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LASER BEAMS

The production of polarized and monochromatic gamma radiation by means of Compton scattering at electron synchrotrons and linear accelerators

by

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*This work was done during the 1964 Summer Study program at SLAC.

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I. INTRODUCTION

The possibility of scattering energetic electrons off an intense laser-generated photon beam has been discussed in a number of articles.[1,2,3] In summary, when the electrons are extremely relativistic (multi-GeV), doubly-Doppler shifted scattered photons emerge having energies in a relatively flat spectrum which itself extends up to one or more GeV. This spectrum tends to peak at its highest energy, the more so the higher the electron energy, leading to an intrinsic monochromatization when beams from ultra-energetic, not-yet-constructed accelerators are used. At lower energies the beam can be monochromatized by angular selection of a portion of the photon beam. By polarizing the laser beam, the high-energy end of the photon spectrum can be made to consist of highly polarized photons, to a degree decreasing with increasing energy, but above ninety percent even up to about 8 GeV. The total cross section for the basic process is of the order of one-half barn. Table I summarizes typical kinematic parameters and polarizations expected at representative primary electron energies. More detailed results on the energy and polarization spectra are to be found in References 1, 2, and 3.

Because the experimental advantage of knowing the polarization state or energy (or both) of a beam of energetic photons is very attractive in high energy physics, it is worthwhile to make a detailed examination of the practical aspects of creation of such a beam by the Compton process, and in particular to estimate the yields which might be expected at contemporary accelerators. The resulting beams may then be compared with those generated by other methods of polarization and monochromatization.[4,5,6]

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In the sections following I shall attempt to make such an examination. Specific reference will be made to applications at the 6-GeV Cambridge Electron Accelerator, "CEA," (an experiment already in progress) and to the forthcoming 20-GeV Stanford Linear Accelerator, "SLAC." Because of the markedly different geometric and duty-cycle characteristics of the two machines, the problems of "laser" beam extraction turn out to be complementary in many respects. Representative extractor designs will be presented and discussed. Since the characteristics of an acceptable beam depend on what is to be done with it, some typical experimental applications will be mentioned. The crucial element in the production of a "laser" beam being the laser itself (both its physical characteristics and its cost), I shall include therefore some relevant features of the present state of the rapidly expanding laser technology, and shall speculate on some potential ways in which pulsed laser systems suitable for application at electron accelerators might be realized in practice.

TABLE I

ENERGY AND POLARIZATION FOR HIGHEST ENERGY PHOTONS PRODUCED BY RUBY LASER PHOTONS SCATTERED ON ELECTRONS OF ENERGY E

Electron Energy E	Peak Output Photon Energy	Peak Polarization
1.02 GeV	28 MeV	1.00
2.92	216	1.00
4.16	426	0.99
4.60	515	0.99
5.11	628	0.99
5.48	715	0.99
5.84	806	0.99
6.21	903	0.99
6.57	1.00 GeV	0.99
8.76	1.69	0.98
11.69	2.83	0.96
20.80	7.55	0.91
41.60	22.10	0.77
58.40	35.90	0.67

II. THE TECHNICAL PROBLEM

The basic problem is to produce beams of counter-moving electron and light pulses, each of maximum intensity, such that the spatial and temporal overlap is maximized. With beams containing finite members of electrons and photons, respectively, the maximum intensities are to be achieved by minimizing the cross-sectional areas and durations of the pulses in each. These conditions are not independently controllable, being subject to certain requirements of phase space conservation in each beam. In addition there are, among others, numerous practical constraints:

1. The available particle output and repetition rate of both laser and electron sources; also the emittance in phase space of such sources, and their pulse lengths and timing.

2. The geometrical limitations on the injection optics required to bring the light and electron beams together; these arising both from the aperture limitations of the beam transport systems, and the presence of a destructively intense electron beam.

3. Shielding requirements requiring separation of laser and electron sources, for convenience in maintenance.

4. Minimization of gas scattering as a source of competing background photons.

5. Avoidance of the deleterious effect of intense synchrotron radiation on optical components.

6. Duty cycle limitations on acceptable output beams, and requirements on the degree of focusing of such beams.

7. Compatibility with other modes of machine operation, insofar as the "laser" beam is to run in a subsidiary or parasitic fashion.

8. Stability of the electron and light beams in space and time, and the need for practical alignment procedures.

The manifestations of these requirements will become apparent in the discussion of the design of specific systems for CEA and SLAC and the estimation of the probable beams to be extracted therefrom.

III. INTERACTION RATE FORMULAE

We must compute the rate with which the colliding beams produce output quanta. This problem may be solved, in its simplest form, by considering a parallel beam of N_e particles per unit time, velocity v_e, traveling in a beam of cross-sectional area A_e. The volume density of such a beam is $N_e/v_e A_e$, and a length z of it will present $\Sigma = N_e z/v_e A_e$ particles per unit area. A stationary particle of a second species, "p," swept over by this beam for a time interval t_p, will pass through a length $z = v_e t_p$ and will interact with a probability $\sigma\Sigma$, where σ is the interaction cross section.

If "p" is moving with velocity v_p counter to v_e, then $z = (v_e + v_p)\, t_p$, instead. It will take "p" a time $t_p = L/v_p$ to cross through a stationary interaction region of length L, so $z = (v_e + v_p)L/v_p$. Substituting this in the expression for Σ one sees that N_p particles per unit time of the species "p" will interact with a net rate

$$\Gamma = \frac{v_e + v_p}{v_e\, v_p}\; N_e N_p\; \frac{L\sigma}{A_e} = \lambda\sigma$$

where λ is defined as the luminosity, and Γ is the number of interaction per unit time within a length L. The spatial distribution of the second beam does not enter as long as it lies entirely within the first and the first is homogeneous. If the area A_p of the second beam is larger than A_e, then A_p appears in the denominator instead. The time overlap requirements may be similarly analyzed with the assumption of simple square pulses of length T_e and T_p, containing $W_e = N_e T_e$ and $W_p = N_p T_p$ particles. Then the net yield in length L can be written symmetrically

$$W_\gamma = \frac{v_e + v_p}{v_e\, v_p}\; W_e W_p\, \frac{L\sigma}{AT} \qquad (1)$$

where $A = \max(A_e, A_p)$ and $T = \max(T_e, T_p)$, and the W_γ output photons arrive during time: $\min(T_e, T_p)$. For present applications the velocity ratio $(v_e + v_p)/v_e v_p = 2/c$, where c is the velocity of light. Of course it is assumed in this formula that both the time intervals and the areas are similar and are centered each on the other in their respective spaces, and that the quantity T is much longer than the transit time through the interaction region. Because practical interaction lengths work out to one or two meters at most, the time condition would in general be satisfied with beam pulses longer than 20-30 nsec.

The rate Eq. (1) is appropriate to geometries in which the length of the interaction region L that is available is the factor limiting the overall reaction rate. The conditions under which this occurs depend upon the natural beam divergence, as will be discussed presently, and may be roughly stated in the form: The interaction is length-limited provided the larger diameter beam no more than doubles this diameter between the center and the ends of the region of length L. The rate formula used for this case takes the form

$$W = 2W_e W_p \frac{L}{cAT} \sigma . \quad \text{(length-limited)} \qquad (2)$$

Profit is to be gained by increasing L as much as possible, up to the point at which an ultimate limit arising from beam phase space precludes further gain. Alternatively, the larger diameter beam may be focused to produce (effectively by decreasing A) a greater yield with given L, if need be also focusing the smaller beam to keep it inside the larger. At CEA both L and beam divergence are severely restricted for mechanical reasons and neither of the above procedures is very practical. Equation (2) is applicable to and will be used for calculations pertaining to the CEA.

The output beam of a linear accelerator such as SLAC is relatively accessible as well as long, and permits one to attain easily a geometry maximizing the yield W_γ to that allowed by the ultimate phase-space limit. Equation (2) must therefore be generalized. Assume the beam "p" is brought to a focus in the form of a circle of radius r_p, and that this spot radiates uniformly across its surface and in solid angle, for polar angles $\theta < \theta_p$ with respect to the axis of the disk. The time rate of particles passing through area dA_p at radius r from the axis into solid angle $d\omega_p$ at polar angle θ is thus $I\,(r,\theta)\cos\theta\, dA_p d\omega_p = [N_p/(r_p \sin\theta_p)^2]\,\delta(r,\theta)\cos\theta\, dA_p d\omega_p$, where $\delta(r,\theta) = 0$ unless $r \leq r_p$ and $\theta \leq \theta_p$. The quantity $(r_p \sin\theta_p)$ is an invariant in linear focusing systems for particles and light, being essentially the constant quantity of the Abbe sine condition. In the present case θ_p is generally small compared to unity and $\sin\theta_p$ will be replaced by θ_p. This representation of both electron and laser beams as emanating from black bodies (of rather high temperature!) appears to be a reasonable approximation for the purpose of discussion.

At a distance $2r_p/\theta_p$ away from the focal plane the beam will have approximately doubled its diameter. Thus at the end of a region of length $4r_p/\theta_p$ the particle density will be down to one-fourth its central value, and will decrease roughly quadratically with distance from there on. If one neglects the contributions from these outer regions, which a more exact calculation shows amount to about one-third of the total for interactions occurring along the axis (see Appendix A), and puts $L = 4r_p/\theta_p$ into Eq. (2), together with $A_p = \pi r_p^2$, one obtains

$$W_\gamma = \frac{2W_e W_p \sigma}{cT} \cdot \frac{4}{\pi r_p \theta_p} \tag{3}$$

A more exact derivation of this formula is made in Appendix A with generalizations to allow for a finite second beam and a focusing system whose central zone is absent. Let beam "e" be antiparallel to beam "p" but displaced laterally from it by a radial distance $r_e = \beta r_p$.

Beam "e" is still assumed of negligibly small radius and divergence. Also assume that the focusing system for beam "p" is such as to restrict its components to those diverging with polar angles greater than some minimum angle $\theta_m = \alpha\theta_p$, where $\alpha < 1$. The more general result is

$$W_\gamma = \frac{2W_e W_p \sigma}{cT} \cdot \frac{J_o(\beta)}{(1+\alpha) r_p \theta_p} \qquad (4)$$

where W_p is the number of particles actually present in the focused beam. The function $J_o(\beta)$ is plotted against β in Fig. 1, and is seen to be more or less constant as long as beam "e" lies within the central two-thirds of the focused beam "p". A more useful plot is also given to describe the ratio of $[J_o(\beta)/J_o(0)]_{avg}$ averaged over a circle of radius r_e. Thus as long as beam "e" (assumed completely parallel) has a diameter less than 80 percent of that of the focused beam "p", and is centered thereon, the value of J_o in Eq. (4) may as well be approximated by $J_o(0) = 4/\pi$, as in Eq. (3). In practice the invariant quantity $r_e\theta_e$ for a linear accelerator beam will be one or two orders of magnitude smaller than that for a high-power laser, so that the latter may be taken as beam "p" in Eq. (4) and the electron beam focused down so as to lie completely within the laser beam, both in spot size and divergence angle.

It is also shown in Appendix A that the interactions may be considered to lie within a finite length of interaction region. Specifically, a fraction $P < 1$ of the interactions may be considered to occur approximately within a region of total length

$$L_p \doteq \frac{2}{(1-P+\alpha P)} \left(\frac{r_p}{\theta_p} \right) \qquad (5)$$

within about a factor of thirty percent either way.

Although these formulae are based upon highly idealized beams and are otherwise approximate, they do indicate the dependence of the yield upon the geometry of the interaction region and the time structure of the two beams. One may draw some general conclusions:

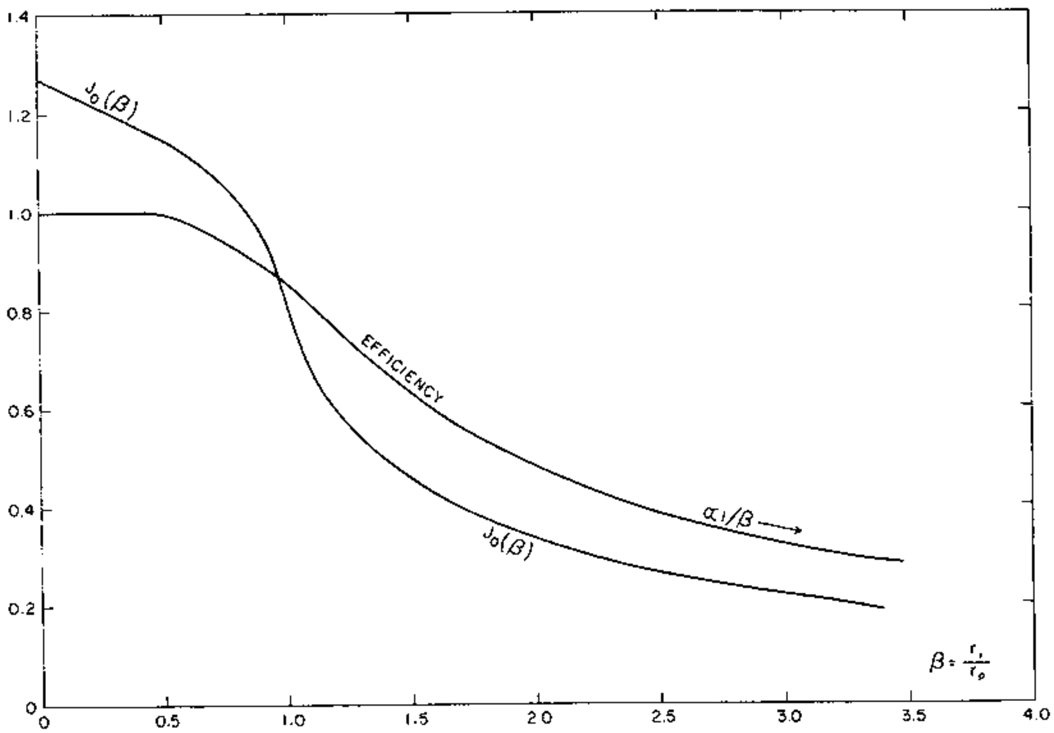

FIG 1 GRAPH OF $J_0(\beta)$ AND $\left[J_0(\beta)/J_0(0)\right]$ 177-1-8

(1) The limiting geometrical factor is the larger of the two phase space factors $(r_p\theta_p)$ associated with the two beam sources. This quantity establishes the maximum yield, through Eqs. (3) and (4). One cannot beat the game by use of lenses since $(r_p\theta_p)$ is an invariant in a lossless system. Because the quantity W_p in these equations represents the total number of particles arriving at the focus, stopping down one's optics to reduce θ_p in $(r_p\theta_p)$ reduces W_p itself as $(\theta_p)^2$, and hence the yield W_γ as θ_p, so one can only lose. The product $(r_p\theta_p)$ is actually a measure of the absolute source temperature of the laser (or linear accelerator) and the restrictions just described may be considered thermodynamic.

