theory has solved another, very different, long-standing problem in the subject.

This result led to much further study. It was found that in addition to the

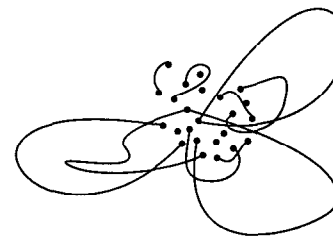


Fig. 8. A state of many D0-branes, with attached strings.

agreement of the entropy of BPS states with the Bekenstein–Hawking entropy, agreement was also found between the general relativity calculation and the D-brane calculation for the entropies of near-BPS states, and for various dynamical quantities such as absorption and decay amplitudes. This goes beyond the adiabatic continuation argument used to justify the entropy calculation, and in late 1997 these results were understood as consequences of a new duality, the *Maldacena duality*. This states that, not only are the weakly coupled D-branes the adiabatic continuation of the black hole, but that *the D-brane system at all couplings is dual (physically equivalent) to the black hole*. In effect, D-branes are the atoms from which certain black holes are made, but for large black holes they are in a highly quantum state while the dual gravitational field is in a very classical state. A precise statement of the Maldacena duality requires a low energy limit in the D-brane system, while on the gravitational side one takes the limit of the geometry near the horizon.

6.2 The Information Paradox

This new duality has two important consequences. The first is for another of the nagging problems of quantum gravity, the black hole information paradox. A black hole emits thermal Hawking radiation, and will eventually decay completely. The final state is independent of what went into the black hole, and incoherent. In other words, an initially pure state evolves into a mixed state; this is inconsistent with the usual rules of quantum mechanics. Hawking argued that in quantum gravity the evolution of states must be generalized in this way.

This has been a source of great controversy. While most physicists would be pleased to see quantum mechanics replaced by something less weird, the particular modification proposed by Hawking simply makes it uglier, and quite possibly inconsistent. But 20 years of people trying to find Hawking’s “mistake,” to identify the mechanism that preserves the purity of the quantum state, has only served to sharpen the paradox: because the quantum correlations are lost behind the horizon, either quantum mechanics is modified in Hawking’s way, or the locality of physics must break down in a way that is subtle enough not to infect most of physics, yet act over long distances.

The duality conjecture above states that the black hole is equivalent to an ordinary quantum system, so that the laws of quantum evolution are unmodified. However, to resolve fully the paradox one must identify the associated nonlocality in the spacetime physics. This is hard to do because the local properties of spacetime are difficult to extract from the highly quantum D-brane system: this is related to the *holographic principle*. This term refers to the property of a hologram, that the full picture is contained in any one piece. It also has the further connotation that the quantum state of any system can be encoded in variables living on the *boundary* of that system, an idea that is suggested by the entropy–area connection of the black hole. This is a key point where our ideas are in still in flux.

6.3 Black Holes and Gauge Theory

Dualities between two systems give information in each direction: for each system there are some things that can be calculated much more easily in the dual description. In the previous subsection we used the Maldacena duality to make statements about black holes. We can also use it in the other direction, to calculate properties of the D-brane theory.

To take full advantage of this we must first make a generalization. We have said that D-branes can be points, strings, sheets, and so on: they can be extended in p directions, where here $p = 0, 1, 2$. Thus we refer to Dp -branes. The same is true of black holes: the usual ones are local objects, but we can also have black strings — strings with event horizons — and so on. A black p -brane is extended in p directions and has a black hole geometry in the orthogonal directions. The full Maldacena duality is between the low energy physics of Dp -branes and strings

in the near-horizon geometry of a black p -brane. Further, for $p \leq 3$ the low energy physics of N Dp -branes is described by $U(N)$ Yang–Mills theory with $\mathcal{N} = 16$ supersymmetries. That is, the gauge fields live on the D-branes, so that they constitute a field theory in $p + 1$ “spacetime” dimensions, where here spacetime is just the world-volume of the brane. For $p = 0$, this is the connection of matrix quantum mechanics to Yang–Mills theory that we have already mentioned below (19).

