Physical Elements

Symbolic Constants

TRANSPORT recognizes the mathematical and physical constants listed in the table below. The use of recognizably named constants is included for compatability with the MAD program. The list of constants is essentially the same as is found in the MAD program. Users should avoid using the names of such symbolic constants for any other purpose. The use of a symbolic constant name for a user-defined label will cause the user definition to supersede the symbolic constant meaning. Further use as a symbolic constant may then be inconsistent with the new meaning and cause unpredictable results. However, if a symbolic constant is redefined in the data as a parameter with the same value (next page), there will be no problem. Decks of data based on versions of the program before the inclusion of symbolic constants may have such defined parameters.

The symbolic constants recognized are:

Symbol	Name	Value Used	Unit
π	PI	3.14159265358979323	
2π	TWOPI	2.0*π	
$180/\pi$	DEGRAD	180.0/π	
$\pi/180$	RADDEG	π/180.0	
e	E	2.718281828459045	
m_e	EMASS	$.510099906 * 10^{-3}$	GeV
m_p	PMASS	.93827231	GeV
c	CLIGHT	$2.99792458 * 10^{8}$	m/s
	MTOIN	.0254	
	INTOM	1.0/MTOIN	

Further constants can be defined using the parameter statement defined below.

PARAMETERS

The PARAMETER element is taken from the MAD program. There is no positional or original style TRANSPORT input.

A numerical value to be used many times over in the definition of elements can be specified a single time by the use of a parameter statement. The parameter is given a proper name via the parameter element. It is then referred to by that proper name on all other elements. For example, a drift length LD1 which is to be .5 meters can be defined by

```
LD1:  = .5 ;
```

or alternatively

```
PARAM LD1 = 0.5 ;
```

In defining a drift space D1 whose length is LD1, one can write

```
D1:  DRIFT, L = LD1 ;
```

It is also possible to define a parameter by its use on a given element. On a later element it is represented by the label of the first element followed by the keyword of the parameter in brackets. The length of the drift space D1 may now be given directly on the drift specification as:

```
D1:  DRIFT, L = 0.5 ;
```

A second drift space, whose length is always to be the same as that of the first can be given as

```
D2:  DRIFT, L = D1[L] ;
```

The symbol D1 after the equals sign indicates that the length of element D2 is to be taken from element D1. The L in the brackets indicates that the parameter taken from D1 is the length.

A parameter may be varied as part of a fitting problem. Details are given on page 259.

Algebraic Expressions

Any algebraic expression made up from previously defined parameters can also serve as a parameter. This algebraic expression can be evaluated either on another PARAMETER element, or as one of the physical parameters of a physical element. In the example above, a second drift space can be defined in terms of the parameter LD1

```
D2:  DRIFT, L = 10.0 - LD1 ;
```

The value of the parameter LD1 may be varied, but the sum of the two drift lengths will be held fixed at 10.0 meters. The operations which may be used in the algebraic expression are:

+	Addition
-	Subtraction
*	Multiplication
/	Division
SQRT	Square root
ALOG	Natural logarithm (written Ln in engineering notation).
EXP	Exponential function
SIN	Sine function
COS	Cosine function
SINH	Hyperbolic sine function
COSH	Hyperbolic cosine function
ASIN	Inverse sine function
ACOS	Inverse cosine function

For example, the lengths of three drift spaces in a system can be made to increase in geometric progression. The first two drift spaces D1 and D2 can be made free parameters and given as:

```
D1:  DRIFT, L = LD1 ;
D2:  DRIFT, L = LD2 ;
```

A third drift space can be defined as

```
D3:  DRIFT, L = LD2*LD2/LD1
```

DRIFT

A drift space is a field-free region through which the beam passes. It is specified by a single parameter, which is its length. The length may be varied in first-, second-, or third-order fitting. The drift space is a MAD compatible element and can be expressed in either MAD notation or original TRANSPORT notation.

MAD Notation

A single keyword is used in specifying a drift space. It is `L` which indicates the length of the drift space. A specification for a drift space `D1` of length 6.0 meters is written as

```
D1:  DRIFT,  L = 6.  ;
```

Original TRANSPORT Notation

There are two parameters:

1. `DRIF(T)` or type code `3.0` (specifying a drift length).
2. (Effective) drift length (meters).

Typical input format for a `DRIFT`:

```
                          Label (if desired)
                          (not to exceed fifteen
                          spaces between quotes).
DRIFT   6.   'D1'   ;
```

Drift Space Matrix

The first-order R matrix for a drift space is as follows:

$$\begin{bmatrix} 1 & L & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & L & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where

$$L = \text{the length of the drift space} .$$

The dimensions of L are those chosen for longitudinal length. In any of the global units specifications (**UMAD**, **UMETER**, **UTRANS**, etc.) this unit is meters. An alternate longitudinal unit may be chosen via a

```
                    ┌──────── units symbol
                    │   ┌──────── scale factor (if needed)
                    ↓   ↓
UNIT  8.        '  '       ;
```

element preceding the **BEAM** element. Details about unit changes can be found on page 75.

For a normal drift space, the first-order transformation is the <u>exact</u> solution of the equations of motion. There are no higher-order terms.

A normal drift space is one which begins and ends at a plane which is perpendicular to the reference trajectory. It is sometimes desirable to have a drift space end at a plane which makes a different angle to the reference trajectory. One example might be the focal plane of a dispersive focusing system. The foci at the different momenta are spread out transversely because of the dispersion and longitudinally because of the chromatic aberration of the focusing devices (quadrupoles or their optical equivalents). A drift space can be specified to end at such a tilted focal plane. For details of implementation, see page 86.

BEND – Sector Bending Magnet

A sector bending magnet implies that the central trajectory of the beam enters and exists perpendicularly to the pole-face boundaries. In previous editions of this manual, the term "wedge" has been used to describe such a bending magnet. There is no intended distinction between the terms "sector" and "wedge". The term "sector" has now been adopted for reasons of compatability with MAD, especially with the MAD element SBEND, meaning sector bend.

The BEND element is used to represent the interior field of a bending magnet. No fringe field effects are included in the BEND element. It is especially useful when segmenting a magnet longitudinally to determine the trajectory transformation at interior points. To include fringing-field effects and non-perpendicular entrance or exit boundaries with the bend element see the ROTATE and SPECIAL elements. The ROTATE element (see page 139) represents the fringe field itself. The SPECIAL element (see page 79) may be used to define certain parameters used to describe the fringe field.

First-, second, and third-order matrix elements can be calculated by TRANSPORT for the BEND element. The order actually calculated will depend on the ORDER command as described on page 91. With no ORDER element the first-order transformation will be calculated. The number of parameters necessary to specify a BEND element depends on the order of calculation requested.

The element BEND is not found in MAD, but can be described by the same set of parameters as is used for the elements RBEND and SBEND in MAD. It can also be described by several other sets of parameters as well as in original TRANSPORT notation.

To represent a bending magnet complete with fringing fields as a single element one should use the RBEND or SBEND elements described on page 144. A more complete description of the set of parameters used to represent the magnetic field is given on page 2. Some of this description is repeated here for the user's convenience.

MAD-like Notation

The BEND element, without fringing fields, is not found in the MAD program. It is, however, a TRANSPORT element which can be described in original TRANSPORT notation. The parameterization of it, described here as MAD-like, has been implemented for consistency with the RBEND and SBEND elements. To first order, a BEND element is specified by three real attributes.

```
label: BEND, L = value , ANGLE = value , K1 = value;
```

A sample BEND element might look like:

```
BR: BEND, L = 10., ANGLE = 0.17453, K1 = 0.5E-4 ;
```

The bend angle shown is 0.17453 radians or 10 degrees. The value in radians is shown since radians are the standard MAD units for bending angle. The use of MAD units is indicated by the choice of the global unit set UMAD. The same element, but with the bending angle expressed in degrees would be:

```
BR:  BEND,  L = 10.,  ANGLE = 10.,  K1 = 0.5E-4  ;
```

The bend angle is expressed in degrees in the UTRANS data set. Other global unit sets are described on page 71

The MAD-like keywords which can be used in specifying a sector bending magnet are

Symbol	Keyword	Description
L	L	The magnet length (normally meters)
α	ANGLE	The angle through which the reference trajectory is bent (degrees with unit set UTRANS, radians with UMAD, default 0.0).
K_1	K1	The quadrupole coefficient (normally meters^{-2}, default 0.0), defined by

$$K_1 = \frac{1}{B\rho}\,\frac{\partial B_y}{\partial x}$$

Symbol	Keyword	Description
K_2	K2	The sextupole coefficient (normally meters^{-3}, default 0.0), defined as

$$K_2 = \frac{1}{2B\rho}\,\frac{\partial^2 B_y}{\partial x^2}$$

Symbol	Keyword	Description
K_3	K3	The octupole coefficient (normally meters^{-4}, default 0.0), defined as

$$K_3 = \frac{1}{6B\rho}\,\frac{\partial^3 B_y}{\partial x^3}$$

In terms of these parameters, the field expansion on the magnetic midplane is given by

$$B_y = (B_o\rho)\sum K_n x^n$$

$$B_x = 0$$

The definition of the multipole components α, K_1, K_2, and K_3 allows the orbit or a particle to be computed without knowledge of its momentum. The normalization of K_1, K_2, and K_3 is such that a multipole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory. In the

MAD program itself the values of K_1, K_2, and K_3 are defined as the x derivatives of the field divided by $B_0\rho$. The value of K_2 in MAD is then a factor of 2 greater than the value defined above. The MAD value of K_3 is a factor of 6 (= 3!) greater than that used here. When the global unit set UMAD is used, the MAD definition of the multipoles is also used.

The units for any of the parameters may be changed as desired. The units of the parameters K_1, K_2, and K_3 are in inverse powers of the unit of longitudinal length. Global unit sets are described on page 71. Individual units changes are described on page 75.

The parameters L, α, and K_1 may be varied in first-order fitting. The parameter K_2 may be varied in second-order fitting. The parameter K_3 may be varied in third-order fitting. The VARY command is described on page 259 in the section on the FIT element.

Other Keyword Options

To allow keyword representation of data in the original TRANSPORT variables and to permit other options for specifying a bending magnet, the following keywords are also recognized. They may be used instead of or combined with the MAD-like keywords.

Symbol	Keyword	Description
B	B	The magnetic field (normally kilogauss)
ρ	RADIUS	The radius of curvature of the reference trajectory (normally meters).
n	N	The normalized field derivative (dimensionless).
ϵ	EPS	The quadratic field dependence (in inverse squared units of horizontal beam width – meters in UMAD, cm in UTRANS.
ϵ_3	EPS3	The cubic field dependence (in inverse cubed units of horizontal beam width).

The complete set of variables available for defining a bending magnet contains a great deal of redundacy. The bending magnet, used as an example earlier, may be described to first order in any of ten ways. We assume the beam momentum to be 600 GeV/c. The following five lines are then all equivalent.

```
BEND,  L = 10.,  B = 20.,  N = 0.5 ;

BEND,  L = 10.,  ANGLE = 10.,  N = 0.5 ;

BEND,  L = 10.,  RADIUS = 1000.,  N = 0.5 ;

BEND,  B = 20.,  ANGLE = 10.,  N = 0.5 ;

BEND,  RADIUS = 1000.,  ANGLE = 10.,  N = 0.5 ;
```

Here the angle is assumed to be in milliradians. The angular unit specified by the global unit set UMAD is radians. The angular unit specified by UTRANS, is degrees. Other units may be specified by use of an alternate global units specification (UMETER, UMM, etc.) or a UNITS element.

In addition the field gradient may be specified by K_1 instead of n. The two are related by

$$K_1 = -\frac{n}{\rho^2} = \frac{1}{B\rho}\frac{\partial B_y}{\partial x}$$

The expansion of the magnetic field on the magnetic midplane is given by

$$B_y = B_o(1 + r_s)(1 - nhx + \beta\, h^2x^2 + \gamma\, h^3x^3 + \ldots)$$

$$B_x = 0$$

The sextupole component β is related to the coefficients ϵ and K_2 by

$$K_2 = \frac{\epsilon}{\rho} = \frac{\beta}{\rho^3} = \frac{1}{2B\rho}\frac{\partial^2 B_y}{\partial x^2}$$

Only one of the two quantities ϵ and K_2 should be specified. If neither is specified, the sextupole component is set equal to the default value given by the special parameters (SPEC element). If the default value is also unspecified, it is taken to be zero.

The octupole component γ is related to the cubic variation of the magnetic field by.

$$K_3 = \frac{\epsilon_3}{\rho} = \frac{\gamma}{\rho^4} = \frac{1}{6B\rho}\frac{\partial^3 B_y}{\partial x^3}$$

The default value for the octupole component ϵ_3 is also set by the special parameters (SPEC element). Otherwise it is taken to be zero.

Any of the first-order parameters L, B, ρ, α, and n can be varied in first-order fitting. The parameter ϵ can be varied in second-order fitting. The parameter ϵ_3 can be varied in third-order fitting.

Mispowering and Violations of Midplane Symmetry

The vertical field in a bending magnet along the reference trajectory may be mispowered so that the reference particle will be bent away from the reference trajectory. This can happen simply by increasing the actual field in the magnet without changing the reference field. A physical bending magnet may also have a horizontal field component on what is taken as the magnetic midplane. A set of parameters to describe both the effects of excess bend field and violation of midplane symmetry follows. The multipole components which violate midplane symmetry are also known as skew multipoles. For a discussion of the magnetic field components, see page 2.

Some of these parameters are MAD-like, and should be used if the midplane-symmetric field is given a MAD-like parameterization. Specifically the parameters K_1' and K_2' are MAD-like. The parameters n' and ϵ' are traditional- TRANSPORT-like. The two scaling parameters r_s and r_a can be used with either representation.

Symbol	Keyword	Description
r_s	RMPS	The fractional excess magnetic field (unitless).
r_a	RNMS	Scaling factor for the skew field components (unitless).
v_r	VR	The normalized skew dipole magnetic field (unitless).
K_1'	K1P	The skew quadrupole coefficient (normally meters^{-2}).
K_2'	K2P	The skew sextupole coefficient (normally meters^{-3}).
n'	NP	The midplane-symmetry-violating normalized field derivative (dimensionless).
ϵ'	EPSP	The midplane-symmetry-violating quadratic field dependence (same units as EPS).

In terms of these new parameters, the field expansion on the nominal magnetic midplane is given by

$$B_y = (B_o\rho)(1 + r_s) \sum K_n x^n$$

$$B_x = (B_o\rho) r_a \sum K_n' x^n$$

where

$$B_0 v_r = (B_0\rho) K_0'$$

or

$$B_y = B_o(1 + r_s)(1 - nhx + \beta\ h^2x^2 + \gamma\ h^3x^3 + \ldots)$$

$$B_x = B_o r_a (v_r - n'hx + \beta' h^2 x^2 + \ldots)$$

The quantity r_s represents the fractional excess field. The reference trajectory is defined as if r_s equalled zero. The field of the magnet may then be increased or decreased to steer the beam about the reference trajectory. The quantity r_a exists only to facilitate simultaneous scaling of all midplane symmetry violating multipoles.

If r_s and r_a are not specified for a particular BEND element, they are set equal to the default values given on the special parameter element (SPEC element – see page 89). If the default values are also unspecified, they are taken to be zero.

If the magnet in the example just above is mispowered so that the field is excessively strong by 1%, it might be specified as:

```
BR:  BEND,  L = 10.,  ANGLE = 10.,  K1 = 0.5E-4,  RMPS = 0.01  ;
```

If there is a skew quadrupole component which is whose strength is 1% that of the midplane symmetric quadrupole component, the bend magnet could be described as:

```
BR: BEND, L = 10., ANGLE = 10., K1 = 0.5E-4, RNMS = 0.01, K1P = 0.5E-4 ;
```

or as:

```
BR: BEND, L = 10., ANGLE = 10., K1 = 0.5E-4, RNMS = 1.0, K1P = 0.5E-6  ;
```

Any of the parameters, r_s, r_a, v_r, K_1', K_2', n', and β' may be set on the individual bend element, or left to equal the default value. The default values may be set on the special parameter element. Otherwise they are all zero. The parameters K_1' and K_2' are related to n', β', and ϵ' respectively in the same way that K_1 and K_2 are related to n, β, and ϵ.

Any of the first-order parameters r_s, r_a, v_r, K_1', and n' may be varied in first-order fitting. The parameters K_2' and ϵ' can be varied in second-order fitting. There are no third-order midplane-symmetry-violating transfer matrix elements in TRANSPORT.

Original TRANSPORT Notation

There are four first-order parameters to be specified for the sector magnet:

1. `BEND` (or type code 4 (specifying a sector bending magnet)).

2. The (effective) length L of the central trajectory (normally in meters).

3. The central field strength $B(0)$ in kG,

   $$B(0) = 33.35641(p/\rho_0) \quad ,$$

   where p is the momentum in GeV/c and ρ_0 is the bending radius of the central trajectory in meters.

4. The field gradient (n-value, dimensionless); where n is defined by the equation

   $$B_y(x,0,t) = B_y(0,0,t)(1 - nhx + \ldots) \quad ,$$

where

$$h = 1/\rho_0 \ . \quad ^*$$

The standard units for L and B are meters and kG. If desired, these units may be changed by `UNIT` elements preceding the `BEAM` element. The quantities L, $B(0)$, and n may be varied for first-order fitting (see the `FIT` element for a discussion of vary codes).

The bend radius in meters and the bend angle in degrees are printed in the output.

A typical first-order TRANSPORT input for a sector magnet is

Label (not to exceed fifteen spaces)

BEND L B n ' ' ;

If n is not included in the data entry, the program assumes it to be zero. The example used in the previous sections, now in positional notation, would look like:

```
BEND  10.   20.   0.5  ;
```

Even in positional notation additional parameters may be specified for a bending magnet in keyword notation. For example, a quadratic dependence of the field of the above bend magnet can be given by adding the `EPS` parameter to the element to get:

BEND L B n, EPS = 1.E-4 ;

Fringing Fields

If fringing field effects are to be included, a `ROTAT` element must immediately precede and follow the pertinent `BEND` element (even if there are no pole-face rotations). Thus a typical TRANSPORT input for a bending magnet including fringing fields might be:

Labels (not to exceed fifteen spaces) if desired

ROTAT. 0. ' ' ;
BEND L $B(0)$ n ' ' ;
ROTAT 0. ' ' ;

For non-zero pole-face rotations a typical data input might be

ROTAT 10. ; BEND L $B(0)$ n ; ROTAT 20. ;

*See SLAC-75 [4] (page 31).

Note that the use of labels is optional and that all data entries may be made on one line if desired.