(2) Assuming the electron beam has the smaller $(r_e\theta_e)$, as is the case from a linear accelerator, and that its focal state has been chosen to yield a beam of suitable divergence for experimental purposes, one may focus the laser beam in such a way as to minimize the interaction length given in Eq. (5). The limits are given by the requirement that the radius of the laser focus be no less than about 1.2 times that of the electron beam passing through it. The profit in this procedure, as will be seen, may come from the possibility of reducing the quantity α representing a missing central zone in the beam focusing system. Also it may permit operating in the realm of Eq. (4) rather than in the less efficient domain of length-limited Eq. (2). Of course, θ_p should not be increased so much that it is limited by an aperture stop.

(3) It may happen, as in the case of the CEA internal beam, that the geometry is intrinsically length-limited, through finite interaction path and small optical injection aperture, to the realm of Eq. (2). Then, through focusing with due regard for the shape and location of the aperture stops present, one must maximize the ratio of injected photons to focal area, W_p/A_p. Again, no further gain is to be realized once the sizes of the electron and photon beams become commensurate. One may have further restrictions on the possible location and size of focusing elements.

(4) Other things being equal, the time duration T of the longer of the two interacting beam pulses should be made as short as possible without reduction of the corresponding W. However, in a synchrotron one has the possibility of repetitive passes of the same electrons through a given long light pulse so that detailed analysis must be made with reference to each specific machine.

(5) The numbers of injected particles per pulse, W_e and W_p, should obviously be made as large as possible. However, the overall yield, W_γ, is to be integrated over a long time period, hours or days for typical experiments, so that high <u>average</u> power lasers are required, those capable of sustaining high pulse-output energy at a high repetition rate.

With these principles in mind I shall discuss specific systems applicable to the CEA and SLAC and shall estimate yields expected with laser sources of contemporary and foreseeable existence.

IV. LASER CHARACTERISTICS

For the present geometric computation the pulsed laser source will be considered to be a black box emitting a beam of optical photons into a small solid angle in pulses of short duration, and this with some finite repetition rate. Output pulses of light with energies up to 1500 joules are available from commercially available lasers, and pulse lengths extending from about 600 μsec down to 1 nsec have been obtained. Repetition rates in the range of 10-20 pps have been obtained in some devices, and average power outputs extending up to the order of 50 watts have been obtained at lower repetition rates. Not all of these characteristics pertain to one and the same laser system. In particular, high single-pulse energies, very short pulses, and high average power outputs tend to some extent to be mutually exclusive. The outlook on this problem will be discussed more extensively in Section X.

For the sake of discussion we shall make rate estimates in terms of a one-joule ruby laser, radiating a wavelength 6943 Å , or photons of energy 1.79 eV each. Thus,

$$W_p = 3.50 \times 10^{18} \text{ photons per joule output} \qquad (6)$$

A typical, not very select commercial ruby laser is found to radiate a spot of about 0.5 cm diameter into a cone of total angle about 10^{-2} radians. Thus, taking $r_p = 0.25$ cm and $\theta_p = 0.5 \times 10^{-2}$, we get for such a laser the phase space factor

$$(r_p \theta_p) = 1.25 \times 10^{-3} \text{cm} \quad \text{(laser phase space)} \qquad (7)$$

This number will be used in our rate estimates. It should be noted that this number will vary markedly depending upon the laser construction, mode of operation, and ruby quality. To some extent the laser output W_p may be increased by using a larger ruby and increasing r_p as well, thus nullifying any increase in overall yield W_γ, Eq. (4). The main benefit in such a case would then be the possibility of using a shorter interaction length, L_p, Eq. (5), which may or may not be advantageous, depending on electron beam size and machine geometry. Conversely, reducing r_p and perhaps θ_p, together with W_p, as occurs in some Q-switching arrangements to obtain short pulses, may not necessarily be deleterious providing one has leeway in the choice of the interaction length, L_p. This point of view becomes more palatable if one views the ratio $W_p/r_p\theta_p$ as reflecting the source temperature of the laser atoms and of a virtual source at the focal point in the interaction region.

The intrinsic diffraction limit for a laser may be calculated from the Rayleigh formula to be $(r_p\theta_p) = 0.61\lambda = 4.25 \times 10^{-5}$ cm for a ruby, thirty times smaller than empirical values. Multimode operation is to blame.

Because extrapolations are readily made, we shall for the present base our considerations on the parameters of Eqs. (6) and (7). Other kinds of source lasers will be discussed later.

V. APPLICATION TO STANFORD LINEAR ACCELERATOR (SLAC)

A. Basic Yield

The output beam of a linear accelerator being much less restricted with regard to interaction regions than, say, the internal beam of a synchrotron, I shall discuss the former first.

Assume the following SLAC parameters:

Electron energy	20 GeV
Average beam current	15 μA
Pulse repetition rate	360 pps
Electron beam pulse length (each pulse with 2856 five-psec bunches)	1 μsec
Phase space factor $(r_e\theta_e)$	3×10^{-5}cm*

---

*For the maximum aperture available in beam switchyard arm A. Extrapolating data from the Mark III linac, this parameter is likely to be nearer 1.5×10^{-6} cm at 20 GeV.

The electron phase space factor is so much smaller than that obtainable from lasers that it poses no great restriction unless monochromatic beams are desired. Were lasers able to achieve their diffraction limit, about 4×10^{-5}cm, a more careful estimate for the electron beam would be needed. The electron rate is thus

$$W_e = 2.6 \times 10^{11} \text{ electrons/1-}\mu\text{sec pulse.} \qquad (8)$$

Using $T = 10^{-6}$ sec in Eq. (3) we may calculate an upper limit to the output photon yield

$$\frac{W_\gamma}{\sigma} = \frac{8W_pW_e}{\pi cTr_p\theta_p} = \frac{8\times3.5\times10^{18}\times2.6\times10^{11}}{3.14\times3\times10^{10}\times10^{-6}\times1.25\times10^{-3}} \times 10^{-27} \text{ photons/laser joule/pulse/mb}$$

$$\frac{W_\gamma}{\sigma} = 62 \text{ photons/laser joule/pulse/mb.} \qquad (9)$$

This presupposes no geometric losses or other inefficiencies. For a 20-GeV primary electron beam energy, the total cross section is about 460 mb, and photons in the top tenth of the output spectrum, about 6.8-7.3 GeV, account for about 44 mb of this. Thus, one may estimate an overall yield of about

$$W_\gamma = 2.8 \times 10^4 \text{ photons/laser joule/1-}\mu\text{sec pulse} \quad (10)$$

with about 2700 in the top tenth of the spectrum.

B. Timing

Before considering the geometric requirements for such a yield at SLAC, one should note that in practice when one uses a Q-switch device to obtain a short laser pulse, what one gets has a length of the order of 50-100 nsec or less. Could one crowd the $W_e = 2.6 \times 10^{11}$ electrons from SLAC into a shorter time interval, such as 0.1 μsec instead of 1 μsec, the yield of Eq. (10) would be increased by a factor of 10. The virtue of this gain depends upon the use to be made of the beam, since the duty cycle will be worsened by the same factor of ten. The coincident timing of laser and electron pulses would probably be impossible with rotating mirror Q-switches but should be a simple matter using Kerr or Pockel cells. Note that insofar as the laser pulse is forced to be less than 100 nsec long anyway, one can only gain by reducing the electron pulse to the same length.

C. SLAC Geometry for Laser Injection

Although in principle the yield of Eq. (10) is independent of the focal state of the laser beam, in fact a number of constraints tend to govern the specific choice of r_p and θ_p, including

(1) Limitation of available interaction length L, arising from
   (a) Only a finite uncluttered run of electron beam at the desired state of focus.

(b) The need to reduce background bremsstrahlung from gas by swerving the electron beam into and out of a short interaction region.

(c) The finite length in space of short electron or laser bunches.

These place an upper limit of about 10 meters (33 nsec) upon L, and less if the gas background is severe.

(2) The necessity to place the laser source apparatus in a habitable area with tolerable radiation background, that is, outside some main shield barrier. At SLAC this means about 16 meters away from the beam axis above the beam switchyard, unless a special shielded room is buried in the earth fill.

(3) The requirement of a mirror adjacent to the electron beam to inject the light in at a small angle nearly antiparallel to the electron motion. This mirror must either be offset from the beam or, if in a symmetric configuration, have a hole through it to permit passage of the electron beam. If polarized output beams are desired, the mirror reflection must not modify the polarization state in an irremediable manner. Its surface must be either proof against or shielded from deleterious radiations coming along with the electron beam, as, for example, synchrotron radiation from previous beam deflections.

As a starting point, consider a laser beam with $r_p = 0.25$ cm and $\theta_p = 0.5 \times 10^{-2}$ focused one-to-one in the desired interaction region. Then, by Eq. (5), 90 percent of the interactions will occur within a total length of about 10 meters. Thus an unaltered laser beam would give a yield on the margin of being limited by the interaction length available. At one end of the interaction region, 500 cm from the center, the laser beam would have a diameter of about $2 \times (\theta_p \times 500 \text{ cm} + r_p) = 5.5$ cm, and a mirror of this diameter must be provided to encompass it. However, this mirror must have a hole in it to allow the electron beam to pass through. At the moment it is difficult to estimate what sort of halo might surround a beam presumed to be of diameter of the order of 0.5 cm, the laser focus diameter (electrons outside this diameter will have a reduced reaction rate anyway, as seen from Fig. 1).

Let us say that a 1-inch/diameter (2.54 cm) hole is required in the mirror (Fig. 2). This has the approximate effect of presenting the system with a minimum angle of beam divergence, making $\alpha = 0.46$ in Eq. (4). Because the laser may be presumed to illuminate a circular area uniformly, the photons entering the interaction are reduced in number by $(1-\alpha^2)$. Hence the output yield W_γ is reduced by a factor $(1-\alpha^2)/(1+\alpha)=(1-\alpha)=0.64$. By Eq. (5), the 90-percent interaction length becomes about two meters, but cannot be exploited without further increase of α.

Improving upon this requires additional information about the state of the desired output beam. There are two characteristic possibilities.

1. Divergent Output Beam

In this case it is assumed that the electron beam may be focused to a very small spot in the interaction region, a spot smaller in any case than that to which the laser is focused. For example, assume the electron beam is focused to a spot of radius $r_e = 0.03$ cm. With a phase space of 3×10^{-5} cm, this beam will diverge with half-angle $\theta_e = 10^{-3}$, or six inches in 500 feet, which may be desirable for track chamber experiments. The laser beam may now be focused to $r_p = 0.04$ cm, giving $\theta_p = 3.14 \times 10^{-2}$. The 90-percent interaction length $L_{c.9} = 25$ cm. A mirror a convenient two meters away can have a diameter of 12.6 cm, or about five inches. A one-inch hole in this mirror results in $\alpha = 0.2$, and efficiency loss by a factor of only 0.8.

Another circumstance requiring focusing is one in which the laser beam happens to fall within a phase space considerably smaller than $r_p \theta_p = 1.25 \times 10^{-3}$ cm, as assumed above. In such circumstances focusing down to the electron beam size would be mandatory to get the interaction length within bounds.

In conclusion, it is evident that to the extent that the electron beam may itself be focused to keep it within the laser beam, the latter may be efficiently transferred into a short interaction region with only a minor reduction of the basic yield of 2.8×10^4 photons/laser joule/1-μsec pulse, Eq. (10). A specific illustrative design to accomplish this at SLAC is given in Section IX.

