CHAPTER 5 LUMINOSITY MEASUREMENT

This chapter will describe the method of measuring luminosity at SLD, and will present the measurements for the 1993-1998 SLD datasets. Luminosity at SLD is measured with the LUM, which was described in section 4.2.6 above. We employ the same luminosity measurement procedure as described in[53], so a detailed description will not be repeated here. However, the important features of the general method will be described in this chapter in addition to detailed descriptions of those parts of the method which were unique or were improved in our analysis. In particular, several sources of systematic error from the earlier analysis were identified and improved. The LUM had an original design goal for a 3% relative systematic error[43], but through the previous analysis as well as our own it has been reduced to 0.76%.

5.1 Measuring Luminosity With Bhabha Scattering

All e^+e^- colliders measure luminosity using equation (4.1) by measuring the rate of small-angle Bhabha scattering ($e^+e^- \rightarrow e^+e^-$) for three main reasons:

1. The method is simple and straightforward. For any physics process $v = \mathcal{L}_{\text{Integrated}} \sigma$, where v is the number of events, $\mathcal{L}_{\text{Integrated}} = \int \mathcal{L} dt$ is the integrated luminosity and σ is the cross section. By calculating the Bhabha scattering cross section σ for a detector and counting the number of events seen in the detector, the integrated luminosity is easily calculated. Therefore, measuring luminosity is in principle a simple measurement relying only on identifying and counting electrons and positrons within a well-defined region.

- 2. The physics of small-angle Bhabha scattering is extremely well understood since it is almost entirely a QED process dominated by t-channel photon exchange, which means that the cross section σ can be calculated to extremely high precision.
- The Bhabha scattering cross section increases rapidly as ~ ¹/_{θ³} for small polar angles near the beam line, making this cross section the dominate physics process in e⁺e[−] collisions. Therefore, Bhabha scattering allows for high-statistic measurements.

The SLD LUM was designed specifically to identify small-angle Bhabha scattered events and to provide precision energy and position measurements of the outgoing electron and positron. This process involves triggering on all Bhabha event candidates, identifying the Bhabha events from background and classifying the events for counting purposes. These will be covered in the following sections

5.2 Trigger

Since small-angle Bhabha scattering is dominated by t-channel photon exchange, the outgoing electron and positron from the elastic collision will each carry the full beam energy of 45 GeV and will be back-to-back with an acolinearity near zero. Initial and final state radiation modify this picture slightly, but the effects are small and easily accounted for in both the trigger and subsequent event selection criteria.

Since the LUM was designed to fully absorb the energy from an incident 45 GeV electron, the LUM trigger is in principle trivial. During the 1991 through 1993 SLD runs the

LUM trigger consisted of simple threshold suppressed energy sums for the total energy in each module. Experience in the earliest runs found that summing only EM2 towers above 1.25 GeV, and requiring a minimum of 12.5 GeV of energy in both the North and South module provided a trigger that was 99.6% efficient[60]. This trigger was calculated at 120 Hz on every SLC beam crossing, and was therefore dead-timeless.

Although this was an ideal physics trigger for finding Bhabha events and could run at the full SLC repetition rate, bad SLC beam conditions would occasionally blast the LUM with enough beam-related background that satisfied the trigger at a high enough repetition rate that the internal data acquisition buffers would fill and begin to drop events.

To Protect against this, a new trigger was implemented in 1994 with the same 1.25 GeV tower threshold, but the energy sums were formed only over φ wedges in EM2 that spanned 1/16th (or 22.5°, which is two single-wide towers in φ as seen in Figure 4-20) of the transverse plane of the detector. The energy in each pair of adjacent wedges was summed to form an overlapping octant energy sum. The trigger would evaluate true if the sum of any octant energy sum in the North with the diametrically opposed octant in the South was above 12.5 GeV. This arrangement effectively imposed an acolinearity cut into the trigger, which was loose enough to accept all Bhabha events but stringent enough to reject most SLC beam related background since the background is uncorrelated in φ . The new trigger lead to a dramatic increase in purity with no loss in efficiency.

5.3 Event Selection

The Bhabha event selection criteria is basically a tightening of the trigger since the Bhabha events are very well separated from the background. The size of a typical electromagnetic shower in the LUM is about 1cm², which is about the size of a LUM tower in the transverse plane. Therefore, simple clusters are formed in each EM layer by combining the tower with the most energy with all of its nearest neighbors. Detailed studies with GEANT[61] and EGS4[57] showed that clusters of this size contained 90% of the electromagnetic shower energy on average.

Data for runs prior to 1996 used the two-layer LUM, which was comprised of layers EM1 and EM2. For the 1996 run and beyond the four-layer LUM was used which was comprised of layers EM1, EM2, EM3 and EM4. Because there was no overwhelming advantage in incorporating the extra information from the four layers, and since the Bhabha analysis technique and software package was mature and stable, the EM2, EM3 and EM4 layers were combined into an effective layer that mimicked the old EM2 layer. This allowed the Bhabha analysis for the entire SLD dataset to be treated in a consistent and simple way. In the following discussion, when we refer to EM2 we mean the effective EM2 which is comprised of EM2+EM3+EM4, and there will be no further mention of the physical EM2, EM3 and EM4 layers.

For each event there are four clusters, one in each layer (EM1 and EM2) in each module (North and South). The average positions $\overline{\theta}$ and $\overline{\varphi}$ of the incident particle are calculated using simple energy weighted means. Both the EM1 and EM2 clusters are used to calculate these average positions unless they are separated by more than 6 towers in azimuth (or 67.5°), in which case only the EM2 cluster is used. This requirement simply requires that the EM1 and EM2 clusters form one logical cluster, since 6 towers in azimuth would represent a physically distinct cluster since clusters have a fixed shape that are 3×3 towers in size. Although it is possible to calculate more precise positions using detailed Monte Carlo simulations[62], it is unnecessary for our purposes as long as all pertinent cuts are placed

along tower boundaries where the position resolution is best (which is about 300 μ m). Our intent is not to measure the Bhabha differential cross section $\partial_{\theta}\sigma$, it is simply to count Bhabha events within a well defined region.

The primary source of LUM background comes from a continuous profile of SLC electromagnetic radiation falling rapidly with radius from the beam line. To insure a highly pure Bhabha sample, clusters are rejected for EM1 clusters below 1.25 GeV and EM2 clusters below 2.50 GeV. Clusters which pass these cuts are then required to satisfy the following criteria

20 GeVEM1+EM2 < 125 GeV
$$\left|\pi - (\varphi_{\text{North}} - \varphi_{\text{South}})\right| < 0.5 \text{ rad.}$$
 (5.1)

Notice that there is no cut on the observable θ . The θ observable is used to classify events for the luminosity measurement technique to be presented next.

5.4 Classification

Since the Bhabha cross section is such a rapidly varying function of θ , small misalignments can have a large impact on the parts of the Bhabha cross section sampled by the two LUM modules. Therefore, to reduce the sensitivity of the luminosity measurement on calorimeter alignment we make use of the gross-precise method[63-66]. This method logically divides each LUM module into a tight *precise* region and a looser *gross* region. The regions are defined along tower boundaries where the position resolution is best. Referring to Figure 4-21, the innermost ring ($\theta_{bin} = 56$) and the two outermost rings ($\theta_{bin} = 52,51$) define the gross region. The central shaded rings with $\theta_{bin} = 55,54,53$ define the precise

region. Events where both the North and South cluster are in the precise region are classified as precise events. Events where one cluster is in the precise region and the other is in the gross region are labeled gross events. Other events where both clusters are in the gross region are not used for the luminosity analysis, but this does not mean the event is not a Bhabha.

Using this classification scheme, precise events are given a weight of 1 and gross events are given a weight of ¹/₂, and an effective number of events is defined as

$$\mathbf{n}_{\rm eff} = \mathbf{n}_{\rm precise} + \frac{1}{2} \mathbf{n}_{\rm gross} \tag{5.2}$$

The power of the gross-precise method is that the number of effective Bhabhas given by equation (5.2) is a constant for small displacements. For the LUM this means transverse displacements as much as 2 mm and displacements along the z-axis as much as several centimeters are possible while still keeping n_{eff} constant. This can be understood qualitatively by observing that any misalignment causes a net loss of precise events and a net gain of gross events.

5.5 Accounting

The number of precise and gross Bhabhas for the various SLD run periods are listed in Table 5-1. The accounting is broken down into consecutive blocks of runs that were treated separately due to potentially significant changes in running conditions and LUM configurations where the Bhabha cross section sampled by the LUM may be different. The accounting is also broken down according to the polarization of the electron beam, which is important for measuring the left and right-handed luminosity used in the measurement of \overline{g}_{V}^{e} and \overline{g}_{A}^{e} presented in Chapter 7 below. The number of effective Bhabha events, n_{eff} , for each SLD run period are listed in Table 5-2.

Table 5-1 Number of precise and gross Bhabha events for each SLD run period as measured by the LUM. The number of events are listed separately according to the polarization of the electron beam for the event; Left means left-handed polarization, Right means right-handed polarization.

Name	Description	Precise Left	Gross Left	Precise Right	Gross Right
1993 Run		54,875	9,169	55,406	9,115
1994 Run	pre-September	17,369	3,237	17,367	3,188
1994 Run	Fall: before LUM noise period	26,177	4,571	26,191	4,768
1994 Run	Fall: during LUM noise period	1,898	321	1,918	316
1994 Run	Fall: after LUM noise period	27,694	4,725	28,102	4,651
1995 Run		35,057	6,452	35,273	6,488
1996 Run	before R20 translation #1	1,919	351	1,806	362
1996 Run	during R20 translation #1	14,077	2,775	14,034	2,723
1996 Run	during R20 translation #2	32,888	6,692	32,486	6,573
1996 Run	during R20 rotation #1	7,821	1,570	7,751	1,651
1997 Run		113,543	24,620	114,968	24,725
1998 Run	before off-energy Z-peak scans	171,890	36,115	171,393	36,401
1998 Run	after Z-peak scan	78,814	16,912	78,747	16,634

Table 5-2 Number of effective Bhabhas for each SLD run period for left-handed beams and right-handed beams.

Voor	Effective LUM Bhabhas			
Ital	Left	Right		
1993	59,459.5	59,963.5		
1994	79,565.0	80,039.5		
1995	38,283.0	38,517.0		
1996	62,399.0	61,731.5		
1997	125,853.0	127,330.5		
1998	277,217.5	276,657.5		

5.6 Cross Section Calculation

The cross section for small-angle Bhabha scattering into the LUM is calculated from simulating electromagnetic showers from Monte Carlo event generators which model the small-angle Bhabha scattering process. Although small-angle Bhabha scattering is based on the well known physics of QED, radiative corrections modify the tree level differential cross section by a few percent. Therefore, special precision Monte Carlo generators which take into account these higher order corrections have been written specifically for the cross section calculations required to measure luminosity at e^+e^- colliders. We use two Monte Carlo generators for consistency cross checks, BHLUMI [67, 68] and BABAMC[69]. The BABAMC generator includes only single photon Bremsstrahlung in the initial and final state. Two versions of BHLUMI were used, version 2.01[67] uses the Yennie-Frautschi-Saura (YFS) $O(\alpha)$ exponentiation, and version 4.04[68] which improves on version 2.01 by including missing second-order leading-logarithmic (LL) corrections and QED corrections to the Z⁰ contribution. BHLUMI 2.01 quotes an overall precision of 0.25% and BHLUMI 4.04 quotes an overall precision of 0.11%.

The BABAMC generator was used simply for a consistency cross check of BHLUMI 2.01, and we found the two generators to agree within 0.1%. BHLUMI 2.01 was used to calculate the cross section for the 1993-1995 run periods and BHLUMI 4.04 was used to calculate the cross sections for the 1996-1998 run periods.

Events from the Monte Carlo generators are simulated with GEANT[61] using parameterized electromagnetic showers based on GFLASH[70]. Parameterization of electromagnetic showers is necessary because of the time consuming nature of full shower simulations, which took about an hour per event at the time this dissertation was written. Everything that is known about the configuration of the real LUM and the running conditions are put into the simulation, including actual beam energies, beam energy uncertainties, beam energy widths, position and variation of the interaction point, known dead channels in the LUM electronics and masking of the LUM by inner beam line components.

The simulated Monte Carlo events are passed through the same Bhabha filter used for the data, the number of precise and gross events are tabulated and the effective number of events n_{eff} is calculated according to equation (5.2). An effective cross section is then defined according to the formula

$$\sigma_{\rm eff} \equiv \frac{n_{\rm eff}}{n} \sigma_{\rm MC}$$
(5.3)

where n is the number of events generated by the Monte Carlo generator and σ_{MC} is the cross section calculation of the event generator. The quantity σ_{eff} is the cross section for small-angle Bhabha scattering into the two LUM modules, and will be used in section 5.7 below for calculating integrated luminosity.

During the course of SLD's data acquisition history careful records were kept of changes in running conditions and LUM periods which could have affected the luminosity measurement. These run blocks are listed in Table 5-3, which list the beginning and ending run numbers for each block and the effective cross section σ_{eff} for small-angle Bhabha scattering into the LUM.