Direction of Bend

The sign conventions for bending magnet entries are illustrated in the following figure. For TRANSPORT a positive bend is to the right looking in the direction of particle travel. To represent a bend in another sense, the coordinate rotation matrix (SROT element) should be used as follows:

A bend up is represented by a rotation of the (x,y) coordinates by -90.0 degrees about the (z) beam axis as follows:

Labels (not to exceed fifteen spaces) if desired

```
SROT   -90.  ' ' ;
ROTATE  β(1) ' ' ;
BEND  L  B  n ' ' ;
ROTATE  β(2) ' ' ;
SROT   +90.  ' ' ; (returns coordinates to their initial orientation)
```

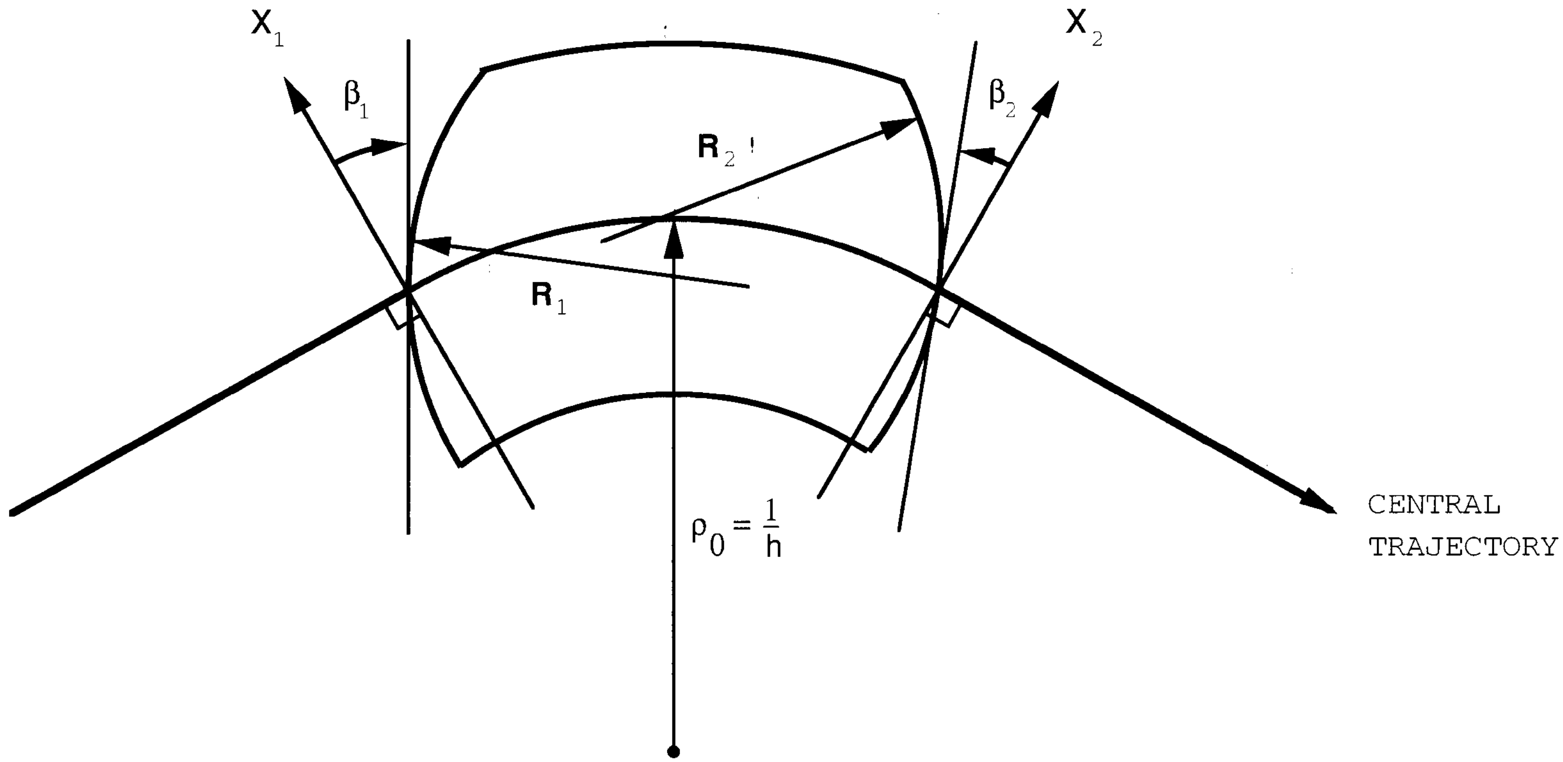
A bend down is accomplished via:

```
SROT   +90.  ' ' ;
ROTATE
BEND
ROTATE
SROT   -90.  ' ' ;
```

A bend to the left (looking in the direction of beam travel) is accomplished by rotating the x,y coordinates by 180 degrees, e.g.

```
SROT   +180.  ' ' ;
ROTATE
BEND
ROTATE
SROT   -180.  ' ' ;
```

The SROT need be used only with the BEND element. For the RBEND and SBEND elements the coordinate rotation may be specified on the element itself as the ROLL parameter.



Field Boundaries for Bending Magnets

The TRANSPORT sign conventions for x, β, R and h are all positive as shown in the figure. The positive y direction is out of the paper. Positive β's imply transverse focusing. Positive R's (convex curvatures) represent negative sextupole components of strength $S = (-h/2R)\ sec^3\beta$. (See SLAC-75 [4], page 71 or SLAC-PUB-3381 [11], page 22.)

First-Order Sector Bending Magnet Matrix

$$
\begin{bmatrix}
\cos k_x L & \frac{1}{k_x}\sin k_x L & 0 & 0 & 0 & \frac{h}{k_x^2}(1-\cos k_x L) \\
-k_q \sin k_x L & \cos k_x L & 0 & 0 & 0 & \frac{h}{k_x}\sin k_x L \\
0 & 0 & \cos k_y L & \frac{1}{k_y}\sin k_y L & 0 & 0 \\
0 & 0 & -k_y \sin k_y L & \cos k_y L & 0 & 0 \\
\frac{h}{k_x}\sin k_x L & \frac{h}{k_x^2}(1-\cos k_x L) & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
$$

Definitions: $h = 1/\rho$, $k_x^2 = (1-n)h^2$, $k_y^2 = nh^2$
$\alpha = hL =$ the angle of bend
$L =$ path length of the central trajectory

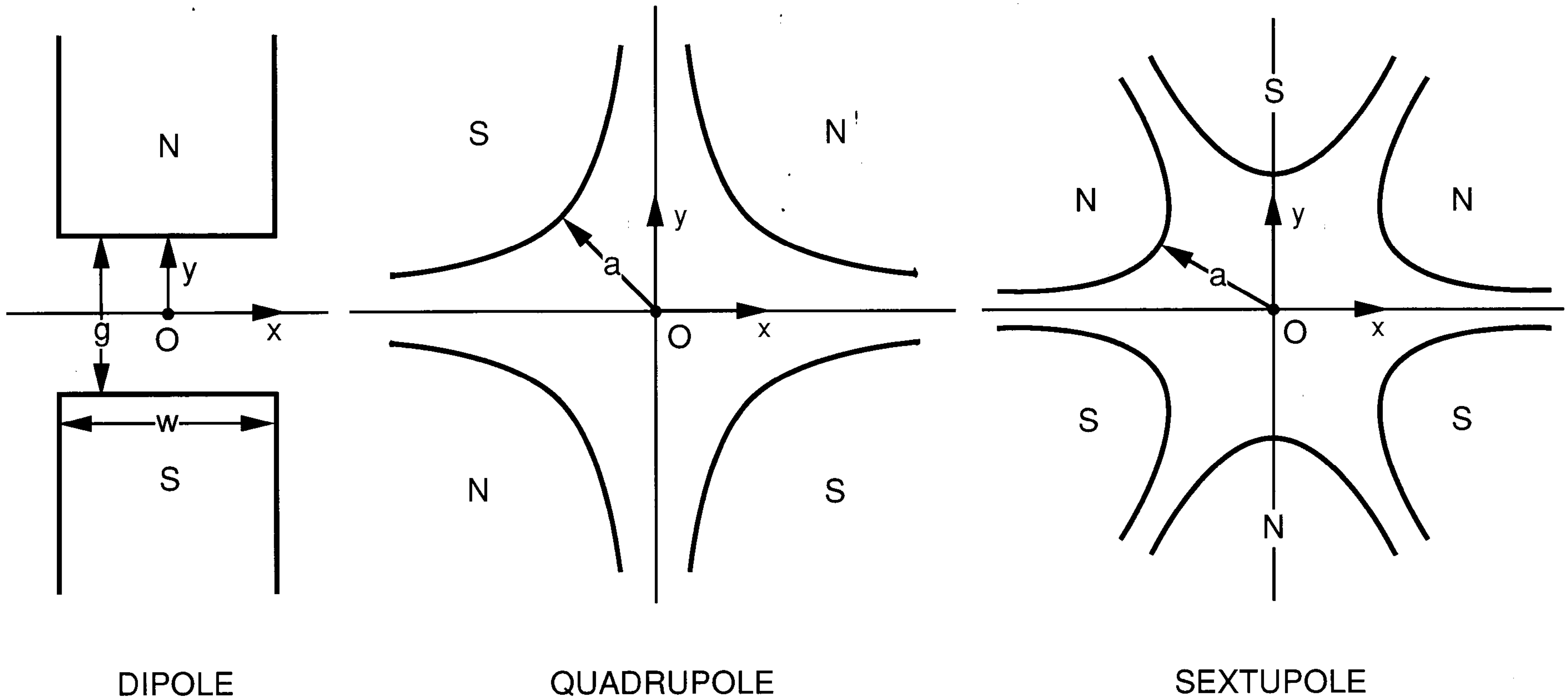


Illustration of the magnetic midplane (x axis) for dipole, quadrupole and sextupole elements. The magnet polarities indicate multipole elements that are positive with respect to each other. The coordinate system is right-handed, so that the beam particles will be emerging from the paper and moving into the face of the reader.

ROTATION — Fringing Fields and Pole-Face Rotations for Bending Magnets

To provide for fringing fields and/or pole-face rotations on bending magnets, the `ROTATION` element is used. The combined use of the `BEND` and the `ROTATION` elements allows the user to separate the effects of the fringing fields from those of the central body of a bending magnet.

First-, second-, and third-order matrix elements are included in TRANSPORT for the `ROTATION` element. In a third-order calculation, an extended fringe field (not an abrupt beginning or end) is required for the matrix elements to be finite (not infinite). For the fringe field to be extended, it is necessary to give a nonzero value to the magnet half gap (explained below).

Like the `BEND` element, the `ROTATION` element is not found in MAD. The combination of `BEND` and `ROTATION` elements may be replaced in MAD with the single `RBEND` or `SBEND` element. The `ROTATION` element can be expressed either in keyword or original TRANSPORT notation.

MAD-like Notation

The `ROTATION` element is not found in the MAD program. It is, however, a TRANSPORT element which can be described in the original TRANSPORT notation. The parameterization of it described here as MAD-like has been implemented to accompany the `BEND` element. The keyword notation has been chosen for consistency with the `RBEND` and `SBEND` elements. These two latter named elements include both the central and the fringe fields of bending magnets.

A first-order specification of a `ROTATION` element will typically have a value for just the rotation angle.

```
label: ROTAT,  ANGLE = value ;
```

A pole face rotation of 5 degrees might be represented as:

```
ROTAT,  ANGLE = 5.0  ;
```

The units for the pole face rotation angle will be radians in the global unit set `UMAD` and degrees with `UTRANS`. Other global unit sets are described on page 71. Other individual unit changes are listed on page 75.

To represent the entrance face rotation the `ROTATION` element should be placed immediately <u>before</u> the `BEND` element. To represent the exit face rotation the `ROTATION` element should be placed immediately <u>after</u> the `BEND` element. Additional parameters (see below) are available for higher-order calculations or in situations where greater precision is required is describing the configuration of the fringing field.

In the above example we omit the optional label. ROTATION is not a MAD element and is typically not used when a beam line is assembled from previously defined elements. Labels used for other purposes, such as simple identification, are generally attached to the accompanying BEND element.

The mnemonic used in specifying a pole face rotation is ROTATION. The MAD-like keywords which can be used are

Symbol	Keyword	Description
β	ANGLE	Angle of pole-face rotation (radians in UMAD, degrees with UTRANS).
$g/2$	HGAP	Half gap (meters with UMAD, cm with UTRANS, default 0.0).
κ_1	FINT	Fringe field integral (unitless, default 0.5).
$1/R_{1,2}$	H	Pole face curvature (normally meters^{-1}, default 0.0).

None of these parameters need be specified. The default value of the pole-face rotation angle is zero.-The keywords HGAP, FINT, and H allow the user to override temporarily the default values. The values specified by these three keywords on any ROTAT element will apply only to that element. The default values of these three parameters may be changed by the special parameter element described on page 79.

The pole-face rotation angle β can be varied in first order. The pole face curvature $1/R_{1,2}$ can be varied in second order. The procedure for indicating that a parameter is to be varied is given on page 259 in the section on the VARY command.

Original TRANSPORT Notation

There are two parameters:

1. ROTA(TE) (or type code 2.).
2. Angle of pole-face rotation (radians witn UMAD, degrees in UTRANS).

The units for the pole-face rotation angle can be changed to any desired with a UNITS command. (See page 75.)

The input format is:

```
                 ┌──── Label (if desired)
                 ↓
ROTAT    β.    'RO'   ;
```

A symmetrically oriented rectangular bending magnet whose total bend is 10 degrees would be represented by the three entries

```
ROTAT  5. ;    BEND  - - - ;    ROTAT  5. ;
```

In original TRANSPORT notation, only the pole-face rotation angle is given in the ROTAT element. The other three parameters are given by the special parameter element. The keywords HGAP, FINT, and H now allow the user to override temporarily the default values given by the special parameter cards. The values specified by these three keywords on any ROTAT element will apply only to that element. Default values which assumed if not specified through the special parameter cards. These default values are given on page 79 and following.

The use of keywords to override the defaults for special parameters allows the user to specify the characteristics of a pole face rotation on the card for that element alone. A complete bending magnet with fringe fields and pole face rotations and curvatures might be written as

```
ROTAT  .5,  H = 0.01  ;
BEND  10.   20.   'BM'  ;
ROTAT  .5,  H = -0.1  ;
```

Here a bending magnet 10 meters long with a field of 20 kG has the entrance and exit pole face rotations specified also. Both pole face rotation angles are .5 degrees. The entrance pole face is convex and has a curvature of 0.01 inverse meters, or a radius of curvature of 100 meters. The exit pole face is concave, and the radius of curvature is 10 meters. The units are based on the assumption that the global unit set UTRANS is employed.

The angle of rotation may be varied. For example, the element ROTAT.1 5. ; would allow the angle to vary from an initial guess of 5 degrees to a final value which would, say, satisfy a vertical focus constraint imposed upon the system. See the page 266 for a complete discussion of vary codes.

Usage

The ROTATION element must either immediately precede a bending magnet (BEND element) element (in which case it indicates an entrance fringing field and pole-face rotation) or immediately follow a BEND element (exit fringing field and pole-face rotation) with no other data entries between.* A positive sign of the angle on either entrance or exit pole-faces corresponds to a non-bend plane focusing action and bend plane defocusing action.

**It is extremely important that no data entries be made between a* ROTAT *element and a* BEND *element* entry. If this occurs, it may result in an incorrect matrix multiplication in the program and hence an incorrect physical answer. If this rule is violated, an error message will be printed.

Even if the pole-face rotation angle is zero, (`ROTAT 0.  ;`) entries must be included in the data set before and after a **BEND** element entry if fringing-field effects are to be calculated.

A single **ROTAT** element entry that follows one bending magnet and precedes another will be associated with the latter.

Should it be desired to misalign such a magnet, an update element or alignment marker must be inserted immediately before the first **ROTATION** element. Then the convention appropriate to misalignment of a set of elements should be applied, since, indeed, three separate transformations are involved. See page 195 for a discussion of misalignment calculations and page 223 element for a discussion of updates or page 227 for the alignment marker.

Pole-Face Rotation Matrix

The first-order R matrix for a pole-face rotation used in a TRANSPORT calculation is as follows:

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \dfrac{tan\beta}{\rho_0} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -\dfrac{tan(\beta-\psi)}{\rho_0} & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

where † ‡

$$\psi = \kappa_1 \left(\frac{g}{\rho_0}\right) \left(\frac{1+sin2\beta}{cos\ \beta}\right) \left[1 - \kappa_1\kappa_2 \left(\frac{g}{\rho_0}\right)\ tan\ \beta\right]$$

Definitions:

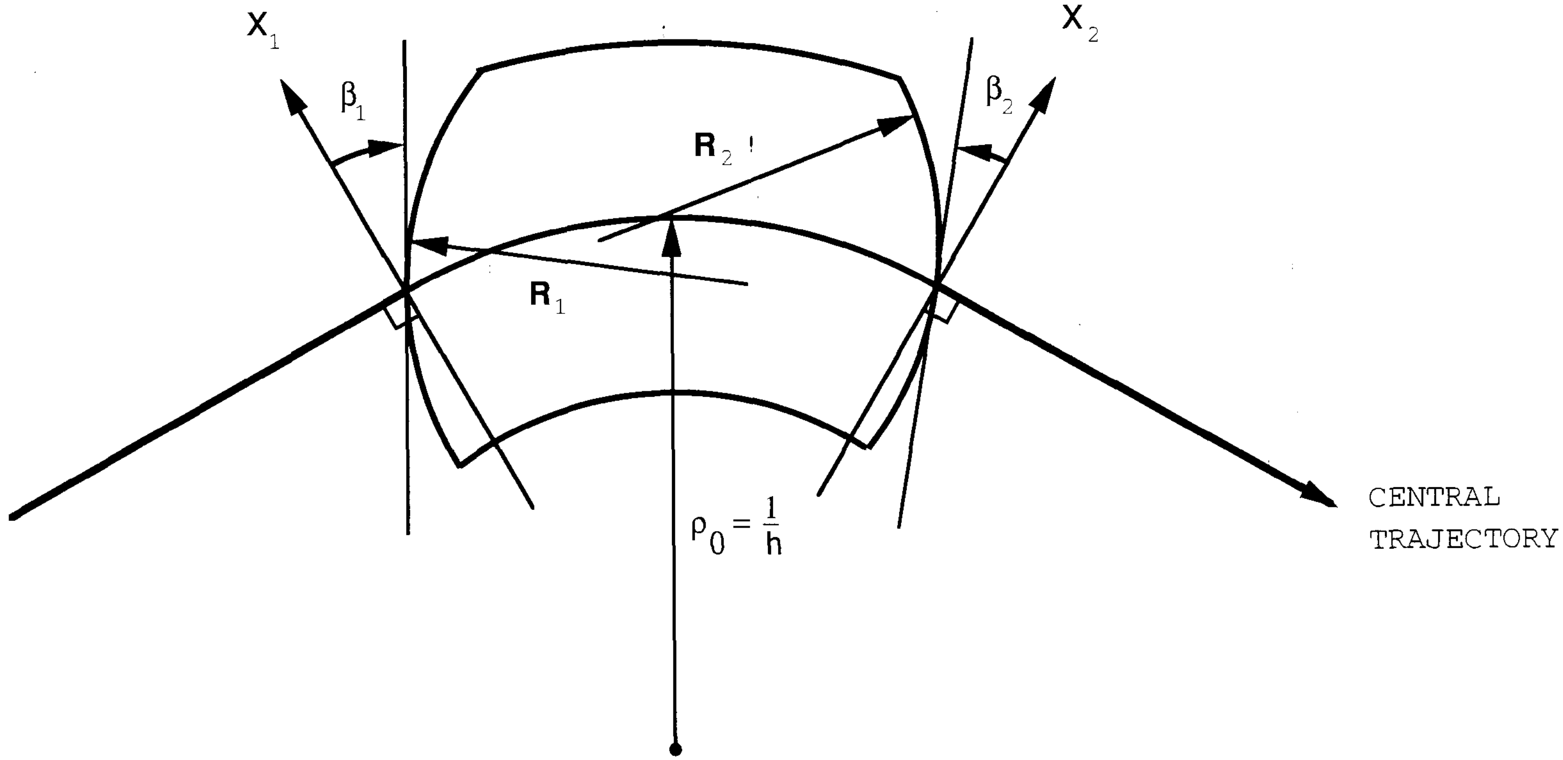
β	=	Angle of rotation of pole-face (see figure on following page for sign convention of β).
ρ_0	=	Bending radius of central trajectory.
g	=	Total gap of magnet.
ψ	=	Correction term resulting from spatial extent of fringing fields.