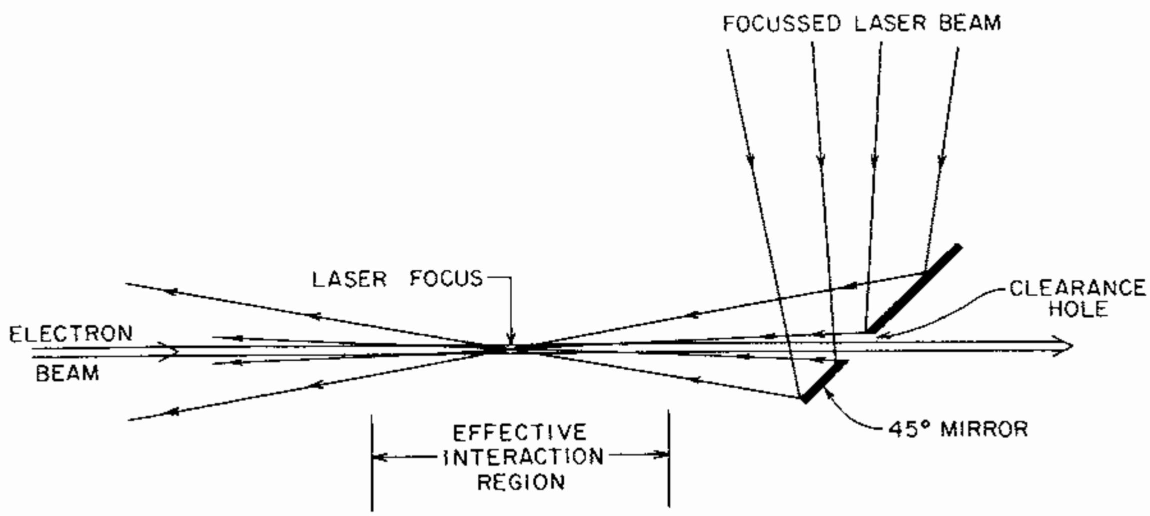

FIG. 2 SKETCH OF INTERACTION REGION GEOMETRY 177-2-A

2. Quasi-Parallel Output Beam - Monochromatic Beams

The monochromatization process is discussed in Appendix B. It is shown, in Fig. 3, that to restrict the output spectrum to the top 10 percent or so requires, at 20 GeV, that polar laboratory angles less than 10^{-5} be segregated. For a point source this results in a spot of 0.15 cm radius were only 500 feet available for such a beam run. This short distance would at best permit only a rough monochromatization of the "laser" beam. A run of 1000 feet should, on the other hand, make monochromatization definitely feasible. At the same energy and distance (500 feet), a spot of 0.30 cm radius will include roughly the top 30 percent of the output spectrum (5.2 - 7.3 GeV). Thus at the interaction region the electron beam must have a radius of the order of 0.1 cm or less for useful monochromatization to occur at these energies, and thereby a phase space factor $(r_e\theta_e)$ no larger than, say, about 10^{-6} cm. Considering that the observed 1-GeV Stanford Mark III linear accelerator factor is about 3×10^{-5}cm, and assuming the theoretical inverse dependence with energy, SLAC at 20 GeV should have a factor of 1.5×10^{-6}cm. This is, if it is realistic, barely satisfactory. A 1000-foot beam run would relax $(r_e\theta_e)$ to 2×10^{-6}, which is then consistent with 1.5×10^{-6} from the accelerator. Should the accelerator factor turn out to be larger than 1.5×10^{-6} cm, part of the electron beam would have to be collimated away (a formidable process, it would appear, in view of the fearful power densities in the electron beam!), with concomitant loss of yield. At lower machine energies, both the phase space factor and the collimation angle increase inversely proportional to the energy so that the same monochromatization should be achievable at lower energies. Indeed, the electron focal spot size need not be quite so small, so that either a more fully utilized beam or a sharper slice of the energy spectrum might be obtained by appropriate increases in the quantity r_e. In any case, it is evident that monochromatization with efficient use of all or most of the available electrons requires electron beams of radii 0.1 - 0.15 cm. The laser may thus be focused down to $r_p = 0.15$ cm or less. With our assumed laser having $(r_p\theta_p) = 1.25 \times 10^{-3}$cm, one gets $\theta_p = 0.83 \times 10^{-2}$ requiring optics of aperture diameter 8.3 cm at a

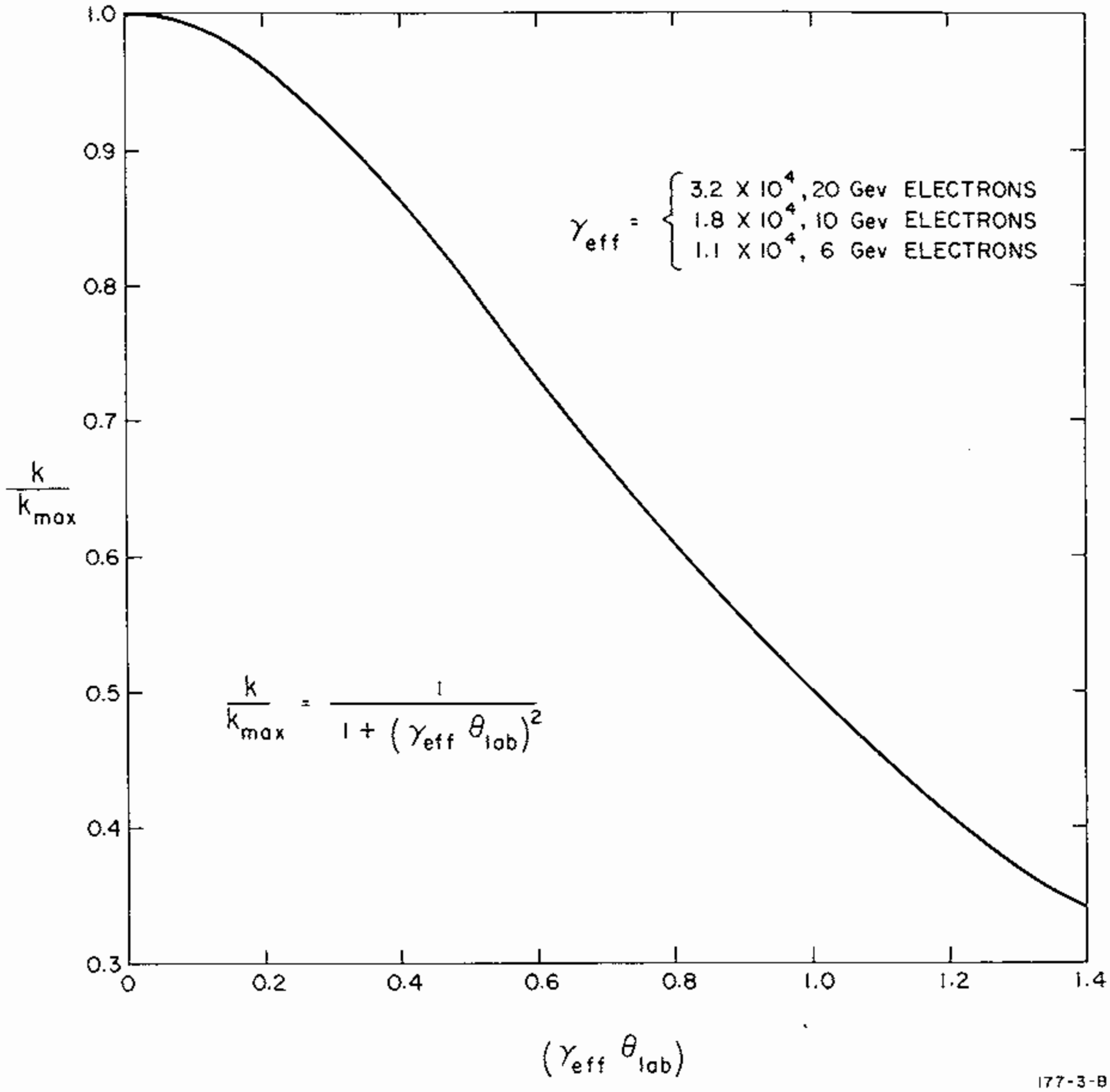

FIG. 3 OUTPUT PHOTON ENERGY vs. LABORATORY ANGLE

distance of five meters. A one-inch hole through this device to pass the electron beam gives $\alpha = 0.306$. The reduction of yield Eq. (10) from this cause will be by a net factor $(1-\alpha) = 0.69$, which is not too bad. The 90-percent interaction length, Eq. (5), will be $L_{0.9} = 0.96$ meters, which is consistent with a reasonable geometry. A five-meter optical throw, as presumed above, is as large as one could conveniently obtain.

One must consider the consequences of having a laser whose phase space factor $(r_p \theta_p)$ is smaller than presumed above. With constant r_p the factor $(1-\alpha)$ would have to diminish, vanishing when the beam diameter at the injection mirror equals the size of the hole passing the electron beam. On the other hand, θ_p and hence the quantity of α could be maintained by reducing r_p. This, according to the efficiency graph of Fig. 1, will reduce the yield approximately by the factor (r_p/r_e) when $r_p < r_e$. However, the yield per incident photon increases as r_p^{-1}, according to Eq. (4), so that essentially no decrease of yield will be expected. The effective interaction length $L_{0.9}$, however, will decrease as r_p. This is not too useful because the large (five meters) optical throw must be maintained to keep $(1-\alpha)$ large. Of course, if an interaction region longer than ten meters were available, a reduction in $(r_p \theta_p)$ could be compensated by removing this region further from the mirror so that no change in $(1-\alpha)$ need occur. The yield would then increase as $1/(r_p \theta_p)$.

It is simply shown that if $(1-\alpha)$ is less than 0.5, increase of yield will result from increasing the mirror aperture, and thus θ_p, at the expense of decreasing r_p below r_e. Specifically, if with $r_p = r_e$ the efficiency $(1-\alpha) \leqq e_o < 0.5$, then increasing the angle θ_p through focusing to raise α to 0.5 will yield a final efficiency factor $0.25/(1-e_o)$ multiplying Eq. (3) for the yield. Thus, at worst, one is down to 25 percent of the optimum yield. Of course, if one starts out with α effectively greater than 1, and cannot extend the optical throw or reduce the beam-hole size, a greater loss is to be expected.

In conclusion, assuming that the entire electron beam can be utilized in the monochromatization process, it should be possible to achieve at least 50 percent of the overall yield figure of Eq. (10), that is, of the order of 1.4×10^4 photons/laser joule/1-μsec pulse in the whole output spectrum. An illustrative design for a SLAC monochromatic beam will also be given in Section IX.

VI. APPLICATION TO CAMBRIDGE ELECTRON ACCELERATOR (CEA)

The electron beam from a circular accelerator may be utilized either by injecting the laser beam into the accelerator itself, tangent to the circulating beam, or by extracting the electron beam and using the same procedures just described for a linear accelerator beam.

A. External Beam

In the latter case one sees at once that everything depends upon the spill time achieved for the extracted beam. If the entire electron beam can be extracted in a time comparable to one orbit revolution, say in a pulse about 1 μsec long, one may obtain immediate results by scaling from the estimates developed for SLAC. The phase space factor for the electron beam will be expected to be approximately that for the circulating beam, namely about $\theta_e = 10^{-3}$ and $r_e = 0.4$ cm, giving $r_e\theta_e = 4 \times 10^{-4}$ cm, approximately. This is sufficiently less than that of the probable laser beam so that both could be focused down, as in the SLAC case, to $r_p = r_e = 0.1$ cm. The output photon beam would then diverge with $\theta_e \approx 4 \times 10^{-3}$, or into a spot about ten inches in diameter 100 feet away. This would be suitable for a large track chamber. The photon angle $\theta_p = 1.25 \times 10^{-2}$ would require a mirror about 5 cm in diameter, if located two meters upstream. With a 2.5-cm-diameter hole in such a mirror, to pass the electron beam, one has $\alpha \approx 0.5$. The 90-percent interaction length will then be about 15 cm so that the whole injection apparatus need occupy only about ten feet of electron beam. No rapid CEA ejector now exists so that the electron yield W_e is problematical. Assuming 25-percent extraction of an optimum 10^{-11}-electron circulating beam, one gets a yield of 2.5×10^{10} electrons/1-μsec pulse, about 1/10 that from SLAC, Eq.(8). With $\alpha = 0.5$, the resultant yield would then be about 1.6×10^{3} photons/laser joule/1-μsec pulse into the whole spectrum. This would be sufficient to saturate a large hydrogen bubble chamber studying polarized photon interactions up to the 850-MeV photon energy available from the 6-GeV CEA. A higher yield would be obtained were the same number of electrons injected into a shorter, say 0.1 μsec, bunch in the CEA orbit, as is probably possible, and the laser pulse suitably timed to meet the electrons. The worsened duty cycle will not affect bubble chamber operation. Bubble chamber operation could be parasitic upon other use of the residual electron beam provided the focusing of the electron beam were not intolerable or uncorrectable, and the poor duty cycle acceptable. The last is unlikely.

Clearly a long-spill external beam, say, 1 msec, so agreeable for counter experiments, would diminish the output photon yield to an impossible value in the above arrangement. It is also evident that even in the one-turn extraction arrangement the electron beam phase space precludes monochromatization.

B. Internal Beam

In this case the laser beam is injected into the synchrotron proper, tangent to the electron beam, preferably in a "straight section" between deflecting magnet segments. This procedure is of immediate interest because it is the most feasible procedure to bring high energy "laser" beams out of presently operating electron accelerators. Use of an internal beam presents two distinct differences from the use of an external or linear accelerator beam, one being the severe geometrical constraints imposed upon injection geometry by the accelerator magnet and chamber configuration, and the other being the possibility of efficiently obtaining long output pulses through multiple traversals of the same electrons through a light pulse hundreds of microseconds long. The geometrical constraints are only a nuisance, but the long-pulse possibility offers the opportunity of using lasers in free-running, as opposed to Q-switched, operation, wherein considerably higher output energies may be obtained, and no tricky timing problems are encountered. Bremsstrahlung from residual gas in the interaction region does become more severe in effect with long pulses, however, but this is in principle, at least, to be ameliorated by pumping. In addition, one may reasonably consider using counter techniques to measure the output beam intensity and perhaps secondary particles as well.

The geometrical constraints are simple to describe, the most obvious being the fact that the CEA straight section is only about one meter long and that the state of focus of the internal beam is not an adjustable parameter, being of order of magnitude 1/8-inch high and 1/2-inch wide at 6 GeV. Beam currents of about 5 mA peak are presently obtained, corresponding to 2.4×10^{10} electrons circulating at 1.32×10^{6} sec^{-1}. In addition, the entrance aperture along a tangent to the beam is constrained vertically to an opening of 5/8-inch, and horizontally to about two inches, at a distance of 150 inches from the center of a straight section. There is no room to mount an injection mirror much closer than 220 inches from the straight section. Altering these constraints would involve in general formidable reconstruction of basic machine components, although there is a good possibility of increasing the vertical aperture from 5/8-inch to one inch.

