EXPERIMENTAL ASPECTS of CP VIOLATION*

HARRY N. NELSON
Department of Physics
University of California
Santa Barbara, California 93106-9530

ABSTRACT

We review the experimental aspects of CP violation, from the past, present, and into the future, with particular emphasis on those experiments most likely to elucidate the mechanism of CP violation.

Presented at the SLAC Summer Institute on Particle Physics,
Stanford, California, July 13 - July 24, 1992

* Supported by the U.S. Department of Energy Award No. DE-FG03-91-ER40618L.

© H. N. Nelson 1993

1. Introduction and Overview

It is an honor to give lectures on the experimental aspects of CP violation at this year's SLAC Summer Institute, which includes a symposium that commemorates the discovery of the τ lepton. How are CP violation and the τ lepton related to one and other?

First, if you accept the most conventional explanation of CP violation, namely the Standard Model, the discovery of CP violation was the first, albeit *implicit*, evidence for a third generation of fermions. The quark mixing (Cabibbo-Kobayashi-Maskawa, or CKM) matrix cannot support a CP violating phase if there are only two generations, but has exactly one pigeon-hole for such a phase if there are three generations. Of course, the tau lepton was the first *explicit* evidence of a third generation. It is quite possible that study of the quark sibling of the tau in the third generation, the b-quark, will for once and for all elucidate the mechanism of CP violation.

Second, the discoveries of CP violation in the neutral kaon system and the τ lepton were made largely as a result of the experimenter's initiative. It seems today that the particle experimenter's role is to act out a script written by our theoretical colleagues. To be sure, the observation of the W, the characterization of the Z^0, and the probing of the symmetry breaking sector at the SSC, all involved, or will involve, impressive initiative, ingenuity, and originality by experimenters. But, the spark of initiative has come largely from the theoretical side in those cases. It is not always so, as CP violation and the τ lepton show. As I have struggled through a week of owl shifts, or made my way through the construction and calibration of hundreds of channels of detector, that thought has often kept me going! A quote, attributed to I. I. Rabi, puts it well:

> "The last person a young experimenter should ask, in my belief, about an experimental program is a theoretical physicist."
>
> "Not that theoretical physicists are stupid, but they have their ideas, and they want the answers to their own problems in their own terms. We would not have advanced very far in basic discoveries with a concentration on theory alone..."
>
> "If you follow a theorist, you come in and say, 'Now what shall I do?' And then you do it and say, 'What have I done?' I don't know why people work that way."
>
> "My own view is that you take these things personally. You do an experiment because your own philosophy makes you want to know the result. It's too hard, and life is too short, to spend your life doing something because someone else has said it's important. You must feel the thing yourself—feel that it will change your outlook and your way of life."

However, failure never excuses ignorance, and although I am not a historian, I believe the discoverers of CP violation and the τ had a strong *logical*, if not *theoretical*, motivation for their work. In the case of the τ, perhaps the logic was, "Nature gave us the muon. Did Nature stop there?" Nowadays, when we know that the Z^0 decays into only three light neutrinos, initiation of a search for a fourth sequential lepton would be a bit ignorant. Perhaps for CP violation, the logic was "Nature disrespects parity (P). What about CP?" It is logical to expect P and C violation, as two experimenters, Purcell and Ramsey, noted in 1951, well before parity violation was observed[1]:

> "The argument against electric dipoles, in another form, raises directly the question of parity. A nucleon with an electric dipole moment would show an asymmetry between left- and right- handed coordinate systems; in one system the dipole moment would be parallel to the angular momentum and in the other, antiparallel. But there is no compelling reason for excluding this possibility. **It would not be the only asymmetry of particles of ordinary experience, which already exhibit conspicuous asymmetry in respect to electric charge.**"

Today, we view the predominance of positively charged protons in our galaxy as a logical *consequence* of CP violation, but the clever experimentalists of the past viewed that predominance as a logical *reason* to search for CP violation.

I would like to give two views of the 'logic' of today's experimental situation in CP violation. The first view is that the Standard Model is the source of CP violation. This view may seem like blind homage to the conventional, but really it isn't; the Standard Model has one huge success in describing CP violation: it gets the order of magnitude of $|\eta_{+-}|$ correct:

$$|\eta_{+-}| \equiv \left[\frac{\mathrm{B}(K_L^0 \to \pi^+\pi^-)}{\tau_L} \frac{\tau_S}{\mathrm{B}(K_S^0 \to \pi^+\pi^-)}\right]^{1/2} = (2.27 \pm 0.02) \times 10^{-3} \quad . \quad (1.1)$$

If CP violation is characterized by a simple Fermi-like coupling constant G_{CP} that describes the transition $K_L^0 \to \pi^+\pi^-$, then $G_{CP}/G_F \approx |\eta_{+-}|$, and naively, the gauge boson corresponding to G_{CP} would have mass $\approx M_W/\sqrt{|\eta_{+-}|} \approx$ 1.6 TeV. From Yosef Nir's lectures, the Standard Model (through the second-order box diagrams, not first order Fermi-like coupling) gives:

$$|\eta_{+-}| \approx \frac{s_{23}s_{13}}{s_{12}} \sin\delta \approx 7 \times 10^{-4} \sin\delta \quad (1.2)$$

where s_{23}, s_{13}, s_{12}, and δ are from the usual CKM parameterization (see Nir's lectures). This is the success. If it were to turn out that CP violation were caused by some 'new interaction' described by G_{CP}, one would view it as an incredible coincidence that G_{CP} happened to sit right in the range easily described by the Standard Model.

The next logical step, in this view, is to find the 'smoking gun' that *proves* that the Standard Model is the source of CP violation. Although $K_L^0 \to \pi^+\pi^-$ was first observed twenty-eight years ago, we still lack an incontrovertible second piece of evidence. The Standard Model was once thought to make the prediction[2]:

$$\frac{\mathrm{B}(K_L^0 \to \pi^0\pi^0)}{\mathrm{B}(K_L^0 \to \pi^+\pi^-)} \neq \frac{\mathrm{B}(K_S^0 \to \pi^0\pi^0)}{\mathrm{B}(K_S^0 \to \pi^+\pi^-)}. \quad (1.3)$$

Indeed, the most experimental effort has gone in to trying to probe that inequality. The latest two experiments, NA31 and E731, each with a sensitivity to deviations from equality of the two sides of (1.3) at the 0.4% level, give a mild indication that the inequality is correct, but the significance is low. In the meantime, revision of the Standard Model prediction, mostly to account for the large top quark mass, has allowed equality in (1.3) within the Standard Model. A number of experimental groups are still pushing on to test (1.3) to even higher precision, and also to search for the 'smoking gun' in other kaon decay modes, such as $K_L^0 \to \pi^0 e^+ e^-$, $K_L^0 \to \pi^0 \nu \overline{\nu}$, or $K^+ \to \pi^+ \pi^+ \pi^-$. In every case the hope is to dredge up a small effect via use of huge statistics, or to see a small signal on a large background. If we get proof of the Standard Model from those experiments, that proof will not startle with its clarity.

The Standard Model makes a more startling prediction outside of the neutral kaon system, in the neutral B system.[3] A difference of order 10% is predicted between the probability that a B^0 will end up in a $J/\psi K_S^0$ final state, and the probability that a $\overline{B^0}$ will end up in the same final state. Observation of such a large asymmetry would, in this view, provide the crucial proof that CP violation is part of the Standard Model. If the asymmetry is large, a number of venues, including hadron colliders, symmetric e^+e^- colliders, and asymmetric e^+e^- colliders, all have a fair chance of seeing it first. For subsequent studies of other asymmetries in the B hadron system, studies that allow precision tests of the self-consistency of the pattern of CP violation predicted by the Standard Model, an asymmetric e^+e^- collider is probably the best bet.

The second view is contrarian. Here one dismisses the successful computation of $|\eta_{+-}|$ within the Standard Model as a coincidence; $\sin\delta$ might be quite small, for example. Second, the origin of $K_L^0 \to 2\pi$ is the following process, or 'state mixing':

$$K_2^0 \to K_1^0 \to 2\pi, \tag{1.4}$$

and so a computation of $G_{\rm CP}$ could be modified to account for this second order process, where the CP violation occurs in the transition $K_2^0 \to K_1^0$. Under the assumption that a Fermi-type interaction produces CP violation in the $\Delta S = 2$ transition, $s\overline{d} \to \overline{s}d$, one can estimate

$$\langle K_1^0 | H | K_2^0 \rangle \approx G'_{\rm CP} f_{\rm K}^2 M_{\rm K}$$

and

$$G'_{\rm CP} \approx \frac{\sqrt{2}\Delta M_K}{f_{\rm K}^2 M_{\rm K}} |\eta_{+-}| = 7 \times 10^{-11} G_{\rm F} \quad . \tag{1.5}$$

This is the superweak hypothesis of Wolfenstein.[4] The point is, $K_L^0 \to 2\pi$ renders up remarkable sensitivity to a new Fermi-type $\Delta S = 2$ interaction.

The superweak hypothesis predicts that CP-violating processes in the kaon system are caused solely by the transition $K_2^0 \to K_1^0$. All final states should show the same amount of CP violation, so Eq. (1.3) should be an equality, in agreement with contemporary Standard Model predictions. Rates for $K_L^0 \to \pi^0 e^+ e^-$ and $K_L^0 \to \pi^0 \nu \overline{\nu}$ are predictable from $K_S^0 \to \pi^0 e^+ e^-$ and $K_S^0 \to \pi^0 \nu \overline{\nu}$, and there would be no effects in $K^+ \to \pi^+ \pi^+ \pi^-$. The logic, then, in pursuing all of these modes of CP violation, is, *any one* deviation from superweak rejects the superweak hypothesis, independent of Standard Model considerations.

