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(EXPI)

HIGH-SPEED SWITCHING PROPERTIES OF THE
EMITTER-COUPLED TRANSISTOR-PAIR

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ABSTRACT

High-speed switching properties of the emitter-coupled transistor-pair are analyzed by means of a digital computer. Waveforms and risetimes are computed for a wide range of parameters. The resulting risetimes are interpreted in terms of the gain-bandwidth products of the transistors, external capacitances, and the risetime of the input signal.

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I. INTRODUCTION

The emitter-coupled transistor pair of Fig. 1 has found many uses in high-speed switching circuits. When components are suitably chosen, the transistors do not saturate and switching times in the nanosecond region are readily attainable. There are many variations of the circuit: Both bases may be driven, one of the two collector resistors, R_{C1} or R_{C2} , may be omitted, current source Q_3 may be replaced by a resistor.

In the following it will be assumed that the circuit of Fig. 2 provides a reasonable approximation to the actual circuit. Transistors Q_1 and Q_2 are characterized by a single fixed parameter $\tau_0 \triangleq 1/(2\pi f_\tau)$ where f_τ is the gain-bandwidth product of both transistors, ohmic base resistances are included in R_g , and all capacitances are lumped into C_{ext} . This approximation is reasonably good if one has the circuit of Fig. 1 with $R_{C1} = 0$: In this case R_{C1} of Fig. 2 is chosen zero and the stray capacitance on the base of Q_1 and its collector-to-base capacitance are included in C_{ext} .

II. COMPUTATION OF THE TRANSIENT

The collector current $i_{C1}(t)$ will be computed for the generator voltage signal $v_g(t)$ of Fig. 3. The hybrid equivalent circuit of Fig. 4 will be used for each transistor with $\alpha \approx 1$, i. e., $\beta \rightarrow \infty$. With these assumptions the circuit shown in Fig. 5 results. It can be seen that the circuit enclosed in the box of broken lines is grounded only via R_B , hence the value and location of R_B is arbitrary; in the following an $R_B = \infty$ will be taken. Also, observing the nodes at B_1 and B_2 it is apparent that all of i_{B1} flows into C_{e1} and all of i_{B2} into C_{e2} . Thus, Fig. 5 can be redrawn as Fig. 6 where C_{ext} has been included in C_{e1} and C_{e2} .

Now, the transient of the circuit can be computed solely from the loop of v_g , R_g , C_{e1} and C_{e2} .

Defining

$$v_{B1E} \triangleq v_{B1} - v_E \quad (1)$$

and

$$v_{B2E} \triangleq v_{B2} - v_E \quad (2)$$

the collector currents are given by the diode equations as

$$i_{C1} = I_0(e^{v_{B1E}/V_T} - 1) \quad (3)$$

and

$$i_{C2} = I_0(e^{v_{B2E}/V_T} - 1), \quad (4)$$

where

$$i_{C1} + i_{C2} = I_{DC}. \quad (5)$$

Here I_0 is the saturation current (in the vicinity of nanoamperes); $V_T = n \frac{kT}{q}$, where k is the Boltzmann constant $k = 1.38 \times 10^{-23}$ Ws/ $^{\circ}$ K, T is the absolute temperature in $^{\circ}$ K, and q is the charge of the electron $q = 1.6 \times 10^{-19}$ As.

Constant n is dimensionless, $n \approx 1$ to 1.5 for germanium, $n \approx 1.5$ to 2 for silicon diodes.

The value of kT/q at room temperature is ≈ 25 mV, thus V_T is typically between 25 mV and 50 mV.

Capacitances C_{e1} and C_{e2} are given by

$$C_{e1} \triangleq \frac{dq_{B1E}}{dv_{B1E}} = \frac{d(i_{C1}\tau_0)}{dv_{B1E}} = \frac{\tau_0}{V_T} I_0 e^{v_{B1E}/V_T} \quad (6)$$

and

$$C_{e2} \triangleq \frac{dq_{B2E}}{dv_{B2E}} = \frac{d(i_{C2}\tau_0)}{dv_{B2E}} = \frac{\tau_0}{V_T} I_0 e^{v_{B2E}/V_T}. \quad (7)$$

Also

$$C_{e1}' = C_{e1} + C_{\text{ext}} \frac{C_{e1} + C_{e2}}{C_{e2}}, \quad (8)$$

$$C_{e2}' = C_{e2} + C_{\text{ext}} \frac{C_{e1} + C_{e2}}{C_{e1}}, \quad (9)$$

and

$$i_{B1} = \frac{v_g + v_{B2E} - v_{B1E}}{R_g}. \quad (10)$$

The base-emitter voltages are given by the integrals

$$v_{B1E} = \int \frac{i_{B1}}{C_{e1}} dt \quad (11)$$

and

$$v_{B2E} = \int \frac{i_{B2}}{C_{e2}} dt = - \int \frac{i_{B1}}{C_{e2}} dt. \quad (12)$$

Unfortunately i_{B1} , C_{e1} , and C_{e2} vary with time and the integrals have to be evaluated numerically. Equation (11) can be approximated as

$$v_{B1E} = \int \frac{i_{B1}}{C_{e1}} dt \approx \sum \frac{i_{B1}}{C_{e1}} \Delta t,$$

which can be also written as

$$v_{B1E}(t + \Delta t) \approx v_{B1E}(t) + \Delta v_1, \quad (13)$$

where

$$\Delta v_1 \triangleq \frac{i_{B1}(t)}{C_{e1}(t)} \Delta t. \quad (14)$$

Similarly (12) becomes

$$v_{B2E}(t + \Delta t) \approx v_{B2E}(t) + \Delta v_2, \quad (15)$$

with

$$\Delta v_2 \triangleq \frac{-i_{B1}(t)}{C_{e2}(t)} \Delta t. \quad (16)$$

The initial values of v_{B1E} and v_{B2E} can be computed from (3), (4), and (5) as

$$v_{B1E}(t=0) = v_g(t<0) + V_T \ln \frac{2 + I_{DC}/I_0}{1 + e^{v_g(t<0)/V_T}} \quad (17)$$

$$v_{B2E}(t=0) = V_T \ln \frac{2 + I_{DC}/I_0}{1 + e^{v_g(t<0)/V_T}} \quad (18)$$

and i_{C1} can be computed by substituting (17) into (3).

Equations (3), (8), (9), (10), (13), and (15) are solved by a digital computer using the flow chart of Fig. 7 with $\Delta t_{\max} = 0.01\tau_0$, $\Delta t_{\min} = 10^{-5}\tau_0$, and $\Delta v_{\max} = 0.01V_T$. The Fortran IV computer programs are shown in Fig. 10, 11, and 12.

III. RESULTS

Representative waveforms of $i_{C1}(t)$ are shown in Fig. 8 and Fig. 9. The risetimes between the 10% and 90% points of i_{C1}/I_{DC} are summarized in Table 1, together with those obtained from the approximation

$$t_r \approx \sqrt{(t_{r7} + t_{rc})^2 + t_{rg}^2}, \quad (19)$$

where

$$t_{r7} \triangleq 0.8 \frac{R_g I_{DC}}{v_{g1} - 0.4V_T} \tau_0, \quad (20)$$

$$t_{rc} \triangleq \frac{V_T^2 \ln 9}{v_{g1} - 0.8V_T} R_g C_{\text{ext}}, \quad (21)$$

and

$$t_{rg} \triangleq \frac{V_T^2 \ln 9}{v_{g1} - v_{g0}} t_g. \quad (22)$$

There are three contributions to the risetime: 1), t_{r7} of (20) results from a finite τ_0 (finite gain - bandwidth product), 2), t_{rc} of (21) results from the finite C_{ext} , and 3), t_{rg} of (22) from the finite risetime t_g of the input signal.

1), t_{rT} . In the limiting case when C_{ext} and t_g are zero, the risetime is given by (20). For $v_{g1} \gg V_T$ this is the current gain $R_g I_{DC} / v_{g1}$ multiplied by τ_0 and by a factor of 0.8 for a risetime computed between 10% and 90%. The term $0.4 V_T \approx 20 \text{ mV}$ in the denominator of (20) represents a voltage "used up" for dc switching, which has to be taken account if $v_{g1} \gg V_T$.

2), t_{rc} . In addition to the charge $I_{DC} \tau_0$ in the base emitter junction, the source has to provide a charge into capacitor C_{ext} which results in the risetime t_{rc} of (21). The dc voltage swing on the bases between the 10% and 90% points of i_{C1} (from (3), (4), and (5) with $i_{C1} \gg I_0$ and $i_{C2} \gg I_0$) is $\approx V_T 2 \ln 9$. Assuming a voltage of $0.8 V_T$ "lost" from v_{g1} for dc current transfer, t_{rc} represents the charge supplied to C_{ext} during a voltage swing of $V_T 2 \ln 9$.

3), t_{rg} . Equation (22) represents the risetime of the input waveform during the voltage swing of $V_T 2 \ln 9$. Since this risetime t_{rg} is independent of that of the circuit, $t_{rT} + t_{rc}$, the squares of the two risetimes are added in (19).

As can be seen in Table 1, Eq. (19) provides a rather good approximation, therefore it can be utilized to obtain risetimes for parameter values not listed in the table.

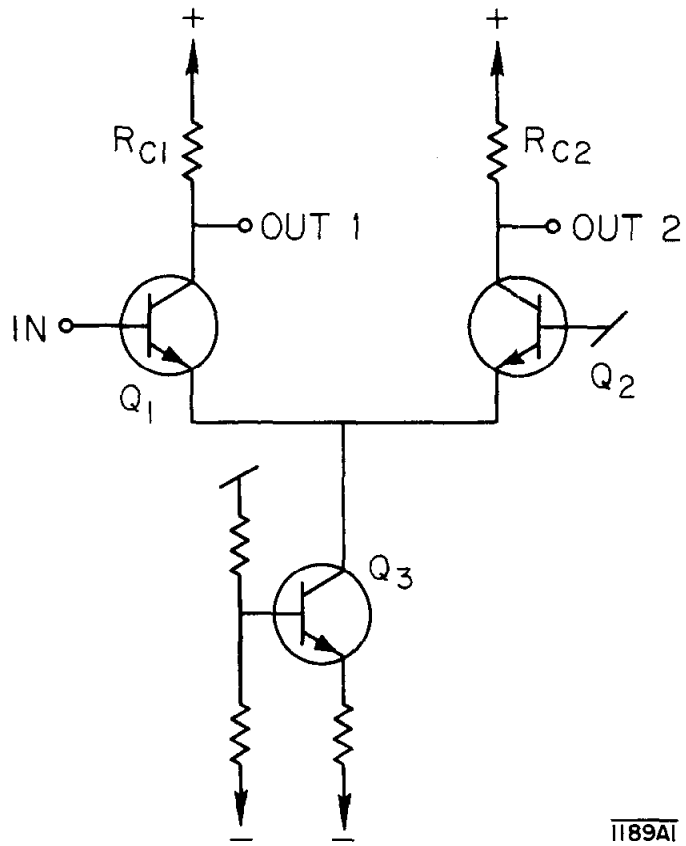
TABLE 1

10% to 90% risetime of i_{C1} for various values of v_{g0} , v_{g1} , R_g , C_{ext} , and t_g , using the flow-chart of Fig. 7 ("EXACT"), and the approximation of Eq. (19) ("APPROX"). Legend: $R = R_g I_{DC} / V_T$, $C = C_{ext} V_T / \tau_0 I_{DC}$.

$\frac{-v_{g0}}{V_T}$	$\frac{v_{g1}}{V_T}$	R	C	$t_g/\tau_0 = 0$		$t_g/\tau_0 = 0.5$		$t_g/\tau_0 = 1$		$t_g/\tau_0 = 2$		$t_g/\tau_0 = 5$		$t_g/\tau_0 = 10$		$t_g/\tau_0 = 20$		$t_g/\tau_0 = 50$		$t_g/\tau_0 = 100$			
				EXACT	APPROX	EXACT	APPROX	EXACT	APPROX	EXACT	APPROX	EXACT	APPROX	EXACT	APPROX	EXACT	APPROX	EXACT	APPROX	EXACT	APPROX	EXACT	APPROX
				3	3	1C	0.0	3.1	3.1	3.2	3.1	3.3	3.2	3.7	3.4	5.2	4.8	8.2	7.9	15.1	15.0	36.8	36.7
3	3	10	0.1	5.0	5.1	5.0	5.1	5.0	5.1	5.3	5.3	6.4	6.3	9.0	8.9	15.5	15.5	37.0	37.0	73.4	73.4		
3	3	1C	1.0	21.7	23.1	21.7	23.1	21.7	23.1	21.7	23.1	21.8	23.3	22.8	24.2	26.2	27.3	41.6	43.3	75.2	76.8		
3	3	1C	10.0	189.7	202.8	189.7	202.8	189.7	202.8	189.7	202.8	189.7	202.9	189.7	203.0	189.7	203.4	191.1	206.1	201.5	215.6		
3	3	3C	0.0	9.5	9.2	5.5	9.2	9.5	9.3	9.7	9.3	10.7	9.9	13.0	11.8	18.5	17.3	38.5	37.8	74.0	73.8		
3	3	3C	0.1	15.0	15.2	15.0	15.2	15.1	15.2	15.1	15.3	15.7	15.7	17.3	16.9	21.8	21.1	39.8	39.7	74.9	74.8		
3	3	3C	1.0	65.4	65.2	65.4	69.2	65.4	69.2	65.4	69.2	65.4	69.3	65.4	69.5	66.5	70.7	75.0	78.3	97.0	100.7		
3	3	3C	10.0	561.3	608.5	561.3	608.5	561.3	608.5	561.3	608.5	561.2	608.5	561.2	608.5	561.1	608.6	560.7	609.6	560.3	612.9		
3	10	1C	0.0	0.8	0.8	0.9	0.9	1.1	0.9	1.5	1.1	2.7	1.9	4.5	3.5	7.7	6.8	17.3	16.9	34.0	33.8		
3	10	1C	0.1	1.2	1.3	1.3	1.3	1.4	1.4	1.7	1.5	3.0	2.1	4.8	3.6	8.1	6.9	17.7	17.0	34.2	33.8		
3	10	1C	1.0	5.2	5.6	5.2	5.6	5.2	5.6	5.3	5.7	5.8	5.9	7.4	6.5	11.3	8.8	21.6	17.8	37.5	34.3		
3	10	1C	10.0	45.4	48.6	45.4	48.6	45.4	48.6	45.4	48.6	45.4	48.6	45.4	48.7	45.8	49.1	51.9	51.5	68.5	59.2		
3	10	3C	0.0	2.4	2.5	2.4	2.5	2.5	2.5	2.9	2.6	4.1	3.0	6.3	4.2	10.0	7.2	19.9	17.1	35.8	33.9		
3	10	3C	0.1	3.7	3.9	3.7	3.9	3.8	3.9	3.9	4.0	4.9	4.3	6.9	5.2	10.8	7.8	21.0	17.4	37.1	34.0		
3	10	3C	1.0	15.8	16.8	15.8	16.8	15.8	16.8	15.8	16.8	15.8	16.9	16.4	17.2	19.0	18.1	30.3	23.9	48.2	37.8		
3	10	3C	10.0	126.3	145.8	126.3	145.8	126.3	145.8	126.3	145.8	126.3	145.8	126.3	145.8	126.3	146.0	136.7	146.8	143.8	149.7		
3	30	1C	0.0	0.3	0.3	0.4	0.3	0.6	0.3	0.9	0.4	1.5	0.7	2.3	1.4	3.7	2.7	7.6	6.7	13.9	13.3		
3	30	1C	0.1	0.4	0.4	0.5	0.4	0.7	0.4	1.0	0.5	1.7	0.8	2.6	1.4	4.0	2.7	8.0	6.7	14.3	13.3		
3	30	1C	1.0	1.7	1.8	1.7	1.8	1.7	1.8	2.0	1.8	2.9	1.9	4.3	2.2	6.4	3.2	11.2	6.9	18.1	13.4		
3	30	1C	10.0	14.9	15.3	14.9	15.3	14.9	15.3	14.9	15.3	14.9	15.3	14.9	15.3	15.4	17.5	15.5	26.7	16.7	39.4	20.3	
3	30	3C	0.0	0.8	0.8	0.9	0.8	1.0	0.8	1.4	0.9	2.4	1.0	3.5	1.6	5.4	2.8	9.9	6.7	16.5	13.3		
3	30	3C	0.1	1.2	1.3	1.3	1.3	1.4	1.3	1.7	1.3	2.7	1.4	4.0	1.8	6.0	2.9	10.7	6.8	17.5	13.4		
3	30	3C	1.0	5.2	5.3	5.2	5.3	5.2	5.3	5.2	5.3	5.7	5.4	7.1	5.5	10.2	6.0	17.2	8.5	26.1	14.3		
3	30	3C	10.0	44.8	46.0	44.8	46.0	44.8	46.0	44.8	46.0	44.8	46.0	44.8	46.0	44.9	46.0	50.0	46.4	64.4	47.8		
10	10	1C	0.0	0.8	0.8	0.9	0.8	0.9	0.9	1.2	0.9	2.0	1.4	3.2	2.3	5.4	4.5	11.7	11.0	22.3	22.0		
10	10	1C	0.1	1.2	1.3	1.2	1.3	1.2	1.3	1.3	1.4	1.9	1.7	3.2	2.6	5.5	4.6	12.0	11.1	22.6	22.0		
10	10	1C	1.0	5.2	5.6	5.2	5.6	5.2	5.6	5.2	5.6	5.2	5.7	5.2	6.0	6.0	7.1	12.3	12.3	23.2	22.7		
10	10	1C	10.0	45.4	48.6	45.4	48.6	45.4	48.6	45.4	48.6	45.4	48.6	45.4	48.6	45.4	48.8	45.4	49.8	45.4	53.3		
10	10	3C	0.0	2.4	2.5	2.4	2.5	2.4	2.5	2.6	2.5	3.2	2.7	4.7	3.3	7.3	5.1	14.2	11.3	24.6	22.1		
10	10	3C	0.1	3.7	3.9	3.7	3.9	3.7	3.9	3.7	4.0	3.8	4.1	4.5	4.5	7.1	5.9	14.4	11.7	25.3	22.3		
10	10	3C	1.0	15.8	16.8	15.8	16.8	15.8	16.8	15.8	16.8	15.8	16.9	15.8	17.0	15.8	17.4	16.8	20.1	26.3	27.7		
10	10	3C	10.0	135.7	145.8	135.6	145.8	135.7	145.8	135.6	145.8	135.6	145.8	135.6	145.8	135.4	145.9	135.1	146.2	134.5	147.4		
10	10	3C	0.0	0.3	0.3	0.4	0.3	0.5	0.3	0.8	0.3	1.3	0.6	2.0	1.1	3.2	2.2	6.5	5.5	11.7	11.0		
10	10	3C	0.1	0.4	0.4	0.4	0.4	0.5	0.4	0.7	0.4	1.2	0.7	1.9	1.2	3.2	2.2	6.6	5.5	12.0	11.0		
10	10	3C	1.0	1.7	1.8	1.7	1.8	1.7	1.8	1.7	1.8	1.7	1.9	2.3	2.1	3.5	2.8	6.9	5.8	12.3	11.1		
10	10	3C	10.0	14.9	15.3	14.9	15.3	14.9	15.3	14.9	15.3	14.9	15.3	14.9	15.4	14.9	15.5	15.0	16.3	20.3	18.9		
10	10	3C	0.0	0.8	0.8	0.8	0.8	0.9	0.8	1.2	0.8	2.0	1.0	3.0	1.4	4.7	2.3	8.6	5.6	14.2	11.0		
10	10	3C	0.1	1.2	1.3	1.2	1.3	1.2	1.3	1.2	1.3	1.8	1.4	2.7	1.7	4.3	2.5	8.4	5.6	14.4	11.1		
10	10	3C	1.0	5.2	5.3	5.2	5.3	5.2	5.3	5.2	5.3	5.2	5.4	5.2	5.4	5.5	5.8	9.5	7.7	15.4	12.2		
10	10	3C	10.0	44.8	46.0	44.8	46.0	44.8	46.0	44.8	46.0	44.8	46.0	44.8	46.0	44.8	46.0	44.8	46.3	44.8	47.3		