The Maldacena duality then implies that various quantities in the gauge theory can be calculated more easily in the dual black p -brane geometry. This method is only useful for large N , because this is necessary to get a black hole which is larger than string scale and so described by ordinary general relativity. Of course we have a particular interest in gauge theories in $3 + 1$ dimensions, so let us focus on $p = 3$. The Maldacena duality for $p = 3$ partly solves an old problem in the strong interaction. In the mid-’70s ‘t Hooft observed that Yang Mills theory simplifies when the number of colors is large. This simplification was not enough to allow analytic calculation, but its form led ‘t Hooft to conjecture a duality between large- N gauge theory and some unknown string theory. The Maldacena duality is a precise realization of this idea, for supersymmetric gauge theories.[§] For the strong interaction we need of course to understand nonsupersymmetric gauge theories. One can obtain a rough picture of these from the Maldacena duality, but a precise description seems still far off. It is notable, however, that string theory, which began as an attempt to describe the strong interaction, has now returned to its roots, but only by means of an excursion through black hole physics and other strange paths.

6.4 Spacetime Topology Change

This subsection is not directly related to black holes, but deals with another exotic question in quantum gravity. Gravity is due to the bending of spacetime. It is an old question, whether spacetime could not only bend but break: does its topology as well as its geometry evolve in time?

Again, string theory provides the tools to answer this question. The answer

[§]For $p = 3$ the near-horizon geometry is the product of an anti-de Sitter space and a sphere, while the supersymmetric gauge theory is conformally invariant (a conformal field theory), so this is also known as the *AdS–CFT correspondence*.

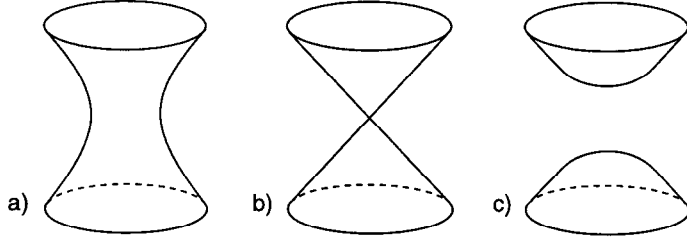


Fig. 9. An example of spacetime topology change.

is “yes” — under certain controlled circumstances the geometry can evolve as shown schematically in figure 9. It is interesting to focus on the case that the topology change is taking place in the compactified dimensions, and to contrast the situation as seen by the short-distance and long-distance observers of figures 2a and 2b. The short distance observer sees the actual process of figure 9. The long distance observer cannot see this. Rather, this observer sees a *phase transition*. At the point where the topology changes, some additional particles become massless and the symmetry breaking pattern changes. Thus the transition can be analyzed with the ordinary methods of field theory; it is this that makes the quantitative analysis of the topology change possible.

Incidentally, topology change has often been discussed in the context of spacetime foam, the idea that the topology of spacetime is constantly fluctuating at Planckian distance scales. It is likely that the truth is even more strange, as in matrix theory where spacetime becomes “non-Abelian.”

7 Unification and Large Dimensions

My talks have been unapologetically theoretical. The Planck length is far removed from experiment, yet we believe we have a great deal of understanding of the very exotic physics that lies there. In this final section I would like to discuss some ways in which the discoveries of the last few years might affect the physics that we see.

Let me return to the unification of the couplings in figure 10a, and to the failure of the gravitational coupling to meet the other three exactly. There are