The transverse shift in the reference trajectory is given by

$$\Delta x = \frac{1}{cos^2\beta}\ \frac{g^2}{\rho}\ \kappa_0$$

†See **SPECIAL** element on page 79 for input formats for g, κ_1, κ_2 TRANSPORT entries.

‡See SLAC-75 [4] (page 74) for a discussion of ψ.



Field Boundaries for Bending Magnets

The TRANSPORT sign conventions for x, β, R and h are all positive as shown in the figure. The positive y direction is out of the paper. Positive β's imply transverse focusing. Positive R's (convex curvatures) represent negative sextupole components of strength $S = (-h/2R)\ sec^3\beta$. (See SLAC-75 [4], page 71 or SLAC-PUB-3381 [11], page 22.)

RBEND and SBEND – Bending Magnets with Fringing Fields

The elements RBEND and SBEND indicate bending magnets including the fringing fields. The mnemonic RBEND indicates a rectangular bending magnet. The rectangular shape pertains to the entire magnet, as seen from above (plan view). The default value for both the entrance and exit pole face rotation angles is half the bend angle.

The mnemonic SBEND indicates a sector bending magnet. Here the reference trajectory enters and exits from the magnet perpendicular to the magnet pole faces. Illustrations of both the RBEND and the SBEND magnets can be found at the end of this section. These mnemonics are the ones used to describe a bending magnet in the MAD program.

The term "wedge magnet" has also been used to describe an SBEND element. We intend no distinction between the terms "sector" and "wedge". The default values for the entrance and exit pole face rotation angles are therefore both zero.

For both RBEND and SBEND the default values for both the entrance and exit pole face curvatures are zero. The default values for the quadrupole, sextupole, and octupole components of the field are also zero. Any of these parameters can be set to whatever value is desired. A more complete description of the set of parameters used to represent the magnetic field is given on page 2.

First-, second-, and third-order matrix elements are included in TRANSPORT for the RBEND and SBEND elements. In a third-order calculation, and extended fringe field (not an abrupt beginning or end) is required for the matrix elements to be finite (not infinite). For the fringe field to be extended, it is necessary to specify a nonzero value to the magnet half gap (explained below).

An extended fringe field also results in a transverse displacement of the floor coordinates in comparison to a sharply cut off field. Details, including a description of when this effect is important, can be found in the ROTATE section starting on page 139.

RBEND and SBEND are MAD elements. They may be represented in MAD notation, other keyword notation, or in original TRANSPORT notation. Strict MAD notation requires the use of the UMAD unit set. In TRANSPORT other unit sets, such as UTRANS can also be used. In the immediately following example, we first use the UMAD unit set but also mention the UTRANS unit set. The unit sets are described on page 75.

MAD Notation

To first order, a BEND element might be specified by three real attributes.

label: RBEND, L = value , ANGLE = value , K1 = value;

A sample SBEND element might look like:

```
BR:  SBEND,  L = 10.,  ANGLE = 0.17453,  K1 = 0.5E-4  ;
```

The bend angle shown is 0.17453 radians or 10 degrees. The value in radians is shown since radians are the standard MAD units for bending angle. The same element, but with the bending angle expressed in degrees would be

```
BR:  SBEND,  L = 10.,  ANGLE = 10.0,  K1 = 0.5E-4  ;
```

The bend angle is expressed in degrees in the `UTRANS` unit set.

The keywords used in the MAD program for RBEND and SBEND are

Symbol	Keyword	Description
L	L	The magnet length (normally meters).
α	ANGLE	The angle through which the reference trajectory is bent (radians with the unit set UMAD, degrees with UTRANS).
K_1	K1	The quadrupole coefficient (normally meters^{-2}, default 0.0), defined by

$$K_1 = \frac{1}{B\rho} \frac{\partial B_y}{\partial x}$$

β_1	E1	The rotation angle for the entrance pole face (radians in UMAD, degrees in UTRANS, default 0.0).
β_2	E2	The rotation angle for the exit pole face (radians in UMAD, degrees in UTRANS, default 0.0).
K_2	K2	The sextupole coefficient (normally meters^{-3}, default 0.0), defined as

$$K_2 = \frac{1}{2B\rho} \frac{\partial^2 B_y}{\partial x^2}$$

	H1	The curvature of the entrance pole face (normally meters^{-1}, default 0.0).
	H2	The curvature of the exit pole face (normally meters^{-1}, default 0.0).
K_3	K3	The octupole coefficient (normally meters^{-4}, default 0.0), defined as

$$K_3 = \frac{1}{6B\rho} \frac{\partial^3 B_y}{\partial x^3}$$

$g/2$	HGAP	The half gap of the magnet (meters in UMAD, cm in UTRANS, default 0.0).

FINT	The field integral (given the symbol κ_1 in TRANSPORT).
TILT	The rotation angle about the entrance reference trajectory. (radians in UMAD, degrees in UTRANS, default 0.0).

In terms of these parameters, the field expansion on the magnetic midplane is given by

$$B_y = (B_o\rho)\sum K_n x^n$$

$$B_x = 0$$

The definition of the multipole components α, K_1, K_2, and K_3 allows the orbit or a particle to be computed without knowledge of the reference momentum. The normalization of K_1, K_2, and K_3 is such that a multipole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory. In the MAD program itself the values of K_1, K_2, and K_3 are defined as the x derivatives of the field divided by $B_0\rho$. The value of K_2 in MAD is then a factor of 2 greater than the value defined above. The MAD value of K_3 is a factor of 6 (= 3!) greater than that used here. When the global unit set UMAD is used, the MAD definition of the multipoles is also used.

The units of any of the parameters may be changed as described on page 75. The units of the parameters K_1, K_2, and K_3 are in inverse powers of the unit of longitudinal length.

The keyword TILT indicates that the bending magnet is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. The value given is the angle of rotation in degrees. If the word TILT appears alone without a value, the angle of rotation is taken to be 90 degrees, corresponding to a downward bend. A bend upward can be obtained simply by reversing the sign of the bend angle.

An example of the same bend as above, but vertically bending in the downward direction would be:

```
BR:  SBEND,  L = 10.,  ANGLE = 10.0,  K1 = 0.5E-4,  TILT ;
```

An example of the same bend but rotated by only 10° to provide a slight downward bend would be:

```
BR:  BEND,  L = 10.,  ANGLE = 10.0,  K1 = 0.5E-4,  TILT = 10.0 ;
```

Here both bend angle and rotation angle about the beam axis are assumed to be in degrees. The units for the rotation about the beam axis may be changed as described on page 75. Rotations about the longitudinal axis are in degrees for all globally specified units sets (UTRANS, UMETER, etc.) except UMAD, for which it is in radians.

The parameters L, α, and K_1, β_1, β_2, and the TILT may be varied in first-order fitting. The parameters K_2 and $1/R_{1,2}$ may be varied in second-order fitting. The parameter K_3

may be varied in third-order fitting. The VARY command is described on page 259 in the section on the FIT element.

Other Keyword Options

To allow keyword representation of data in original TRANSPORT variables and to permit other options for specifying a bending magnet, the following keywords are also recognized. Some may be used instead of the MAD variables, others in addition to a MAD specification.

Symbol	Keyword	Description
B	B	The magnetic field (normally kilogauss)
ρ	RADIUS	The radius of curvature of the reference trajectory (normally meters).
n	N	The normalized field derivative (unitless).
ϵ	EPS	The quadratic field dependence (in inverse squared units of horizontal beam width – meters in UMAD, cm in UTRANS.
ϵ_3	EPS3	The cubic field dependence (in inverse cubed units of horizontal beam width).

This set of variables contains a great deal of redundacy. The bending magnet, used as an example earlier in this section, may be described to first order in any of ten ways. For a beam momentum of 600 GeV/c, the following five items are then all equivalent.

```
SBEND,  L = 10.,  B = 20.,  N = 0.5  ;

SBEND,  L = 10.,  ANGLE = 10.0,  N = 0.5  ;

SBEND,  L = 10.,  RADIUS = 1000.,  N = 0.5  ;

SBEND,  B = 20.,  ANGLE = 10.0,  N = 0.5  ;

SBEND,  RADIUS = 1000.,  ANGLE = 10.0,  N = 0.5  ;
```

Here the angle is assumed to be in milliradians. The units of bend angle for UMAD are radians and those of UTRANS are degrees. Units other than the normal ones may be specified by use of a UNITS element.

In addition the field gradient may be specified by K_1 instead of n. The two are related by

$$K_1 = -\frac{n}{\rho^2} = \frac{1}{B\rho}\frac{\partial B_y}{\partial x}$$

The expansion of the magnetic field on the magnetic midplane is given by

$$B_y = B_o(1 + r_s)(1 - nhx + \beta\, h^2x^2 + \gamma\, h^3x^3 + \ldots)$$

$$B_x = 0$$

The sextupole component β is related to the coefficients ϵ and K_2 by

$$K_2 = \frac{\epsilon}{\rho} = \frac{\beta}{\rho^3} = \frac{1}{2B\rho}\,\frac{\partial^2 B_y}{\partial x^2}$$

The octupole component γ is related to the cubic variation of the magnetic field by.

$$K_3 = \frac{\epsilon_3}{\rho} = \frac{\gamma}{\rho^4} = \frac{1}{6B\rho}\,\frac{\partial^3 B_y}{\partial x^3}$$

The default value for the octupole component ϵ_3 is also set by the special parameters (`SPEC` element). Otherwise it is taken to be zero.

The multipole structure of a magnet should be specified in a consistent manner. One should use either K_1, K_2, and K_3 and the mid-plane symmetry violating counterparts or use n, β, and ϵ and their counterparts. One should not attempt to mix the two forms of field expansion. The two normalization parameters r_s, and r_a can be used with either type of specification.

Any of the first-order parameters L, B, ρ, α, and n can be varied in first-order fitting. The parameter ϵ can be varied in second-order fitting. The parameter ϵ_3 can be varied in third-order fitting.

Mispowering and Violations of Midplane Symmetry

The vertical field in a bending magnet along the reference trajectory may be mispowered so that the reference particle will be bent away from the reference trajectory. This can happen simply by increasing the field in the magnet without redefining the reference momentum. A physical bending magnet may also have a horizontal field component on what is taken as the magnetic midplane. A set of parameters to describe both the effects of excess bend field and violation of midplane symmetry follows. The multipole components which violate midplane symmetry are also known as skew multipoles. For a discussion of the magnetic field components, see page 2.

Some of these parameters are MAD-like, and should be used if the midplane-symmetric field is given a MAD-like parameterization. Specifically the parameters K_1' and K_2' are MAD-like. The parameters n' and ϵ' are traditional- TRANSPORT-like. The two scaling parameters r_s and r_a can be used with either representation.

Symbol	Keyword	Description
r_s	RMPS	The fractional excess magnetic field (unitless).
r_a	RNMS	Scaling factor for skew field components (unitless).
v_r	VR	The normalized skew dipole magnetic field (unitless).
K_1'	K1P	The skew quadrupole coefficient (normally meters^{-2}).
K_2'	K2P	The skew sextupole coefficient (normally meters^{-3}).
n'	NP	The skew normalized field derivative (unitless).
ϵ'	EPSP	The skew quadratic field dependence (same units as EPS).

In terms of these new parameters, the field expansion on the nominal magnetic midplane is given by:

$$B_y = (B_o\rho)(1 + r_s)\sum K_n x^n$$

$$B_x = (B_o\rho)r_a \sum K_n' x^n$$

where

$$B_0 v_r = (B_0\rho)K_0'$$

or

$$B_y = B_o(1 + r_s)(1 - nhx + \beta\ h^2x^2 + \gamma\ h^3x^3 + \ldots)$$

$$B_x = B_o r_a(v_r - n'hx + \beta' h^2 x^2 + \ldots)$$

The quantity r_s represents the fractional excess field. The reference trajectory is defined as if r_s equalled zero. The field of the magnet may then be increased or decreased to steer the beam about the reference trajectory. The quantity r_a exists only to facilitate simultaneous scaling of all midplane symmetry violating multipoles.

The four parameters, r_a, v_r, n', and β' may also be set on the individual bend element, or left to equal the default value. The default values may be set on the special parameter element. Otherwise they are all zero. The parameters K_1' and K_2' are related to n', β', and ϵ' respectively in the same way that K_1 and K_2 are related to n, β, and ϵ.

If any of this set of parameters is not specified it is set equal to the default value given on the special parameter element (SPEC element). If the default value is also unspecified, it is taken to be zero.

If the magnet in the example above is mispowered so that the field is excessively strong by 1%, it might be specified as:

```
BR: SBEND, L = 10., ANGLE = 10.0, K1 = 0.5E-4, RMPS = 0.01 ;
```

If there is a skew quadrupole component which is whose strength is 1% that of the midplane symmetric quadrupole component, the bend magnet could be described as:

```
BR: SBEND, L = 10., ANGLE = 10.0, K1 = 0.5E-4, RNMS = 0.01, K1P = 0.4E-4 ;
```

or as:

```
BR: SBEND, L = 10., ANGLE = 10.0, K1 = 0.5E-4, RNMS = 1.0, K1P = 0.4E-6 ;
```

In the use of these keywords, the elements RBEND and SBEND are identical.

Any of the first-order parameters r_s, r_a, v_r, K_1', and n' may be varied in first-order fitting. The parameters K_2' and ϵ' can be varied in second-order fitting. There are no third-order midplane-symmetry-violating transfer matrix elements in TRANSPORT.

Original TRANSPORT Notation

There are four first-order parameter to be specified for RBEND or SBEND:

1. RBEND (specifying a rectangular bending magnet) or SBEND (specifying a wedge bending magnet). Both include fringing fields.
2. The (effective) length L of the central trajectory in meters.
3. The central field strength $B(0)$ in kG,

   $$B(0) = 33.35641(p/\rho_0) \quad ,$$

   where p is the momentum in GeV/c and ρ_0 is the bending radius of the central trajectory in meters.
4. The field gradient (n-value, dimensionless); where n is defined by the equation

   $$B_y(x,0,t) = B_y(0,0,t)(1 - nhx + \ldots) \quad ,$$

   and

   $$h = 1/\rho_0 \ . \quad ^*$$

*See SLAC-75 [4] (page 31).

A sector bending magnet of length 10 meters and magnetic field of 20 kilogauss might be written as follows.

```
SBEND  10.   20.   'SB'  ;
```

Additional parameters may be inserted by using the keyword notation. A sector bend with slight pole face rotation angles of 2.0 and 3.0 degrees might then be written as

```
SBEND  10.   20.   'SB',  E1 = 2.0,  E2 = 3.0  ;
```

Any keyword parameter may be set to a value different from the default by use of the keyword notation. If, as above, the specification of the element is begun in original TRANSPORT notation, then any keyword specifications must follow the last parameter specified in original TRANSPORT notation.

A `BEND` element indicates the central body of a bending magnet, without either entrance or exit fringing fields. The original TRANSPORT notation for `RBEND` and `SBEND` has been made to be the same as that for the element `BEND` (or type code `4.`). A bending magnet may now be given fringing fields merely by adding either of the letters `R` or `S` to the mnemonic `BEND` to become `RBEND` or `SBEND`.

The `SBEND` defined just above, with zero pole-face-rotation angles, is equivalent to:

```
ROTAT  0.   ;
BEND  10.   20.   'SB'  ;
ROTAT  0.   ;
```

The quantities L, B(0), and n may be varied for first-order fitting (see the `FIT` element for a discussion of vary codes).

The bend radius in meters and the bend angle in degrees are printed in the output.

A typical first-order TRANSPORT input for a wedge magnet is

```
                   ┌──── Label
                   ↓
SBEND  L  B  n  '   '  ;
```

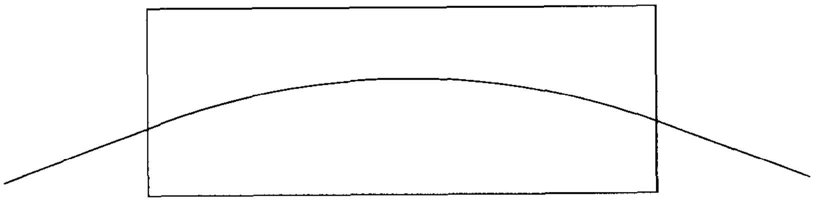
A typical first-order TRANSPORT input for a rectangular magnet is

```
                   ┌──── Label
                   ↓
RBEND  L  B  n  '   '  ;
```

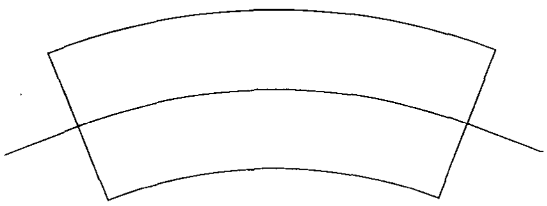
The `RBEND` example above cannot be recast into an equivalent example using `BEND` (the central bending field) and `ROTAT` (the pole face rotations with fringing fields). An `RBEND` with a given set of parameters will be rectangular no matter what the reference momentum, since the pole face rotation angle is calculated by the program. For the `ROTAT`, `BEND`, `ROTAT` combination the pole face rotation angle must be given explicitly and will depend on the reference momentum. If the magnetic field is changed as a result of a fitting process, and the pole face rotation angle is not correspondingly adjusted by hand, the specified magnet may no longer be rectangular.

Note that the use of labels is optional and that all data entries may be made on one line if desired.

The sign conventions for bending magnet entries are illustrated in the following figure. For TRANSPORT a positive bend is to the right looking in the direction of particle travel. To represent a bend in another sense, the `TILT` keyword should be used to rotate the bend about the entrance reference trajectory.



An RBEND Type of Bending Magnet



An SBEND Type of Bending Magnet

HKICK or VKICK – Horizontal and Vertical Steering Magnets

The difference between a steering magnet and a bending magnet is that the steering magnet does not affect the reference trajectory. If the beam line is perfectly aligned with no errors, and the beam centroid follows the reference trajectory, the field of the steering magnets is zero. The steering magnets are used to compensate for errors and misalignments in the beam line. If they are given a nonzero field, the beam line reference trajectory is not affected. However, the beam is steered either toward or away from the reference trajectory.

In TRANSPORT the elements `HKICK` and `VKICK` should always be used for steering magnets. The centroid shift element `CENTROID` should never be used for that purpose. The steering magnet elements contain a complete set of first-, second-, and third-order transfer matrix elements. This includes the effect of having the angle of deflection vary inversely with the momentum of the particle. The centroid shift element contains none of these effects, but simply deflects the beam by a fixed distance or angle.

A positive angle bends to the right for the `HKICK` element or down for the `VKICK` element. This convention is consistent with that used for bending magnets. In accelerator usage, a steering magnet is often described by its "kick". A kick is considered to be positive if it increases p_x or p_y, or x' or y'. It is then opposite in sense but equal in magnitude to the angle.

There is no provision for non-uniform field in this element. The fringe fields are included automatically. The pole-face rotation angle is automatically set so that the magnet is rectangular. The entrance and exit pole faces are still both perpendicular to the reference trajectory, since the reference trajectory is considered to be undeflected by the kicking magnet. There is also no provision for pole face curvature.

Both `HKICK` and `VKICK` are MAD elements. They may also be expressed in other keyword notation or in original TRANSPORT notation.

MAD Notation

The `HKICK` and `VKICK` elements are specified by two real attributes:

```
label:   HKICK,  L = value,  KICK = value  ;
```

or

```
label:   VKICK,  L = value,  KICK = value  ;
```

An example of a horizontal steering magnet element would then be:

```
HKICK,  L = 1.0,  KICK = -0.0001  ;
```

Here the units for the kick are assumed to be in radians. This choice is made for consistency with standard accelerator usage. The angle is then the same as in subsequent examples in this section.

