# SPIN STRUCTURE MEASUREMENTS FROM E143 AT SLAC

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#### ABSTRACT

Measurements have been made of the proton and deuteron spin structure functions,  $g_1^p$  and  $g_1^d$  at beam energies of 29.1, 16.2, and 9.7 GeV, and  $g_2^p$  and  $g_2^d$  at a beam energy of 29.1 GeV. The integrals  $\Gamma_p = \int_0^1 g_1^p(x, Q^2) dx$  and  $\Gamma_d = \int_0^1 g_1^d(x, Q^2) dx$  have been evaluated at fixed  $Q^2 = 3$  (GeV/c)<sup>2</sup> using the 29.1 GeV data to yield  $\Gamma_p =$  $0.127 \pm 0.004(\text{stat.}) \pm 0.010(\text{syst.})$  and  $\Gamma_d = 0.041 \pm 0.003 \pm 0.004$ . The  $Q^2$  dependence of the ratio  $g_1/F_1$  has been studied and is found to be small for  $Q^2 > 1$  (GeV/c)<sup>2</sup>. Within experimental precision the  $g_2$  data are well-described by the twist-2 contribution,  $g_2^{WW}$ . Twist-3 matrix elements have been extracted and are compared to theoretical predictions. The asymmetry  $A_2$  has also been measured and is found to be significantly smaller than the positivity limit  $\sqrt{R}$  for both targets.  $A_2^p$ is found to be positive and inconsistent with zero.

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### 1 Introduction

Measurements of nucleon spin-dependent structure functions are valuable tools used to understand the complex nature of nucleon structure. These structure functions are probes of the longitudinal and transverse quark and gluon polarization distributions inside the nucleons. Measurements of these structure functions allow us to test sum rules, quark model predictions, and QCD predictions.

The spin dependent structure functions  $g_1(x, Q^2)$  and  $g_2(x, Q^2)$  are measured by scattering longitudinally polarized leptons from a target which is polarized either longitudinally or transversely. The longitudinal  $(A_{\parallel})$  and transverse  $(A_{\perp})$ asymmetries are formed from combining data taken with opposite beam helicity, and the structure functions are determined from these asymmetries:

$$g_{1}(x,Q^{2}) = \frac{F_{1}(x,Q^{2})}{d} [A_{\parallel} + \tan(\theta/2)A_{\perp}],$$
  

$$g_{2}(x,Q^{2}) = \frac{yF_{1}(x,Q^{2})}{2d} \left[\frac{E + E'\cos\theta}{E'\sin\theta}A_{\perp} - A_{\parallel}\right],$$
(1)

where E is the incident electron energy, E' is the scattered electron energy,  $\theta$  is the scattering angle, x is the Bjorken scaling variable,  $Q^2$  is the four-momentum transfer squared, y = (E - E')/E,  $d = [(1 - \epsilon)(2 - y)]/[y(1 + \epsilon R(x, Q^2))]$ ,  $\epsilon^{-1} =$  $1 + 2[1 + \gamma^{-2}]\tan^2(\theta/2)$ ,  $\gamma = 2Mx/\sqrt{Q^2}$ , M is the nucleon mass,  $F_1(x, Q^2)$  is one of the spin-averaged structure functions, and  $R(x, Q^2) = \sigma_L/\sigma_T$  is the ratio of the longitudinal to transverse virtual photon absorption cross sections. Also of interest are the virtual photon absorption asymmetries

$$A_1 = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}} \quad \text{and} \quad A_2 = \frac{2\sigma_{TL}}{\sigma_{1/2} + \sigma_{3/2}}, \tag{2}$$

where  $\sigma_{1/2}$  and  $\sigma_{3/2}$  are the virtual photon-nucleon absorption cross sections for total helicity between photon and nucleon of 1/2 and 3/2 respectively, and  $\sigma_{TL}$ is an interference term between the transverse and longitudinal photon-nucleon amplitudes. These asymmetries are also determined from the measured asymmetries:

$$A_{1} = \frac{1}{d} \left[ A_{\parallel}(1 + xM/E) - A_{\perp} \frac{xM}{E \tan(\theta/2)} \right],$$
  

$$A_{2} = \frac{\gamma(2 - y)}{2d} \left[ A_{\perp} \frac{y(1 + xM/E)}{(1 - y)\sin\theta} + A_{\parallel} \right].$$
(3)

#### **1.1** Physical Interpretation of $g_1$

The structure function  $g_1(x)$  is interpreted in the naive parton model as the charge weighted difference between momentum distributions for quarks and nucleon helicities aligned parallel ( $\uparrow$ ) and anti-parallel ( $\downarrow$ ):

$$g_1(x) = \frac{1}{2} \sum_i e_i^2 [q_i^{\uparrow}(x) - q_i^{\downarrow}(x)] \equiv \sum_i e_i^2 \Delta q_i(x), \qquad (4)$$

where  $e_i$  is the charge of quark flavors i, and  $q_i^{\uparrow(\downarrow)}(x)$  are the quark plus antiquark momentum distributions. The quantity  $\int_0^1 \Delta q_i(x) dx = \Delta i$  refers to the helicity of quark species i = u, d, s in the proton, and  $\Delta q = \Delta u + \Delta d + \Delta s$  is the net helicity of quarks. Using measurements of  $\int_0^1 g_1(x) dx$ ,  $g_A/g_V$  and F/D as well as the QCD corrections to the sum rules, one can separately extract the quantities  $\Delta_i$ !