The effect of these constraints is, first that they cause the yield to be limited by the interaction length, as described by Eq. (2). An injected laser beam clearing even a 1-inch-diameter aperture 150 inches from the interaction region will subtend a half-angle $\theta_p = 3.3 \times 10^{-3}$. If focused to a spot 1/8-inch in diameter (the electron beam height), $r_p = 0.16$ cm, so that the 90-percent interaction length, given by Eq.(5) with $\alpha = 0$, is 9.7 meters, much longer than the available straight section of electron beam. Moreover, the product $(r_p\theta_p) = 0.53 \times 10^{-3}$ cm, which is less than half that available from normal lasers. A certain amount of beam vignetting is inevitable unless efficient lasers of smaller phase space can be used. The design problem is thus to maximize W_p and minimize A_p in Eq. (2).

Write $W_p = (3.50 \times 10^{-18} \times E)$ photons per laser joule, where $E < 1$ is an efficiency factor describing the vignetting of the laser beam. With $W_e = 2.4 \times 10^{10}$ electrons per CEA beam revolution passing the interaction region, $L = 100$ cm, and $\sigma = 575$ mb total cross section at 6 GeV, Eq. (2) can be written

$$W_\gamma = 3.2 \times 10^{-4} \frac{E}{T(\text{sec}) \times A(\text{cm}^2)} \text{ photons/laser joule/revolution} \qquad (11)$$

As the laser beam is focused down, its focal area decreases as r_p^2, so that from this effect W_γ is proportional to $(1/r_p^2)$. Vignetting begins as soon as the conical beam begins to hit the one-inch vertical aperture limitation, at which point E begins to decrease proportional to r_p itself. The result is that W_γ will still increase as $(1/r_p)$ up to the point at which the two-inch horizontal aperture begins to cut off the beam. Sharper laser focusing will not increase W_γ any more. It is assumed that the laser focus is larger than the electron beam height so that A in Eq. (11) is that of the former beam. Limiting the laser beam to an aperture of two-inch-diameter, 150 inches from the interaction region corresponds to $\theta_p = 6.6 \times 10^{-3}$. This, with a laser having $(r_p\theta_p) = 1.25 \times 10^{-3}$cm as assumed, gives $r_p \approx 0.19$ cm, or a spot of about 5/32-inch diameter. This is a little larger than the presumed 1/8-inch electron beam height so our procedure is appropriate. The resulting area in Eq. (11) then is $A = 0.113$ cm^2. Insofar as the external optics can project a two-inch-diameter laser beam on the one-inch by two-inch entrance aperture, the factor $E = 0.6$. Clearly not much is to be gained by making a smaller focal spot as the electron beam area is limiting anyway. In fact, the approximate half-inch width of this beam (which is not a well-determined quantity) would impose another factor of about $\frac{(1/8 \text{ inch})}{(1/2 \text{ inch})} = 0.25$ upon E, reducing it in effect to $E = 0.15$. On the other hand, a laser with an intrinsic emittance in phase space of about 5×10^{-4} cm would raise E up again to 0.25. Putting E = 0.15 and $A = 0.113$ cm^2 into Eq. (11) gives

$$W_\gamma = \frac{4.3 \times 10^{-4}}{T\ (\mathrm{sec})} \text{ photons/laser joule/revolution} \qquad (12)$$

To discuss T one should consider separately two domains, according to whether the length of the laser pulse is greater or less than one CEA beam revolution period (0.76 μsec).

1. Long Laser Pulse (>> 1 μsec)

In this case the number of beam revolutions approximately equals the revolution frequency (1.32×10^6 sec^{-1}) times the length of the laser pulse, T. This assumes that the accelerator can hold the electron beam in a stable orbit position for the requisite time. The total yield is then substantially independent of the laser pulse length and amounts to a nominal $W_\gamma = 600$ photons/laser joule. The laser may then be run with arbitrary timing to achieve a maximum yield. Were it not for the geometric inefficiencies imposed by the accelerator entrance aperture and rectangular beam cross section, this figure would be closer to 4000 photons/joule. However, a beam of 600 photons per pulse would just about saturate a large hydrogen bubble chamber with pairs and Compton electrons.

2. Short Laser Pulse (< 1 μsec)

If the electrons just filled the CEA orbit, one would have $W_\gamma = 600$ photons/laser joule no matter what the laser pulse length. The orbiting electrons constitute a target of constant density. On the other hand, if the electrons are bunched into a fraction $f < 1$ of the orbit, then, as long as the laser pulse is of shorter duration than the electron bunch, the yield becomes $W_\gamma = 600/f$ photons/laser joule. One might expect to gain a factor of 10-20 in intensity this way. Because bubble chambers have sensitive times of the order of 500-1000 usec, the length of the photon beam pulse is not directly critical; the major background source is bremsstrahlung arising from residual gas in the accelerator straight section. This background will be discussed more fully in the next section.

It is to be observed that more efficient use of the sub-microsecond laser pulse can be made with an external electron beam, primarily because of the greater flexibility and lack of spatial constraints with an external beam. However, such a technique will not be useful until an efficient one-turn electron beam ejector is developed. The internal beam has the additional advantage that its use is almost completely parasitic on the beam itself.

VII. BACKGROUND FROM GAS BREMSSTRAHLUNG

Even in the absence of a laser beam, the electron beam from a linear accelerator or synchrotron will be accompanied by an appreciable quantity of energetic photons. Most of these will be synchrotron radiation associated with magnetic deflection of the beams. These photons in sufficient number will have a deleterious effect upon optical surfaces and lenses in their path but, being less than a few tens of keV in energy, will not appreciably influence experiments using multi-MeV output beams. A potentially more serious background consists of bremsstrahlung arising from electrons scattered off the walls and obstructions in the beam vacuum chamber. One may at this point only assume a sufficiency of clippers, magnetic deflections, collimators, etc., to keep any detector from looking at a surface illuminated by stray electrons.

Bremsstrahlung by electrons scattering against the residual gas in the interaction region forms the worst threat. We may estimate its severity by considering that air at STP has a radiation length of 330 meters.[7] At a pressure of 10^{-6} mm Hg, the radiation length is about 2.5×10^{11} meters. A pressure of 10^{-6} mm Hg is a rather good vacuum for a large system in which no special cleaning or pumping measures have been taken, and in any case is quite typical of the operating CEA, with all pumps working well and before the machine heats up appreciably. In each revolution through a one-meter straight section, the CEA's 2.4×10^{10}

by 6-GeV electrons will radiate about 600 MeV into a bremsstrahlung spectrum extending up to 6 GeV, about 0.1 equivalent quanta per turn. Even with beam hardening, background photons between 20 MeV and 6 GeV will emerge numbering 0.57 per beam revolution. On the other hand, a typical long (400 μsec) laser burst will yield, according to Eq. (12), about one photon/laser joule/revolution. Thus a one-joule laser will just barely yield a signal above noise level in this mode of operation. For the CEA an improvement could be expected from

(1) Using a laser more energetic than one joule (such lasers are presently available).

(2) Pumping the interaction region down to a vacuum of 10^{-7}mm Hg or better; probably a feasible procedure using titanium getter pumps or like fast systems.

(3) Reducing the laser pulse length, while maintaining its energy, and at the same time gating the detector; or in the case of a bubble chamber, dumping the electron beam, so that the number of photons produced in the gas is diminished.

(4) Using a sub-microsecond laser pulse together with a tight bunching of the electrons in orbit to improve the intrinsic signal-to-noise ratio.

It is to be noted that an increase of the electron beam intensity will not influence the fractional amount of gas-induced background.

At SLAC a similar gas problem exists, but is not so serious. The increased electron beam intensity will yield about 7.5 photons per meter, between 20 MeV and 20 GeV, in a 10^{-6}mm Hg vacuum. Unfortunately, the beam run from the linear accelerator is likely to be straight and long, up to nearly 100 meters, so that gas background could very well be appreciable, especially were lasers of less than one joule used. In addition to the steps (1), (2) and (4) above, it might prove possible magnetically to put a small directional zig-zag in the electron beam, bracketing the interaction region, to divert the desired laser beam photons away from the majority of the background. It should be relatively easy to pump the restricted region to a 10^{-7} vacuum or better.

VIII. THE INJECTION MIRROR

The mirror which deflects the laser beam into the oncoming electron beam is a crucial element in the system. It must, first of all, have a very high reflectivity for the laser wavelength, 6943 Å for a ruby laser. Reflectivities above 90 percent are possible with silver surfaces, and higher still using multilayer dielectric coatings.

Insofar as the polarization of the injected beam is to be preserved, care must be taken in the choice of mirror geometry. For a non-normal reflection, polarizations lying in and those lying perpendicular to the reflection plane suffer in general both different reflection coefficients and a relative phase shift. These will alter the polarization state of the reflected light. Two corrective courses are open:

(1) Minimize the effect by the use of a nearly normal reflection. For example, a negligible phase shift is induced by a silver surface when the angle of incidence is less than about 10 degrees, permitting a 20-degree deflection.* Such an injector is used in present experiments at the CEA, providing about a 15-degree deflection. Similar results are to be had from dielectric coatings provided the incidence angle is much smaller than the Brewster angle.

(2) If a large angle of incidence is required, as at SLAC where 90-degree deflection is mandatory, one may accept the mirror's phase shift and make up for it ahead of time by means of a quartz compensator. Amplitude compensation might also be needed to correct for the different reflection coefficients of the two polarization components. Such compensation requires the introduction of a number of extra optical surfaces with the attendant possibility of losses. However, a device to produce polarized high energy photon beams would almost inevitably require the presence of such a compensator to control the output beam polarization so that modification to make up for the mirror would be relatively simple.

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*To achieve a phase shift of 90° an angle of incidence of about 77° is required.

It should be noted that even with an unpolarized laser beam a 90-degree deflection will partially polarize the output beam. This might be an annoyance if unanticipated in an experiment.

The mirror faces another problem in that it is the element most subject to radiation damage, both from high energy particles and x-rays, and from the relatively soft synchrotron radiation. The latter has been found to be a severe problem at the CEA. By a process not fully understood, a reflecting surface in the median accelerator plane is darkened through a sequence of rainbow hues until it becomes black. This phenomenon is clearly identifiable with the synchrotron radiation, and may have something to do with the presence of hydrocarbons in the adjacent residual gas. Both the cause and the cure of this deleterious phenomenon are essentially unknown. Meanwhile, it will be difficult to utilize for an extended period a reflector embracing a region about 3/8-inch wide near the CEA median plane. Fortunately, the possibility of focusing the laser beam allows the remainder of the mirror, outside this strip, to be used.

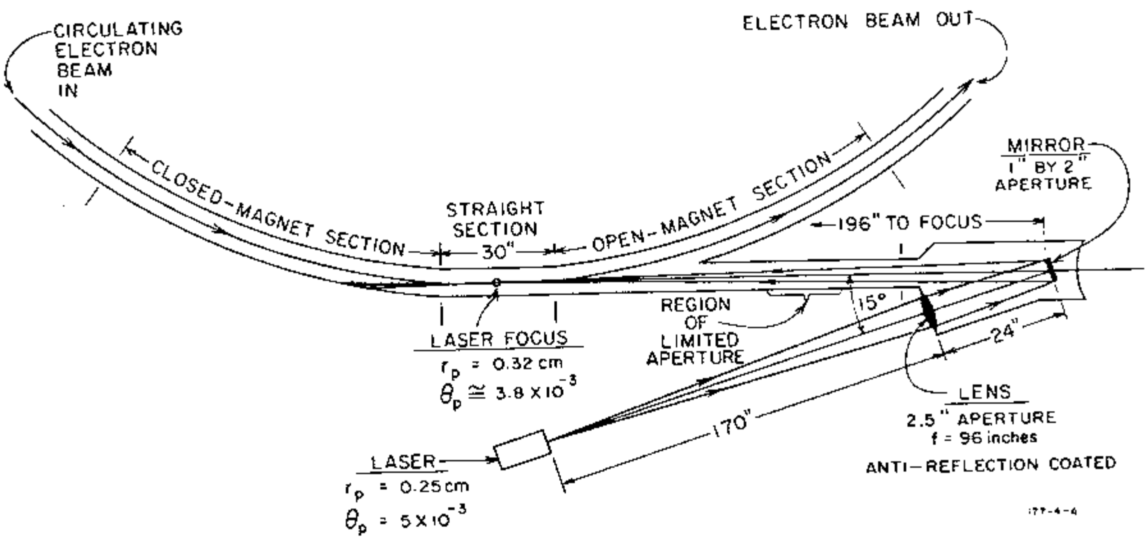
F-center darkening of most glasses and many other dielectrics by radiation may cause similar difficulties in dielectric multilayers and totally-reflecting prisms, should such be used as injector mirrors. Of course, one may succeed in shielding the mirror and isolating it from strong radiation sources, perhaps by accepting some loss in geometric efficiency. This procedure would probably be simpler at SLAC where the space available is more flexible, and the central mirror zones would be removed to clear the electron beam. The high probability of periodic replacement for radiation damage of one form or another suggests that the mirror consist of a simple flat, with the focusing of the beam left to a refractive element farther away from harmful influences. Next to the primary laser source itself, the injection mirror seems to present the most serious technical challenge in extracting a "laser" beam.

IX. REPRESENTATIVE DESIGNS

A. Cambridge Electron Accelerator

In Fig. 4 is shown a design applicable to the CEA, based upon calculations similar to those of Section VI, but restricted to a 5/8-inch vertical aperture. This design is in fact being used in initial experiments to bring a "laser" beam out of the CEA. The optical system is especially simple. The mirror is oblong, a region about 2 inches wide by 3/4-inch high being used to reflect the laser beam into the accelerator tangent to the electrons in a straight section. The central regions of this mirror are subject to synchrotron radiation damage, which has in fact been observed. The produced high energy photons must likewise pass through this mirror, but are expected to be of negligible quantitative effect. The latter photons will be degraded somewhat by having to pass through about 0.2-inch of fused silica or 0.1-inch of silver, depending on the mirror used. The laser is focused by a lens of 2.5 inches clear aperture and 96 inches focal length. The lens is antireflection coated. The image of the laser is about 220 inches from the lens and hence slightly magnified. The lens also forms the accelerator vacuum wall. The laser and the mirror are attached to mechanisms permitting adjustment of their positions and orientations.

