# FILMPLANE RECONSTRUCTION OF TRAJECTORIES IN A NONUNIFORM MAGNETIC FIELD: THE COMPUTER PROGRAM SYBIL 

DIETRICH E. C. FRIES<br>STANFORD LINEAR ACCELERATOR CENTER<br>STANFORD UNIVERSITY<br>Stanford, California

PREPARED FOR THE U.S. ATOMIC ENERGY COMMISSION UNDER CONTRACT NO. AT(04-3)-515

December 1969

Reproduced in the USA. Available from the Clearinghouse for Federal Scientific and Technical Information, Springfield, Virginia 22151.
Price: Full size copy $\$ 3.00$; microfiche copy $\$ .65$.

## ACKNOWLEDGEMENTS

I gratefully acknowledge the programming assistance I received from Mess. J. Ahern, R. Ching, and M. Lieber.

## TABLE OF CONTENTS

Page
I. Introduction ..... 1
A. Special problems in streamer chamber analysis ..... 1
B. Reconstruction in the film plane ..... 1
II. Mathematical Formulation ..... 4
A. Representation of particle trajectories ..... 4
B. Coordinate systems ..... 6
C. Corrections to the parameters by minimizing of chisquare ..... 9
D. Differential relations between image plane and real space ..... 10
III. Computational Details ..... 14
A. Computation flow ..... 14
B. Starting values for the parameters ..... 14
C. Assignment of point weights and measurement errors ..... 16
D. Bookkeeping procedures and convergence tests ..... 17
E. Track integration and computation of residuals ..... 19
F. Optical distortions and the representation of the magnetic field ..... 20
G. Setting of the parameters ..... 21
IV. Input-Output Routines ..... 25
A. Input of measured data ..... 25
B. Output facilities ..... 28
C. Tape control ..... 34
References ..... 36

## LIST OF FIGURES

Page

1. Cross section of the magnet streamer chamber system, as it was used in a photoproduction experiment (Ref. 1) ..... 2
2. The master coordinate system and the definition of angles. ..... 7
3. Coordinate systems in the three film planes $(u, v)$ and the master system ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) ..... 8
4. An integration step projected into the film plane ..... 13
5. Flow diagram ..... 15
6a. Example of detailed printout for a 5-prong event observed in the streamer chamber ..... 29
6b. Example of detailed printout for a 5-prong event observed in the streamer chamber ..... 30

## I. INTRODUCTION

A. Special Problems in Streamer Chamber Analysis

It is a recent development that a large streamer chamber is being used in experimental high-energy physics. ${ }^{1}$ The photographic recording of particle interactions combined with the possibility to activate the chamber with a fast selective trigger gives this particle detection device a place between bubble chamber and wire spark chambers. Together with magnetic analysis, a streamer chamber offers high resolution in a large solid angle. For the technical details concerning the chamber and the drive system, the reader is referred to Ref. 2. An account of various performance tests is given in Ref. 3.

Owing to inherent properties of the streamer chamber and the way they may be used in an experiment, special consideration had to be given to the problems of data reduction and analysis. In order to recognize this we will first take a brief look at some details and problems of an experiment with a large streamer chamber which has been carried out at SLAC (loc. cit. ${ }^{1}$ ). We studied hadronic interactions of high-energy photons on protons by letting a pencil photon beam collide with gaseous $\mathrm{H}_{2}$, compressed in a plastic tube. The tube was suspended inside of a large streamer chamber ( $220 \times 150 \times 60 \mathrm{~cm}$ ). The streamer chamber was placed in the gap of a 500 -ton, $14-\mathrm{kG}$ magnet with circular pole piece of 200 cm diameter. The upper iron pole piece of the magnet was removed, thus permitting viewing of the chamber with three cameras from the top through the upper coils of the magnet (Fig. 1). Because of the rather open construction of the magnet, the magnetic field can have radial components up to $30 \%$ of $|\mathrm{B}|_{\max }$ within the chamber. The interaction vertex being inside of the target tube was not visible (in with the present status of techniques a useful streamer mode has not been achieved in hydrogen gas).

Faced with these particular problems it became apparent that a new geometrical analysis procedure was needed, which optimizes physical parameters such as particle momenta and angle, including the unobserved vertex in the presence of a nonuniform, essentially arbitrary, magnetic field.

## B. Reconstruction in the Film Plane

A first effort has been to compute in a conventional way spatial track coordinates from film measurements and to use for the least square optimization of the parameters our computer procedure CIRCE, ${ }^{4}$ which, provided the input are spatial coordinates, is well matched to the fitting problem discussed above.


FIG. 1--Cross section of the magnet streamer chamber system, as it was used in a photoproduction experiment (Ref. 1).

It was recognized, however, and it can be shown with simulated events that this method has in principal and for the case of a nonuniform field in particular, some shortcomings, such as the following:
a. Spatially reconstructed points have neither uncorrelated nor gaussian distributed errors.
b. The reconstruction with the help of almost corresponding points becomes rather inaccurate when the projected trajectory (or parts of $i t)$ can not be represented very well by a circle.
c. Usually when corresponding points are being calculated the number of points used in the fit does not correspond to the number of points measured.
d. The choice of an appropriate stereo system for the reconstruction makes the optimal use of a three-camera system difficult.
An alternative approach which is not affected by the disadvantages of the spatial reconstruction method is the least square optimization of the geometrical parameters directly in the three film planes. ${ }^{5}$ We therefore developed a novel computer procedure, with the name SYBIL, which possesses the following main features.
a. The projection of the particle trajectories are optimized with respect to the measurements in three film planes by employing a least square method.
b. The fit is performed simultaneously for all charged tracks originating from a vertex.
c. The optimization renders the vertex coordinates in real space and the inverse momenta and spatial angles of the particles.
d. The fit makes use of all measured points in three film planes independently to the extent that for any track at least two views are measured.
e. No "corresponding" point or spatially reconstructed points are used for the fit. No stereo system or corresponding transformation is needed.
f. The variance-covariance matrix of fitted parameters contains the full correlations between all parameters of all tracks.
g. The magnet field used in SYBIL has to be locally defined and can be arbitrary.

In this report we want to describe in some detail our computer procedure SYBIL which is being used for the analysis of streamer chamber experiments at SLAC.

Much use has been made of concepts and the logic developed in our earlier procedure CIRCE, about which a detailed documentation has been made. We do not wish to be too repetitious and restrict in this report our discussion more or less to the points which are essential to the functions characteristic for the procedure SYBIL. We will therefore refer the reader to a number of details in our earlier report. In particular the SUBROUTINES ESTIM, BWRITE and the SUBROUTINES SOLVE, INVEQ and FORM have been taken over from the procedure CIRCE with very little changes. In fact the main changes are in the COMMON blocks and in the dimension statements.