Symbol	Keyword	Description
L	L	The effective length of the magnet (meters).
$\Delta x'$ or $\Delta y'$	KICK	The kick angle (radians in UMAD, degrees in UTRANS).

The units illustrated are those specified by one of the global unit sets UMAD or UTRANS. Other global unit sets are described on page 71. The kick angle may be varied in first-order fitting.

Other Keyword Options

For consistency with bending magnet specifications, it is also possible to parameterize a KICK in terms of its length and bend angle. Recall that for a BEND, RBEND, or SBEND a positive bend is one which deflects the reference trajectory toward negative x. The bend angle is then equal in magnitude and opposite in sense from the kick value described above.

label: HKICK, L = value, ANGLE = value ;

The same KICK element as above can then be expressed in terms of the ANGLE keyword as

```
HKICK, L = 1.0, ANGLE = 0.0001 ;
```

The keyword format can also be used to specify a steering magnet in terms of parameters which are more like the original TRANSPORT parameters for a bending magnet.

label: HKICK, L = value, B = value ;

The keywords available are:

Symbol	Keyword	Description
L	L	The effective length of the magnet (meters).
α	ANGLE	The deflection angle (radians in UMAD, degrees in UTRANS).
B	B	The magnetic field (kilogauss).

The units for either the length, the bend angle, or the magnetic field may be changed by use of the UNIT element. Here we assume that the angle units are in milliradians. The same horizontal steering magnet element as shown above would then be:

```
HKICK, L = 1.0, ANGLE = 0.1 ;
```

Assuming a 600 GeV beam, the following element is also equivalent to the previous examples.

```
HKICK,  L = 1.0,  B = 2.0  ;
```

The deflection angle α or the magnetic field B may be varied in first-order fitting.

Original TRANSPORT Notation

Using original positional TRANSPORT input, the description of `HKICK` and `VKICK` is much the same as that of a bending magnet. There are three items needed.

1. The mnemonic `HKICK` or `VKICK`
2. Effective length (meters).
3. The central field strength (kilogauss).

The units given, meters and kilogauss, can be changed to any desired by a `UNITS` command. (See page 75.) The central magnetic field strength can be varied in first-order fitting.

An example of a steering magnet element in original TRANSPORT notation might then be.

```
HKICK  1.0  2.0  'KICK'  ;
```

This steering magnet is then the same as in the examples given above.

KICKER – A Steering Magnet in Both Planes

The KICKER element is a steering magnet that steers in both planes simultaneously. The MAD standard does not have a TILT parameter on either HKICK or VKICK. Instead steering in both planes simultaneously is done using the KICKER element. All angle conventions applying to HKICK or VKICK apply to KICKER also.

The KICKER element exists only in MAD notation It also contains first-, second-, and third-order matrix elements. The fringe fields are included automatically.

MAD Notation

The KICKER element is specified by three real attributes:

```
label: KICKER, L = value , HKICK = value, VKICK = value ;
```

A simultaneous kick in the horizontal plane of $-.0002$ radians and in the vertical plane of 0.00012 radians could be represented as:

```
KICKER, L = 1.0, HKICK = -.0002, VKICK = .00012 ;
```

Symbol	Keyword	Description
L	L	The effective length of the magnet (meters).
$\Delta x'$	HKICK	The horizontal kick angle (radians in UMAD, degrees in UTRANS).
$\Delta y'$	VKICK	The vertical kick angle (radians in UMAD, degrees in UTRANS).

Other unit sets are also allowable and are described on page 71. Both the horizontal and vertical kick angles are variable in first-order fitting.

Output

A physical steering magnet which bends in both planes is a rotated dipole magnet. To facilitate installation of such a magnet in a beam line, the TRANSPORT output describes a KICKER as a rotated dipole. The rotation angle and the net magnetic field are calculated and printed.

QUADRUPOLE

A quadrupole provides focusing in one transverse plane and defocusing in the other. The reference trajectory through a quadrupole is a straight line. If a quadrupole is misaligned (via an `ALIGN` element) a dipole component will be introduced, steering the beam away from the aligned reference trajectory.

A quadrupole has no second-order geometric aberrations. The only second-order aberrations are chromatic, representing the variation in focusing strength of the quad with momentum.

A quadrupole has geometric aberrations of third and higher orders. When TRANSPORT performs a third-order calculation, the quadrupole fringe fields* are included automatically. The longitudinal field profile is assumed to be rectangular, meaning that the field begins and ends abruptly and is otherwise uniform. No current provision exists for extended fringing fields.

The `QUAD` is a MAD element. It may be expressed in MAD notation, other keyword notation, or original TRANSPORT notation.

MAD Notation

For the MAD program, a magnetic quadrupole is specified by three real attributes.

label: `QUAD(RUPOLE)`, `L` = value, `K1` = value, `TILT` = value ;

A typical quadrupole specification might then look like:

```
Q1:  QUAD,  L = 5.,  K1 = .01  ;
```

Symbol	Keyword	Description
L	`L`	The quadrupole length (normally in meters, default = 0.0)
K_1	`K1`	The quadrupole coefficient (defined below). A positive quadruole strength indicates horizontal focussing of positively charged particles. (The units are normally meters^{-2})
	`TILT`	The roll angle about the longitudinal axis (radians with `UMAD`, degrees with `UTRANS`).

The units of any of the physical parameters can be changed to any desired. Other unit set specifications are described on page 71. Individual unit changes are described on page 75.

The keyword `TILT` indicates that the quadrupole is rotated in a clockwise sense about

*G.E.Lee-Whiting, Nuclear Instruments and Methods 83, 232 (1970).

the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. *If the word* TILT *appears alone without a value, the angle of rotation is taken to be 45 degrees.* This configuration is known to accelerator designers as a skew quadrupole.

The length L, the quadrupole coefficient K_1, and the TILT parameter may be varied in first- or higher-order fitting.

Other Keyword Options

The same quadrupole can also be specified by other sets of variables. The first of the two following options corresponds to the original TRANSPORT variables expressed in keyword notation.

```
label: QUADRUPO,  L = value ,  B = value,  APER = value ;

label: QUADRUPO,  L = value ,  GRAD = value ;
```

The TILT parameter may also be used with either of these two options.

Symbol	Keyword	Description
L	L	The magnet length (normally meters).
B_o	B	The magnetic field (normally kilogauss).
a	APER(TURE)	The magnetic half aperture (meters with UMAD, cm with UTRANS).
g	GRAD(IENT)	The gradient of the magnetic field (kG/m with UMAD, kG/cm with UTRANS).

Global unit set specifications other than UMAD or UTRANS may also be selected and are described on page 71. Individual units changes are described on page 75.

The following three elements are all equivalent.

```
Q1:  QUAD,  L = 5.,  B = 10.,  APER = 5.   ;

Q1:  QUAD,  L = 5.,  GRAD = 2.   ;

Q1:  QUAD,  L = 5.,  K1 = .01  ;
```

Here we have assumed a beam momentum of 600 GeV/c. We have also assumed that the aperture is measured in the UTRANS unit of cm. The description in terms of K_1 has the advantage that it does not require a separate specification of the beam momentum. The quantities K_1 and g are related to the pole tip field B_o and the aperture a by:

$$K_1 = \frac{g}{(B\rho)} = \frac{1}{(B\rho)} \frac{B_o}{a}$$

The length L, pole-tip magnetic field B, and gradient g of a quadrupole may be varied in first- or higher-order fitting. The aperture may not be varied in fitting of any order..

Original TRANSPORT Notation

There are four parameters to be specified.

1. `QUAD(RUPO)` or type code `5.0` (specifying a quadrupole).
2. (Effective) magnet length L (in meters).
3. Field at pole tip B (in kG). A positive field implies horizontal focusing; a negative field, vertical focusing.
4. Half-aperture a (meters in `UMAD`, cm in `UTRANS`). Radius of the circle tangent to the pole tips.

The input format for a typical quadrupole element is then:

```
                              ┌────── Label (if desired)
                              ↓
QUAD   L   B   a   ' '   ;
```

The same quadrupole as in the above examples can now be represented as

```
QUAD 5.  10.  5.  'Q1' ;
```

The standard TRANSPORT units (selected with the global units specification `UTRANS`) for L, B and a are meters, kG, and cm, respectively. The MAD units (selected with `UMAD`) are meters, kG, and meters respectively. If other units are desired they must be chosen via other global units specifications or the appropriate `UNIT` element preceding the `BEAM` element.

The strength of the quadrupole is computed from its field, aperture and length. The length and field of a quadrupole may be varied in first-order fitting. The aperture may not be.

Output

The horizontal focal length is printed in parentheses as output. A positive focal length indicates horizontal focusing and a negative focal length indicates horizontal defocusing. The quantity actually printed is the reciprocal of the $(x'|x)$ transfer matrix element $(1/R_{21})$

for the quadrupoles. Thus two identical quadrupoles of opposite polarity will have different horizontal focal lengths due to the difference between the sine and the hyperbolic sine.

First-Order Quadrupole Matrix

$$\begin{bmatrix} \cos k_q L & \frac{1}{k_q} \sin k_q L & 0 & 0 & 0 & 0 \\ -k_q \sin k_q L & \cos k_q L & 0 & 0 & 0 & 0 \\ 0 & 0 & \cosh k_q L & \frac{1}{k_q} \sinh k_q L & 0 & 0 \\ 0 & 0 & k_q \sinh k_q L & \cosh k_q L & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

These elements are for a quadrupole which focuses in the horizontal (x) plane (B positive). A vertically (y-plane) focusing quadrupole (B negative) has the first two diagonal submatrices interchanged.

Definitions:

	L	=	the effective length of the quadrupole.
	a	=	the radius of the aperture.
	B_0	=	the field at the radius a.
	k_q^2	=	$(B_0/a)(1/B_0\rho_0)$, where $(B\rho_0)$ = the magnetic rigidity (momentum) of the central trajectory.

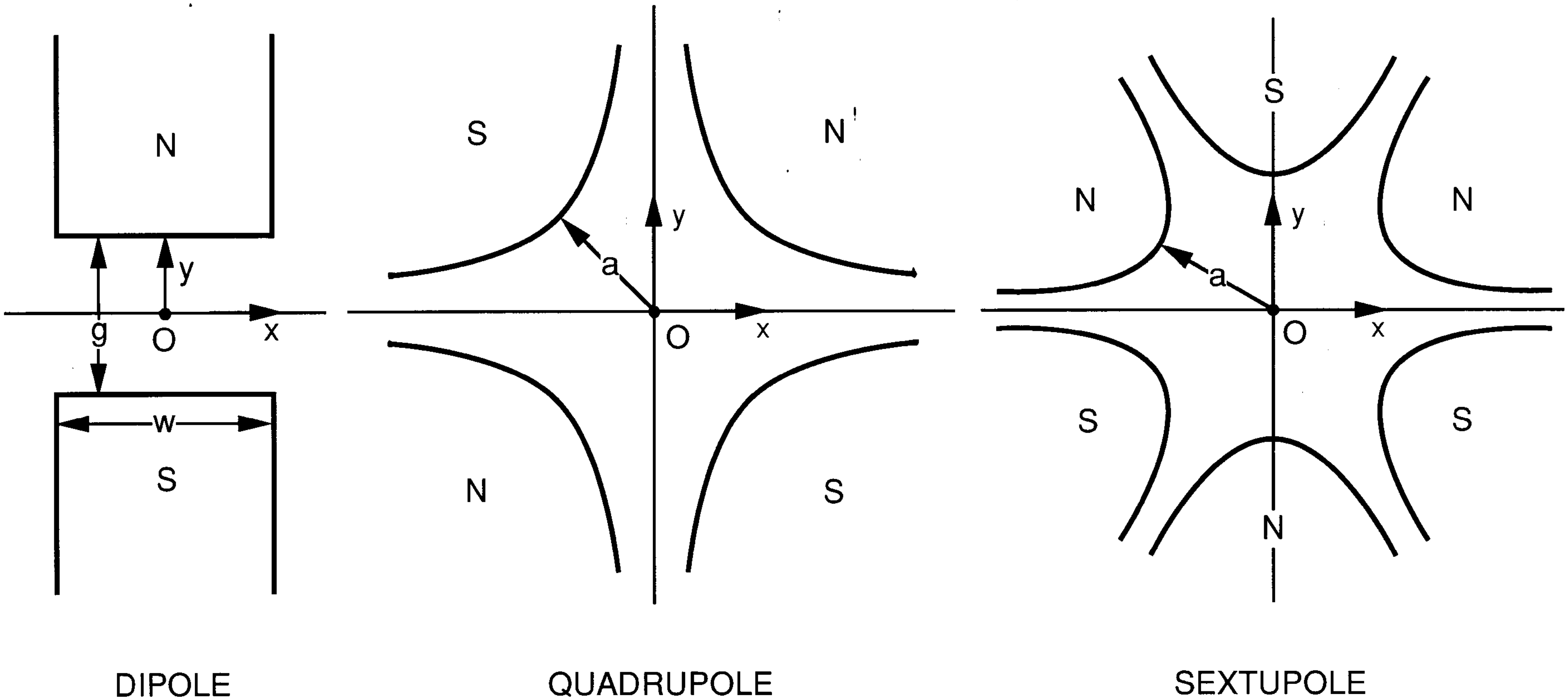


Illustration of the magnetic midplane (x axis) for dipole, quadrupole and sextupole elements. The magnet polarities indicate multipole elements that are positive with respect to each other. The coordinate system is right-handed, so that the beam particles will be emerging from the paper and moving into the face of the reader.

SEXTUPOLE

Sextupole magnets are used to modify second-order aberrations in beam transport systems. The reference trajectory through a sextupole is a straight line. The action of a sextupole on beam particles is a second- and higher-order effect. In first order runs (absence of the **ORDER** element) a correctly aligned sextupole will act as a drift space. A misaligned sextupole will produce effective quadrupole and dipole components in addition to the sextupole field. For a more complete description of the effects of misalignments on sextupoles, see the **ALIGN** element on page 195.

First-, second-, and third-order matrix elements for a sextupole are included in TRANSPORT. Third-order terms arise from the momentum dependence of the second-order terms and from the coupling between second-order terms at different longitudinal locations within the same sextupole. For the second- or third-order terms to have any effect, a higher-order calculation must be specified with the **ORDER** command described on page 91.

The **SEXTUPOLE** is a MAD element. It may be expressed in MAD notation, other keyword notation, or original TRANSPORT notation.

MAD Notation

For the MAD program, a magnetic sextupole is specified by three real attributes.

```
label: SEXTUPOLE, L = value, K2 = value , TILT = value ;
```

A typical sextupole specification might be:

```
S1:    SEXT,  L = 5.,  K2 = 0.2  ;
```

Symbol	Keyword	Description
L	L	The magnet length (normally meters).
K_2	K2	The normalized sextupole strength (in meters^{-3}).
	TILT	The angle of rotation about the optic axis (radians with **UMAD**, degrees with **UTRANS**).

The units of any of the physical parameters can be changed to any desired. Units are shown above for the **UMAD** and **UTRANS** global unit set specifications. Other unit set specifications are described on page 71. Individual unit changes are described on page 75.

The quantity K_2 is related to the pole tip field B_o and the aperture a by:

$$K_2 = \frac{1}{(B\rho)} \frac{B_o}{a^2}$$

The normalization of K_2 is such that an sextupole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory. It should be noted that in the MAD program itself, the parameter K_2 is defined to be the third derivative of the field divided by $B_0\rho$. The MAD value of K_2 is then greater by a factor of 2 (= 2!) than the value defined here. Specification of the global unit set UMAD will cause TRANSPORT to use the MAD definition of multipole strength.

The keyword TILT indicates that the sextupole is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. The value given is the angle of rotation (degrees in UTRANS, radians in UMAD). *If the word* TILT *appears alone without a value, the angle of rotation is taken to be 30 degrees.*

The normalized sextupole strength K_2 can be varied in second- or third-order fitting. The variation of a physical parameter is indicated by the VARY command, described on page 259.

Other Keyword Options

The same sextupole can be specified by another set of variables. The following parameterization corresponds to the original TRANSPORT variables expressed in keyword notation.

label: SEXTUPOL, L = value , B = value , APER = value ;

Symbol	Keyword	Description
L	L	The magnet length (normally meters).
B_o	B	The magnetic field (normally kilogauss).
a	APER(TURE)	The magnetic half-aperture or pole-tip radius (meters with UMAD, cm with UTRANS).

Global unit set specifications other than UMAD or UTRANS may also be selected and are described on page 71. Individual units changes are described on page 75.

For a beam momentum of 600 GeV/c, the following two elements are equivalent.

```
S1:   SEXT,  L = 5.,  B = 10.,  APER = 5.   ;

S1:   SEXT,  L = 5.,  K2 = 0.2  ;
```

Here we assume that the aperture is measured in cm. The description in terms of K_2 has the advantage that it does not require a separate specification of the beam momentum. The quantity K_2 is related to the pole tip field B_o and the half aperture a by:

$$K_2 = \frac{1}{(B\rho)} \frac{B_o}{a^2}$$

The normalization of K_2 is such that a sextupole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory. It should be noted that in the MAD program itself, the parameter K_2 is defined to be the second derivative of the field divided by $B_0\rho$. The MAD value of K_2 is then greater by a factor of 2 than the value defined here.

The pole-tip magnetic field B can be varied in second- or third-order fitting. It may also be constrained not to exceed a certain specified maximum field. The aperture may not be varied. The variation of a physical parameter is indicated by the `VARY` command, described on page 259.

Original TRANSPORT Notation

There are four parameters:

1. `SEXT(UPOLE)` (or type code `18.0`)
2. Effective length (metres).
3. Field at pole tips (kG). Both positive and negative fields are possible (see figures below).
4. Half-aperture (meters with `UMAD`, cm with `UTRANS`). Radius of circle tangent to pole tips.

The normal orientation of a sextupole is shown in the following figure. Other orientations of the sextupole may be obtained using the beam rotation element (`SROT`).

As above, we assume that the aperture is measured in cm. The standard TRANSPORT units (selected with the global units specification `UTRANS`) for L, B and a are meters, kG, and cm, respectively. The MAD units (selected with `UMAD`) are meters, kG, and meters respectively. If other units are desired they must be chosen via other global units specifications or the appropriate `UNIT` element preceding the `BEAM` element.

The pole tip field may be varied in second-order fitting. It may also be constrained not to exceed a certain specified maximum field. (See the explanation of vary codes on page 266.) Such a constraint allows one to take into account the physical realities of limitations on pole tip fields. The half aperture may not be varied.

