Predicting Quark and Neutrino Masses and Mixings*

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1 Introduction

It is no exaggeration to say that the majority of particle physics experiments are designed to measure either the masses of the quarks and leptons, or their couplings to the W boson. There is no mystery about why this is so: we are most interested in learning about the fundamental parameters of the Standard Model, and 13 out of 18 of these correspond to quark and lepton masses and mixing†. I am not trying to minimize the importance of the five parameters of the gauge sector, which can be taken as $\alpha, \alpha_s, G_F, M_Z$ and M_H; but it is a simple fact that the large majority of the fundamental parameters belong to the flavor sector.

Each of the 18 fundamental parameters is represented in the Standard Model by a coupling constant. Conventional wisdom in particle physics has it that theory got way ahead of experiment, and consequently became a victim of its own success. What do you do after successfully predicting the existence and masses of the W and Z particles? This masks an important point; the triumph of particle theory was the construction of the Standard Model, not the understanding of the values of the 18 fundamental coupling constants. The prediction of the Z mass was possible because the four observables $\alpha, G_F, \sin^2\theta$ and M_Z depend on only three of the fundamental independent couplings, giving a prediction

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†A simple extension of the Standard Model is to allow for dimension five operators of the form $\ell_i \ell_j HH$ to generate neutrino masses. Here ℓ_i is a lepton doublet and H the Higgs. In this case there are an additional 9 observables: the three neutrino masses together with the three neutrino mixing angles and the three phases of the leptonic mixing matrix. I ignore $\overline{\theta}$.

$M_Z^2 = \pi\alpha/\sqrt{2}G_F \sin^2\theta \cos^2\theta$. The real problem with theory is that it has failed to calculate any of the 18 fundamental coupling constants, while experiments have measured, to varying levels of accuracy, all 18. Particle theory has hit a brick wall. It is a victim of its present failures not its previous successes.

I do not know how to construct a fundamental theory which would allow a first principles calculation of coupling constants. Does this mean I have no hope of making predictions? **No.** It is always possible to obtain predictions by *reducing the number of free parameters.* The Balmer formula provides a superb illustration of this. A large number of observables (the hydrogenic spectral wavelengths) are described by a single free parameter (the Rydberg constant). Twenty-eight years after this incredibly successful formula was written down, it played a dominant role in leading Bohr to his atomic model in which he could compute the Rydberg, $\mathcal{R} = 2\pi^2 m Z^2 e^4/h^3$. This crowning achievement was the birth of the quantum theory of atomic structure. It may will be that a predictive scheme for fermion masses, depending on far fewer than the 13 flavor couplings of the Standard Model, is a prerequisite for the development of a fundamental theory of fermion masses. Indeed such a predictive scheme for fermion masses would start looking very much like a fundamental theory if it involved few enough parameters.

Progress has been made in reducing the number of parameters in the gauge sector. In grand unified theories (GUTs) the three independent gauge couplings become related [1]. This implies predictions for the weak scale gauge couplings $g_i(M_W)$, i = 1...3, of the form [2]:

$$g_i(M_W) = C_i\, \eta_i\, g_G \tag{1}$$

where g_G is the GUT gauge coupling, C_i are numerical group theory constants and the η_i, which are radiative corrections computed with the renormalization group, depend on mass ratios such as M_W/M_G, where M_G is the GUT scale. Let me define the number of predictions of any sector of a theory by

$$\text{Predictions} = (\text{Independent observables}) - (\text{Free parameters})\,. \tag{2}$$

How many predictions occur in the gauge sector of GUTs? While the C_i are purely numerical group theory constants, the η_i depend on ratios of various mass scales. If there are two or more mass ratios on which the η_i depend, then there are no predictions: together with g_G there are three or more free parameters for the three observables g_i. The only hope is for the maximally predictive possibility that the η_i depend only on the single mass ratio M_W/M_G, in which case there will be one prediction, usually chosen to be the weak mixing angle $\sin^2\theta$.

There are many possible GUTs which have no new scale other than M_G. How many different predictions for $\sin^2\theta$ can they give? The answer is just two: .211 without supersymmetry and .233 with weak-scale supersymmetry [3]. What is the accuracy of these predictions? There are GUT/supersymmetric model-dependent corrections which are typically around .002 [4]. Since the Standard Model is *consistent* with any value of $\sin^2\theta$ from 0 to 1, I think that it is very significant that the minimal supersymmetric scheme predicts precisely the experimental value of .233 ± .001. Many people shrug this off, but let's face it, it *is* significant.