Namo	Description	Begin Run	End Run	Cross
INAIIIC	Description	Number	Number	Section
1993 Run		15807	23700	66.748
1994 Run	pre-September	26844	28187	60.42544
1994 Run	Fall: before LUM noise period	28518	29407	66.748
1994 Run	Fall: during LUM noise period	29455	29494	66.748
1994 Run	Fall: after LUM noise period	29518	30077	66.748
1995 Run	-	30320	31226	66.748
1996 Run	before R20 translation #1	33383	33445	67.307
1996 Run	during R20 translation #1	33446	34267	67.307
1996 Run	during R20 translation #2	34268	35253	67.307
1996 Run	during R20 rotation #1	35254	35522	67.307
1997 Run		37418	40724	67.307
1998 Run	before off-energy Z-peak scans	41098	43153	67.307
Z-peak scan	on peak	42786	43153	67.307
Z-peak scan	high energy point	43166	43202	
Z-peak scan	low energy point	43203	43258	
1998 Run	after Z-peak scan	43269	43934	67.307

Table 5-3 SLD Run blocks of significant note throughout SLD's data acquisition history. Listed are the beginning and ending run numbers for each block and the effective cross section σ_{eff} for small-angle Bhabha scattering into the LUM.

5.7 Integrated Luminosity Measurement

Actually calculating the integrated luminosity is a straightforward process once the Bhabha events are properly identified and counted and the cross section for Bhabha scattering into the LUM σ_{eff} is calculated. The formula for calculating integrated luminosity is given by

$$\mathcal{L}_{\text{Integrated}} \equiv \int \mathcal{L} dt = \frac{n_{\text{eff}}}{\sigma_{\text{eff}}}$$
(5.4)

Using this formula, the total integrated luminosity for all run periods is measured by calculating the effective number of Bhabha events n_{eff} for each run block listed in Table 5-3

using equation (5.2) and the precise and gross event counts in Table 5-1, and the effective cross sections σ_{eff} listed in Table 5-3. The total integrated luminosity for all run periods is given in Table 5-4. The total systematic error for the luminosity measurement is 0.76%, and the various contributions to this error will be presented in section 5.8.

Table 5-4 Total integrated luminosity measurements for each SLD run period for left-handed electron beams, right-handed electron beams, and the total integrated luminosity (the sum of the two previous columns). The error on each measurement is the combined statistical error and systematic error. The systematic error is fixed at 0.76% for each run period and helicity except for a small part of the 1994 run which is treated separately in the text..

Run Period	Integrated Luminosity (inverse nanobarns)			
	Left	Right	Total	
1993	891 ± 8	898 ± 8	1,789 ± 15	
1994	1,222 ± 10	1,229 ± 10	2,451 ± 20	
1995	574 ± 5	577 ± 5	1,151 ± 10	
1996	927 ± 8	917 ± 8	1,844 ± 15	
1997	1,870 ± 15	1,892 ± 15	3,762 ± 30	
1998	4,130 ± 32	4,121 ± 32	8,251 ± 64	
Total	9,613 ±74	9,634 ± 74	19,247 ± 147	

5.8 Systematic Errors

A detailed study and description of the LUM systematic errors appears in [53, p. 81-97] and will not be repeated here. However, since the aforementioned study of systematic errors was performed only for the early 1992 SLD data, we identified those sources of systematic error that we believed either would have changed for the later data presented in this dissertation, or that could be improved. The complete list of LUM systematic errors appears in Table 5-5; the first column lists the original systematic error measurements from[53], while the second column lists the systematic errors used in this dissertation, some if which were reanalyzed and updated.

As can be seen in Table 5-5 below, the major source of LUM signal contamination comes from SLC beam related background in the form of electromagnetic radiation. To measure this effect, events were selected which passed all LUM Bhabha cuts except for the cut on the opening angle between the North and South clusters, and of these events only those which have an opening angle near zero radians (instead of π radians) are tagged as beam related contamination. As noted in[53], this method of estimating the SLC beam background is actually an overestimate of the effect since the background events are correlated. We ran the LUM Bhabha filter MBHFLT[71] with its default set of cuts, but with the modified opening angle cut as described above, on the entire 1993 SLD raw triggers and found 14 precise-precise events and 214 gross-precise events that passed the LUM Bhabha cuts, which corresponds to 121 effective LUM Bhabhas. Since there were 119,488 actual LUM Bhabhas in this dataset¹⁸, we estimate the beam background to be 121/119,488 =0.101%. The same exercise was run on smaller samples of the SLD raw triggers for later runs which found similar results. For simplicity we take the 0.101% as the size of the systematic error due to beam related background. It should be noted that applying the 0.101% as a correction to the data instead of assigning it as the systematic error is another possibility which would potentially lower the error due to beam related background. However, this would have required running the modified Bhabha filter on all SLD raw data tapes which was unfeasible due to the large number of raw data tapes.

¹⁸ The 119,488 LUM Bhabha events for the 1993 run period differs from 119,423 as listed in Table 5-2 because the so-called *default cuts* were used instead of the *analysis cuts*. The default cuts are a looser set of cuts from the original MBHFLT Bhabha filter software package[71] The analysis cuts are a slightly tighter set of cuts used for the precision luminosity measurement.

The theoretical uncertainty for the small-angle Bhabha event generator BABMC, written by Berends, Hollik and Kleiss[72], is 0.5%. As already noted the BHLUMI smallangle Bhabha event generators had theoretical uncertainties of 0.25% and 0.11% for version 2.01 and version 4.04, respectively, and the BABAMC event generator was found to agree with the BHLUMI 2.01 generator to within 0.1%. For the simplicity of having one global systematic error for the LUM measurement, we conservatively estimate the theoretical uncertainty of the Monte Carlo event generators as the simple average of the individual theoretical uncertainties, which is 0.3%.

Finally, the smallest dataset of simulated Monte Carlo events passed through the LUM Bhabha filter found 226,082 precise events and 33,492 gross events, which corresponds to 242,828 effective LUM Bhabhas. The statistical error for this dataset is therefore 0.196%. All of these improvements are listed in column 2 of Table 5-5 below.

Systematic Error Source	Systematic Error		
<i>,</i>	Original	Updated	
contamination from $e^+e^- \rightarrow e^+e^-X$	0.010%	0.010%	
contamination from $e^+e^- \rightarrow \gamma\gamma$	0.050%	0.050%	
contamination from $e^+e^{L,R} \rightarrow Z^0 \rightarrow e^+e^-$	0.001%	0.001%	
contamination SLC background	0.320%	0.101%	
energy scale uncertainty	0.170%	0.170%	
125 GeV upper energy cut	0.020%	0.020%	
tower-to-tower calibration effects	0.220%	0.220%	
dead towers between calibrations	0.050%	0.050%	
E_{cm} uncertainty and E_{cm} spread.	0.080%	0.080%	
e ⁺ energy asymmetry	0.060%	0.060%	
IP position uncertainty	0.030%	0.030%	
finite beam crossing angle	0.060%	0.060%	
Monte Carlo generator technical precision	0.100%	0.100%	
Mote Carlo theoretical uncertainty	0.500%	0.300%	
Monte Carlo statistics	0.500%	0.196%	
GEANT/GFLASH simulation accuracy	0.430%	0.430%	
snout modeling uncertainty	0.004%	0.004%	
uncertainty in not modeling pseudo-projective towers	0.190%	0.190%	
1mm IP offset due to θ cut uncertainty	0.080%	0.080%	
LUM module misalignment in transverse plane	0.085%	0.085%	
uncertainty in distance between triplets	0.300%	0.300%	
Total	1.019%	0.759%	

Table 5-5 Luminosity measurement systematic error contributions. The first column lists the systematic errors originally calculated in [53] for the 1992 SLD run periods. The right column lists the systematic errors for the later SLD run periods, 1993-1998.

CHAPTER 6 WIDE-ANGLE BHABHA EVENT SELECTION

The wide-angle Bhabha (WAB) events used in this analysis are measured solely by the SLD LAC calorimeter subsystem. We begin the chapter by describing the entire SLD data-flow starting from an individual beam crossing and follow the data through the LAC digitization and readout process, the Energy Trigger and other downstream filter processes, and the reconstruction of the raw data into physical observables suitable for interactive data analysis. Next, the selection criteria that separate wide-angle Bhabha events from other physics processes and sources of background are described. Finally, the procedure to correct the final selection of WAB events for effects such as detector inefficiencies and contamination by other physics processes is described. The pseudo-event method is presented and described in some detail, which we use to overcome the limitations of the GEANT[61] software simulation of the SLD detector.

6.1 The Data Acquisition Phase

The data-flow of the SLD experiment is logically and physically broken into two steps; the data acquisition on a per beam-crossing basis handled by the Below Line¹⁹ subsystem, followed by an event reconstruction and post-analysis process handled by the

¹⁹ The so-called Below Line is really the SLD Data Acquisition subsystem that is responsible for directly reading out digitized data from the entire SLD detector, calculating trigger quantities, acting on this trigger information, and finally handing the data off to the so-called Online subsystem for writing the data to magnetic tape. Most experiments have just a single Online subsystem that subsumes all of these responsibilities, but the SLD physicists responsible for the data acquisition drew a clear line of distinction between their domain, which was electronics focused, and the rest of the Online subsystem, which was software focused, and coined the term "Below Line" to make explicit this subversive distinction[73]

Offline subsystem. Although the entire SLD Data Acquisition Subsystem is a fascinating topic in and of itself, the current chapter will describe only those parts of the data acquisition that pertains to the SLD LAC calorimeter.

The analog signal of each LAC Tower is digitized on every SLC beam crossing (120 Hz) and the resulting ADC values are stored by a set of custom FastBUS[52] modules, named Calorimeter Data Modules or CDMs[74] for short. The CDMs store the entire KAL data in memory while also calculating the quantities used by the SLD Energy Trigger. If the criteria of the Energy Trigger are satisfied, the CDMs forward their ADC data to a robotic tape silo where the data is written to magnetic tape.

The design of any trigger should be as efficient as possible for capturing any relevant physics information while rejecting as much as possible all forms of background noise. The SLD Energy Trigger is no different, and its sole purpose is to reject bad beam pulses and SLC muon background showers in order to reduce the amount of data written to tape to as low a value as possible while maintaining the full fidelity of any potential physics information which the SLD detector was designed to measure.

The CDMs form two energy sums named E_{LO} and E_{HI} , which are simple sums over all LAC towers subject to two simple threshold values,

$$E_{LO} = \sum_{i=1}^{n_{LO}} E_{i}^{layer} > E_{electronics noise} = 8 \text{ ADC for EM, } 12 \text{ ADC for HAD}$$

$$E_{HI} = \sum_{i=1}^{n_{HI}} E_{i}^{layer} > E_{mip} = 60 \text{ ADC for EM, } 120 \text{ ADC for HAD}$$
(6.1)

Here n_{LO} and n_{HI} are the number of towers with energy above $E_{electronics noise}$ and E_{mip} , respectively. The value $E_{electronics noise}$ is set just above the ambient electronic noise and is

therefore chosen empirically, although it is clear that it will depend on the size of a KAL tower because electronics noise is proportional to capacitance, and capacitance is proportional to a tower's area, as defined by the lead tiles, divided by the gap distance between the lead tiles and lead plates. This is why $E_{electronics noise}$ has different values for the EM and HAD sections of the LAC, with the value for the HAD sections being larger. The value of E_{mip} is chosen to be just above the energy of a minimum ionizing particle, and is designed to reject the background muons generated by the SLC beams scraping beamline components on their way to the IP.

The conversion from ADC to energy for the LAC is 524 MeV/128 ADC for the EM1 and EM2 sections, and 1384 MeV/128 ADC for the HAD1 and HAD2 sections, and is the energy scale for a minimum ionizing particle (mip). The mip energy scale is used consistently throughout this dissertation, meaning that no e/μ correction factor is applied. All quoted energies, and all plots involving energy, use the mip energy scale. For example, any plot that involves energy on an axis will appear "low" compared to an absolute energy scale.

The SLD Energy Trigger is designed to be a general-purpose physics trigger, and is implemented using a simple energy threshold requiring $E_{HI} > 12$ GeV and $n_{LO} < 1000$. These two thresholds simply require that there be enough energy in the calorimeter in not too many towers so that the energy is somewhat localized to specific regions of the detector and not so diffuse as to be SLC beam related noise. When this trigger condition is satisfied all towers above 2 ADC in EM1, 3 ADC in EM2 and 6 ADC in HAD1 or HAD2 are written to tape. For WAB events the trigger is nearly 100% efficient, since a WAB event contains no invisible energy and will deposit nearly the entire center-of-mass energy into a few localized regions of the LAC.

6.2 The Pass 1 Filter

The entire reconstruction phase is broken up into a pipeline of several independent passes that collectively reads the raw ADC values from magnetic tape and performs several filtering and data reduction passes to reduce the raw data into a manageable form more convenient for interactive data analysis. These passes are not unique to the WAB events, and are simply general-purpose physics filters designed to reduce the amount of data that is attributable to background noise.

The first of these filters is called Pass 1, and is simply a more restrictive application of the ideas behind the Energy Trigger. The real-time nature of the data-acquisition environment where the Energy Trigger is applied demands a very loose set of requirements since there will not be an opportunity to revisit an event again if it is not written to tape. During offline processing we can be more discriminating, since if we later discover that our filtering procedures are wrong, we can simply fix the problem and rescan the raw data tapes. The energy and tower count thresholds used for the Pass 1 filter are given in equation (6.2).

$$E_{LO} < \begin{cases} \frac{2}{3}E_{HI} + 70 \text{ GeV} \\ 140 \text{ GeV} \end{cases} \text{ whichever quantity is less} \\ E_{HI} > 15 \text{ GeV} \\ n_{HI}^{\text{EM layers}} > 10 \end{cases}$$
(6.2)

The new ideas here beyond the requirements of the Energy Trigger are that the energy is even more localized to specific regions of the calorimeter (as specified by the threshold on E_{LO}), and at least some of the energy must come from electromagnetic particles (which is specified by the threshold on $n_{HI}^{EM layers}$).