See SLAC-75 [4] for a tabulation of sextupole matrix elements. The original TRANS-

PORT input format for a typical sextupole element is:

```
                              ┌──── Label (if desired)
                              ↓
SEXT  L  B  a  '  ' ;
```

The same sextupole as in the above examples can now be represented as

```
SEXT 5. 10. 5. 'S1' ;
```

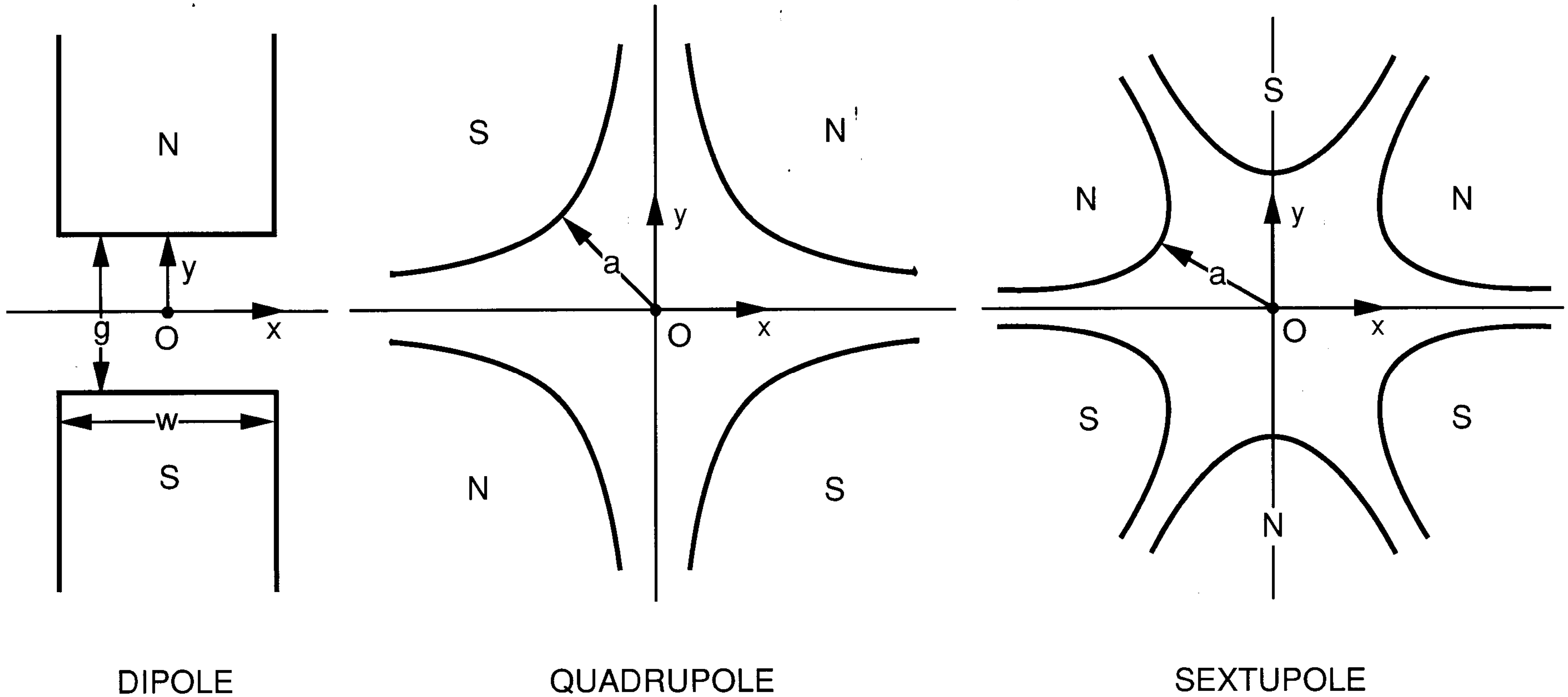


Illustration of the magnetic midplane (x axis) for dipole, quadrupole and sextupole elements. The magnet polarities indicate multipole elements that are positive with respect to each other. The coordinate system is right-handed, so that the beam particles will be emerging from the paper and moving into the face of the reader.

OCTUPOLE

Octupole magnets are used to affect third-order aberrations in charged particle optical systems. The effect of an octupole is limited to third and higher orders. In a first- or second-order calculation, a correctly aligned octupole will act as a drift space. A misaligned octupole will have effective sextupole, quadrupole, and dipole components also.

Matrix elements up to and including third order can be calculated by TRANSPORT for an octupole. The second-order terms for an octupole are all zero. The difference between an octupole and a drift space begin at third order. In order to activate the third-order matrix elements for an octupole a third-order calculation must be specified via the `ORDER` element, described on page 91.

The `OCTUPOLE` is a MAD element. It may be expressed in MAD notation, other keyword notation, or original TRANSPORT notation.

MAD Notation

For the MAD program, a magnetic octupole is specified by three real attributes.

```
label: OCTUPOLE,  L = value ,  K3 = value ,  TILT = value ;
```

A typical octupole specification might then look like:

```
O1:   OCTUPOLE,  L = 5.,  K1 = 4.0  ;
```

Symbol	Keyword	Description
L	`L`	The magnet length (normally meters).
K_3	`K3`	The normalized octupole component (normally meters^{-4}, default 0.0).
	`TILT`	The angle of rotation about the optic axis (radians in `UMAD`, degrees in `UTRANS`).

The units of any of the physical parameters can be changed to any desired. Other unit set specifications are described on page 71. Individual unit changes are described on page 75.

The quantity K_3 is related to the pole tip field B_o and the aperture a by:

$$K_3 = \frac{1}{(B\rho)}\;\frac{B_o}{a^3}$$

The normalization of K_3 is such that an octupole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory.

It should be noted that in the MAD program itself, the parameterK_3 is defined to be the third derivative of the field divided by $B_0\rho$. The MAD value of K_3 is then greater by a factor of 6 ($=3!$) than the value defined here. Specification of the global unit set UMAD will cause TRANSPORT to use the MAD definition of multipole strength.

The keyword TILT indicates that the octupole is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. The value given is the angle of rotation in degrees. *If the word* TILT *appears alone without a value, the angle of rotation is taken to be 22.5 degrees.*

The normalized octupole strength K_3 can be varied in third-order fitting. The variation of a physical parameter is indicated by the VARY command, described on page 259.

Other Keyword Options

The same octupole can also be specified by another set of variables. The following option exists for consistency with the original TRANSPORT set of variables

label: OCTUPOLE, L = value , B = value , APER = value ;

Symbol	Keyword	Description
L	L	The magnet length (normally meters).
B_o	B	The magnetic field (normally kilogauss).
a	APER(TURE)	The magnetic aperture (meters in UMAD, cm in UTRANS).

The TILT parameter may also be used with this option. Global unit set specifications other than UMAD or UTRANS may also be selected and are described on page 71. Individual units changes are described on page 75.

For a beam momentum of 600 GeV/c, the following two elements are equivalent.

```
O1:    OCTUPOLE,  L = 5.,  B = 10.,  APER = 5.   ;

O1:    OCTUPOLE,  L = 5.,  K1 = 4.0  ;
```

Here we assume that the aperture is specified in cm. The description in terms of K_3 has the advantage that it does not require a separate specification of the beam momentum.

The pole-tip magnetic field B can be varied in third-order fitting. The aperture a may not be varied. The variation of a physical parameter is indicated by the VARY command, described on page 259.

Original TRANSPORT Notation

Using the original positional TRANSPORT notation, four items are needed to describe an octupole.

1. The Mnemonic `OCTU(POLE)`.
2. The effective length (meters).
3. Field at pole tips (normally kilogauss). Both positive and negative fields are possible. A positive field is defined by the same convention as for dipoles, quadrupoles, and sextupoles. A positive field will cause a beam particle crossing the positive x axis to be deflected to the right.
4. The half aperture (meters in `UMAD`, cm with `UTRANS`).

A typical octupole element will then be:

```
                 ┌──── Label (if desired)
                 ↓
OCTU  L  B  a  '   '  ;
```

The same octupole as in the above examples can now be represented as

```
OCTU 5. 10. 5. '01' ;
```

As above, we assume that the aperture is measured in cm. The standard TRANSPORT units (selected with the global units specification `UTRANS`) for L, B and a are meters, kG, and cm, respectively. The MAD units (selected with `UMAD`) are meters, kG, and meters respectively. If other units are desired they must be chosen via other global units specifications or the appropriate `UNIT` element preceding the `BEAM` element.

The pole tip field may be varied in third-order fitting. The half aperture may not be varied. In Original TRANSPORT notation, the variation of a parameter is indicated by vary codes, which are described on page 266.

PLASMALENS – Plasma (Lithium) Lens

A plasma lens has focusing characteristics similar to those of a quadrupole, except that the plasma lens can focus in both transverse planes simultaneously. The original TRANSPORT specification for a plasma lens is the same as that for a quadrupole.

First-, and second-order matrix elements for a plasma lens are included in TRANSPORT. The second-order terms arise from the momentum variation of the first order terms (chromatic aberration). Third-order terms for a plasma lens are not in TRANSPORT at this time. A higher-order calculation is requested with the `ORDER` element, described on page 91.

The plasma lens is not a MAD element. However, it can be described using the same parameters as a quadrupole. It therefore can be expressed in MAD-like notation, other keyword notation, or in original TRANSPORT notation.

MAD-like Notation

The only reason that the notation is described as MAD-like is that this element is not in the MAD program. The keywords which can be used in specifying a plasma lens are also the same as for a quadrupole. They are here repeated for convenience. A plasma lens is specified by two real attributes.

label: `PLASMA,  L` = value, `K1` = value ;

A typical plasma lens specification might then look like:

```
P1:   PLAS,  L = 5.,  K1 = .01  ;
```

Symbol	Keyword	Description
L	`L`	The magnet length (normally meters).
K_1	`K1`	The normalized quadrupole strength (normally in meters^{-2}).

The units of any of the physical parameters can be changed to any desired. Other unit set specifications are described on page 71. Individual unit changes are described on page 75. The length and normalized quadrupole strength K_1 of the plasma lens can be varied in first-order fitting.

Other Keyword Options

The same plasma lens can also be specified by other sets of variables. The first of the two following options corresponds to the original TRANSPORT variables expressed in keyword notation

label: `PLASMA,  L` = value , `B` = value , `APER` = value ;

```
label: PLASMA, L = value , GRAD = value ;
```

Symbol	Keyword	Description
L	L	The magnet length (normally meters).
B_o	B	The magnetic field (normally kilogauss).
a	APER(TURE)	The magnetic aperture (meters with UMAD, cm with UTRANS).
g	GRAD(IENT)	The gradient of the magnetic field (kg/m with UMAD, kg/cm with UTRANS).

Global unit set specifications other than UMAD or UTRANS may also be selected and are described on page 71. Individual units changes are described on page 75.

For a beam momentum of 600 GeV/c, the following three elements are all equivalent.

```
P1:   PLAS,  L = 5.,  B = 10.,  APER = 5.   ;

P1:   PLAS,  L = 5.,  GRAD = 2.   ;

P1:   PLAS,  L = 5.,  K1 = .01  ;
```

Here we assume that the aperture is measured in cm. The description in terms of K_1 has the advantage that it does not require a separate specification of the beam momentum. The quantities K_1 and g are related to the pole tip field B_o and the aperture a by:

$$K_1 = \frac{g}{(B\rho)} = \frac{1}{(B\rho)} \frac{B_o}{a}$$

The length L, magnetic field B, and gradient g of a plasma lens may all be varied in first-order fitting. The aperture a may not be. The variation of a physical parameter is indicated by the VARY command, described on page 259.

Original TRANSPORT Notation

There are four parameters to be specified.

1. PLAS(MA).
2. (Effective) magnet length L (in meters).
3. Field at pole tip B (in kG). A positive field implies focusing; a negative field, defocusing.
4. Half-aperture a (meters in UMAD, cm in UTRANS). A normalizing radius used to calculate the gradient.

Once again, the aperture is assumed here to be in cm. If the global unit set **UTRANS** is specified, the units for L, B and a are meters, kG, and cm, respectively. If the unit set **UMAD** is specified, the corresponding units are meters, kG, and meters respectively. Other units may be chosen via the appropriate **UNIT** element preceding the **BEAM** element.

The input format for a typical plasma lens element is then:

```
                        ┌──── Label (if desired)
                        ↓
PLAS   L   B   a   '   '   ;
```

The same plasma lens as in the above examples can now be represented as

```
PLASMA 5. 10. 5. 'Q1' ;
```

The length L and field B or gradient g of a plasma lens may be varied in first-order fitting. The normalizing radius a may not be. In original TRANSPORT notation, the variation of a physical parameter is indicated by a vary code, described on page 266.

SOLENOID

The solenoid is most often used as a focusing element in systems passing low-energy particles. The solenoid transformation is also used in conjunction with particle detectors in interaction regions of particle colliders.

Particles in a solenoidal field travel along helical trajectories. The solenoid fringing field effects necessary to produce the focusing are included.

First- and second-order matrix elements for a solenoid are included in TRANSPORT. The second-order matrix elements arise from the momentum variation of the first-order elements. Third-order matrix elements for a solenoid have yet to be included in TRANSPORT. The order of the calculation is indicated by the `ORDER` element described on page 91.

The `SOLENOID` is a MAD element. It can be expressed in MAD notation, other keyword notation, or original TRANSPORT notation.

MAD Notation

For the MAD program, a solenoid is specified by two real attributes.

```
label: SOLENOID, L = value , KS = value ;
```

A typical solenoid specification might then look like:

```
SOL1:   SOLEN,  L = 1.0,  KS = .00025  ;
```

Symbol	Keyword	Description
L	L	Effective length of the solenoid (meters).
θ	KS	The solenoid strength in terms of the reference momentum rotation angle (radians with `UMAD`, degrees with `UTRANS`).

The units of any of the physical parameters can be changed to any desired. Other units may be selected either by use of a different global unit specification (page 71) or individual unit changes (page 75).

The length L and solenoid strength θ may be varied in first-order fitting.

Other Keyword Options

A solenoid may alse be specified by its length and field.

```
label: SOLENOID, L = value , B = value ;
```

The same example as above would then be written as:

```
SOL1:   SOLEN,  L = 1.0,  B = 10.0  ;
```

Here we have assumed a beam momentum of 600 GeV/c. This is a higher value than than the momentum used in most solenoid applications. However, we continue to use it here for consistency with the other sections.

Symbol	Keyword	Description
L	L	Effective length of the solenoid (normally meters).
B	B	The field (normally kilogauss).

The units of either physical parameter can be changed with a `UNIT` command as described on page 75.

The relation among its length and field and its rotation angle is

$$\frac{BL}{(B\rho)} = 2\theta$$

If the reference coordinate system is rotated by the angle θ after the solenoid, then the transfer matrix will take the block diagonalized form similar to that of a quadrupole.

The solenoid length L and magnetic field B may be varied in first-order fitting

Original TRANSPORT Notation

There are three parameters:

1. `SOLE(NOID)` (or type code `19.0`)
2. Effective length of the solenoid (meters).
3. The field (kG). A positive field, by convention, points in the direction of positive z for positively charged particles.

The units are meters and kG in both global unit sets `UMAD` and `UTRANS`. If other units are desired they can be chosen via other global units specifications or the appropriate `UNIT` element preceding the `BEAM` element.

The length and magnetic field may be varied in first-order fitting. In original TRANSPORT notation, the variation of a physical parameter is indicated by a vary code, described on page 266.

A typical input format is:

```
                       ┌──── Label (if desired)
                       ↓
SOLENOID  L  B   '   '   ;
```

The example used earlier now appears as:

```
SOLEN  1.0  10.0  ;
```

First-Order Solenoid Matrix *

Definitions: L = effective length of solenoid.
$K = B_0/(2B\rho_0)$, where B_0 is the field inside the solenoid and $(B\rho_0)$ is the magnetic rigidity (momentum) of the central trajectory.
$C = \cos KL$
$S = \sin KL$

$$R(\text{Solenoid}) = \begin{pmatrix} C^2 & \frac{1}{K}SC & SC & \frac{1}{K}S^2 & 0 & 0 \\ -KSC & C^2 & -KS^2 & SC & 0 & 0 \\ -SC & -\frac{1}{K}S^2 & C^2 & \frac{1}{K}SC & 0 & 0 \\ KS^2 & -SC & -KSC & C^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

*For a derivation of this transformation see report SLAC-4 by R. Helm [14].

Alternate Forms of Matrix Representation of the Solenoid:

Rotating the transverse coordinates about the z axis by an angle $= -KL$ decouples the x and y first-order terms, i.e.

$$R(-KL) \cdot R(\text{Solenoid}) = \begin{pmatrix} C & \frac{1}{K}S & 0 & 0 & 0 & 0 \\ -KS & C & 0 & 0 & 0 & 0 \\ 0 & 0 & C & \frac{1}{K}S & 0 & 0 \\ 0 & 0 & -KS & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where $KS = \frac{1}{F}$, and $F =$ focal length of solenoid.

ACCELERATION

An energy gain is reflected in both the divergence and the width of the beam. This element provides a simulation of a travelling wave linear accelerator energy gain over a field free drift length (i.e. no externally applied magnetic field).

The energy of the reference trajectory is assumed to increase linearly over the entire accelerator length. If this is not the case, an appropriate model may be constructed by combining separate `ACCEL` elements. An `ACCEL` element with a zero energy gain is identical to a drift length.

First-order matrix elements are included in TRANSPORT for the accelerator section. Second- or third-order matrix elements have not been incorporated in the program.

The `ACCEL` element is MAD compatible. It can be expressed in MAD notation, other keyword notation, or in original TRANSPORT notation.

MAD Notation

For the MAD program, a travelling wave linear accelerator element is specified by four real attributes.

label: `ACCEL,  L = value ,  VOLT = value ,  LAG = value ,  FREQ = value ;`

A typical accelerator element might then look like:

```
ACCEL,  L = 10.0,  VOLT = 0.1,  LAG = 30.0,  FREQ = 30.   ;
```

Symbol	Keyword	Description
L	L	Accelerator length (meters).
	VOLT	Energy gain (GeV) at crest of wave.
ϕ	LAG	Phase lag (degrees).
ν	FREQ	The RF frequency in MHz (no default value).

The units of only the length and the energy gain may be changed. The different unit set specificiations are described on page 71. Individual unit changes are described on page 75. None of the parameters can be varied.

Other Keyword Notation

The MAD parameters are very close, but not identical to the original TRANSPORT parameters. The original TRANSPORT set of parameters can also be expressed in keyword notation by four attributes.

label: `ACCEL,` `L` = value , `VOLT` = value , `LAG` = value , `WAVE` = value ;

The following two parameterizations of the accelerator element are equivalent.

```
ACCEL, L = 10.0, VOLT = 0.1, LAG = 30.0, FREQ = 30.    ;

ACCEL, L = 10.0, VOLT = 0.1, LAG = 30.0, WAVEL = 1000.0  ;
```

Here we have taken the units of wavelength to be cm.

Symbol	Keyword	Description
L	`L`	Accelerator length (meters).
	`VOLT`	Energy gain (GeV) at crest of wave.
ϕ	`LAG`	Phase lag (degrees).
λ	`WAVELENGTH`	Wavelength (meters in `UMAD`, cm in `UTRANS`).

The units for the length, the energy gain, and the wavelength may be changed. The beam unit of longitudinal separation is used for the wavelength and may be changed by the `UNIT` command described on page 75. None of the parameters can be varied.

Original TRANSPORT Notation There are five parameters:

1. `ACCE(LERA)` (or type code `11.0`)
2. Accelerator length (meters).
3. Energy gain (GeV) at crest of wave.
4. ϕ (phase lag in degrees).
5. λ (wavelength – meters in `UMAD`, cm in `UTRANS`).

The units given for the five parameters are those given by the global units specification `UTRANS`. Other units sets may be specified by the use of `UMAD`, `UMETER`, `UMM`, etc. (See page 71.) The units of parameters 2, 3 and 5 may be changed individually by (`UNIT 8.`), (`UNIT 11.`), and (`UNIT 5.`) elements respectively. (For details, see page 75.)

The new beam energy is taken as the new energy of the reference particle and is printed as output. The energy gain of the reference particle is $\Delta E \cos\phi$.

None of the parameters can be varied.