In many experiments it is essential to continue the geometrical analysis with a kinematical fit using energy momentum conservation to test the hypothesis about the mass assignment of the tracks, or to fit unmeasured parameters. A corresponding least square fitting procedure, named 'TEUTA', ${ }^{6}$ matched to the SYBIL output information (including the complete covariance matrix) was developed at SLAC and has been used together with SYBIL as the standard analysis program for the analysis of streamer chamber pictures. ${ }^{1}$

## II. MATHEMATICAL FORMULATION

## A. Representation of Particle Trajectories

Our method to optimize geometrical parameters makes essential use of the fact that a trajectory of a charged particle in a magnetic field is completely determined by the initial value solution of the set of differential equations

$$
\begin{equation*}
\frac{\mathrm{d} \overrightarrow{\mathrm{u}}}{\mathrm{ds}}=\frac{\mathrm{c}}{\mathrm{p}} \overrightarrow{\mathrm{u}} \times \overrightarrow{\mathrm{B}} \tag{II.1}
\end{equation*}
$$

where the following definitions are used

$$
\begin{align*}
& \overrightarrow{\mathrm{u}}=\left(\frac{\mathrm{dx}}{\mathrm{ds}}, \frac{\mathrm{dy}}{\mathrm{ds}}, \frac{\mathrm{dz}}{\mathrm{ds}}\right) \\
& \mathrm{s}=\text { arclength of the trajectory with }  \tag{II.2}\\
& d \mathrm{~s}=\left\{d \mathrm{x}^{2}+\mathrm{dy}{ }^{2}+d z^{2}\right\}^{1 / 2} \text { (meter) }
\end{align*}
$$

and

$$
B=\left(B_{x}(x, y, z), B_{y}(x, y, z), B_{z}(x, y, z)\right)
$$

representing the magnetic field in kilogauss
$\mathrm{p}=\mathrm{p}(\mathrm{s})$ momentum of the particle in $\mathrm{GeV} / \mathrm{c}$
$\mathrm{C}=2.99791 \times 10^{-2}$

In the absence of an electric field (II.1) describes the path of a point charge for arbitrary velocities, all quantities being defined in the laboratory system. The initial values for the differential equation (II. 1) are related to the parameters which we have set out to optimize.

Considering now the simultaneous optimization of $\mathrm{N}_{\mathrm{R}}$ tracks emerging from the same (unmeasured) vertex we have $3 \mathrm{~N}_{R}+3$ parameters which represent our multiprong event.

Introducing a vector notation they are

$$
\vec{\alpha}=\left(k_{1}, \lambda_{1}, \phi_{1}, \mathrm{k}_{2}, \lambda_{2}, \phi_{2}, \ldots \mathrm{k}_{\mathrm{N}_{\mathrm{R}}}, \lambda_{\mathrm{N}_{\mathrm{R}}}, \phi_{\mathrm{N}_{\mathrm{R}}}, \mathrm{x}_{0}, \mathrm{y}_{0}, \mathrm{z}_{0}\right)
$$

they are related to the initial values of the set of differential equation (II.1) by

$$
\begin{aligned}
& \left(\frac{d x}{d s}\right)_{s=0}=\cos \lambda \cos \phi \\
& \left(\frac{d y}{d s}\right)_{s=0}=\cos \lambda \sin \phi \\
& \left(\frac{d z}{d s}\right)_{s=0}=\sin \lambda
\end{aligned}
$$

$\lambda$ is the dip angle and $\phi$ is the azimuthal angle of the trajectory at the vertex. (For the exact definition, see Section II. B.) $k=1 / \mathrm{p}$. We use $k=1 / \mathrm{p}$ instead of $p$ as a fit parameter, assuming that the dependence on $k$ is to a lesser degree nonlinear than the dependence on p . This is obvious when the trajectory is a helix. We follow in that the example of most major geometry analysis procedures.

In the present version of SYBIL we regard $p=$ const although one could specify $p=p(s)$. The numerical solution of Eq. (II.1) have been discussed in greater detail in Ref. 4.

## B. Coordinate Systems

We envisage a track chamber which is viewed by three cameras. The system is specified in the following way: The camera axes are parallel to each other and perpendicular to the ( $\mathrm{x}, \mathrm{y}$ ) plane of the main coordinate system in space (master system). The focal length, height and position of each camera can be choosen arbitrarily. (A system offering large stereo angles is of course advantageous in view of accuracy.)

Distinguishing coordinate systems in the image plane (film plane) and in real space we have
a. The master coordinate system ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ) (see Fig. 2). $\mathrm{x}, \mathrm{y}$ defines a (horizontal) plane perpendicular to the main component of the magnetic field. The $x$ axis is a horizontal symmetry axis through the magnet and usually parallel to and in direction of the incident beam. The $z$ axis points upwards. The right-handed master system is located such that the center of the gap of the magnet (intersection of vertical and horizontal symmetry axis of the magnet) has coordinates ( $\mathrm{x}=1.00, \mathrm{y}=0, \mathrm{z}=0$ meter).
b. The coordinate system in the film plane ( $u, v$ ). Each film plane has its own coordinate system. The origin ( $u=0, v=0$ ) is defined at the intersection of the optical axis of the camera (through the nodal and focal point) with the film plane (see Fig. 3).

The relation between film coordinates and the master coordinate system is established by introducing the central projection which transforms spatial points into points in the image plane (pin hole transformation):

$$
\begin{align*}
& u^{(k)}=\frac{d_{k}}{D_{k}-z}\left(x-L_{x}^{(k)}\right) \\
& v^{(k)}=\frac{d_{k}}{D_{k}-z}\left(y-L_{y}^{(k)}\right) \tag{II.3}
\end{align*}
$$

where we define

$$
\mathrm{k}=1,2,3 \text { referring to different views. }
$$

$\mathrm{d}_{\mathrm{k}}=$ perpendicular distance between nodal point and film plane (here approximately the film lense focal length)


FIG. 2--The master coordinate system and the definition of angles.


THREE CAMERA SYSTEM

SPECIAL EXAMPLE OF CAMERA POSITIONS $L_{a}=L_{b}$

$$
L_{1}=-L_{2}
$$

$$
L_{c}=L_{3}=0
$$

I, II, III INDICATES POSITION OF CAMERA I'NODAL POINTS IN X Y PLANE

$$
\overline{629-1-c}
$$

$$
\begin{aligned}
& \left(u_{i}, v_{i}\right) \text { FILM PLANE COORDINATE SYSTEM } \\
& X, Y, Z=\text { COORDINATE SYSTEM IN REAL } \\
& \text { SPACE (RIGHT HANDED) } \\
& \begin{array}{ll}
u_{i}=\frac{d_{i}}{D_{i}-Z}\left(X-L_{\alpha}\right) & i=1,2,3 \\
\alpha=a, b, c
\end{array} \\
& \begin{array}{rlrl}
v_{i}=\frac{d_{i}}{D_{i}-z}\left(Y-L_{i}\right) & d_{i} & =\text { FOCAL LENGTH } \\
D_{i} & =z-\text { COORD OF } \\
& \text { NODAL POINT }
\end{array}
\end{aligned}
$$

FIG. 3--Coordinate systems in the three film planes ( $u, v$ ) and the master system ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ).

$$
\mathrm{D}_{\mathrm{k}}=\mathrm{z} \text { coordinate of the camera nodal point in the }
$$ master system

$$
\begin{aligned}
L_{x}^{(k)}= & x \text { coordinate of the camera nodal point in the } \\
& \text { master system }
\end{aligned}
$$

$L_{y}^{(\mathrm{k})}=\mathrm{y}$ coordinate of the camera nodal point in the master system

The transformation (II. 3) determines the direction of $u$ and $v$ as the opposite direction of $x, y$, respectively. The convenience gained by this definition is the fact that a particle moving in positive $x$ direction will appear to move in positive u direction.

The relations (II.3) actually refer to ideal film coordinates. The specification of parallel camera axis which are perpendicular to the xy plane may not always be met exactly. When the tilt of the camera axis is determined one can correct $u$ and $v$ for this 'keystone' effect. Furthermore the camera objectives have considerable distortions at large angles because of the large aperture of the camera objective which is required for faint streamer chamber tracks. Again the measured coordinates have to be corrected before (II. 3) can be applied (see Section III. F).

## C. Corrections to the Parameters by Minimizing of Chisquare

The corrections to the parameters $\vec{\alpha}$ are being computed by minimizing the chisquare formed from the measured film coordinates ( $u, v$ ). The matrix formulation of $X^{2}$ is the following

$$
x^{2}=\overrightarrow{\mathrm{d}}^{\mathrm{T}} \mathrm{~W} \overrightarrow{\mathrm{~d}}
$$

T stands for transposed
The measured quantities $\vec{d}$ are defined as the perpendicular distance of a measured point in the film plane and the projection of the computed trajectory

$$
\begin{equation*}
|\overrightarrow{\mathrm{d}}|=\left\{\left(\mathrm{u}_{\mathrm{m}}-\mathrm{u}_{\mathrm{c}}\right)^{2}+\left(\mathrm{v}_{\mathrm{m}}-\mathrm{v}_{\mathrm{c}}\right)^{2}\right\}^{1 / 2} \tag{II.4}
\end{equation*}
$$

$u_{m}, v_{m}$ are measured point coordinates in the film plane. $u_{c}, v_{c}$ are the computed corresponding coordinates of the trajectory (in the film plane).