6.3 The Pass 2 Filter

Events that satisfy the Pass 1 filter are pipelined to the Pass 2 filter where the raw data is reconstructed into physics observables. It is in this filter that derived quantities are calculated and pattern recognition algorithms come into play. As a particle enters the LAC it will begin to interact and create secondary particles, which in turn cascades into yet more particles. This process is called showering, of which there are two types; electromagnetic showers caused by radiating electromagnetic particles, and hadronic showers caused by interacting hadrons. In our case we are concerned only with electromagnetic showers, since Bhabha scattering only involves electrons and positrons in the final state. The dimensions of these electromagnetic showers are extremely well contained, being almost pencil-like in shape and size.

Collections of LAC towers with energy above the thresholds given in Table 6-1 and physically adjacent to one another are grouped together and identified as a cluster. These clusters are three-dimensional entities that span adjacent towers in each layer and extend over each of the four calorimeter layers EM1, EM2, HAD1 and HAD2. The idea is to associate each cluster with a single incident particle, although due to fluctuations within each shower and the inherent energy resolution of the LAC it is sometimes necessary to merge clusters in order to associate the properties of a cluster with a specific incident particle. For example, tracking information can be used to associate a group of clusters as belonging to a single track.

As show in Table 6-1, the LAC tower ADC thresholds were lowered for the 1996-1998 run period compared to the 1994-1995 run period. The reason was to increase the efficiency of associating CDC tracks with LAC clusters, since the lower ADC thresholds allowed more LAC clusters to be identified. Although tracking information is not used in this analysis, the lower tower thresholds produce an explosion in the number of clusters per event compared to the 1994-1995 run period that could potentially affect a WAB analysis. As will be show in section 6.4 below, the WAB selection criteria are designed to take this increased number of clusters into account.

by the Pass 2 filter and any further processing.						
Dar Voor			ADC Thresholds			
Kull Tear		EM1	EM2	HAD1	HAD2	
1994	-	1995	7	7	7	9
1996	-	1998	2	3	6	6

Table 6-1 LAC tower thresholds for readout by the CDMs. LAC towers below these ADC values are completely ignored by the Pass 2 filter and any further processing.

Each cluster is assigned six derived quantities: two angular coordinates $\cos\theta$ and ϕ , and the energy sums in each of the four LAC layers of the towers comprising the cluster. For simplicity, the two angular coordinates, $\cos\theta$ and ϕ are calculated as simple energy-weighted mean values of the absolute positions of the LAC towers.

6.4 Wide-Angle Bhabha Event Selection Criteria

At this stage in the processing, we have, for each event, a collection of calorimeter clusters which we need to analyze to determine if the event was due to Bhabha scattering. The method for identifying WAB events makes use of the fact that all of the final state particles for each event are electromagnetic, and therefore deposit the majority of the centerof-mass energy into the electromagnetic section of the calorimeter. Additionally, there will be exactly one electron, one positron, and relatively few photons²⁰, if any, in the final state. Since this analysis uses only calorimeter information, it is not known which particle is associated with which $cluster^{21}$. However, the order α^2 Monte Carlo we use in this analysis shows that the electron and positron will be the two most energetic clusters in the event more than 99% of the time, and that these two clusters will have an acolinearity near zero, meaning that they are nearly back-to-back. It is this last observation that makes the identification of WAB events easily separable from non-Bhabha physics processes and other sources of background.

Briefly, the selection criteria that follow identify events with relatively few clusters (but at least two) which deposit most of their energy in the electromagnetic section of the calorimeter, and where the two highest energy clusters are nearly back-to-back and contain the majority of the center-of-mass energy.

6.4.1 Cluster Quality Cuts

First, a subset of the original cluster list is selected by imposing the following cluster quality cuts:

$$E_{EM1+EM2} > 0 \text{ GeV}$$

$$E_{Total} > 1 \text{ GeV}$$
(6.3)

²⁰ As will be shown in the following sections, the order α^2 Bhabha Monte Carlo we use in this analysis shows that 92% of the time there are only two clusters in the final state that pass all of our selection criteria. ²¹ It may be possible to determine whether a cluster is associated with the electron or positron from the curvature of the CDC track(s) associated with the cluster, since each particle's transverse component of momentum to the magnetic field will cause them to bend in opposite directions due to their opposite electric charges.

These very loose energy requirements simply reject clusters that are due to known background sources with negligible impact on the WAB signal. The benefit of the cluster quality cuts is that the number of clusters per event is greatly reduced, as well as outright rejecting anywhere between 15% and 30% (depending on the beam conditions of a given run) of the original set of events due to their being less than two clusters in the event.

Figure 6-1 and Figure 6-2 show the distribution of clusters before (above) and after (below) imposing the cluster quality cuts given in equation (6.3) on the raw data for the 1994 through 1995 runs (the VXD2 era) and the 1996 through 1998 runs (the VXD3 era). No other selection criteria have been applied other than there are at least two clusters in the event. Notice that the cluster quality cuts make no difference when the higher tower thresholds were used in the 1994 through 1995 era (see Figure 6-1), but that the cuts make a tremendous difference for the later runs (see Figure 6-2). Following the cluster quality cuts, the data samples for each era are similar, and therefore the same cuts can be applied in the subsequent analysis.



Figure 6-1 Number of clusters present before (above) and after (below) the cluster quality cuts in equation (6.3) for the 1994 run, which is representative of all 1994-1995 data (this is the VXD2 era). No other criteria have been applied other than there are at least two clusters in the event. There are no differences between the two plots, as expected.



Figure 6-2 Number of clusters present before (above) and after (below) the cluster quality cuts in equation (6.3) for the 1997 data, which is representative of all 1996-1998 data (this is the VXD3 era). No other criteria have been applied other than there are at least two clusters in the event. Comparing the bottom plot to the plots in Figure 6-1 shows that the cluster quality cuts establish distributions which are similar for the two eras.

6.4.2 Cluster Energy Cuts

From the Unibab Monte Carlo[19] generator we know that over 99% of the time the two highest energy clusters will be the final state electron and positron. We therefore place the following stringent energy requirements on these two clusters:

$$\begin{split} E_{\rm EM1+EM2}^{\rm cluster 1} &> 10 \ {\rm GeV} & E_{\rm EM1+EM2}^{\rm cluster 2} > 10 \ {\rm GeV} \\ E_{\rm HAD1}^{\rm cluster 1} &< 3 \ {\rm GeV} & E_{\rm HAD1}^{\rm cluster 2} < 3 \ {\rm GeV} \\ E_{\rm HAD2}^{\rm cluster 1} &< 0.5 \ {\rm GeV} & E_{\rm HAD2}^{\rm cluster 2} < 0.5 \ {\rm GeV} \end{split}$$

$$(6.4)$$

These selection criteria demand that these two highest energy clusters deposit the majority of their energy into the EM sections of the LAC, with any energy leaking into the HAD sections falling off rapidly with depth.

The distributions for this set of selection criteria are shown in Figure 6-3 (for the highest energy cluster) and Figure 6-4 (for the second highest energy cluster). For each plot, the selection criterion for the plotted quantity has not been applied. For example, the plot of the EM energy for cluster 1 (the top plot of Figure 6-3) does not include the requirement that $E_{EM1+EM2} > 10$ GeV, but it does include the requirements of the other five selection criteria listed in equation (6.4).



Figure 6-3 Cluster energy cuts for the highest energy cluster. The only criteria applied to select events in each plot are the cluster quality cuts of equation (6.3) and five of the six criteria in equation (6.4), the missing criterion being the quantity actually plotted so that it is clear which events are being discarded by the cut (accepted events appear in the darker shaded region). Data is from the 1997 run and is representative of the entire SLD dataset.



Figure 6-4 Cluster energy cuts for the second highest energy cluster. The only criteria applied to select events in each plot are the cluster quality cuts of equation (6.3) and five of the six criteria in equation (6.4), the missing criterion being the quantity actually plotted so that it is clear which events are being discarded by the cut (accepted events appear in the darker shaded region). Data is from the 1997 run and is representative of the entire SLD dataset.

Recall from section 6.1 that the energy scale used is that for a minimum ionizing particle, and therefore the true energies of the electron and positron are greater than represented in the plots.

Notice that the top plots in Figure 6-3 and Figure 6-4. both show two identical peaks, one at 32 GeV and one at 25 GeV. These two peaks are really just the single energy distribution of the WAB events, but they appear as two peaks due to two regions of the LAC which have different energy responses; the peak at 32 GeV are events contained in the barrel region ($|\cos \theta| < 0.7$), while the peak at 25 GeV are events confined to the endcap region ($|\cos \theta| > 0.7$). Again, as described in section 6.1, these energies are calibrated to the response of a minimum ionizing particle, and are therefore lower that the true electron or positron energy. The peak at 5 GeV in the top plot of Figure 6-4 is from background events, which are removed by the selection criterion $E_{EM1+EM2} > 10$ GeV for cluster 2. It is this last cut that is the most effective cut out of all of the selection criteria, as only about 17% of all Pass 2 events pass this one cut.

6.4.3 Angle Dependent Energy Cut

The next most effective cut is an angle dependent energy cut on the two highest energy clusters that addresses the different energy responses of the barrel and endcap regions of the LAC noted earlier. The values of the selection criteria are given in equation (6.5) and are displayed visually in Figure 6-5. As can be seen in this figure, the cut at 55 GeV in the barrel region is placed well below the band of WAB events at 65 GeV. As we enter the endcap region beginning at $|\cos \theta| \approx 0.68$ the amount of material in front of the LAC increases which causes the energy response of the LAC to decrease.

$$0 \leq |\cos \theta| < 0.68 \qquad E_{Total}^{cluster 1} + E_{Total}^{cluster 2} > 55 \text{ GeV}$$

$$0.68 \leq |\cos \theta| < 0.80 \qquad E_{Total}^{cluster 1} + E_{Total}^{cluster 2} > 168.3 - 166.7 |\cos \theta_{thrust}| \qquad (6.5)$$

$$0.80 < |\cos \theta| \qquad E_{Total}^{cluster 1} + E_{Total}^{cluster 2} > 20 \text{ GeV}$$



Figure 6-5 Angle dependent energy cut given in equation (6.5) for the sum of the energy of the two highest energy clusters. The band at 64 GeV are the WAB events. Material in front of the LAC begins degrading the energy response at $|\cos \theta| \approx 0.68$, which the cuts take into account. All cuts have been applied except for the angle dependent energy cut, which is shown in the shaded region. The data is from the 1997 run and is representative of the entire SLD dataset.

6.4.4 Global Event Cuts: Total Energy and Energy Imbalance

Thus far, all of the selection criteria have focused on selecting events with at least two high-energy clusters, but we have not applied any criteria to the event as a whole. Since all of the final state particles of a WAB event are electromagnetic, all of the particles within the acceptance of the LAC will deposit nearly all of their energy into the LAC. We therefore place two requirements on the total energy of the event; the event must have a certain minimum energy, and this energy must be symmetrically distributed. To quantify these two requirements, we define the quantity E_{Total} as the sum of the total energy of all clusters satisfying the cluster quality cuts of equation (6.3), and $E_{Imbalance}$ as the magnitude of the vector sum of these clusters:

$$\begin{aligned}
\vec{v} &\equiv (\mathbf{E}_{\text{cluster}}, \boldsymbol{\theta}_{\text{cluster}}) \\
\mathbf{E}_{\text{Total}} &= \sum \left| \vec{v} \right| \\
\mathbf{E}_{\text{Imbalance}} &= \frac{\left| \sum \vec{v} \right|}{\mathbf{E}_{\text{Total}}}
\end{aligned}$$
(6.6)

Clearly, for events with final state particles of negligible mass, those events with no invisible energy will have $E_{Imbalance} = 0$ since conservation of momentum will balance the energy symmetrically. However, events with missing or invisible energy (e.g. events with neutrinos in the final state) will have $E_{Imbalance} > 0$. In the limiting case where an event has just one particle with visible energy in the final state (as can occur with beam background) then $E_{Imbalance} = 1$.

Since the final state particles of WAB events all have negligible mass and have no invisible energy (they deposit nearly all of their energy into the EM section of the LAC), we require events to satisfy the following criteria, which has a negligible impact on WAB event selection efficiency while greatly reducing hadronic events, tau events and beam related background:

$$E_{Total} > 15 \text{ GeV}$$

 $E_{Imbalance} < 0.6$
(6.7)

The distribution of E_{Total} is shown in Figure 6-6 for all Pass 2 events satisfying the cluster quality cuts of equation (6.3) for the 1997 Data. Notice the peaks at 65 GeV in the barrel region and 52 GeV in the endcap region, which are exactly twice the value of the two peaks in the top plots of Figure 6-3 and Figure 6-4, demonstrating that the events in these peaks carry the majority of their energy almost entirely in the two highest energy clusters. The 65 GeV peak are due to events in the barrel region of the LAC ($|\cos \theta| \le 0.82$), while the 52 GeV peak are due to events in the endcap region of the LAC ($|\cos \theta| \le 0.82$). The other peaks at lower energies in the plots are various kinds of background²² to the WAB signal, and are removed by cuts presented in later sections.

²² These background events come from both physics processes as well as beam related sources.



Figure 6-6 Total energy distribution of all Pass 2 events satisfying only the cluster quality cuts of equation (6.3). No other criteria have been applied other than there are at least two clusters in the event. The top plot shows events in the barrel region of the LAC ($|\cos \theta| < 0.82$) while the bottom plots show events in the endcap region of the LAC ($|\cos \theta| > 0.82$). The peak at 65 GeV in the barrel region and the peak at 52 GeV in the endcap region are the WAB events. The darker shaded region shows events satisfying the cut on total energy. Data is from the 1997 run and is representative of the entire SLD dataset.