Accelerator Section Matrix for Effectively Massless Particles

This matrix assumes that $E_0 \gg m_0 c^2$.

$$
\begin{bmatrix}
1 & \left[L \dfrac{E_o}{\Delta E \cos\phi} \log\left(1 + \dfrac{\Delta E \cos\phi}{E_o}\right)\right] & 0 & 0 & 0 & 0 \\[2ex]
0 & \dfrac{E_o}{E_o + \Delta E \cos\phi} & 0 & 0 & 0 & 0 \\[2ex]
0 & 0 & 1 & \left[L \dfrac{E_o}{\Delta E \cos\phi} \log\left(1 + \dfrac{\Delta E \cos\phi}{E_o}\right)\right] & 0 & 0 \\[2ex]
0 & 0 & 0 & \dfrac{E_o}{E_o + \Delta E \cos\phi} & 0 & 0 \\[2ex]
0 & 0 & 0 & 0 & 1 & 0 \\[2ex]
0 & 0 & 0 & 0 & \left(\dfrac{\Delta E \sin\phi}{E_o + \Delta E \cos\phi}\right)\left(\dfrac{2\pi}{\lambda}\right) & \dfrac{E_o}{E_o + \Delta E \cos\phi}
\end{bmatrix}
$$

Definitions:

- L = effective length of accelerator sector.
- E_0 = particle energy at start of sector.
- ΔE = Energy gain over sector length at crest of accelerating wave.
- ϕ = phase lag of the reference particle behind the crest of the accelerating wave, i.e. if ϕ is positive then for some $\ell > 0$ the particles having this value are riding the crest of the wave; the units of ϕ are degrees.
- λ = Wavelength of accelerating wave; the units of λ are those of ℓ (normally cm).

Accelerator Section Matrix for Massive Particles

The nonrelativistic case involves more complicated matrix elements, so the expressions for the elements must be given individually: The case for massless particles can be solved exactly. The massive particle case can be solved only approximately. The solution given here is based on the WKB approximation of quantum mechanics.

$$R_{11} = R_{33} = \left(\frac{\eta_f}{\eta_i}\right)^{\frac{1}{4}} \cosh\alpha_1 - \frac{1}{4}\,\frac{\eta_i'}{\eta_i}\,\frac{\eta_i^{\frac{3}{2}}}{\sqrt{Q}}\left(\frac{\eta_f}{\eta_i}\right)^{\frac{1}{4}} \sinh\alpha_1$$

$$R_{12} = R_{34} = \frac{\eta_i^{\frac{3}{2}}}{\sqrt{Q}}\left(\frac{\eta_f}{\eta_i}\right)^{\frac{1}{4}} \sinh\alpha_1$$

$$R_{21} = R_{43} = \frac{1}{4}\left[\frac{\eta_f'}{\eta_f^{\frac{3}{4}}\eta_i^{\frac{1}{4}}} - \frac{\eta_i'\eta_i^{\frac{1}{4}}}{\eta_f^{\frac{5}{4}}}\right]\cosh\alpha_1 + \left[\left(\frac{\eta_f}{\eta_i}\right)^{\frac{1}{4}}\frac{\sqrt{Q}}{\eta_f^{\frac{3}{2}}} - \frac{1}{16}\,\frac{\eta_f'\eta_i'\eta_i^{\frac{1}{4}}}{\eta_f^{\frac{3}{4}}\sqrt{Q}}\right]\sinh\alpha_1$$

$$R_{22} = R_{44} = \frac{1}{4}\,\frac{\eta_i^{\frac{3}{2}}}{\sqrt{Q}}\,\frac{\eta_f'}{\eta_f^{\frac{3}{4}}\eta_i^{\frac{1}{4}}}\sinh\alpha_1 + \left(\frac{\eta_i}{\eta_f}\right)^{\frac{5}{4}}\cosh\alpha_1$$

$$R_{55} = \frac{\beta_f}{\beta_i}\left(\frac{\eta_i}{\eta_f}\right)^{\frac{3}{4}}\left[\cos\alpha_2 + \frac{3}{4}\,\frac{\eta_f'\eta_f^{\frac{1}{2}}}{\sqrt{Q_L}}\sin\alpha_2\right]$$

$$R_{56} = -\frac{\beta_f\lambda}{2\pi}\,\frac{L}{\Delta E\sin\phi_s}\left\{\frac{3}{4}\left[\left(\frac{\eta_i}{\eta_f}\right)^{\frac{1}{4}}\eta_f' - \left(\frac{\eta_i}{\eta_f}\right)^{\frac{3}{4}}\eta_i'\right]\beta_i\cos\alpha_2\right.$$

$$\left. - \left[\frac{9}{16}\,\frac{\eta_i'\eta_f'}{\eta_f^{\frac{1}{4}}\eta_i^{\frac{1}{4}}}\,\frac{\eta_i}{\sqrt{Q_L}} + \left(\frac{\eta_i}{\eta_f}\right)^{\frac{1}{4}}\frac{\sqrt{Q_L}}{\eta_f^{\frac{1}{2}}}\right]\beta_i\sin\alpha_2\right\}$$

$$R_{65} = \frac{\eta_i^{\frac{3}{4}}}{\eta_f^{\frac{1}{4}}\beta_i\beta_f}\sqrt{Q_L}\sin\alpha_2$$

$$R_{66} = \left(\frac{\eta_i}{\eta_f}\right)^{\frac{1}{4}}\frac{\beta_i}{\beta_f}\left[\cos\alpha_2 - \frac{3}{4}\,\frac{\eta_i^{\frac{1}{2}}\eta_i'}{\sqrt{Q_L}}\sin\alpha_2\right]$$

Definitions: L = effective length of accelerator sector.

E_0 = particle energy at start of sector.

ΔE = energy gain over sector length at crest of accelerating wave.

ϕ = phase lag of the reference particle behind the crest of the accelerating wave, i.e. if ϕ is positive then for some $\ell > 0$ the particles having this value are riding the crest of the wave; the units of ϕ are degrees.

λ = wavelength of accelerating wave; the units of λ are those of ℓ (normally cm).

$$\alpha_1 = \sqrt{Q} I_\eta$$

$$\alpha_2 = \sqrt{Q_L} I_\eta$$

$$Q = \frac{\Delta E \sin\phi}{L} \frac{\pi}{\lambda} \frac{1}{m_p c^2} , \quad Q_L = 2Q$$

$$I_\eta = \int_0^L \frac{dz}{\eta^{\frac{3}{2}}}$$

$$\approx \frac{1}{\eta_i^{\frac{3}{2}} \gamma_f'} \left[\Delta\gamma - \frac{3}{4} \frac{\gamma_i}{\eta_i^2} (\Delta\gamma)^2 - \frac{1}{4\eta_i^2} (\Delta\gamma)^3 + \frac{7}{8} \frac{\gamma_i^2}{\eta_i^4} (\Delta\gamma)^3 \right]$$

$$\Delta\gamma = \gamma_f - \gamma_i$$

$$\eta = \beta\gamma$$

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

The index i indicates initial values. The index f indicates final values, so that

$$\gamma_i = \frac{E_0}{m_p}$$

$$\gamma_f = \frac{E_0 + \Delta E \cos\phi}{m_p}$$

SEPTUM

An electrostatic septum is used to split a particle beam into two portions differently directed. Wires are strung transversely on the element $(x - z)$ midplane. The electric field above the wires is in the positive y direction and deflects the particles upward. The electric field below the wires is in the negative y direction and deflects the particles downward. There is therefore a sudden discontinuity in field direction at the plane of the wires. Such a discontinuity is not consistent with the formalism of TRANSPORT.

The `SEPTUM` element is included in TRANSPORT for compatability with the program TURTLE. In TRANSPORT this element is treated by assuming that the field above the wires defines the beam we wish to follow. The field above the wires is then considered to fill the entire space through which the beam passes. The discontinuity in the field at the wire plane is ignored.

The septum is assumed to be aligned along the incoming coordinate axes. The septum element is treated like a kick. The reference trajectory is defined as if the electric field is not present. It is a straight line continuation of the entrance reference trajectory. The beam centroid will be displaced from the reference trajectory in the direction of the electric field. The septum element will also produce a vertical dispersion. Unlike the case with magnetic elements, this dispersion will depend on the particle mass in addition to its central momentum.

Only first-order transfer matrix elements are included in TRANSPORT for the septum element.

The `SEPTUM` element is not a MAD element. It can be expressed only in keyword notation.

Keyword Notation

An electrostatic septum is specified by the following real attributes.

Symbol	Keyword	Description
L	`L`	The septum length (meters, default: 0.0 m)
V	`VOLTAGE`	The voltage across the septum gap (megavolts, default: 0.0 MV)
a	`APERTURE`	The septum gap (meters in `UMAD`, cm in `UTRANS`)
E	`E`	The electric field (Mv/m in `UMAD`, MV/cm in `UTRANS`)
Δy	`OFFSET`	The vertical offset of the wire plane (meters in `UMAD`, cm in `UTRANS`)
d	`WIRE`	The wire diameter (meters in `UMAD`, cm in `UTRANS`)
ΔL	`SPACING`	The wire spacing (normally in meters)
r	`RADL`	The radiation length in the wire (used for scattering calculations) (normally in meters)

TILT — The roll angle about the longitudinal axis (radians in UMAD, degrees in UTRANS). The septum is rotated in a clockwise sense about the entrance reference trajectory. The positive sense appears clockwise looking downstream since the z axis also points downstream. If the word TILT appears alone without a value, the angle of rotation is taken to be 90 degrees.

Global unit set specifications other than UMAD or UTRANS may also be selected and are described on page 71. Individual units changes are described on page 75.

Several of the parameters are redundant, and others are useful only for TURTLE. For TRANSPORT, a complete description of an unrotated septum can be given by:

label: SEPTUM, L = value , VOLT = value , APER = value ;

or by

label: SEPTUM, L = value, E = value ;

Assuming the UTRANS set of units, a typical septum specification might appear as:

```
SEPTUM, L = 3.0, VOLT = 0.1, APER = 1.0 ;
```

or

```
SEPTUM, L = 3.0, E = 0.1 ;
```

indicating a septum of length 3 meters and a voltage of 0.1 megavolts on a 1 cm aperture.

None of the parameters of a SEPTUM element may be varied.

MARKER – Location markers

In the MAD program, beam lines are defined by explicit commands which assemble the beam lines from previously defined components. Marker elements may be among those components. Their sole purpose is to demarcate a place in the beam line. Operations, which refer to those markers, can be defined after the beam line specification is complete. By the use of markers a beam line can be specified in an uninterrupted manner in MAD notation. The operations, not necessarily in MAD notation, will be all together after the beam line specification.

The `MARKER` is a MAD element. A marker has no additional keywords but is completely given by the word `MARKER` plus a label. Any print statements, constraints, matrix updates or other operations may refer to the markers placed in the beam line. When TRANSPORT runs through the beam line, it executes the operations when it encounters the corresponding marker.

A marker element with the label "`MAR1`" would have the appearance:

```
MAR1:   MARKER ;
```

These markers may then be inserted into a beamline or subline defined by the `LINE` statements using the MAD procedure for assembling beam lines. This procedure is described on page 215. Reference to the markers can be made by any of the elements specifying operations. These operations include the `UPDATE` element, fitting constraints (the `FIT` element), input-output options (the `PRINT` element), the storage of matrix elements (`STORE` element), the alignment marker (`ALMARK`), and the plot command (`PLOT`).

Other Transformations

MATRIX — Arbitrary Transformation Input

To allow for the use of empirically determined fringing fields and other specific (perhaps nonphase-space-conserving) transformations, provision has been made for reading in an arbitrary transformation matrix. First-, second-, and third-order matrix elements may be included. The order that will be used in the program is determined by the order parameters on the `ORDER` element described on page 91.

The first-order 6×6 matrix is read in row by row.

The arbitrary matrix element is specifiable only in original TRANSPORT notation.

Original TRANSPORT Notation

There are eight parameters for each row of a first-order matrix entry:

1	`MATR(IX)` (or type code `14.0`)
2 to 7	The six numbers comprising the row.
8	Row number (1. to 6.)

The units must be those used to print the transfer matrix; in other words, consistent with the `BEAM` input/output. The `BEAM` element and the units used for the specification of the input phase space are described on page 105.

A complete matrix must be read and applied one row at a time. Rows that do not differ from the unit transformation need not be read.

For example,

```
MATRIX  -.1  .9  0.   0.   0.   0.   2.   ;
```

introduces a transformation matrix whose second row is given but which is otherwise a unit matrix. Note that this transformation does not conserve phase space because $R_{22} = 0.9$, i.e. the determinant of $R \neq 1$.

Usage

Any of the components of a row may be varied; however, there are several restrictions.

`MATRIX` elements that immediately follow one another will all be used to form a single transformation matrix. If distinct matrices are desired, another element must be inserted to separate the **MATRIX** elements. Several do-nothing elements are available; for example, a zero length drift (`DRIFT 0. ;`) is a convenient one.

When the last of a sequence of **MATRIX** cards is read, the assembled transformation matrix will be printed in the output. Note that

$$\begin{pmatrix} 1 & 0 \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} a_{11} & a_{12} \\ 0 & 1 \end{pmatrix} \neq \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

Hence, a matrix formed by successive (`MATRIX ; DRIFT 0. ; MATRIX ;`) – elements is not always equal to the one formed by leaving out the (`DRIFT 0. ;`) element.

If components of a **MATRIX** card are to be varied, they must be specified on the last **MATRIX** card for the given matrix. Components to be varied can occur in any single row, as the rows do not have to be specified in order (i.e. the second row can be specified before the first, etc.) If more than one row is to have variable components, the user must therefore split the matrix into factors .

Second Order

If it is desired to include the second-order matrix coefficients for the i^{th} row, then the following 22 additional numbers may be read in.

9 continuation code `0.`

10 to 30 The 21 coefficients:

T_{i11} T_{i12} T_{i13} T_{i14} T_{i15} T_{i16}
T_{i22} T_{i23} T_{i24} T_{i25} T_{i26} T_{i33}
T_{i34} T_{i35} T_{i36} T_{i44} T_{i45} T_{i46}
T_{i55} T_{i56} T_{i66}

in that order, where i is the row number. It is necessary to read in the first-order matrix row which corresponds to the second-order matrix row being read in.

As in the first-order case, full rows not different from the identity matrix (i.e., $R_{ii} = 1$, all other $R_{ij} = 0$, and all $T_{ijk} = 0$) need not be read in. The second-order arbitrary matrix elements can not be varied.

Third Order

It is also possible to include the third-order matrix coefficients for the i^{th} row by reading in the following 57 additional numbers.

31 continuation code `0.`

32 to 87 The 56 coefficients:

U_{i111} U_{i112} U_{i113} U_{i114} U_{i115} U_{i116}
U_{i122} U_{i123} U_{i124} U_{i125} U_{i126} U_{i133}

U_{i134} U_{i135} U_{i136} U_{i144} U_{i145} U_{i146}
U_{i155} U_{i156} U_{i166} U_{i222} U_{i223} U_{i224}
U_{i225} U_{i226} U_{i233} U_{i234} U_{i235} U_{i236}
U_{i244} U_{i245} U_{i246} U_{i255} U_{i256} U_{i266}
U_{i333} U_{i334} U_{i335} U_{i336} U_{i344} U_{i345}
U_{i346} U_{i355} U_{i356} U_{i366} U_{i444} U_{i445}
U_{i446} U_{i455} U_{i456} U_{i466} U_{i555} U_{i556}
U_{i566} U_{i666}

in that order, where i is the row number. It is now necessary to read in both the first- and second-order matrix rows which correspond to the third-order matrix row being read in.

The third-order arbitrary matrix elements can not be varied.

SROT — Coordinate Rotation

The transverse coordinates x and y may be rotated through an angle α about the z axis (the axis tangent to the central trajectory at the point in question.* Thus a rotated bending magnet, quadrupole, or sextupole may be inserted into a beam transport system by preceding and following the element with the appropriate coordinate rotation. (See examples below.) The positive sense of rotation is clockwise looking in the direction of the positive z axis, i.e. the direction of beam travel.

The coordinate rotation is an exact first-order transformation. There are no higher order terms. However, the coordinate rotation will affect the higher-order terms from the elements that precede it.

The `SROT` element is used in MAD. Since there is only parameter to be specified, there is no other keyword notation. The `SROT` element can be expressed in either MAD notation or in original TRANSPORT notation.

MAD Notation

For the MAD program, a rotation about the longitudinal axis is specified by a single physical attribute:

```
label: SROT,  ANGLE=value ;
```

A rotation of 90° expressed in radians (using the global unit set `UMAD`) would then be:

```
SROT,  ANGLE = 1.570796327  ;
```

and in degrees (using the global unit set `UTRANS`) would be

```
SROT,  ANGLE = 90.   ;
```

Symbol	Keyword	Description
ψ	`ANGLE`	The angle of rotation (radians in `UMAD`, degrees in `UTRANS`, default: 0.0 rad)

The units of the rotation angle can be changed to any desired. Other unit set specifications are described on page 71. Individual unit changes are described on page 75.

The angle of rotation may be varied in first-order fitting. The variation of physical parameters is specified in keyword notation with the `VARY` command. (For details, see page 259.)

*See SLAC-75 [4], page 45 and page 12, fig. 4, for definitions of x, y, and z coordinates.

Original TRANSPORT Notation

There are two parameters to be specified for a coordinate rotation:

1. `SROT` (or type code 20) (signifying a beam coordinate rotation).
2. The angle of rotation α (radians with **UMAD**, degrees with **UTRANS**).

Other global unit sets or individual units changes are described on pages 71 and 75 respectively.

The angle of rotation may be varied in first-order fitting. In original TRANSPORT, the variation of a parameter is specified by a vary code. (See the **VARY** section on page 266).

Note:

This transformation assumes that the units of (x and y) and (x' and y') are the same. This is always true unless a (`UNIT 3.0`) or a (`UNIT 4.0`) element has been used.

Printing Transfer and Beam Matrices

In early versions of TRANSPORT the transfer and beam matrices and the floor coordinates were always printed in the rotated system. The transfer matrix elements would have mysterious gaps or regions of value zero when traversing a vertically bending magnet. These gaps occurred because the transfer matrix temporarily became block off-diagonalized when the coordinate system was rotated by 90 degrees.

It is now possible to have all these quantities printed in the unrotated system. Then the directions of vertical and horizontal will remain fixed even if the beam line contains many magnets rotated about the beam axis at various angles. The transfer matrix elements and the accelerator functions β, α, and η will now have the physical meaning one would normally expect them to have. The command

```
PRINT, REFER ;
```

placed at the beginning of the data, will cause the various matrices and coordinates to be printed in the unrotated system. The constraints will also pertain to the unrotated system.

Examples:

For a bending magnet, the beam rotation matrix may be used to specify a rotated magnet. In all these examples, original TRANSPORT notation is either used or implied. With keyword notation, the rotation of certain magnetic elements can also be done by the use of the TILT parameter on the element specification. Bending magnets, including fringing fields, can be represented as a single RBEND or SBEND element. (See page 144) These two elements, along with the QUAD, SEXT, OCT, and SEPTUM elements, all can be rotated with a TILT parameter.

Example No. 1

A bend up is represented by rotating the x,y coordinates by -90.0 degrees as follows:

```
              ┌──── Label if desired
              ↓
SROT  -90.   ' '  ;
ROTAT β1  ' '  ;
BEND L  B  n  ' '   ;
ROTAT β2  ' '  ;
SROT  +90.   ' '  ;   (returns coordinates to their initial orientation)
```

A bend down is accomplished via a $+90$ degree rotation.

```
SROT  +90.  ' ' ;
ROTAT
BEND
ROTAT
SROT  -90.   ' '  ;
```

A bend to the left (looking in the direction of beam travel) is accomplished by rotating the x,y coordinates by 180 degrees, e.g.

```
SROT  +180.  ' '  ;
ROTAT
BEND
ROTAT
SROT  -180.  ' '  ;
```

Example No. 2

A quadrupole rotated clockwise by 60 degrees about the positive z axis would be specified as follows:

```
SROT  +60.  ' '  ;
QUAD  L B a ' '   ;
SROT  -60.  ' '  ;
```

Beam rotation matrix

$$R = \begin{pmatrix} C & 0 & S & 0 & 0 & 0 \\ 0 & C & 0 & S & 0 & 0 \\ -S & 0 & C & 0 & 0 & 0 \\ 0 & -S & 0 & C & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

where $C = \cos\alpha$,
$S = \sin\alpha$,
α = angle of coordinate rotation about the beam axis.

e.g. for $\alpha = +90$ degrees, this matrix interchanges rows 1 and 2 with 3 and 4 of the accumulated R matrix as follows:

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} R(11) & R(12) & R(13) & R(14) \\ R(21) & R(22) & R(23) & R(24) \\ R(31) & R(32) & R(33) & R(34) \\ R(41) & R(42) & R(43) & R(44) \end{bmatrix} = \begin{bmatrix} R(31) & R(32) & R(33) & R(34) \\ R(41) & R(42) & R(43) & R(44) \\ -R(11) & -R(12) & -R(13) & -R(14) \\ -R(21) & -R(22) & -R(23) & -R(24) \end{bmatrix}$$

(The rest of the matrix is unchanged.)