The sign of the square root is determined from the cross product of the local tangential vector of the trajectory and the local distance vector $\vec{d}$ (see also Section II.D). The vectors $d$ are functions of the measured points in the film plane of
three views. $W$ is a weight matrix which should be proportional to the inverse of the covariance matrix of the measurements. To the extent that the measurements of different points are independent we assume $W$ to be diagonal and supply usually a constant weight for all measured points (see also Section III). A first order Taylor expansion of the chisquare with respect to the parameters $\vec{\alpha}$ renders a relation linear in $\Delta \vec{\alpha}$

$$
\begin{aligned}
\chi^{2} & =\left(\overrightarrow{\mathrm{d}}_{0}+\mathrm{A} \Delta \vec{\alpha}\right)^{\mathrm{T}} \mathrm{~W}\left(\overrightarrow{\mathrm{~d}}_{0}+\mathrm{A} \Delta \vec{\alpha}\right) \\
\mathrm{A} & =\left(\frac{\partial \overrightarrow{\mathrm{d}}}{\partial \vec{\alpha}}\right)_{\mathrm{d}=\mathrm{d}_{0}}
\end{aligned}
$$

$\Delta \vec{\alpha}$, can be considered as the correction to the parameters $\vec{\alpha}_{0}$. The index ' 0 ' labels the first estimates of the parameters (starting values) or after each iteration the current value of $\vec{\alpha}$.

The minimum condition $\partial X^{2} / \partial \vec{\alpha}=0$ renders a system of equations

$$
\mathrm{A}^{\mathrm{T}} \mathrm{~W} \mathrm{~A} \Delta \vec{\alpha}=-\mathrm{A}^{\mathrm{T}} \mathrm{~W} \overrightarrow{\mathrm{~d}}
$$

from which $\Delta \vec{\alpha}$ is evaluated (SUBROUTINE SOLVE) by a GAUSS-JORDAN reduction. By correcting $\vec{\alpha}_{0}$ by $\Delta \vec{\alpha}$ one obtains new parameters $\vec{\alpha}$. After each iteration the convergence (or nonconvergence) of the procedure is tested (see Section III.D).

Necessary conditions for this procedure to converge and to yield an unbiased estimate of the parameters is the assumption that the perpendicular distance d of a measured point from the trajectory has approximately a normal distribution and that the parameter space is sufficiently smooth in the neighborhood of the final fit parameters $\vec{\alpha}$; that is to say that the partial derivatives $\partial \vec{d} / \partial \vec{\alpha}$ guide the iterative process in the direction to the $\chi^{2}$ minimum.

## D. Differential Relations Between Image Plane and Real Space

Since the optimization of the parameters proceeds by defining residuals in the film plane whereas the actual numerical integration of the track is carried out in the three-dimensional space we have to derive appropriate relation, which connect distances and angles of both space.

Rewriting the transformation (II.3) as

$$
\begin{align*}
& \xi=\lambda\left(\mathrm{x}-\mathrm{L}_{\mathrm{x}}\right)  \tag{II.5}\\
& \eta=\lambda\left(\mathrm{y}-\mathrm{L}_{\mathrm{y}}\right) \\
&-10-
\end{align*}
$$

with

$$
\lambda=\frac{\mathrm{d}_{\mathrm{f}}}{\mathrm{D}-\mathrm{z}} \quad \vec{\xi}=(\xi, \eta)
$$

We define a unit tangent vector in the image plane

$$
\begin{equation*}
\vec{\omega}=\frac{\mathrm{d} \vec{\xi}}{\mathrm{~d} \sigma}=\left(\frac{\mathrm{d} \xi}{\mathrm{~d} \sigma}, \frac{\mathrm{~d} \eta}{\mathrm{~d} \sigma}\right) \tag{II.6}
\end{equation*}
$$

where $\mathrm{d} \sigma$ is an arclength element of the projection of the trajectory into the image plane.

From the total differential in the $(\xi, \eta)$ space one obtains

$$
\begin{equation*}
\vec{\omega}=\frac{\mathrm{d} \vec{\xi}}{\mathrm{~d} \sigma}=\frac{\partial \vec{\xi}}{\partial \mathrm{x}} \frac{\mathrm{dx}}{\mathrm{ds}} \frac{\mathrm{ds}}{\mathrm{~d} \sigma}+\frac{\partial \vec{\xi}}{\partial y} \frac{\mathrm{dy}}{\mathrm{ds}} \frac{\mathrm{ds}}{\mathrm{~d} \sigma}+\frac{\partial \vec{\xi}}{\partial z} \frac{\mathrm{dz}}{\mathrm{ds}} \frac{\mathrm{ds}}{\mathrm{~d} \sigma} \tag{II.7}
\end{equation*}
$$

we insert into (II. 7)

$$
\begin{aligned}
& \frac{\partial \xi}{\partial z}=\frac{x-L_{x}}{D-z} \lambda \\
& \frac{\partial \eta}{\partial z}=\frac{y-L_{y}}{D-z} \lambda
\end{aligned}
$$

and recognize the components of the unit tangent vector of the trajectory in threedimensional space present in (II. 7):

$$
\vec{u}=\left(u_{x}, u_{y}, u_{z}\right)=\left(\frac{d x}{d s}, \frac{d y}{d s}, \frac{d z}{d s}\right)
$$

Thus we can define a vector

$$
\begin{equation*}
\overrightarrow{\mathrm{Q}}=\left(\mathrm{Q}_{\xi}, \mathrm{Q}_{\eta}\right) \tag{II.8}
\end{equation*}
$$

with

$$
\begin{aligned}
& Q_{\xi}=u_{x}(D-z)+u_{z}\left(x-L_{x}\right)=\lambda^{-1}\left(u_{x} d_{f}+u_{z} \xi\right) \\
& Q_{\eta}=u_{y}(D-z)+u_{z}\left(y-L_{y}\right)=\lambda^{-1}\left(u_{y} d_{f}+u_{z} \eta\right)
\end{aligned}
$$

which is related to $\vec{\omega}$ (II. 6) by

$$
\begin{equation*}
\vec{\omega}=\frac{\lambda^{2}}{d_{f}} \vec{Q} \tag{II.9}
\end{equation*}
$$

Since $|\vec{\omega}|=1$ (per definition) we obtain the equation which relates the arclength element in the image plane to the arclength element in the three-dimensional space

$$
\begin{equation*}
\left|\frac{\mathrm{d} \vec{\xi}}{\mathrm{~d} \sigma}\right|=1=\left|\frac{\mathrm{ds}}{\mathrm{~d} \sigma}\right| \frac{\lambda^{2}}{\mathrm{~d}_{\mathrm{f}}}\left\{\mathrm{Q}_{\xi}^{2}+\mathrm{Q}_{\eta}^{2}\right\}^{1 / 2} \tag{II.10}
\end{equation*}
$$

or

$$
\begin{equation*}
|\mathrm{d}|=|\mathrm{d} \sigma| \frac{\mathrm{d}_{\mathrm{f}}}{\lambda^{2} \sqrt{\mathrm{Q}_{\xi}^{2}+\mathrm{Q}_{\eta}^{2}}} \tag{II.11}
\end{equation*}
$$

The relation connecting directional cosines in both spaces is established by (II. 3).
We can now express the perpendicular distance $d$ of a measured film point $\vec{\xi}$ from the projected trajectory (we refer to it also as the closest distance point). Consider the particular integration step which passes the measured film point $\vec{\xi}$ in a closest distance d. The situation is illustrated in Fig. 4.