The distribution of $E_{Imbalance}$ for the same subset of events is shown in Figure 6-7, and is plotted against E_{Total} in Figure 6-8. Notice the two clusters of events in this last figure at 65 GeV and 52 GeV with $E_{Imbalance}$ near 0 corresponding to the two peaks pointed out earlier in the top plots of Figure 6-3 and Figure 6-4, which is again consistent with the hypothesis that these are indeed WAB events.



Figure 6-7 Energy imbalance of all Pass 2 events satisfying only the cluster quality cuts of equation (6.3). No other criteria have been applied other than there are at least two clusters in the event. The darker shaded region sows events satisfying the cut on energy imbalance. Data is from the 1997 run and is representative of the entire SLD dataset.



Figure 6-8 Total Energy vs. Energy Imbalance for Pass 2 events satisfying only the cluster quality cuts of equation (6.3). No other criteria have been applied other than there are at least two clusters in the event. The cluster of events at 65 GeV are events in the barrel region of the LAC ($|\cos \theta| < 0.82$), while the cluster of events at 52 GeV are events in the endcap region of the LAC ($|\cos \theta| > 0.82$). Both peaks are almost entirely WAB events. The darker shaded region shows events which satisfy the total energy and energy imbalance cuts in equation (6.7). Only 20,000 events are plotted from the 1997 SLD dataset in order to make the plot legible, although the data is representative of the entire SLD dataset.

6.4.5 Multiplicity Cut

The multiplicity of an event is defined as the number of calorimeter clusters in the event. The multiplicity of WAB events will be small since they have only an electron and positron in the final state, and so will have a typical value near two. Hadronic decays of the Z^0 , on the other hand, will have larger multiplicity values because of the much larger number of particles in the final state due to effects such as hadronization. The multiplicity distribution is show in Figure 6-9 where these features can clearly be seen. The narrow distribution which peaks at 6 clusters are primarily the low multiplicity WABs, while the

broad distribution which peaks at 25 clusters are primarily the hadronic events. We therefore require WAB candidate events to lie within the following small event multiplicity range:

$$2 \le n_{\rm clus} \le 11 \tag{6.8}$$



Figure 6-9 The number of clusters in the event (event multiplicity) for Pass 2 events satisfying only the cluster quality cuts of equation (6.3). No other criteria have been applied other than there are at least two clusters in the event. The narrow distribution which peaks at 6 clusters are primarily WAB events, while the broad distribution which peaks at 25 clusters are primarily hadronic events. The darker shaded region shows events satisfying the multiplicity cut in equation (6.8). Data is from the 1997 run and is representative of the entire SLD dataset.

6.4.6 Rapidity Cut

Finally, the last criterion imposed to select the final WAB event sample is the defacto signature of a WAB event, namely that the two final state leptons are back to back, or nearly so, in the center-of-mass (or CMS) frame of reference. In the absence of radiative corrections there will be no initial state radiation and the CMS frame will be the same as the LAB frame (i.e. the SLD LAC calorimeter in our case). In this scenario the clusters corresponding to these two leptons will always be back to back in the LAB frame and a simple hard cut on the acolinearity of the two highest energy clusters would suffice.

In the real world, however, radiative corrections are a fact of life and constitute large corrections of about 30% to the WAB cross section[75]. The effect of radiation on the kinematics of the event can be understood using the collinear radiation approximation[76] in which the differential cross section is convolved with two electron structure functions which give the probability for the incoming leptons to radiate away a fraction of their energy into photons which are collinear to the incoming leptons[77, 78]. In this approximation, the Bhabha scattering process is viewed as occurring in three steps:

- Emission of collinear photons from the incoming electron and positron, known as Initial State Radiation or ISR. When there is initial state radiation there will typically be only one photon since this is a purely QED process²³. Before radiation, each incoming lepton has energy E_{beam}, and therefore the center-of-mass energy is √s = 2 E_{beam}. After initial state radiation, but before the hard scattering process (step 2. below), the electron (positron) has a fraction x₋(x₊) of E_{beam}, such that 0 ≤ x_± ≤ 1.
- 2. The actual hard scattering process $e^+e^- \rightarrow (Z^0, \gamma) \rightarrow e^+e^-$ occurring at a fractionally reduced collision-frame invariant energy squared $s' = s \cdot x_- \cdot x_+$.

²³ The probability of a QED vertex goes like α , where $\alpha = \frac{1}{137.03599976}$ at $Q^2 = 0$. Therefore, the probability to emit more than one photon is small.
Emission of mostly collinear photons from the final state electron and positron, known as Final State Radiation or FSR. Most of the time the FSR photons are so close to the outgoing final state leptons as to be indistinguishable from them²⁴.

The problem, therefore, is how to identify the back-to-back nature of WAB events measured in the SLD LAB frame without knowing a priori the CMS system of the hard scattering process (step 2. above). We can qualitatively understand the nature of the problem from the preceding description of the collinear radiation approximation. If we assume one of the leptons radiates a photon and the other does not (by far the most probably scenario), then we can immediately see that when the energy of the radiated photon is small, the CMS system isn't boosted very much relative to the LAB frame and therefore the acolinearity between the final state leptons will be small. On the other hand, when the radiated energy of the photon is large then the CMS frame will be highly boosted relative to the LAB frame and the acolinearity between the final state leptons will be large. This situation is shown graphically in Figure 6-10.

²⁴ To quantify this, 1 Million Unibab events were generated and passed through the Bhabha selection criteria presented in this chapter. Of these, 525,053 pass the selection criteria and fall within the acceptance of the LAC, and of these events only 42,109, or 8% of the total, have a third cluster that has > 10 GeV and are separated from the nearest electron or positron by more than 0.02 in $\cos\theta$, the size of a typical LAC tower.



Figure 6-10 Kinematic diagram showing the LAB frame of reference (top diagram) and the center-of-mass (CMS) frame of reference (bottom diagram) when one photon is radiated during initial state radiation, as in the collinear radiation approximation.

From Figure 6-10 we define the acolinearity of the final state electron and positron in the LAB (i.e. LAC calorimeter) frame of reference:

$$\zeta = 180^{\circ} - (\theta_{-} + \theta_{+}) \tag{6.9}$$

Here θ_{-} and θ_{+} are the LAB frame scattering angles of the electron and positron, respectively²⁵. Similarly, we can relate the LAB frame scattering angles to the center-of-mass scattering angle:

$$\cos\theta_{\rm CMS} = \frac{\sin\frac{\theta_--\theta_+}{2}}{\sin\frac{\theta_-+\theta_+}{2}} \tag{6.10}$$

We also define the rapidity as a simple relation between the fractional energies left to the electron and positron after initial state radiation:

$$y = \ln \sqrt{\frac{x_+}{x_-}} \tag{6.11}$$

In the limit of the collinear radiation approximation (i.e. assuming only one radiated photon in the initial state) this definition of rapidity is related to the LAB frame scattering angles of the final state particles[79]:

$$y = \sqrt{\frac{E + p_Z}{E - p_Z}} = \ln \sqrt{\frac{\sin \theta_+ (1 + \cos \theta_-) + \sin \theta_- (1 + \cos \theta_+)}{\sin \theta_+ (1 - \cos \theta_-) + \sin \theta_- (1 - \cos \theta_+)}}$$
(6.12)

Here E and p_z are the energy and longitudinal component of momentum, respectively, of the center-of-mass system as measured in the LAB frame.

Figure 6-11 shows the acolinearity as a function of the center-of-mass scattering angle for three values of x_{-} , the electron's fractional energy after initial state radiation,

²⁵ Although we label the angles separately for the electron and positron, none of the relations actually depend on the particle's type. All that matters are the two angles of the two final state particles independent of their actual type.

assuming that the positron did not radiate ($x_{+} = 1$). From this figure it is clear that a cut on rapidity (or equivalently, a cut on the amount of initial state radiation) acts as an angledependent acolinearity cut.



Figure 6-11 Acolinearity vs. $\cos \theta_{\rm CMS}$ when one photon is radiated in the initial state. We have arbitrarily chosen to vary x_{-} , the electron's fractional energy after initial state radiation, and assume that the positron does not radiate ($x_{+} = 1$). The value of $x_{-} = 55\%$ corresponds to a rapidity value of y = 0.30, the rapidity cut used for the WAB selection criteria.

We chose the following cut on rapidity, which corresponds to limiting a photon from initial state radiation to carry 45% or less of the beam energy down the beam pipe (i.e. $x_{-} = 55\%$). The distribution of rapidity and the photon's fractional energy are shown in Figure 6-12.

$$y < 0.3$$
 (6.13)



Figure 6-12 Rapidity distribution (top plot) and calculated fractional energy of a single photon from initial state radiation (bottom plot) for events passing all of the selection criteria except for the cut on rapidity. The rapidity cut, placed at |y| < 0.3, corresponds to a photon from initial state radiation carrying 45% or less of the beam energy. The darker shaded regions have all of the WAB selection cuts applied. Data is from the 1997 run and is representative of the entire SLD dataset.

6.4.7 Event Selection Summary

The wide-angle Bhabha event selection criteria take advantage of the unique topology of the relatively simple and clean final state particle distribution of Bhabha scattering. The 11 cuts used to select WAB events, and their effectiveness, are summarized below in Table 6-2. This table shows the percentage of events which pass each cut, which is listed separately for each SLD run period.

Table 6-2 Summary of the effectiveness for each cut (named in the first column) for each SLD run period (listed along the top). The numbers in the table specify the percentage of events passing the cut listed in the left-most column. For example, the single most effective cut for every run period is EM(cluster 2) > 10 GeV where only 12%-15% of the entire SLD dataset pass this single cut. The row labeled "Passing All Cuts" is the logical AND of all of the cuts, showing that only 6%-7% of the entire SLD dataset are WAB events. The sum of the individual cuts don't add to 100% because all of the cuts are correlated with one another. The table is sorted by the column for the 1997 dataset, although the general trend for cut effectiveness is the same for all run periods.

Cut Name Year					
	1994	1995	1996	1997	1998
Passing All Cuts	7%	6%	6%	7%	8%
EM(cluster 2) > 10 GeV	13%	12%	12%	14%	15%
$2 \le n_{clus} \le 11$	29%	29%	43%	32%	32%
EM(cluster 1) > 10 GeV	30%	30%	29%	33%	34%
Angle Dependent Energy Cut	36%	36%	44%	37%	39%
HAD_2 (cluster 1) > 10 GeV	61%	63%	64%	58%	60%
HAD_2 (cluster 2) > 10 GeV	72%	73%	76%	71%	72%
HAD_1 (cluster 1) > 10 GeV	75%	77%	78%	74%	75%
y < 0.3	71%	69%	67%	74%	76%
$E_{Total} > 15$	78%	75%	66%	78%	80%
HAD_1 (cluster 2) > 10 GeV	87%	88%	89%	87%	87%
$E_{Imbalance} < 0.6$	86%	86%	79%	87%	88%

It is clear from the table that some cuts are more effective than others. For example, the most effective cut is for the electromagnetic energy of the second most energetic cluster to be EM(cluster 2) > 10 GeV, as this rejects over 85% of all events. This makes perfect sense, since there are not many physics processes of the form $e^+e^- \rightarrow X$ which produce two such high energy electrons or photons in the final state (see section 6.5.2). Also note that the multiplicity cut is the next most effective selection criterion which rejects nearly 70% of all events, which is due to rejecting events from hadronic decays of the Z^0 .

This final tabulation of all events passing the WAB selection criteria are listed in Table 6-3. The last column shows the number of WAB events used in the log-likelihood fits (see Chapter 7) which are within the angular acceptance $|\cos \theta_{\text{Thrust}}| < 0.9655$ and for which a polarization quality cut is applied.

Table 6-3 Number of events in the entire SLD dataset, and the number of events which pass all of the WAB selection criteria described in the preceding sections. The last column is a subset of the "Pass All Cuts" column where the thrust angle is constrained to be within the acceptance used for the log-likelihood fits described in Chapter 7 below.

year	Total SLD Dataset	Pass All Cuts	Pass All Cuts and $ \cos\theta_{\text{Thrust}} < 0.9655$
1994	123,280	8,208	5,478
1995	57,587	3,462	2,497
1996	100,754	6,530	4,812
1997	175,564	12,461	9,126
1998	375,554	29,567	21,309
Total	832,739	60,228	43,222

6.5 Correction Factors

Since we will eventually perform a simultaneous fit of the selected WAB events for both \overline{g}_{V}^{e} and \overline{g}_{A}^{e} it is important to understand the efficiency of both our detector (e.g. the LAC) and our selection criteria, and possible contamination from other physics processes. The reason this is important is due to \overline{g}_{A}^{e} being a sensitive function of the shape of the WAB angular distribution. If any inefficiencies or contamination exist that have an angular dependence (and there is a strong detector inefficiency angular dependence in our case as will be shown below), then the measurement of \overline{g}_{A}^{e} (and \overline{g}_{V}^{e} through its correlation²⁶ with \overline{g}_{A}^{e}) will be affected. Additionally, since we use the luminosity measurement to provide an absolute normalization for the WAB angular distribution, it is important that the WAB inefficiencies be corrected.

6.5.1 Efficiency

For an ideal detector with 100% detection efficiency and with no material between it and the interaction point, the energy response as a function of angle for WAB events would be a perfect band at the center-of-mass energy, about 91 GeV (or in our case about 65 GeV since, as noted in section 6.1, we are using the minimum-ionizing-particle energy scale). However, as can be seen in Figure 6-13 below, the energy response of the LAC as a function of angle for WAB events is anything but uniform. Although there are large regions of the LAC which do have a uniform flat angular dependence (e.g. for $0 \le |\cos \theta| < 0.42$ and $0.50 \le |\cos \theta| < 0.65$), other regions do not, and it is these regions which are highly likely to be inefficient which are the most critical to correct the data for, and the most difficult to

 $^{^{26}}$ The correlation coefficient between \overline{g}_V^e and \overline{g}_A^e is about 0.25 from our log-likelihood fits.

model. Regions such as $|\cos \theta| \ge 0.65$ are so critical because the LAC inefficiency is changing most rapidly and dramatically as a function of $\cos \theta$, which overlaps the region where the WAB cross section is also changing most rapidly.



representative of the entire SLD dataset.