The pure superweak hypothesis makes no prediction about the magnitude of CP violation in the B system. A good benchmark is to assume universality between the $s\overline{d} \to \overline{s}d$, and $b\overline{d} \to \overline{b}d$ interactions, in which case the largest CP violating asymmetries will be of order of magnitude:

$$\approx \frac{\frac{\Delta M_K}{f_{\rm K}^2 M_{\rm K}}}{\frac{\Delta M_B}{f_{\rm B}^2 M_{\rm B}}} |\eta_{+-}| \approx \frac{1}{10} |\eta_{+-}| \approx 10^{-4} \tag{1.6}$$

or much smaller than the Standard Model predicts. If the superweak interaction is not of universal strength, asymmetries as large as those of the Standard Model might result; to do so, however, the superweak interaction must dominate the mass splitting ΔM_B in the B^0 system. One must dismiss as coincidence the easy accommodation of ΔM_B by the Standard Model as coincidence. The distinctive prediction of the superweak hypothesis is that the magnitude of all asymmetries in the B system will be identical, so to rule out superweak, *two* asymmetries must be measured.

I find the first-order superweak explanation of CP violation more elegant and simple than the rather convoluted, second order, CKM explanation. I also prefer to hope that there is new physics, not a desert, on the mass scale of $M_W / \sqrt{7 \times 10^{-11}} \approx 1000\,\text{TeV}$ implied by the superweak hypothesis. It makes good experimental sense to choose the most clear and startling phenomena predicted by the alternate hypothesis, namely the Standard Model, and pursue it. Observation of the large and varying asymmetries in the B hadron system that are predicted by the Standard Model would really change my outlook of particle physics.

The remainder of these lectures are split into four sections. First, I give a heuristic view of the phenomenology of CP violation. Second, I address why the *discovery* of CP violation was so easy, and why has the rest been so hard? Third, I discuss the contemporary experiments that seek CP violation in the kaon system, including E731 and NA31. Finally, I address in some detail the prospects for observation of CP violation in the B hadron system.

2. Phenomenology

Yosef Nir has given an excellent account of the theory of CP violation; I follow the same notation whenever possible. Here, I'd like to underscore some physical connections.[5]

2.1 Two Component Formalism, CPT and CP

The time development of an arbitrary K^0 state,

$$a|K^0\rangle + b|\overline{K^0}\rangle$$

is governed by the Schrödinger equation,

$$i\begin{bmatrix}\dot{a}\\ \dot{b}\end{bmatrix} = H\begin{bmatrix}a\\ b\end{bmatrix} = \begin{bmatrix}M_{11}-\frac{i}{2}\Gamma_{11} & M_{12}-\frac{i}{2}\Gamma_{12}\\ M_{12}^*-\frac{i}{2}\Gamma_{12}^* & M_{22}-\frac{i}{2}\Gamma_{22}\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix} \tag{2.1}$$

Here M_{ij} are usually called the 'mass matrix' and Γ_{ij} are the 'decay matrix.' If there is CP violation in the time evolution governed by (2.1), that is called CP violation in state-mixing; it is not so easy to distinguish whether such CP violation originates in the mass matrix or the decay matrix.

It is a consequence of CPT invariance that separately $M_{11}=M_{22}$ and $\Gamma_{11}=\Gamma_{22}$. If we accept CPT, then the interesting physics of lifetime splitting, mixing, and CP violation comes from the off-diagonal terms. The only physics lost by dropping the diagonal terms of (2.1) is the common-mode lifetime, $|a|^2 \propto e^{-\Gamma_{11}t} \propto |b|^2$. The remaining description of the neutral kaon system is similar to the standard treatment of a spin-1/2 particle in a B field that has only x and y components.

How the remaining Hamiltonian transforms under CP depends on some phase conventions. The 'standard' convention is:

$$\begin{aligned} &\mathrm{P}|K^0\rangle = -|K^0\rangle, \quad \mathrm{P}|\overline{K^0}\rangle = -|\overline{K^0}\rangle \quad \text{(pseudoscalar mesons)}\\ &\mathrm{C}|K^0\rangle = -|\overline{K^0}\rangle, \quad \mathrm{C}|\overline{K^0}\rangle = -|K^0\rangle\\ \text{so,}\ &\mathrm{CP}|K^0\rangle = |\overline{K^0}\rangle, \quad \mathrm{CP}|\overline{K^0}\rangle = |K^0\rangle \quad . \end{aligned} \tag{2.2}$$

In the 2×2 subspace of (2.1), the combined operation CP is just:

$$\mathrm{CP} = \begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} = (\mathrm{CP})^\dagger \quad .$$

The test of CP invariance through state-mixing in (2.1) is whether:

$$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & M_{12}-\frac{i}{2}\Gamma_{12}\\ M_{12}^*-\frac{i}{2}\Gamma_{12}^* & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} =$$

$$\begin{bmatrix}0 & M_{12}^*-\frac{i}{2}\Gamma_{12}^*\\ M_{12}-\frac{i}{2}\Gamma_{12} & 0\end{bmatrix} \stackrel{?}{=} \begin{bmatrix}0 & M_{12}-\frac{i}{2}\Gamma_{12}\\ M_{12}^*-\frac{i}{2}\Gamma_{12}^* & 0\end{bmatrix}$$

or, simply,

$$M_{12}-\frac{i}{2}\Gamma_{12} \stackrel{?}{=} M_{12}^*-\frac{i}{2}\Gamma_{12}^* \quad . \tag{2.3}$$

According to (2.3), CP is conserved when both M_{12} and Γ_{12} are real. Actually, CP is conserved whenever

$$|M_{12}-\frac{i}{2}\Gamma_{12}| = |M_{12}^*-\frac{i}{2}\Gamma_{12}^*| \quad . \tag{2.4}$$

The reason here is that no known physical process is sensitive to the phase change[6,2,7]

$$|K^0\rangle \to e^{-i\zeta}|K^0\rangle \ , \quad |\overline{K^0}\rangle \to e^{+i\zeta}|\overline{K^0}\rangle \ . \tag{2.5}$$

The choice of

$$\zeta = -\frac{1}{4}\left[\arg(M_{12}-\frac{i}{2}\Gamma_{12}) - \arg(M_{12}^*-\frac{i}{2}\Gamma_{12}^*)\right]$$

causes the equality in (2.3) to follow from (2.4), with the consequence that M_{12} and Γ_{12} are both real. In the spin-1/2 analogy, this change just corresponds to use of a coordinate system rotated by 2ζ about the z axis from the original one. As Nir has described, it is also possible to 'rephase' the CP operator such that in the new basis, the standard convention (2.2) still holds.

For CP violation in state-mixing, it is necessary to have a difference in *magnitude* between the two off-diagonal elements of H. A nice physical picture of what this type of CP violation is a difference between the *rates* for $K^0 \to \overline{K^0}$ and $\overline{K^0} \to K^0$. The rate difference will be present when there is a phase difference between M_{12} and Γ_{12}. The standard kaon *convention* is to keep Γ_{12} real, and describe the observed CP violation by a small imaginary part in M_{12}.

2.2 Physical Features

2.2.1 Lifetime Difference, $\Delta I = 1/2$

In the neutral kaon system, the most striking physical feature is the difference in lifetime between K_L^0 and K_S^0:

$$\frac{\tau_L}{\tau_S} = \frac{51.7\,\mathrm{ns}}{0.0892\,\mathrm{ns}} = 580 \quad . \tag{2.6}$$

If a K^0 is produced in some typical hadronization process, it appears to decay with two wildly different lifetimes! This is not due to CP violation. This difference is described in the phenomenology by the real part of Γ_{12}, which from Nir is given to second order:

$$\Gamma_{12} = 2\pi\sum_n \rho_n\langle K^0|H_{\Delta S=1}|n\rangle\langle n|H_{\Delta S=1}|\overline{K^0}\rangle = 2\pi\sum_n \rho_n A_n^*\overline{A}_n \tag{2.7}$$

where n are the final states accessible in the decays of *both* the K^0 and the $\overline{K^0}$, ρ_n is their density, and the decay amplitudes are $A_n = \langle n|H_{\Delta S=1}|K^0\rangle$,

$\overline{A}_n = \langle n|H_{\Delta S=1}|\overline{K^0}\rangle$. The most important final states for the neutral kaon system are those of two pions with no relative orbital angular momentum, then: $CP|2\pi\rangle = +|2\pi\rangle$. A state need not be a CP eigenstate to be included in the sum (2.7); an example is $\pi^+\pi^-\pi^0$. Only when all three pions have no relative angular momentum is $\pi^+\pi^-\pi^0$ an eigenstate of CP with $CP|\pi^+\pi^-\pi^0\rangle = -|\pi^+\pi^-\pi^0\rangle$; we denote this $\pi^+\pi^-\pi^0$(S-wave). This final state is favored in K^0_L decay.

In the same formalism,

$$\Gamma_{11} = 2\pi \sum_m \rho_m |A_m|^2 = 2\pi \sum_m \rho_m |\overline{A}_m|^2 = \Gamma_{22} \quad .$$

The sum over m now encompasses *all* final states available to the K^0. In the kaon system, the 2π final states account for all but one part in 580 of Γ_{11}, which gives rise to (2.6).

The physical picture follows from first considering the limit of CP conservation. The CP eigenstates K^0_1 and K^0_2 are defined by:

$$\begin{aligned} |K_1^0\rangle &= \frac{1}{\sqrt{2}}\left[|K^0\rangle + |\overline{K^0}\rangle\right], \quad CP|K_1^0\rangle = +|K_1^0\rangle \\ |K_2^0\rangle &= \frac{1}{\sqrt{2}}\left[|K^0\rangle - |\overline{K^0}\rangle\right], \quad CP|K_2^0\rangle = -|K_2^0\rangle \,. \end{aligned} \tag{2.8}$$

For $K^0_2 \to 2\pi$, $K^0 \to 2\pi$ and $\overline{K^0} \to 2\pi$ interfere destructively, and delete the 2π final state from the K^0_2's total width.

$$|\langle 2\pi|H_{\Delta S=1}|K_2^0\rangle|^2 = \frac{1}{2}\left[|A_{2\pi}|^2 + |\overline{A}_{2\pi}|^2 - 2\mathrm{Re}[A_{2\pi}\overline{A}^*_{2\pi}]\right] \quad . \tag{2.9}$$

When CP is conserved,

$$A_{2\pi} = \langle 2\pi|H_{\Delta S=1}|K^0\rangle = \langle 2\pi|(CP)^\dagger H_{\Delta S=1}(CP)|K^0\rangle = +\langle 2\pi|H_{\Delta S=1}|\overline{K^0}\rangle = \overline{A}_{2\pi}$$

and

$$A_{2\pi}\overline{A}^*_{2\pi} = |A_{2\pi}|^2 \quad . \tag{2.10}$$

So $|\langle 2\pi|H_{\Delta S=1}|K^0_2\rangle|^2$ vanishes. Similarly, $A_{3\pi} = -\overline{A}_{3\pi}$ results in the suppression through destructive interference of $K^0_1 \to \pi^+\pi^-\pi^0$(S-wave). These pieces of physics are described in the Schrödinger formalism by Γ_{12}.