According to Section VI, the system would be more efficient were the laser moved farther away, to demagnify its image, and a lens of 125-inch focal length used. This possibility depends critically upon the laser phase space factor. As earlier calculated, one would predict that the latter, more efficient arrangement would yield about 600 photons per laser joule when used with a 5 mA circulating electron beam and a normal long laser output pulse. The system used in initial experiments should yield about one-third of this. The 15-degree mirror deflection should preserve the laser polarization state and yield a polarized output beam (in the sense of the highest energy photons being nearly 100 percent polarized).



NOTE: This system is one actually made at CEA. The system discussed in the text would mount the laser 290 inches from a lens of 125-inch focus, 3.2-inch aperture and would achieve a yield about three times larger than that of the system drawn, but only if the accelerator entrance aperture widened to 1-inch vertically.

FIG. 4 CEA LASER BEAM INJECTOR

(NOT TO SCALE)

B. Stanford Linear Accelerator Center

In Fig. 5 is proposed a design applicable to SLAC, one which would yield the maximum output in the form of a slightly divergent beam forming a spot less than a foot in diameter at 500 feet, (in fact, less than an inch in diameter insofar as one may extrapolate Mark III phase space to 20 GeV). The laser input is assumed to be of a well defined polarization, say circular, and to originate in a source above the SLAC beam switchyard fill, about 16 meters above the electron beam axis. The laser beam is passed through a quartz compensating element which in turn forms the first element of a Cassegrain system. In effect this lens adjusts the phase space of the laser to yield a spot which fills the aperture of the injection mirror. Near the mirror is a large aperture lens, which also forms the beam tube vacuum wall, to focus the laser beam to the desired size. In the design shown the focal characteristics are those used in the calculations of Section V. Collimators might be used to protect the mirror from stray radiation. In this case, assuming that optical losses are rendered negligible by the use of antireflection coatings, one predicts a yield, over the whole spectrum, of 2.8×10^4 photons/laser joule/1-μsec-pulse, this with 15μA/average electron beam current. Of the order of 10 times as many would be available if 0.1-μsec pulses were acceptable. If a repetition rate of 360 1-μsec laser pulses a second could be achieved, by means such as will be suggested in the next section, the yield would be about 10^7 photons/laser joule/second (or 10^8, with shorter pulses), a respectable number for track chamber experiments.

Figure 6 shows an alternative design appropriate to production of a partially monochromatic beam from SLAC. The differences from the normal beam are that lenses of different focal lengths are used and the required interaction path and focal lengths are much longer. The locations of the laser and the optical elements are the same in both cases so that the two arrangements are essentially interchangeable. The entire output from this system is expected to be no more than about half that of the previous system, putting about 1.4×10^4 photons/laser joule/1-μsec pulse into the whole spectrum, and, of course, only 30-40 percent of this into the higher-energy, monochromatized portion of it. Were a repetition

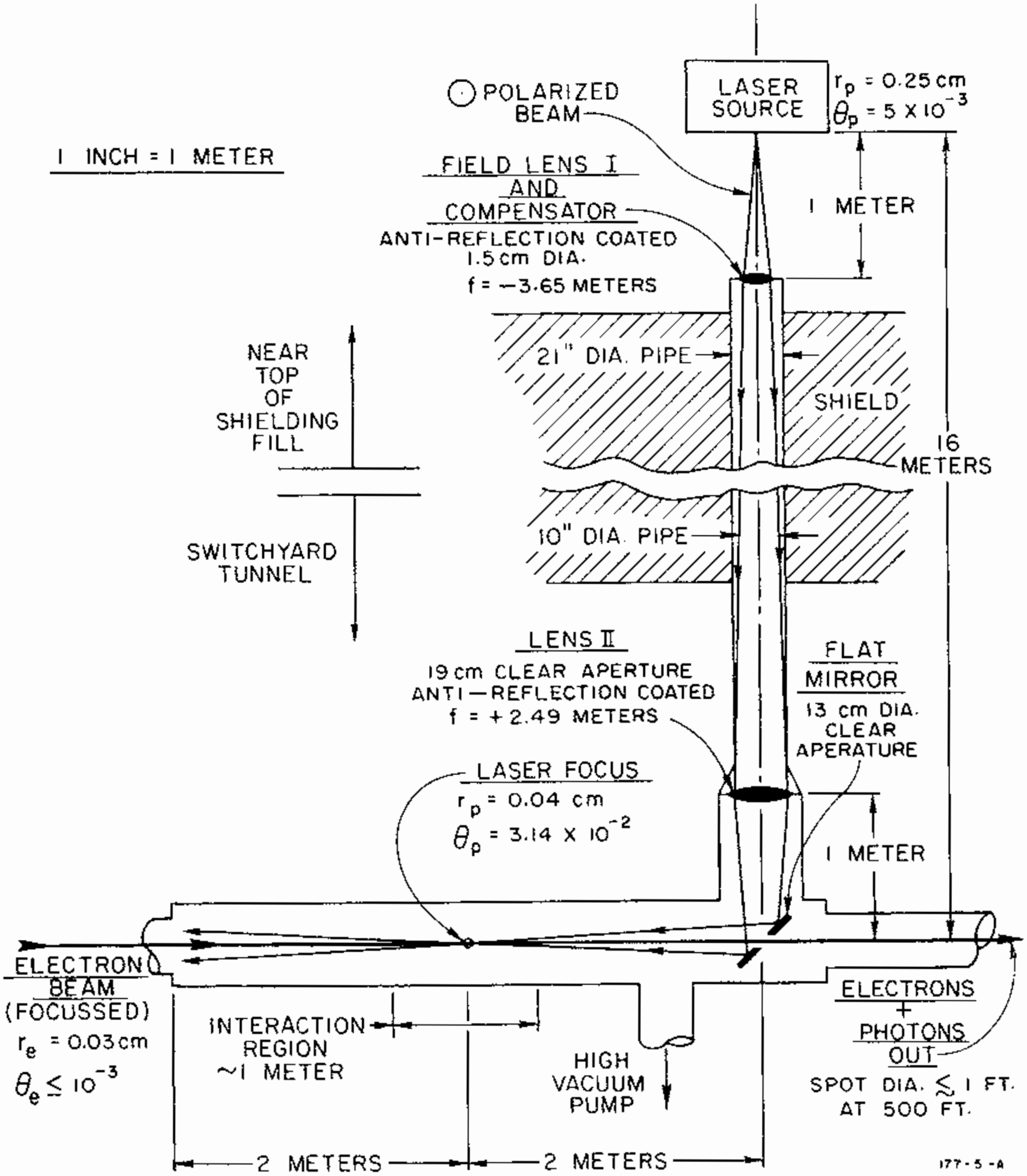


FIG. 5 SLAC LASER BEAM INJECTOR

(FOCUSED ELECTRON BEAM)

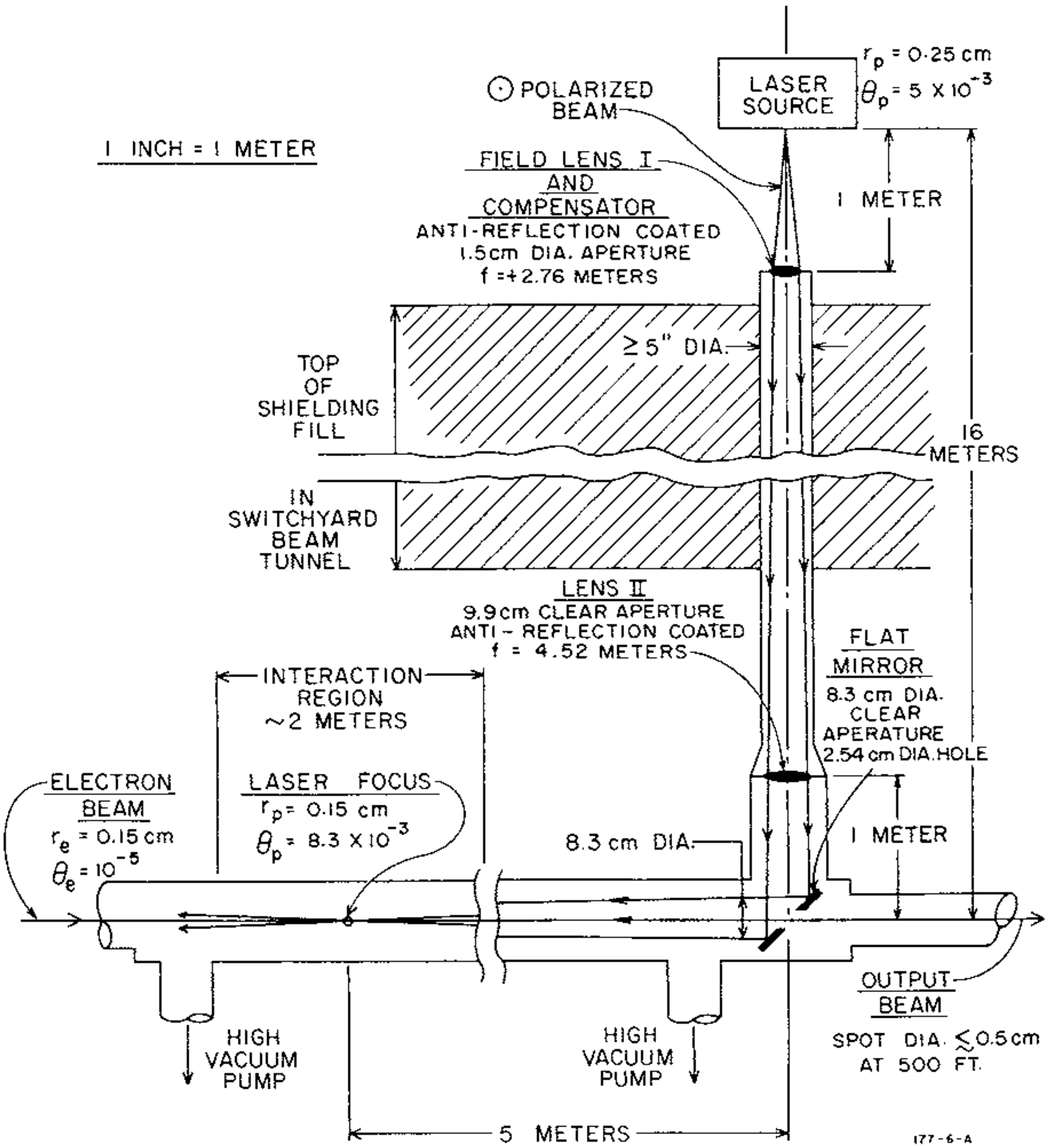


FIG. 6 SLAC LASER BEAM INJECTOR

(MONOCHROMATIC BEAM)

rate of 360 pps attained, the yield of monochromatized photons might optimistically be expected to be 1.6×10^6/laser joule/second. These photons would, of course, be polarized were the original laser likewise, and optical compensation for the effects of the mirror duly made.

Naturally these predictions are somewhat optimistic, for the totally efficient optical system does not exist and there are other places for the inevitable factors less-than-one to creep in. However, the nominal "one-joule" laser assumed throughout is of relatively conservative magnitude. The major development problem for the production of a "laser" beam is to produce such a laser source, capable of being Q-switched to produce joule-sized pulses 1 μsec long or less, with accurate phasing, at repetition rates of 60-360 pps. A brief summary of possibilities in this direction will be given in the next section.

X. SOURCE LASER SYSTEMS

The crucial element in the generation of a laser beam is the laser itself. The necessary specifications for this source depend markedly upon the use to be made of the output beam, its flux, pulse length, polarization and, in particular, repetition rate. A number of possible applications will be discussed in order of increasing complexity and probable expense of realization.

A. Bubble Chamber Beam

The simplest system would be one for use with a bubble chamber experiment. Because of electromagnetic pair background, bubble chamber exposures to more than a few hundred incident photons per pulse are seldom considered, and pulse repetition rates less than two per second are the rule. At the CEA the calculations of Section VI suggest that a one-joule laser would produce about 600 photons per pulse, which is the desired order of magnitude. Were a short, Q-switched pulse necessary, in conjunction with some beam dumping arrangement, to minimize gas bremsstrahlung background, the required laser would doubtless have to be more powerful to make up for the inefficiencies of Q-switching, say a nominal 5-10 joules output (un-Q-switched). Thus one is concerned with laser systems whose average optical power capability is at most around 20 watts at a repetition rate of two pulses per second. This is well within the range of commercial systems currently available. These systems have been developed primarily for welding applications and a number of them, at least, utilize water-cooled rubies, lamps and cavities. For example, one commercial system is advertised as producing 6 joules at 1 pulse per second, or 3 joules at 4 per second, with 10 kW input. Another system claims 20 joules at 1 pulse per second, 5 joules at 10 per second, or 0.5 joules at 20 per second, this at 30 kW input power.[8] Both systems purport to be compatible with Q-switching, though the output under these conditions is not specified. Providing one does not crowd the power limit, lamp lives of the order of 10^5 pulses are claimed. This would give a large part of a day's operation between shutdowns. The two lasers cost about $6000, for the ruby-lamp system, and about $4400 and $11,000 for the 10 kW and 30 kW supplies, respectively. These systems are only to be taken as examples, as a number of other companies are working on high-average-power systems with welding applications in mind. It seems evident, however, that single laser systems already exist with the capability of generating a laser beam for use with bubble chambers. Since polarized beams are obtained as a matter of course using X-cut rubies and Brewster-angle end faces, one can anticipate useful bubble chamber exposures to polarized beams, plane and circular, up to the energy limit of the basic Compton process - 850 MeV at the CEA and 7.3 GeV at SLAC.