### **1.2** Physical Interpretation of $g_2$

Unlike  $g_1$ , the interpretation of  $g_2$  in the naive parton model is ambiguous? A more advanced light-cone parton model<sup>3,4</sup> as well as an operator product expansion (OPE) analysis<sup>5</sup> indicate that there are three components contributing to  $g_2$ . These components include the leading twist-2 part,  $g_2^{WW}(x, Q^2)$ , coming from the same set of operators that contribute to  $g_1$ , another twist-2 part coming from the quark transverse polarization distribution  $h_T(x, Q^2)$ , and a twist-3 part coming from quark-gluon interactions  $\xi(x, Q^2)$ :

$$g_2(x,Q^2) = g_2^{WW}(x,Q^2) - \int_x^1 \frac{\partial}{\partial y} \left(\frac{m}{M} h_T(y,Q^2) + \xi(y,Q^2)\right) \frac{dy}{y}.$$
 (5)

The quark mass is denoted by m, and the  $g_2^{WW}$  expression of Wandzura-Wilczek<sup>6</sup> is given by

$$g_2^{WW}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{g_1(y,Q^2)}{y} dy.$$
 (6)

### 2 Sum Rules

#### 2.1 Bjorken Sum Rule

A sum rule developed by Bjorken<sup>7</sup> relates the integral over the proton minus neutron spin structure functions to the nucleon beta decay weak coupling constants. It is believed to be strictly valid at infinite  $Q^2$ :

$$\int \left(g_1^p(x) - g_1^n(x)\right) dx = \frac{1}{6} \frac{g_A}{g_V} \qquad Q^2 = \infty,$$
(7)

where  $g_A$  and  $g_V$  are the nucleon axial-vector and vector coupling constants and  $g_A/g_V = 1.2573 \pm 0.0038^8$ . The advent of QCD corrections has brought this sum rule into the regime where it and thus the QCD corrections can be experimentally tested. These non-singlet corrections<sup>9</sup> to order three for three quark flavors are  $C_{NS} = \left[1 - \alpha_s/\pi - 3.58 (\alpha_s/\pi)^2 - 20.22 (\alpha_s/\pi)^3\right]$  where  $\alpha_s(Q^2)$  is the strong coupling constant.

#### 2.2 Ellis-Jaffe Sum Rule

Other sum rules of interest for  $g_1$ , although less rigorous than the Bjorken sum rule, are the Ellis-Jaffe sum rules<sup>10</sup> which were derived using SU(3) symmetry and assuming the strange sea in the nucleons is unpolarized.

$$\Gamma_1^p(Q^2) = \int_0^1 g_1^p(x, Q^2) dx = \frac{1}{18} \left[ C_{NS}(3F + D) + 2C_S(3F - D) \right]$$
  
$$\Gamma_1^n(Q^2) = \int_0^1 g_1^n(x, Q^2) dx = \frac{1}{9} \left[ -DC_{NS} + C_S(3F - D) \right], \tag{8}$$

where F and D are weak hyperon decay constants extracted from data<sup>11</sup>  $F/D = 0.575 \pm 0.016$ ,  $F + D = g_A/g_V$ , and the second order singlet QCD corrections<sup>12</sup> are given by  $C_S = \left[1 - 0.3333\alpha_s/\pi - 0.5495(\alpha_s/\pi)^2\right]$ .

#### 2.3 OPE Sum Rules

The OPE<sup>2,5,13</sup> is a useful technique within QCD because it separates the physics into a perturbative part which is easily treatable and a nonperturbative part which is parameterized in terms of unknown matrix elements of Lorentz-covariant operators. The OPE analysis of  $g_1$  and  $g_2$  yields an infinite number of sum rules

$$\int_{0}^{1} x^{n} g_{1}(x, Q^{2}) dx = \frac{a_{n}}{2}, \qquad n = 0, 2, 4, \dots$$

$$\int_{0}^{1} x^{n} g_{2}(x, Q^{2}) dx = \frac{1}{2} \frac{n}{n+1} (d_{n} - a_{n}), \qquad n = 2, 4, \dots$$
(9)

where  $a_n$  are the twist-2 and  $d_n$  are the twist-3 matrix elements of the renormalized operators. The OPE only gives information on the odd moments of the spin structure functions. The Wandzura-Wilczek relation in Eq. (6) can be derived from these sum rules by setting  $d_n = 0$ .

#### 2.4 Burkhardt-Cottingham Sum Rule

The Burkhardt-Cottingham sum rule<sup>14</sup> for  $g_2$  at large  $Q^2$ ,

$$\int_0^1 g_2(x) dx = 0,$$
 (10)

has been derived from from virtual Compton scattering dispersion relations. This sum rule does not follow from the OPE since the n = 0 sum rule is not defined for  $g_2$  in Eq. (9). The Burkhardt-Cottingham sum rule relies on  $g_2$  obeying Regge theory which may not be a good assumption. A non-Regge divergence of  $g_2$  at low x would invalidate this sum rule<sup>2,5</sup> and such a divergence could be very difficult to detect experimentally.