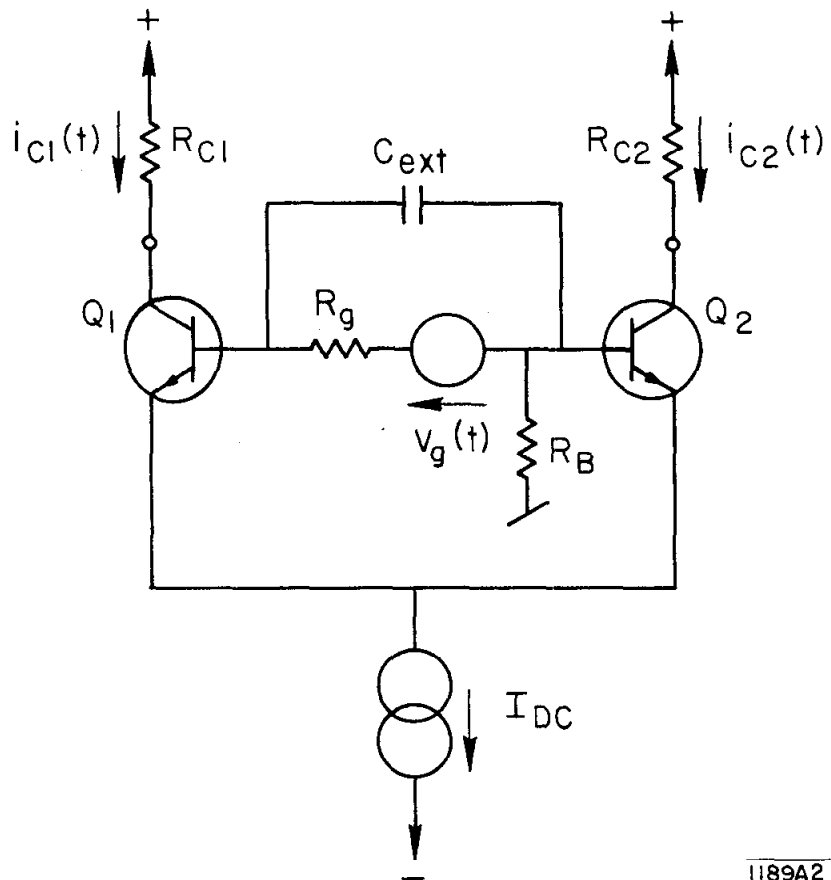
FIGURE CAPTIONS

1. Emitter-coupled transistor pair.
2. An approximation of the emitter-coupled transistor pair of Fig. 1.
3. Generator voltage for Fig. 2.
4. Hybrid transistor equivalent circuit.
5. The circuit of Fig. 2 with the transistor equivalent circuit of Fig. 4.
6. Simplification of the circuit of Fig. 5.
- 7a and 7b. Flow-charts of the computer program.
- 8a through 8s. Waveforms of $i_{C1}(t)$ for $C_{ext} = 0$ and various values of v_{g0} , v_{g1} , R_g , and t_g .
- 9a through 9h. Waveforms of $i_{C1}(t)$ for $t_g = 0$ and various values of v_{g0} , v_{g1} , R_g , and C_{ext} .
10. Fortran-IV computer program resulting in Fig. 8.
11. Fortran-IV computer program resulting in Fig. 9.
12. Fortran-IV computer program resulting in Table 1.



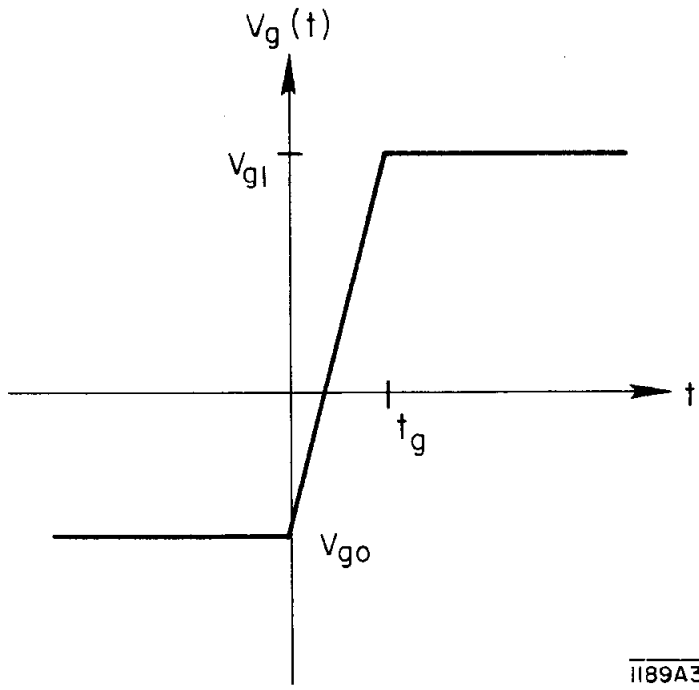
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FIG. 1



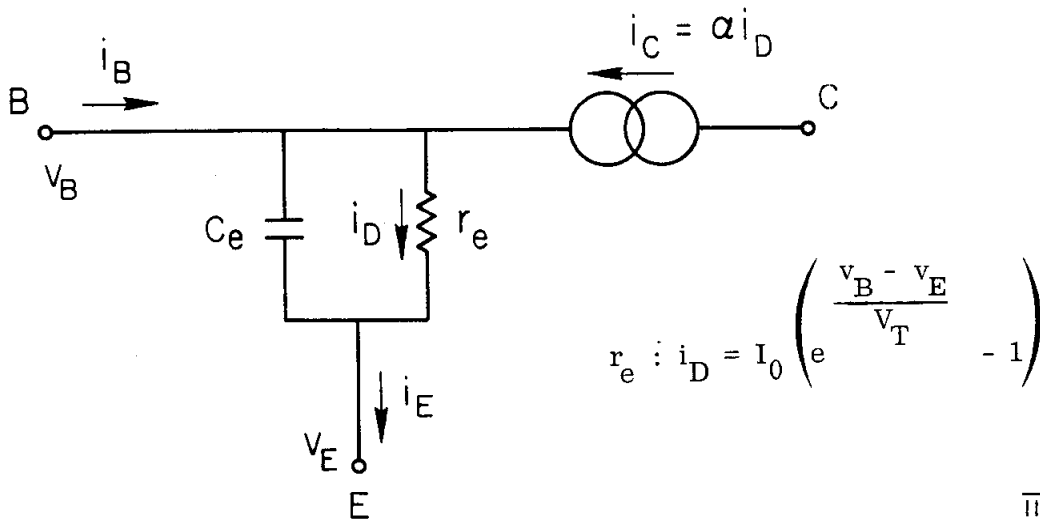
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FIG. 2



1189A3

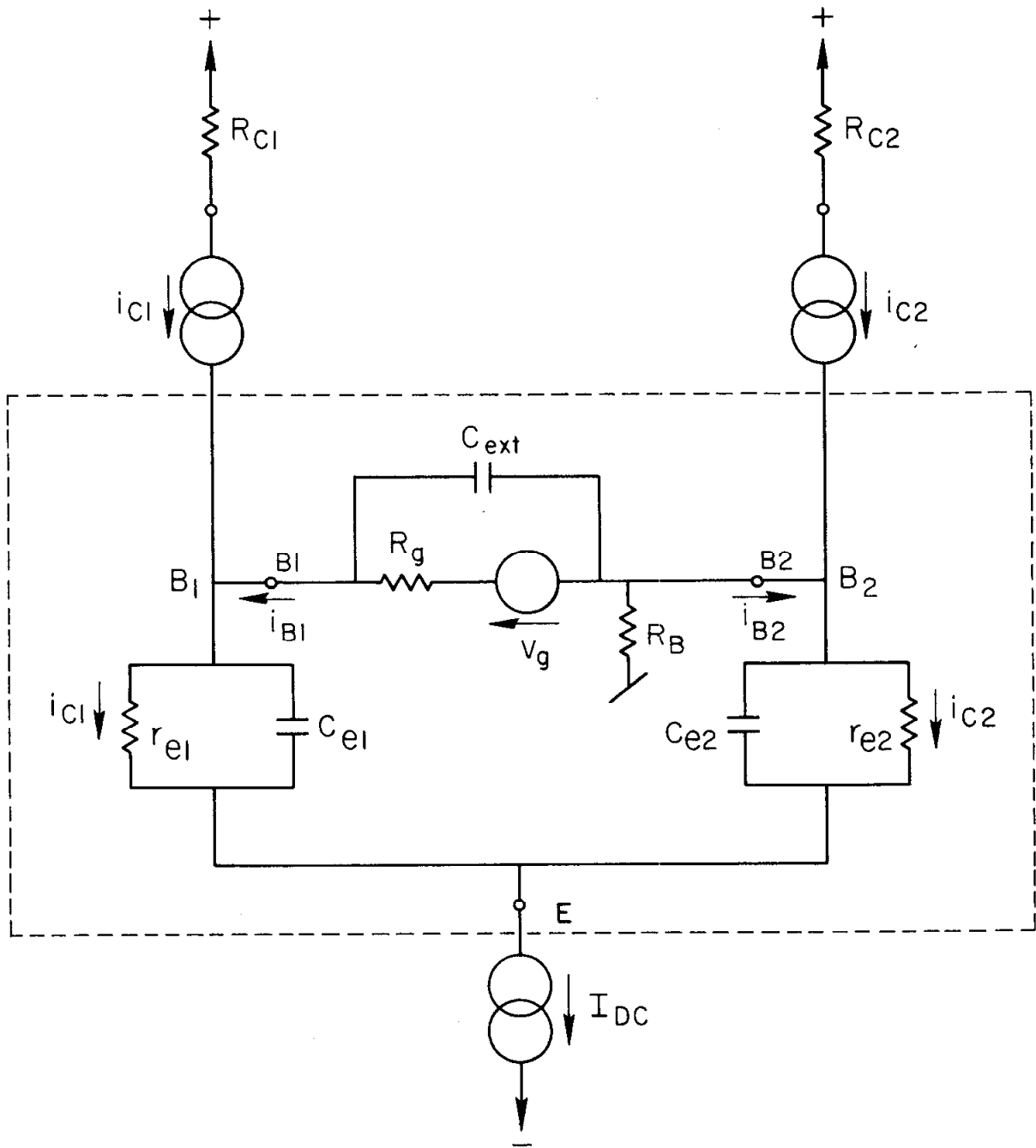
FIG. 3



$$r_e : i_D = I_0 \left(e^{\frac{V_B - V_E}{V_T}} - 1 \right)$$

1189A4

FIG. 4

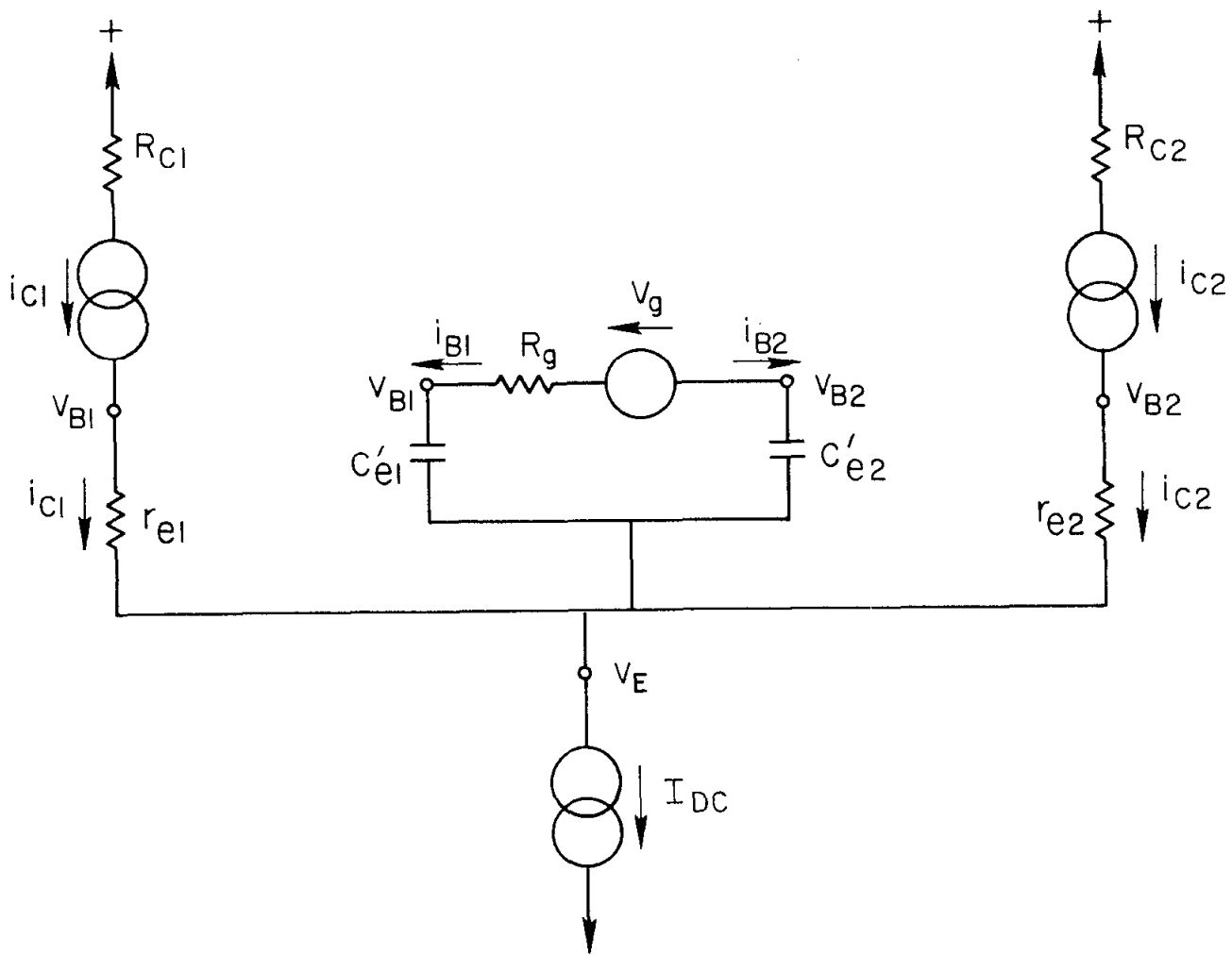


$$r_{e1} : i_{C1} = I_0 \left(e^{\frac{v_{B1} - v_E}{V_T}} - 1 \right)$$

$$r_{e2} : i_{C2} = I_0 \left(e^{\frac{v_{B2} - v_E}{V_T}} - 1 \right)$$

1189A5

FIG. 5



$$r_{e1} : i_{C1} = I_0 \left(e^{\frac{v_{B1} - v_E}{V_T}} - 1 \right)$$

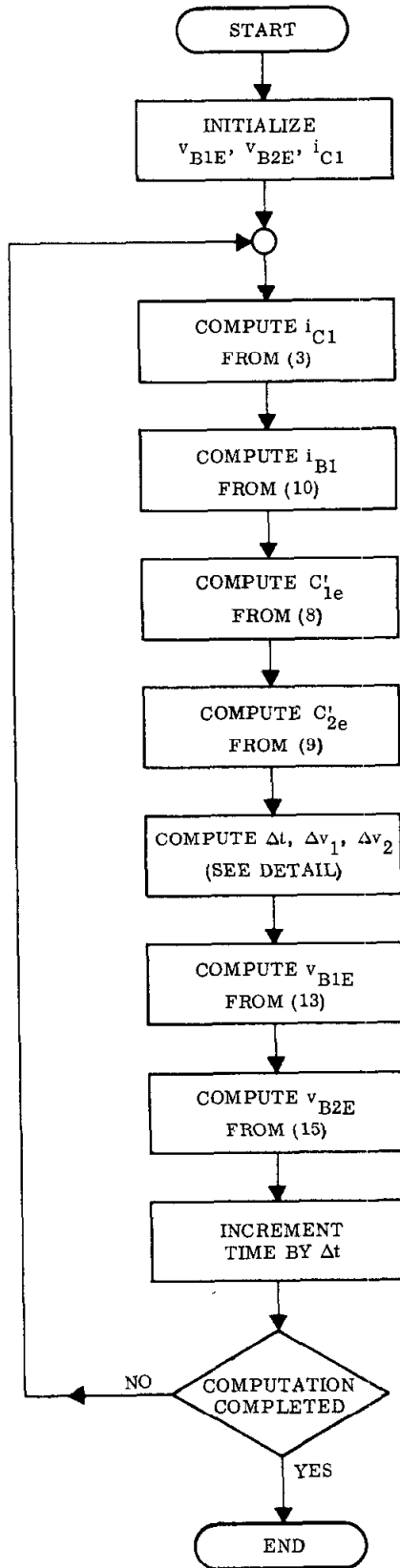
$$r_{e2} : i_{C2} = I_0 \left(e^{\frac{v_{B2} - v_E}{V_T}} - 1 \right)$$

$$C'_{e1} = C_{e1} + C_{ext} \frac{C_{e1} + C_{e2}}{C_{e2}}$$

$$C'_{e2} = C_{e2} + C_{ext} \frac{C_{e1} + C_{e2}}{C_{e1}}$$

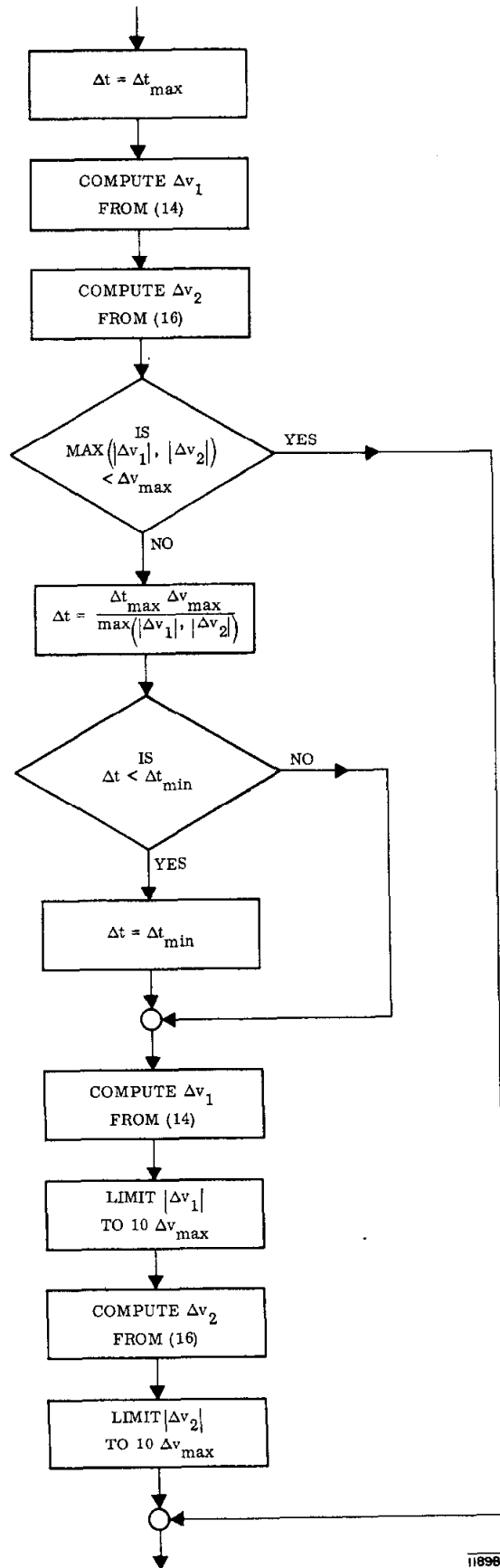
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FIG. 6