In the limit of CP conservation, the K^0_1 has a total width of $\Gamma_{11} + \Gamma_{12} \approx \Gamma_S$, and the K^0_2 a width of $\Gamma_{11} - \Gamma_{12} = \Gamma_L$. The greater density of final states available to 2π compared to the other, mostly three-body, final states is reason why the 2π states so dominate Γ_{11}. The dominance of the 2π final states is more surprising when stated as the following ratio, which has little correction for density of states:

$$\frac{\Gamma(K^0_S \to 2\pi)}{\Gamma(K^+ \to 2\pi) + \Gamma(K^- \to 2\pi)} = 325 \quad . \tag{2.11}$$

This is one of the experimental underpinnings of the $\Delta I = 1/2$ rule. Two pion states with total isospin of both 0 and 2 are accessible from the K^0_S, which itself has isospin 1/2; two pion states with total isospin 1 are forbidden by generalized Bose symmetry. Only $(2\pi)_{I=2}$ is accessible from the K^+, which also has isospin 1/2. The $\pi^+\pi^0$ must have total isospin of at least 1, because the third component of isospin is 1. So, the ratio (2.11) can be described by a huge enhancement of the $\Delta I = 1/2$ transition relative to the $\Delta I = 3/2$. This description predicts a value of two for what is known in neutral kaon physics as the 'single ratio':

$$\frac{B(K^0_S \to \pi^+\pi^-)}{B(K^0_S \to \pi^0\pi^0)} = 2.19. \tag{2.12}$$

The prediction comes heuristically from noting that in the $(2\pi)_{I=0}$ state:

$$|(2\pi)_{I=0}\rangle = \sqrt{\frac{1}{3}}\left[|\pi_1^+\pi_2^-\rangle - |\pi^0\pi^0\rangle + |\pi_1^-\pi_2^+\rangle\right] \tag{2.13}$$

there is, after Bose symmetrization, twice as much $\pi^+\pi^-$ as $\pi^0\pi^0$. The isospin 2,

$$|(2\pi)_{I=2}\rangle = \sqrt{\frac{1}{6}}\left[|\pi_1^+\pi_2^-\rangle + 2|\pi^0\pi^0\rangle + |\pi_1^-\pi_2^+\rangle\right] \tag{2.14}$$

has four times as much $\pi^0\pi^0$ as $\pi^+\pi^-$, after Bose symmetrization. It is useful to remember this result when considering $\mathrm{Re}[\epsilon'/\epsilon]$.

One physical visualization of the $\Delta I = 1/2$ enhancement starts with the observation that the $I = 1$ π triplet is lower in mass than the $I = 0$ η singlet, largely because the η picks up rest energy from annihilation through gluons.[14] The $(2\pi)_{I=2}$ quintuplet should be degenerate at mass $\approx 2m_\pi$ with the $\pi^+\pi^+$, and cannot raise its mass through annihilation. Transitions to $(2\pi)_{I=2}$ are then unlikely to be enhanced from final state annihilations. The pions in $(2\pi)_{I=0}$ presumably do annihilate, so it is easy to visualize lots of resonant enhancement of this channel. The phase shift for $(2\pi)_{I=0}$ scattering at $\sqrt{s} = M_K$, $\delta_0 = 46^\circ \pm 5^\circ$, is large, while that for $(2\pi)_{I=2}$ scattering is small, $\delta_2 = -7.2 \pm 1.3$, supporting this simple physical picture. So final state rescattering probably contributes to the $\Delta I = 1/2$ enhancement. Other $\Delta I = 1/2$ enhancements, particularly from penguin diagrams,[2] are theoretically expected, but are a challenge to visualize.

No one expects a dramatic lifetime split in the neutral B system. The reason is that the overwhelming majority of final states expected from B^0 decay are not accessible, in the Standard Model, from $\overline{B^0}$. A consequence for CP violation is that the condition of (2.4) is very nearly satisfied, from smallness of Γ_{12}. On the empirical side, current experiments have not addressed Γ_{12}/Γ_{11}, but they probably would have noticed if it were much greater than 1/2.

2.2.2 Flavor Oscillation and Mass Difference

The second distinctive physical feature of the neutral kaon system is that a K^0 eventually turns into a $\overline{K^0}$, through time evolution described by (2.1). The same mechanism produces the mass splitting between the K^0_S and K^0_L, which was measured prior to the discovery CP violation. The physics of flavor oscillation and mass difference is the primary consequence of M_{12}. If CP is conserved, it is straightforward to obtain the probability of detecting a $\overline{K^0}$ at a later proper time t, if the initial state at $t = 0$ was a K^0, $P(\overline{K^0}, t; K^0)$ from (2.1):

$$\begin{aligned} P(\overline{K^0}, t; K^0) &= \frac{1}{4}\left[e^{-(\Gamma_{11}+\Gamma_{12})t} + e^{-(\Gamma_{11}-\Gamma_{12})t}\right] - 2\,e^{-\Gamma_{11}t}\cos(2M_{12}t) \\ &= \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2\,e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta M_K t)\right] \\ &= P(K^0, t; \overline{K^0}) \ . \end{aligned} \tag{2.15}$$

It is possible to directly 'sample' (2.15) through decays to final states that are not common to K^0 and $\overline{K^0}$, such as $\pi^{\mp} l^{\pm} \nu$.[15] Such data is shown in Fig. 1. An interesting feature is that ΔM_K is the 'beat frequency' in the interference between K^0_S and K^0_L. In the neutral kaon system ΔM_K is measured explicitly through that and other interferences, yielding

$$\Delta M_K \equiv M_L - M_S = -2M_{12} = (3.522 \pm 0.016)\,\mu\text{eV}.$$

Explicit interference has not yet been observed in the B system, but measurements of like and unlike sign dileptons from $\Upsilon(4S)$ decay can be related to the integral of $P(\overline{B^0}, t; B^0)$ over all time. The implication is:

$$|\Delta M_B| = (360 \pm 70)\,\mu\text{eV}. \tag{2.16}$$

This is for the B^0 system, and is usually quoted in a different notation:

$$x_d \equiv \frac{|\Delta M_B|}{\Gamma_b} = 0.71 \pm 0.14 \ .$$

The Standard Model predicts that for the B^0_s system, the mass splitting is expected to be about an order of magnitude higher than (2.16), while the lifetime is unchanged; so $x_s \approx 8$.

We have as yet no experimental measure phase of ΔM_B in the complex plane. It is possible in the Standard Model that ΔM_B is *purely* imaginary, a manifestation of the innately 'large' CP violation in the B system.

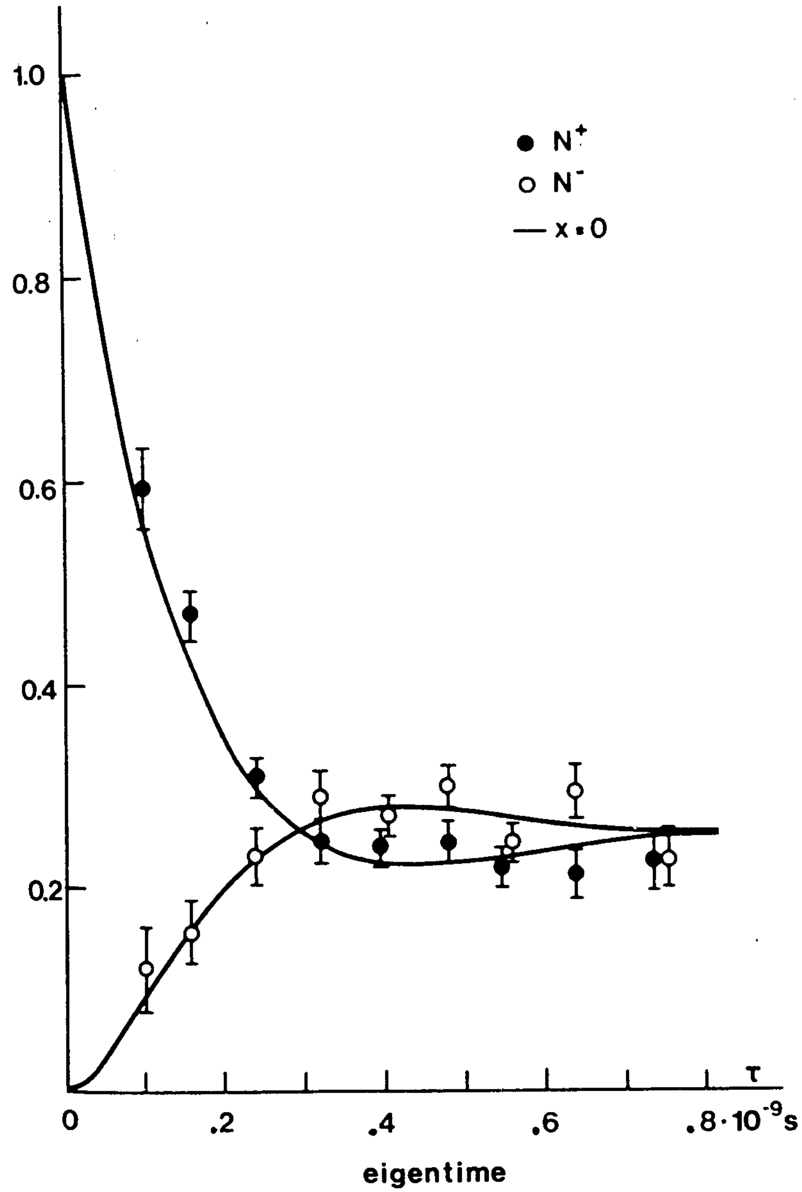

Figure 1. A K^0 was produced and tagged at $\tau = 0$; N^+ is the number of $\pi^- e^+ \nu$ decays, N^- is $\pi^+ e^- \nu$. A K^0_S lifetime is slightly less than one division.[15]

2.3 CP VIOLATION

The first manifestation of CP violation was the observation of $K_L^0 \to \pi^+\pi^-$. Terminology has changed here. Prior to the observation of $K_L^0 \to \pi^+\pi^-$, one would have assumed that the long-lived kaon was the K_2^0; after, the more empirical K_L^0 became appropriate. Mathematically, K_L^0 denotes the longer-lived eigenstate of the Hamiltonian.