The successful prediction of $\sin^2\theta$ resulted from requiring a larger symmetry than dictated by experiment. It is well known that this same enlargement of the gauge symmetry can also yield predictions in the flavor sector. Flavor observables at the weak scale, $F_a(M_W)$, can be given by predictions of the form

$$F_a(M_W) = C_a\, \eta_a \tag{3}$$

where C_a are again purely numerical group theory constants, while the dynamical factors η_a depend on several parameters, including α_s and mass ratios such as M_W/M_G. The first such prediction was for m_b/m_τ[5]. However, we now know that in this case η_a depends on m_t and α_s, leading to uncertainties of 30% and 10% respectively. Hence this successful prediction is much less significant than $\sin^2\theta$, especially as one successful prediction out of so many flavor parameters is not convincing.

Recently Savas Dimopoulos, Stuart Raby and I have constructed a scheme with only eight independent flavor parameters [6]: we predict 14 of the 22 quark and neutrino masses and mixings.[‡] Our scheme is based on two sets of symmetries: an S0(10) supersymmetric gauge symmetry and the family symmetry of Georgi and Jarlskog [7]. We have used these two types of symmetries, GUT and family,

‡In fact since the theory is supersymmetric there is an extra flavor parameter: $\tan\beta$, the ratio of VEVs. We predict 15 of the 23 total flavor parameters.

because they are the only known tools available for obtaining predictive flavor theories, other than just phenomenological guesswork. Our scheme is by far the most predictive that has ever been written down. It may not be *the* most predictive, and it may not be correct, but it *can* be tested.

What is the level of accuracy of our predictions? This is determined by the experimental uncertainties of the inputs used to determine our free parameters. For example we use $\sin\theta_c$, m_c and m_u/m_d as inputs, and these are known only to 1%, 10% and 30% respectively. Hence our predictions have accuracies which are typically 1-30% depending on which inputs they are sensitive to.

Six of our 14 predictions occur in the charged fermion sector. Our scheme may well be probed via the top mass. We are unable to give a very precise determination of m_t because it depends on inputs α_s, m_c and V_{cb} which all have 0(10%) uncertainties. However, we will need to rethink if m_t is outside the range 165 ± 25 GeV. A crucial and definitive test of our scheme will occur if the angles α, β, γ of the unitarity triangle of the KM matrix are accurately determined through CP violating decays of neutral B mesons at a B factory. These predictions are discussed further in Section III below.

The neutrino masses and mixings are completely determined in terms of the charged fermion masses and mixings, with the one exception of the overall mass scale of the neutrino masses. We do not know any way of predicting this scale. As far as we know, this is the first time anything about the neutrino masses and mixings has been predicted using the known quark and charged lepton masses and mixings as input. We predict every element of the 3×3 lepton mixing matrix, and both neutrino mass ratios m_{ν_τ}/m_{ν_μ} and m_{ν_μ}/m_{ν_e}. In constructing our scheme for neutrino masses we have made several assumptions, each motivated by the desire to obtain a maximum number of predictions. The assumptions concern our choice of symmetries and how these symmetries are broken. Before giving more details of our predictions, I will now discuss some of the history of attempts to predict fermion masses in gauge theories.

2 A Brief Historical Review

A prerequisite for a predictive gauge theory of fermion masses is a proof that gauge theories are renormalizable. This is because it is renormalizability which leads to a gauge theory being completely determined by a finite set of parameters. It is interesting to note that in the very paper of 1971 in which spontaneously broken gauge theories were shown to be renormalizable [8] it was also pointed out that certain mass ratios could be calculated.

The first attempts to obtain quark or lepton mass predictions in gauge theories concentrated on m_e/m_μ [9]. In the Standard Model m_e and m_μ are independent; they arise from the two independent Yukawa couplings λ_e and λ_μ. The idea of reference [9] was to increase the electroweak gauge group in such a way that there was only a single Yukawa coupling, and such that electroweak symmetry breaking gave rise only to a muon mass at tree level. The electron mass was then to be understood as a finite and calculable radiative correction. For example, with an electroweak gauge group $SU(3) \times SU(3)$ the electron and muon leptons could be arranged as $\psi_L = (e, \nu_e, \mu^+)_L \sim (3,1)$ and $\psi_R = (\mu^+, \overline{\nu}_\mu, e^-)_R \sim (1, \overline{3})$. The single Yukawa interaction $\lambda\overline{\psi}_L\phi\psi_R$ involves scalar mesons in the (3, 3) representation. A vacuum expectation value $\langle\phi_{31}\rangle = v$ leads to tree level masses $m_\mu = \lambda v$, $m_e = 0$. Note that $SU(2)_L$ lies in the first 2×2 subspace of the vector $SU(3)_{L+R}$. This means that ϕ_{31} is just the neutral component of an $SU(2)_L$ doublet, it is the Higgs of the Standard Model. The crucial point is that the gauge symmetry has been arranged so that this Higgs has no coupling to electrons. Such a coupling is induced by ϕ_{13}, but that is in a different $SU(2)_L$ doublet which is assumed not to acquire a VEV.