A region of the LAC with a non-uniform energy response as a function of $\cos \theta_{\text{Thrust}}$ does not necessarily mean that the region is inefficient, although it is highly likely that detection inefficiencies do exist in the region. The reason for the non-uniform energy response of the LAC is largely due to a non-uniform distribution of material between the interaction point and LAC towers, since other reasons for the degraded energy response such as calibration errors and argon impurity have been accounted for[80]. Since we know there is more material in front of those regions of the LAC with the more degraded energy

response, we know that particles entering those regions will be more likely to begin showering in this material and the LAC will not detect the early energy deposition of these showers. Additionally, longitudinal shower fluctuations will be an even greater effect for these showers.

Many of the inefficient regions of the LAC are understood well enough that they can be correctly modeled with GEANT. For example, the dip in response for the region $0.435 \le |\cos \theta| < 0.493$ (see Figure 6-13) is due to the washer where the barrel LAC sections are joined together. Since the material and geometry of the washer are well known, the energy loss of particles passing through this region of the SLD is well modeled.

Other inefficient regions of the LAC are qualitatively understood but are more difficult to model. The energy response for the region $0.65 \le |\cos \theta| < 0.85$ falls off sharply and almost linearly. The material in front of the LAC in this region includes plumbing, electronics and cables for the endcap Drift chamber, barrel CRID and endcap CRID, in addition to the increased amount of aluminum dewar for the LAC. The energy response recovers a bit for the region $0.85 \le |\cos \theta| \approx 0.9$, but then rapidly falls off again for $|\cos \theta| > 0.9$ as the cryogenics, electronics and cables for the Vertex Detector (and for the 1994-95 runs, the cables and connectors for the MASC) get in the way.

These observations have made modeling the LAC notoriously difficult for SLD physicists, and have limited many SLD physics analyses to the barrel region of the LAC. One possible solution is to try to correctly model all of the extra material in the region $|\cos \theta| \ge 0.65$ by including it as part of the GEANT description of the SLD. This method was used by Pitts[53] (see section 7.5.1) where the author empirically added approximations of material in various configurations until the LAC energy response as a function of angle for the simulated Monte Carlo WAB events matched that of the real data.

We have chosen another, independent method to correct the WAB data for inefficiencies. Our new method, which we call the *Pseudo Event Method*, uses the data itself to measure the LAC inefficiency as a function of $\cos \theta$. This method takes advantage of our knowledge of the unique physics topology of a WAB event. We determined and used a special set of selection criteria which selected a subset of all SLD events which would have passed the WAB selection criteria described in section 6.4 above except for the cluster selection criteria (i.e. we only kept the selection criteria for the global event quantities). In place of the cluster selection criteria, we required only an electron or positron in the final state with energy above an angle dependent hard energy threshold, which are listed in Table 6-4.

Table 6-4 Angle dependent energy thresholds for selecting Pseudo Events. Any SLD event with $|\cos \theta_{\text{Low}}| \le |\cos \theta_{\text{Thrust}}| < |\cos \theta_{\text{High}}|$ for which $\mathbf{E}_{\text{cluster}} \ge$ the energy threshold listed in the table is a candidate for one-half of a Pseudo Event.

$ \cos \theta_{\rm Low} $	$\cos heta_{ ext{High}}$	Energy Threshold (GeV)
0	0.435	30.0
0.435	0.464	22.0
0.464	0.493	26.0
0.493	0.6985	30.0
0.6985	0.7335	27.5
0.7335	0.765	25.0
0.765	0.793	22.0
0.793	0.8185	20.0
0.8185	0.8465	18.0
0.8465	0.872	23.0
0.872	0.897	25.0
0.897	0.9655	26.0

The energy thresholds are chosen empirically to select WAB events for an ideal detector with 100% detection efficiency. The idea is to chose a stiff enough energy threshold so that when we find an event with a LAC cluster above this energy threshold (which we call a *Gold Cluster*) we know that if the detector were 100% efficient then there must be another cluster with this much energy, but in the opposite direction, in order to balance energy and momentum for the event. We create two lists of events by dividing the LAC into two hemispheres of North and South. Event information for Gold Clusters that are in the North hemisphere are written to the North Cluster List, and event information for events with a Gold Cluster in the South hemisphere are written to the South Cluster List. Each list contains the four quantities Run Number, Event Number, LAC Tower Index (determined by $\cos \theta_{\text{Cluster}}^{\text{Gold}}$) and $\phi_{\text{Cluster}}^{\text{Gold}}$. With these two lists in hand we create yet a third list which is a combination of the North and South Cluster Lists. This third list contains random permutations of each list's entries with the same LAC Tower Indices, and is called the *Pseudo Event Index Table*.

The Pseudo Event Index Table is a lookup table that allows us to create the pseudoevents. For each entry in this table, we take only the clusters from the South hemisphere for the event tagged with the North Gold Cluster and add these clusters into a new event structure with the clusters from the North hemisphere tagged by the South Gold Cluster. The angle ϕ_{Cluster} for each cluster in these hemispheres is rotated by $\pi - \phi_{\text{Cluster}}^{\text{Gold}}$ so that if the event with the Gold Cluster were a WAB, the clusters would line up in the proper back-toback fashion. For this rotation, we arbitrarily chose to rotate the South Clusters. This new event structure, which is the combination of clusters from independent events, is appropriately called a Pseudo Event. Finally, this new set of Pseudo Events is passed through the normal full WAB selection criteria described in section 6.4 above. For a detector with 100% efficiency we would expect all of the Pseudo Events to pass; any Pseudo Events that do not pass must be due to inefficiency. The inefficiencies are independently measured and calculated for each SLD run period and for each KAL tower. The efficiency for each KAL tower for the 1997 data, which is representative of all of the SLD run periods, is shown in Figure 6-14.



Figure 6-14 Efficiency as a function of $|\cos \theta_{\text{Thrust}}|$ as determined by the Pseudo Event method. Data is from the 1997 run and is representative of the entire SLD dataset.

6.5.2 Contamination

Other non-WAB physics processes could potentially slip through our WAB selection criteria to make our final selection of WAB events impure by some factor. The primary sources are $e^+e^- \rightarrow \gamma\gamma$ and $e^+e^- \rightarrow Z^0 \rightarrow \tau^+\tau^- \rightarrow e^+\nu_e\overline{\nu}_r e^-\overline{\nu}_e\nu_\tau$. These processes were measured by Pitts[53] (see section 7.5.3) and found to be relatively small, with an overall contamination of 1.25% and 0.28% of the WAB yield, respectively. Pitts measured the contamination of these two processes as a function of angle, which is important since the differential cross section angular distributions are both different from the WAB differential cross section angular distribution. Additionally, Pitts performed Monte Carlo studies of the contamination from the hadronic decays of the Z^0 and found the contamination to be < 1%, which is negligible. We simply reuse these small correction factors to correct our WAB data.

6.5.3 Summary of Correction Factors

The correction factors for both efficiency and contamination are used to correct the selected WAB events for each SLD run period on a per LAC tower basis. Plots of all of the data showing the final angular distributions of selected WAB events both before and after all of the correction factors are show in Figure 6-15 through Figure 6-19. These datasets will be used in the log-likelihood fits in Chapter 7.



Figure 6-15 Angular distribution of selected WAB events corrected for efficiency and contamination. Data is from the 1994 run.



Figure 6-16 Angular distribution of selected WAB events corrected for efficiency and contamination. Data is from the 1995 run.



Figure 6-17 Angular distribution of selected WAB events corrected for efficiency and contamination. Data is from the 1996 run.



Figure 6-18 Angular distribution of selected WAB events corrected for efficiency and contamination. Data is from the 1997 run.



Figure 6-19 Angular distribution of selected WAB events corrected for efficiency and contamination. Data is from the 1998 run.

CHAPTER 7 DATA ANALYSIS AND RESULTS

In this chapter, we present the analysis techniques used to extract the electron coupling parameters \overline{g}_{V}^{e} and \overline{g}_{A}^{e} from the angular distribution of the polarized wide-angle Bhabha scattered events and the luminosity measurement. First, we describe the Extended Maximum Likelihood method. Next, the function that is minimized is described in some detail, as it is here that careful attention must be paid to incorporating the radiative corrections into the tree-level analytical expression that describes polarized wide-angle Bhabha scattering. Finally, we present the results of the fit, followed by a discussion of the systematic errors.

7.1 The Extended Maximum Likelihood Method

Given a probability distribution function $P(x, \tau)$ (hereafter called the p.d.f.) which describes the distribution of a random variable x for a specified value of a parameter τ , the method of maximum likelihood is used to estimate the best value of the parameter τ for a given finite sample of data. Under the assumption that each measurement x_i is independent of every other measurement, the probability for a set of n measurements of the x_i to occur, where each observation i is measured to be between x_i and $x_i + dx_i$, is given by

$$\prod_{i=1}^{n} P(x_i, \tau) dx_i \tag{7.1}$$

If the hypothesis $P(x, \tau)$ correctly describes the physics we measure in the laboratory, then one expects a higher probability for values of τ that are closer to its true value than for values of τ that are farther away from its true value. Since the dx_i do not depend on the parameter τ , the same line of reasoning holds for the following function $L(\tau)$, called the likelihood function:

$$L(\tau) = \prod_{i=1}^{n} P(x_i, \tau)$$
(7.2)

Therefore, determining the value of $\tau = \tau_{MAX}$ for which the likelihood function $L(\tau)$ is a maximum will provide the best estimate of τ for the finite sample of observations x_1, \dots, x_n . Finding the value of τ_{MAX} is straightforward, as it is simply the solution to the equation

$$\partial_{\tau} L(\tau) = 0 \tag{7.3}$$

Therefore, the requirements on $L(\tau)$ are really quite general, requiring only that the function be differentiable w.r.t. the parameter τ . Without loss of generality, we can extend the procedure to allow each measurement of the random variable x_i to be a collection of observables, so that each x_i is a vector of measurements and not just a single measurement. Likewise, there is no reason to limit the hypothesized p.d.f. to be a function of just one parameter, as allowing the p.d.f. to be a function of multiple parameters is straightforward. For the case of *m* such parameters equation (7.3) generalizes to

$$\partial_{\tau_j} L(\tau_j) = 0 \qquad j = 1, \cdots, m$$
(7.4)

This technique for determining the best estimate of a model's parameters τ_j from a finite sample of data is known as the maximum likelihood method.

Up to now, the size of the data sample n has been fixed. However, in our case we not only have a collection of n polarized wide-angle Bhabha events, but the SLD Luminosity Monitor also tells us how many events we should expect in our polarized wideangle Bhabha sample by way of the luminosity measurement of small-angle Bhabhas. Recall that

$$\nu = \mathcal{L}_{\text{Integrated}} \sigma \tag{7.5}$$

Where ν is the expected number of polarized wide-angle Bhabha events, $\mathcal{L}_{\text{Integrated}}$ is the integrated luminosity and σ is the integrated wide-angle Bhabha cross section of our polarized wide-angle Bhabha hypothesis. Therefore, the luminosity tells us, within Poisson statistics, what the sample size n of our polarized wide-angle Bhabha events should be. Thus, the procedure is clear: we simply modify equation (7.1) by multiplying it by the probability that n polarized wide-angle Bhabha events are seen for a given luminosity $\mathcal{L}_{\text{Integrated}}$, which is simply given by the well-known Poisson distribution, so that our new likelihood function becomes

$$L(x_{i},\tau) = \frac{\nu^{n}}{n!} e^{-\nu} \prod_{i=1}^{n} P(x_{i},\tau)$$
(7.6)

The explicit construction of this likelihood function is the topic of the next section.

7.2 The Likelihood Function for Polarized Bhabha Scattering

For the case of wide-angle Bhabha scattering, the vector of measurements consists of the scattering angle $x = \cos \theta$ and the polarization of the incident electron beam P_e , where the polarization may be negative (for left-handed events) or positive (for right-handed events). Thus, for a set of *n* independent measurements of these two observables, we have $\{(x_1, P_{e,1}), (x_2, P_{e,2}), \dots, (x_n, P_{e,n})\}$.

The model for the hypothesis of polarized wide-angle Bhabha scattering is simply the polarized differential cross section $\partial_x \sigma(x, P_e; \overline{g}_V^e, \overline{g}_A^e)$ for this set of measurements, and is given by the sum of the analytic expressions in equations (3.1) through (3.10) in Chapter 3 above. Recall that this equation describes the differential cross section for the combined process $e^+e^-_{I,R} \rightarrow Z^0$, $\gamma \rightarrow e^+e^-$, where $e^-_{I,R}$ represents an initial state electron that is either left-handed (meaning its spin is anti-parallel to its momentum vector) or right-handed (meaning its spin is parallel to its momentum vector). Therefore, the differential cross section $\partial_x \sigma(x, P_e; \overline{g}_V^e, \overline{g}_A^e)$ may be written as

$$\partial_{\mathbf{x}}\boldsymbol{\sigma}(\mathbf{x},\mathbf{P}_{e};\overline{\mathbf{g}}_{V}^{e},\overline{\mathbf{g}}_{A}^{e}) = \mathbf{p}_{L}\partial_{\mathbf{x}}\boldsymbol{\sigma}_{L}(\mathbf{x};\overline{\mathbf{g}}_{V}^{e},\overline{\mathbf{g}}_{A}^{e}) + \mathbf{p}_{R}\partial_{\mathbf{x}}\boldsymbol{\sigma}_{R}(\mathbf{x};\overline{\mathbf{g}}_{V}^{e},\overline{\mathbf{g}}_{A}^{e})$$
(7.7)

Where $p_{L,R}$ is the probability that the initial sate electron is either left-handed or right-handed, respectively. These two probabilities must be assigned to the initial state electron because the SLC experimental apparatus does not prepare single, individual electrons with a specific energy and helicity for scattering off a single, individually prepared positron with a specific energy. Instead, approximately 10^{10} electrons, known as a bunch, are made to pass through another bunch of positrons in the hope that just one electron will interact with one positron. This process is called a beam crossing and is repeated 120 times each second. For each beam crossing, the energy distribution is measured for each electron and positron bunch. Additionally, the average polarization P_e is measured for each electron bunch.