We will focus first on description of $K_L^0 \to 2\pi$. The other important physical manifestation of CP violation is the distribution in time of decays, through which one 'samples' $P(\overline{K^0}, t; K^0)$ or $P(K^0, t; \overline{K^0})$; inequality between these is a clean proof of CP violation in state-mixing.

2.3.1 $K_L^0 \to 2\pi$

Consider state-mixing CP violation, that is, CP violation in the Hamiltonian; then through (2.4), we know that the off-diagonal elements of H differ in *magnitude*. The eigenvalues and eigenvectors of H, denoted by Nir $\Delta\mu/2$, are found from:

$$\begin{vmatrix} -\Delta\mu/2 & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & -\Delta\mu/2 \end{vmatrix} = 0 \quad \Rightarrow \Delta\mu/2 = \pm\left[(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)\right]^{1/2}$$

$$\begin{bmatrix} -\Delta\mu/2 & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & -\Delta\mu/2 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = 0 \quad \Rightarrow \frac{q}{p} = \pm\left[\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right]^{1/2} \neq e^{2i\zeta} \quad . \tag{2.17}$$

The inequality at right is a reminder of (2.4); were that equality true, with (2.5) $p = q = \pm 1/\sqrt{2}$, (2.8) is recovered, and state-mixing CP violation is absent.

The usual overall phase convention is:

$$\begin{aligned} |K_S^0\rangle &= \left[p|K^0\rangle + q|\overline{K^0}\rangle\right], \quad \text{eigenvalue } \Delta\mu/2 \\ |K_L^0\rangle &= \left[p|K^0\rangle - q|\overline{K^0}\rangle\right], \quad \text{eigenvalue } -\Delta\mu/2 \quad . \end{aligned} \tag{2.18}$$

Consider the rate for $K_L^0 \to 2\pi$, in analogy with (2.9):

$$|\langle 2\pi | H_{\Delta S=1} | K_S^0\rangle|^2 = |pA_{2\pi}|^2 + |q\overline{A}_{2\pi}|^2 - 2\mathrm{Re}(pq^* A_{2\pi}\overline{A}_{2\pi}^*) \quad . \tag{2.19}$$

An important physical concept underlies (2.19): CP violation experiments always measure CP violation via decays, so the interpretation of their results always entangles state-mixing (p and q) with CP violation in the decay amplitudes (A and $\overline{A}$).

The usual supposition for the kaon system, which corresponds to Nir's case (*ii*) in his 'Manifestations of CP violation,' is that the CP violation arises from state-mixing ($|p| \neq |q|$), then, by (2.10):

$$|\langle 2\pi | H_{\Delta S=1} | K_S^0\rangle|^2 = |A_{2\pi}|^2 \, |p - q|^2 \quad . \tag{2.20}$$

The factorization in (2.20) implies that *all* CP-forbidden decays rates will be in a universal ratio to their CP-allowed counterparts:

$$\begin{aligned} &\left[|\eta_{+-}|^2 \equiv \frac{\Gamma(K_L^0 \to \pi^+\pi^-)}{\Gamma(K_S^0 \to \pi^+\pi^-)}\right] = \left[|\eta_{00}|^2 \equiv \frac{\Gamma(K_L^0 \to \pi^0\pi^0)}{\Gamma(K_S^0 \to \pi^0\pi^0)}\right] = \\ &\left[|\eta_{000}|^2 \equiv \frac{\Gamma(K_S^0 \to 3\pi^0)}{\Gamma(K_L^0 \to 3\pi^0)}\right] = \left[\ldots\right] = \left|\frac{p-q}{p+q}\right|^2 \equiv |\epsilon|^2 \; . \end{aligned} \tag{2.21}$$

The parameter ϵ is introduced to quantify the deviation, small in the kaon system, of q/p from unity.

$$\frac{q}{p} = \frac{1-\epsilon}{1+\epsilon} = \left[\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right]^{1/2} \quad .$$

The phase of ϵ is defined in harmony with the superweak hypothesis, by *the convention* [through use of (2.5)]: $\mathrm{Im}[\Gamma_{12}] = 0$; then with

$$z \equiv i\mathrm{Re}M_{12} + \frac{1}{2}\Gamma_{12} = -\frac{i}{2}\Delta M_K + \frac{1}{4}(\Gamma_S - \Gamma_L) = (2.55\,\mu\mathrm{eV})e^{-43.7°}$$

here, $\phi_\epsilon = \tan^{-1}(2\Delta M_K/(\Gamma_S - \Gamma_L)) = 43.7° \pm 0.2°$ is often called the 'superweak phase'; then

$$\frac{1-\epsilon}{1+\epsilon} = \left[\frac{1 + \mathrm{Im}[M_{12}]/z}{1 - \mathrm{Im}[M_{12}]/z}\right]^{1/2}$$

so approximately

$$\epsilon = -\left[\frac{\mathrm{Im}[M_{12}]}{5.09\,\mu\mathrm{eV}}\right] e^{43.7°} \quad . \tag{2.22}$$

From (1.1) and (2.21), $\mathrm{Im}[M_{12}] = -11.6\,\mathrm{neV}$. This one number best summarizes the hypothesis of state-mixing as the source of CP violation in the kaon system. Both the Standard Model and the superweak hypothesis are consistent with the value. It must be accidental that $\mathrm{Im}[M_{12}]$ is near the full width of the K_L^0, $\Gamma_L = 12.7\,\mathrm{neV}$.

One of the paradoxes of the neutral B system that although the Standard Model predicts $|\mathrm{Im}[M_{12}]| \approx |\Delta M_B| \approx 360\,\mu\mathrm{eV}$, four orders of magnitude larger than $\mathrm{Im}[M_{12}]$ in the kaon system, the observable consequences are less evident. One reason is that the lifetime splitting in the B system is expected to be only at the level of $|\Gamma_{11} - \Gamma_{12}|/|\Gamma_{11} + \Gamma_{12}| \approx 10^{-3} - 10^{-2}$, so the superweak phase is near 90°, and q/p is again unity.

For the kaon system, is the description that $\mathrm{Im}[M_{12}] \neq 0$ the complete story? What might come from $\mathrm{Im}[\Gamma_{12}]$, or $A \neq \overline{A}$ for some decay amplitude? Non-negligible contributions to Γ_{12} come from decay amplitudes to 2π and 3π final states. The contribution of the 3π states to Γ_{12} is at most:

$$|2\pi\rho_{3\pi}A^*_{3\pi}\overline{A}_{3\pi}| < \frac{1}{2}\frac{\mathrm{B}(K^0_L \to 3\pi^0) + \mathrm{B}(K^0_L \to \pi^+\pi^-\pi^0)}{\tau_L} = 2.2\,\mathrm{neV} \quad .$$

This is too small to give all of the observed CP violation. Only the 2π amplitudes have sufficient magnitude to vie with an explanation of pure $\mathrm{Im}[M_{12}]$. The usual phenomenology casts the amplitude for $K^0 \to \pi^+\pi^-$, A_{+-}, and that for $K^0 \to \pi^0\pi^0$, A_{00}, in terms of amplitudes to two pion states of total isospin 0, A_0, and total isospin 2, A_2:

$$\begin{aligned} A_{+-} &= \sqrt{\frac{2}{3}}A_0e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2e^{i\delta_2} \\ A_{00} &= -\sqrt{\frac{1}{3}}A_0e^{i\delta_0} + \sqrt{\frac{2}{3}}A_2e^{i\delta_2} \quad . \end{aligned} \tag{2.23}$$

It is a consequence of CPT invariance that the decay amplitudes of the $\overline{K^0}$ can be written

$$\begin{aligned} \overline{A}_{+-} &= \sqrt{\frac{2}{3}}A^*_0e^{i\delta_0} + \sqrt{\frac{1}{3}}A^*_2e^{i\delta_2} \\ \overline{A}_{00} &= -\sqrt{\frac{1}{3}}A^*_0e^{i\delta_0} + \sqrt{\frac{2}{3}}A^*_2e^{i\delta_2} \quad . \end{aligned} \tag{2.24}$$

That A_0 and A_2, but not the strong phase shifts, are complex conjugated is known as 'Watson's Theorem' and is subtle.[16]

From the $\Delta I = 1/2$ rule, we know $|A_2| << |A_0|$. Imagine $A_2 = 0$; then $|A_{+-}| = |\overline{A}_{+-}|$ and $|A_{00}| = |\overline{A}_{00}|$. It is part of the standard convention to *pick* A_0 as real, employing the mathematical artifice of (2.5) if necessary. **All** of the kaon CP violation could physically come through a phase in A_0, but the description is transferred to $\mathrm{Im}[M_{12}]$. In the Standard Model, for example, penguin diagrams do contribute to state-mixing CP violation through A_0.[2]

The only viable alternative source of kaon CP violation is a non-zero imaginary part of A_2 relative to A_0, which forces $|A_{+-}| \neq |\overline{A}_{+-}|$ *and* $|A_{00}| \neq |\overline{A}_{00}|$. Analysis of the ratio (2.11) indicates $|A_2/A_0| \approx 0.05$, so CP violation in A_2 is large enough to produce the entire $K^0_L \to 2\pi$ rate. The prime physical effect of CP violation exclusively through A_2 would be a change in the ratio of $K^0_L \to \pi^0\pi^0$ to $K^0_L \to \pi^+\pi^-$, relative to that for K^0_S. The contribution that is lowest order in A_2/A_0 (2.19) is, from (2.23) and (2.24):

$$\begin{aligned} \langle\pi^+\pi^-|H_{\Delta S=1}|K^0_L\rangle &= \frac{1}{\sqrt{2}}\left[\sqrt{\frac{1}{3}}(A_2 - A^*_2)e^{i\delta_2}\right] = \frac{1}{\sqrt{2}}\left[\sqrt{\frac{1}{3}}\,2\,i\mathrm{Im}A_2e^{i\delta_2}\right] \\ \langle\pi^+\pi^-|H_{\Delta S=1}|K^0_S\rangle &= \frac{1}{\sqrt{2}}\left[\sqrt{\frac{2}{3}}\,2\,A_0e^{i\delta_0} + \sqrt{\frac{1}{3}}(A_2 + A^*_2)e^{i\delta_2}\right] \quad . \end{aligned}$$