1185A7

FIG. 7a



118986

FIG. 7b

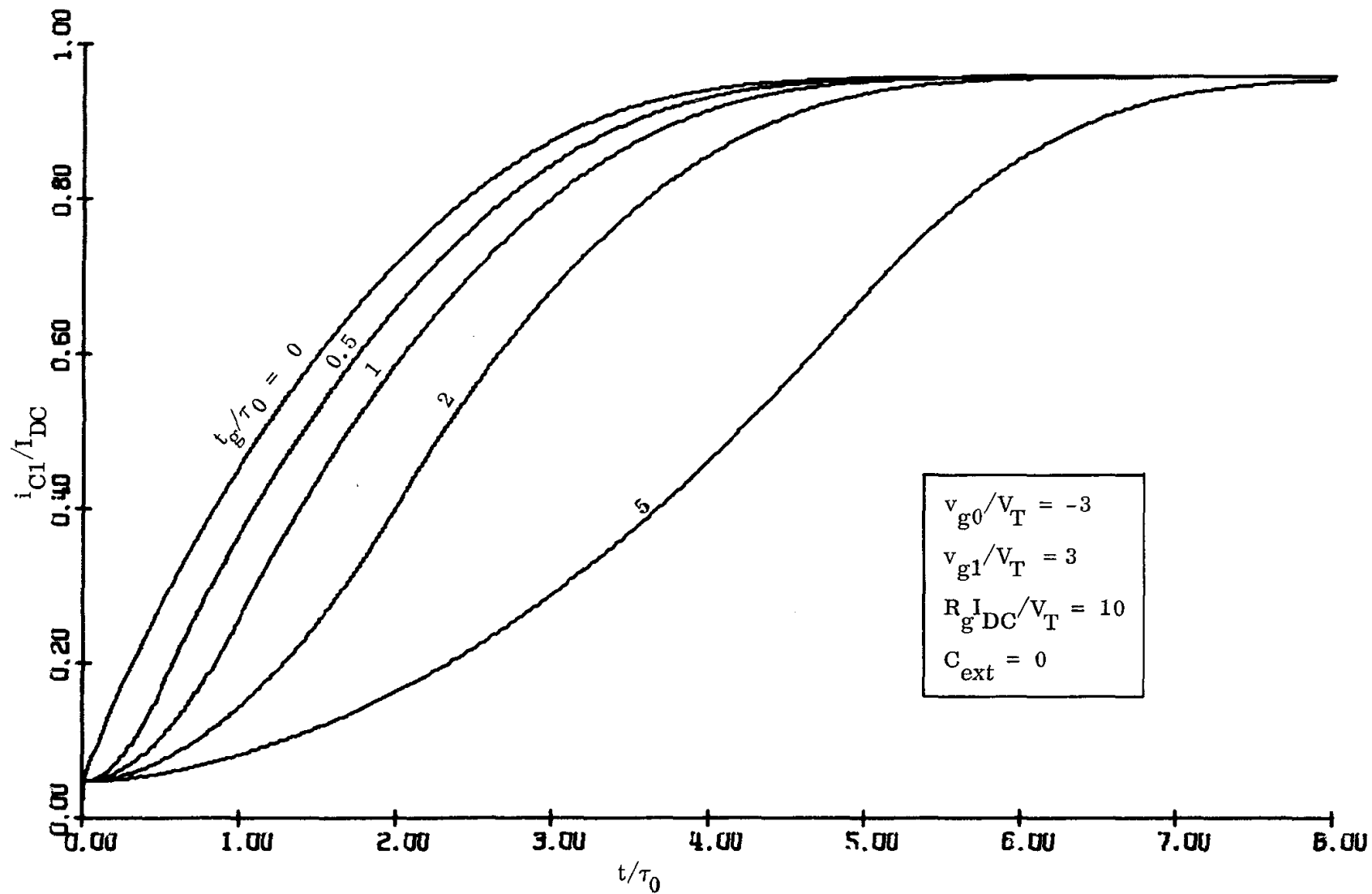


FIG. 8a

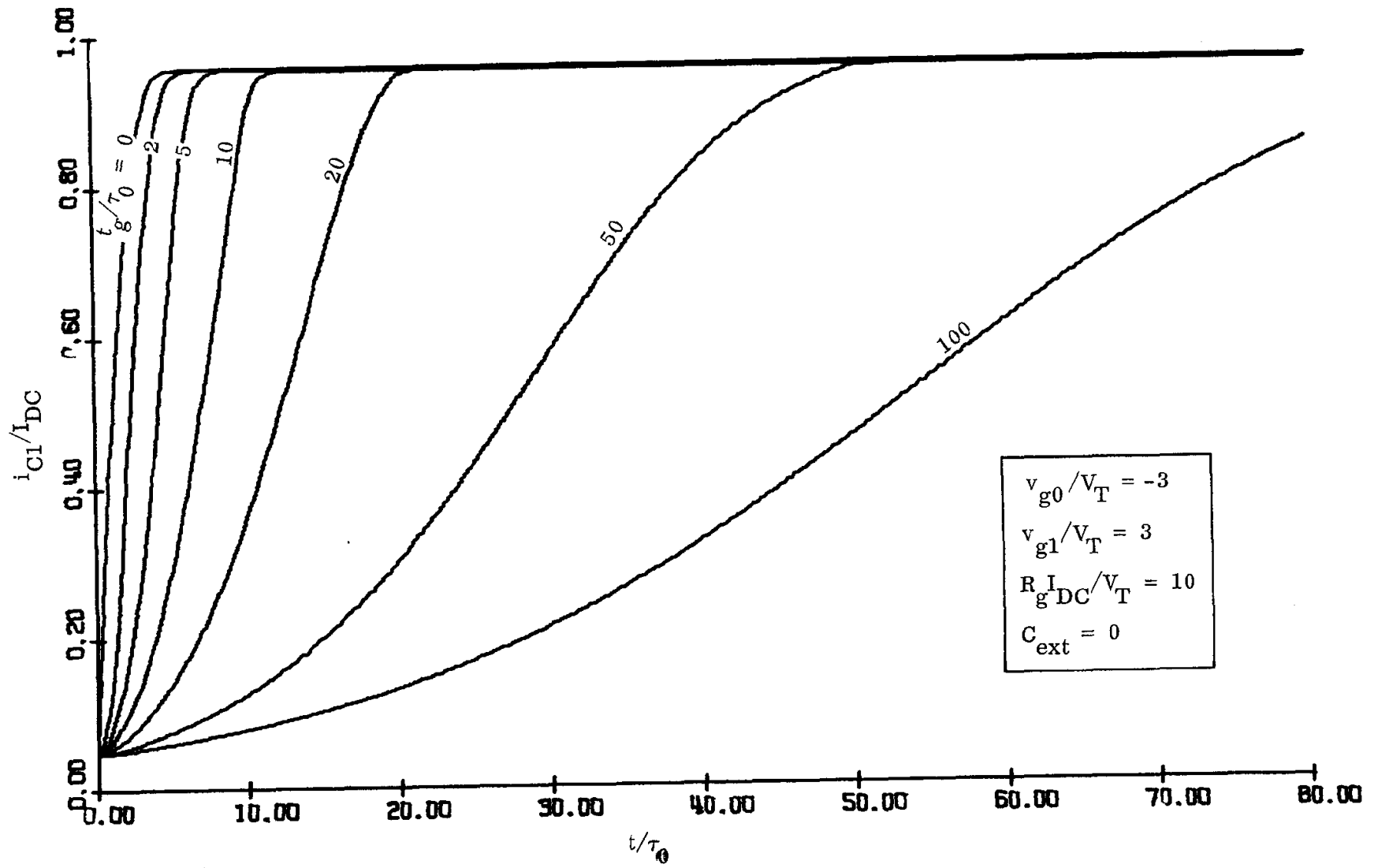


FIG. 8b

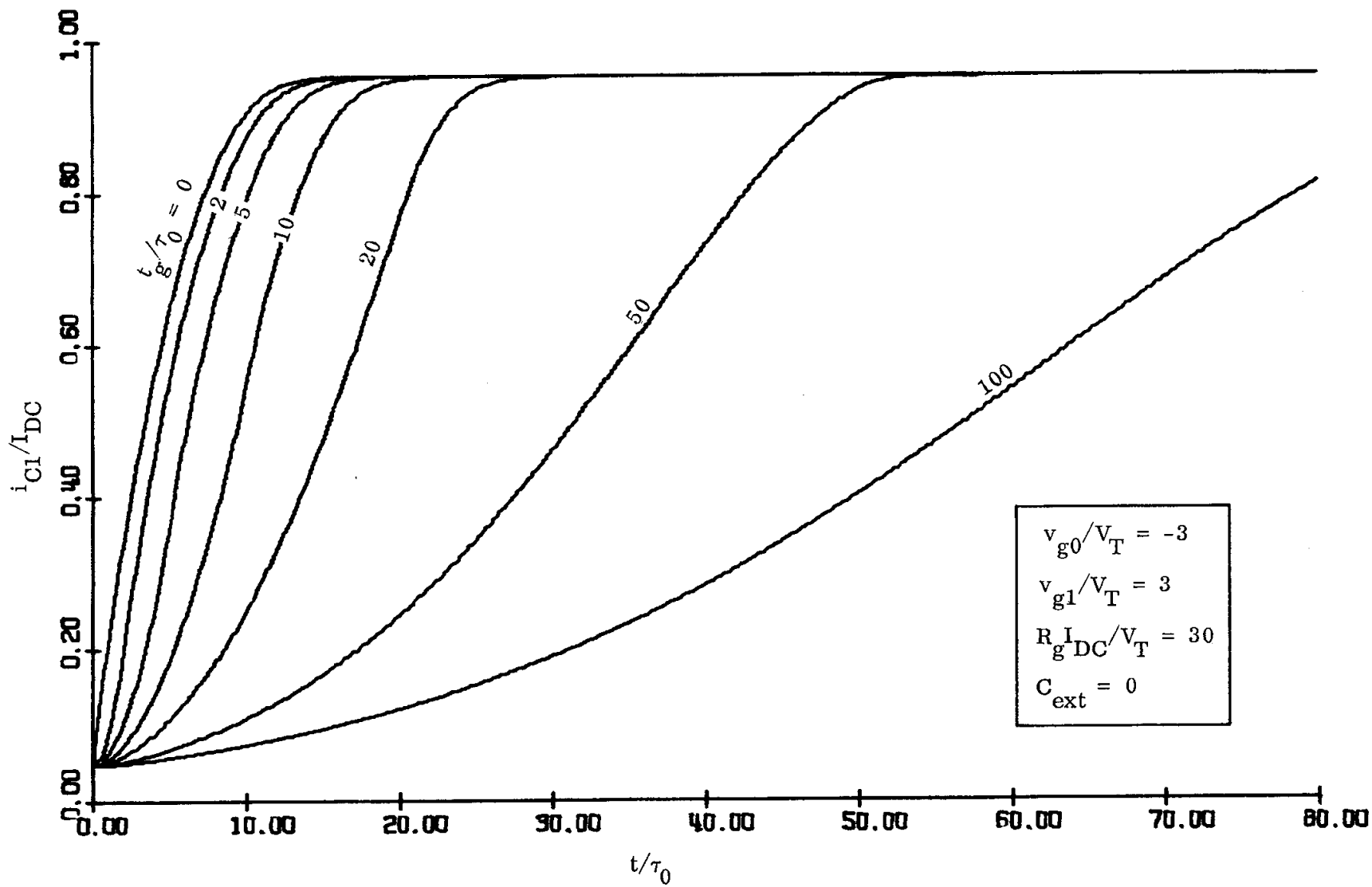


FIG. 8c

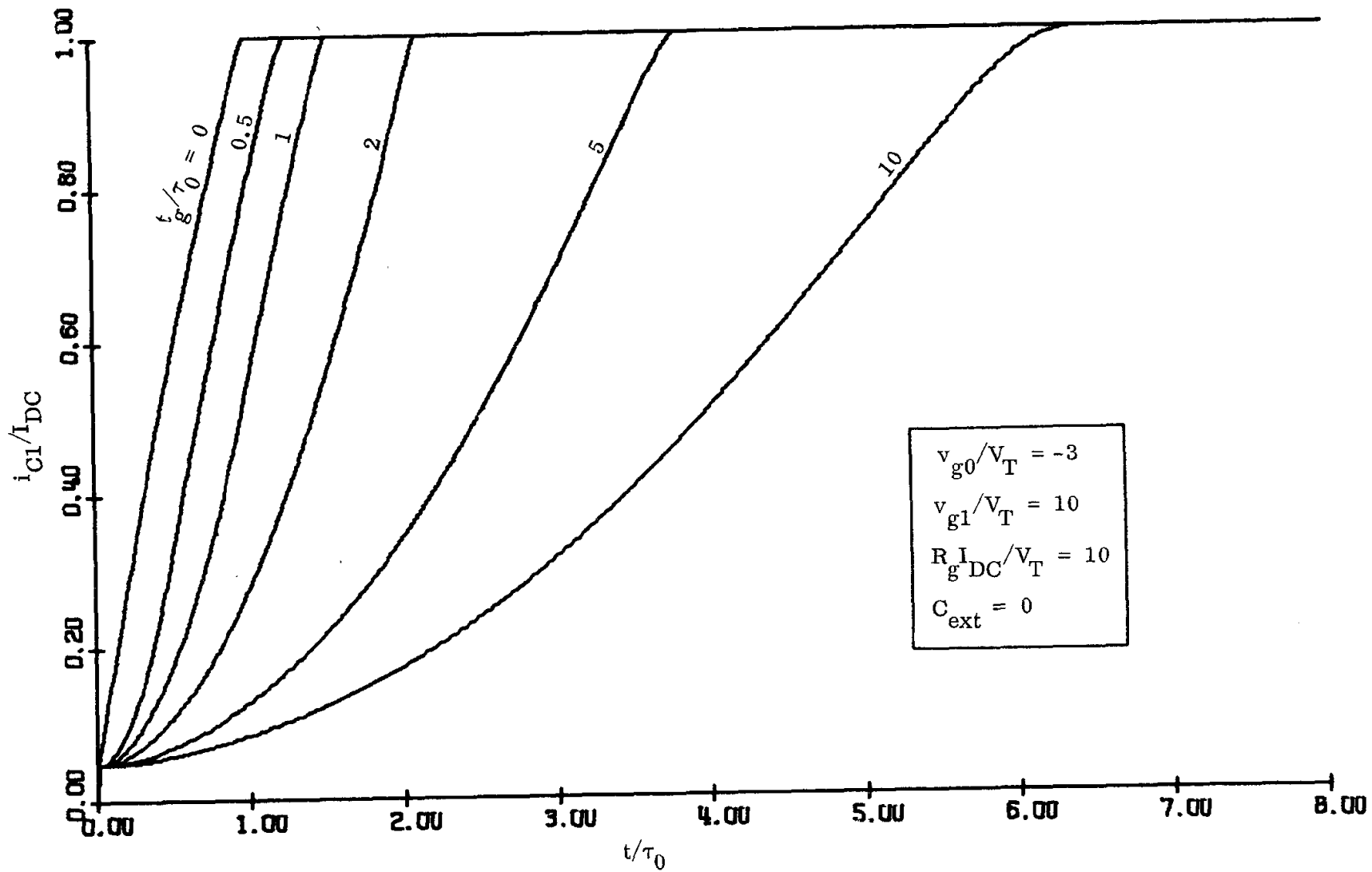


FIG. 8d

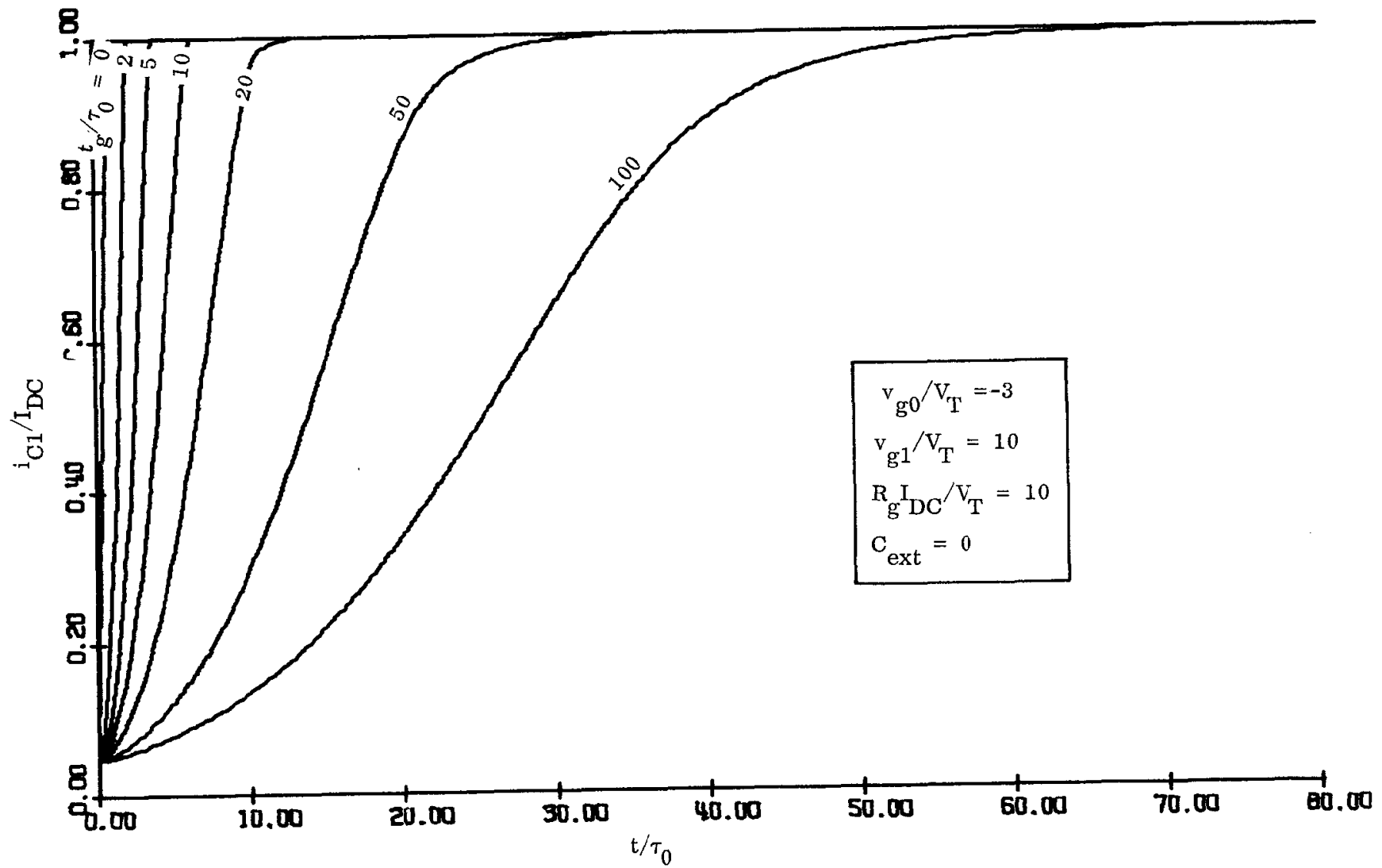


FIG. 8e

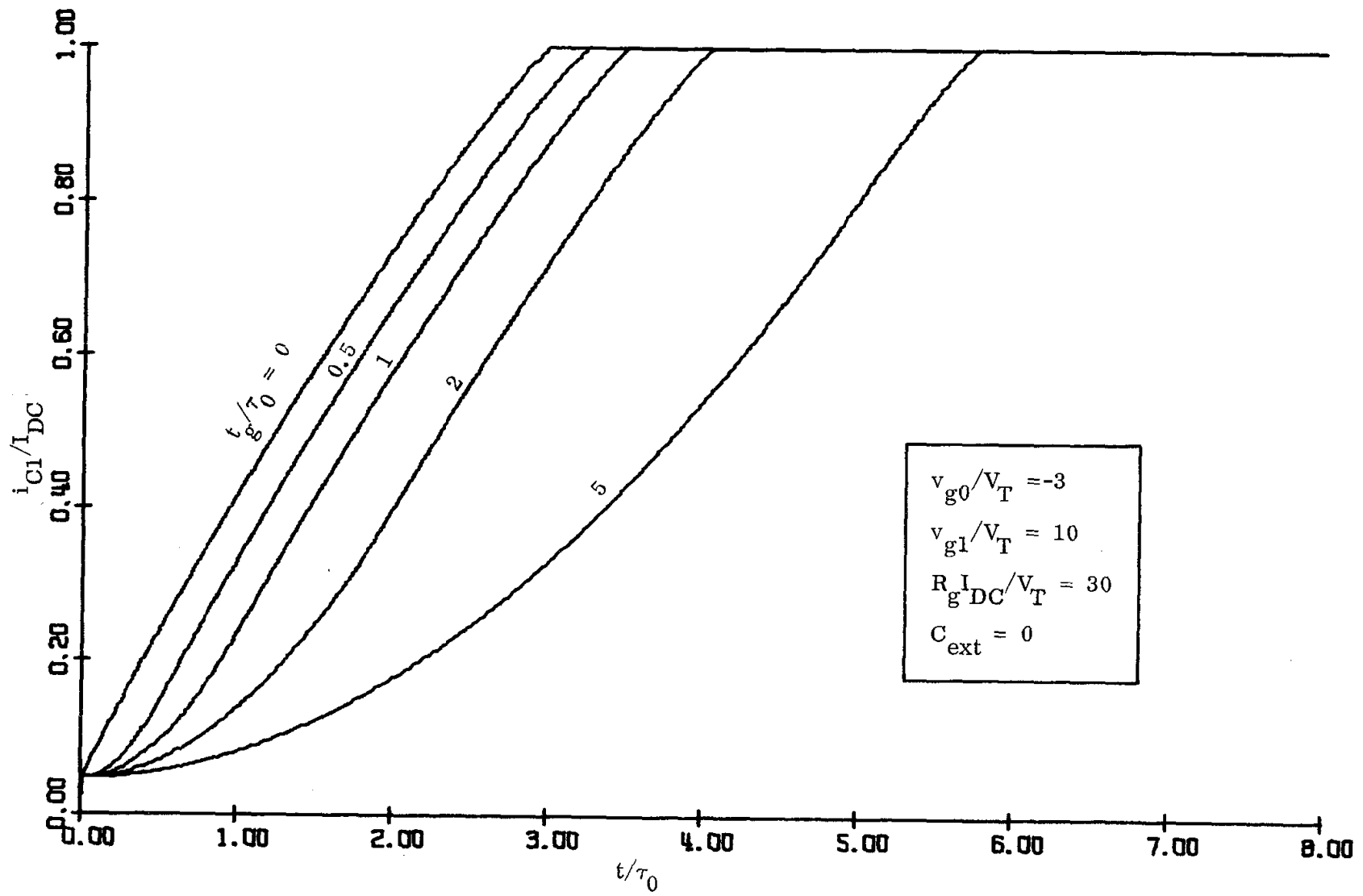


FIG. 8f

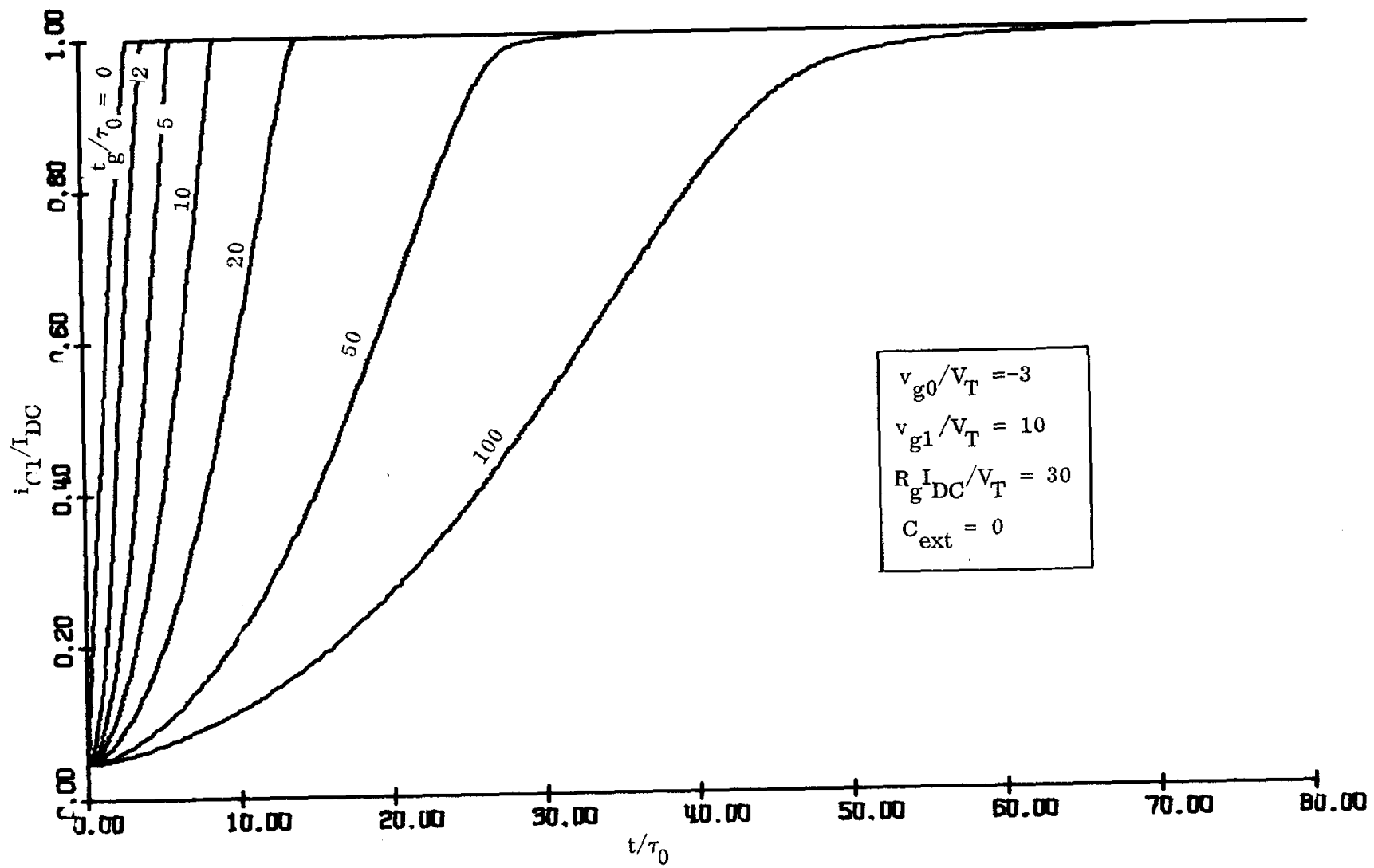


FIG. 8g

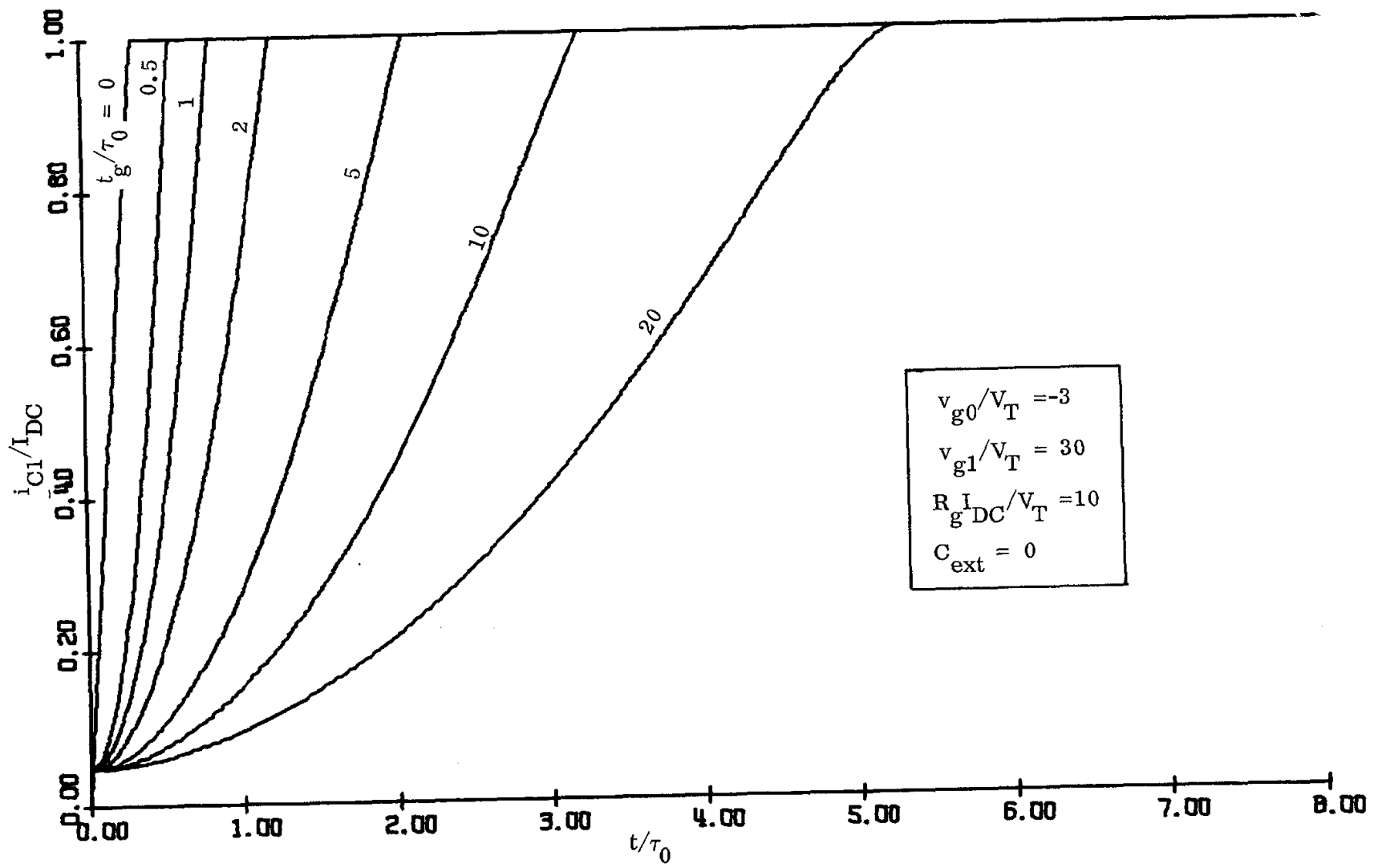


FIG. 8h

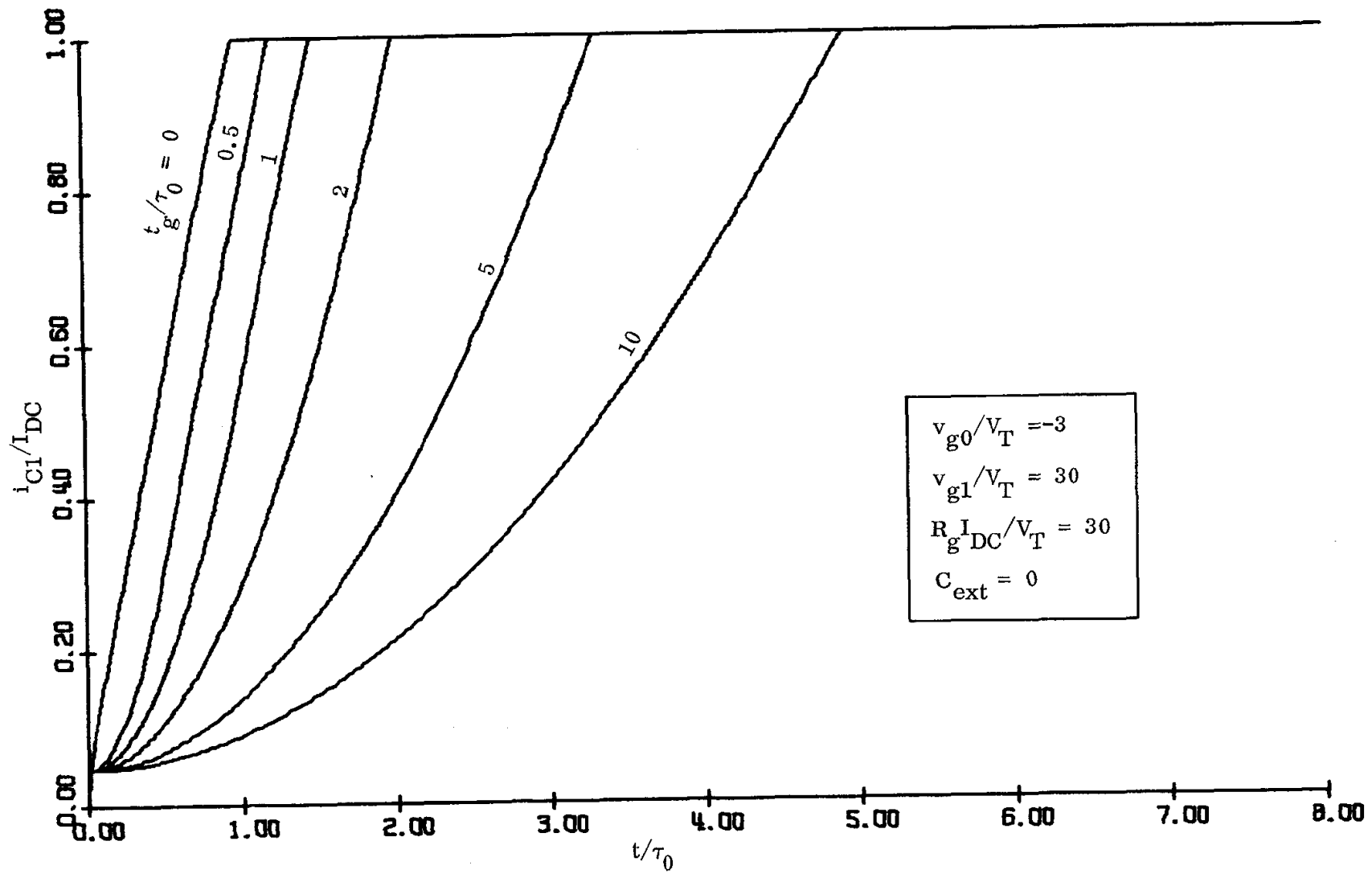


FIG. 8i

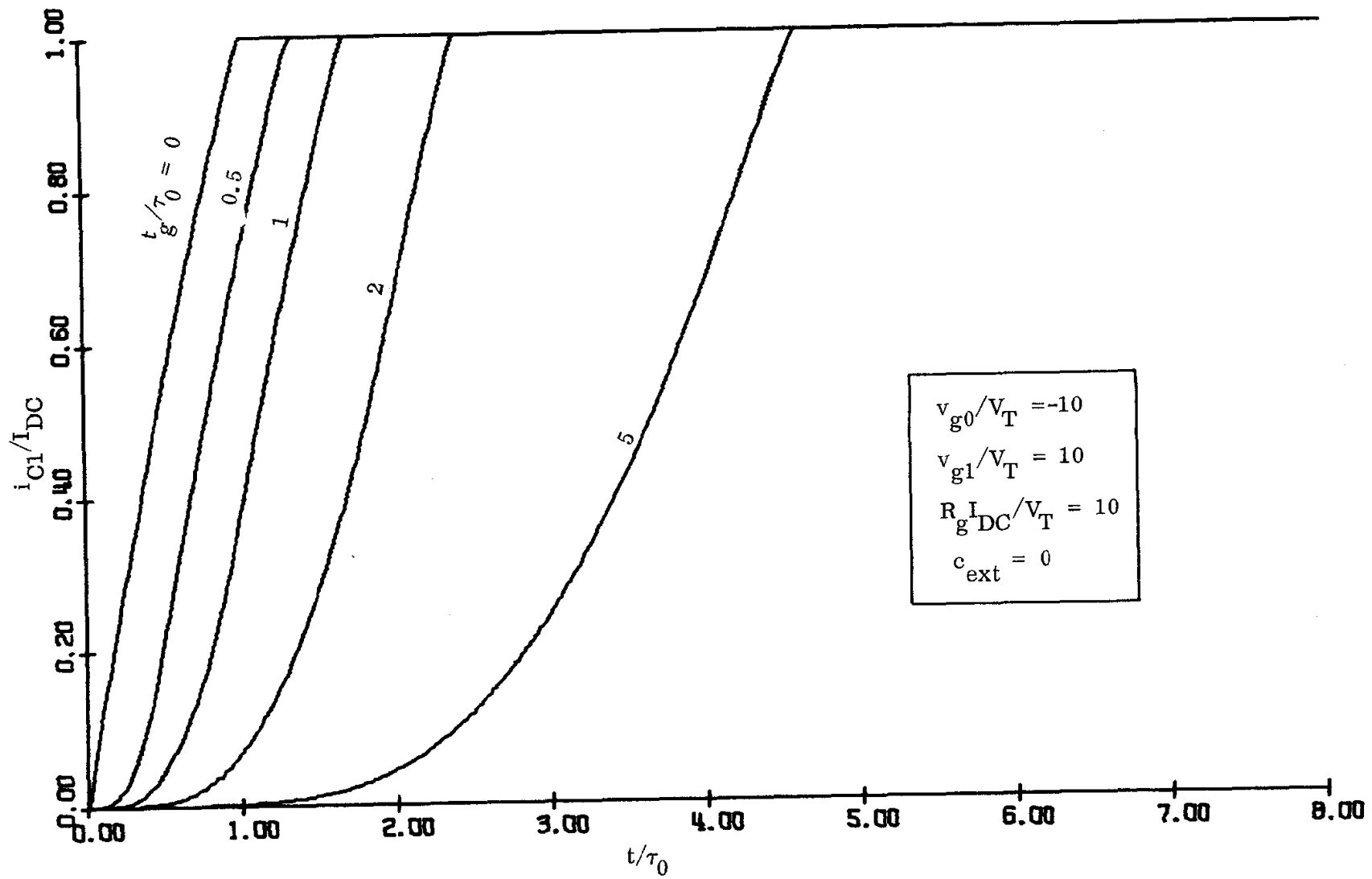


FIG. 8j

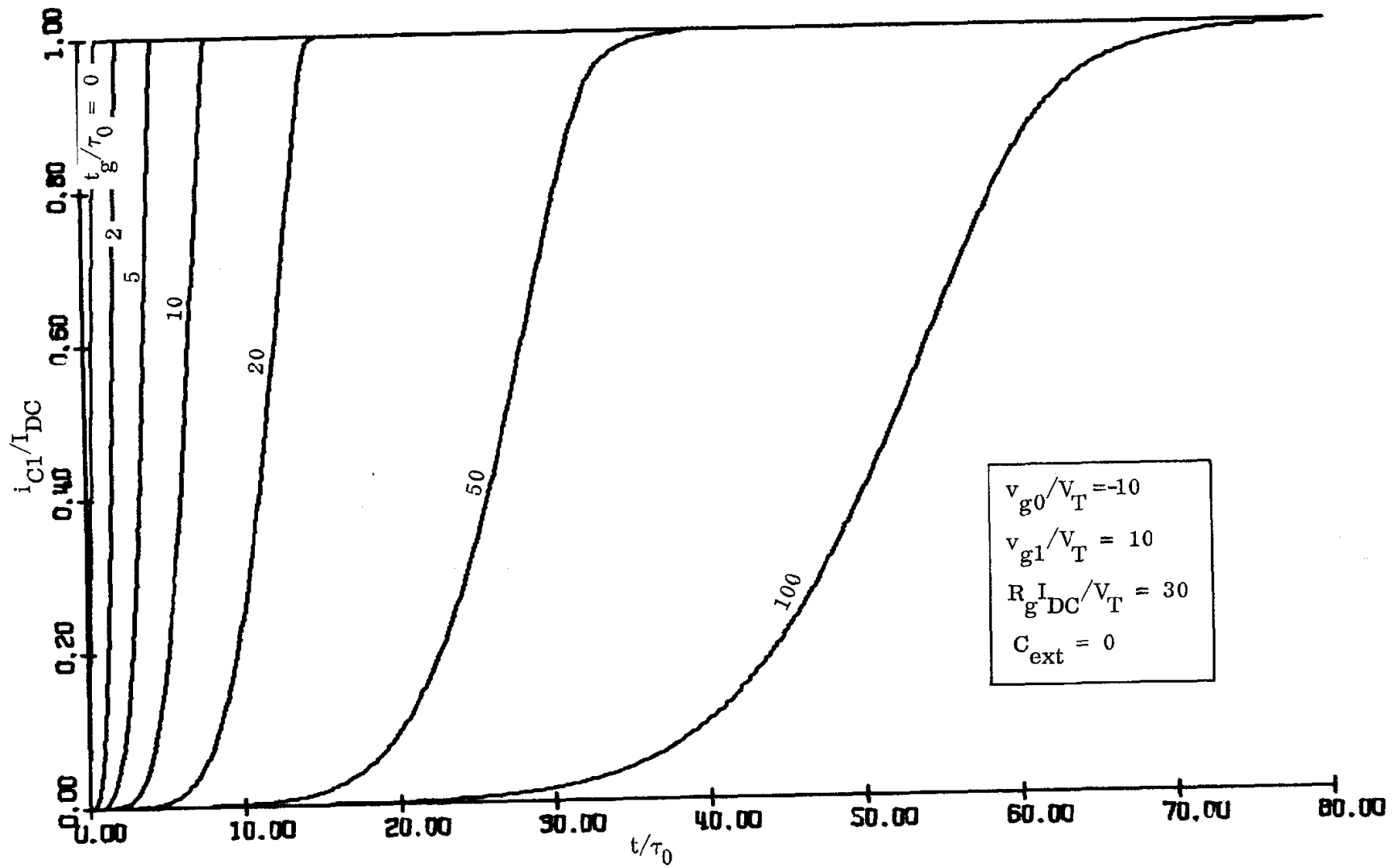


FIG. 8k

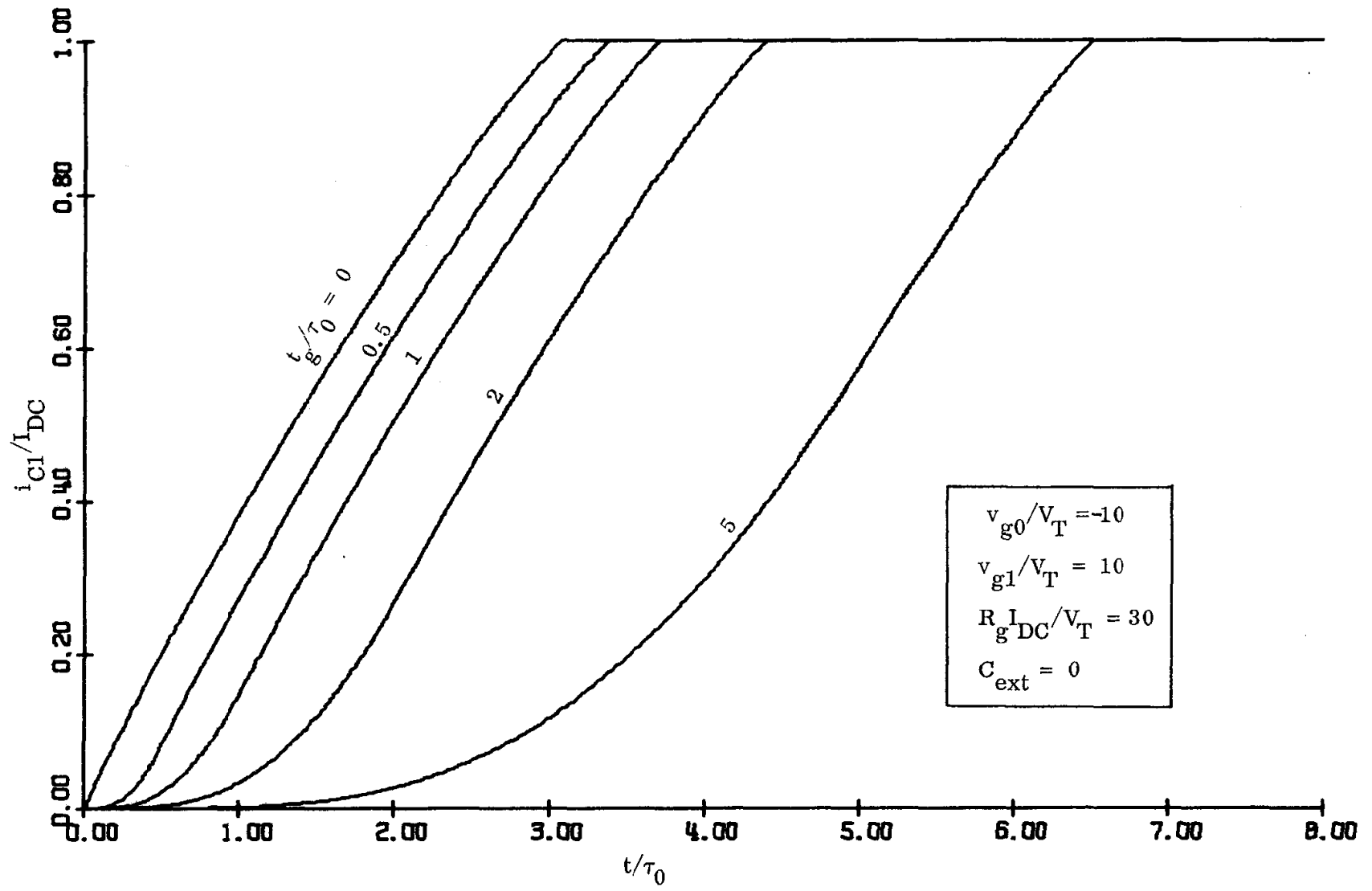


FIG. 81

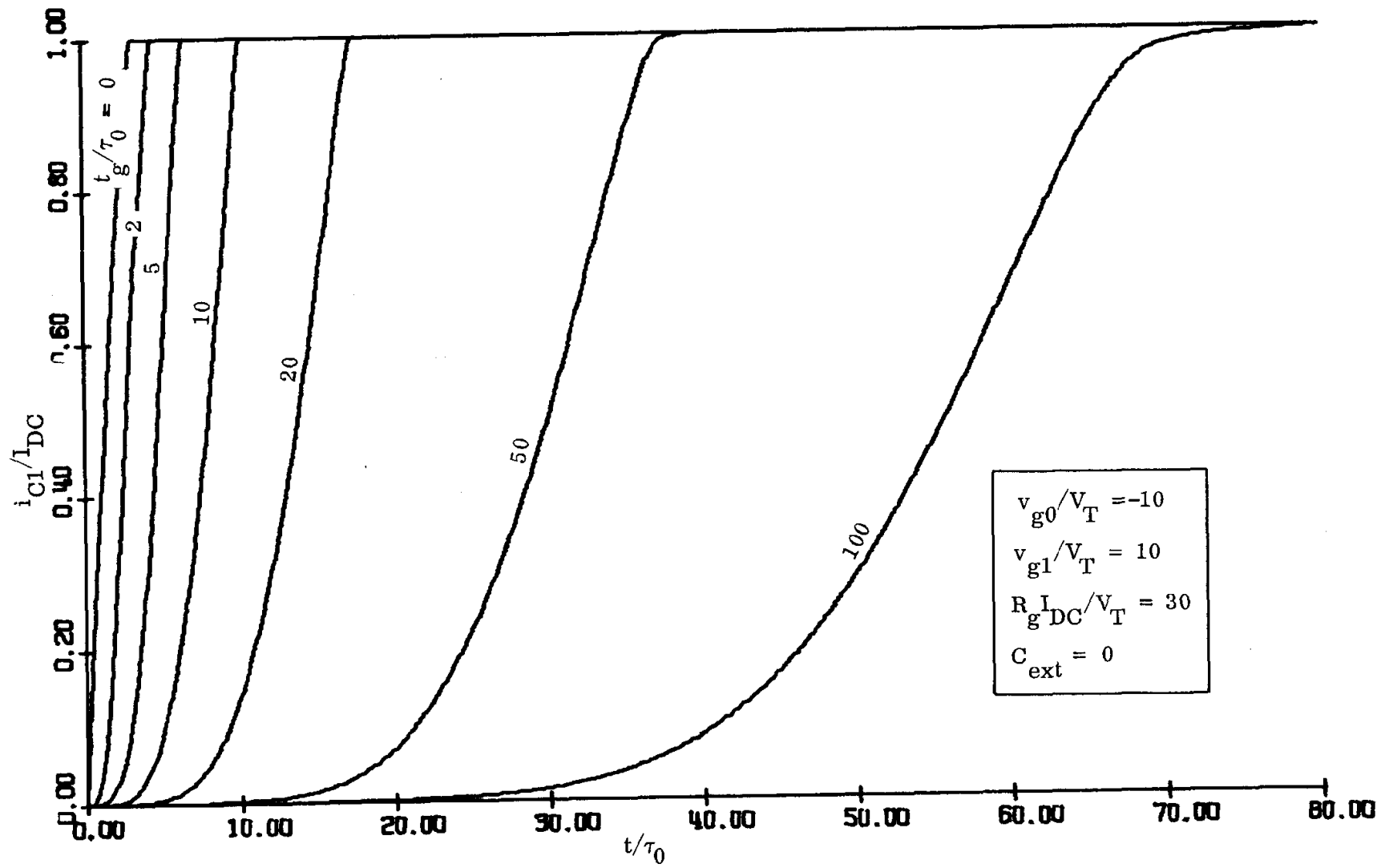


FIG. 8m

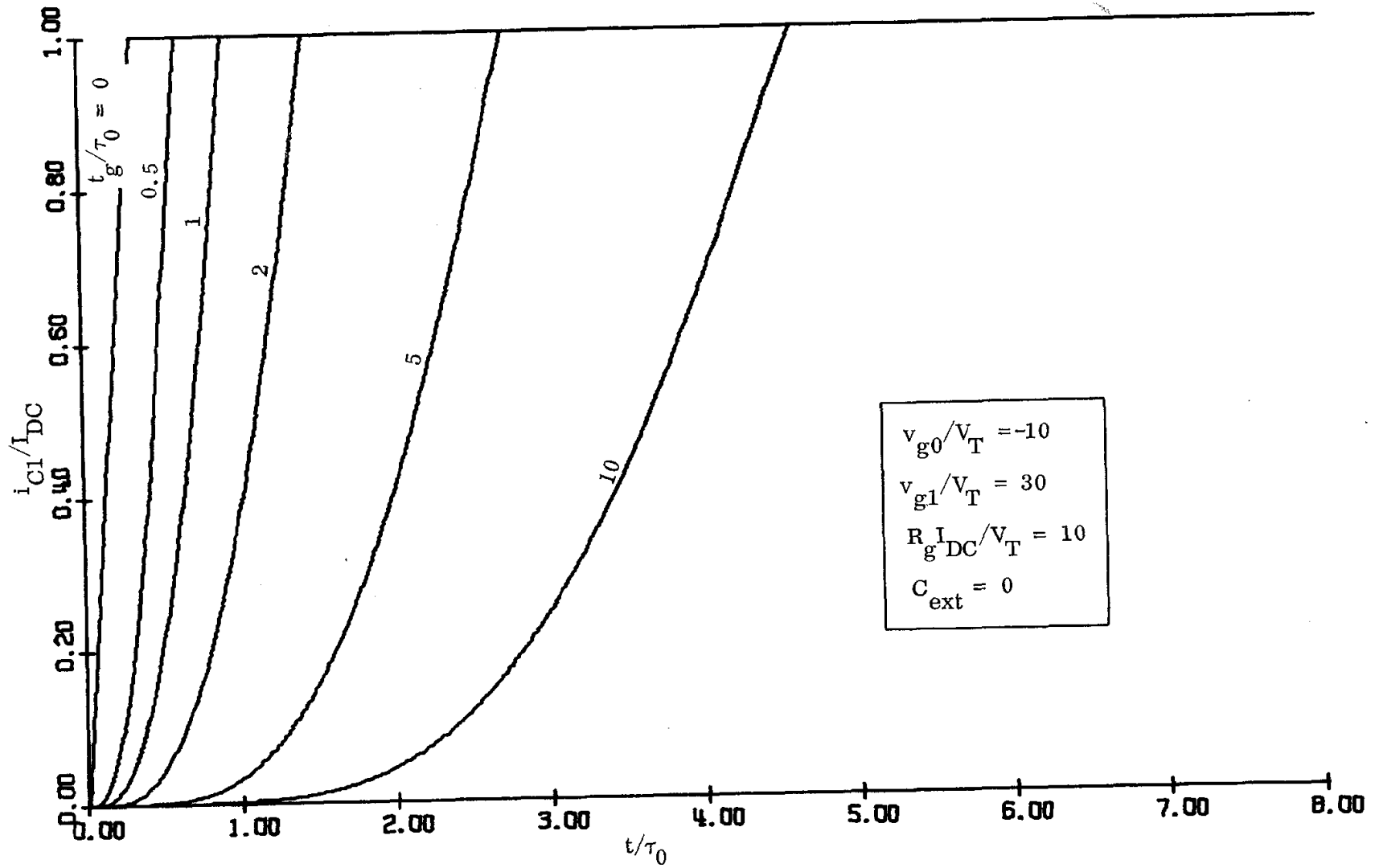


FIG. 8n

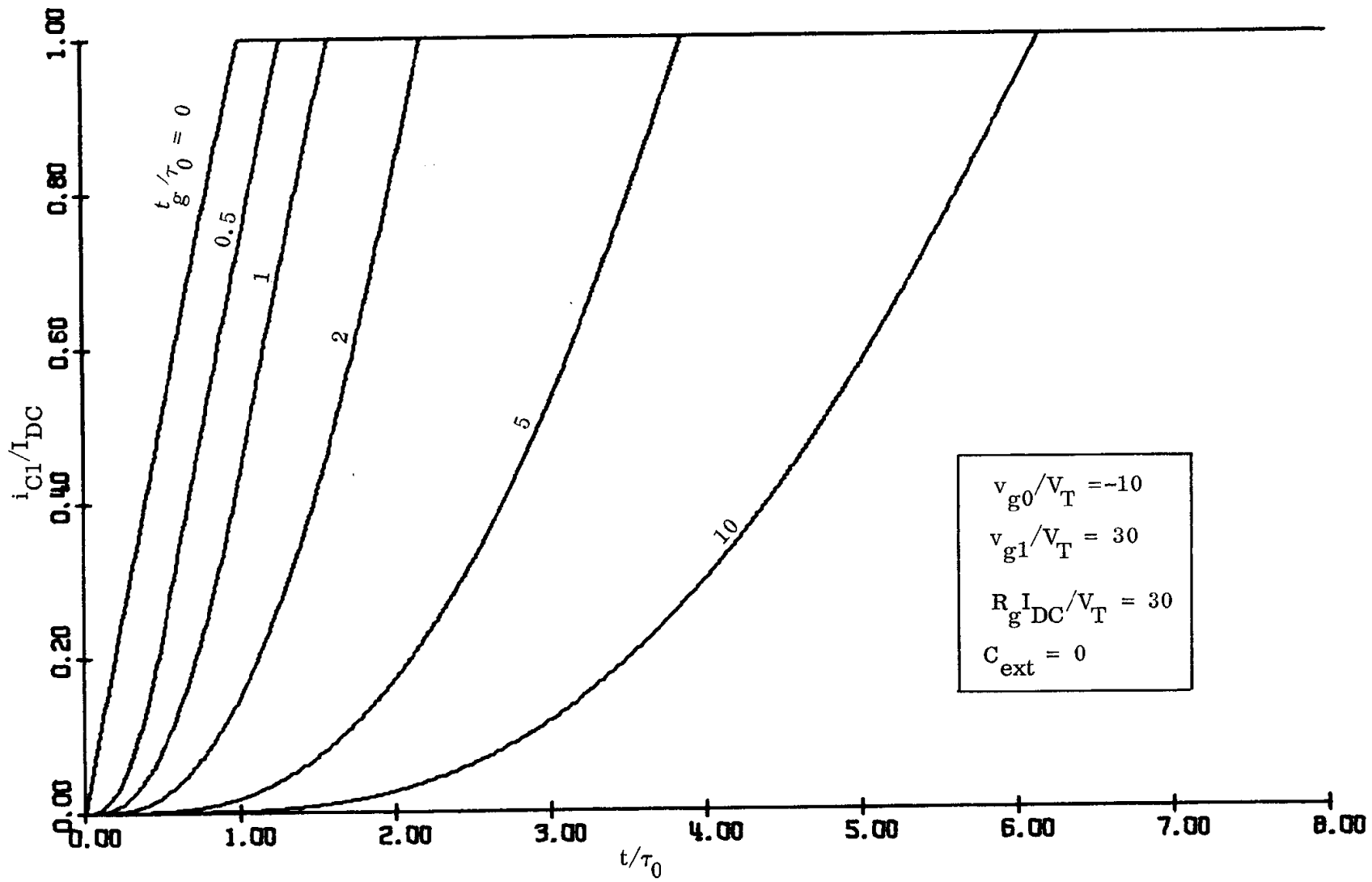


FIG. 8p

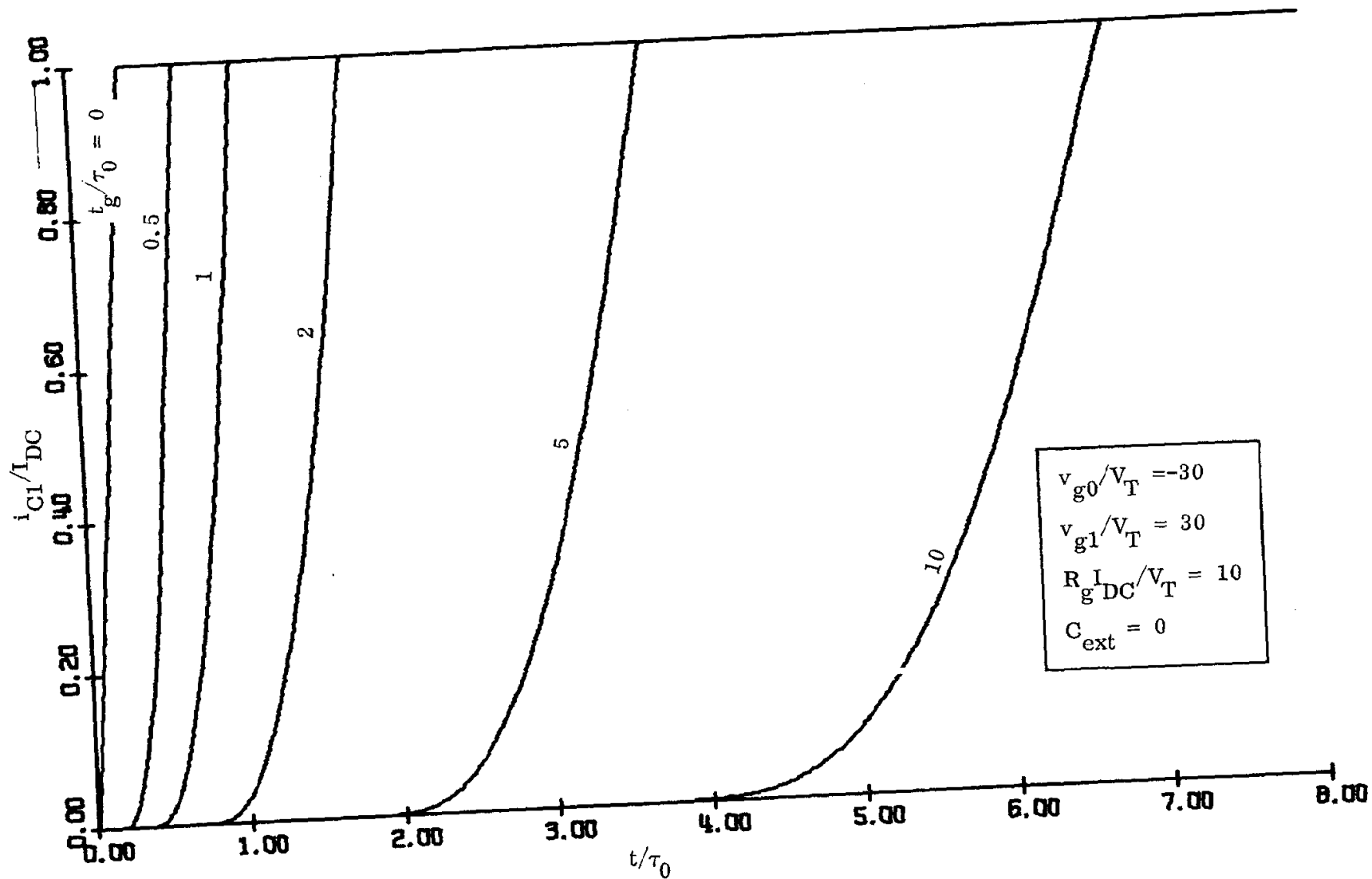


FIG. 8r

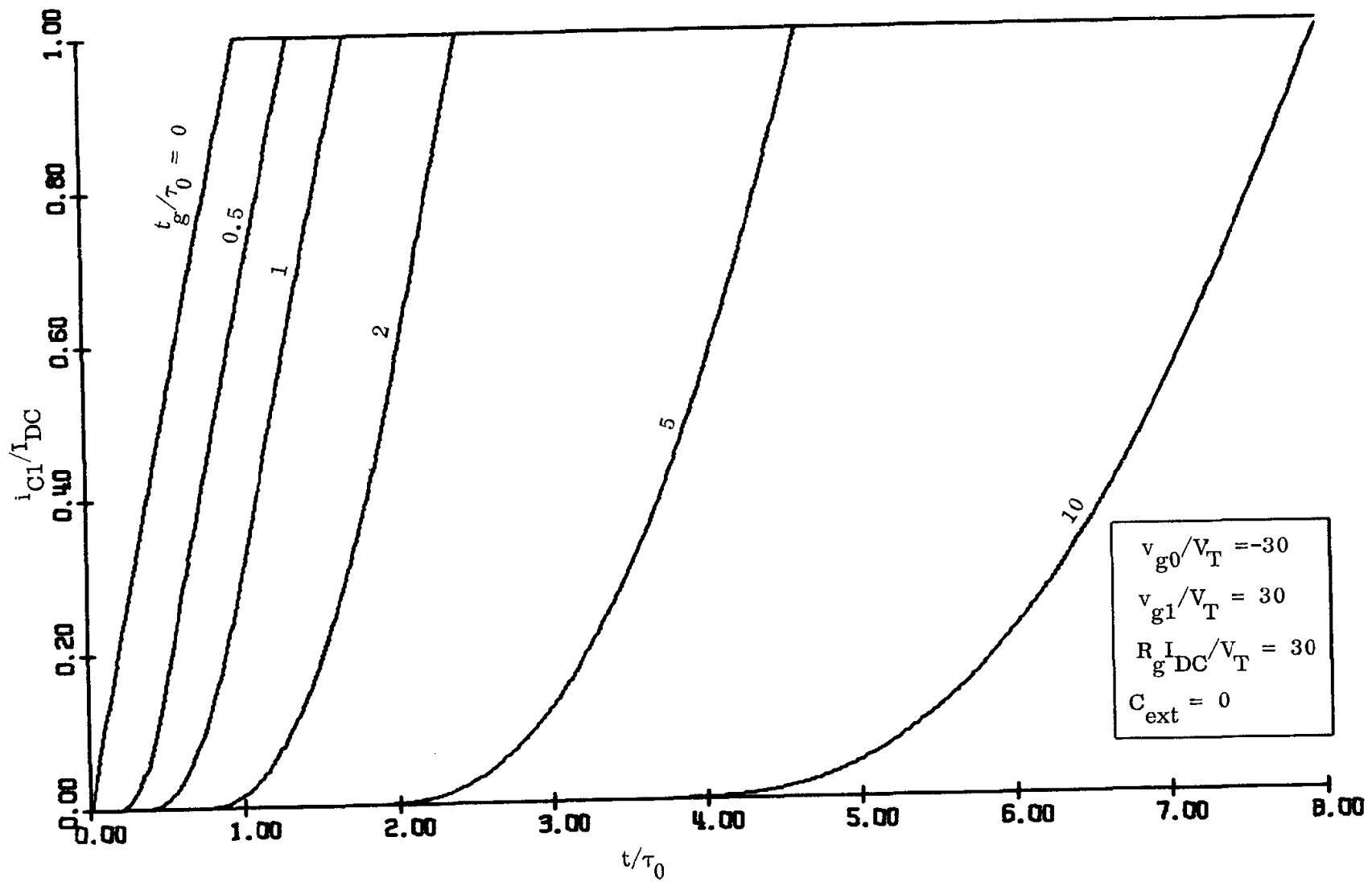


FIG. 8s

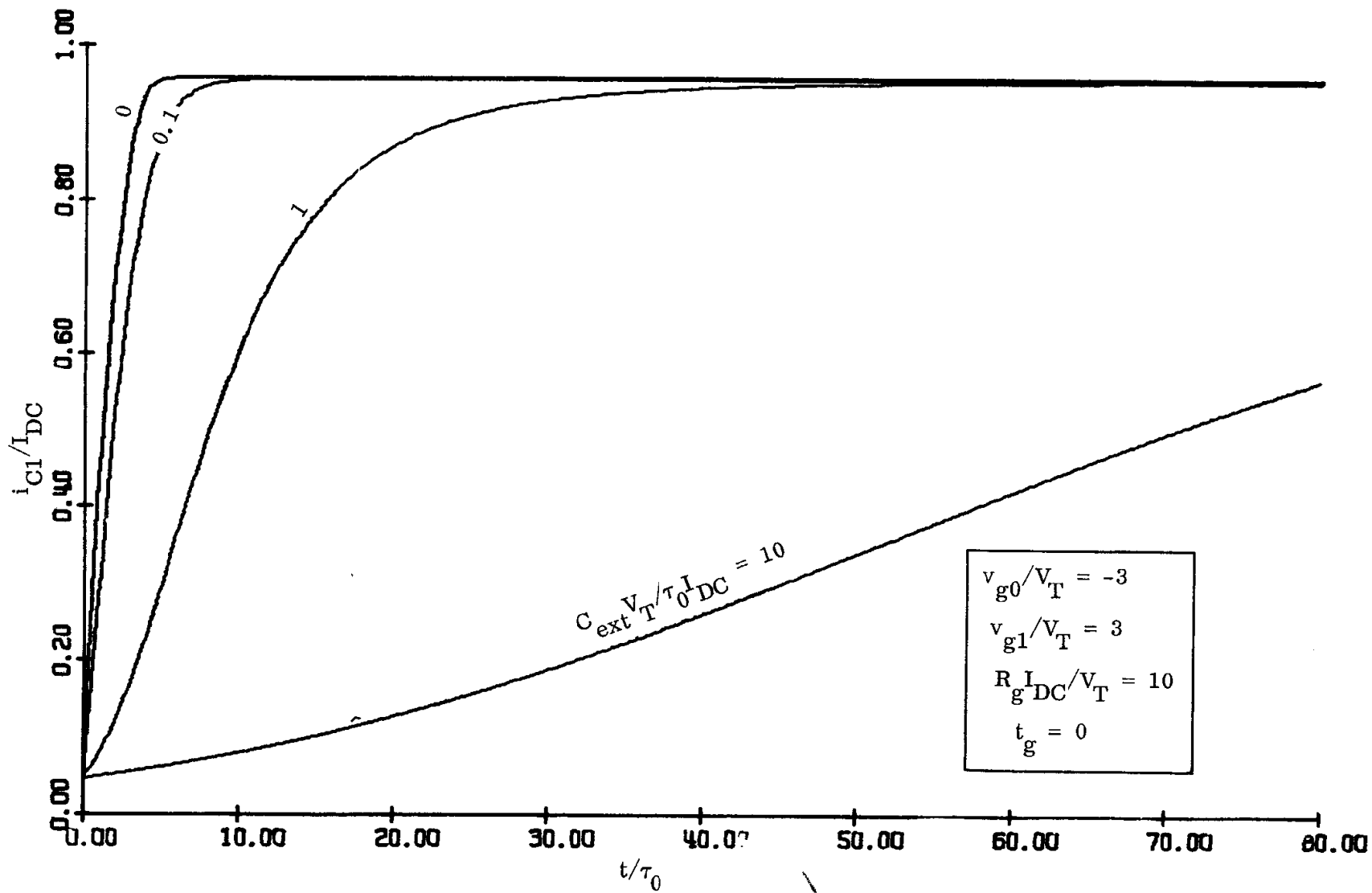


FIG. 9a

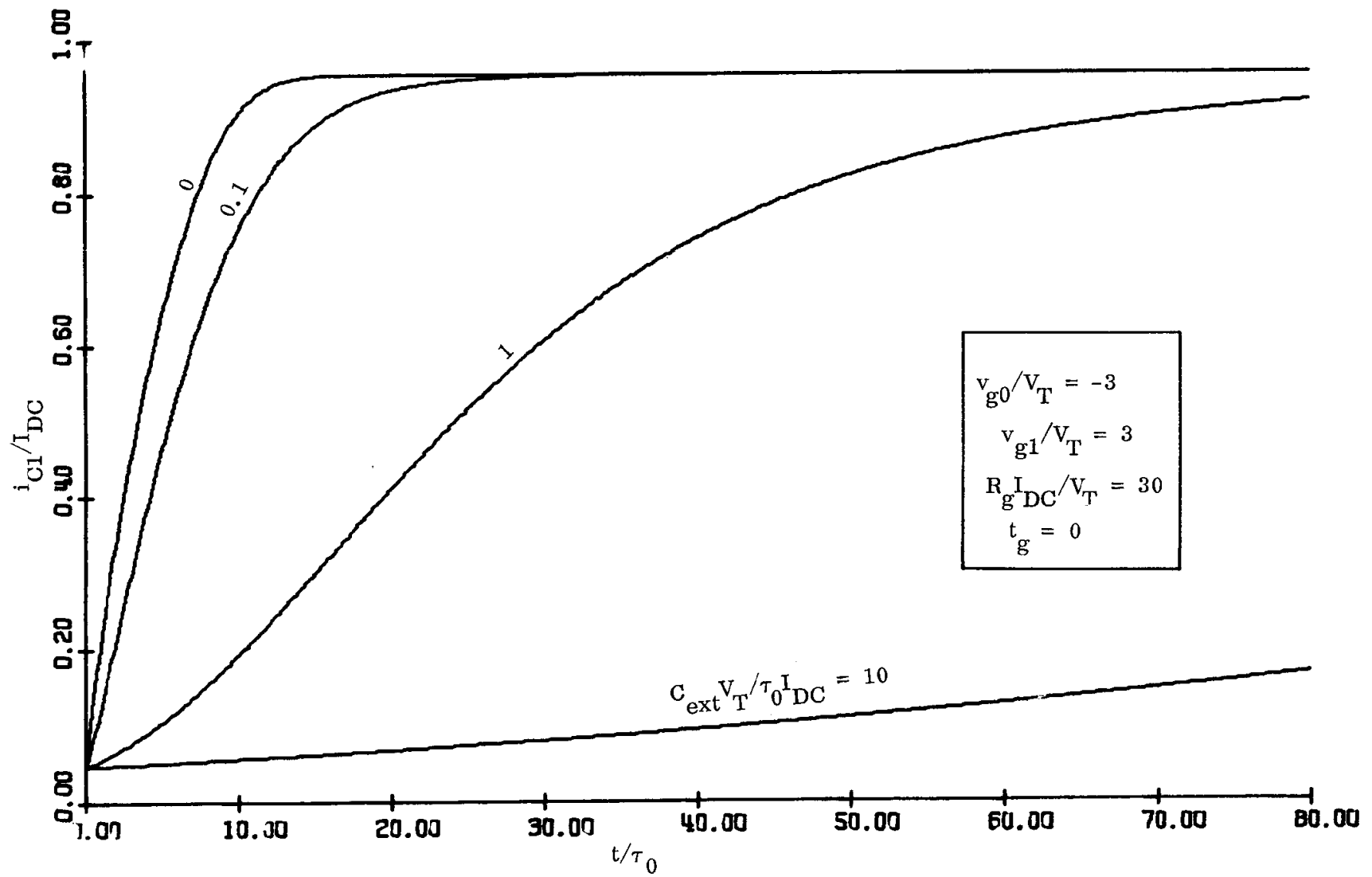


FIG. 9b

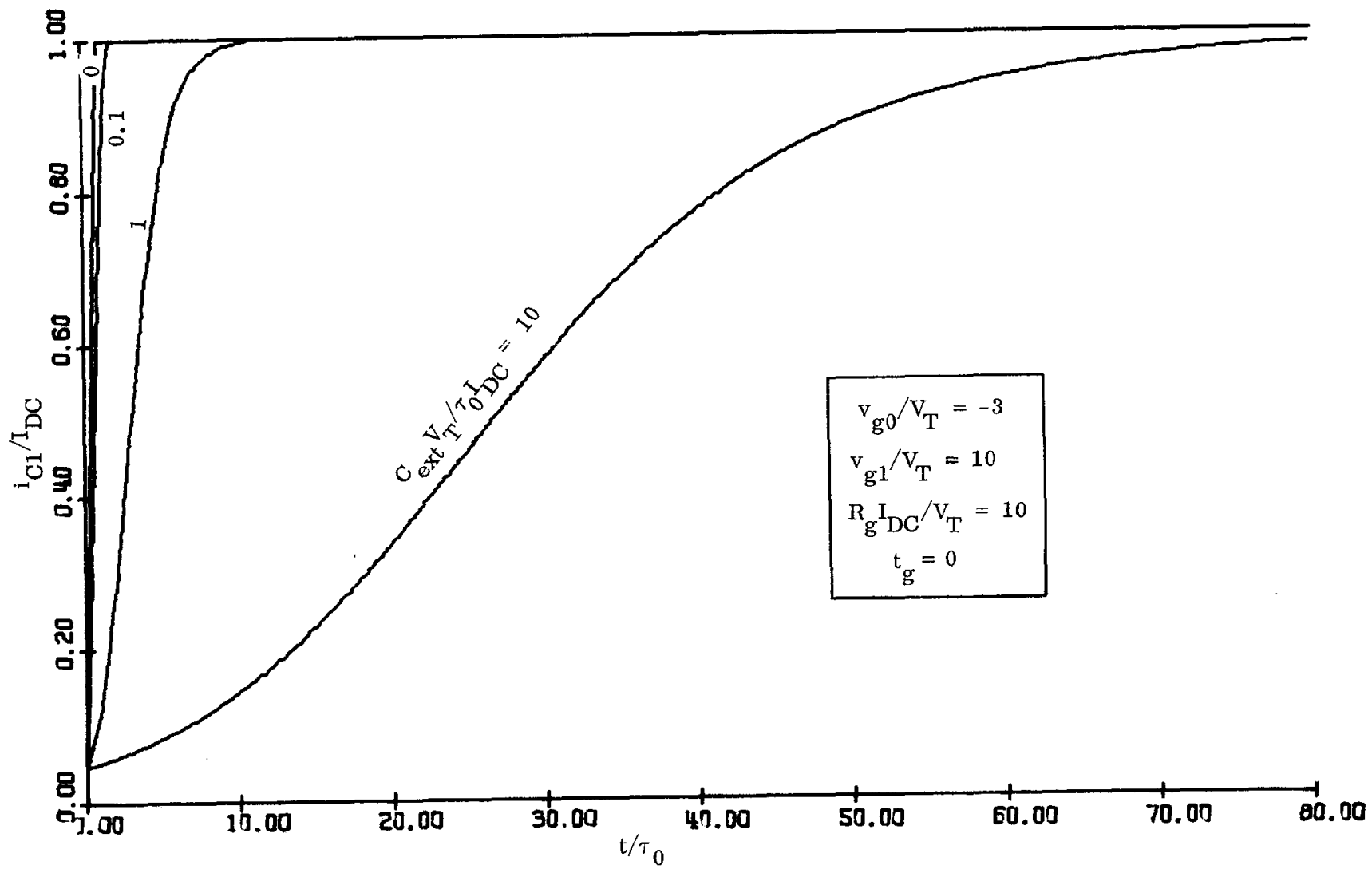


FIG. 9c

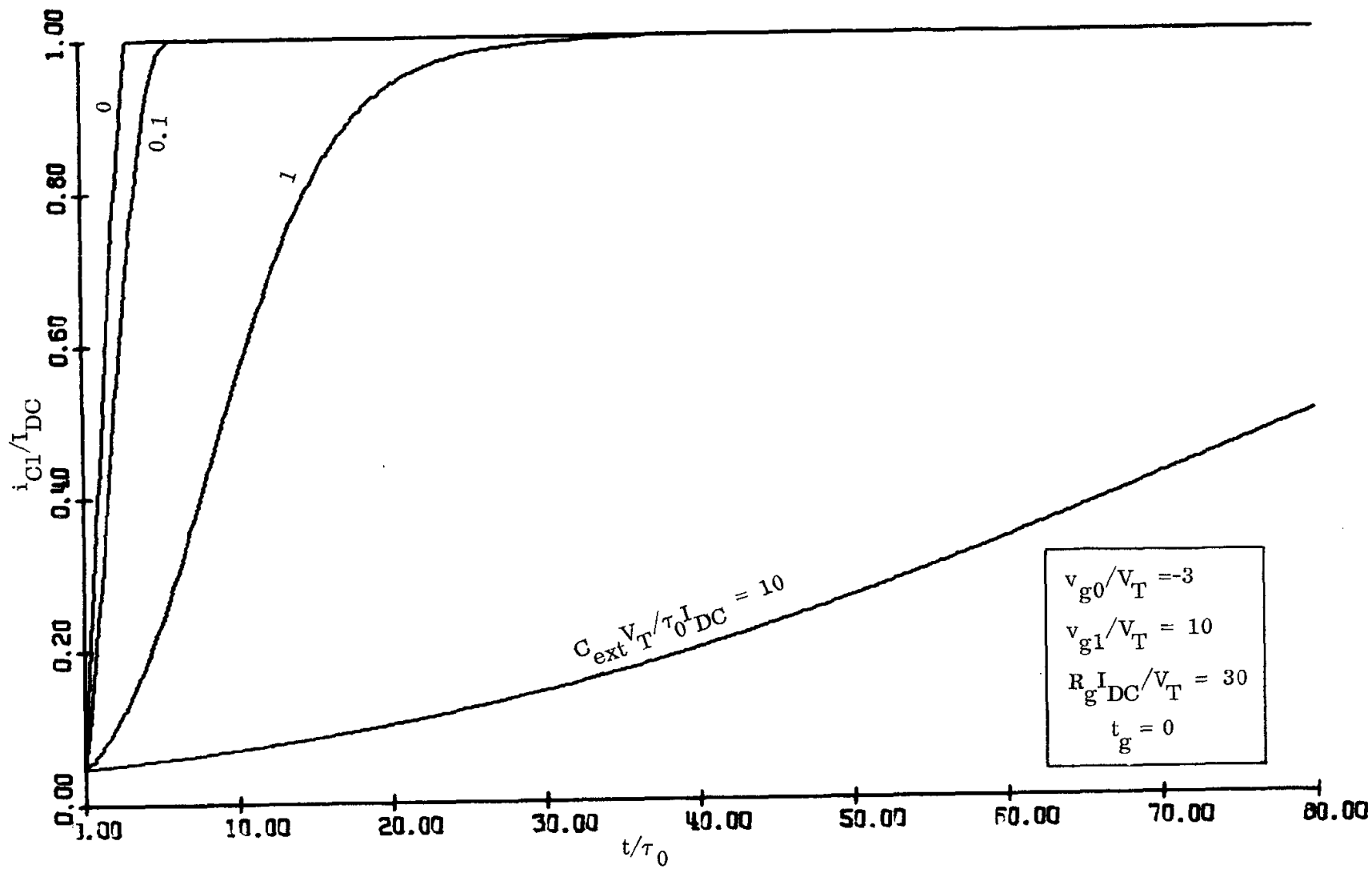


FIG. 9d

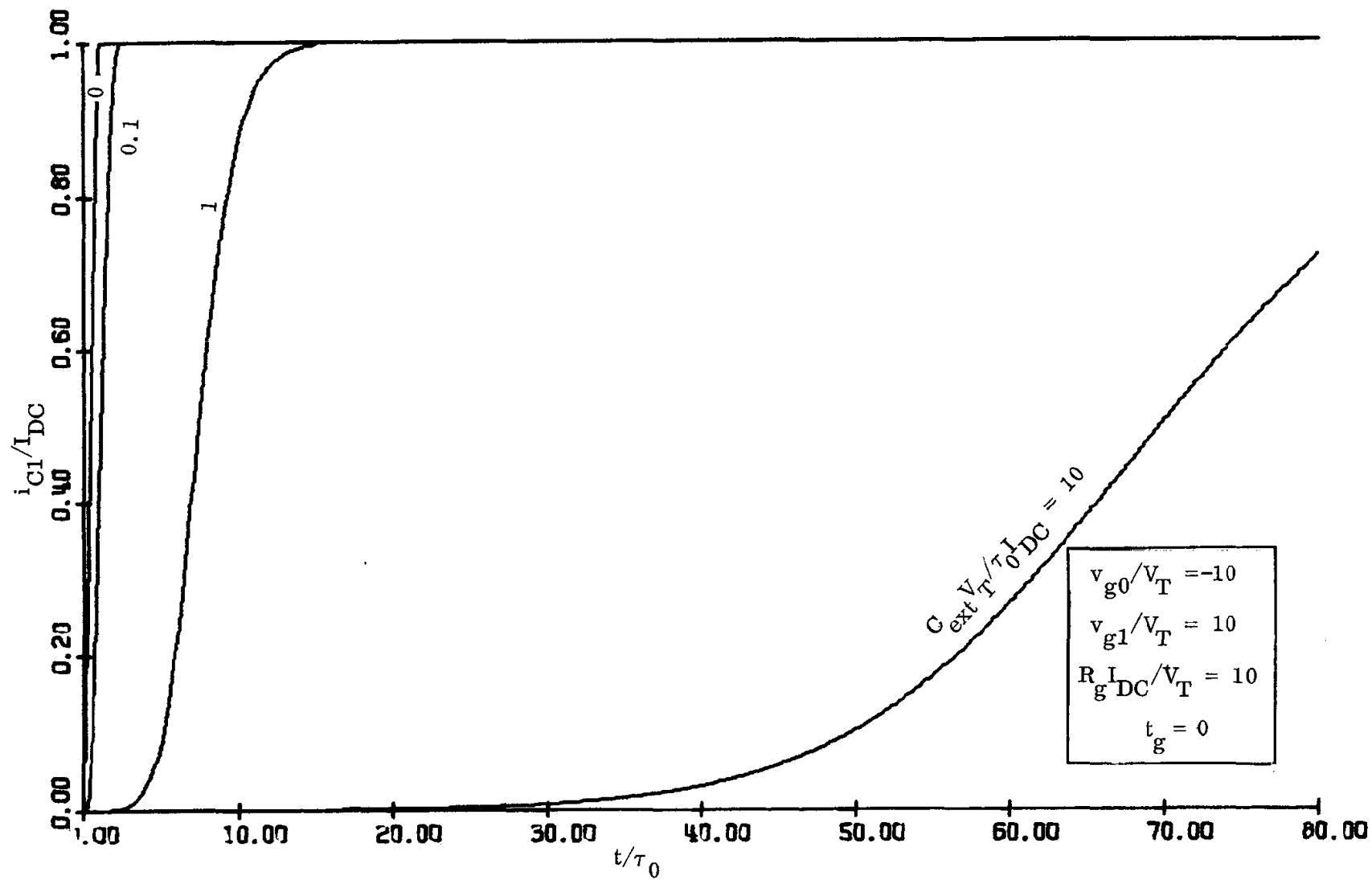


FIG. 9e

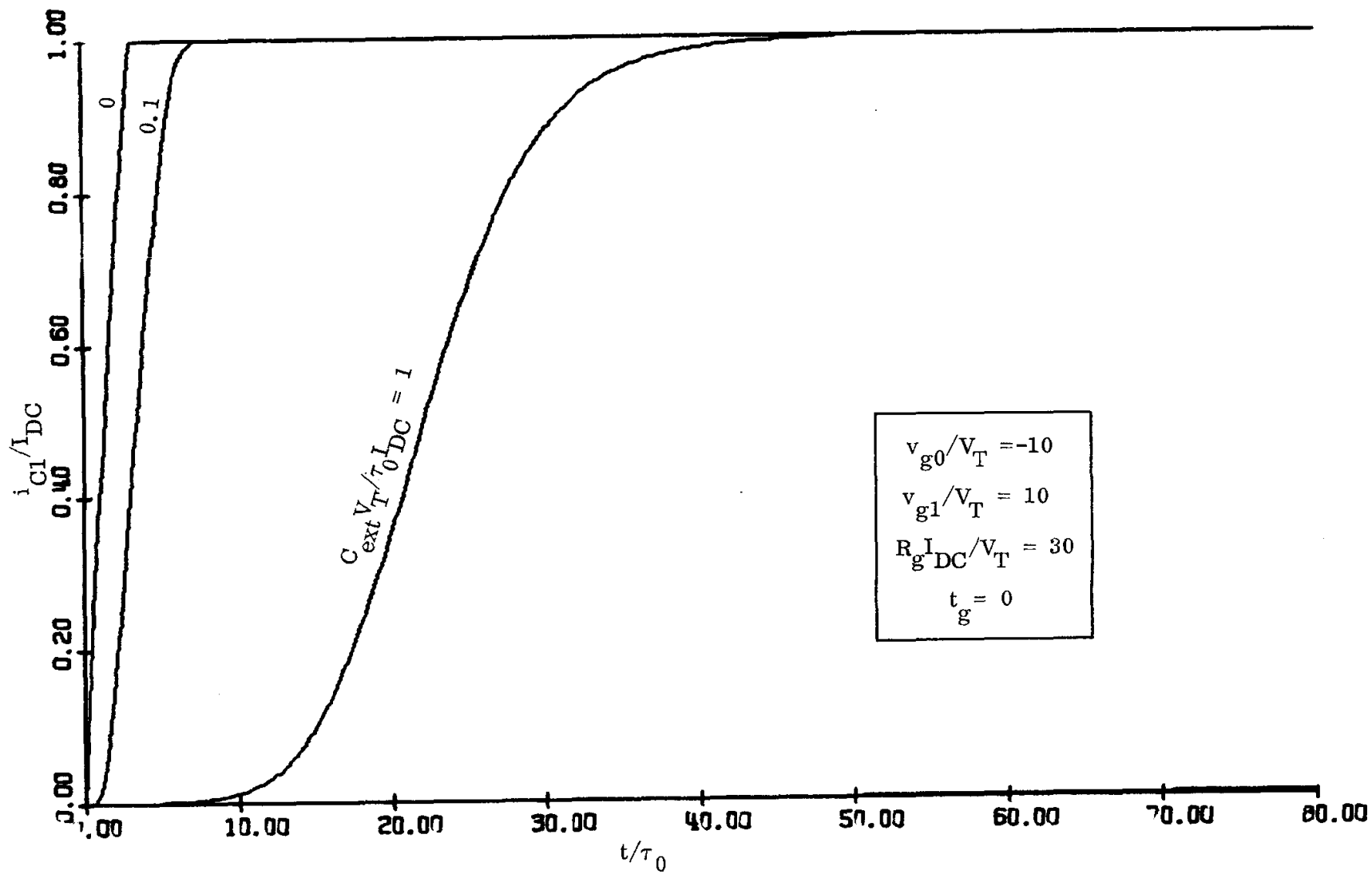


FIG. 9f

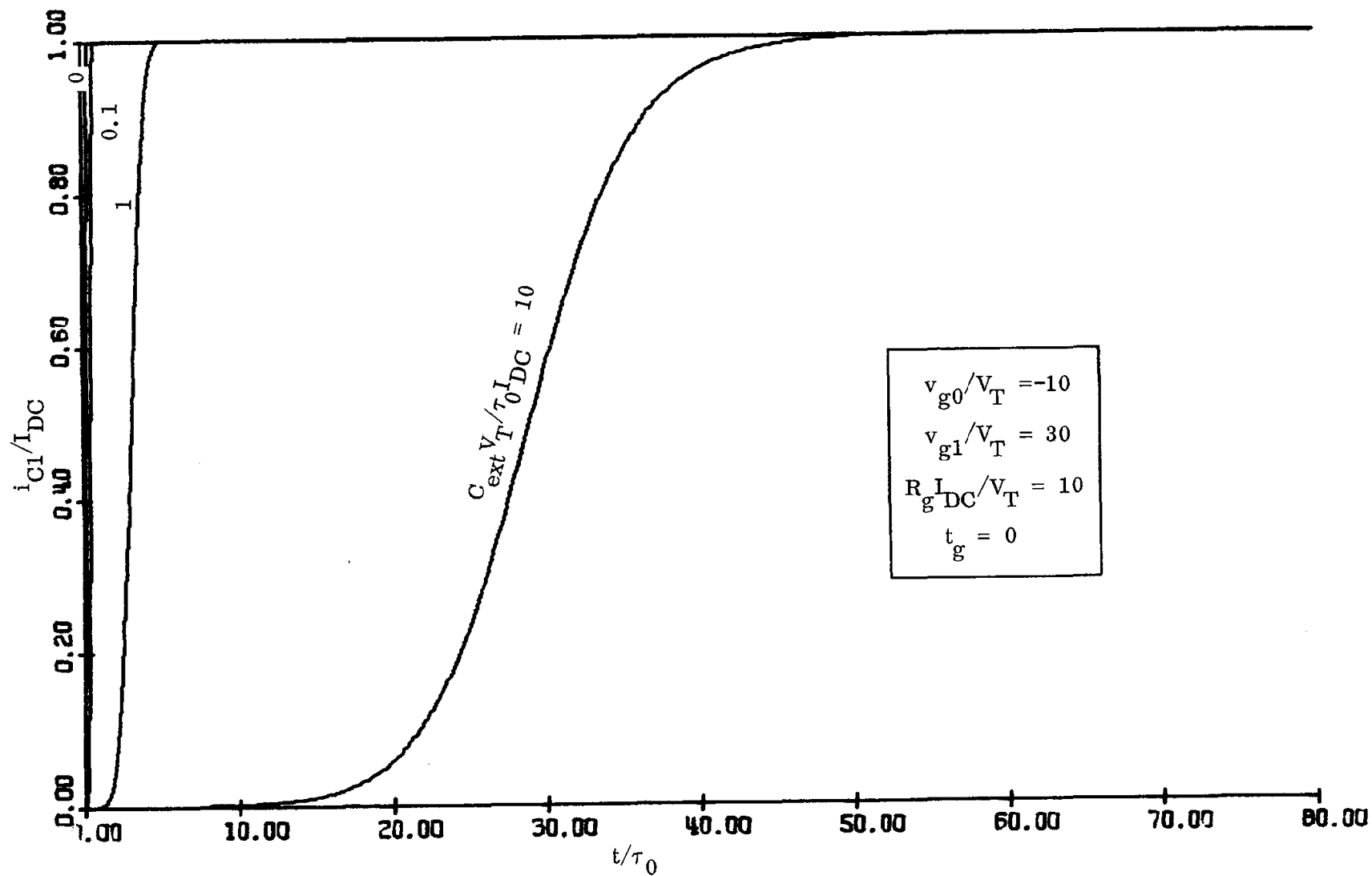


FIG. 9g

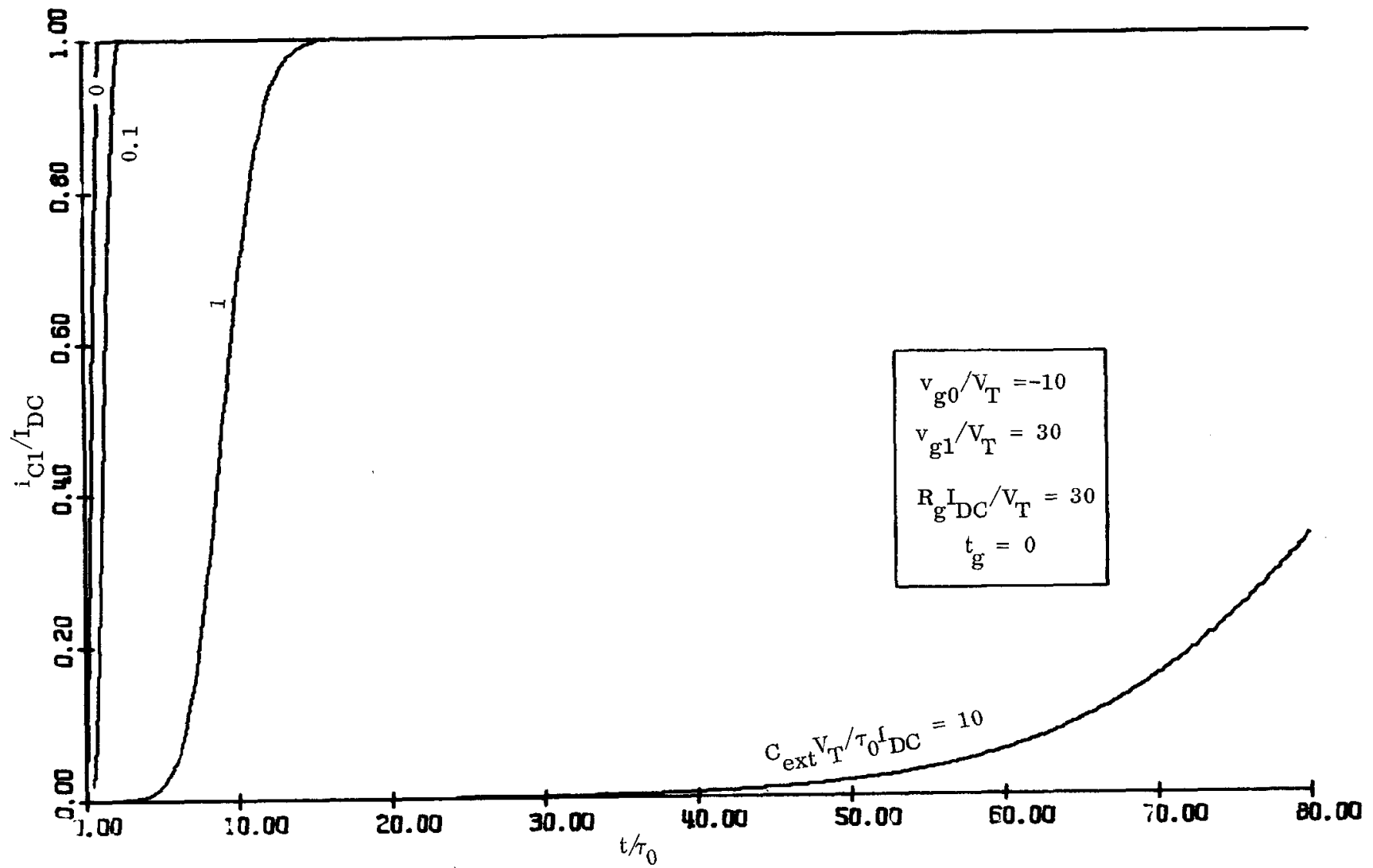