The distance between the measured point $\vec{\xi}^{*}$ and the point $\vec{\xi}^{(2)}$ on the projected trajectory is

$$
\begin{equation*}
\vec{\delta}=\vec{\xi}^{*}-\vec{\xi}^{(2)} \tag{II.12}
\end{equation*}
$$

$\xi^{(2)}$ is the endpoint of this particular integration step which has an arclength $\Delta \sigma$.

The location of the closest distance point $\vec{\xi}$ can now approximately be calculated from the scalar product

$$
\begin{equation*}
\Delta \sigma^{*}=\vec{\delta} \cdot \vec{\omega}^{(2)} \tag{II.13}
\end{equation*}
$$

( $\vec{\omega}$ being a unit vector) which gives the distance $\Delta \sigma^{*}$ between $\vec{\xi}^{(2)}$ and $\vec{\xi}$.
Equation (II. 11) can be used to compute the corresponding distance along the trajectory in real space

$$
\begin{equation*}
\Delta \mathrm{s}^{*}=\frac{\mathrm{d}_{\mathrm{f}}}{\lambda^{2}} \frac{\vec{\delta} \cdot \vec{\omega}^{(2)}}{\sqrt{\mathrm{Q}_{\xi}^{2}+\mathrm{Q}_{\eta}^{2}}} \tag{II.14}
\end{equation*}
$$

The numerical integration of (II. 1) which is carried out in the SUBROUTINE PRTRACK provides all quantities necessary to compute the right-hand side of (II. 13). The distance $\Delta s^{*}$ is then used as a corrective integration step taken from $\vec{\xi}^{(2)}$ in reverse direction and will render the coordinates of $\vec{\xi}$.


FIG. 4--An integration step projected into the film plane.

Clearly one can think of using a second order approximation of $\Delta s$ which will take in account the local curvature of the trajectory; that is to say one can utilize second order derivatives which are as well available from the algorithm of the numerical integration. In our experience the second order correction did not change noticeably our results when the integration steplength has been taken sufficiently small.

## III. COMPUTATIONAL DETAILS

## A. Computation Flow

Before discussing the function and details of the more important subroutines we survey the diagrams of the general computational flow (Fig. 5). The input and output facilities tape positioning and the control over the amount of data to be processed is prepared in SETLOG, which is called from MAIN. The parameters not to be changed during the processing are being set in CONSET, where also optical constants, the fiducial coordinates and the coefficients of the magnetic field representation are stored.

The measured film coordinates are read into buffers by the SUBROUTINE TAPIN; always an entire event. The format and tape addresses are listed in Section IV. For the transformation of the data from the digitizer system, the routines LEASTQ, ROTA and UNDIST are being used.

Starting values for the parameters are obtained by reconstructing a few spatial points of each track. Spatial track coordinates are computed in the SUBROUTINES SELECT and TWOVIE; the starting values are computed in ESTIM.

The SUBROUTINE ASSWET takes care of the assignment of weights to each measured point.

In FIT the input information for the least square algorithm is assembled and the fit procedure is initiated and controlled. SUBROUTINES used to obtain partial derivatives and residuals are FUNC and PRTRACK the correction to the parameters are computed in the least square package SOLVE.

The output information on tape is written in MAIN. The standard printed output is provided by the SUBROUTINE WSFORM.

## B. Starting Values for the Parameters

Starting values for the parameters $\alpha$ are being computed in ESTIM by using spatial track coordinates and assuming that the trajectories near the vertex can be approximated by circles. The SUBROUTINE ESTIM has been described in

## FLOW DIAGRAM



FIG. 5--Flow diagram.
some detail in a previous report. ${ }^{4}$ Spatial track coordinates, reconstructed from two views, are calculated in TWOVIE. Since the spatially reconstructed points are only used for the first estimate of the parameters we reconstruct only the first few points of each track (IPH). In TWOVIE the standard method of computing almost corresponding points is applied. The stereo system is chosen such that the angle between a second to the circle (track projected into xy plane) and the stereo axis is a maximum. The same criterion is used to select the principle view. A SUBROUTINE SELECT renders for a given sequence of points the selected principle and secondary view (IPV, ISV) for the spatial reconstruction. TWOVIE returns the spatial reconstructed spatial points in arrays XSTORE ( $\mathrm{J}, \mathrm{I}$ ), YSTORE ( $\mathrm{J}, \mathrm{I}$ ), ZSTORE ( $\mathrm{J}, \mathrm{I}$ ). The number of reconstructed points can be found in $\mathrm{NO}(J)$.

In general it is not necessary to reconstruct spatial track points, since starting values can be also computed using only the measured film points in two views. (The relations given in Section II.D provide a starting point to derive the necessary formulae.)

Since in many cases certain parameters vary little, the starting values can be supplied as fixed values. Vertex coordinates are one example.

## C. Assignment of Point Weights and Measurement Errors

In the SUBROUTINE ASSWET we assign to each measured point in the film plane a weight WPOINT (I, J, L) where L refers to the view, J refers to the track number and I refers to the sequence number of the measured points of the track J in view L.

The standard choice for the weight is the inverse squared measuring error which we assumed to be constant. The constant set error $\operatorname{SGR}(\mathcal{J})$ is defined in CONSET as the square root of the measuring error in the two coordinate directions: SGU and SGV. However ASSWET provides the facility to alter any point weight if the analysis of the measured data requires it. In particular the following procedures are provided:

Firstly, a test procedure, active for tracks with momenta of less than a number ZZA10 (usually 1 GeV ), eliminates track points in any view when the sagitta of the projected track exceeds a number ZZA11. In fact the number ZZA11 is replaced by a linear momentum dependence which insures that the test becomes the more restrictive the smaller the momentum is. This is done
because for large sagittas the error computed for the fitted parameters are smaller than the systematic uncertainties of the parameters. The point elimination is discontinued when a track has only four measured points left (in a view). If the sagitta is still too large all point weights are reduced.

Secondly, we decrease the weight of all film points which - in a particular view - correspond to an optical projection angle of more than 20 degrees, since our correction functions for optical distortions are unreliable at large angles (due to the absence of fiducials there).

It should be pointed out here that the introduction of weights or rejection of points will alter in a most natural way the covariance matrix of the fitted parameters (which has to be used for kinematical optimization) without further introduction of scale factors and floors. The signal array INOTE records changes of weights and elimination of points.

## D. Bookkeeping Procedures and Convergence Tests

In the SUBROUTINE FIT essentially four different takks are being pursued. First, by looping sequentially through all measured points of all tracks and in each available view a general bookkeeping is established. The control of the sequence of points and the order of tracks serves to collect the necessary residuals and partial derivatives for the least square algorithm. The actual computation of the residuals and the partial derivatives is organized in the SUBROUTINE FUNC which is called for each point of all tracks from FIT. The number of points per track and view are stored in NV1(J), NV2(J), NV3(J) where J refers to the track number. Bookkeeping with regard to the iterations and to the corrections of the parameters is also maintained in FIT.

A second function performed in FIT is the controlling of the partial chisquare which is defined as that part of the chisquare sum which refers to one track projection in one view. When during iterations the convergence of the overall fit is slow because one of the partial chisquare is not decreasing at the same rate as the others, the following weighting procedure will be activated.

After a chosen number of iteration (IZZ8) we compute the partial chisquare for the projections of all tracks in each view. The chisquares are normalized to the number of measured points. The identifying names of the chisquares for the three views are HE LP1(J), HELP2(J), HELP3(J), where J refers to the track number. If one of the partial chisquare exceeds a number ZPE, the point
weight of this track in the particular view will automatically decrease by a factor, equal to the partial chisquare. This weighting will be repeated in each following iteration in case the test ZPE requires it.