### 2.5 Efremov-Teryaev Sum Rule

The Efremov-Teryaev sum rule<sup>15</sup> is derived in leading order QCD in which quarkgluon correlators have been included. This sum rule relates the  $g_1$  and  $g_2$  structure functions:

$$\int_0^1 x [2g_2(x) + g_1(x)] dx = 0$$

## **3** Other Experiments

The earliest spin structure experiments, E80,<sup>16</sup> E130,<sup>17</sup> and EMC<sup>18</sup> measured  $A_{\parallel}$  for the proton only. Using the assumption that  $g_1 \simeq F_1 A_1$ , the EMC extracted  $g_1^p(x, Q^2)$  with sufficient precision to test the Ellis-Jaffe sum rule which was violated and the so-called "spin crisis" was born. In the naive quark model this was interpreted to mean that the total quark helicity was small and consistent with zero while the strange quark helicity was negative and inconsistent with zero. This unexpected result has generated a lot of interest in the physics community. Many theoretical papers have surfaced to explain the data, better QCD corrections have been calculated bringing predictions closer to experimental results, and extensive experimental programs at SLAC, CERN and HERA were begun to learn more about nucleon spin structure. Results are now available from the SMC<sup>19-22</sup> experiment at CERN, and the E142<sup>23</sup> experiment at SLAC. These data include significantly more precise proton data, measurements on deuterons and <sup>3</sup>He (neutrons), and the first measurement of the transverse asymmetry  $A_2$  for the proton.

These experiments have confirmed the Bjorken sum rule, and have shown that the Ellis-Jaffe sum rules for both the proton and neutron are violated.

### 4 This Experiment

For this experiment, E143,<sup>24–27</sup> longitudinally polarized electrons were scattered from polarized protons and deuterons into two independent spectrometers at angles of 4.5° and 7°. The beam polarization, typically  $P_b = 0.85 \pm 0.02$ , was measured with a Møller polarimeter. Measurements were made at three beam energies of 29.1, 16.2, and 9.7 GeV. The target cells were filled with granules of either <sup>15</sup>NH<sub>3</sub> or <sup>15</sup>ND<sub>3</sub>, and were polarized using the technique of dynamic nuclear polarization. The targets could be polarized longitudinally or transversely relative to the beam by physically rotating the polarizing magnet. Target polarization  $P_t$ , measured by a calibrated NMR, averaged around 0.65  $\pm$  0.017 for protons and 0.25  $\pm$  0.011 for deuterons.

The experimental asymmetries  $A_{\parallel}$  and  $A_{\perp}$  were determined from

$$A_{\parallel} \text{ (or } A_{\perp}) = C_1 \left( \frac{N_L - N_R}{N_L + N_R} \frac{1}{f P_b P_t} - C_2 \right) + A_{RC}, \tag{11}$$

where  $N_L$  and  $N_R$  are the number of scattered electrons per incident electron for negative and positive beam helicity, where corrections have been made for charge-symmetric backgrounds and deadtime; f is the dilution factor representing the fraction of measured events originating from polarizable protons or deuterons within the target;  $C_1$  and  $C_2$  correct for the polarized nitrogen nuclei and for residual polarized protons in the ND<sub>3</sub> target; and  $A_{RC}$  are the radiative corrections, which include internal<sup>28</sup> and external<sup>29</sup> contributions.

# 4.1 Longitudinal results at E = 29 GeV

From the measured values of  $A_{\parallel}$  and  $A_{\perp}$  we calculated the ratios  $g_1^p/F_1^p$  and  $g_1^d/F_1^d$ using the definition given in Eq. (1). For  $F_1(x, Q^2) = F_2(x, Q^2)(1 + \gamma^2)/[2x(1 + R(x, Q^2))]$  we used the NMC<sup>30</sup> fits to  $F_2(x, Q^2)$  data and the SLAC fit<sup>31</sup> to  $R(x, Q^2)$ , which was extrapolated to unmeasured regions for x < 0.08. These results<sup>24,25</sup> are shown in Fig. 1. Also included in the plots are the data from other experiments,<sup>16-18,20,22</sup> which are all in good agreement with the E143 results.



Figure 1: Measurements of  $g_1/F_1$  for (a) proton and (b) deuteron for all experiments. The E143 data are in good agreement with all other data. Uncertainties for the E143 data include statistical contributions only.



Figure 2: Measurements of  $xg_1$  for (a) proton and (b) deuteron from experiment E143 at a constant  $Q^2 = 3 \, (\text{GeV/c})^2$ . The uncertainties include statistical contributions only.

Values of  $xg_1^p$  and  $xg_1^d$  at the average  $Q^2 = 3 \text{ (GeV/c)}^2$  of this experiment are shown in Fig. 2. The evaluation at constant  $Q^2$  is model dependent, and we have made the assumption that  $g_1/F_1$  is independent of  $Q^2$  which is believed to be reasonable for the kinematics of this experiment (see discussion on  $Q^2$  dependence below).

Values of  $xg_1^p$  and  $xg_1^n$  from several experiments at an average  $Q^2 = 5 (\text{GeV/c})^2$ are shown in Fig. 3. The data were evolved to constant  $Q^2$  assuming  $g_1/F_1$  is independent of  $Q^2$ . The neutron results from this experiment<sup>25</sup> and from SMC<sup>22</sup> were extracted from proton and deuteron data using  $g_1^d = \frac{1}{2}(g_1^p + g_1^n)(1 - \frac{3}{2}\omega_D)$ , where  $\omega_D$  is the probability that the deuteron is in a D state. Both experiments used  $\omega_D = 0.05 \pm 0.01^8$  We see from Fig. 3 that the data sets are in good agreement when evolved to the same  $Q^2$ .