FIG. 9h

```

FUNCTION CS(V)
  IF(V.LT.-100.0)V=-100.0
  IF(V.GT.+100.0)V=+100.0
  CS=EXP(V)
  RETURN
END
FUNCTION CIGCE(V)
  IF(V.LT.-100.0)V=-100.0
  IF(V.GT.+100.0)V=+100.0
  CIGCE=EXP(V)-1.0
  RETURN
END
REAL*8 I1, I2, IZ, V1E, V2E, VE, VGZERO
CALL STFTP1(10)
1  FORMAT(5F10.5,11,F9.5)
2  FORMAT(' ',5F10.5,11,F9.5)
3  FORMAT(' ',1PE10.3,5(1PC10.3),4(1PE10.3))
4  FORMAT(' ',*R ISET(HE=' ',F10.4)
5  FORMAT(F10.5)
6  FORMAT('I',F10.5)
  REAC(5,5)DVL
  WRITE(6,6)DVL
99 CONTINUE
  REAC(5,1)VGZERO, VG, R1, U1, DELT, NEWPLT, A
  IF(U1.LE.0)GO TO 100
  DELT2=0.001*DELT
  WRITE(6,2)VGZERO, VG, R1, U1, DELT, NEWPLT, A
  IF(NEWPLT.EC.0)GO TO 11
  CALL PLCT1(20.0,0.0,-3)
  CALL AXIS1(0.0,0.0,'T',-1.8,0.0,0.0,1.0,10.0)
  CALL AXIS1(0.0,0.0,'I',+1.5,0.0,0.0,0.2,10.0)
11 CONTINUE
  XPLCT=C.0
  YPLCT=0.0
  CALL PLCT1(XPLOT, YPLCT, +3)
  VGZERO=-VGZERO
  TG=1.0/A
  IZ=EXP(-U1)
  I2=0.0
  T1=0.0
  T2=0.0
  VE=DLCG((1.0+DEXP(VGZERO))/(2.0+1.0/IZ))
  V1E=VGZERO-VE
  V2E=-VE
  T=0.0
  XP=-0.02
  YPLOT=-0.1
12 CONTINUE
  I1=I2*CIGCE(V1E)
  IF(T1.EC.0.0.ANC.I1.GE.0.1)T1=T
  IF(T2.EC.0.0.ANC.I1.GE.0.9)T2=T
  IF(T.LT.TG)V=VGZERO+A*T*(VG-VGZERO)
  IF(T.GE.TG)V=VG
  I2=(V-V1E+V2E)/R1
  C1=CS(V1E)*IZ
  C2=CS(V2E)*IZ
  DELT1=DELT
  DELIC1=C1*DELT1
  CV1=DELIC1/C1
  CV2=DELIC1/C2
  ABSDV1=ABS(CV1)
  ABSDV2=ABS(CV2)