In the Standard Model we could also arrange for the electron to be massless at tree level by setting $\lambda_e = 0$. However, in this case the chiral symmetry of the electron field is unbroken so that the electron remains massless to all orders in perturbation theory. The crucial point about these extended electroweak gauge models is that the masslessness of the electron is an accidental consequence of the tree level structure of the theory. It is not guaranteed by a symmetry and hence radiative corrections induce an electron mass. The point is that the chiral symmetry of the electron is the same as that of the muon as they appear in the same multiplet. This means that the muon mass term breaks electron chiral symmetry! The way to communicate this chiral symmetry breaking from the muon to the electron is via the $SU(3)_L$ and $SU(3)_R$ gauge bosons $X_{L,R}$ which couple electrons to muons. Hence the one loop diagram for the electron mass is as shown in Figure 1. The electron mass is finite and calculable precisely

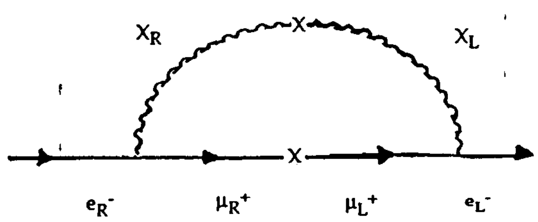

Figure 1: A one loop contribution to the electron mass. $X_{L,R}$ are exotic gauge bosons.

because the theory is renormalizable. Furthermore the electron mass $m_e \propto \alpha m_\mu$ looks as if it might be numerically in the right ballpark.

Although there is much which smells right about this approach it has a crucial flaw; the electron mass depends upon the masses of the $X_{L,R}$ gauge bosons. This is a disaster because until such bosons are discovered, and their masses measured, the theory has no predictivity in the sense of equation (2). There is a vast literature on radiative fermion masses. However, I will not discuss it further in these lectures: I simply have not seen such a scheme which was predictive. While the radiative hierarchy is frequently quite appealing, there are no hard numbers being predicted, which would render the theory testable. In my opinion, the most important aspect of this early attempt at understanding fermion masses is that it was the first example of how the number of independent Yukawa couplings could be reduced by an increase in the symmetry. This is the crucial tool which has led to theories which can make actual numerical predictions for fermion masses and mixings. There are two symmetries which have proved useful in this regard: grand unified gauge symmetries and global family (or generation) symmetries. It is astonishing that it took a further five years before either of these symmetries was used to make predictions in the flavor sector, but in 1977 each of these types of symmetries was used to make an apparently successful prediction.

The relationship $\tan\theta_c = \sqrt{m_d/m_s}$ had been noticed in the 1960s [10]. In the Standard Model the elements of the Kobayashi-Maskawa matrix are independent of the quark masses. However, by using both a left-right extended gauge symmetry and global family symmetries, it can appear as a tree-level mass relation [11]. To obtain this mass relation we need only study the lightest two generations. To see how $\tan\theta_c = \sqrt{m_d/m_s}$ arises, consider the following 2×2 mass matrix:

$$\left(\bar{d}_L \bar{s}_L\right) \begin{pmatrix} C & B \\ B' & A \end{pmatrix} \begin{pmatrix} d_R \\ s_R \end{pmatrix}. \qquad (4)$$

Since we do not wish to consider the up-quark mass matrix there is no loss of generality in taking the parameters A, B, B', C to be real. Assume that the Cabibbo angle arises from the diagonalization of this down quark mass matrix, with only a negligible correction from the up sector. Since there are four parameters for three observables (m_s, m_d, θ_c) there are in general no predictions.

The mass relation follows only after symmetries have been introduced which set $B' = B$ and $C = O$. In this case, taking $A >> B$, $m_s \propto A$, $m_d \propto B^2/A$ and $\tan\theta_c = B/A = \sqrt{m_d/m_s}$. Since the Cabibbo angle is known to 1% accuracy, the significance level of this relation is governed by the 20% uncertainties in the determinates of m_d/m_s. Thus this prediction is good at the 10% level.