Use of a "laser" beam with a bubble chamber at SLAC is evidently even more feasible, because of the larger electron current and more efficient optical coupling possible there. In fact, the predicted yield of the order of 10^4 photons/laser joule/1-μsec pulse is considerably in excess of the tolerance of foreseeable bubble chambers, and would have to be cut down. This is best done by reducing the electron beam intensity by a factor of 30 or so. The laser intensity should be kept high to minimize the relative background from gas bremsstrahlung. Reducing the electron beam intensity might be done by reducing the electron phase space in order to facilitate monochromatization. The only disadvantage with this latter process is that it results in a final beam only a few millimeters in diameter entering the bubble chamber. Although determining the scattering angle and hence the energy of photons in this beam should be fairly simple, involving measurements of about 1 mm precision, the concentration of beam would effectively saturate the chamber at a much smaller flux than the incident photons were spread over a few hundred square centimeters. However, this saturating flux would consist entirely of useful photons within the monochromatized energy range under study, were a suitable collimating aperture installed, since the desired high-energy end of the energy spectrum would be concentrated in a small circular spot. Although it is unlikely that less than the top 5-10 percent of the energy range could be resolved in this way, the reduction of useless background events would in part make up for the forced reduction in incident photon flux. Thus the overall data yield would be reduced only moderately, if at all, from that with a non-monochromatized, spread-out beam. In such a case the gain would be in the reduction of ambiguous events through the additional kinematic constraint. Of course the polarization feature is unchanged by the monochromatization.

B. Rapid Repetition Rate Beams Using Commercial Lasers

Much more powerful laser systems are required if one is to utilize the full repetition rate of either the CEA (60 pps) or SLAC (360 pps). Even with only one-joule laser output per pulse, one is talking in terms of average optical power outputs of 60-360 watts. Since the commercially available systems appear to have a power efficiency of the order of 0.07 percent, extrapolation suggests that electrical power inputs upwards of 82-520 kilowatts are involved, virtually all of which is turned into heat within the few cubic centimeters of active lamp and ruby volume in the laser.

The main problem with high power inputs to a laser is in the heating of the ruby itself and of its lamp. Increase in temperature increases the ruby's threshold and decreases its output; lamps fail rapidly if run too hot. The glamorous, very high energy lasers which produce upwards of 300 joules output per pulse at present require many tens of seconds, and even minutes, before refiring. The high average power lasers described earlier seem to give their best outputs with less than 10 joules output each firing. One might expect, with such devices, to obtain of the order of one joule per pulse when fired with a Q-switch to obtain a submicrosecond pulse. Repetition rates of 60-360 pulses per second are then to be obtained by using many laser elements, perhaps fed from a common power supply, and developing an optical commutator to feed the laser outputs in turn into the beam injector. For example, one might place 36 of the water-cooled lasers previously described, each producing 5 joules output at 10 pps, arranged in a circle with their beams aimed at a 90-degree reflector rotating at 10 revolutions per second (Fig.7). The output beam would emerge parallel to the axis of rotation. A compensator on the prism base converts incident plane polarized light to circular so that the prism rotation does not affect the polarization state. Each laser would be provided with its own Q-switch, Kerr or Pockel cell, and the whole system phased to SLAC electron pulses. Allowing about $1000 each for the rubies, cavities and the Q-switch elements, and a remaining $30,000 for the circular mount, cooling system and commutator, the whole laser source would cost about $138,000. A power supply of the order of 1 megawatt average (and upwards of 10 megawatt peak) output would be required, and would probably cost a comparable amount. It should be observed that power supplies of similar performance are commonly used to drive large klystrons. One might hope, then, for a price of the order of magnitude of $300,000, to bring out at SLAC a polarized "laser" beam of the order of 10^7 photons per second (somewhat more if submicrosecond pulses are acceptable, somewhat less if monochromatization is required).

At CEA the price would be considerably less, because with a repetition rate of only 60 per second, only 6 of the aforementioned lasers would be needed. The price would probably not be linear in the number of lasers, but one might hope to get it under $100,000. The output, based upon a yield of 600 photons/joule/pulse as calculated earlier, would be 36,000/sec if the Q-switch were used, and more like 180,000/sec if the long, 5-joule laser output were used. In the latter case, omission of the Q-switch would lead to great simplification in both timing and expense problems, but would necessitate maintenance of a 10^{-7}mm Hg vacuum in the interaction region.

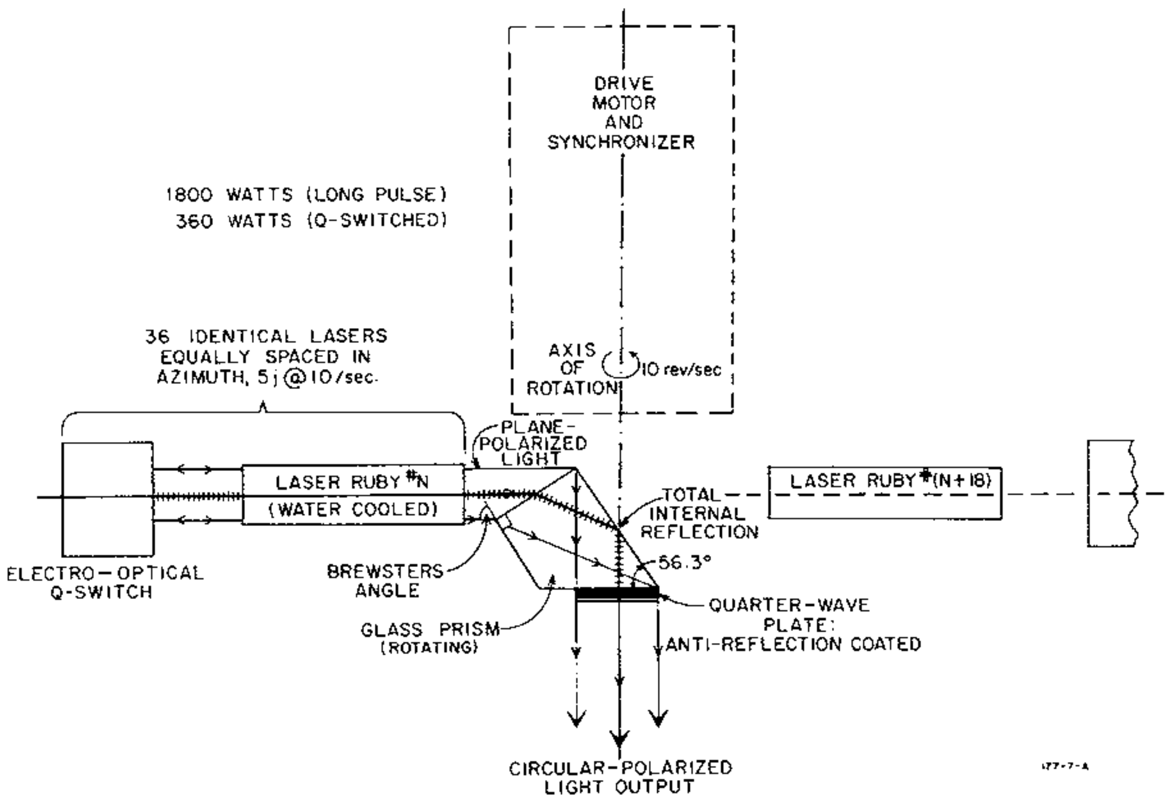


FIG. 7 36 — RUBY LASER SOURCE
(NOT TO SCALE)

One may contemplate a number of alternative schemes for accomplishing the same results as the above system. For example, one might be able to combine the rotating prism commutator with a rotating mirror Q-switch, revolving at 370 revolutions per second, to commutate and fire 36 ruby lasers at 10 per second each. Q-switches commonly work at speeds such as this. A possible design is shown in Fig. 8. Alternatively, since it is difficult to extract more than one joule from a Q-switched laser in the first place, it has been suggested[9] that one might try to produce a rapid repetition rate device of output only a few millijoules, Q-switched, and then commutate around a series of water-cooled amplifying rubies to yield multi-joule outputs. (See Fig. 9.) This last system has the *prima facie* advantage that only one Q-switch needs to be phased to the accelerator, while the phasing of the rotating mirror is much less critical than were it the Q-switching element itself. Although the amplification scheme is frequently used for achieving high pulse output powers, one may question whether a rapid pulsing primary laser could operate at more than 100 per second.

It is evident that some sort of high power, rapid-repetition-rate laser source could be put together by just stacking up existing systems in parallel and making a commutator. On the other hand, considerable simplification and economy might be obtained with a reasonable application of intelligence and development effort. That such effort has not yet apparently been made in the laser industry is probably a testimony to the youth of the art. Present publicized achievements have been concerned with making spot welds and fine holes in extraordinarily refractory materials. When this is accomplished the natural next step is to make laser flame cutters and mills, and this will require just the sort of high repetition rate, high power density laser source needed for a strong "laser" beam.

C. Possible Improvements

A number of incidental comments may be made concerning possible developments in the laser art appropriate to the present discussion. For one thing, there is some evidence that the available laser energy can be channeled into a smaller phase space by controlling the modes into

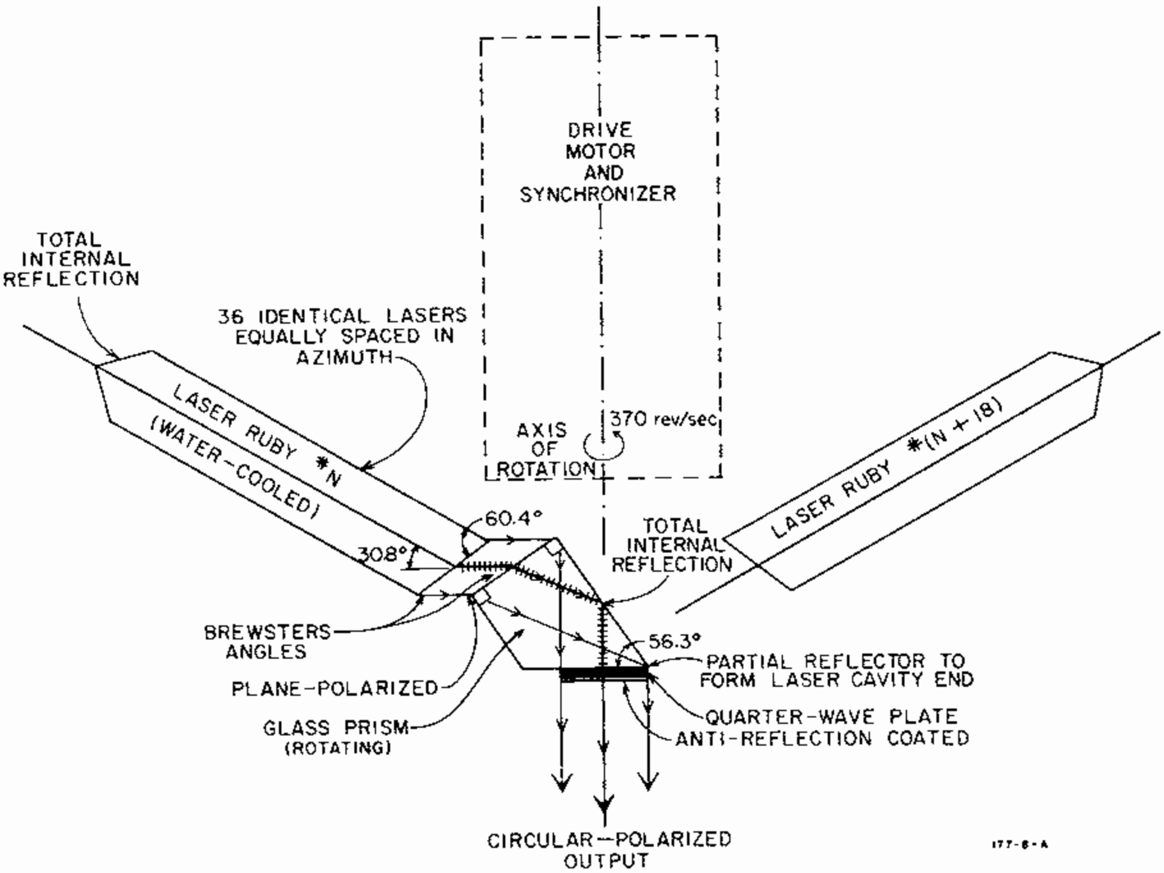


FIG. 8 36- RUBY SOURCE INTEGRAL Q-SWITCH
(NOT TO SCALE)