If A_2 were to dominate, to lowest order in $|A_2/A_0|$:

$$\eta_{+-} = \frac{\langle\pi^+\pi^-|H_{\Delta S=1}|K^0_L\rangle}{\langle\pi^+\pi^-|H_{\Delta S=1}|K^0_S\rangle} = \frac{i}{\sqrt{2}}\mathrm{Im}\left[\frac{A_2}{A_0}\right]e^{i(\delta_2-\delta_0)} \equiv \epsilon' \quad . \tag{2.25}$$

This second to last expression is the definition of ϵ'. For the transitions to $\pi^0\pi^0$:

$$\begin{aligned} \langle\pi^0\pi^0|H_{\Delta S=1}|K^0_L\rangle &= \frac{1}{\sqrt{2}}\left[\sqrt{\frac{2}{3}}(A_2 - A^*_2)e^{i\delta_2}\right] = \frac{1}{\sqrt{2}}\left[\sqrt{\frac{2}{3}}\,2\,i\mathrm{Im}A_2e^{i\delta_2}\right] \\ \langle\pi^0\pi^0|H_{\Delta S=1}|K^0_S\rangle &= \frac{1}{\sqrt{2}}\left[-\sqrt{\frac{1}{3}}\,2\,A_0e^{i\delta_0} + \sqrt{\frac{2}{3}}(A_2 + A^*_2)e^{i\delta_2}\right] \\ \eta_{00} &= \frac{\langle\pi^0\pi^0|H_{\Delta S=1}|K^0_L\rangle}{\langle\pi^0\pi^0|H_{\Delta S=1}|K^0_S\rangle} = -2 \times \frac{i}{\sqrt{2}}\mathrm{Im}\left[\frac{A_2}{A_0}\right]e^{i(\delta_2-\delta_0)} = -2\epsilon' \quad . \end{aligned} \tag{2.26}$$

Were all kaon CP violation to come from A_2, the branching ratio for $K^0_L \to \pi^0\pi^0$ would be four times as large as that for $K^0_L \to \pi^+\pi^-$, as one would have expected from (2.14), in contrast to (2.13). Experimentally, however, the latest experiments NA31 and E731 find nearly the same excess of charged pions as in (2.11). The dominant source of kaon CP violation is **not** A_2, but is state-mixing.

Nevertheless, the Standard Model predicts a small violation of CP through A_2; the amplitudes (2.25) and (2.26) add coherently to that of state mixing:

$$\begin{aligned} \eta_{+-} &= \epsilon + \epsilon' \\ \eta_{00} &= \epsilon - 2\epsilon' \quad . \end{aligned} \tag{2.27}$$

The quantity that is accessible with small experimental systematic error is the 'double ratio', R:

$$R \equiv \frac{\mathrm{B}(K^0_L \to \pi^0\pi^0)}{\mathrm{B}(K^0_L \to \pi^+\pi^-)}\frac{\mathrm{B}(K^0_S \to \pi^+\pi^-)}{\mathrm{B}(K^0_S \to \pi^0\pi^0)} = \frac{|\eta_{00}|^2}{|\eta_{+-}|^2} \quad . \tag{2.28}$$

Very naively, one expects CP violation in A_2 will favor $R > 1$, because A_2 favors transitions of K^0_L to neutral pions. However, the relative minus sign between the $|\pi^0\pi^0\rangle$ terms in (2.13) and (2.14) inverts this expectation, as the first factor in (2.26) shows. Plugging (2.27) in, one gets to lowest order:

$$\mathrm{Re}(\epsilon'/\epsilon) = \frac{1}{6}[1 - R] \quad . \tag{2.29}$$

Accidental agreement of the phase of ϵ' (from strong interactions, $\pi/2 + \delta_2 - \delta_0 \approx 90° - (-7.2°) + 46° = 51° \pm 5°$) and that of ϵ means $\mathrm{Re}[\epsilon'/\epsilon] \approx \epsilon'/\epsilon$.

It is known as 'direct' CP violation when two decay amplitudes such as A_0 and A_2 interfere, and introduce a rate difference in decay. Direct CP violation corresponds to Nir's case (i) in his 'Manifestations of CP violation.'

The Standard Model should induce both state-mixing and direct CP violation in the B^0 system, but only at the same level as that observed in the kaon system, 10^{-3}.[17] The dominant feature in the experimentally attractive decay modes for detection of CP violation in the B^0 system, such as $B^0 \to J/\psi K^0_S$, is neither state-mixing nor direct CP violation: it is interference between the two, which corresponds to Nir's case (*iii*). For the B^0 system, $|q/p| \approx 1$, as discussed after (2.22), and as well $|\overline{A}_{J/\psi K^0_S}/A_{J/\psi K^0_S}| \approx 1$. However, the quantity

$$\lambda_{J/\psi K^0_S} = \frac{q}{p}\frac{\overline{A}_{J/\psi K^0_S}}{A_{J/\psi K^0_S}} = e^{i2\beta} \neq 1 \quad .$$

Here, β is an angle of the most common 'unitary triangle,' and can be of order unity. The point is, if one measures the asymmetry in decays to $J/\psi K^0_S$ between initial B^0 and $\overline{B^0}$, one sees an effect of order $\sin 2\beta$.

2.3.2 Asymmetries

If the probability of seeing a $\overline{K^0}$ at proper time t, given an initial K^0, $P(\overline{K^0}, t; K^0)$ differs from $P(K^0, t; \overline{K^0})$, that is evidence of CP violation. Experimentally, the first job is to tag the initial flavor; the second job is to detect a decay of the K^0 or $\overline{K^0}$ to a final state f, and record its time. A situation that cleanly selects only state-mixing CP violation arises when the decay f is not common to both K^0 and $\overline{K^0}$, such as $\pi^\mp e^\pm \nu$. If the final state f is common to both K^0 and $\overline{K^0}$, state-mixing CP violation, direct CP violation, or interference between the two will cause an interference pattern from the indistinguishable processes $K^0 \to \overline{K^0} \to f$ and $K^0 \to K^0 \to f$.

If state-mixing causes $K^0_L \to 2\pi$ one expects a rate asymmetry between $K^0_L \to \pi^+ e^- \overline{\nu}$ and $K^0_L \to \pi^- e^+ \nu$, and this is observed. Common final states in both the kaon and B systems are usually CP eigenstates, such as $\pi^+\pi^-$, $\pi^0\pi^0$, and $J/\psi K^0_S$, and it is by fitting the interference pattern that one can extract the phase of η_{+-} and η_{00}. The interference pattern is precisely how the asymmetry of order $\sin 2\beta$ is generated $B^0 \to J/\psi K^0_S$.

Call $\Psi(t)[\overline{\Psi}(t)]$ the state that evolves from an initial $K^0(\overline{K^0})$. Decomposition into eigenstates of H yields:

$$\begin{aligned} |\Psi(t)\rangle &= \frac{1}{2}\left[(e^{-it\Delta\mu/2} + e^{it\Delta\mu/2})|K^0\rangle + \frac{q}{p}(e^{-it\Delta\mu/2} - e^{it\Delta\mu/2})|\overline{K^0}\rangle\right] \\ |\overline{\Psi}(t)\rangle &= \frac{1}{2}\left[(e^{-it\Delta\mu/2} + e^{it\Delta\mu/2})|\overline{K^0}\rangle + \frac{p}{q}(e^{-it\Delta\mu/2} - e^{it\Delta\mu/2})|K^0\rangle\right] \end{aligned} \tag{2.30}$$

where $\Delta\mu/2$ is the positive eigenvalue. It is evident from (2.30) that $P(K^0, t; K^0)$ and $P(\overline{K^0}, t; \overline{K^0})$ are both unaffected by CP violation, and remain as they were in its absence:

$$\begin{aligned} P(K^0, t; K^0) &= \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2\,e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta M_K t)\right] \\ &= P(\overline{K^0}, t; \overline{K^0}) \quad . \end{aligned}$$

To order ϵ, however,

$$\begin{aligned} P(\overline{K^0}, t; K^0) &= (\frac{1}{4} - \mathrm{Re}[\epsilon])\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2\,e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta M_K t)\right] \\ \mathrm{P}(K^0, t; \overline{K^0}) &= (\frac{1}{4} + \mathrm{Re}[\epsilon])\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2\,e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta M_K t)\right] \quad . \end{aligned} \tag{2.31}$$

Plots of these expressions are shown in Figure 2. These equations express what state-mixing CP violation does; note their simple relationship to (2.15). One 'samples' (2.31) with distinct final states: only $K^0 \to \pi^- e^+ \nu$ and only $\overline{K^0} \to \pi^+ e^- \overline{\nu}_e$, for example. The form of (2.31) means ***no tag is necessary***, so for an arbitrary initial superposition of K^0 and $\overline{K^0}$, the following asymmetry[18]:

$$\frac{N(\pi^- e^+ \nu) - N(\pi^+ e^- \overline{\nu})}{N(\pi^- e^+ \nu) + N(\pi^+ e^- \overline{\nu})} \overset{t\to\infty}{\longrightarrow} \equiv \delta_e = 2\mathrm{Re}[\epsilon]. \tag{2.32}$$

The data used to measure (2.32) is shown in Fig. 3.