SHIFT — Shift in the Reference Coordinate System

If a particle beam is transferred from one optical system to another, there may be a discontinuity in the reference coordinate systems used. The centroid coordinates may change at a given point due to this discontinuity. The effect may be parameterized by the change in the centroid coordinates. The `SHIFT` element is specified in exactly the same manner as the `CENTROID` element, except that the keyword here is `SHIFT` instead of `CENTROID`.

The `SHIFT` element does affect the floor coordinates. This property is the main difference between the `CENTROID` and the `SHIFT` elements.

The `SHIFT` element is not in MAD. It may be expressed in either keyword notation or in original TRANSPORT notation.

Keyword Notation

The mnemonic used in specifying a shift in the reference coordinate system is `SHIFT`. The keywords which can be used are

Symbol	Keyword	Description
x	X	The shift in the x coordinate (meters with UMAD, cm with UTRANS)
x'	XP	The shift in x' (radians with UMAD, mr with UTRANS)
y	Y	The shift in y (meters or cm)
y'	YP	The shift in y' (radians or mr)
ℓ	L	The shift in ℓ. (meters or cm)
δ	DEL	The shift in δ (fractional with UMAD, percent with UTRANS).

The six parameters correspond exactly with the six positional parameters used in the original style TRANSPORT input. The units are the same as for the `BEAM` element (page 105. Other unit sets in addition to those mentioned above can be used and are described on page 71. Individual units changes are described on page 75.

An example of a downward kink in the reference trajectory increasing the vertical angle by 1.0 milliradians might be

```
SHIFT,  YP = 1.0  ;
```

Any of the parameters can be varied in first-order fitting. The floor coordinates or centroid position at any subsequent point in the beam line can be constrained.

Original TRANSPORT Notation

Seven parameters are required:

1	SHIFT
(2 to 7)	The coordinates x, x', y, y', ℓ, and δ defining the shift in the location of the beam centroid with respect to its previous position.

The units for x, x', y, y', ℓ, δ are the same as those chosen for the BEAM element. With the global unit set UMAD the units are meters, radians, meters, radians, meters, and fractional. With the global unit set UTRANS they are cm, mr, cm, mr, cm, and percent. Other global unit sets are described on page 71.

Another input representing the same vertical kink in the beam might be

```
SHIFT  0.0  0.0  0.0  1.0  0.0  0.0  ;
```

Any or all of the six reference coordinate system shift parameters may be varied in first-order fitting. The centroid position or reference trajectory floor coordinates can then be constrained at any later point in the beam line.

Usage

As with a centroid shift, the presence of a reference coordinate system shift in the data will automatically cause the transfer matrix to be expanded about the new beam centroid position. Both the order used in calculating the transfer matrix, and the order to which it is expanded about the new reference trajectory are specified by the ORDER element.

ALIGN — Magnet Alignment Tolerances

The first-order effects of the misalignment of a magnet or group of magnets are a shift in the centroid of the beam and a change in the beam focusing characteristics. The transverse displacement of any focusing device, such as a quadrupole, will cause a deflection of the beam centroid, and therefore a downstream transverse shift in the position of the beam ellipse. A longitudinal shift of a quadrupole, or a rotation of a quadrupole about its optic axis, or a transverse displacement of a sextupole will cause a change in the beam focusing characteristics. This change will appear as a change in the transfer matrix or in the dimensions of the beam ellipse. More on this subject will be found later in this section under "The Effects of Misalignments."

Three varieties of misalignment simulation are commonly needed:

(1) The physical position of the magnet is uncertain within a given tolerance.
or (2) The magnet is displaced and/or rotated by a known amount;
or (3) The magnet is displaced and/or rotated by a definite but randomly chosen amount.

The results may be displayed in either of two ways:

(1) In the printed output of the beam (σ) matrix, including the beam centroid.
or (2) Tabulated in a special misalignment table (described below).

Three types of units may be misaligned:

(1) A single bending magnet, quadrupole, sextupole, or octupole
or (2) A section of a beam line
or (3) All bending magnets and/or quadrupoles)

In addition, there is some choice among coordinate systems to which the misalignment is to be referred.

The element `ALIGN` is a TRANSPORT element but is not a MAD element. In TRANSPORT it may be expressed either in keyword notation or in original TRANSPORT notation.

Keyword Notation

The mnemonic for magnet misalignments is `ALIGN`. The keywords which can be used in specifying the parameters describing the misalignments are given below. The parameters consist of three displacements and three rotations plus a code number indicating the type of misalignment. There is one parameter for a displacement along and one parameter for a rotation about each of the coordinate axes x, y, and z.

Global unit set specifications other than `UMAD` or `UTRANS` may also be selected and are described on page 71. Individual units changes are described on page 75.

Symbol	Keyword	Description
Δx	X	The magnet displacement in the x direction (meters with UMAD, cm with UTRANS).
Δr_x	RX	A rotation about the x axis (radians with UMAD, mr with UTRANS).
Δy	Y	The magnet displacement in the y direction (meters or cm).
Δr_y	RY	A rotation about the y axis (radians or mr).
Δz	Z	The magnet displacement in the z (beam) direction (meters or cm).
Δr_z	RZ	A rotation about the z axis (radians or mr).
	CODE	A three-digit code number (defined below) specifying the type of misalignment among the options listed above.

The seven parameters correspond exactly with the seven positional parameters used in the original TRANSPORT input. Any parameters not specified are taken to equal zero. The CODE parameter will be described separately below. Any of the physical parameters may be varied to satisfy some constraint later in the beam line. (See page 259 a description of the procedure for varying parameters.)

An example of an element in keyword notation which specifies an uncertainty of 2 mrad in roll of the preceding magnet is

```
MIS1:   ALIGN,  RZ = 2.0,  CODE = 0  ;
```

A roll is a rotation about the longitudinal or z axis.

Original TRANSPORT Notation

There are eight parameters to be specified. The parameters consist of three displacements and three rotations plus a code number indicating the type of misalignment. There is one parameter for a displacement along and one parameter for a rotation about each of the coordinate axes x, y, and z.

1. **ALIGN** (or type code 8.0) (specifying a misalignment).
2. The magnet displacement in the horizontal direction (meters in UMAD, cm in UTRANS).
3. A rotation about the horizontal axis (radians in UMAD, mr in UTRANS).
4. A displacement in the vertical direction (meters or cm).
5. A rotation about the vertical axis (radians or mr).
6. A displacement in the beam direction (meters or cm).

7. A rotation about the beam direction (radians or mr).
8. A three-digit code number (defined below) specifying the type of misalignment among the options listed above.

The units shown are those for either the `UMAD` global set of units or the `UTRANS` global set of units. Other global unit sets are described on page 71. Individual units changes are described on page 75. If the units are changed, the units of the misalignment displacements are those determined by the (`UNIT 1. `) element; the units for the misalignment rotations are those determined by the (`UNIT 2. `) element.

An element in original TRANSPORT notation which specifies an uncertainty of 2 mrad in roll of the preceding magnet is

```
ALIGN  0.0  0.0  0.0  0.0  0.0  2.0  000  'MIS1'  ;
```

A roll is a rotation about the longitudinal or z axis.

The tolerances may be varied. Thus, type-vary code `ALIGN.111111` permits any of the six parameters (2 through 7 above) to be adjusted to satisfy whatever beam constraints may follow. The fitting capability allows the user to reverse the procedure of the calculation of tolerances. Given the allowable displacements of the beam centroid at some downstream point, he (or she) can determine the magnitude of misalignment that will keep the centroid displacement within the given tolerance.

For fitting, a misalignment specification must pertain to a single magnet or single section of the beam line, and the results must be displayed in the beam (σ) matrix. (See page 266 for a discussion of the use of vary codes.)

The Effects of Misalignments

The transverse displacement of any focusing device will cause a deflection of the beam centroid. The deflection will then result in a transverse shift in the position of the beam ellipse at all points downstream. Focusing devices include quadrupoles, sextupoles, and octupoles. A bending magnet serves as a focusing device with two exceptions: (1) A uniform field sector magnet (with the pole faces perpendicular to the reference trajectory) does not focus vertically, and (2) A uniform-field rectangular bending magnet does not focus horizontally.

A horizontally displaced quadrupole will induce a horizontally bending dipole component. The beam centroid will suffer a horizontal deflection due to this bending component. A vertically displaced quadrupole will induce a vertically bending dipole component. A displacement of a quadrupole in any direction does not affect the quadrupole component of the magnetic field, nor does it induce field components of order higher than first.

The deflection of the centroid by a transversely displaced sextupole is an effect of second-order in the field expansion of the sextupole. Therefore, if a displaced sextupole is to have a visible effect in the program output, the order of the calculation, as specified by the ORDER element, must be two or greater. The order of the calculation is given by the first parameter on the ORDER element. (See page 91.)

The displaced sextupole induces both dipole and quadrupole components about the aligned reference trajectory. The effect of the induced field components will be exhibited both as a shift in the beam centroid and as a change in the first-order transfer matrix due to the focusing effect of the induced quadrupole. There will be no change in the second or higher-order transfer matrix.

A displaced octupole will similarly induce dipole, quadrupole, and sextupole effects. For these effects to be displayed in the output, the first parameter on the ORDER element must be at least three. For the induced sextupole field component to affect the printed output, the second parameter on the ORDER element must be at least two.

The rotation of any magnet about the x axis at the entrance face is equivalent to holding the position of the entrance face fixed and vertically shifting the exit face. Similarly, the entrance face rotation about the y axis is equivalent to fixing the entrance face position and horizontally shifting the exit face. The rotation of any magnet about either axis at the longitudinal midpoint consists of holding the magnet fixed at the longitudinal midpoint and displacing the entrance and exit faces in opposite directions.

The rotation of a bending magnet about either the x or y axis causes a break in the reference trajectory and therefore a downstream displacement of the beam centroid. The rotation of a quadrupole, sextupole, or octupole about the x or y axis at the entrance face causes a transverse displacement of the downstream end of the magnet. The effect is a displacement of the beam centroid which is smaller than if the magnet were transversely displaced without rotation.

The rotation of a quadrupole, sextupole, or octupole about a transverse axis (x or y) at the longitudinal midpoint also causes a deflection of the beam centroid. However, this deflection is typically much smaller than is caused by a rotation about a a transverse axis at the magnet entrance or exit face.

The quadrupole component which is induced by a displaced sextupole causes a change in the first-order transfer matrix and a corresponding alteration in the dimensions of the beam ellipse. The horizontal displacement of a sextupole will induce a midplane-symmetric quadrupole component. The midplane-symmetric quadrupole component may be corrected by the placement of another midplane-symmetric quadrupole elsewhere in the beam line lattice.

A vertically displaced sextupole will induce a skew quadrupole component (a quadrupole rotated by 45 degrees) and a midplane symmetric dipole component. The skew quadrupole component changes the focal characteristics of the beam line by causing a first-order mixing of the horizontal and vertical planes. The effect of the skew induced quadrupole may be correctable by another skew quadrupole placed at the appropriate phase position in the beam line.

The quadrupole component induced by a displaced octupole will cause a change in the first-order transfer matrix. The similarly induced sextupole component will cause a change in the second order transfer matrix. Both induced components will alter the dimensions of the calculated beam ellipse. A horizontally displaced octupole will induce midplane symmetric dipole, quadrupole, and sextupole components. A vertically displaced octupole will induce a vertically bending dipole, a midplane-symmetric quadrupole component, and a skew sextupole component (a sextupole rotated by 30 degrees).

In general, a horizontally shifted midplane-symmetric multipole of any order will induce midplane-symmetric multipoles of all lower orders, but of no higher order. A vertically shifted midplane-symmetric multipole of any order will also induce multipoles of all lowers, but of no higher order. In the vertically shifted case, the lower multipoles will alternate in being midplane symmetric and skew (midplane antisymmetric). This alternation will include the original multipole.

A longitudinal shift or a rotation about the reference trajectory of a bending magnet will again cause a break in the reference trajectory and a corresponding downstream shift in the beam centroid. The bending magnet is a special case in this respect since it is the only element through which the aligned reference trajectory is not a straight line.

The longitudinal shift or a rotation about the axis of a quadrupole, sextupole, or an octupole does not have an effect on a centroid which is initially on the optic axis. It will emerge from the magnetic element still aligned on the optic axis. However, the misaligned element will have an effect on the focal characteristics of the beam. A roll of a quadrupole about its axis will cause a first-order coupling in the two transverse planes. This effect will appear in both the transfer matrix and the phase ellipse.

The effect of a roll in a sextupole will also be to cause a mixing of the two transverse planes, but of second order. Similarly, the roll of a octupole about its axis will cause a mixing of transverse planes in third order. Quadrupoles, sextupoles, and octupoles also have higher-order terms in their individual transfer matrices. Rotation of any of these elements about their longidutinal axes will therefore cause mixing of transverse planes in higher orders also.

Known Misalignments

A "known" misalignment means that the magnets have definite positions which are different from the design position for the beam line. These positions may have been determined by surveying the beam line. They may also be hypothetical, so as to determine the sensitivity of the centroid position or the beam ellipse size to the magnet alignment tolerances.

A definite but random misalignment means that that the parameters on the misalignment element are taken to specify the range of possible positions and angles describing the misalignment. The actual positions and angles are taken to be definite values within the given range. Each value on the `ALIGN` is multiplied by a random number to determine the actual magnitude of the misalignment. The random numbers are distributed uniformly from -1 to $+1$. The random number generator is the same as the one used for the `RANDOM` element, described on page 96.

The definite but random misalignment feature of TRANSPORT is used to give simultaneous but independent misalignments to the components of the beam line. It obviously makes little sense to use this feature if only one component of the beam line is being misaligned. The known but random feature should be used either when a number of components are being given separate alignment specifications or when one misalignment element is being used to misalign all subsequent bending magnets and/or quadrupoles. (This last feature is described below under "What Can Be Misaligned.")

A known misalignment, as explained above, will typically affect the beam centroid, and possibly the beam dimensions as well. The phase ellipse will represent the actual phase space volume of the beam particles. Therefore one should place on the `BEAM` card the actual dimensions of the beam entering the system. The definite but random misalignment is interpreted in the same manner as the known misalignment.

If the user specifies that the results of a known misalignment are to be displayed in the beam matrix, then the transfer matrix will be affected also. The effect of the misalignment at the entrance and exit faces of the misaligned magnet or section is treated as a known element. The resulting centroid displacement and change in the transfer matrix will be calculated and included in the accumulated transfer matrix for the beam line. The transfer matrix may be accumulated to first, second, or third order. If the floor coordinates are printed, the misaligned coordinates will be shown inside the misaligned section.

Uncertain Misalignments

An uncertain misalignment means that the magnet position is given a certain probability distribution. Rather than the magnet displacement or rotation being known, the magnet has uniform probability of being anywhere within a certain range of displacements or rotations. The results of a calculation using this type of misalignment is similarly a range of possible centroid positions.

The interpretation of the set of numbers describing what we call the "beam ellipse" depends on the nature of the misalignment being imposed. For an uncertain alignment the most common use for the beam ellipse is to represent the uncertainty in the position of the beam centroid. The initial beam dimensions should be set to zero, i.e., the beam element at the beginning of the system should appear as follows:

```
BEAM 0.   0.   0.   0.   0.   0.   p(0).
```

At positions downstream of the misaligned magnet, the actual beam centroid will be at an unknown position within the region described by the beam ellipse. The printed centroid will be the average value of the unknown centroid range. The printed centroid will be unaffected by the misalignment. The printed beam ellipse dimensions will give the range of possible positions for the unknown centroid.

The effects of each misalignment of each misaligned element is added in quadrature to the effect of the previous elements. The elements are then represented as having been misaligned independently. If the parameters on the misalignment element are taking as representing the standard deviation of a probabilistic distribution, then the dimensions of the resulting beam ellipse represent the standard deviation of a similar distribution for the position of the beam centroid.

Some types of misalignment do not show an effect on the beam centroid. For example, the roll of a quadrupole about its axis leaves the centroid undisturbed, but affects the focusing of the beam. For the quadrupole roll to have a perceptable effect a nonzero initial phase space must be specified. The effect of the roll of the magnet is then shown by a broadening of the phase ellipse. The effect on the phase ellipse is caused by an $x - y$ coupling of the rotated quadrupole. The beam ellipse at this point becomes a confused mess containing both the transformed initial phase space and the effect of the uncertainty in the roll angle of the quadrupole.

The effects of an uncertain misalignment are limited to first order in TRANSPORT. The average change in any element of the first-order transfer matrix is zero. Since the beam ellipse is a quadratic function of the transfer matrix, results of an uncertain misalignment may be seen in the beam ellipse. The alterations in the beam ellipse can come from three sources:

1. A direct shift in the beam centroid caused by the transverse displacement of a magnetic element or a rotation about a transverse axis.
2. A change in the first-order transfer matrix caused by the misalignment, such as the mixing of planes resulting from a rotation of a quadrupole about the longitudinal axis.
3. A change in the first-order transfer matrix induced by a higher-order component. An example is the quadrupole component induced by a transversely displaced sextupole magnet.

An uncertain alignment pertaining to a single magnet or section of the beam line updates (reinitializes) the R2 transfer matrix, but not the R1 matrix. The R2 matrix is an auxiliary transfer matrix to supplement R1, the normal transfer matrix. (For a more detailed explanation, see page 225.) The R2 matrix is used to calculate the beam matrix, and is also made available to users to impose constraints on portions of the beam line.

A misalignment element may also indicate that uncertain misalignments are to be given to all subsequent bending magnets and/or quadrupoles. In such cases the R2 matrix will be updated after each such misaligned magnet. If the magnets to be misaligned include bending magnets then the R2 matrix will be updated before each bending magnet for which the presence of fringing fields is indicated. Such bending magnets consist of the RBEND element, the SBEND element, and the BEND element with an adjacent ROTAT element.

Parameters Describing the Misalignment

The three displacements and three rotations described above comprise the six degrees of freedom of a rigid body and are used as the six misalignment coordinates. The displacements are taken to be along each of the three coordinate axes x, y, and z in turn. The rotations are taken to be about the same three coordinate axes. For the displacements the interpretation is straightforward. However, the interpretation of the rotations requires some care.

A rotation about the or x (horizontal) axis will not affect the x coordinate or the angle x' in the $x - z$ plane of any trajectory. It may affect the y (vertical) coordinate and will affect the (vertical) angle y'. For an element with a positive (counterclockwise) rotation about the x axis, y' will experience a sudden increase at the entrance of the misaligned element. The increase is caused by the transition from the external aligned coordinate system to the internal misaligned system. The internal misaligned system is the coordinate system that is attached to the misaligned magnet. The coordinates and angles will then be altered by the transition through the misaligned magnet.