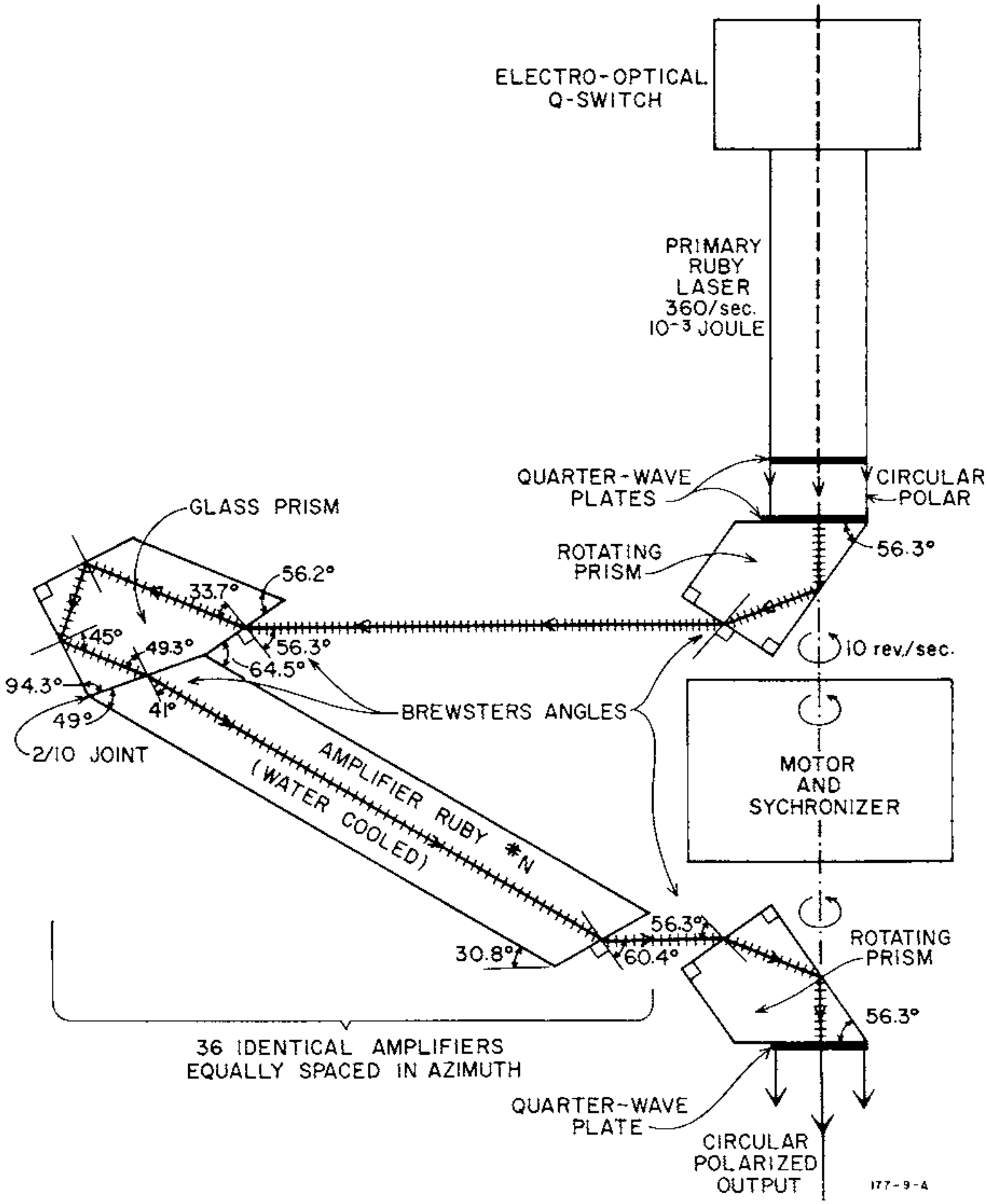


FIG. 9 SKETCH SHOWING OPTICAL DESIGN OF 36-RUBY COMMUTATING POWER AMPLIFIER (NOT TO SCALE)

which an amplifying de-excitation of the inverted ruby levels can occur.[10] As a result, it has been found possible to increase the effective brightness of a ruby laser, proportional to $W_p/(r_p\theta_p)^2$, by a factor of about 6. One may infer that the ratio $W_p/(r_p\theta_p)$ in Eq. (4) is roughly doubled. Thus mode control, say be increasing the ruby resonant path length, might provide a cheap way of increasing the output yield.

Other experimenters have found that since much of the input energy to a three-level laser goes into achieving the minimal inversion of the level populations, a number of short Q-switched pulses can be developed from a single pumping pulse.[11] This may represent a technique which improves the efficiency of producing Q-switched pulses, say for use at SLAC, and moreover might permit developing Q-switched pulses of microsecond lengths. At present Q-switched pulses are generally found to be less than 0.2 μsec in length and would impose a duty cycle limitation on the use of the produced energetic photon beam.

Another possible method of increasing the yield of the final photon-electron process would possibly be by reusing the laser beam after its initial pass through the interaction region. A mirror arrangement such as sketched in Fig. 10 might be used. Although the efficiency would doubtless be less than 100 percent, even a gain of a factor of, say 1.7 by this means would be cheap compared to a commensurate intensification by 700 input-kilowatts of a primary laser source. The procedure might well be iterated several times.

Were it possible to find a high average power laser comparable to the ruby but generating output in the blue region of the spectrum, the upper energy limit of the final beam would be increased to about 11 GeV (with 77 percent polarization) for SLAC and about 1.4 GeV (with 96 percent polarization) for the CEA. Particularly in the CEA case this is a very attractive proposition as strange particle photoproduction becomes feasible. Second harmonic generation with approximately 20 percent power efficiency has been achieved using very high peak power pulses in KDP and ADP crystals.[12] These experiments were with a ruby whose second harmonic appeared at 3470 Å. One may question whether the absolute output intensity required for a "laser" beam may be obtained by this means, since

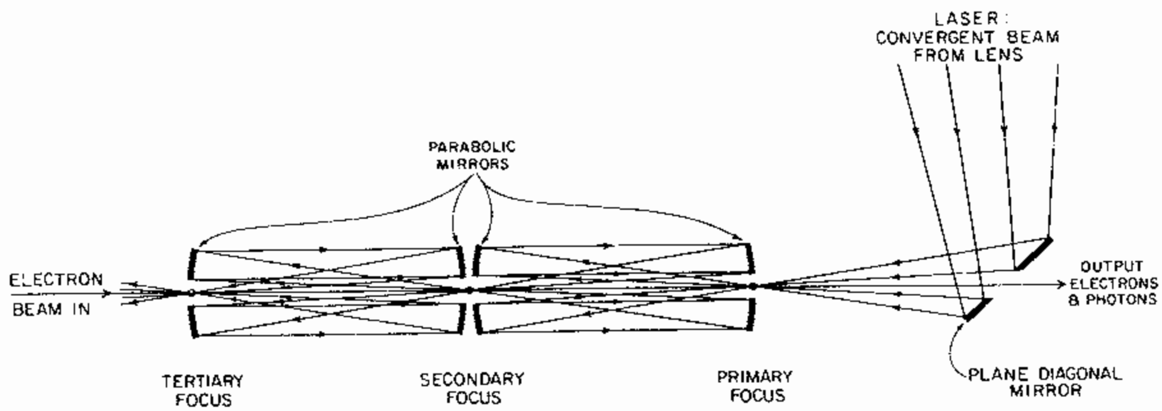


FIG. 10 SCHEME TO RE-USE LASER BEAM
(SHOWING 3 INTERACTION REGIONS)

there was an experimental tendency for the KDP crystal to be destroyed by the high peak powers necessary to achieve harmonic generation of 6 millijoules. However, in view of the significance of a "blue" laser beam for bubble chamber experiments at the CEA, and the relatively modest beam required for such experiments, developments along these lines should be kept in view.

XI. EXPERIMENTS WITH POLARIZED PHOTONS

It is interesting to consider at this point the kinds of experiments that might be done using polarized photons. The following brief survey is in no way to be viewed as either sophisticated or comprehensive.

A. Phenomenological Experiments

A number of papers have considered the general characterization of the information to be derived from photoproduction experiments, limited only by certain invariance principles. Programs of this sort are doubtless motivated by the relative success of the determination of the Wolfenstein parameters in nucleon-nucleon scattering. Moravcsik, for example, has discussed in detail experiments by which spin-zero bosons are photoproduced on spin-1/2 fermions.[13] Starting with the most general transition matrix complying with rotation and reflection invariance, appropriate to a given incident energy and production angle, he found that there are only seven independent real parameters describing the process once the parity of the outgoing boson was decided. These will be measured, perhaps with some ambiguities, by various sets of seven appropriately chosen experiments. Most such sets involve polarized targets as well as polarized photons, but Moravcsik shows that one class of sets does not. This one requires, along with the two polarized differential cross sections, five measurements involving all three components of the recoil fermion polarization in correlation with incident photons both plane and circularly polarized. Noting that both forms of polarization are expected to be readily available from a "laser" beam, and that proton polarization measurements are readily made at lower energies, one may infer that a complete phenomenological description of single pion photoproduction is subject only to the still formidable experimental problem of obtaining adequate intensities for a practical measurement. Although one may perhaps question the need for such a general approach below the N^* (1238) resonance, where only a couple of angular momentum states are involved and models based upon this have been relatively successful, the confusion at the higher pion-nucleon resonances suggests that a complete set of model-independent data would be of considerable utility, especially to theorists with new models.

The phenomenological approach has been generalized by other workers. Eriksson and Möller have studied proton Compton scattering in this manner and discuss the experiments required to determine the eleven real parameters involved in that process.[14] A somewhat different development has been given by Berman.[15] He compares in a uniform way photoproduction and electroproduction, both being described in terms of a set of real form factors, functions of the energy, momentum transfer, and virtual photon mass. In the photoproduction case the determination of the two relevant form factors involves the use of polarized photons. He does not generalize his theory to include fermion spin measurements, but one would think that such an extension would be feasible.

It should be evident that while the determination of complete sets of phenomenological coefficients in photoproduction is not likely to be made in the immediate future, the basic means thereto are essentially at hand. It is also clear that the intrinsic complexity of photoproduction matrix elements requires, for a complete description, the use of polarized photon beams, or their equivalent in electroproduction. Good electroproduction measurements must include recoil particle detection and are non-trivial, in any case.

Above the strange particle production threshold the use of a polarized photon beam becomes even more interesting because some of the hyperons, through their weak decays, efficiently analyze their own polarizations. Extracting the various contributing photoproduction matrix elements would, _a priori_, be expected to yield data on the electromagnetic moments of the baryon system.

B. Model Experiments

In this category are included those experiments that are designed to determine a limited number of specific parameters associated with various simplified models of photoproduction and analogous processes. Experiments of this sort have already been performed using partially polarized photon beams to study the relative contributions of low-order multipole absorptions to pion photoproduction in the vicinity of $N^*(1238)$.[5,16,17,18] These experiments have essentially measured the differential cross section for neutral and charged pion photoproduction with incident photons polarized parallel and perpendicular to the production plane. For 90° center-of-mass angle this ratio would be 1:4 with only magnetic dipole absorption contributing. Deviations from this ratio are observed and are interpretable in terms of an electric quadrupole contribution or, more probably, the presence of a pole term arising from ρ or ω meson exchange.[16,19] These experiments were done with polarizations in the vicinity of 20 percent and 30-35 percent for the normal and coherent bremsstrahlung methods, respectively. It would be interesting, if possible, to make an analogous study in the vicinity of higher pion-nucleon resonances whose structures are less well known than the one at 1238 MeV.

C. Other Experiments

A number of other applications of polarized beams have been suggested by various authors. Moravcsik has proposed using photons polarized perpendicular to the plane of production to eliminate the photoelectric term in charged pion photoproduction.[20] This term confuses the interpretation of angular distribution data at higher energies and forward angles, requiring, at present, removal by a delicate extrapolation process applied to the data.[21]

Deuteron photodisintegration also exhibits an empirical polarization dependence at energies of 100 MeV or so, a dependence presently at odds with theory.[22,23] It has been predicted that proton Compton scattering below the $N^*(1238)$ resonance will show a marked dependence upon photon polarization according to the contribution of the eta-meson, and might be used to estimate the eta's lifetime.[24]

It is evident from the survey of Berman and Drell that the photoproduction of vector mesons, by either exchange or diffraction processes, will depend upon the photon polarization.[25] One's intuition suggests that beams of high intrinsic plane and circular polarization may be of use in segregating the various diagrams involved, and perhaps in measuring the electromagnetic moments of the vector mesons. A theoretical investigation into possibilities of this sort would be of considerable interest.

D. Comparison with Other Polarization Methods

One might expect that a "laser" beam would be competitive with the other modes of polarized photon production to some extent, in spite of the difficulty of obtaining a high intensity laser source, for several reasons.

1) The "laser" beam spectrum is substantially flat in energy, and the useful, highly polarized photons are at the top end of this spectrum. (This presupposes there is negligible background from gas bremsstrahlung.) Thus in working, say, at the N^* (1512) resonance, one could hope to use kinematic constraints in the elimination of two-pion production without experiencing excessive background from the sort of high-energy tail associated with coherent and normal bremsstrahlung sources. Moreover, the unwanted low-energy end of the incident photon spectrum, producing Compton electrons and pairs to fill up one's detectors, is much smaller in the case of the "laser" beam.

2) The polarizations of the useful portions of the "laser" beam spectrum are expected to be upwards of 90 percent, both plane and circular, even at SLAC. Since the precision for a polarization experiment is roughly proportional to the square of the polarization, statistically, and systematic errors are also of less moment, one would expect that fewer beam photons would suffice from a "laser" beam than from a less polarized beam, in a typical experiment. This would make up for, perhaps, a factor of ten or more in the relatively feeble "laser" beam intensity, compared to other methods, and would render feasible for some experiments the use of detectors such as bubble chambers whose rate of data acquisition is intrinsically slow.

In any case, in view of the markedly different systematic errors associated with the "laser" beam process, it would be worthwhile, if possible, to repeat some of the low-energy pion photoproduction experiments using it, and then to extend these to higher energies where its unique characteristics might prove indispensable. This is not to minimize the difficulty of conducting such experiments. Although we have seen that it should be relatively easy to saturate a hydrogen bubble chamber with a "laser" beam, one may quickly calculate that use of a bubble chamber becomes very tedious when it is desired to obtain reasonable statistics on processes of less than 10-microbarn cross section. Thus measurements of the first N^* resonance are surely feasible, but of the second are at best just within the margin of practicality. In principle, the intensities of "laser" beams to be expected from SLAC, using high-repetition rate sources, will permit the analysis of processes having at least an order of magnitude lower cross sections.[26] Measuring such one-microbarn and lower cross sections will be essential if strange particle photoproduction work is to be done effectively at SLAC. One may hope that large solid angle track chambers, or super-efficient bubble chambers, will be developed in any case for observation of multi-body events. Insofar as this is done, use of a polarized, and perhaps partially monochromatized, beam will give a great deal more information per experiment than an unpolarized beam. Particularly with those hyperons that analyze their own spins, a knowledge of photon polarization comes close in principle to yielding complete information on the transition matrix elements.