The integrals over x of  $g_1$  for the proton  $(\Gamma_1^p)$ , deuteron  $(\Gamma_1^d)$ , and neutron  $(\Gamma_1^n)$  were evaluated at a constant  $Q^2 = 3 \, (\text{GeV/c})^2$ . The measured x region was 0.029 < x < 0.08. The extrapolation from x = 0.8 to x = 1 was done assuming



Figure 3: Measurements of  $xg_1$  for (a) proton and (b) neutron for E143,<sup>24,25</sup> E142,<sup>23</sup> and SMC<sup>20,22</sup> at a constant  $Q^2 = 5 \text{ (GeV/c)}^2$ . The data sets are in agreement. Uncertainties for the E143 data include statistical only.

that  $g_1$  varies as  $(1-x)^3$  at high x. The extrapolation from x = 0 to x = 0.029was determined by fitting the low x data to a Regge<sup>32</sup> motivated form  $g_1 = Cx^{-\alpha}$ . An alternate form<sup>33</sup>  $g_1 = C\ln(1/x)$ , which provides a good fit to the low- $x F_2$  data from NMC and HERA, gives consistent results within the uncertainties. Table 1 gives a summary of the measured and extrapolated contributions to  $\Gamma_1^p$  and  $\Gamma_1^d$ . Table 2 shows the E143 measurements for  $\Gamma_1^p$ ,  $\Gamma_1^d$ ,  $\Gamma_1^n$  and  $\Gamma_1^p - \Gamma_1^n$ , as well as the corresponding Ellis-Jaffe and Bjorken sum rule predictions for  $Q^2 = 3$  (GeV/c)<sup>2</sup>. The data consistently demonstrate that the Ellis-Jaffe sum rule is violated. The most precise determination is given by the deuteron measurement which is more than 3  $\sigma$  away from the prediction. Note that the E143 results agree with the E142 results<sup>23</sup> for  $\Gamma_1^n = -0.022 \pm 0.011$  at  $Q^2 = 2$  (GeV/c)<sup>2</sup>, and the SMC<sup>20,22</sup> results for  $\Gamma_1^p = 0.136 \pm 0.016$  and  $\Gamma_1^d = 0.034 \pm 0.011$  at  $Q^2 = 10$  (GeV/c)<sup>2</sup>. The estimated  $Q^2$  dependence of these quantities for  $2 < Q^2 < 10$  (GeV/c)<sup>2</sup> is within the errors on all the experiments. Table 3 is a summary of the dominant systematic error contributions to the E143 measured integrals shown in Table 2.

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systematic.					
The uncertainty on the extrapolated contributions is assumed					
sured contribution has a statistical and systematic uncertainty.					
up into the measured and extrapolated contributions. The mea-					
Table 1: Results f	for $\Gamma_1^p$ and $\Gamma_1^d$ from ex	periment E143, broken			

x Region	$\Gamma^p_1$	$\Gamma_1^d$
0 < x < 0.029	$0.006\pm0.006$	$0.001\pm0.001$
0.029 < x < 0.8	$0.120 \pm 0.004 \pm 0.008$	$0.040 \pm 0.003 \pm 0.004$
0.8 < x < 1	$0.001\pm0.001$	$0.000\pm0.001$
Total	$0.127 \pm 0.004 \pm 0.010$	$0.042 \pm 0.003 \pm 0.004$

Table 2: Summary of E143 $g_1$ sum rule tests			
	Measured	Prediction	Sum Rule
$\Gamma_p$	$0.127 \pm 0.004 \pm 0.010$	$0.160\pm0.006$	Ellis-Jaffe
$\Gamma_d$	$0.042 \pm 0.003 \pm 0.004$	$0.069 \pm 0.004$	Ellis-Jaffe
$\Gamma_n$	$-0.037 \pm 0.008 \pm 0.011$	$-0.011 \pm 0.006$	Ellis-Jaffe
$\Gamma_p - \Gamma_n$	$0.163 \pm 0.010 \pm 0.016$	$0.171\pm0.008$	Bjorken

Table 3: Summary of the systematic error contri-				
butions to the E143 $g_1$ integrals				
source	$\Gamma_p$	$\Gamma_d$	$\Gamma_n$	$\Gamma_p$ - $\Gamma_n$
beam polarization	0.003	0.001	0.001	0.004
target polarization	0.003	0.002	0.005	0.007
dilution factor	0.004	0.002	0.006	0.008
radiative corrections	0.002	0.002	0.006	0.007
$F_2, R$	0.004	0.001	0.002	0.005
Extrapolation	0.006	0.001	0.004	0.006
TOTAL	0.010	0.004	0.011	0.016

The violation of the Ellis-Jaffe sum rule implies that the assumption that the strange quark is unpolarized within the nucleon may be false. This can be seen by extracting the net quark helicity within the proton using the naive quark model<sup>1</sup> (See Eq. (4) and related discussion). Table 4 gives the extracted quark helicities as determined from the measurements of  $\Gamma_1^p$  and  $\Gamma_1^d$  and the SU(3) coupling constants F and D. The data include third order non-singlet and second order singlet QCD corrections. We see that the net quark helicity  $\Delta q$  is significantly less than a prediction<sup>10</sup> that  $\Delta q = 0.58$  assuming zero strange quark helicity and SU(3) flavor symmetry in the baryon octet. Also,  $\Delta s$  is negative and significantly different from zero. Figure 4 shows a plot of  $\Delta q$  versus  $\Delta s$  as extracted from various experimental measurements at the appropriate  $Q^2$ . We see that all experiments are consistent with a small  $\Delta q$  and a  $\Delta s$  which is negative and inconsistent with zero.