The analogous situation in the B^0 system is simplified because the lifetime splitting is expected to be negligible, so the analog of (2.31) is:

$$\begin{aligned} P(\overline{B^0}, t; B^0) &= (1 - 4\mathrm{Re}[\epsilon_B])e^{-\Gamma_b t}\sin^2(\Delta M_B t/2) \\ P(B^0, t; \overline{B^0}) &= (1 + 4\mathrm{Re}[\epsilon_B])e^{-\Gamma_b t}\sin^2(\Delta M_B t/2) \\ P(B^0, t; B^0) &= e^{-\Gamma_b t}\cos^2(\Delta M_B t/2) \\ &= P(B^0, t; B^0) \ . \end{aligned} \tag{2.33}$$

The experimental procedure to observe state-mixing in the B system would differ from that in the kaon system; one would look for evidence that $P(\overline{B^0}, t; B^0) \neq P(B^0, t; \overline{B^0})$ through unequal numbers of like-sign positively charged dilepton events and like-negatively charged dilepton events. If only incoherent $B^0\overline{B^0}$ pairs are made, for example,

$$\frac{N(l^+l^+) - N(l^-l^-)}{N(l^+l^-) + N(l^+l^+) + N(l^-l^+) - N(l^-l^-)} = 2\mathrm{Re}[\epsilon_B]\,\frac{x_d^2(2 + x_d^2)}{[1 + x_d^2]^2} \quad . \tag{2.34}$$

To compute the probability that an initially tagged K^0 decays to $\pi^+\pi^-$, the interference between $K^0 \to \pi^+\pi^-$ and $\overline{K^0} \to \pi^+\pi^-$ must be accounted for. The phase ϕ_{+-} defined through $\eta_{+-} = |\eta_{+-}|e^{i\phi_{+-}}$, is experimentally accessible

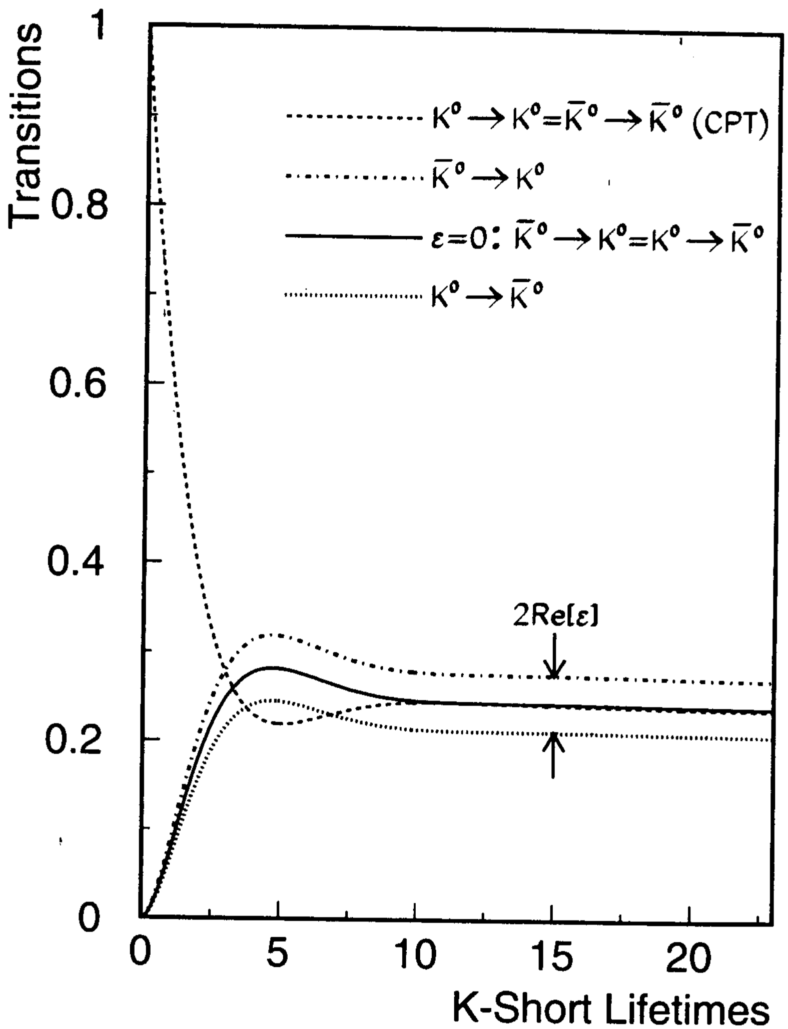

Figure 2. CP violation in state-mixing only modifies mixing.

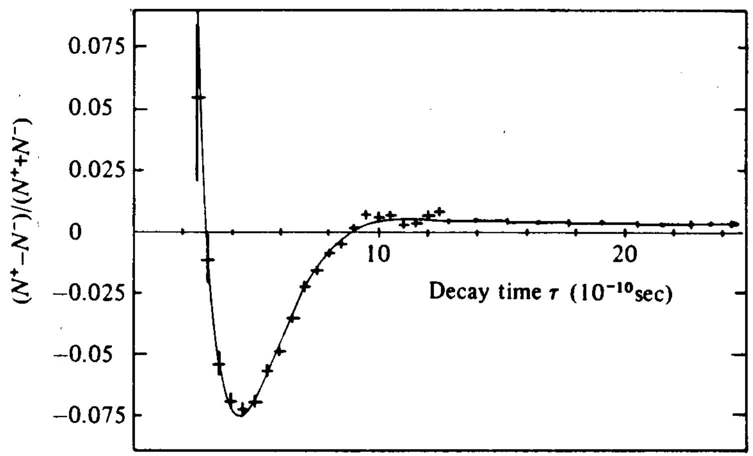

Figure 3. The charge asymmetry in the K^0 system. The asymmetry approaches a constant as $t \to \infty$, as the system becomes pure K^0_L.

through this interference. Calculation of the interference pattern is straightforward if (2.30) is rewritten in terms of the eigenstates of the Hamiltonian:

$$\langle \pi^+\pi^- | H_{\Delta S=1} | \Psi(t) \rangle = \langle \pi^+\pi^- | H_{\Delta S=1} \frac{1}{2p} \left[e^{-i\Delta\mu/2t} | K^0_S \rangle + e^{i\Delta\mu/2t} | K^0_L \rangle \right]$$
$$= \frac{\langle \pi^+\pi^- | H_{\Delta S=1} | K^0_S \rangle}{2p} \left[e^{-it\Delta\mu/2} + \eta_{+-} e^{it\Delta\mu/2} \right] \quad .$$

The probability that one observes a $\pi^+\pi^-$ from an initial K^0 or $\overline{K^0}$ is[19]

$$\begin{aligned} P(\pi^+\pi^-, t; K^0) \propto (\frac{1}{2} - \mathrm{Re}[\epsilon]) \Big[& e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t} \\ & + 2|\eta_{+-}| e^{-(\Gamma_S+\Gamma_L)t/2} \cos(\Delta M_K t - \phi_{+-}) \Big] \\ P(\pi^+\pi^-, t; \overline{K^0}) \propto (\frac{1}{2} + \mathrm{Re}[\epsilon]) \Big[& e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t} \\ & - 2|\eta_{+-}| e^{-(\Gamma_S+\Gamma_L)t/2} \cos(\Delta M_K t - \phi_{+-}) \Big] \quad . \end{aligned} \tag{2.35}$$

The key point is that the interference term flips sign for initial antimatter. The expressions (2.35) are shown in Fig. 4, and experimental data is shown in Fig. 5.

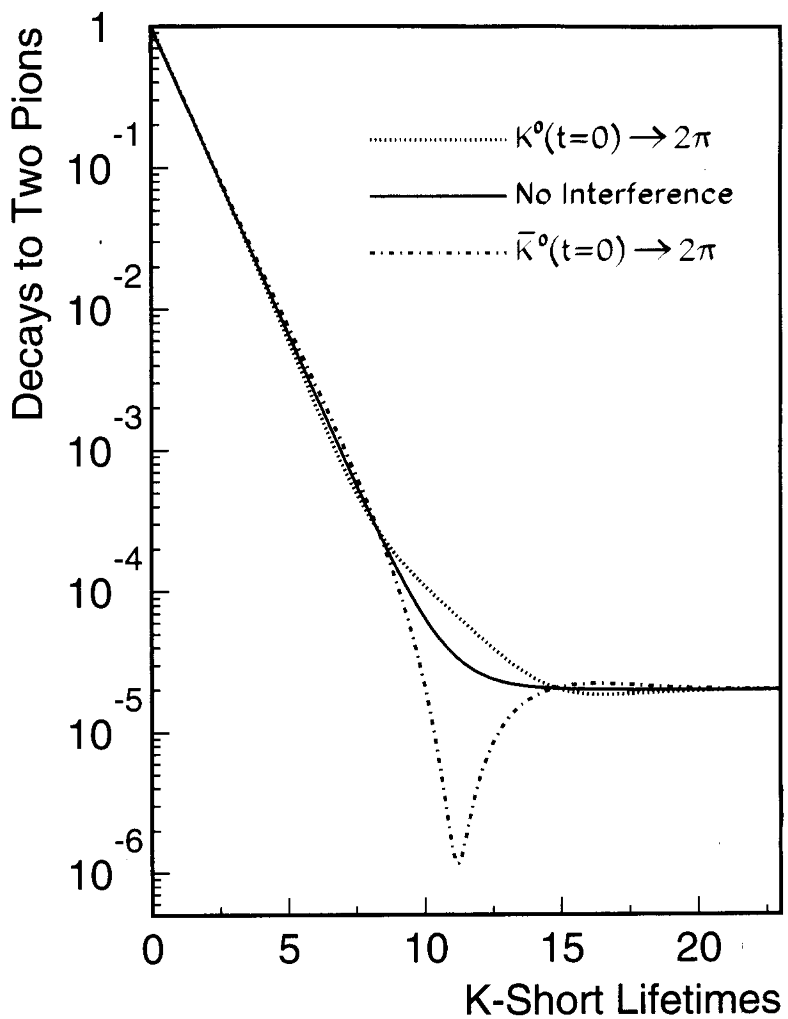

Figure 4. The distributions of 2π as a function of time, for an initial K^0, $\overline{K^0}$, and if there was no interference.