  IF(ABSDV1.LT.DVL.ANC.ABSDV2.LT.DVL)GO TO 15
  ABSMAX=AMAX1(ABSDV1,ABSDV2)
  DELT1=CVL*DELT/ABSMAX
  IF(DELT1.LT.DELT2)DELT1=DELT2
  DELIC1=IC1*DELT1
  CV1=DELIC1/C1
  IF(CV1.GT.0.1)DV1=0.1
  IF(DV1.LT.-0.1)DV1=-0.1
  DV2=DELIC1/C2
  IF(DV2.GT.0.1)DV2=0.1
  IF(DV2.LT.-0.1)DV2=-0.1
15 CONTINUE
  T=T+DELT1
  V1E=V1E+DV1
  V2E=V2E+DV2
  XPLCT=T
  IF(XPLCT-XP.LT.0.01)GO TO 14
  XP=XPLCT
  IF(11.GE.0.998)GO TO 98
  YPL=5.0*I1
  IF(YPLCT.GE.YPL.ANC.YPL.GE.4.95)GO TO 98
  YPLCT=YPL
  IF(YPLCT.LT.0.0)YPLCT=0.0
  IF(YPLCT.GE.10.0)YPLCT=10.0
  CALL PLCT1(XPLOT, YPLCT, +2)
14 CONTINUE
  IF(XPLCT.LE.8.0)GO TO 12
  TR=T2-T1
  WRITE(6,4)TR
  GO TO 99
98 CONTINUE
  CALL PLCT1(XPLOT, 5.0, +2)
  CALL PLCT1(8.0, 5.0, +2)
  TR=T2-T1
  WRITE(6,4)TR
  GO TO 99
100 CONTINUE
  CALL PLCT1(20.0,0.0,-3)
  CALL ENCP1
  STOP
END

```

FIG. 10

C

```

FUNCTICA CSIV)
IF(V.LT.-100.0)V=-100.0
IF(V.GT.+100.0)V=+100.0
CS=EXP(V)
RETURN
END
FUNCTICA CIODE(V)
IF(V.LT.-100.0)V=-100.0
IF(V.GT.+100.0)V=+100.0
CIODE=EXP(V)-1.0
RETURN
END
REAL*8 I1,I2,V1E,V2E,VE,VZERO
DIMENSION CE(5)
DATA CE/0.0,0.1,1.0,10.0,C.C1/
CALL STRP1(10)
1 FORMAT(5F10.5,11,F9.5)
2 FORMAT(' ',5F10.5,11,F9.5)
3 FCRMAT(' ',1PE10.3,5(1PC10.3),4(1PE10.3))
4 FORMAT(' ',R1SETIME='F10.4)
5 FCRMAT(F10.5)
6 FCRMAT('1',F10.5)
READ(5,5)DVL
WRITE(6,6)DVL
99 CONTINUE