Table 4: Extracted quark helicities				
from	from experiment E143.			
	$\Gamma_p$	$\Gamma_d$		
$\Delta u$	$0.81\pm0.04$	$0.83\pm0.02$		
$\Delta d$	$-0.44\pm0.04$	$-0.43\pm0.02$		
$\Delta s$	$-0.10\pm0.04$	$-0.09\pm0.02$		
$\Delta q$	$0.27 \pm 0.11$	$0.30\pm0.06$		

#### 4.2 Transverse results at E = 29 GeV

From the measured values of  $A_{\parallel}$  and  $A_{\perp}$  at E = 29 GeV we have calculated  $g_2^p$ ,  $g_2^d$ ,  $A_2^p$  and  $A_2^d$  using Eqs. (1) and (3). The results for  $A_2$  for the proton and



Figure 4: The quark helicity content of the proton as extracted from various measurements is shown for  $\Delta q$  versus  $\Delta s$ . The data include third order non-singlet and second order singlet QCD corrections.



Figure 5: Measurements of (a)  $A_2^p$ , and (b)  $A_2^d$  from E143<sup>26</sup> and SMC<sup>21</sup> Systematic errors are indicated by bands. The curves show the  $\sqrt{R}^{31}$  positivity constraints for the three data sets. The solid, dashed and dotted curves correspond to the 4.5° E143, 7.0° E143, and SMC kinematics, respectively. Overlapping data have been shifted slightly in x to make errors clearly visible.

deuteron are shown in Fig. 5. The systematic errors, dominated by radiative correction uncertainties, are indicated by bands for the two spectrometers used in the experiment. The data agree within errors despite the differences in  $Q^2$  of the measurements (nearly a factor of two). Also in Fig. 5 are the proton results from SMC,<sup>21</sup> and the  $\sqrt{R}$ <sup>31</sup> positivity limits for each data set. The data are much closer to zero than the positivity limit. Results for  $A_2^p$  are consistently > 0, and since  $A_2$  is expected to be zero at high  $Q^2$  (because  $R \to 0$ ), these data indicate that  $A_2$  must have  $Q^2$  dependence. A comparison of the data with the hypothesis  $A_2 = 0$  yields  $\chi^2 = 73$  for the proton and  $\chi^2 = 44$  for the deuteron for 48 degrees of freedom.

The results for  $xg_2$  for the proton and deuteron are shown in Fig. 6. The  $g_2^d$  results are per nucleon. The systematic errors are indicated by bands. Also shown is the  $g_2^{WW}$  curve evaluated using Eq. (6) at E = 29 GeV and  $\theta = 4.5^{\circ}$ . The same



Figure 6: Spin structure function measurements for (a)  $xg_2^p$ , and (b)  $xg_2^d$  from E143. Systematic errors are indicated by bands. Overlapping data have been shifted slightly in x to make errors clearly visible. The solid curve shows the twist- $2 g_2^{WW}$  calculation for the kinematics of the 4.5° spectrometer. The same curve for 7° is nearly indistinguishable. The bag model calculations at  $Q^2 = 5.0 \, (\text{GeV/c})^2$ by Stratmann<sup>34</sup> (dotted) and Song and McCarthy<sup>35</sup> (dashed) are indicated.

curve for  $\theta = 7^{\circ}$  is nearly indistinguishable. The values for  $g_2^{WW}$  were determined from  $g_1(x, Q^2)$  evaluated from a fit to world data of  $A_1^{27}$  and assuming negligible higher-twist contributions. Also shown are the bag model predictions of Stratmann<sup>34</sup> and Song and McCarthy,<sup>35</sup> which include both twist-2 and twist-3 contributions for  $Q^2 = 5$  (GeV/c)<sup>2</sup>. At high x the results for  $g_2^p$  indicate a negative trend consistent with the expectations for  $g_2^{WW}$ . The deuteron results are less conclusive because of the larger errors.

We can look for possible quark mass and higher twist effects by extracting the quantity  $\overline{g_2}(x, Q^2) = g_2(x, Q^2) - g_2^{WW}(x, Q^2)$ , If the term in Eq. (5) which depends on quark masses can be neglected then  $\overline{g_2}(x, Q^2)$  is entirely twist-3. Our results can be seen in Fig. 7. Within the experimental uncertainty the data are consistent with  $\overline{g_2}$  being zero but also with  $\overline{g_2}$  being of the same order of magnitude as  $g_2^{WW}$ . Also shown in Fig. 7 are the bag model predictions of Stratmann<sup>34</sup> and Song and McCarthy<sup>35</sup> for  $Q^2 = 5$  (GeV/c)<sup>2</sup> which compare favorably with the data given the large experimental uncertainties.



Figure 7: E143 results<sup>26</sup> for (a)  $x\overline{g_2}^p$ , and (b)  $x\overline{g_2}^d$  Overlapping data have been shifted slightly in x to make errors clearly visible. The bag model calculations at  $Q^2 = 5.0 \, (\text{GeV/c})^2$  by Stratmann<sup>34</sup> (solid) and Song and McCarthy<sup>35</sup> (dashed) are indicated.