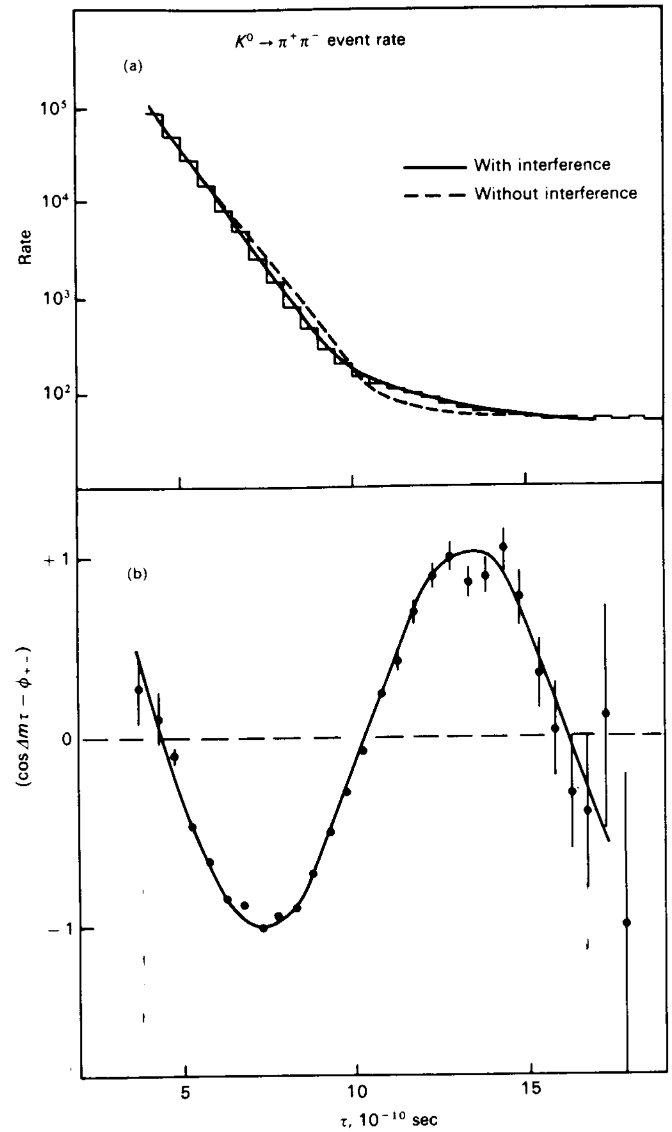

Figure 5. An experimental measure of the interference between $K^0 \to \pi^+\pi^-$ and $\overline{K^0} \to \pi^+\pi^-$. More K^0 are made at the target at $\tau = 0$, eliminating the need for tagging.[5]

These distributions have been measured many times, most recently by NA31. The E731 experiment uses an initial state prepared by regeneration of a K_S^0 beam from a K_L^0 beam. This makes a *coherent* superposition of K^0 and $\overline{K^0}$, so adds a 'regeneration phase' to ϕ_{+-}, and modulates the interference term.

In the neutral B system, there is not expected to be a dramatic lifetime splitting, so the analog of (2.35) is a bit simpler. One obtains, for example,

$$
\begin{aligned}
\mathrm{P}(J/\psi K_S^0, t; B^0) &\propto (\frac{1}{4} - \mathrm{Re}[\epsilon_B])e^{-\Gamma_B t}\Big[\cos^2(\Delta M_B t/2) + |\lambda_{J/\psi K_S^0}|^2 \sin^2(\Delta M_B t/2) \\
&\qquad + \mathrm{Re}(i\lambda_{J/\psi K_S^0})\sin(\Delta M_B t)\Big] \\
&\propto e^{-\Gamma_B t}\Big[1 - \mathrm{Im}(\lambda_{J/\psi K_S^0})\sin(\Delta M_B t)\Big] \qquad |\epsilon_B| \ll 1,\ |\lambda_{J/\psi K_S^0}| = 1 \\
\mathrm{P}(J/\psi K_S^0, t; \overline{B^0}) &\propto (\frac{1}{4} + \mathrm{Re}[\epsilon_B])e^{-\Gamma_B t}\Big[\cos^2(\Delta M_B t/2) + |\lambda_{J/\psi K_S^0}|^2 \sin^2(\Delta M_B t/2) \\
&\qquad - \mathrm{Re}(i\lambda_{J/\psi K_S^0})\sin(\Delta M_B t)\Big] \\
&\propto e^{-\Gamma_B t}\Big[1 + \mathrm{Im}(\lambda_{J/\psi K_S^0})\sin(\Delta M_B t)\Big] \qquad |\epsilon_B| \ll 1,\ |\lambda_{J/\psi K_S^0}| = 1 \ .
\end{aligned}
\tag{2.36}
$$

When $\mathrm{Im}(\lambda_{J/\psi K_S^0})$ is near unity, as is possible in the Standard Model, a large asymmetry between an initial B^0 and a $\overline{B^0}$ should occur. The cost is a small branching ratio, $\mathrm{B}(B^0 \to J/\psi K_S^0) \approx 4 \times 10^{-4}$.

For the B system, it is an experimental challenge to reconstruct the time evolution pattern in (2.36). Only recently have vertex detectors begun to see the exponential decay in the decay of B-hadrons. The time integrated asymmetry still shows evidence of CP violation:

$$
\begin{aligned}
A_{J/\psi K_S^0} &\equiv \frac{\Gamma(B^0 \to J/\psi K_S^0) - \Gamma(\overline{B^0} \to J/\psi K_S^0)}{\Gamma(B^0 \to J/\psi K_S^0) + \Gamma(\overline{B^0} \to J/\psi K_S^0)} = -\frac{x_d}{1+x_d^2}\mathrm{Im}(\lambda_{J/\psi K_S^0}) \\
&\approx -0.47\sin(2\beta) \ .
\end{aligned}
\tag{2.37}
$$

2.3.3 Tagging

To measure an asymmetry, the initial state must be tagged as matter or antimatter. These are some of the methods:

1. Build the experiment out of matter, not equal parts matter and antimatter.
2. Reconstruct both flavors in the event.
3. Pair produce in a $C = +1$ state.

The recent kaon experiments, NA31 and E731, both used the first method. NA31 produced their kaons by steering a proton beam onto a target. Pairs of $s\bar{s}$ quarks are produced, but two to three times as many K^0 as $\overline{K^0}$ emerge from hadronization. The difference likely comes about because s quarks hadronize with the abundant u and d quarks to form baryons, whereas $\bar{u}$ and $\bar{d}$ quarks are less common. In E731 a $K^0/\overline{K^0}$ asymmetry is produced by a regenerator made of matter. Both experiments then see the interference term contained in (2.35), even though they make no effort, on an event-by-event basis, to determine the K^0 strangeness. This is an extremely important practical point, as it allows gathering of high statistics.

Recent data shows little, if any, matter/antimatter asymmetry in charm hadroproduction.[20] Probably any asymmetry in B hadroproduction is negligible. Very slight asymmetries have been discussed in considerations of SSC B production.

The fact that experiments are made of matter, and not equal parts matter and antimatter, can result in *fake* CP-like asymmetries. For example, the probability that a K^+ will fake a muon signal tends to be larger than the probability that a K^- will do so, simply because the $\bar{u}$ quark in the K^- causes the K^- to suffer a hadronic interaction more often than the K^+. Similarly, since matter is slightly richer in neutrons than protons, the π^- should fake a muon signal more often than a π^+. If CP violation is reported in the B system through observation of a muon charge asymmetry, *caveat emptor*.

The second method of tagging is to exploit flavor pair production. A clear example is the strategy of the CPLEAR experiment at CERN. They exploit the exclusive reactions:

$$
p\bar{p} \to \begin{cases} \pi^- K^+ K^0 \\ \pi^+ K^- \overline{K^0} \end{cases} \ . \tag{2.38}
$$

The sign of the charged kaon tags whether the neutral kaon state was initially K^0 or $\overline{K^0}$. There is a loss of acceptance relative to NA31 or E731 due to the reconstruction of the $K^\pm\pi^\mp$, but background rejection is gained through kinematic constraints.

In most experimental proposals for the B system, only a partial reconstruction of the 'tag' hadron is made. For example, in a hadron collider, most proposals employ:

$$
p\bar{p} \to \begin{cases} X\overline{B}_{\rm tag} B^0 \\ X B_{\rm tag} \overline{B^0} \end{cases} \ .
$$

Then the sign of the lepton in the semileptonic decay of the $B_{\rm tag}$ is used to get the flavor of the tagged B. Full kinematic reconstruction is not made, as in (2.38).

The process $e^+e^- \to \Upsilon(4s) \to B^0\overline{B^0}$ suffers a peculiar tagging deficiency. Even if CP violation is present in the B system, it cannot manifest itself with a *rate* asymmetry between initially tagged B^0 and $\overline{B^0}$, if the B production is from the $\Upsilon(4s)$. The reason is that the Υ has C=-1, and the strong decay to

$B^0\overline{B^0}$ conserves C, so the $B^0\overline{B^0}$ product state ends up exclusively in

$$\frac{1}{\sqrt{2}}\left[|B^0\rangle|\overline{B^0}\rangle - |\overline{B^0}\rangle|B^0\rangle\right] .$$

If an analogy with two spin-1/2 particles is made, this state is like the singlet. Then if the time evolution of the B^0 flavor is regarded as analogous to the precession of a spin 1/2 particle, then just as the singlet state of a pair of spin 1/2 particles cannot precess due to absence of a magnetic moment, it is difficult to see a time averaged CP violating effect.

In contrast, the C=+1 state is analogous to the spin triplet, and CP violating rate asymmetries are enhanced by a naive factor of 2 (spin-1/spin-1/2, the ratio of the 'precession frequencies') if $B^0\overline{B^0}$ can be produced in the C=+1 state. This is the third method of tagging, and can be achieved through the process $e^+e^- \to \overline{B}B^* \to \overline{B}B\gamma$. A careful evaluation includes the effect of mixing, and yields, for example, in place of (2.37),

$$A_{J/\psi K^0_S} \equiv \frac{\Gamma(B^0 \to J/\psi K^0_S) - \Gamma(\overline{B^0} \to J/\psi K^0_S)}{\Gamma(B^0 \to J/\psi K^0_S) + \Gamma(\overline{B^0} \to J/\psi K^0_S)} = -\left[\frac{2}{1+x_d^2}\right]\frac{x_d}{1+x_d^2}\mathrm{Im}(\lambda_{J/\psi K^0_S})$$

$$\approx -0.63\sin(2\beta) \ . \tag{2.39}$$

As we will discuss, the deficiency of the C=-1 $B\overline{B}$ is overcome by the introduction of an energy asymmetry between the e^+ and e^- beams, thereby producing a moving $B\overline{B}$ system. Vertex reconstruction allows explicit reconstruction of the CP-violating interference as a function of time. The larger cross section of $e^+e^- \to \Upsilon(4S)$ ends up favoring the $\Upsilon(4S)$.