READ(5,1)VZERO,VG,R1,U1,DELT,NEWPLT,A
IF(U1.LE.0)GO TO 100
DELT2=0.001*DELT
WRITE(6,2)VZERC,VG,R1,U1,DELT,NEWPLT,A
CALL PLCT1(0.0,0.0,+3)
IF(NEWPLT.EC.0)GO TO 11
CALL PLCT1(20.0,0.0,-3)
CALL AXIS1(0.0,0.0,'T',-1,0.0,0.0,0.0,10.,1C.0)
CALL AXIS1(0.0,0.0,'I',+1,5.C,90.0,0.0,0.2,1C.0)
11 CONTINUE
VZERC=-VZERO
TG=1.0/A
IZ=EXP(-U1)
DC 999 J1=1,4
CEXT=CE(J1)
XPLECT=0.0
YPLCT=0.0
CALL PLCT1(XPLECT,YPLCT,+3)
I1=0.0
T1=0.0
T2=0.0
VE=DLOG((1.0+DEXP(VZERO))/(2.0+1.0/IZ))
V1E=VZERC-VE
V2E=-VE
T=0.0
XP=-0.C2
YPLCT=-C.1
12 CONTINUE
I1=IZ*CIODE(V1E)
IF(I1.EC.0.AND.I1.GE.0.1)T1=T
IF(T2.EC.0.AND.T1.GE.0.9)T2=T
IF(T.LT.T6)V=VZERC+A*T*(V-VZERC)
IF(T.GE.T6)V=VG
IC1=(V-V1E+V2E)/R1
C1A=CS(V1E)*T
C2A=CS(V2E)*T
C1=C1A+CEXT*(C1A+C2A)/C2A
C2=C2A+CEXT*(C1A+C2A)/C1A
DELT1=DELT
DELIC1=IC1*DELT1
CV1=DELIC1/C1
CV2=DELIC1/C2
ABSDV1=ABS(DV1)
ABSDV2=ABS(DV2)
IF(ABSDV1.LT.DVL.AND.ABSDV2.LT.DVL)GO TO 15
ABSMAX=MAX1(ABSDV1,ABSDV2)
DELT1=CV1*DELT/ABSMAX
IF(DELT1.LT.DELT2)DELT1=DELT2
DELIC1=IC1*DELT1
CV1=DELIC1/C1
IF(CV1.CT.0.1)DV1=0.1
IF(CV1.LT.-0.1)DV1=-0.1
CV2=DELIC1/C2
IF(CV2.CT.0.1)DV2=0.1
IF(DV2.LT.-0.1)DV2=-0.1
15 CONTINUE
T=T+DELT1
V1E=V1E+DV1
V2E=V2E+DV2
XPLECT=T*0.1
IF(XPLECT-XP.LT.0.01)GO TO 14
XP=XPLECT
IF(I1.GE.0.998)GO TO 98
YPL=5.0*11
IF(YPLCT.GE.YPL.AND.YPL.GE.4.95)GO TO 98
YPLCT=YPL
IF(YPLCT.LT.0.0)YPLCT=0.0