Using our results for the longitudinal spin structure functions  $g_1^p$  and  $g_1^d$ , we have computed the first few moments of the OPE sum rules, and solved for the twist-3 matrix elements  $d_n$ . These moments are defined to be  $\Gamma_1^{(n)} = \int_0^1 x^n g_1(x) dx$ and  $\Gamma_2^{(n)} = \int_0^1 x^n g_2(x) dx$ . For the measured region 0.03 < x < 0.8, we evaluated  $g_1$  and corrected the twist-2 part of  $g_2$  to fixed  $Q^2 = 5$  (GeV/c)<sup>2</sup> assuming  $g_1/F_1$ is independent of  $Q^2$ , and have averaged the two spectrometer results to evaluate the moments. Possible  $Q^2$  dependence of  $\overline{g_2}$  has been neglected. We neglect the contribution from the region  $0 \le x < 0.03$  because of the  $x^n$  suppression factor. For  $0.8 < x \le 1$ , we assume that both  $g_1$  and  $g_2$  behave as  $(1 - x)^3$ , and we fit the data for x > 0.56. The uncertainty in the extrapolated contribution is taken to be the same as the contribution itself. The results are shown in Table 5a. For comparison, in Table 5b we quote theoretical predictions<sup>34-37</sup> for  $d_2^p$  and  $d_2^d$ . For  $d_2^d$ the proton and neutron results were averaged and a deuteron D-state correction was applied. Our extracted values for  $d_n$  are consistent with zero, but the errors

Table 5a: Results for the moments $\Gamma_1^{(n)}$ and $\Gamma_2^{(n)}$ evaluated at $Q^2 = 5$				
(Ge	$(\text{GeV/c})^2$ , and the extracted twist-3 matrix elements $d_n$ for proton			
(p)	and deut	teron (d) targets.	The errors include	statistical (which
dom	ninate) ai	nd systematic cont	ributions.	
	n	$\Gamma_1^{(n)}$	$\Gamma_2^{(n)}$	$d_n$
р	2	$0.0121 \pm 0.0010$	$-0.0063 \pm 0.0018$	$0.0054 \pm 0.0050$
	4	$0.0032 \pm 0.0004$	$-0.0023 \pm 0.0006$	$0.0007 \pm 0.0017$
	6	$0.0012 \pm 0.0002$	$-0.0010 \pm 0.0003$	$0.0001 \pm 0.0008$
d	2	$0.0040 \pm 0.0008$	$-0.0014 \pm 0.0030$	$0.0039 \pm 0.0092$
	4	$0.0008 \pm 0.0003$	$0.0000 \pm 0.0010$	$0.0017 \pm 0.0026$
	6	$0.0002 \pm 0.0001$	$0.0001 \pm 0.0005$	$0.0006 \pm 0.0011$
Table 5b: Theoretical predictions for the twist-3 matrix element $d_2^p$				
for proton and $d_2^d$ for deuteron. The values for $Q^2$ are in $(\text{GeV/c})^2$ .				
	Bag models		QCD sum rules	
	$\operatorname{Ref.}^{35}$	${ m Ref.}^{34}$	${ m Ref.}^{36}$	${ m Ref.}^{37}$
$Q^2$	5	5	1	1
$d_2^p$	0.0176	0.0060	$-0.006 \pm 0.003$	$-0.003 \pm 0.006$
$d_2^d$	0.0066	0.0029	$-0.017 \pm 0.005$	$-0.014 \pm 0.006$

are large. The results do not have sufficient precision to distinguish between the model predictions.

We have also evaluated the integrals  $\int_{0.03}^{1} g_2(x) dx$  and  $\int_{0.03}^{1} x [2g_2(x) + g_1(x)] dx$ for both the proton and deuteron structure functions. We do not attempt a low xextrapolation due to the theoretical uncertainty on the low x behavior of  $g_2$ . For the latter integral, the low x region is suppressed by x, so it is not unreasonable to assume that the low x extrapolation is negligible. The high-x extrapolation is done as discussed above. The results are given in Table 6, and are all consistent with zero within their large errors as expected from the Burkhardt-Cottingham and Efremov-Teryaev sum rules. Of course, we cannot really test the Burkhardt-Cottingham sum rule due to the uncertainty in the unmeasured low x behavior.

Table 6: Summary of E143 $g_2$ sum rule tests. The			
predictions for both sum rules are zero.			
	$\int_{0.03}^{1} g_2(x) dx$	$\int_{0.03}^{1} x [2g_2(x) + g_1(x)] dx$	
proton	$-0.013 \pm 0.028$	$0.008 \pm 0.008$	
deuteron	$-0.033 \pm 0.082$	$-0.001 \pm 0.014$	

## **4.3** $Q^2$ **Dependence of** $g_1$

Data for  $g_1$  measured at a fixed energy of 29 GeV were discussed above. These data cover the range  $1 < Q^2 < 10 \, (\text{GeV/c})^2$  where the lower values of  $Q^2$  are at the lower values of x. In order to evaluate sum rules at some fixed  $Q^2$  it is necessary to extrapolate the data from the measured kinematics. Since this is a model-dependent procedure (e.g., assuming  $g_1/F_1$  is independent of  $Q^2$ ), it is useful to measure the  $Q^2$  dependence by taking data at multiple beam energies. In E143 we made measurements at beam energies of 29.1, 16.2, and 9.7 GeV. The kinematic coverage of all these data sets where a  $Q^2$ -dependent measurement has been made is 0.03 < x < 0.6 and  $0.3 < Q^2 < 10 \, (\text{GeV/c})^2$ .