3. Easy, Hard, and Classic CP Violation Experiments

Why was the 'first' exploration of CP violation in kaon system so easy? Why has 'all the rest' been so hard?

More specifically, why was it so easy to

1. Observe $K^0_L \to \pi^+\pi^-$?
2. Measure the phase ϕ_{+-}?
3. Measure the semileptonic charge asymmetry?

I'm sure the people who did those experiments wouldn't call them 'easy,' but the experiments were pretty much all wrapped up by the mid-seventies, with relatively primitive technology compared to what is available today. Contrast with the situation concerning $\mathrm{Re}[\epsilon'/\epsilon]$; now, in 1992, after a decade of intense experimentation, there is still controversy over whether $\mathrm{Re}[\epsilon'/\epsilon]$ is non-zero, and another decade-long round of experiments has been initiated. Contrast with the situation in the B system: despite the fact that Standard Model CP violating effects are in some sense 'large' in the B system (e.g., ΔM_B might be completely

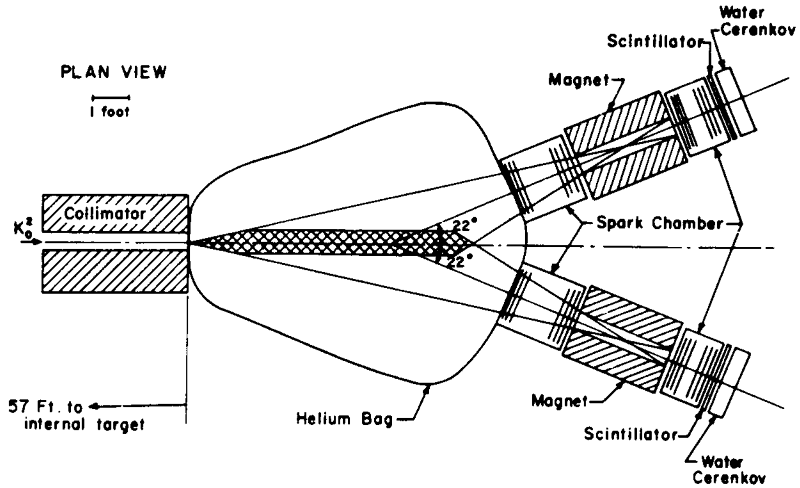

Figure 6. A schematic of the experiment that first observed $K^0_L \to 2\pi$.

imaginary; $\epsilon_B \approx \epsilon$) more time has now elapsed between the discovery of the b quark and the present than elapsed between the discovery of strangeness and the first observation of CP violation.

Why ask why? Maybe there is an 'easy' method of making the next CP violation measurement that has not been elucidated.

The fundamental reasons that the early CP violation experiments were easy can be abstracted from the second section on phenomenology. They are:

1. The enormous lifetime splitting in the kaon system.
2. The dominant CP violation in the kaon system involves a *rate* difference between matter and antimatter transitions, so tagging is inessential.
3. Both K^0 production from a target and K^0_S regeneration introduce an inclusive $K^0/\overline{K^0}$ asymmetry, allowing the measurement of *interference* effects, such as ϕ_{+-}.

To see how these fundamental aspects influence practical experimental considerations, let's look at the experiment that first observed $K^0_L \to \pi^+\pi^-$, that of Christenson, Cronin, Fitch, and Turlay (CCFT).[21] Figure 6 has a schematic of the experiment. After considering that experiment, we'll consider where one loses in the B system.

In the CCFT experiment, 30 GeV protons were incident on a beryllium target; a neutral beam was defined at an angle of 30° from the proton direction by collimators and sweeping magnets. At the target, more K^0's than $\overline{K^0}$'s were made, as discussed previously. Half of the K^0's and $\overline{K^0}$'s evolved into K_S^0 and decayed well before the region where K_L^0 decays were accepted by the detector, which started 17.4 meters downstream from the target, and extended for about 3 meters. The mean momentum of the K_L^0's that decayed in the acceptance was about measured with regeneration to be 1.1 GeV, so the distance from the target corresponded to only $\approx \gamma c\tau_L/2$. This mean momentum is *systematically lower* than that of the produced K^0 and $\overline{K^0}$, because, by time dilatation, fast K_L^0 systematically escape rather than decay. This effect is important in understanding contemporary Re$[\epsilon'/\epsilon]$ experiments. Let's *guess* that the mean momentum of the initial K^0 and $\overline{K^0}$ was 2 GeV.

We can then estimate several interesting numbers. First, the distance from the target to the start of the K_L^0 acceptance was $\approx 160\gamma c\tau_S/2$. The possibility that a K_S^0 survived into the apparatus, and gave a false $K_L^0 \to \pi^+\pi^-$, was of order $e^{-160} \approx 10^{-70}$. This is the great practical benefit yielded by the lifetime splitting in the K^0 system; the CP+ component dies away, leaving a pure CP- beam. Second, only about $3/(2\gamma c\tau_L) \approx 1/40$ of the K^0 and $\overline{K^0}$ that were produced decayed in the acceptance.

About 5300 K_L^0 decays were reconstructed by CCFT, and most of those were from $K_L^0 \to \pi^\mp l^\pm \nu$ and $K_L^0 \to \pi^+\pi^-\pi^0$ decays, and so suffered apparent missing momentum and energy. About 60 lay within what would be termed today the 'search region' for $K_L^0 \to \pi^+\pi^-$, and the background from $K_L^0 \to$ three-body, extrapolated from the remaining 5240 decays, was 15. The resulting branching ratio for $K_L^0 \to \pi^+\pi^-$ of $(2.6 \pm 0.5)\times10^{-3}$ is close to today's PDG value of $(2.03 \pm 0.04)\times10^{-3}$.

To understand how point (2) helps make this observation 'easy.' The *magnitude* of the amplitude for $K^0 \to \overline{K^0} \to \pi^+\pi^-$ differs from that for $\overline{K^0} \to K^0 \to \pi^+\pi^-$. In absence of CP violation, the K_L^0 achieves its long life through the precise cancellation of $K^0 \to \pi^+\pi^-$ and $\overline{K^0} \to \pi^+\pi^-$ amplitudes; the amplitude difference slightly upsets this cancellation, and allows the K_L^0 to 'leak' slightly into $\pi^+\pi^-$. The rate at which it does so is independent of whether the initial state was K^0 or $\overline{K^0}$ [see (2.35)], so *no tag is necessary*.

Now, imagine the B^0 system; suppose one wants to see state mixing through the lepton asymmetry (2.34) of order of magnitude 10^{-3}. Where are the factors lost relative to the CCFT experiment?

The first price one must pay comes from not having large lifetime splitting. Whereas CCFT only needed to reconstruct about 1/40 of the K^0 and $\overline{K^0}$ they produced, and let the rest decay either upstream or downstream of their apparatus, a B experiment must reconstruct all the B^0 and $\overline{B^0}$ it produces. Second, the natural division into short-lived CP+ state and long-lived CP- state that occurred in the K^0 system never happens in the B system. One is therefore obliged to tag the flavor of the initial B. Pair production of $b\bar{b}$ will most likely be exploited to do so; then the most realistic tag is the semileptonic decay of the $\bar{b}$; only 1/10 of the $\bar{b}$'s will undergo a semileptonic decay into the acceptance of a typical experiment. The same factor of 1/10 will be lost to obtain the flavor of the b.

The final factor that must be paid is the biggest. In the CCFT experiment they searched for a final state, $K_L^0 \to \pi^+\pi^-$, which was rather distinct from its background. In a B lepton asymmetry experiment, one searches for a tiny systematic difference between two large numbers (the numbers of like-signed positive and negative lepton pairs, N^{++} and N^{--}, respectively). One must accumulate enough events to overcome statistical fluctuations, and that is a drastic penalty. If one demands that an asymmetry of 10^{-3} be observed above fluctuations with at least s standard deviations, then:

$$\frac{1}{\sqrt{N^{++}+N^{--}}} < \frac{1}{s}\times10^{-3} \quad . \tag{3.1}$$

Putting the three factors together, one can estimate the number of $B^0\overline{B^0}$ events $N_{B^0\overline{B^0}}$ needed for three-standard observation of a 10^{-3} lepton asymmetry, relative to the number reconstructed in the CCFT experiment, $N_{\rm CCFT} = 5300$:

$$\begin{aligned} N_{B^0\overline{B^0}} &\approx [40] \times [10^2] \times [10^3] \times [s^2 \times 10^3 = 1.7 N_{\rm CCFT}] \\ &\approx 7 \times 10^6\, N_{\rm CCFT} \\ &\approx 4 \times 10^{10} \quad . \end{aligned} \tag{3.2}$$

That is more B hadrons than is reasonable to produce, so some trick must be employed to succeed in the B system. As we will discuss in the last section, that trick is to exchange the relatively high semileptonic branching ratio and small CP asymmetry for the tiny branching ratio but large CP asymmetry of $B^0 \to J/\psi K_S^0$.

Some good questions to ask at this point are:

A. To measure ϕ_{+-} in the kaon system, one is also *obliged* to tag the initial kaon flavor, [see (2.35)]. Why then has it been straightforward to measure ϕ_{+-}?

B. Why has the measurement of the semileptonic charge asymmetry in the kaon system been possible?

C. One uses kaons to measure Re$[\epsilon'/\epsilon]$, why has that been difficult?

The answer to (A) is point (3) above: fixed target experiments are constructed of matter, and there is an inclusive excess of K^0 when kaons are produced in manners such as used in the CCFT experiment. The 'price' of the tag has been avoided in kaon physics; however, the precision experiments of ϕ_{+-} have systematically started from K^0 initial states. The CPLEAR experiment will able to get a high statistics sample of initial $\overline{K^0}$. The current PDG value is:

$$\phi_{+-} = (46.6 \pm 1.2)^\circ \ .$$

If all CP violation in the kaon system is state mixing, then from (2.27), it should be that $\phi_{+-} = \phi_\epsilon = (43.7 \pm 0.2)^\circ$. There is a mild discrepancy, of order two standard deviations, but much too large to be accounted for by ϵ'; CPT