IF(YPLCT.GE.10.0)YPLCT=10.0
CALL PLCT1(XPLECT,YPLCT,+2)
14 CONTINUE
IF(XPLECT.LE.0.0)GO TO 12
GO TO 97
98 CONTINUE
CALL PLCT1(XPLECT,5.0,+2)
CALL PLCT1(8.0,5.0,+2)
97 CONTINUE
TR=T2-T1
WRITE(6,4)TR
999 CONTINUE
GC TC 95
100 CONTINUE
CALL PLCT1(20.0,0.0,-3)
CALL ENCP1
STCP
END

```

3.0 3.0 10.0 20.0 0.C1 1900.0

FIG. 11

```

C TRANSISTOR PAIR WITH EXTERNAL CAPACITANCE***
C
FUNCTION CS(V)
IF(V.LT.-100.0)V=-100.0
IF(V.GT.+100.0)V=+100.0
CS=EXP(V)
RETURN
END

C
FUNCTION CICE(V)
IF(V.LT.-100.0)V=-100.0
IF(V.GT.+100.0)V=+100.0
CICE=EXP(V)-1.0
RETURN
END

C
FUNCTION TR(VGZERO,VG,R1,U1,DELTA,A,CEXT)
DOUBLE PRECISION I1,IC1,IZ,V1E,V2E,VE,VGZERO
DVL=0.01
DELTA2=0.001*DELTA
VGZERC=-VGZERO
TG=1.0/A
IZ=EXP(-U1)
IC1=0.0
I1=0.0
I2=0.0
VE=DLCG((1.0+DEXP(VGZERO))/(2.0+1.0/I2))
V1E=VGZERC-VE
V2E=-VE
T=0.0
12 CONTINUE
I1=IZ*(IICE(V1E))
IF(I1.EC.0.0.ANC.I1.GE.0.1)I1=T
IF(I2.EC.0.0.ANC.I1.GE.0.9)GO TO 101
IF(T.LT.TG)V=VGZERC+A*T*(VC-VGZERO)
IF(T.GE.TG)V=VG
IC1=(V-V1E+V2E)/R1
C1A=CS(V1E)*IZ
C2A=CS(V2E)*IZ
C1=C1A+CEXT*(C1A+C2A)/C2A
C2=C2A+CEXT*(C1A+C2A)/C1A
DELTA=DELTA
DELIC1=IC1*DELTA
CV1=DELIC1/C1
CV2=DELIC1/C2
ABSDV1=ABS(CV1)
ABSDV2=ABS(CV2)
IF(ABSDV1.LT.DVL.ANC.ABSDV2.LT.CVL)GO TO 15
ABSMAX=AMAX1(ABSDV1,ABSDV2)
DELTA=CVL*DELTA/ABSMAX
IF(DELTA.LT.DELTA2)DELTA=DELTA2
DELIC1=IC1*DELTA
CV1=DELIC1/C1
IF(DVL.CT.0.1)DV1=0.1