The effect of this weighting is a speed up of the convergence and an enlargement of the errors of the corresponding parameters. The activation of this weighting is recorded through the signal words INOTE.

The third function performed in FIT is the test of the convergence of the iterations and the correcting of the parameters. As we have discussed in Ref. 1 possible convergence criteria are the following:
a. $\Delta \sqrt{\bar{X}_{\mathrm{n}}^{2}}$ is smaller than a number (such as 0.1 ) where $\bar{X}_{\mathrm{n}}^{2}$ is the normalized chisquare of the $n^{\text {th }}$ iteration. $\bar{X}_{\mathrm{n}}^{2}=\left(x^{2} / \mathrm{n}_{\mathrm{f}}\right) \mathrm{n}^{\text {th }}$ iteration and $\Delta$ denotes the difference of $\sqrt{\bar{X}^{2}}$ from the iterations n and $\mathrm{n}+1$.
b. $\Delta \vec{\alpha}^{T}\left(\mathrm{~A}^{\mathrm{T}} \mathrm{WA}\right) \Delta \vec{\alpha} \leq 1$. This relation tests whether the corrections are already within the bounds of the error ellipsoid of the parameters $\alpha$. It is assumed then that $W$, the weight matrix of the measurements is the inverse of the covariance matrix of the measurements.
c. $\sqrt{\bar{X}_{\mathrm{n}}^{2}}$ is smaller than a number (such as 0.9 ). One could use this condition when the convergence statement must not be very exact (since one wants to continue with a further kinematical optimization) or when one wants to save computer time.
The convergence conditions are tested in the order whether anyone of them is satisfied. Thus one of the following decisions will be made

1. The convergence is satisfactory; the fit will be terminated and output procedures will be activated (IFAIL=0).
2. The convergence is not yet satisfactory and a further iteration is started.
3. The case appears to be hopeless. The fit is terminated as a 'failure ${ }^{9}$ (IFAIL=2).
Finally in FIT the corrections $\Delta \vec{\alpha}$ are being added to the current parameters $\vec{\alpha}$. The corrections are returned from SOLVE to FIT in the first column of the matrix $A(I, L)$ in exactly the same order in which the partials $\partial d / \partial \alpha_{i}$ enter $A$ in FUNC.

## E. Track Integration and Computation of Residuals

The SUBROUTINE PRTRAC being called for the initial values of a given track computes the numerical solution of the Eq. (II.1) in real space. The numerical method employed is a third order RUNGE KUTTA method. The explicit mathematical formulation for the system of differential equations (II.1) has been given in our previous report. 4 The initial parameters K(J), LAMBDA(J), and PHI(J) are transmitted through the calling sequence of PRTRACK. The COMMON blocks contain the vertex parameter XIN, YIN, ZIN and the arrays of point measurements transformed into the 'ideal film plane ${ }^{\mathrm{p}}$ (see Section II. B) UR(I), VR(I), US(I), $\mathrm{VS}(\mathrm{I}), \mathrm{UT}(\mathrm{I}), \mathrm{VT}(\mathrm{I})$ referring to the u and v measurements in three different film views.

Starting from the vertex and progressing by numerical integration steps we project each point on the trajectory into the three planes. In order to compute the closest distance $\mathrm{d}_{\mathrm{m}}$ in the film planes between the projected trajectory and the measured point $\vec{\xi}_{\mathrm{m}}^{*}$ we have to test whether $\Delta \sigma$ in Eq. (II. 13) is smaller or larger than zero. This involves the evaluation of the relations discussed in Section II.D, especially the computation of (II.9) and (II.12). Clearly (II.13) has to be computed in each view separately. The case $\Delta \sigma^{(k)}<0$, in any particular view k , signals that the point of closest distance in that view was overstepped by the integration procedure. A corrective integration step $\Delta s^{(k)}$ will be computed from (II. 14) which reaches the approximate point of closest distance in view k . The perpendicular distance $\mathrm{d}_{\mathrm{m}}^{(\mathrm{k})}$ in view k is then obtained straight forwardly.

The $\mathrm{d}_{\mathrm{m}}^{(\mathrm{k})}$ are stored in the array $\operatorname{DCAL}(\mathrm{I})$. The measured point $\vec{\xi}_{\mathrm{m}}$ in view k will be updated, hence the next test of $\Delta \sigma^{(\mathrm{k})}$ will pertain to the next measured point $\vec{\xi}_{\mathrm{m}+1}$ in the sequence of points measured in view k .

The test on $\Delta \sigma$ is applied in all views subsequentially. The updating procedure ensures that all measured points will be considered independently and no point in the sequence is omitted. The computed distances $d$ in the three different views are stored in the same array DCAL ordered according to views. They have been given a sign which is equal to the sign of the vector product $\vec{\omega} \times \vec{\delta}$ (see Fig. 4). The d's are therefore oriented distances in a coordinate system which follows the projected trajectory.

In each view the exact number of points specified by NV1(J), NV2(J), NV3(J) is expected. If all the points have been found the control returns to the SUBROUTINE FUNC.

All necessary information is returned to FUNC by LABEL COMMON, including the directional cosines $\vec{\omega}$ of the projected trajectories, the spatial end points of the tracks and the arclength.

In order to stop the integration going astray because of very wrong initial conditions an IFAIL flag is set when the number of integration steps exceeds a number such as 1000 . This can happen when a trajectory starts to spiral, due to a very small momentum, with very small integration steps, or when a trajectory propagates into an area where the magnetic field is not defined. The encounter of physical boundaries is checked in the SUBROUTINE FUNC.

For the choice of the stepsize $\Delta \mathrm{s}$ the same scaling rules have been used as described in Ref. 4, which is simply a constant stepsize for momenta above 1 Bev and a linearly reduced stepsize for momenta below 1 BeV . If the tracks traverse areas with very small magnetic field then one has to use a more economical scaling.
F. Optical Distortions and the Representation of the Magnetic Field

The film coordinates used in Eq. (II.3) are coordinates of the "ideal" film plane. The real coordinates have to be corrected for optical distortions. For the present choice of camera lenses (ZEISS PLANAR F-number 2.0) we used distortion functions of the form

$$
\begin{aligned}
& u_{\text {ideal }}=u_{\text {real }}+\beta \cos \gamma \sin \alpha_{0} \\
& v_{\text {ideal }}=v_{\text {real }}+\beta \sin \gamma \sin \alpha_{0}
\end{aligned}
$$

with

$$
\begin{aligned}
& \tan \theta=\mathrm{d} /\left(\mathrm{u}_{\text {real }}^{2}+\mathrm{v}_{\text {real }}^{2}\right)^{1 / 2} \\
& \tan \gamma=\mathrm{v}_{\text {real }} / \mathrm{u}_{\text {real }}
\end{aligned}
$$

d distance of camera nodel point to the film plane
$\beta \approx-10^{-2}$
$\alpha \approx 2.85-3$
The corrections to the film coordinates are computed in the SUBROUTINE UNDIST.
The main component of magnetic field $B_{Z}(R, Z)$ is computed for coordinates $Z$ and $R$ by the function $F L D B Z$. Similarly the radial component $B_{R}(R, Z)$ is computed by the function $F L D B R, R$ being the radial coordinate measured in the $x y$ plane from the center of the magnet. The coefficients used for the functions representing the magnetic field are stored in CONSET.

## G. Setting of the Parameters

A special SUBROUTINE CONSET is provided in which all fixed parameters, tests and switches for the entire procedure are collected. CONSET being called only once before starting the actual analysis of events sets switches which control input-output facilities and logical and procedural details of the analysis. In particular the constants entering the various tests are being set here.

Most of the constants are commented in order to make the list of CONSET selfexplanatory. We list here the more important ones of the constants. Fixed point words IZZ1, IZZ2, .... are mostly used to activate the print of certain quantities.