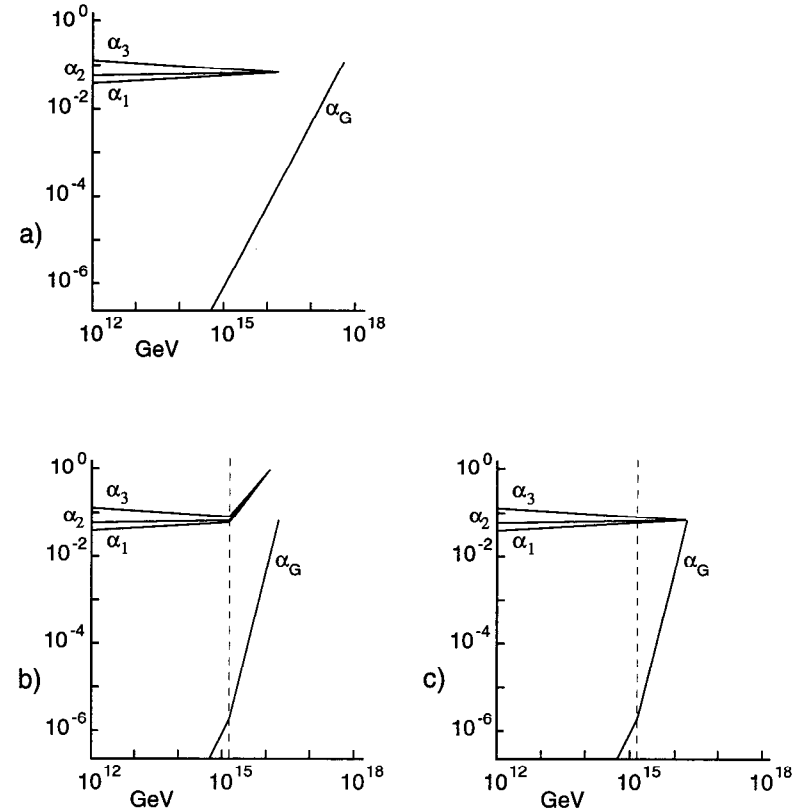


Fig. 10. a) Running of the three gauge couplings and the dimensionless gravitational coupling with energy. b) Effect of a fifth dimension below the unification scale. c) Effect of a fifth dimension of the Horava-Witten type.

many ideas to explain this. There could be additional particles at the weak, intermediate, or unified scales, which change the running of the gauge couplings so as to raise the unification point. Or it may be that the gauge couplings actually do unify first, so that there is a normal grand unified theory over a small range of scales before the gravitational coupling unifies. These ideas focus on changing the behavior of the gauge couplings. Since these already unify to good approximation, it would be simpler to change the behavior of the gravitational coupling so that it meets the other three at a lower scale — to lower the Planck scale. Unfortunately this is not so easy. The energy-dependence of the gravitational coupling is just dimensional analysis, which is not so easy to change.[¶]

There is a way to change the dimensional analysis — that is, to change the dimension! We have discussed the possibility that at some scale we pass the threshold to a new dimension. Suppose that this occurred below the unification scale. For both the gauge and the gravitational couplings the units change, so that both turn upward as in figure 10b. This does not help; the couplings meet no sooner.

There is a more interesting possibility, which was first noticed in the strong coupling limit of the $E_8 \times E_8$ heterotic string. Of the five string theories, this is the one whose weakly coupled physics looks most promising for unification. Its strong-coupling behavior, shown in figure 11, is interesting. A new dimension appears, but it is not simply a circle. Rather, it is bounded by two walls. Moreover, all the gauge fields and the particles that carry gauge charges move only in the walls, while gravity moves in the bulk. Consider now the unification of the couplings. The dynamics of the gauge couplings, and their running, remains as in 3 + 1 dimensions; however, the gravitational coupling has a kink at the threshold, so the net effect can be as in figure 10c. If the threshold is at the correct scale, the four couplings meet at a point.

As it stands this has no more predictive power than any of the other proposed solutions. There is one more unknown parameter, the new threshold scale, and one more prediction. However, it does illustrate that the new understanding of string theory will lead to some very new ideas about the nature of unification. Figure 11 is only one example of a much more general idea now under study, that

[¶]It was asked whether the gravitational coupling has additional β -function type running. Although this could occur in principle, it does not do so because of a combination of dimensional analysis and symmetry arguments.

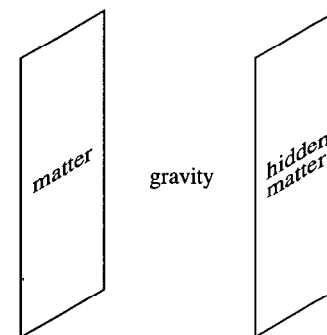


Fig. 11. A Horava–Witten spacetime. The two planes represent 3 + 1 dimensional walls, in which all the Standard Model particles live, while gravity moves in the 4 + 1 dimensional bulk between the walls. In string theory there are six additional dimensions, which could be much smaller and are not shown. The wall is then 9 + 1 dimensional in all, and the spacetime 10 + 1 dimensional.

the Standard Model lives in a brane and does not move in the full space of the compact dimensions, while gravity does do so.

This new idea leads in turn to the possibility of radically changing the scales of new physics in string theory. To see this, imagine lowering the threshold energy (the kink) in figure 10c; this also lowers the string scale, which is where the gravitational coupling meets the other three. From a completely model-independent point of view, how low can we go? The string scale must be at least a TeV, or else we would already have seen string physics. The five-dimensional threshold must correspond to a radius of no more than a millimeter, or else Cavendish experiments would already have shown the four-dimensional inverse square law turning into a five-dimensional inverse cube. Remarkably, it is difficult to improve on these extreme model-independent bounds. The large dimension in particular might seem to imply a whole tower of new states at energies above 10^{-4} eV, but these are very weakly coupled (gravitational strength) and so would not be seen. It may be that construction of a full model, with a sensible cosmology, will raise these scales, but that they will still lie lower than we used to imagine.