According to the GLAP equations<sup>38</sup> which give the predicted  $Q^2$  dependence of the nucleon polarized and unpolarized quark and gluon distribution functions,  $g_1$ is expected to evolve logarithmically in a similar way as the unpolarized structure functions  $F_1(x, Q^2)$  and  $F_2(x, Q^2)$ . The  $Q^2$  dependence of the ratio  $g_1/F_1$  may be independent of  $Q^2$  to a first approximation, but the precise behavior is sensitive to the underlying spin-dependent quark and gluon distribution functions. Measurements will help pin down this behavior. Fits have been made<sup>39,40</sup> of  $g_1(x, Q^2)$  data using next-to-leading-order (NLO) GLAP equations.<sup>41</sup> The results indicate that NLO fits are more sensitive to the strength of the polarized gluon distribution function  $\Delta G(x, Q^2)$  than previous leading-order (LO) fits.<sup>41-45</sup> In addition our understanding of the  $Q^2$  dependence of  $g_1$  is complicated by possible higher twist contributions which are not part of the GLAP equations. These terms are expected to behave as  $C(x)/Q^2$ ,  $D(x)/Q^4$ , etc., where C(x) and D(x) are unknown functions.

The ratio  $g_1/F_1$  has been extracted from the data taken in this experiment<sup>27</sup> as well as from other available data for the proton<sup>16-18,20</sup> and the deuteron<sup>22</sup> using the relations given in Eq. (1). The twist-2 model of Wandzura and Wilczek<sup>6</sup> given in Eq. (6) was used to describe  $g_2$  for all data since the E143  $g_2$  data discussed above are in agreement with this model. The results for  $g_1^p/F_1^p$  and  $g_1^d/F_1^d$  are shown in Figs. 8 and 9, respectively, at eight values of x. Improved radiative corrections have been applied to the  $E80^{16}$  and  $E130^{17}$  results. Only statistical uncertainties have been plotted. For the present experiment, most systematic uncertainties (beam polarization, target polarization, fraction of polarizable nucleons in the target) for a given target are common to all data and correspond to an overall normalization error of about 5% for the proton data and 6% for the deuteron data. The remaining point-to-point systematic uncertainties (radiative corrections, model uncertainties for  $R(x, Q^2)$ , resolution corrections) vary over x from a few percent to 15%, and are consistently less than the statistical uncertainties for all data. We see in Figs. 8 and 9 that  $g_1/F_1$  is approximately independent of  $Q^2$  at fixed x, although there is a noticeable trend for the ratio to decrease for  $Q^2 < 1 \; ({\rm GeV}/c)^2.$ 

We have performed several simple global fits<sup>27</sup> to the data, in order to have a practical parameterization (needed, for example, in making radiative corrections to the data), and to study the possible  $Q^2$  dependence of the first moments of  $g_1$ . The fits are of the general form  $g_1/F_1 = ax^{\alpha}(1 + bx + cx^2)[1 + Cf(Q^2)]$  where  $a, \alpha, b, c$ , and C are fit parameters and  $f(Q^2)$  is defined to be either  $1/Q^2$ , or  $\ln(1/Q^2)$ . Cuts were applied to some of the fits to include only data with  $Q^2 > 1$  $(\text{GeV/c})^2$ , and C was forced to be zero (no  $Q^2$  dependence) for some fits. The results indicate that when all the data are included, the fits where  $C \neq 0$  have significantly better  $\chi^2$  per degree of freedom than those where C = 0. However, good fits to the data are obtained when C = 0 and the  $Q^2 > 1$  (GeV/c)<sup>2</sup> cut is applied to the data (fit II). Two of these global fits<sup>27</sup> are shown in Figs. 8 and 9:



Figure 8: Ratios  $g_1^p/F_1^p$  extracted from experiments assuming  $g_2 = g_2^{WW}$ . The uncertainties are statistical only. Data are from E143<sup>27</sup> (solid circles), E80<sup>16</sup> (diamonds), E130<sup>17</sup> (triangles), EMC<sup>18</sup> (squares), and SMC<sup>20</sup> (open circles). The dashed and solid curves correspond to global fits<sup>27</sup> II ( $g_1^p/F_1^p$  Q<sup>2</sup>-independent) and III ( $g_1^p/F_1^p$  Q<sup>2</sup>-dependent), respectively. Representative NLO pQCD fits from Ref.<sup>39</sup> and Ref.<sup>40</sup> are shown as the dot-dashed and dotted curves respectively.



Figure 9: Ratios  $g_1^d/F_1^d$  from E143<sup>27</sup> (solid circles) and SMC<sup>22</sup> (open circles). The curves are as in Fig. 8

fit II and fit III which assumes  $f(Q^2) = 1/Q^2$  and the data at all  $Q^2$  are fit.

Also shown in Figs. 8 and 9 are representative global NLO pQCD fits<sup>39,40</sup> to available structure function data excluding those measured at the 9.7 GeV and 16.2 GeV beam energies of this experiment. These fits are indicated as the dot-dashed curves<sup>39</sup> and the dotted curves.<sup>40</sup> Both sets of predictions<sup>39,40</sup> indicate that  $g_1^p/F_1^p$  decreases with  $Q^2$  at lower x, in agreement with the trend of our E = 9.7 and E = 16.2 results.