Thus, for each event produced by a beam crossing, the exact energy and helicity of the initial state electron and positron are not known, as only the bulk properties of each electron and positron bunch are known. Therefore, only the probabilities are known as to whether the electron or positron had a specific energy, and whether the electron was left or right-handed. Thus, to turn the expression for the cross section into a probability that can be used for a maximum likelihood fit, the average polarization P_e for each electron bunch must be turned into a probability that the interacting electron was either left or right-handed.

To begin, we define the electron bunch polarization to be

$$P_{e} = \frac{N_{L} - N_{R}}{N_{L} + N_{R}}$$

$$(7.8)$$

where N_L and N_R are the number of left-handed and right-handed electrons in a given bunch, respectively.

To calculate the likelihood function for the data $\{(x_1, P_{e,1}), (x_2, P_{e,2}), \dots, (x_n, P_{e,n})\}$ and for a given set of values for \overline{g}_V^e and \overline{g}_A^e , we must know the probability $P(x_i, P_{e,i}; \overline{g}_V^e, \overline{g}_A^e)$ that a given event *i* scattered at an angle $x_i = \cos \theta_i$ and was produced by an incoming beam of electrons with average polarization P_e . This probability is just the ratio of the differential wide-angle Bhabha cross section to the total wide-angle Bhabha cross section integrated over the acceptance of the SLD calorimeter

$$P(\mathbf{x}_{i}, \mathbf{P}_{e,i}; \overline{\mathbf{g}}_{V}^{e}, \overline{\mathbf{g}}_{A}^{e}) = \mathbf{p}_{L} \frac{\partial_{\mathbf{x}} \sigma_{L}(\mathbf{x}_{i}, \mathbf{P}_{e,i}; \overline{\mathbf{g}}_{V}^{e}, \overline{\mathbf{g}}_{A}^{e})}{\int_{LAC} \partial_{\mathbf{x}} \sigma_{L}(\mathbf{x}_{i}, \mathbf{P}_{e,i}; \overline{\mathbf{g}}_{V}^{e}, \overline{\mathbf{g}}_{A}^{e}) d\mathbf{x}} + \mathbf{p}_{R} \frac{\partial_{\mathbf{x}} \sigma_{R}(\mathbf{x}_{i}, \mathbf{P}_{e,i}; \overline{\mathbf{g}}_{V}^{e}, \overline{\mathbf{g}}_{A}^{e})}{\int_{LAC} \partial_{\mathbf{x}} \sigma_{R}(\mathbf{x}_{i}, \mathbf{P}_{e,i}; \overline{\mathbf{g}}_{V}^{e}, \overline{\mathbf{g}}_{A}^{e}) d\mathbf{x}}$$
(7.9)

The probabilities $p_{I,R}$ can be determined from P_e , as equation (7.8) can be written as

$$P_{e} = \frac{N_{L} - N_{R}}{N_{L} + N_{R}} = \frac{p_{L} - p_{R}}{p_{L} + p_{R}}$$
(7.10)

Since $p_L + p_R = 1$ (as they are probabilities), we can rewrite equation (7.10) as

$$P_{e} = p_{L} - (1-p_{R}) = 2p_{L} - 1$$
(7.11)

Therefore, we can write p_L and p_R in terms of P_e :

$$p_{\rm L} = \frac{1}{2} (1 + P_{\rm e})$$

$$p_{\rm R} = \frac{1}{2} (1 - P_{\rm e})$$
(7.12)

To summarize these definitions and formalism and form the actual log-likelihood function, we substitute the tree-level analytical differential cross section equations (3.1) through (3.10) into equation (3.11) along with the coefficients from Table 3-4 which gives us $\partial_x \sigma$, the differential cross section for polarized wide-angle Bhabha scattering that includes radiative corrections. The expression for $\partial_x \sigma$ includes 10 parameters defined in Table 3-2, and we substitute the values from this table into $\partial_x \sigma$ except for P_e (the initial state electron polarization) and $s = E_{CM}^2$ (the square of the center-of-mass energy), and the two parameters we fit for, \overline{g}_V^e and \overline{g}_A^e . The average values for $|P_e|$ and E_{cm} are measured experimentally by SLD for each run period and are given in Table 4-1 and Table 4-2. We substitute the appropriate values from these tables into $\partial_x \sigma$ depending on the SLD dataset we fit for. The sign of P_e is measured on an event-by-event basis and is substituted during the fitting process to be described in the next section. The average values for $|P_e|$ are also used to determine p_L and p_R using equation (7.12). The values for $\partial_x \sigma$, p_L and p_R are then substituted into equation (7.9) to give $P(x_i, P_{e_i}; \overline{g}_V^e, \overline{g}_A^e)$, the probability that a given event has $|\cos \theta| = x_i$ and signed initial state electron polarization P_{e,i}. We use $|\cos \theta| < 0.9655$ for the limits of integration for the angular acceptance of the LAC. The polarized luminosity measurements from Table 5-4 and $\int_{LAC} \partial_x \sigma$ are substituted into equation (7.5) to determine ν , the expected number of wide-angle Bhabha events. Finally the likelihood function is formed for each SLD dataset containing *n* wide-angle Bhabha events by substituting *n*, ν and $P(x_i, P_{e_i}; \overline{g}_V^e, \overline{g}_A^e)$ into equation (7.6).

7.3 <u>Fitting the Polarized Bhabha Distribution for \overline{g}_{V}^{e} and \overline{g}_{A}^{e} </u>

To minimize equation (7.6) we use RooFit[81], a C++ class library designed for minimizing and plotting multivariate probability distribution functions. RooFit is itself built on top of ROOT[82], which is also a C++ class library and is designed for large scale data analysis and provides core data analysis services such as histograms, plotting and fast access to large datasets stored in its own proprietary format. The core minimization engine provided by ROOT, and therefore RooFit, is a wrapper around the MINUIT [83] program. The results of the Maximum Likelihood fits coming from RooFit for all SLD run periods are given in Table 7-1. The systematic errors for these results are presented in section 7.4 below.

Run Period	$\overline{\mathbf{g}}_{\mathrm{v}}^{\mathrm{e}}$	$\overline{\mathbf{g}}_{\mathrm{A}}^{\mathrm{e}}$		
1994	-0.0337 ± 0.0064	-0.4901 ± 0.0026		
1995	-0.0498 ± 0.0097	-0.4803 ± 0.0039		
1996	-0.0539 ± 0.0070	-0.5245 ± 0.0028		
1997	-0.0527 ± 0.0053	-0.5038 ± 0.0020		
1998	-0.0467 ± 0.0036	-0.5054 ± 0.0014		

Table 7-1 Maximum Likelihood fit results for all SLD run periods. The errors are statistical only and derive from $\frac{1}{2}$ a unit of log-likelihood from the point of maximum likelihood.

Table 7-2 lists the number of unweighted, weighted (corrected for efficiency and contamination) and expected wide-angle Bhabha events for each SLD run period. The number of expected events v is calculated using the polarized luminosities from Table 5-4 and the integrated wide-angle Bhabha cross section used in the likelihood function.

Table 7-2 Number of unweighted, weighted (corrected for efficiency and contamination) and expected wide-angle Bhabha events for each SLD run period. The number of expected events v is calculated using the polarized luminosities from Table 5-4 and the integrated wide-angle Bhabha cross section used in the likelihood function. The fitted value for \bar{g}_{V}^{e} and \bar{g}_{A}^{e} from Table 7-1 are used in the cross section calculation.

SLD Run	Unweighted Events		Weighted Events		Expected Events	
	Left	Right	Left	Right	Left	Right
1994	2,885	2,593	3,936	3,544	3,858	3,517
1995	1,338	1,159	1,848	1,599	1,796	1,561
1996	2,603	2,209	3,757	3,192	3,500	2,933
1997	4,837	4,289	6,627	5,822	6,464	5,630
1998	11,294	10,015	15,253	13,478	14,213	12,424

The results of the fit are overlaid with the data in Figure 7-1 through Figure 7-10. The inset figures of the distribution of residuals all show a mean near zero and a standard deviation near one, as expected for a good fit. Figure 7-11 shows the wide-angle Bhabha polarized differential cross section for the data from all SLD run periods, 1994-1998, overlaid with the analytical expression for the polarized wide-angle differential cross section evaluated using the final fit results for \overline{g}_{V}^{e} and \overline{g}_{A}^{e} given in section 7.5 below.



Figure 7-1 1994 left-handed data overlaid with the theoretical model. The top plot shows the binned data with statistical error bars using LAC tower boundaries. The solid line is the analytical expression of the theoretical wide-angle Bhabha differential cross section using the results from the extended log-likelihood fit. The inset shows the residuals as a function of $|\cos \theta_{\text{Thrust}}|$. The bottom plot shows the distribution of residuals as points with associated error bars overlaid with a Gaussian fit (solid line). The results of the Gaussian fit show a mean near zero and a standard deviation near one, as expected.



Figure 7-2 1994 right-handed data overlaid with the theoretical model. The top plot shows the binned data with statistical error bars using LAC tower boundaries. The solid line is the analytical expression of the theoretical wide-angle Bhabha differential cross section using the results from the extended log-likelihood fit. The inset shows the residuals as a function of $|\cos \theta_{\text{Thrust}}|$. The bottom plot shows the distribution of residuals as points with associated error bars overlaid with a Gaussian fit (solid line). The results of the Gaussian fit show a mean near zero and a standard deviation near one, as expected.



Figure 7-3 1995 left-handed data overlaid with the theoretical model. The top plot shows the binned data with statistical error bars using LAC tower boundaries. The solid line is the analytical expression of the theoretical wide-angle Bhabha differential cross section using the results from the extended log-likelihood fit. The inset shows the residuals as a function of $|\cos \theta_{\text{Thrust}}|$ The bottom plot shows the distribution of residuals as points with associated error bars overlaid with a Gaussian fit (solid line). The results of the Gaussian fit show a mean near zero and a standard deviation near one, as expected.



Figure 7-4 1995 right-handed data overlaid with the theoretical model. The top plot shows the binned data with statistical error bars using LAC tower boundaries. The solid line is the analytical expression of the theoretical wide-angle Bhabha differential cross section using the results from the extended log-likelihood fit. The inset shows the residuals as a function of $|\cos \theta_{\text{Thrust}}|$. The bottom plot shows the distribution of residuals as points with associated error bars overlaid with a Gaussian fit (solid line). The results of the Gaussian fit show a mean near zero and a standard deviation near one, as expected.



Figure 7-5 1996 left-handed data overlaid with the theoretical model. The top plot shows the binned data with statistical error bars using LAC tower boundaries. The solid line is the analytical expression of the theoretical wide-angle Bhabha differential cross section using the results from the extended log-likelihood fit. The inset shows the residuals as a function of $|\cos \theta_{\text{Thrust}}|$. The bottom plot shows the distribution of residuals as points with associated error bars overlaid with a Gaussian fit (solid line). The results of the Gaussian fit show a mean near zero and a standard deviation near one, as expected.



Figure 7-6 1996 right-handed data overlaid with the theoretical model. The top plot shows the binned data with statistical error bars using LAC tower boundaries. The solid line is the analytical expression of the theoretical wide-angle Bhabha differential cross section using the results from the extended log-likelihood fit. The inset shows the residuals as a function of $|\cos \theta_{\text{Thrust}}|$. The bottom plot shows the distribution of residuals as points with associated error bars overlaid with a Gaussian fit (solid line). The results of the Gaussian fit show a mean near zero and a standard deviation near one, as expected.



Figure 7-7 1997 left-handed data overlaid with the theoretical model. The top plot shows the binned data with statistical error bars using LAC tower boundaries. The solid line is the analytical expression of the theoretical wide-angle Bhabha differential cross section using the results from the extended log-likelihood fit. The inset shows the residuals as a function of $|\cos \theta_{\text{Thrust}}|$. The bottom plot shows the distribution of residuals as points with associated error bars overlaid with a Gaussian fit (solid line). The results of the Gaussian fit show a mean near zero and a standard deviation near one, as expected.



Figure 7-8 1997 right-handed data overlaid with the theoretical model. The top plot shows the binned data with statistical error bars using LAC tower boundaries. The solid line is the analytical expression of the theoretical wide-angle Bhabha differential cross section using the results from the extended log-likelihood fit. The inset shows the residuals as a function of $|\cos \theta_{\text{Thrust}}|$. The bottom plot shows the distribution of residuals as points with associated error bars overlaid with a Gaussian fit (solid line). The results of the Gaussian fit show a mean near zero and a standard deviation near one, as expected.



Figure 7-9 1998 left-handed data overlaid with the theoretical model. The top plot shows the binned data with statistical error bars using LAC tower boundaries. The solid line is the analytical expression of the theoretical wide-angle Bhabha differential cross section using the results from the extended log-likelihood fit. The inset shows the residuals as a function of $\cos \theta_{\text{Thrust}}$. The bottom plot shows the distribution of residuals as points with associated error bars overlaid with a Gaussian fit (solid line). The results of the Gaussian fit show a mean near zero and a standard deviation near one, as expected.