Exactly what symmetries are required to obtain the crucial parameter reductions: $B' = B$ and $C = O$? Extend the electroweak gauge group to $SU(2)_L \times SU(2)_R \times U(1)$ and have multiplets $\psi_{1L} = (u,d)_L, \psi_{2L} = (c,s)_L, \psi_{1R} = (u,d)_R$ and $\psi_{2R} = (c,s)_R$ where left (right) handed quarks are doublets of $SU(2)_{L(R)}$. The relevant Yukawa interactions are $\overline{\psi}_{L_i}\lambda_{ij}\psi_{Rj}\phi + h.c.$, where $i,j = 1,2$ are generation indices and ϕ are the Higgs mesons in a (2, 2) representation. So far we haven't got very far because λ_{ij} still contains four independent couplings. To obtain $B' = B$, impose a parity symmetry under $\psi_L \leftrightarrow \overline{\psi}_R$ (this only makes sense if $SU(2)_R$ is gauged). This implies that $\lambda_{ij} = \lambda_{ji}$. It isn't really necessary to go any further, one can simply assume that $\lambda_{11} = C << B^2/A$ and can be ignored. However, it is also possible to force it to zero by introducing two parities and two Higgs meson multiplets: $\phi \to \phi_1, \phi_2$. The most general Yukawas allowed by the symmetries P_2, which changes the sign of fields ψ_{2L}, ψ_{2R} and ϕ_2, and P_1, which changes the sign of fields $\psi_{1L}, \psi_{1R}, \phi_1$ and multiplies ψ_{2L}, ψ_{2R} and ϕ_2 by i, are

$$A\overline{\psi}_{2L}\psi_{2R}\phi_1 + B(\overline{\psi}_{1L}\psi_{2R} + \overline{\psi}_{2L}\psi_{1R})\phi_2 + h.c.$$

These symmetries are not very beautiful. Who cares? We are out to predict numbers which can be compared with experiment. Both terms are allowed by P_1 and P_2, while P_1 forbids $\overline{\psi}_{1L}\psi_{1R}\phi_1$ and P_2 forbids $\overline{\psi}_{1L}\psi_{1R}\phi_2$. There is another set of Yukawa couplings obtained by $\phi_i \to \tilde{\phi}_i$ which generate the up mass matrix of the form

$$\begin{pmatrix} 0 & B' \\ B' & A' \end{pmatrix}.$$

This has two parameters which are fixed by m_u and m_c, and one discovers that the angle necessary to diagonalize this matrix $\theta_u \simeq \sqrt{m_u/m_c}$ is sufficiently small not to substantially contribute to θ_c.

The trick to obtain the θ_c prediction was to assume that $m_d \to 0$ in the limit that $\theta_c \to 0$; i.e., the light quark acquires mass only because of a mixing with the heavier one. This idea can be extended to include a third generation [12], in which case the pattern of the mass matrices is known as the Fritzsch texture:

$$\mathbf{U} = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix} \mathbf{D} = \begin{pmatrix} 0 & F & 0 \\ F & 0 & E \\ 0 & E & D \end{pmatrix}.$$

Fritzsch imposed symmetries such that all six parameters were real. In this case all three of the mixing angles of the Kobayashi-Maskawa matrix can be predicted in terms of the ratios of the quark masses. The six quark masses are just sufficient to determine the six free parameters. Today people have largely forgotten the symmetries Fritzsch used and just remembered the pattern of zeros. This allows one to take all the parameters complex and obtain CP violation. In fact four of the six phases can be removed by field redefinitions. This "Fritzsch" model therefore has eight parameters to describe ten observables. The two predictions are usually taken to be $|V_{ub}|/|V_{cb}| = .07 \pm .01$, which is a good prediction, and the second is taken to be the top mass. One finds that $m_t \lesssim 95$ GeV, which is currently right on the experimental limit. The most elegant fermion mass relations are undoubtedly those given solely from an enlargement of the gauge group, to which we now turn.

In the Georgi-Glashow $SU(5)$ grand unified model with a 5 of Higgs, the down quarks start out degenerate with their corresponding leptons. This works well for the third generation [5] which provided the first GUT mass relation. However it predicts the troublesome relation

$$\frac{d}{s} = \frac{e}{\mu}$$

which appears to be wrong by one order of magnitude. This led H. Georgi and C. Jarlskog to introduce the 45 dimensional Higgs multiplet [7]. This multiplet when coupled to a given family, say the second one, gives $\mu = -3s$ at the GUT scale. Georgi and Jarlskog make use of this multiplet and obtain the following Yukawa matrices for the down quarks and electrons:

$$\mathbf{D} = \begin{pmatrix} 0 & F & 0 \\ F & E & 0 \\ 0 & 0 & D \end{pmatrix} \quad \mathbf{E} = \begin{pmatrix} 0 & F & 0 \\ F & -3E & 0 \\ 0 & 0 & D \end{pmatrix}. \tag{5}$$

The elements D and F arise from the VEV of 5s of Higgs and the entries E and $-3E$ from a 45 of Higgs. The zeros are forced by discrete symmetries. The up matrix has the Fritzsch form

$$\mathbf{U} = \begin{pmatrix} 0 & C & 0 \\ C & 0 & B \\ 0 & B & A \end{pmatrix} \quad . \tag{6}$$

We will refer to the matrices D, E, and U given by Eqs. 5 and 6 as having the Georgi-Jarlskog texture. Harvey, Ramond and Reiss [13] studied this texture in an SO(10) model. They were the first to realize that it led to a prediction for m_t in terms of V_{cb} and that the resulting KM matrix violated CP. However, they did not renormalization-group-scale the Yukawa couplings to obtain predictions for m_t, $|V_{ub}/V_{cb}|$ or the CP violating angle.