I had been somewhat skeptical about this idea, for a reason that is evident in

figure 10c. If the threshold is lowered further, the gravitational coupling meets the other three before they unify and one loses the successful prediction of $\sin^2 \theta_w$. However, it is wrong to pin so much on this one number; the correct prediction might come out in the end in a more complicated way. One should certainly explore the many new possibilities that arise, to see what other consequences there are and to broaden our perspective on the possible nature of unification.

8 Outlook

I will start with the more theoretical problems.

1. The black hole information problem. It seems that the necessary ingredients to solve this are at hand, and that we will soon assemble them correctly. However, it has seemed this way before, and the clock on this problem is at 22 years and counting. Still, our understanding is clearly deeper than it has ever been.

2. The cosmological constant problem. In any quantum theory the vacuum is a busy place, and should gravitate. Why is the cosmological constant, even if nonzero, so much smaller than particle or Planck energies? This is another hard problem, not just in string theory but in any theory of gravity. It has resisted solution for a long time, and seems to require radical new ideas.

The new ideas that I have described have not led to a solution, but they have suggested new possibilities. One important ingredient may be supersymmetry. Throughout the discussion of duality this plays a central role in canceling quantum fluctuations, suggesting that it also does so in the vacuum energy. The problem is that supersymmetry is broken in nature; we need a phase with some properties of the broken theory and some of the unbroken. We have learned about many new phases of string theory, but not yet one with just the right properties.

Another ingredient may be nonlocality. The cosmological constant affects physics on cosmic scales but is determined by dynamics at short distance: this suggests the need for some nonlocal feedback mechanism. Recall that the black hole information problem also seems to need nonlocality; perhaps these are related.

3. Precise predictions from string theory? Our understanding of string dynamics is much improved, but still very insufficient for solving the vacuum selection/stability problem, especially with nonsupersymmetric vacua. It is hard to see how one could begin to address this before solving the cosmological constant

problem, since this tells us that we are missing something important about the vacuum.

An optimistic projection is that we soon solve the information problem, that this gives us the needed idea to solve the cosmological constant problem, and then we can address vacuum selection. When will this happen? Well, if I didn't have to write these d**n notes, More likely, we still are missing some key concepts.

4. What is string theory? We are closer to a nonperturbative formulation than ever before: the things that we have learned in the past few years have completely changed our point of view. It may be that again the ingredients are in place, in that both matrix theory and the Maldacena duality give nonperturbative definitions, and we simply need to extract the essence.

5. Distinct signatures of string theory? Is there any distinctively stringy experimental signature? All of the new physics may lie far beyond accessible energies, but we might be lucky instead. I have discussed the possibility of low energy string theory and large dimensions. I am still inclined to expect the standard picture to hold, but the new ideas are and will remain a serious alternative. Another possibility is a fifth force from the dilaton or other moduli (scalars that are common in string theory). These are massless to first approximation, but quantum effects almost invariably induce masses for all scalars. The resulting mass is likely in the range

$$\frac{m_{\text{weak}}^2}{m_{\text{P}}} < m_{\text{scalar}} < m_{\text{weak}} . \quad (20)$$

The lower limit is interesting for a fifth force, while the whole range is interesting for dark matter.

The most interesting hope is for something unexpected, perhaps cosmological and associated with the holographic principle, or perhaps a distinctive form of supersymmetry breaking.

6. Supersymmetry. Supersymmetry has played a role throughout these talks. In string theory it is a symmetry at least at the Planck scale, but is broken somewhere between the Planck and weak scales. The main arguments for breaking at the weak scale are independent of string theory: the Higgs hierarchy problem, the unification of the couplings, the heavy top quark. In addition, the ubiquitous role that supersymmetry plays in suppressing quantum fluctuations in our discussion of strongly coupled physics supports the idea that it suppresses the quantum corrections to the Higgs mass. The one cautionary note is that the cos-

mological constant suggests a new phase of supersymmetry, whose phenomenology at this point is completely unknown. Still, the discovery and precision study of supersymmetry remains the best bet for testing all of these ideas.

In conclusion, the last few years have seen remarkable progress, and there is a real prospect of answering difficult and long-standing problems in the near future.

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