IZZ $1=0 \quad$ standard short form of printout
IZZ1 = 1 extended printout: measured fiducial coordinates, transformation coefficients, the differences between transformed measured fiducials and the film projections of the real fiducials, the reconstructed spatial track coordinates, the starting values for the parameters, chisquare and corrections to the parameters after each iteration, weights applied to views and tracks, final fit of the parameters, errors and correlations of the parameters.
$\operatorname{IZZ} 2=2 \quad$ prints $\quad d, \frac{\partial d}{\partial k}, \frac{\partial d}{\partial \lambda}, \frac{\partial d}{\partial \phi}, \frac{\partial d}{\partial x_{0}}, \frac{\partial d}{\partial y_{0}}, \frac{\partial d}{\partial z_{0}}$ for all tracks and views, after each iteration (FUNC).

IZZ2 $=3$ prints track lengths, spatial endpoints of the tracks number of measured points in each view for all tracks

IZZ2 $=4 \quad$ prints projected and spatial integration stepsize DSP, DSQ, DSR, DS spatial track points and directional cosines XN , YN, ZN, UX, UY, UZ
all from PRTRAC
IZZ3 maximum number of iterations for a successful fit.

IZZ4 number of iterations for which new partial derivatives $\frac{\partial d}{\partial \vec{\alpha}}$ are being computed.
IZZ8 number of iterations after which for the first time the partial chisquare is being tested for the application of weights

Floating point words ZZA1, ZZA2 ... are used mostly for setting parameters for various tests. ZZA1 and ZZA2 fix the relation between particle momentum and the stepsize in the integration routine PRTRAC.

ZZA3 maximum number of integration steps per track (in PRTRAC)
ZZA4 absolute limit of starting value for XIN
ZZA5 absolute limit of starting value for YIN
ZZA6 absolute limit of starting value for ZIN
ZZA8 sets the permissible absolute difference between the measured and transformed fiducials and the projected image of the real fiducials

ZZA9 maximum $\sqrt{X^{2}}$ for a fit considered successful

ZZA10 maximum track momentum ( $\mathrm{GeV} / \mathrm{c}$ ) for which the sagitta test is applied
ZZA11 maximum accepted sagitta (meter) for track momentum ZZA10

ZZA12 scales the set error of all points of track J for which a flag SGNL (IV,J) was set by the measurer

ZZA13

ZZA14
CPL

CPM $=(\mathrm{DFI} \times \tan (\theta))^{2}$
defines a cone $\theta$ outside of which the point weight is changed (ZZA14)
point weight when optical distortions present tests whether $\Delta \sqrt{\bar{\chi}^{2}}$ (difference between consecutive iterations) is sufficiently small for a successful fit tests whether $\sqrt{\bar{X}^{2}}$ is sufficiently small for a successful fit

In CONSET are furthermore stored and listed the camera parameters which consists of the $x, y, z$ coordinates of the camera nodal points, the distance image plane and nodal point and coefficients pertaining to distortions.

The identifying names are
LA, L1, DZ1, DF1, FAC1 for camera I
LB, L2, DZ2, DF2, FAC2 for camera II
LC, L3, DZ3, DF3, FAC3 for camera III
The coefficients for the representation of the magnetic field are stored in arrays CMC(66), D(66).

The measurement error $\sigma_{u}$ and $\sigma_{v}$ for measured film coordinates are identified by SGU and SGV. The set error actually used for the weights is SGR which is equal to $\sqrt{\sigma_{u}^{2}+\sigma_{v}^{2}}$.

A list for fail messages is provided together with a list of signals coded in the 110 fixed point words of INOTE.

IFAIL LIST SYBIL

| IFAIL $=0$ | Accepted fit |
| :---: | :---: |
| IFAIL $=1$ | Event cannot be fitted because of insufficient number of points (<3) or views (<2) measured for any track. |
| IFAIL $=2$ | $\bar{X}^{2}$ does not decrease in two consecutive iterations, or $\bar{X}^{2}$ did decrease but our convergence criteria were not satisfied, set by constants CPL and CPM. |
| IFAIL $=3$ | Maximum number of permitted iterations exceeded. Set by IZZ3. |
| IFAIL $=10$ | Normalized chisquare exceeded 1500. Probably first estimate very wrong. Track interchange possible. |
| IFAIL $=11$ | Number of integration steps for a track too large ( $\geq$ ZZA3) . |
| IFAIL $=14$ | Less than 3 reconstructed spatial points available. Insufficient for computing a first estimate. Probably too few points measured in one view to compute spatially reconstructed points. |
| IFAIL $=15$ | A first estimate cannot be made because of excessive $\chi^{2}$ for a circle fit. |
| IFAIL $=20$ | First estimate for vertex coordinate \|XIN| exceeds ZZ |


| IFAIL $=21$ | First estimate for vertex coordinate $\mid$ YIN $\mid$ exceeds ZZA4. |
| ---: | ---: |
| IFAIL $=22$ | First estimate for vertex coordinate $\mid$ ZIN $\mid$ exceeds ZZA5. |
| IFAIL $=23$ | End point not in chamber volume defined by $\mid$ X $\mid<$ ZZA6 |
|  | $\|Y\|<$ ZZA4 |
|  | $\mid$ Z $\mid<$ ZZA5. |

IFAIL $=90 \quad$ In two or three views less than 3 fiducials measured.

INOTE
List of signal words
$\operatorname{INOTE}(1) \quad$ Is an array of 110 integer words. They refer to tracks (in blocks of 10 words) as follows ( $\operatorname{INOTE}(10 * \mathrm{~J}+\mathrm{I})$ )

$$
\begin{array}{ll}
J=1,2,3 \quad \text { NR } & \text { track number } \\
\text { I }-1,2,3 \ldots 9 & \begin{array}{l}
\text { are signal words with } \\
\text { the following meaning: }
\end{array}
\end{array}
$$

$\operatorname{INOTE}(10 * J+1)=1$
$\operatorname{INOTE}(10 * J+2)=K \quad$ view $K$ absent for track J (e.g., less than 3 points measured).
$\operatorname{INOTE}(10 * J+3)=1 \quad$ track is weighted due to scanning information (in MAIN).
$\operatorname{INOTE}(10 * J+4)=1 \quad$ weight increased for track J because of partial chisquare (in FIT).

```
INOTE (10*J + 5) = Number of points deleted by sagitta test; View 1
INOTE (10*J + 6) = Number of points deleted by sagitta test; View 2
INOTE(10*J + 7) = Number of points deleted by sagitta test; View 3
INOTE (10*J + 8) = K
Special weight if last four points fail sagitta test in:
    K=1 View 1
    K=2 View 2
    K=3 View 3
    K=4 View 1 and 2
    K=5 View 1 and 3
    K=6 View 2 and 3
    K=7 View 1, 2 and 3
INOTE (10*J +9) = K
```

Special weight for points with optical projections
20 degrees
$\mathrm{K}=1 \quad$ View 1
$\mathrm{K}=2 \quad$ View 2
$K=3 \quad$ View 3
$K=4 \quad$ View 1 and 2
$\mathrm{K}=5 \quad$ View 1 and 3
$K=6 \quad$ View 2 and 3
IV. INPUT-OUTPUT ROUTINES

## A. Input of Measured Data

The present version of SYBIL expects measured data in standard binary record format on magnetic tape. (An exception are Fake events; see below.) The measurements are recorded from the measuring tables always in data blocks of an entire event. The SUBROUTINE TAPIN reads the data into an array $\operatorname{POS}(500)$ by $\operatorname{READ}(8, \operatorname{END}=42) \mathrm{J},(\operatorname{POS}(\mathrm{I}), \mathrm{I}=1, \mathfrak{J})$. The parameters and film
coordinates of a measured event are allocated as follows
TITLE BLOCK *) (if $\mathrm{J}=4$ )
$\operatorname{POS}(1)=\operatorname{IDATE} \quad$ date of measuring
POS(2) = ISHIFT measuring shift
POS(3) = NMESTB $\quad$ code for measuring table
POS(4) = ISCAN
*) not in all versions of SYBIL
DATA BLOCK $\quad$ (if $\mathrm{J} \neq 4$ )
$\operatorname{POS}(1)=\operatorname{IROLL} \quad \mathrm{N}^{0}$ of roll
$\operatorname{POS}(2)=\operatorname{IEVENT} \quad \mathrm{N}^{0}$ of frame
$\operatorname{POS}(3) \quad$ coded information for further event identification