One could imagine choosing the Georgi-Jarlskog texture at the weak scale for U and D alone (ignoring the leptons and any reference to GUTs) as suggested recently [14]. Since the down quark mass matrix is diagonal in the two heaviest generations, one has

$$V_{cb} \simeq \sqrt{\frac{c}{t}} \quad . \tag{7}$$

This implies a very heavy top in the 220-800 GeV region [14], which, when compared to electroweak data, is seen to be unacceptably large.

The unacceptably large top mass is a consequence of Eq. (7) which in turn follows from the Georgi-Jarlskog matrices of Eqs. (5) and (6). In deriving the value of the top mass from Eq. (7), the low energy values of m_c and V_{cb} were used [14]; thus, implicitly assuming that Eqs. (5) and (6) were valid at the electroweak scale. In a Grand Unified Theory [GUT], this assumption is not justified. Thus, the fermion masses have the Georgi-Jarlskog texture of Eqs. (5) and (6) only at M_{GUT} where the theory is defined. At energies below M_{GUT} the form of the mass matrices can change. In particular, zero entries can become nonzero and this can significantly change the connection between masses and mixing angles. For example, a nonvanishing 22 entry in the up mass matrix U will change Eq. (7) and can therefore fix the heavy top mass problem.

The statement that zero entries in the mass matrices can evolve becomes evident when we recall that such zeros originate at the GUT scale as a result of a (typically discrete) symmetry Z. If, as is often the case, Z is spontaneously broken at M_{GUT} then at low energies there is no symmetry to protect the zeros; as a result they become nonvanishing but calculable quantities at the weak scale.

Under what conditions is Z spontaneously broken at M_{GUT}? The implementation of Z requires the existence of several Higgs doublets (belonging to various 5s and a 45 in the Georgi-Jarlskog model); if most of these doublets become superheavy and are not available below M_{GUT} then Z cannot be implemented and is necessarily broken [15]. Such is the case in minimal low energy models where only one Higgs couples to quarks of a given charge.

3 A Recent Framework and Its Predictions

(I) Framework

Our objective is not to focus on a single grand unified theory [GUT] but to propose a general framework which can result from a very large class of theories. Only the features of the framework relevant to predicting the fermion mass spectrum are of interest to us. These are:

1. Grand Unification: We work in the context of GUTs, so that we can relate quark and lepton masses. This leads to an economy of parameters; we save ourselves from having to introduce three additional new parameters to describe the hierarchy of the three lepton masses e, μ and τ.

2. Low Energy Supersymmetry: The successful prediction of $\sin^2\theta_w$ makes it preferable to work in Supersymmetric [SUSY] GUTS. In such a theory, we have two Higgs doublets; thus the fermion mass matrices include a new parameter, $\tan\beta$, the ratio of Higgs VEVs.

3. Georgi-Jarlskog Texture: The mass matrices will have the Georgi-Jarlskog form (Eqs. 5, 6) at M_{GUT}.

4. SO(10): The gauge group will be SO(10) or E(6) instead of SU(5). There are many reasons for this: in SO(10) the mass matrices can be automatically symmetric. This is important since otherwise we are forced to introduce an extra eighth parameter for no fundamental reason and reduce the predictive power of our model. Also in SO(10) we can relate neutrino to quark masses and make predictions about the light majorana neutrino masses. In addition, the Georgi-Jarlskog factor of -3 relating quark and lepton masses can be easily achieved in several ways as a consequence of the Pati-Salam subgroup contained in SO(10).

5. Complex Parameters: To allow for CP-violation we shall start with all the parameters A, B, C, D, E, and F that appear in the mass matrices being complex.

It is immediate to see that in our framework the top is necessarily heavy: recall that we have to avoid the relation $V_{cb} = \sqrt{\frac{c}{t}}$ at low energies. This equation is valid at M_{GUT} since it is a direct consequence of the GUT scale mass matrices given by Eqs. 5, 6. Thus, to avoid it we must ensure that V_{cb} runs between the grand and weak scales; this can only happen if the top Yukawa coupling is large $\lambda_t \sim 1$.