Figure 7-10 1998 right-handed data overlaid with the theoretical model. The top plot shows the binned data with statistical error bars using LAC tower boundaries. The solid line is the analytical expression of the theoretical wide-angle Bhabha differential cross section using the results from the extended log-likelihood fit. The inset shows the residuals as a function of $\cos \theta_{\text{Thrust}}$. The bottom plot shows the distribution of residuals as points with associated error bars overlaid with a Gaussian fit (solid line). The results of the Gaussian fit show a mean near zero and a standard deviation near one, as expected.


Figure 7-11 Wide-angle Bhabha polarized differential cross section as measured by SLD using the combined result for all SLD run periods, 1994-1998. The points with error bars are corrected WAB events normalized by the luminosity as measured by the LUM. The bins are chosen along LAC tower boundaries. The top plot shows WAB events produced by left-handed electrons, while the bottom plot shows WAB events produced by righthanded electrons. The solid line is the analytical expression of the theoretical wide-angle Bhabha differential cross section using the final combined results for \overline{g}_{V}^{e} and \overline{g}_{A}^{e} , a luminosity weighted average polarization $P_{e} = -74\%$ (top plot) and $P_{e} = +0.74\%$ (bottom plot) and a luminosity weighted average center-of-mass energy $E_{cm} = 91.25$ GeV.

7.4 Systematic Errors

In this section we present a study of the systematic effects that could affect the measurement of \overline{g}_{V}^{e} and \overline{g}_{A}^{e} . Systematic effects arise due to both uncertainties in the model (due to uncertainties in the free input parameters), and uncertainties in the data. The method used to measure the systematic effects of the free parameters in equations (3.1) through (3.10) is to vary these free parameters, which are listed in Table 3-2, by the size of the uncertainty of each parameter. Additionally, uncertainties are calculated for the efficiency correction model, radiative correction model and of the various luminosity uncertainties that enter by way of equation (7.6). Each source of systematic error is described in detail.

Many of these uncertainties are different and unique for each SLD run period, so it is important to calculate them individually for each run period. In each case, the extended maximum log-likelihood fits are performed by changing each parameter by the size of its uncertainty, and the maximum change in the values of \overline{g}_{V}^{e} and \overline{g}_{A}^{e} from their central values are taken as the systematic error for that parameter. Since this procedure will result in a unique systematic error for each SLD run period, a global average systematic error is calculated as a luminosity-weighted average of the systematic errors of each SLD run period.

Although we could undertake a study to measure the correlations among each of the input parameters, we instead simply treat them as uncorrelated, which will overestimate their size. This is a reasonable approach since (as will be shown below) each of the systematic errors is small, the approach is conservative and it simplifies the analysis.

The complete list of sources of systematic error and their luminosity weighted average value contribution on the uncertainty on \overline{g}_{V}^{e} and \overline{g}_{A}^{e} are listed in Table 7-3. The following sections describe each of these systematic errors in detail.

Source of Systematic Uncertainty	Systematic Error	
Source of Systematic Uncertainty	$\Delta \overline{\mathbf{g}}_{\mathrm{v}}^{\mathrm{e}}$	$\Delta \overline{\mathbf{g}}_{a}^{e}$
Luminosity	0.00005	0.00189
Luminosity Asymmetry	0.00002	0.00001
P _e	0.00026	0.00002
E _{cm}	0.00012	0.00012
E _{cm} width	0.00003	0.00013
Z^0 mass	0.00001	0.00000
Γ_{z}	0.00002	0.00023
Efficiency	0.00009	0.00077
Radiative Correction Model	0.00024	0.00376
Total Systematic Error	0.00039	0.00429

Table 7-3 Sources of systematic errors and their luminosity weighted average contribution to the uncertainty on \bar{g}_V^c and \bar{g}_A^c .

7.4.1 Luminosity Uncertainty

The uncertainty in the luminosity comes from the error of the luminosity measurement, and enters via the Poisson term in equation (7.6).

To estimate the size of the uncertainty on the fitted values of \overline{g}_{V}^{e} and \overline{g}_{A}^{e} due to the uncertainty in the luminosity measurement, we varied the luminosity by plus and minus the size of the total systematic error listed in Table 5-5 above for all run periods and took the largest deviation of the fitted values for \overline{g}_{V}^{e} and \overline{g}_{A}^{e} as the size of the error for each run period. The luminosity weighted average systematic error due to the uncertainty in the luminosity measurement is ± 0.00005 for \overline{g}_{V}^{e} and ± 0.00189 for \overline{g}_{A}^{e} .

7.4.2 Luminosity Asymmetry

Another source of uncertainty concerning the luminosity measurement is the asymmetry in the luminosity itself between left-handed beams and right-handed beams. If such a luminosity asymmetry existed, it would induce a false left-right asymmetry that would obviously be reflected in the measurements of the coupling constants in a rather significant way. While the SLC goes to great lengths to insure that equal amounts of luminosity are delivered for both left-handed beams and right-handed beams, the luminosity asymmetry can be measured directly by the LUM using the following equation

$$A_{LR}^{LUM} = \frac{N_L^{eff} - N_R^{eff}}{N_L^{eff} + N_R^{eff}}$$
(7.13)

where N_L^{eff} and N_R^{eff} are the number of effective LUM Bhabhas produced by left and right-handed beams, respectively. From the values in Table 5-2 of the effective number of LUM Bhabhas, the luminosity asymmetries for each run period is calculated using equation (7.13) and listed in Table 7-4. From this table it is clear that the measured luminosity asymmetry is very near zero within statistical errors. Another, independent higher-statistics measurement of luminosity asymmetry was performed in which the individual 120 Hz SLC beam records were used[17, 84], which also measured a luminosity asymmetry consistent with zero.

Run Period	$\mathbf{A}_{\mathbf{LR}}^{\mathbf{LUM}}$		
1993	-0.0042 ± 0.0029		
1994	-0.0030 ± 0.0025		
1995	-0.0030 ± 0.0036		
1996	0.0054 ± 0.0028		
1997	-0.0058 ± 0.0020		
1998	0.0010 ± 0.0013		
Total	-0.0011 ± 0.0009		

Table 7-4 Luminosity asymmetry as measured by the LUM for each run period. The errors are statistical only.

Since A_{LR}^{LUM} is so small, we expect the effect on \overline{g}_{V}^{e} to be small and on \overline{g}_{A}^{e} to be negligible. An increase in the luminosity asymmetry will make the likelihood function increase \overline{g}_{V}^{e} slightly since there is no other way for the likelihood function to create an asymmetry since the number of left and right-handed WABs does not change. On the other hand, there is no reason for \overline{g}_{A}^{e} to change at all since \overline{g}_{A}^{e} is proportional to the total number of WABs, and a luminosity asymmetry does not change the total luminosity. To estimate the size of the effect, we varied the luminosity and refit for \overline{g}_{V}^{e} and \overline{g}_{A}^{e} and found that the deviation of the fitted values for \overline{g}_{V}^{e} and \overline{g}_{A}^{e} were small, as expected. We conservatively estimate the luminosity weighted average systematic error due to the luminosity asymmetry to be ± 0.00002 for \overline{g}_{V}^{e} and ± 0.00001 for \overline{g}_{A}^{e} .

7.4.3 Polarization Uncertainty

Since the polarization varies so slowly relative to the time between polarization measurements, there is a unique polarization measurement for each wide-angle Bhabha event. Therefore, it is possible in principle to incorporate the polarization measurements into the log-likelihood fits as an observable instead of a parameter (recall that only the per-event sign of the polarization for each beam crossing is used). However, the method we chose to use in the fits was to multiply the per-event sign of the polarization by the absolute value of the mean polarization for each run period as given in Table 4-1. This will result in a slightly larger error for \overline{g}_{V}^{e} than a full convolution over the polarization distribution, but still much smaller (by nearly a factor a 7) than the statistical error on \overline{g}_{V}^{e} , which is about 5%. This method is also justified since the distribution of polarization measurements are very narrow gaussians and the electron beam polarization enters only linearly in equations (3.1) through (3.10), so that polarization fluctuations above and below the central value for each run period will cancel out.

To estimate the size of the uncertainty on the fitted values of \overline{g}_{V}^{e} and \overline{g}_{A}^{e} due to the polarization uncertainty, we varied the polarization by the size of the errors in Table 4-1 for all run periods and took the largest deviation of the fitted values for \overline{g}_{V}^{e} and \overline{g}_{A}^{e} as the size of the systematic error. The luminosity weighted average systematic error due to the uncertainty in the polarization measurement is ± 0.00026 for \overline{g}_{V}^{e} and ± 0.00002 for \overline{g}_{A}^{e} . The systematic error on \overline{g}_{A}^{e} is near 0% as expected.

We should note that with such a high precision polarization measurement it is possible to perform the log-likelihood fits using the per-event polarization errors. Since the uncertainty on \overline{g}_{V}^{e} due to the polarization uncertainty is the largest systematic error in this analysis, such an approach is certainly called for if the number of events was larger, as will be the case for physics measurements at the Next Linear Collider, for example. The reason it was not done for this analysis was simply that the statistical error on the measurement of \overline{g}_{V}^{e} was so much larger than using the simpler approach, along with a desire to treat all systematic errors simply and uniformly. However, future precision measurements which rely on detectors like the Compton Polarimeter should use per-event errors in the fits, particularly with the advent of tools such as RooFit[81] which make handling per-event errors almost trivial.

7.4.4 Center-of-Mass Energy

There are two uncertainties that arise in measuring the center-of-mass energy: the average energy of each bunch and the energy profile of each bunch. Both of these affects could influence the measurements of \overline{g}_{V}^{e} and \overline{g}_{A}^{e} since the beams are tuned to collide at the peak of the Z⁰ cross section, which is a very strong and broad Breit-Wigner resonance ($\Gamma_{Z} = 2.4952 \text{ GeV}$). Therefore, any deviation of the beam energies from the Z⁰ pole will result in different parts of the Z⁰ distribution being sampled.

The average bunch energy of each electron and positron beam are each individually measured on a per beam crossing basis by the WISRD. Additionally, the finite energy width of each beam's energy profile is measured by periodically scanning a 15-micron graphite wire through a point of high dispersion in the electron beam and measuring the resulting radiation. The energy width is a very stable parameter of the SLC, so only a few energy width measurements are made during any given run.

Unfortunately, the technique we've used up to this point of refitting the data with a modified model by changing the free parameters of interest in the Born level expression given by equation (3.11) cannot be used in this case, because we have been assuming that the center-of-mass energy is fixed. It may be the case that the radiative correction coefficients encapsulated in c_1 , c_2 and c_3 have an energy dependence that would not be taken into account by simple scalar quantities. Therefore, we are not justified in using the technique described in

section 3.2 which used the UNIBAB Monte Carlo since we require a model that explicitly includes the energy dependence in the radiative correction coefficients, $c_1(E_{cm})$, $c_2(E_{cm})$ and $c_3(E_{cm})$.

We therefore use dMIBA[18], a semi-analytical Fortran 77 program that dresses the 10 lowest order Born level terms given in equations (3.1) through (3.10) with all of the important radiative corrections discussed in section 3.2. In many ways it is superior to our method of dressing the 10 lowest order Born level terms with 3 simple constants to incorporate the radiative corrections because in addition to correctly handling the center-of-mass energy, it incorporates our event selection criteria by performing numerical integrations over the phase space of our event selection cuts while still preserving the lowest order analytical expressions. The reasons we did not use dMIBA for the entire wide-angle Bhabha analysis was due to its computationally intensive nature and the appealing simplicity of our tree level method. Even though dMIBA is semi-analytical, computationally it is still an order of magnitude slower than our tree level expression with three simple constants.

The original dMIBA program did not include the effects of polarization when it was written, but due to dMIBA's semi-analytical nature it was straight forward to identify the 10 lowest order Born level terms before they were dressed with radiative corrections (in subroutine sdif) and modify them to include polarization according to equations (3.1) through (3.10).

The center-of-mass energy as measured by the WISRD is given in Table 4-2, from which it is seen that the uncertainty for all of the run periods ranges between 25 MeV and 30 MeV. To estimate the size of the uncertainty on the fitted values of \overline{g}_{V}^{e} and \overline{g}_{A}^{e} due to the center-of-mass energy uncertainty, we varied the center-of-mass energy by the size of the errors in Table 4-2 for the all run periods and took the largest deviation of the fitted values

for \overline{g}_{V}^{e} and \overline{g}_{A}^{e} as the size of the systematic error. The luminosity weighted average systematic error due to the uncertainty in the center-of-mass energy measurement is ± 0.00012 for \overline{g}_{V}^{e} and ± 0.00012 for \overline{g}_{A}^{e} .

The center-of-mass energy width for each run as measured by the wire scans are also given in Table 4-2. We are not justified to use the same technique to estimate the systematic error due to the finite center-of-mass energy width as we did for the center-of-mass energy uncertainty because the beam profile is fixed for every collision. Whereas a colliding electron and positron could have any energy within the range of energies given by the measurement of the energy uncertainty for any given beam crossing, the finite beam energy width profile only tells us the relative amounts of off energy collisions during a given run period. Therefore, the proper way to estimate the size of the uncertainty on the fitted values of \overline{g}_{V}^{e} and \overline{g}_{A}^{e} due to the center-of-mass energy width is to convolve the energy width distribution with the dMIBA p.d.f.. Although this is possible in principle, the computational time needed to perform this convolution would be enormous. We therefore take a simpler approach by simply performing our extended log-likelihood fit using dMIBA at a discrete set of energy points over the gaussian beam energy profile to estimate the convolution.