Another type of fit was made to the data which was motivated by possible differences in the twist-4 contributions to  $g_1$  and  $F_1$ . We fit the data in each xbin (see Figs. 8 and 9) with the form  $g_1/F_1 = a(1 + C/Q^2)$ . The results for the C coefficients are shown in Fig. 10 for fits to all data (circles) and for fits to data with  $Q^2 > 1$  (GeV/c)<sup>2</sup> (squares). The coefficients indicate significantly negative values for C at intermediate values of x when all the data are fit. The errors are much larger when data with  $Q^2 < 1$  (GeV/c)<sup>2</sup> are excluded, and the results are consistent with no  $Q^2$ -dependence to  $g_1/F_1$  (C = 0). The present data do not



Figure 10: Coefficients C for fits to  $g_1/F_1$  at fixed x of the form  $a(1 + C/Q^2)$  for (a) proton and (b) deuteron. Solid circles are from fits to all data, and open squares are from fits to data with  $Q^2 > 1$  (GeV/c)<sup>2</sup> only.

have sufficient precision to distinguish between a logarithmic and power law  $Q^2$ dependence, but can rule out large differences between the  $Q^2$ -dependence of  $g_1$ and  $F_1$ , especially for  $Q^2 > 1$  (GeV/c)<sup>2</sup>.

Using fits<sup>27</sup> II and III described above and a global fit<sup>30,31</sup> to  $F_1$ , we have evaluated the first moments  $\Gamma_1^p$  and  $\Gamma_1^d$ , and the corresponding results for  $\Gamma_1^p - \Gamma_1^n$ and the net quark helicity  $\Delta q$ . The results for  $\Gamma_1^p - \Gamma_1^n$  are shown as a function of  $Q^2$  as the lower (fit II) and upper (fit III) bands in Fig. 11, where the width of the band reflects the combined statistical and systematic error estimate. Both fits are in reasonable agreement with the Bjorken sum rule (solid curve) evaluated using  $\alpha_s(Q^2)$  evolved in  $Q^2$  from  $\alpha_s(M_Z) = 0.117 \pm 0.005^8$  for the QCD corrections.

Our results for  $\Delta q$  evaluated at  $Q^2 = 3 \, (\text{GeV/c})^2$  are shown in Table 7. Note that these results for  $\Delta q$  and for  $\Gamma_1^p - \Gamma_1^n$  have shifted slightly from the



Figure 11: Evaluations of  $\Gamma_1^p - \Gamma_1^n$  from the  $Q^2$ -independent fits II (lower band) and  $Q^2$ -dependent fits III (upper band). The errors include both statistical and systematic contributions and are indicated by the widths of the bands. The solid curve is the prediction of the Bjorken sum rule with third-order QCD corrections.

original results<sup>24,25</sup> at 29 GeV discussed above (See Tables 2 and 4) because of improved radiative corrections, the inclusion of additional data runs, and improved measurements of the beam and target polarizations. Using fits II or III makes little difference at  $Q^2 = 3$  (GeV/c)<sup>2</sup>, but we find  $\Delta q$  (which should be independent of  $Q^2$ ) to vary less with  $Q^2$  for fit III than for fit II, especially for the deuteron fits.

Table 7: Summary of extracted $\Delta q$ results at $Q^2 = 3$			
$(\text{GeV/c})^2$ using fits II and III <sup>27</sup> for $\Gamma_p$ and $\Gamma_d$ .			
Fit	$\Delta q \text{ from } \Gamma_p$	$\Delta q$ from $\Gamma_d$	
II ( $Q^2$ -independent)	$0.34 \pm 0.09$	$0.35\pm0.05$	
III $(Q^2$ -dependent)	$0.36\pm0.10$	$0.34\pm0.05$	

### 5 Summary

Measurements of  $A_{\parallel}$  have been made at beam energies of 29.1, 16.2, and 9.7 GeV and  $A_{\perp}$  at a beam energy of 29.1 GeV for protons and deuterons. The spin structure functions,  $g_1$  and  $g_2$  have been extracted for the 29.1 GeV data. The integrals  $\Gamma_p = \int_0^1 g_1^p (x, Q^2) dx$  and  $\Gamma_d = \int_0^1 g_1^d (x, Q^2) dx$  have been evaluated at fixed  $Q^2 = 3$  (GeV/c)<sup>2</sup>. These results support the Bjorken sum rule predictions, and thus an important test of QCD is passed. The Ellis-Jaffe sum rule predictions for the proton and deuteron, however, are violated. In the context of the quark model, this implies that a non-negligible fraction of the proton helicity is carried by either strange quarks, gluons or both, and that the net quark helicity is smaller than expected. The  $Q^2$  dependence of the ratio  $g_1/F_1$  has been studied and is found to be small for  $Q^2 > 1$  (GeV/c)<sup>2</sup>.

Within experimental precision we find that the  $g_2$  data are well-described by the twist-2 contribution,  $g_2^{WW}$ . Results for  $\overline{g_2}$  are consistent with zero, although  $\overline{g_2}$  about the same order of magnitude as  $g_2^{WW}$  are allowed within the statistical uncertainties. More precise data is needed in the future to provide a more stringent measurement of  $\overline{g_2}$ . Twist-3 OPE matrix elements have been extracted from the moments of  $g_1$  and  $g_2$ . These results have a different sign than the QCD sum rule predictions, although within errors these predictions cannot be ruled out. The asymmetry  $A_2$  has also been measured and is found to be significantly smaller than the positivity limit  $\sqrt{R}$  for both targets.  $A_2^p$  is found to be positive and inconsistent with zero. A number of experimental programs will produce new spin structure function measurements in the future. SMC is continuing to take data. Additional results are expected from SLAC using a 50 GeV incident electron beam. Measurements of the neutron spin structure functions are in progress (E154), and proton and deuterium spin structure functions measurements (E155) will be made in 1996. Also, the HERMES collaboration at HERA is currently measuring spin-dependent structure functions of the proton and neutron. The data from these experiments will improve our understanding of the nucleon spin structure, and should answer many questions that have arisen due to current experimental results.

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