POS(4) $=\operatorname{IETYP} \quad$ event type
POS(5) = ISCAN scanner identification number, coded measuring date; measuring table number
$\operatorname{POS}(6)=\operatorname{IFC}(1) \quad$ address of fiducial 1 (coded)
$\operatorname{POS}(7)=\operatorname{IFC}(2) \quad$ address of fiducial 2 (coded)
$\operatorname{POS}(8)=\operatorname{IFC}(3) \quad$ address of fiducial 3 (coded)
definitions:
$\mathrm{NE}=8$ length of the parameter block
$\mathrm{NR}=\mathrm{IETYP} / 100 \mathrm{~N}^{\circ}$ of charged tracks
NF1, NF2, NF3 $=\mathrm{N}^{\mathrm{O}}$ of measured fiducials new 1, 2,3
**) $\quad \operatorname{POS}(\mathrm{NE}+1)=\mathrm{NF} 1$
$\operatorname{POS}(\mathrm{NE}+2) \quad \mathrm{x}$ coordinate fiducial 1 view 1
$\operatorname{POS}(\mathrm{NE}+3) \quad \mathrm{y}$ coordinate fiducial 1 view 1
$\operatorname{POS}(\mathrm{NE}+4) \quad \mathrm{x}$ coordinate fiducial 2 view 1
$\operatorname{POS}(\mathrm{NE}+5) \quad \mathrm{y}$ coordinate fiducial 2 view 1

```
to
    N1 = NE + 2 * NF1 + 1
POS(N1 + 1) = NP(1,1) NO of points measured for track J = 1 view 1
POS(N1 + 2) x coordinate point, I = 1
                                    track, J = 1
                                    view 1
POS(N1+3) y coordinate I = 1, J=1, view 1
POS(N1+4) x coordinate I= 2, J = 1, view 1
to
    N1 + 2 * NP(1, 1) + 1
POS(N1 + 2 * NP + 2) signal word track 1, view 1
    N2=N1 + 2 * NP(1, 1) + 2
POS(N2 + 1) = NP(1,2) N Nof points track J = 2, view 1
POS(N2+2) x coordinate I =1, J=2, view 1
POS(N2+3) y coordinate I = 1, J = 2, view 1
to
    N2 + 2*NP(1,2) + 1.
POS(N2 + 2 * NP + 2) signal word track 2, view 1
    N3=N2+2 * NP(1,2) +2
POS(N3+1) = NP(1,3) No of points track J = 3, view 1
```

We continue to store this way point coordinates of NR track measured in view 1. Then beginning again with fiducials coordinates measured in view 2 (as from line ${ }^{* *}$ )) one combines to store exactly as before the analoguous information of view 2 and view 3.

The number of measured fiducial coordinates or point coordinates in any particular view can be zero. An error code, if necessary, will be generated automatically.

After all the data due to one event have been read and allocated in arrays, the fiducial addresses are being decoded. The film images of the fiducial positions in the chamber are then supplied in arrays UF1, VF1, UF2, VF2, UF3, VF3. They are projected from real space into the 3 film planes and distorted according to the lens distortion such that real image of the fiducials will be provided.

By a least square fit and a subsequent rotation translation the images of the real fiducials are being matched to the measured fiducials in the film plane. The set of rotation and translation coefficient AHP(1) is then also used to transform the measured track coordinates from the digitizer coordinate system into the film plane coordinate system (Fig. 3). All film coordinates will be corrected for lens distortions (SUBROUTINE UNDIST). The final set of 'ideal' ${ }^{1}$ film coordinates are stored in arrays UST1(J, 1), VST1(J, I), UST2(J, I), VST2(J, 1), UST3(J, I), VST3(J, I) where for 3 views $J$ refers to the track numbers and $I$ is the point counter.

Error flags for insufficient number of measured fiducials or points will be set using the list of IFAIL.

A direct input for FAKE events is provided in MAIN. Since they are supposed to be already 'ideal ${ }^{9}$ film coordinates in the right coordinate system, most of the input preparing subroutines are being by-passed (see IV.C).

## B. Output Facilities

A short form printout which should be used when large blocks of data are to be processed is formated in the SUBROUTINE WSFORM. A more detailed printout is activated by setting IZZ1 = 1. In Fig. $6 \mathrm{a}, \mathrm{b}$ an example is shown for both formats. (The example is a fitted 5 -prong event observed in a photoproduction experiment at SLAC. ${ }^{1}$ )

Going through the output list Fig. 6a we describe briefly the content:
first line (19 2618 ...) the first 4 integer numbers result from the decomposition of the 9 digit fixed point number IREC(3), it follows event type and the real fiducial addresses (7, 9, 19).


FIG. 6a--Example of detailed printout for a 5-prong event observed in the streamer chamber.



FIG. 6b--Example of detailed printout for a 5-prong event observed in the streamer chamber.

Beginning with (-. $247852 \ldots$ ) the rotation translation coefficients AHP. The coordinates of 3 projected real fiducials and of the matched measured fiducials, the differences between corresponding coordinates, in 3 views. Beginning with (. $26100 \times 10^{-5} \ldots$ ) demagnification computed from following the title word EVENT reconstructed, spatial track coordinates $x, y, z$ are listed for all 5 tracks (used for starting values). They are preceded by chisquare information on circle fits to the projected tracks.
It follows the number of measured points per view and per trajectory and the starting values for the parameters.

Following the table word 'CORR' (Fig. 6b) the partial chisquares for each view and track are listed as computed in this iteration and the weights assigned to views and for each iteration one finds (e.g., 37.6, 104, . 02108, . 00301 ...) the squareroot of the normalized chisquare, degrees of freedom, corrections to $k_{1}, \lambda_{1}, \phi_{1}, k_{2}, \lambda_{2}, \phi_{2}$, etc. This is repeated in 3 iterations. The error matrix appears in triangular shape. The left column contains the errors of the parameters. The rest of the triangular matrix contains the correlations which can be identified by imagining the following vertical and horizontal addresses

$$
\begin{array}{lllllll} 
& \mathrm{k}_{1} & \lambda_{1} & \phi_{1} & \mathrm{k}_{2} & \lambda_{2} & \phi_{2} \\
\lambda_{1} & & & & & & \\
\phi_{1} & & & & & & \\
\mathrm{k}_{2} & & & & \cdot & & \\
\lambda_{2} & & & & & & \\
\phi_{2} & & & & & & \\
& & & & & &
\end{array}
$$

After the list of the final fitted parameters which is edited in an obvious way the standard short form output is followed. The last line of integer words INOTE indicates that track 1 and 5 were used to compute the starting value for the vertex.

Format of the Output Tape Record
Since the special SUBROUTINE BWRITE has been used to write the output information on tape, it is helpful to regard the output information rather as a sequence of logical blocks than of physical blocks. The physical block is written as a binary record of variable length. BWRITE orders the information in 3 logical blocks:

20 words IREC for general information transmitted by COMMON/DESCRIPT
block 1 VNF containing the fitted parameters and track information; transmitted through the calling sequence of BWRITE
block 2 CAW
containing the variances and covariances of the parameters; transmitted through the calling sequence
(our - complementary - SUBROUTINE BREAD renders records of this kind immediately in the form of logical blocks).