We refit the dataset for each run period using the dMIBA p.d.f. at the five different energy points given in Table 4-2. The resulting five values of \overline{g}_{V}^{e} and five values of \overline{g}_{A}^{e} as a function of E_{cm} are then each fit to a cubic polynomial which is then convolved with a gaussian with a mean and standard deviation equal to the E_{cm} and $E_{cm} + E_{cm}^{width}$, respectively, for the given run period. The results for the 1997 dataset are shown in Figure 7-12.



Figure 7-12 dMIBA is used to refit values of \overline{g}_{V}^{e} and \overline{g}_{A}^{e} for five different center-of-mass energies: E_{cm}^{0} , $E_{cm}^{0} \pm \delta E_{cm}$ and $E_{cm}^{0} \pm E_{cm}^{width}$. These new values of \overline{g}_{V}^{e} and \overline{g}_{A}^{e} are then fit to a cubic polynomial which is then convolved with a gaussian with $\mu = E_{cm}^{0}$ and $\sigma = E_{cm}^{width}$. The E_{cm} data used is from the 1997 run period, and our results are representative of the entire SLD dataset.

The results of these convolutions for each run period are shown in Table 7-5.

Convolutions with E_{cm} Distributions			
Run Period	$\overline{\mathbf{g}}_{\mathbf{v}}^{\mathbf{e}}$	$\overline{\mathbf{g}}_{\mathbf{A}}^{\mathbf{e}}$	
1994	-0.03410	-0.49806	
1995	-0.05031	-0.48800	
1996	-0.05445	-0.53235	
1997	-0.05285	-0.51065	
1998	-0.04693	-0.51246	

Table 7-5 Results of convoluting $\overline{g}_{V}^{e}(E_{cm})$ and $\overline{g}_{A}^{e}(E_{cm})$ with the gaussian E_{cm} width distribution for each run period.

In principle, we should convolve all of our fits with the E_{cm} distribution in this manner (some 92 independent fits in all). However, using the convolution method just described would require using dMIBA for all of the fits, totaling $92 \times 5 = 460$ fits, which is simply an unreasonable amount of CPU time with the current technology²⁷. Instead, we use the results in Table 7-5 above to correct the final fit results listed in Table 7-1. These corrections are listed in Table 7-6.

²⁷ All data analysis, including the log-likelihood fits, were performed on a Dell Latitude C400 laptop (1 Pentium III CPU clocking at 1.2 GHz) running Microsoft Windows XP Professional.

Table 7-6 Corrections to be applied to \overline{g}_{V}^{e} and \overline{g}_{A}^{e} to account for finite E_{cm} width.

\mathbf{E}_{cm} Width Corrections				
Run Period	$\Delta \overline{\mathbf{g}}_{\mathbf{V}}^{\mathbf{e}}$	$\Delta \overline{\mathbf{g}}_{\mathbf{A}}^{\mathbf{e}}$		
1994	-0.00033	0.00019		
1995	0.00002	0.00017		
1996	0.00001	0.00020		
1997	-0.00019	0.00023		
1998	0.00003	0.00033		

We take half the size of the corrections as the systematic error due to the finite energy width of the beams. This conservative technique will only slightly overestimate the systematic error since we are treating the results of the convolution fits as uncorrelated with the non-convoluted fits. The luminosity weighted average systematic error due to the finite energy width of the beams is ± 0.00003 for \overline{g}_{V}^{e} and ± 0.00013 for \overline{g}_{A}^{e} .

7.4.5 Z⁰ Mass Uncertainty

The Z⁰ mass and its uncertainty come from the LEP Z⁰ line shape fit[7]. To estimate the size of the uncertainty on the fitted values of \overline{g}_{V}^{e} and \overline{g}_{A}^{e} due to the uncertainty of the Z⁰ mass, we varied the Z⁰ mass by ±2.1 MeV for all run periods and took the largest deviation of the fitted values for \overline{g}_{V}^{e} and \overline{g}_{A}^{e} as the size of the systematic error. The luminosity weighted average systematic error due to the uncertainty in the Z⁰ mass is ±0.00001 for \overline{g}_{V}^{e} and ±0.00000 for \overline{g}_{A}^{e} .

7.4.6 Z^0 Width Uncertainty

The Z⁰ width and its uncertainty come from the LEP Z⁰ line shape fit[7]. To estimate the size of the uncertainty on the fitted values of \overline{g}_{V}^{e} and \overline{g}_{A}^{e} due to the uncertainty of the Z⁰ width, we varied the Z⁰ width by ±2.3 MeV for all run periods and took the largest deviation of the fitted values for \overline{g}_{V}^{e} and \overline{g}_{A}^{e} as the size of the systematic error. The luminosity weighted average systematic error due to the uncertainty in the Z⁰ width is ±0.00002 for \overline{g}_{V}^{e} and ±0.00023 for \overline{g}_{A}^{e} .

7.4.7 Radiative Correction Model

To estimate the systematic error due to modeling the radiative corrections presented in section 3.2, the value of \overline{g}_{V}^{e} and \overline{g}_{A}^{e} in Table 7-1 for the Maximum Likelihood fit of the 1997 SLD dataset (the beam parameters of which were used to generate the UNIBAB dataset) were used to recalculate new values of the radiative correction coefficients, the results of which were $c_{1} = -0.0581$, $c_{2} = 0.7180$ and $c_{3} = 0.8597$. These new values of the radiative correction coefficients were then used to refit the SLD datasets for all run periods, and we took the largest deviation of the fitted values for \overline{g}_{V}^{e} and \overline{g}_{A}^{e} as the size of the systematic error. The luminosity weighted average systematic error due to the uncertainty in modeling the radiative corrections is ± 0.00024 for \overline{g}_{V}^{e} and ± 0.00376 for \overline{g}_{A}^{e} .

7.4.8 Efficiency Correction

To estimate the size of the uncertainty on the fitted values of \overline{g}_{V}^{e} and \overline{g}_{A}^{e} due to the uncertainties in the Pseudo Event efficiency modeling technique discussed in section 6.5.1

above, the value of each correction factor was randomly varied by ± 1 standard deviation of the binomial error for each calorimeter tower, and these new values were then used to refit the SLD datasets from all run periods. This approach was performed multiple times for all run periods, and we took the largest deviation of the fitted values for \overline{g}_{V}^{e} and \overline{g}_{A}^{e} as the size of the systematic error. The luminosity weighted average systematic error due to the uncertainties in the Pseudo Event efficiency modeling technique is ± 0.00009 for \overline{g}_{V}^{e} and ± 0.00077 for \overline{g}_{A}^{e} .

7.4.9 Systematic Error Summary

The systematic errors are summarized in Table 7-3.

The largest systematic error for \overline{g}_{V}^{e} , contributing 0.56% to the total systematic error, is due to the uncertainty in the measured value of the polarization. This is understandable as \overline{g}_{V}^{e} is such a sensitive function of the polarization. The SLD Polarimeter has been well modeled to arrive at this error estimation. The next largest contribution for the uncertainty in \overline{g}_{V}^{e} is the uncertainty in the model of the radiative corrections, which contributes 0.50% to the total systematic uncertainty. It might be possible to reduce this error by using dMIBA to perform all of the fits, since then the radiative corrections to the tree-level diagrams are handled explicitly. However, this would require more computational resources.

The largest systematic error for $\overline{g}_{\Lambda}^{e}$, contributing 0.75% to the total systematic uncertainty, is again due to the uncertainty in the model of the radiative corrections. This constitutes 88% of the total systematic error. Clearly, this analysis could benefit from using dMIBA to perform all of the fits. The next largest systematic error comes in at a distant second and is due to the uncertainty in the luminosity measurement, which contributes 0.38% to the total systematic error.

7.5 Final Measurement of \overline{g}_{V}^{e} and \overline{g}_{A}^{e}

The final results for \overline{g}_V^e and \overline{g}_A^e are calculated by applying the corrections due to the finite E_{cm} width in Table 7-6 to the fit results in Table 7-1. By incorporating the systematic errors listed in Table 7-3, our results are

$$\overline{g}_{V}^{e} = -0.0469 \pm 0.0024 \text{ (stat.)} \pm 0.0004 \text{ (sys.)}$$

 $\overline{g}_{A}^{e} = -0.5038 \pm 0.0010 \text{ (stat.)} \pm 0.0043 \text{ (sys.)}$

7.6 Comparison to the Standard Model

Our results for \overline{g}_{V}^{e} and \overline{g}_{A}^{e} from section 7.5 above may be compared to the Standard Model predictions calculated by ZFITTER (see section 3.2 above) of $\overline{g}_{V}^{e} = -0.03657$ and $\overline{g}_{A}^{e} = -0.50134$, which were calculated using $M_{top} = 175$ GeV and $M_{H} = 150$ GeV. A useful way to compare our measurement to the Standard Model is to define an angular dependent version of A_{LR} for wide-angle Bhabha scattering:

$$A_{LR}^{e^+e^-} \left(\left| \cos \theta \right| \right) = \frac{\partial_{\left| \cos \theta \right|} \sigma_L - \partial_{\left| \cos \theta \right|} \sigma_R}{\partial_{\left| \cos \theta \right|} \sigma_L + \partial_{\left| \cos \theta \right|} \sigma_R}$$
(7.14)

where $\partial_{|\cos\theta|}\sigma_{L,R}$ is the differential cross section for wide-angle Bhabha scattering for left (L) and right (R) handed initial state electrons. Equation (7.14) is analogous to equation (2.13) but uses the wide-angle Bhabha differential cross section in place of σ .

Figure 7-13 is a plot of $A_{LR}^{e^+e^-}(|\cos\theta_{Thrust}|)$ vs. $\cos\theta_{Thrust}$ for the entire 1994-1998 WAB dataset, and is overlaid with two curves of $A_{LR}^{e^+e^-}(|\cos\theta|)$ using two different values of \overline{g}_V^e and \overline{g}_A^e . The upper curve uses our final measurement of \overline{g}_V^e and \overline{g}_A^e from section 7.5 above, and the lower dashed curve uses the values of \overline{g}_V^e and \overline{g}_A^e calculated by ZFITTER. The data were binned using KAL tower boundaries and scaled using the luminosity weighted polarization $P_e = 74\%$ according to equation (2.14).



Figure 7-13 $A_{LR}^{e^+e^-}(|\cos\theta_{Thrust}|)$ vs. $|\cos\theta_{Thrust}|$ for the entire 1994-1998 WAB dataset. The upper solid curve uses our final results of $\overline{g}_V^e = -0.0469$ and $\overline{g}_A^e = -0.5038$. The lower dashed curve uses Standard Model predictions from ZFITTER of $\overline{g}_V^e = -0.03657$ and $\overline{g}_A^e = -0.50134$.

Figure 7-14 shows the residual distributions for each curve overlaid with a fit to a gaussian. The middle plot in the figure shows the residual distribution for our final result, which shows excellent agreement with the data. The lower plot in the figure is the residual distribution for the Standard Model prediction, which is inconsistent with the data.



Figure 7-14 Residual distributions for Figure 7-13. The top plot shows the distribution of residuals for our final result, which is fit to a gaussian and shows good agreement with the data, having a mean consistent with zero and a standard deviation near one. The bottom plot shows the residual distribution for the Standard Model prediction, which is inconsistent with the data.

CHAPTER 8 CONCLUSION

This dissertation presented a measurement of the two Z^0 coupling parameters to the electron based on polarized wide-angle Bhabha scattered events ($e^+e^- \rightarrow e^+e^-$) from the 1994-1998 SLD datasets. We developed a Maximum Likelihood fitting technique which allowed the use of all Bhabha scattered events from the full angular acceptance of SLD's calorimeters in a natural way, including the large-angle region where the Z^0 resonance dominates and the small-angle region used for luminosity measurements where t-channel photon exchange dominates.

We measure

$$\overline{g}_{V}^{e} = -0.0469 \pm 0.0024 \text{ (stat.)} \pm 0.0004 \text{ (sys.)}$$
$$\overline{g}_{A}^{e} = -0.5038 \pm 0.0010 \text{ (stat.)} \pm 0.0043 \text{ (sys.)}$$

which, using equation (2.11), represents a measurement of the effective weak mixing angle of

$$\sin^2 \theta_{\rm W}^{\rm eff} = 0.2267 \pm 0.0012 (\text{stat.}) \pm 0.0003 (\text{sys.})$$

The measurement uncertainty of \overline{g}_{A}^{e} is limited by the 0.85% systematic error, which is itself dominated by the uncertainty in the method used to model the radiative corrections (which contribute 0.75% to the total systematic error) and the uncertainty introduced by the luminosity measurement (which contributes 0.38% to the total systematic error).

We also measured the luminosity for the 1993-1998 SLD run period to be

$$\mathcal{L} = 19,247 \pm 17$$
(stat.) ± 146 (sys.) nb⁻¹

This measurement is limited by the 0.76% systematic error, which is composed of 0.70% experimental error and 0.30% theoretical uncertainty. This level of precision is significantly better than the design goal of 3%[43].

The LEP experiments do not have polarized electron beams, but measure \overline{g}_{V}^{e} and \overline{g}_{A}^{e} using a different technique by combining the lepton forward-backward asymmetries, tau polarization and the electron partial width Γ_{ee} to yield a measurement of $\overline{g}_{V}^{e} = -0.0378 \pm 0.0011$ and $\overline{g}_{A}^{e} = -0.50112 \pm 0.00035$ [7]. Our measurement of \overline{g}_{A}^{e} agrees with LEP, but our result for \overline{g}_{V}^{e} differs by over three standard deviations.

Our measurement of \overline{g}_{A}^{e} agrees with the Standard Model, but our measurement of \overline{g}_{V}^{e} differs from the Standard Model by four standard deviations.

APPENDIX A SLD COLLABORATION

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