The parameter counting for the quarks is as follows: the U and D Yukawa matrices have nine nonvanishing entries. We have nine fields at our disposal - three doublets and six singlets – thus eight relative phases that can be used to get rid of all but one of the complex phases. For convenience we use this phase freedom to make A, B, C, D, and E real and keep F complex, and the mass matrices hermitean. Thus we have seven real parameters A, B, C, D, E, the magnitude of F (call it F from now on) and its phase ϕ. A, B and C describe the hierarchy of up masses; D, E and F that of downs or electrons.

The lepton mass matrix E can easily be made real by using the phase freedom of the six fields – three doublets and three charged singlets. We discuss neutrino masses later. Thus the seven parameters A, B, C, D, E, F, and ϕ in the fermion Yukawa matrices, as well as $\tan\beta$, determine the 13 masses and mixing angles and $\tan\beta$, itself; leading to six predictions for the charged fermions.

We will take as inputs and outputs the following quantities:

inputs; m_e, m_μ, m_τ, m_b, $|V_{cb}|$, m_c, m_u/m_d, $|V_{cd}|$,

outputs; m_d, m_s, m_t, $\sin\beta$, $|\frac{V_{ub}}{V_{cb}}|$, ϕ.

(II) Predictions [6]

(α) Top Mass and $\sin\beta$:

the top mass is given by

$$m_t = \frac{m_c m_b}{m_\tau} \frac{1}{V_{cb}^2} f\ (g_3, g_2, g_1)$$

where f is a known function of just gauge couplings. Plugging in we obtain

$$m_t = 179\ GeV \left(\frac{m_b}{4.15\ GeV}\right)\left(\frac{m_c}{1.22\ GeV}\right)\left(\frac{.053}{V_{cb}}\right)^2\left(\frac{1.46}{\eta_b}\right)\left(\frac{1.84}{\eta_c}\right). \quad (8)$$

The η_i are QCD renormalization factors which are plotted as a function of $\alpha_s(m_Z)$ in [20]. Also, a general expression for m_t is

$$m_t = \frac{v}{\sqrt{2}} \sin\beta\, \lambda_t = 174\ GeV\ \sin\beta\, \lambda_t \quad (9)$$

where λ_t is the top Yukawa coupling. Equations 8 and 9 imply that both $\sin\beta$ and λ_t must be near 1. Since the fixed point of λ_t is near 1 ($\lambda_t^{fp} = 1.09$) and is attractive, it follows that λ_t will be at its fixed point for all practical purposes. This was anticipated earlier. Furthermore, combining the above equations it is clear that $\tan\beta$ is large; this has important consequences for SUSY phenomenology. Detailed numerical calculations have now been done for this scheme [21, 20]. The relations between m_t, $\tan\beta$ and V_{cb} are shown for $\alpha_s(M_Z) = 0.126$ in Figures 2, 3 and 4 taken from [20], which also contains figures for other values of $\alpha_s(M_Z)$. These figures illustrate the level of accuracy which may be achieved in

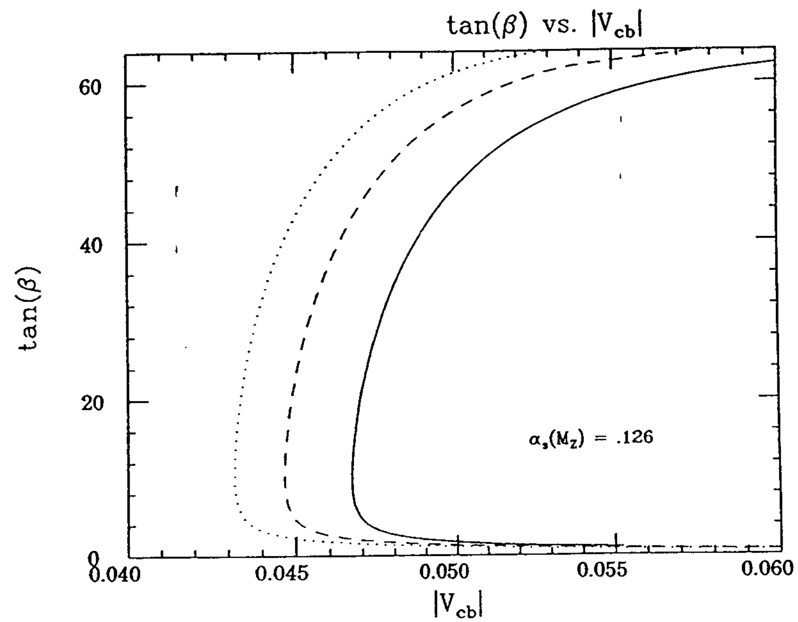

Figure 2: A plot of $\tan\beta$ versus $|V_{cb}|$ for $\alpha_s(m_Z) = .126$. On the solid (dashed) [dotted] curve, the $\bar{MS}$ values of the running quark masses are $m_b(m_b) =$ 4.25 (4.15) [4.086] GeV and $m_c(m_c) = 1.27$ (1.22) [1.186] GeV.