E. Storage Ring Applications

In view of the possible advent of 3-4 GeV electron storage rings it is interesting to speculate on the possibility of injecting into such rings a tangent laser beam. This might be accomplished in a manner similar to that of Fig. 4. The photon energies would extend up to about 220 MeV for a 3 GeV ring. Circulating currents upwards of 0.4 amperes together with a very compact electron beam give a great increase of Compton yield over that expected from the present CEA. Gas bremsstrahlung increases also but the very high vacuum required for a storage ring will minimize this. A rough estimate may be made that a set of commutated lasers yielding 3-joule pulses at 60 per second will generate in such a way $(1-3) \times 10^6$ photons per second between 200 and 220 MeV, with about ten times as many at lower energies. These would appear with the good duty cycle associated with 1-millisecond laser pulses. Noting that proton Compton scattering has a cross section of the order of 0.1 μb in this region, corresponding to a mean free path of 10^7 feet in liquid hydrogen, one sees that a large solid angle spark chamber could yield useful data on this process. The good duty cycle facilitates rejection of the large amount of electromagnetic background by means of counters. A 360 per second laser source suitable for SLAC would be directly applicable to a storage ring as well. Photonuclear processes might also be thereby investigated in the range up to 220 MeV (or 390 MeV, with a 4-GeV ring). It is to be noted that this application would not appreciably influence the beam life in the storage ring, and so could be completely parasitic over a long period of time.

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APPENDIX A

Here is derived the formula describing the effective interaction density presented by a focused, axially symmetric beam of particles of type "p" to a particle of type "e" moving parallel to the axis of "p" but displaced therefrom by a distance "r_1". The results are averaged over radii "r_1" to yield the interaction efficiency when a cylindrical beam of radius "r_1" is brought together with a coaxial focused beam of phase space $(r_p\theta_p)$. The effective interaction length is also discussed.

Assume beam "p" uniformly fills a phase space region described by a focal spot of radius r_p and a half-polar-angle θ_p. Let the z-axis be along the average propagation direction of beam "p". Define $I_p(r,\gamma,z,\theta)$ $\cos\theta dA d\omega$ as the number of particles "p" per unit time passing through area dA located at point (r,γ,z), in cylindrical coordinates, directed into solid angle $d\omega$ along a direction making polar angle θ with the z-axis. (See Fig. 11.) The assumption that the beam "p" uniformly fills a region in phase space may be written in terms of the illumination function $I_p(r,\gamma,z,\theta)$ at the focal spot, z=0, as $I_p(r,\gamma,0,\theta) = D$, a constant, when $r \leq r_p$ and $\theta \leq \theta_p$; and zero otherwise. Calling N_p the total number of particles "p" passing through the focal spot per unit time, one can calculate the constant D and write

$$I_p(r,\gamma,0,\theta) = \frac{N_p}{(\pi r_p \sin\theta_p)^2}\ \delta(r,\theta) \qquad \text{(A-1)}$$

where $\delta(r,\theta) = 1$, if $r \leq r_p$ and $\theta \leq \theta_p$, $= 0$ otherwise. The product $(r_p \sin\theta_p)$ is conserved by first-order focusing systems and is the invariant in the Abbe sine condition.

Consider the flux of particles through area element dA_1 at (r_1,γ_1,z), perpendicular to the z-axis, arising from particles passing through dA at $(r,\gamma,0)$, also perpendicular to the z-axis. One may choose $\gamma_1 = 0$ since a final integration over γ will be effected. Let s be the distance between the points $(r,\gamma,0)$ and $(r_1,0,z)$. Then the number of particles per unit time passing through dA_1 and arising from dA will be

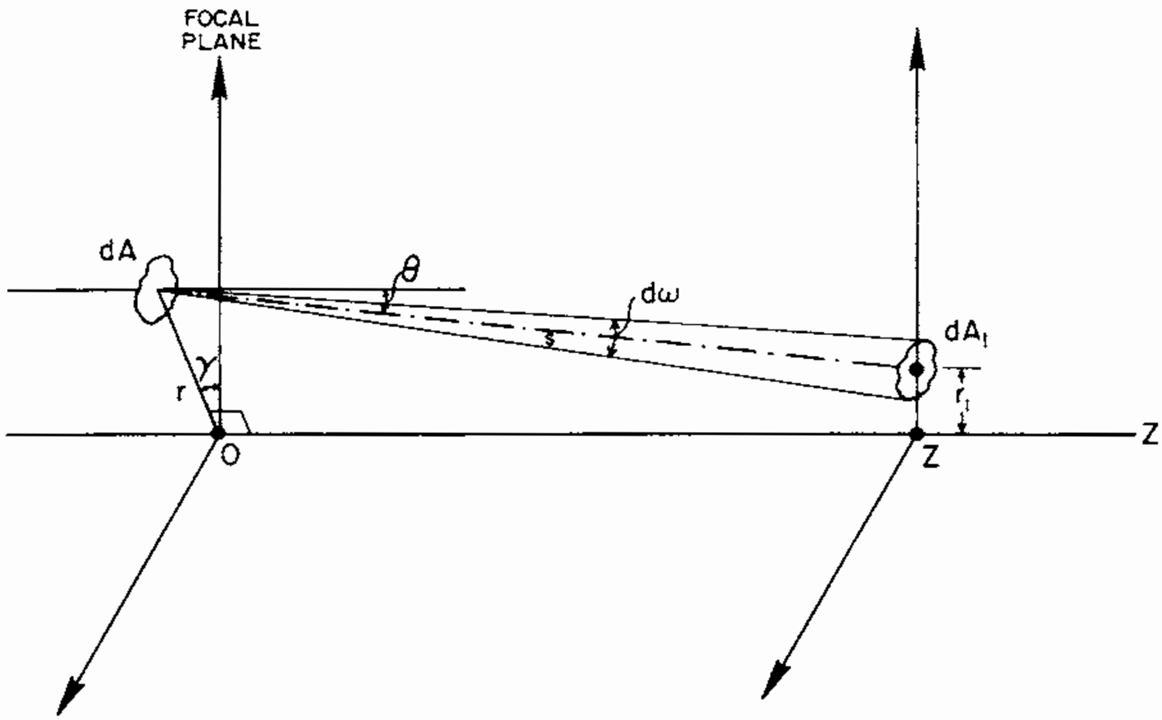

FIG. 11 COORDINATE SYSTEM FOR APPENDIX A.

$I(r_1, 0, z\ ;\ r, \gamma, 0)\ dA dA_1$, where

$$IdAdA_1 = \frac{N_p}{(\pi r_p \sin\theta_p)^2}\ \delta(r,\theta)\ (z^2/s^4)\ dAdA_1 \qquad (A\text{-}2)$$

Here $\cos\theta = z/s$ has been inserted. The volume density $dp(r_1, z)$ of particles at $(r_1, 0, z)$ arising from dA is obtained by dividing Eq. (A2) by the velocity, $v_p \cos\theta$, times dA_1. Hence

$$dp(r_1, z) = \frac{N_p}{v_p(\pi r_p \sin\theta_p)^2} \cdot \delta(r,\theta) \cdot (z/s^3)\ dA\ . \qquad (A\text{-}3)$$

In this equation the polar angle θ is a function of (r, γ, r_1, z).

In all cases of present interest, $\theta_p \ll 5 \times 10^{-2}$ so a small-angle approximation may be made. Thus $z \simeq s$ and $\theta \simeq \sin\theta$, so

$$dp(r_1, z) \cong \frac{N_p}{v_p(\pi r_p \theta_p)^2}\ \delta(r,\theta)\ \frac{dA}{z^2}\ , \qquad (A\text{-}4)$$

with

$$\theta \cong (r^2 + r_1^2 - 2rr_1 \cos\gamma)^{1/2} / |z| \qquad (A\text{-}5)$$

The areal density, $d\Sigma$ (particles per unit area), of beam "p" presented to beam "e" traveling in the $(-z)$ direction at a distance r_1 off axis will be $d\Sigma = \int dz dp\ (r_1, z)$, integrated over those portions of z such that θ in Eq. (A5) lies within the limits $\theta_{min} \le \theta \le \theta_p$. The lower limit θ_{min} allows for a convenient subsequent restriction of the zones contributing to the focus. Thus, since $\delta(r,\theta)$ is a step function

$$d\Sigma = \frac{2N_p\ dA}{v_p(\pi r_p \theta_p)^2} \left[\frac{1}{z_{min}} - \frac{1}{z_{max}}\right] \qquad (A\text{-}6)$$

where the factor of two takes care of the negative-z axis.

Let $\alpha = (\theta_{min}/\theta_p)$, $\beta = (r_1/r_p)$, and $u = (r/r_p)$;

Then

$$d\Sigma = \frac{2N_p(1-\alpha)\; u\; d\gamma\; du}{\pi^2 v_p(r_p\theta_p)\; (u^2 + \beta^2 - 2u\beta\cos\gamma)^{1/2}} \qquad \text{(A-7)}$$

where

$$\left[\frac{1}{z_{min}} - \frac{1}{z_{max}}\right] = \frac{\theta_p(1-\alpha)}{r_p(u^2 + \beta^2 - 2u\beta\cos\gamma)^{1/2}} \qquad \text{(A-8)}$$

has been used.

The net areal density, Σ, is obtained by integrating Eq.(A-7) over $r \leq r_p$ and all γ. The angular integration leads to an elliptic function. Thus

$$\Sigma = \frac{N_p\,(1-\alpha)}{v_p(r_p\theta_p)} \cdot J_o(\beta) \qquad \text{(A-9)}$$

Here

$$J_o(\beta) = \frac{8}{\pi^2}\,\beta \int_o^{1/\beta} \frac{w\;dw}{(1+w)}\; K\left[\frac{2\sqrt{w}}{1+w}\right] \qquad \text{(A-10)}$$

with

$$K(k) = \int_o^{\pi/2} \frac{dx}{(1-k^2\sin^2 x)^{1/2}} \qquad \text{(A-11)}$$

One may readily calculate that $J_o(0) = (8/\pi^2)\,K(0) = 4/\pi$, corresponding to the two beams coaxial. The function $J_o(\beta)$ is plotted in Fig. 1 on the basis of an approximate integration. Also plotted is $J_o(\beta)/J_o(0)$

averaged over a circle of radius β. This is the efficiency of interaction of a cylindrical beam of radius $r_1 = \beta r_p$, compared to a line-beam of zero radius. Note that the efficiency approximately equals 1, $0 \leq \beta \leq 0.8$, and $1/\beta$ when $1.25 \leq \beta$.

In Eq.(A-9) the constant N_p would correspond to the number of particles per unit time were $\alpha = 0$. For non-zero α, the number of particles actually passing through the focus is reduced by the factor $(1-\alpha^2)$. Let N_p' represent the number of particles reaching the focus, so that $N_p' = W_p/T_p$, in the notation of the text. Then Eq.(A-9) becomes

$$\Sigma = \frac{W_p}{T_p} \cdot \frac{J_o(\beta)}{v_p r_p \theta_p (1+\alpha)} . \tag{A-12}$$

From this last equation the yield formula, Eq. (4) of the text, is readily obtained.

The approximate interaction length may be estimated by noting from Eq.(A-6) that a fraction P of the interactions will occur within a region of total length $L_p(r) = 2Z^1$, where

$$\frac{\dfrac{1}{Z_{min}} - \dfrac{1}{Z^1}}{\dfrac{1}{Z_{min}} - \dfrac{1}{Z_{max}}} = P . \tag{A-13}$$

Then, using Eq. (A-5),

$$L_p(r) = \frac{2}{(1-P+\alpha P)} \frac{r_p}{\theta_p} (u^2 + \beta^2 - 2u\beta \cos\gamma)^{1/2} . \tag{A-14}$$

When $\beta = 0$, and $L_p(r)$ is averaged over the focal spot of radius r_p, one obtains $(L_p)_{aver} = 2r_p/(1-P+\alpha P)\theta_p \cdot 2/3$. On the other hand, the maximum value of Eq.(A-14) occurs when $\cos\gamma = -1$ and $r = ur_p = r_p$, in which case $(L_p)_{max} = 2r_p(1+\beta)/(1-P+\alpha P)\theta_p$. It is thus reasonable to take as an approximate measure of interaction length, when r_1 is less than or of order of magnitude r_p,

$$L_p = \frac{2r_p}{(1-P+\alpha P)\ \theta_p} \quad . \tag{A-15}$$

This is the formula used in the text.

APPENDIX B

MONOCHROMATIZATION

The kinematic relationships describing the output photon energy and angle are given in Ref. (1). Let the incident electron energy be E, and photon energy k_i. Define the parameters $\gamma = 1/(1-\beta^2)^{1/2} = E/mc^2$, and $\lambda = 2\gamma k_i/mc^2$. The latter is the ratio of apparent photon energy to electron rest energy in the rest frame of the incident electron. Let $x = \cos\theta_0$, where θ_0 is the photon scattering angle in the electron rest frame. Then the final laboratory photon energy is given by

$$k_f = E\,\frac{\lambda(1-x)}{1+\lambda(1-x)}\,, \tag{B-1}$$

and the laboratory polar angle by

$$2\tan(\theta_f/2) = (1/\gamma)\cot(\theta_0/2), \tag{B-2}$$

For large values of γ virtually all photons of interest have θ_f very small, so that Eq.(B-2) may be written in excellent approximation

$$\theta_f = \frac{(1+x)^{1/2}}{\gamma(1-x)^{1/2}}\ . \tag{B-3}$$

This may be solved for $(1-x)$ and substituted in Eq. (B-1). Writing the maximum output energy as

$$k_{max} = E\,\frac{2\lambda}{1+2\lambda}\,, \tag{B-4}$$

one gets

$$k_f = k_{max}\ \frac{1}{1+(\gamma_{eff}\theta_f)^2} \tag{B-5}$$

where

$$\gamma_{eff} = \frac{\gamma}{(1+2\lambda)^{1/2}} \qquad \text{(B-6)}$$

The small correction to γ is a result of electron recoil. Equation (B-5) is plotted in Fig. 2.