IF(DVL.LT.-0.1)DV1=-0.1
CV2=DELIC1/C2
IF(CV2.CT.0.1)DV2=0.1
IF(CV2.LT.-0.1)DV2=-0.1
15 CONTINUE
T=T+DELTA
V1E=V1E+DV1
V2E=V2E+DV2
GC TO 12
101 CONTINUE
I2=I1
TR=I2-T1
RETURN
END

C
FUNCTION TC(VGZERO,VG,R1,A,CEXT)
ARG1=9.0
ALG9=ALG(ARG1)*2
TR1=R1*(1.8/(VG-0.4)+ALG9*CEXT/(VG-0.8))
TR1SQ=TR1*TR1
TR2=ALG9/(1+VG+VGZERO)*A
TR2SQ=TR2*TR2
TG=SQRT(TR1SQ+TR2SQ)
RETURN
END

C
1 FORMAT(5F10.5,11,F9.5)
2 FORMAT(' ',I2,1X,I3,1X,I2,F5.1,1X,9(' ',F5.1,' ',F5.1,' '))
3 FORMAT('1,')
4 FORMAT('1,')
5 FORMAT(' ',15X,9(' EXACT APPROX'))
WRITE(6,3)
WRITE(6,5)
WRITE(6,4)

C
DDIMENSION CE(4),TRWR(9),TRCW(9),VGZM(16),VG1M(16),R1M(16),AP(9),
1XTRWR(9),XTRCW(9)
DATA CE/0.0,0.1,1.0,10.0/,VGZM/6*3.0,4*10.0,2*30.0/
DATA VG1M/2*3.0,2*10.0,2*30.0,2*10.0,4*30.0/
DATA AM/900.0,2.0,1.0,0.5,0.2,0.1,0.05,C,C2,C.01/
DELTA=0.01
U1=20.0
DO 1001 J11=1,8
R1M(2*J11)=30.0
R1M(2*J11-1)=10.0
1001 CONTINUE

C
DC 1002 J12=1,12
VGZERC=VGZM(J12)
VG=VG1M(J12)
R1=R1M(J12)
DC 1003 J13=1,4
CEXT=CE(J13)
DC 1004 J14=1,9
A=AM(J14)
TRWR(J14)=TR(VGZERO,VG,R1,U1,DELTA,A,CEXT)
XTRWR(J14)=0.1*INT(10.0*TRWR(J14)+0.5)
TRCW(J14)=T(VGZERO,VG,R1,A,CEXT)
XTRCW(J14)=0.1*INT(10.0*TRCW(J14)+0.5)
1004 CONTINUE
IVGZ=INT(VGZERO*0.5)
IVG1=INT(VG*0.5)
IR1=INT(R1*0.5)
WRITE(6,2)IVGZ,IVG1,IR1,CEXT,(XTRWR(I),XTRCW(I),I=1,9)
1003 CONTINUE
1002 CONTINUE
STCP
END

```

FIG. 12