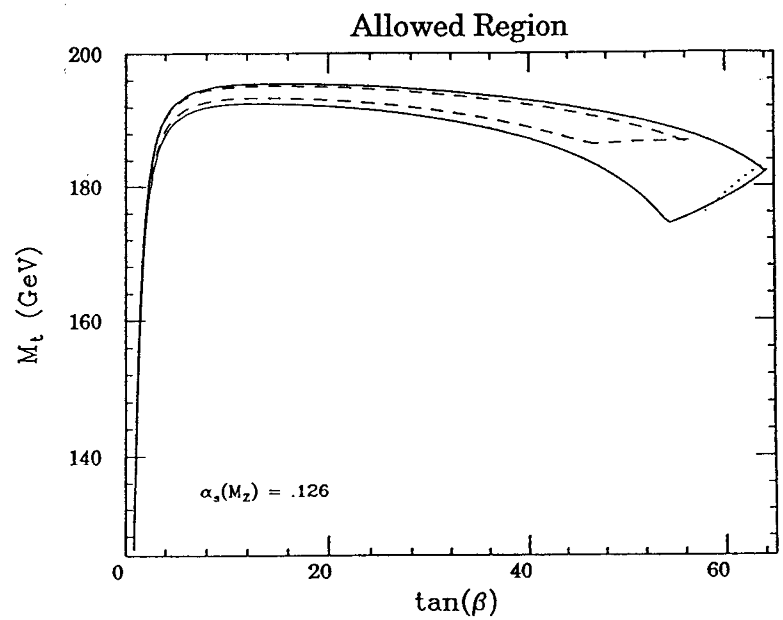

Figure 3: A plot of the top quark's pole mass m_t versus $\tan\beta$ for $\alpha_s(m_Z) = .126$; inside the dashed (solid) curve, $m_b(m_b) = 4.25\pm.1(\pm.164)$ GeV, $m_c(m_c) = 1.27 \pm .05(\pm.082)$ GeV, and $|V_{cb}| \leq .050(\leq .054)$. With these restrictions, the top quark mass is predicted to lie in the range $155.4 < m_t < 195$ $(126 < m_t < 195.5)$, while $\tan\beta$ is restricted to $1.3 < \tan\beta < 56.4$ $(.83 < \tan\beta < 64.3)$. The dotted line gives the prediction for the case $A = D$ with $m_c(m_c) = 1.188$ GeV.

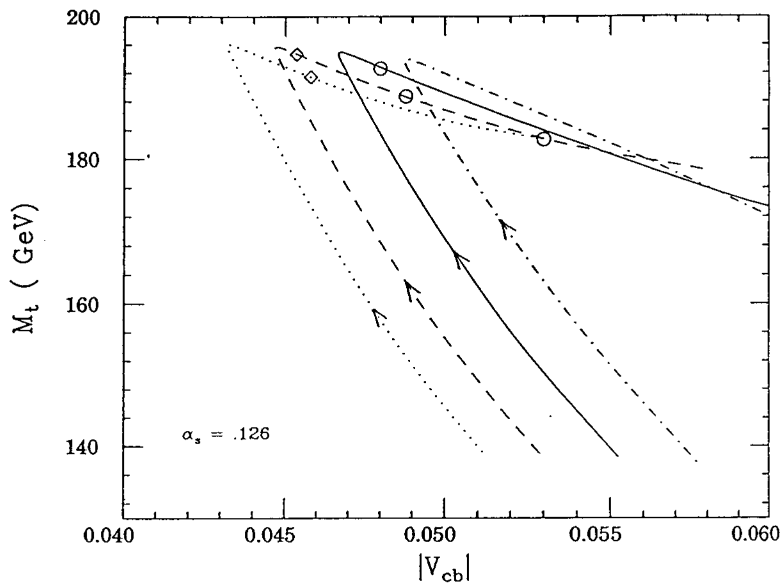

Figure 4: A plot of the pole mass m_t versus $|V_{cb}|$ for $\alpha_s(m_Z) = .126$. The dotted, dashed, and solid curves correspond to those of Figure 2. The additional dotted and dashed curve is for $m_b(m_b) = 4.35$ GeV, and $m_c(m_c) = 1.32$ GeV. On each curve, the arrows indicate the direction of increasing $\tan\beta$ and monotonically decreasing, GUT scale top Yukawa, A. The diamonds (circles) indicate points where $A = 2.5\,(2.0)$.

predictions of this sort. Of particular interest is the fact that an extra prediction can be gained by setting the b and t Yukawas equal at the GUT scale: A = D. In this case m_t is very large and must lie on the dotted line of Figure 3.