A more detailed list of the word addresses follows:

| IREC(1) | experiment label |
| :--- | :--- |
| IREC(2) | event type |
| IREC(3) | event identifying number (roll, frame, <br> zone, etc.) |
| IREC(4) | program version, date |
| IREC(5) | number of charged tracks emerging <br> from the unmeasured vertex |
| IREC(6) | special information (measures identi- <br> fication, table number, etc.) |
| IREC(7) | measuring date |
| (floating point) the normalized chisquare |  |
| of the geometrical fit |  |

IREC(10) length of block 1 (VNF)
IREC(11) length of block 2 (CAW)
block 1

| VNF(1) | $\lambda$ of incident beam |
| :---: | :---: |
| VNF(2) | $\phi$ of incident beam |
| VNF(3) | $k$ of track $\mathrm{N}^{\mathrm{O}} 1$ |
| VNF(4) | $\lambda$ of track $\mathrm{N}^{\circ} 1$ |
| VNF(5) | $\phi$ of track $\mathrm{N}^{\mathrm{O}} 1$ |
| VNF(6) | k of track $\mathrm{N}^{\mathbf{O}} 2$ |
|  |  |
| - |  |
| $\operatorname{VNF}(3 * N R+2-2)$ | $k$ of track $N^{\circ} \mathrm{NR}$ |
| $\mathrm{VNF}(3 * \mathrm{NR}+2-1)$ | $\lambda$ of track $\mathrm{N}^{\mathrm{O}} \mathrm{NR}$ |
| $\mathrm{VNF}(3 \times \mathrm{NR}+2)$ | $\phi$ of track $\mathrm{N}^{\mathrm{O}} \mathrm{NR}$ |
| defining $\mathrm{NT}=3 * \mathrm{NR}+2$ |  |
| $\mathrm{VNF}(\mathrm{NT}+1)$ | x coordinate of fitted vertex |
| $\mathrm{VNF}(\mathrm{NT}+2)$ | $y$ coordinate of fitted vertex |
| $\mathrm{VNF}(\mathrm{NT}+3)$ | z coordinate of fitted vertex |
| $\mathrm{VNF}(\mathrm{NT}+4)$ | not assigned. |
| defining $\mathrm{NM}=\mathrm{NT}+4$ |  |
| * $\mathrm{VNF}(\mathrm{NM}+1)$ | track length of track $\mathrm{N}^{0} 1$ |
| $\mathrm{VNF}(\mathrm{NM}+2)$ | x coordinate of end point track 1 |
| $\mathrm{VNF}(\mathrm{NM}+3)$ | y coordinate of end point track 1 |
| $\mathrm{VNF}(\mathrm{NM}+4)$ | z coordinate of end point track 1 |
| $\mathrm{VNF}(\mathrm{NM}+5)$ | number of measured points view 1, track 1 |
| $\mathrm{VNF}(\mathrm{NM}+6)$ | number of measured points view 2, track 1 |

$$
\begin{array}{ll}
\mathrm{VNF}(\mathrm{NM}+7) & \text { number of measured points view 3, track } 1 \\
\mathrm{VNF}(\mathrm{NM}+8) & \text { INOTE words }(11-20) \text { coded in one fixed } \\
& \text { point word } \\
\mathrm{VNF}(\mathrm{NM}+9)-\mathrm{VNF}(\mathrm{NM}+17) \\
& \text { the same information as in the } 8 \text { words } \\
& \text { starting from } * \text { but for track } 2
\end{array}
$$

This is continued in blocks of 8 words for all NR(=IREC(5)) tracks.
block 2
The variance-covariance matrix is stored in the following sequence
denoting by I subscript in CAW(1)
and by $[\alpha, \beta]_{\mathrm{I}}$ the covariance of the quantities $\alpha$ and $\beta$ (or variance if $\alpha=\beta$ )

CAW(1)
CAW(2)
CAW(3)

$$
\begin{array}{rll} 
& {\left[\mathrm{k}_{1}, \mathrm{k}_{1}\right]_{1}} \\
& & \\
& {\left[\lambda_{1}, \mathrm{k}_{1}\right]_{2}} \\
& {\left[\lambda_{1}, \lambda_{1}\right]_{3}} & \\
{\left[\phi_{1}, \mathrm{k}_{1}\right]_{4}} & {\left[\phi_{1}, \lambda_{1}\right]_{5}} & {\left[\phi_{1}, \phi_{1}\right]_{6}} \\
{\left[\mathrm{k}_{2}, \mathrm{k}_{1}\right]_{7}} & {\left[\mathrm{k}_{2}, \lambda_{1}\right]_{8}} & {\left[\mathrm{k}_{2}, \phi_{1}\right]_{9}}
\end{array} \quad\left[\mathrm{k}_{2}, \mathrm{k}_{2}\right]_{10}
$$

## C. Tape Control

The control of the data flow at the input and output terminals is controlled by 4 external control cards which read in the SUBROUTINE SETLOG.

| 1. data card (input) | FORMAT | (F10.0, D10.0, F10.0) |
| :---: | :---: | :---: |
| column 1-10 | column 11-20 | column 21-30 |
| $x$ | y |  |
| $x=1.0$ | start input tape after event number y (IREC(3)) |  |
| $x=2.0$ | start input tape after y event |  |
| $x=3.0$ | (data card input presently not active) |  |
| $x=4.0$ | start tape at beginning of a tape file |  |
| $x=5.0$ | input tape contains FAKE events (special Format) |  |
| z | number of input tapes to be read |  |
|  | $-34-$ |  |

2. data card (output)

| $x=1.0$ | start output tape after event number $y$ (IREC(3)) |
| :--- | :--- |
| $x=2.0$ | start output tape after last event |
| $x=3.0$ | start output tape $N^{0} y$ with sequence number $z$ |
| $x=4.0$ | card output only |
| $x=5.0$ | printed output only |
| $x=6.0$ | start output tape after sequence number $y$ |

*) checks for both $\operatorname{IREC}(3)=999999$ or end of file mark in BREAD
3. data card (termination of processing)
$x$ is maximum time in minutes,
$y$ is maximum number of events to be processed
4. data card FORMAT (I10)
$x=$ date, one word, begin in column 1
example: 031569
Fake events being entered in MAIN require special formating which corresponds to the output of the FAKE routine CYCLOPE (SLAC experimental group D). The read routine used is BREAD which provides logical blocks, together with 20 IREC words which are transmitted through COMMON block.

## REFERENCES

1. M. Davier, I. Derado, D. Drickey, D. Fries, R. Mozley, A. Odian, F. Villa and D. Yount, "Multibody photoproduction between 2 and 16 GeV ," Report No. SLAC-PUB-613, Stanford Linear Accelerator Center, Stanford University, Stanford, California (1969); to be published in Phys. Rev.
2. F. Bulos, A. Odian, F. Villa and D. Yount, "Streamer chamber development," Report No. SLAC-74, Stanford Linear Accelerator Center, Stanford University, Stanford, California (1967).
3. M. Davier, I. Derado, D. Drickey, D. Fries, M. Liu, R. Mozley, A. Odian, J. Park, W. Swanson, F. Villa and D. Yount, "Performance of a large streamer chamber magnet system, "Stanford Linear Accelerator Center, Stanford University, Stanford, California (1969); to be published in Nucl. Instr. and Methods.
4. D. E. C. Fries, "CIRCE, a general multiprong fitting computer program for spatial coordinates in a nonuniform magnetic field," Report No. SLAC99, Stanford Linear Accelerator Center, Stanford University, Stanford, California (1969).
5. F. T. Solmitz, A. D. Johnson and T. B. Day, "Three-view geometry program," Alvarez Group programming note P-117, Lawrence Radiation Laboratory, Berkeley, California (1965).
6. I. Derado and R. Leedy, "TEUTA kinematical analysis program for colliding beam events," Report No. SLAC-72, Stanford Linear Accelerator Center, Stanford University, Stanford, California (1967).