(β) d, s and ϕ:

We find $d \simeq 6.2$ MeV, $s \simeq 156$ MeV. These should be compared with the Gasser-Leutwyler values of $d = 8.9 \pm 2.6$ MeV and $s = 175 \pm 58$ MeV [16]. Our s/d is a bit on the large side, but not uncomfortably so [17,18]. These predictions are, of course, just a SUSY version of the predictions of Georgi-Jarlskog. The CP violating phase ϕ is given by

$$\sin\phi = .91 \begin{pmatrix} +.05 \\ -.13 \end{pmatrix} \quad .$$

(γ) CKM matrix, $\frac{V_{ub}}{V_{cb}}$:

The CKM matrix is given by

$$V = \begin{pmatrix} c_1 - s_1 s_2 e^{-i\phi} & s_1 + c_1 s_2 e^{-i\phi} & s_2 s_3 \\ -c_1 s_2 - s_1 e^{-i\phi} & c_1 e^{-i\phi} - s_1 s_2 & s_3 \\ s_1 s_3 & -c_1 s_3 & e^{i\phi} \end{pmatrix}$$

where

$$s_1 = \sin\theta_1 = .196$$

$$s_2 = .05\sqrt{\frac{m_u/m_d}{.6}\,\frac{1.25\,GeV}{m_c}} = \left|\frac{V_{ub}}{V_{cb}}\right| \quad .$$

This value of V_{ub}/V_{cb} was in the low end of the acceptable range when the prediction was first made. However, the CLEO collaboration has recently revised their central value for this ratio down by about a factor of two so that the agreement is now quite good. Also there are large uncertainties in extracting V_{ub}/V_{cb} from data.

To obtain predictions in the neutrino sector several more assumptions must be added to the framework, and these are detailed in [19]. Here we just summarize the results. Our predictions for neutrino masses and mixings are shown in the table. The 3×3 mixing matrix has been approximated by rotations $\theta_{e\mu}, \theta_{\mu\tau}$ and $\theta_{e\tau}$, and we have not shown the effects from CP violation. There are two versions of our scheme, which we label I and II.

In Model I, $\theta_{\mu\tau}$ is sufficiently large that the Fermilab E531 results imply that $m_{\nu_\tau} \leq 2.5$ eV. This means that it is unlikely that planned neutrino oscillation experiments will be able to detect the neutrino masses of this model. Although the neutrinos are all too light to be the dark matter, the value of $\theta_{e\mu}$ does allow a resolution of the Cl, Kamiokande and Gallex solar neutrino experiments by MSW oscillations, at the 90% confidence level. Our value of $\theta_{e\mu}$ implies that, as the error bars on the Ga experiments are decreased, a low number of about 50±10 SNUs will result. To test this region of parameter space in the lab would require a long baseline $\nu_\mu \nu_\tau$ oscillation search with sensitivity to smaller mixing angles than the present proposals.

In Model II, $\theta_{\mu\tau}$ is just beyond the E531 limits. This is very exciting because it means that the upcoming $\nu_\mu \nu_\tau$ oscillation searches at CERN will probe a large range of Δm^2 in this model. In particular, if the ν_τ makes a significant contribution to the dark matter in the universe, then O(50) events will be seen and $\sin^2 2\theta_{\mu\tau}$ will be determined to be within 15% of 3.10^{-3}.

	I	II
$\theta_{e\mu}$	$(6.5 \pm .3)10^{-2}$	$.15 \pm .04$
$\theta_{\mu\tau}$	$.081 \pm .008$	$-.027 \pm .003$
$\theta_{e\tau}$	$(5.7 \pm .6)10^{-4}$	$(1.9 \pm 0.2)10^{-4}$
m_{ν_τ}/m_{ν_μ}	208 ± 42	1870 ± 370
m_{ν_μ}/m_{ν_e}	$(3.1 \pm 1.0)10^3$	38 ± 12
$m_{\nu_\tau max}$	2.5 eV	710 eV

Table

Grand unified theories are only interesting if they are testable. The successful weak mixing angle prediction is the first crucial step, but is not sufficient. Observation of proton decay could yield important information about GUT scale physics, but is unlikely to provide a significant numerical test. If the flavor structure of GUTs is simple enough, there can be very many predictions of quark and lepton masses and mixings. This may be the only real hope for definitive progress on GUTs. In these lectures I have given some of the central ideas and history behind making quark and lepton mass predictions, and have provided an explicit example which is the most predictive known to date. While very successful it has some shortcomings. This is a field in rebirth and this example will soon be replaced with models with more symmetry and more predictions.

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