The angle y' will also experience a sudden decrease at the exit of the misaligned element. The decrease is caused by the transition from the internal misaligned coordinate system to the external aligned system. The vertical coordinate y may also experience changes at the magnet entrance and exit faces depending on the position of the origin of the coordinate system for the magnet rotations. Similarly, a rotation about the y or vertical axis will cause a sudden decrease in x' at the entrance of the misaligned element, and a sudden increase in x' at the exit.

The default coordinate system employed is that to which the beam is referred at the point it enters the magnet. For example, a rotation of a bending magnet about the beam direction (parameter 7 above) is referred to the direction of the beam where it enters the magnet. Other reference coordinate systems may be made the default by use of some of the PRINT options described on page 248. The longitudinal midpoint may be used as the origin of the coordinate system, and also the longitudinal coordinate may be placed along the chord of a bending magnet.

What Can Be Misaligned

The misalignment of any physical element or section of a beam line may be simulated. Misaligned sections of a beam line may be nested. A beam line rotation (SROT element) may be included in a misaligned section. Thus, for example, one can simulate the misalignment of magnets that bend vertically. *The arbitrary matrix (*MATRIX *element) may not be included in a misaligned section.*

A misalignment element may indicate that a single magnet or section of the beam line is to be misaligned, or it may indicate that all subsequent quadrupoles and/or bending magnets are to be misaligned. The location of the `ALIGN` element depends on the type of misalignment. The type of misalignment is specified in the three-digit code number.

If a misalignment pertains to a single magnet or a single section of the beam line, then the misalignment element (`ALIGN`) must directly follow that magnet or section of the beam line. If a misalignment element indicates that all subsequent magnets of a given type (quad or bend) are to be misaligned, it must precede the first of such magnets. Further description of the available types of misalignment is given in the table later on page 204..

If a known misalignment is to apply to all subsequent magnets of a given type, then the results should be displayed in the misalignment table (described below). Then the magnets will be misaligned one at a time, and the effect of each will be shown separately in the misalignment table. If all magnets are given a known misalignment and the effects are combined, then the effect is no different from a situation of simply relocating the entire beam line intact. A calculation of this effect is of little use.

Displaying the Results

The results of the misalignment may be displayed in either the beam (σ) matrix or in a misalignment table. If the results are displayed in the beam (σ) matrix, then that matrix is altered by the effects of the misalignment. The effects of additional misalignments cause further alterations, so that at any point along the beam line, the beam (σ) matrix will contain the combined effects of all previous misalignments. If the misalignment is of a known amount, it will also affect the floor coordinates of the magnets within the misaligned section. In addition, a known misalignment displayed in the beam sigma matrix also shows results in the transfer matrix. The transfer matrix may be displayed up to third order, and the effect of the misalignment will be included in the calculated matrix.

The misalignment table can be used to show independently the effect on the beam matrix of a misalignment in each degree of freedom of each misaligned magnet. Each new misalignment to be entered in the table creates a new set of six duplicates of the beam matrix. Printed for each duplicate beam matrix are the centroid displacement and the beam half width in each of the six beam coordinates.

Each of the six misalignment table matrices shows the beam matrix that would result from the misalignment in a single coordinate of a single magnet or section of the beam line. In a single TRANSPORT run the results of misaligning up to ten magnets or sections of the beam line may be included in the misalignment table. Further requests for entry in the misalignment table will be ignored. Examples of such a table and the input which generated it are shown later in this section.

If the misalignment table is selected for the output of a known misalignment, then the centroid displacements will show the effect of the misalignment of any misaligned magnet or section of the beam line in each of the misalignment parameters separately. However, the beam matrix dimensions found in the misalignment table will be calculated from only the first-order terms in the transfer matrices of the elements in the beam line. These first-order terms include those induced from the displacement of higher-order elements, such as a quadrupole component induced by the transverse shift of a sextupole. They will also include those induced from higher-order elements arising from a centroid already off axis. If a misaligned quadrupole causes an off-axis beam centroid, then that off-axis centroid will result in an induced quadrupole component from a downstream aligned sextupole. However, the intrinsic second-order effect from the sextupole itself will not be included in the misalignment table. To include such higher-order effects the effect of the misalignment must be displayed in the beam matrix.

Similarly, if the misalignments are uncertain, then the beam dimensions in the misalignment table will be those resulting from the effect of the uncertainty in position of the individual elements. Only first-order effects, including induced first-order effects, will be shown in the output.

There is no point in directing the output for a known but random misalignment to the misalignment table. In the misalignment table the effect of the misalignment of each magnet of section of the beam line is shown separately. A randomization of the magnitude of the misalignment applied will only cause confusion.

The entries in the centroid column of the misalignment table for a known misalignment are sometimes referred to as the partial derivatives of the centroid position with respect to the misalignment parameters. This description is useful, but not quite correct. The centroid displacements shown are those due to the magnet misalignment. A partial derivative would be a centroid displacement divided by the magnitude of the misalignment causing it.

If a centroid shift is due to a transversely misaligned quadrupole, then the magnitude of the centroid shift will be linear in the displacement of the quadrupole. However, if a centroid shift is due to a transversely misaligned sextupole, then the magnitude of the centroid shift will be quadratic in the displacement of the sextupole. For a quadrupole, transverse displacements in opposite directions will produce centroid shifts in opposite directions. For a sextupole, transverse displacements in opposite directions will produce centroid shifts in the same direction. Care should be taken in the interpretation of the numbers in the misalignment table to insure that the effects observed are of the order expected.

Code Digit

The meaning of the options for each digit of the three-digit code number is given in the following table.

A. The interpretation of the misalignment parameters and of the resulting beam matrix dimensions is given by the hundreds position.

Code Number	Interpretation
0XX	The magnet position is uncertain. The alignment parameters give the range of uncertainty for the magnet position and orientation. The beam ellipse does not give the dimensions of a bunch of particles. It describes the region of phase space in which, for all we know, the particles might be found. If the initial phase space dimensions are all zero, then the beam ellipse gives the range of possible positions of the beam centroid.
1XX	The magnet or section of beam line is misaligned by a known amount. Both the transfer matrix and beam matrix are affected.
2XX.	A third option is a random displacement. The effect of the misalignment will be calculated in the same manner as if the parameters describing the misalignment were known. However, those parameters will be determined by multiplying the parameters on the misalignment element by random numbers. The random numbers are distributed uniformly from −1 to 1. *The parameters on the misalignment element then represent the maximum magnitude of the misalignment.* The random numbers are drawn from the same sequence as those used for errors on the physical parameters of the elements. For a description of such errors, the user should see page 96.

B. The tens position defines the mode of display of the results of the misalignment.

Code Number	Interpretation
X0X	The beam matrix contains the results of the misalignment. The beam matrix is printed wherever a (`PRINT BEAM ;`) element is encountered. The beam matrix will then contain contributions from all previous misalignments.
X1X	A table is used to store the results of misalignments. The effect of up to ten independently misaligned magnets may be shown in the table in a single run. The table is printed via a (`PRINT ALIGN ;`) element, and may be compared with the undisturbed beam matrix (printed by a (`PRINT BEAM ;`) command) at any point. An example of such a table is shown later in this section.

C. The units position specifies the magnet(s) or section of the beam line to be misaligned.

Code Number	Interpretation
XX0	The single magnet (element) immediately preceding the align card is to be misaligned. A bending magnet with fringe fields should be misaligned using one of the options described below.
XX1	The last R1 matrix update (the start of the beam line or a (`UPDATE R1 ;`) element) or alignment marker marks the beginning of the section to be misaligned. The misalignment element itself marks the end. The section is treated as a unit and misaligned as a whole. The misalignments of quadrupole triplets and other combinations involving two or more quadrupoles may be studied using this code digit.
XX2	The last R2 matrix update (see the `UPDATE` element for a list of elements which update R2) or alignment marker marks the beginning of the misaligned section. The misalignment element marks the end. This option makes use of the fact that R2 matrix updates do not affect the R1 matrix. A bending magnet with fringing fields or pole face rotations (`ROTAT` element) should be misaligned using this option. See examples 1 and 2 below for an illustration of this. An array of quadrupoles provides another example of the use of this option. By successive application of align elements, the elements of a quadrupole triplet could be misaligned relative to each other, and then the triplet as a whole could be misaligned. See example 3 below for an illustration of this.
XX3	All subsequent bending magnets and quadrupoles are independently misaligned by the amount specified. This option is useful in conjunction with the tabular display of the misalignment results (see below). A `ALIGN` element with this code number should be placed near the beginning of the data, before any magnets to which it applies. A bending magent, with fringing fields included, is treated as a single unit and misaligned accordingly.
XX4	All subsequent bending magnets, including fringing fields, are independently misaligned by the amount specified. See XX3 above for further comments.
XX5	All subsequent quadrupoles are independently misaligned by the amount specified. See example 4 below for an illustration of this. See XX3 above for further comments.

Any combination of digits may be used to define the exact circumstances intended. However, only some of the combinations are useful. In the above descriptions and in the following examples, we have tried to outline the more useful combinations. A brief summary follows.

The code 000. indicates an uncertainty in the position of a single magnet, with the effects contributing to the beam matrix. The code 100. describes a known misalignment in an otherwise identical situation. The code 003 causes all subsequent quadrupoles and bending magnets to be given an uncertain misalignment, with the results contributing to the beam matrix. Codes 013 and 113 impose respectively uncertain and known misalignment to all subsequent quadrupoles and bending magnets, with the results placed in the misalignment table. Code 203 randomly misaligns all subsequent quadrupoles and bending magnets, with the results tabulated in the beam and transfer matrices.

Any comments applying to a code with the last digit of 0 apply equally well to codes with a last digit of 1 or 2. Similarly any comments applying to a code with the last digit of 3 apply equally well to codes with a list digit of 4 or 5. Thus, code 115. indicates the deliberate displacement of all subsequent quadrupoles (referred to the point where the beam enters each magnet), with the results to be stored in a table.

Examples

Example No. 1: <u>A Bending Magnet with a Known Misalignment</u>

A bending magnet (including fringe fields) misaligned by a known amount might be represented as follows:

```
DRIFT L1 ;
UPDATE R2 ;
ROTATE 0. ; BEND L B n ; ROTATE 0. ;
ALIGN 0. 0. 0. 0. 0. 2. 102. ;
DRIFT L2 ;
```

This represents a known rotation of the bending magnet about the incoming beam direction (z axis) by 2.0 mr. The result of this misalignment will be a definite shift in the beam centroid, and a mixing of the horizontal and vertical coordinates. The use of the (UPDATE R2 ;) transform two update and the misalignment code number XX2 is necessary because the magnetic array (bending magnet + fringing fields) consists of three type code elements instead of one.

The complete magnet including both central field and fringing fields can be represented by a single element using the SBEND element. This single element may then be misaligned using the misalignment code number XX0. The UPDATE is now unnecessary. The above example would then look like:

```
DRIFT L1 ;
SBEND L B n ;
ALIGN 0.  0.  0.  0.  0.  2.  100.  ;
DRIFT L2 ;
```

An RBEND would be a rectangular bending magnet, but would be treated in the same manner.

Example No. 2: A Bending Magnet with an Uncertain Position

A bending magnet having an uncertainty of 2 mrad in its angular positioning about the incoming beam (z axis) would be represented as follows:

```
DRIFT L1 ;
UPDATE R2 ;
ROTATE 0. ; BEND L B n ; ROTATE 0. ;
ALIGN 0. 0. 0. 0. 0. 2.0 002. ;
DRIFT L2 ;
```

To observe the uncertainty in the location of the outcoming beam centroid, the input BEAM card should have zero phase space dimensions as follows:

```
BEAM 0. 0. 0. 0. 0. 0. p(0). ;
```

If the beam dimensions on the input BEAM card are nonzero, the resultant beam (σ) matrix will show the envelope of possible rays, including both the input beam and the effect of the misalignment.

As with example no. 1, the combination of ROTATE and BEND elements can be replaced with a single SBEND element. This example would then be rewritten as:

```
DRIFT L1 ;
SBEND L B n ;
ALIGN 0. 0. 0. 0. 0. 2.0 000. ;
DRIFT L2 ;
```

Example No. 3: A Misaligned Quadrupole Triplet

One typical use of both the R1 and R2 matrices is to permit the misalignment of a triplet. For example, an uncertainty in the positions within the following triplet:

```
QUAD  1.   -8.   10.   ;
QUAD  2.   7.   10.   ;
QUAD  1.   -8.   10.   ;
```

may be induced by appropriate **ALIGN** element as noted:

```
UPDATE  R  ;
QUAD  1.   -8.   10.   ;
UPDATE  R2  ;
QUAD  2.   7.   10.   ;
QUAD  1.   -8.   10.   ;
ALIGN  - - -   - - -   - - -   - - -   - - -   - - -   000.   ;
ALIGN  - - -   - - -   - - -   - - -   - - -   - - -   002.   ;
ALIGN  - - -   - - -   - - -   - - -   - - -   - - -   001.   ;
```

The first **ALIGN** element in the list refers to the misalignment of the third magnet only. The second **ALIGN** element refers to the misalignment of the second and third magnets as a single unit via the R2 matrix update (the (**UPDATE R2 ;**) element). The last **ALIGN** element refers to the misalignment of the whole triplet as a single unit via the R1 matrix update (the (**UPDATE R ;**) entry).

The comments about the **BEAM** card in Example 2 above are applicable here also.

Example No. 4: Misaligned Quadrupoles in a Triplet

Individual uncertainties in the positions of the quadrupoles in the triplet in Example 3 above may be induced by a single misalignment element as follows:

```
ALIGN  - - -   - - -   - - -   - - -   - - -   - - -   015.   ;
QUAD  1.   -8.   10.   ;
QUAD  2.   7.   10.   ;
QUAD  1.   -8.   10.   ;
```

The effect of each misalignment coordinate on each quadrupole will be stored separately in a table. This table is printed wherever a (PRINT, ALIGN ;) command is inserted.

A likely place to insert the (PRINT, ALIGN ;) command is at a focus. The misalignment table, as printed, then will show separately what the beam matrix would be for each quadrupole misplaced in each of six different ways. The entire table will then contain eighteen replications of the set of six beam centroid coordinates and beam half widths. Each may differ from the unaffected set of beam parameters by the effect of a single quadrupole misplaced along a single misalignment coordinate.

This example could equally well have been made up using known misalignments. The alignment code would then be 115. The displacements and rotations of the quadrupoles would cause shifts in the centroid replications in the misalignment table. If the initial beam matrix dimensions were nonzero, then rotations about the quadrupole axes and displacements along the axes would cause a change in the dimensions of the beam dimension columns in the misalignment matrix.

Another meaningful alignment code would be 005. The misalignment of the individual quadrupoles would be uncertain. The effects on the beam ellipse would be added in quadrature, giving a range of positions of the misaligned centroid.

Example No. 5: A Randomly Aligned Quadrupole Triplet

The net effect of random errors in the placement of quadrupoles in a triplet may also be induced by a single misalignment element as follows:

```
ALIGN  ---   ---   ---   ---   ---   ---   205.   ;
QUAD  1.   -8.   10.   ;
QUAD  2.   7.   10.   ;
QUAD  1.   -8.   10.   ;
```

The quadrupoles in the triplet will be given displacements and rotations chosen randomly from the ranges specified on the ALIGN element. The qudrupoles will be misaligned independently of each other. The results of the misalignment will be stored in the beam matrix, including the centroid.

Different configurations can be created by changing the starting point of the random number generation. (See page 86.) The randomly aligned configurations can then be used to investigate different correction schemes for the effect of the misalignments.

```
'RECOMBINED MODE HIGH RESOLUTION BEAM'
0
UNIT  7.  'MR'  ;
UNIT  8.  'FT'  ;
ALIGN  0.02  0.1  0.02  0.1   0.1  1.0  15.0  ;
BEAM  0.05  0.562  0.05  0.867  0.0  0.0 200.0 ;
DRIFT  80.74998 ;
SROT  180.  ;
BEND  10.25  9.18192  'B1'  ;
SROT  -180.  ;
DRIFT  1.75  ;
SROT  180.  ;
BEND  10.25  9.18192  'B1'  ;
SROT  -180.  ;
DRIFT  27.68999  ;
QUAD  10.0  -4.0006  2.54  'Q1' ;
DRIFT  7.7  ;
PRINT, BEAM, TRANS ;
DRIFT  3.6  ;
QUAD  10.0  3.4551  2.54  'Q2' ;
DRIFT  5.0 ;
PRINT, BEAM, TRANS ;
DRIFT  1.5 ;
SROT  180.  ;
ROTAT  8.53  ;
BEND  20.0  18.66998  'BM1' ;
ROTAT  8.53  ;
SROT  -180.  ;
DRIFT  1.5 ;
SROT  180.  ;
ROTAT  8.53  ;
BEND  20.0  18.66998  'BM1' ;
ROTAT  8.53  ;
SROT  -180.  ;
DRIFT  1.5 ;
SROT  180.  ;
ROTAT  8.53  ;
BEND  20.0  18.66998  'BM1' ;
ROTAT  8.53  ;
SROT  -180.  ;
PRINT, BEAM, TRANS ;
DRIFT  5.5 ;
QUAD  10.0  3.551  2.54  'Q3' ;
DRIFT  2.0 ;
QUAD  10.0  -3.6962  2.54  'Q4' ;
DRIFT  289.8396  ;
PRINT, BEAM  ;
PRINT, ALIGN ;
PRINT, TRANS ;
```

Quadrupoles to be misaligned

print instruction for misalignment table

more beam line including more quadrupoles
and misalignment table print instructions if desired

```
SENTINEL
```

Input for a Misalignment Table. Shown is the input for a misalignment run of the early part of a beam line. The misalignment element specified that all subsequent quadrupoles are to be given an uncertain misalignment by the amount specified and the results (for up to ten quadrupoles) are to be entered in a table. The portion of the output produced by the indicated print instruction is shown in the next figure.

548.830 FT

Unperturbed beam ellipse ⟶

0.000	0.086 CM					
0.000	0.329 MR	0.043				
0.000	0.124 CM	0.000	0.000			
0.000	0.351 MR	0.000	0.000	0.031		
0.000	0.209 CM	-0.019	-1.000	0.000	0.000	
0.000	0.000 PC	0.000	0.000	0.000	0.000	0.000

*MISALIGNMENT EFFECT TABLE FOR MISALIGNMENTS OF

0.020 CM	0.100 MR	0.020 CM	0.100 MR	0.100 CM	1.000 MR
*MISALIGNMENT OF Q1 * (130.690 FT TO 140.690 FT) MAGNET ENTRANCE FACE COORDS					
0.000 0.136 CM	0.000 0.086 CM	0.000 0.086 CM	0.000 0.119 CM	0.000 0.086 CM	0.000 0.093 CM
0.000 0.329 MR	0.000 0.329 MR	0.000 0.329 MR	0.000 0.329 MR	0.000 0.329 MR	0.000 0.329 MR
0.000 0.124 CM	0.000 0.164 CM	0.000 0.188 CM	0.000 0.124 CM	0.000 0.124 CM	0.000 0.128 CM
0.000 0.351 MR	0.000 0.351 MR	0.000 0.351 MR	0.000 0.351 MR	0.000 0.351 MR	0.000 0.351 MR
0.000 0.209 CM	0.000 0.209 CM	0.000 0.209 CM	0.000 0.209 CM	0.000 0.209 CM	0.000 0.209 CM
0.000 0.000 PC	0.000 0.000 PC	0.000 0.000 PC	0.000 0.000 PC	0.000 0.000 PC	0.000 0.000 PC
*MISALIGNMENT OF Q2 * (151.990 FT TO 161.990 FT) MAGNET ENTRANCE FACE COORDS					
0.000 0.164 CM	0.000 0.085 CM	0.000 0.085 CM	0.000 0.138 CM	0.000 0.085 CM	0.000 0.093 CM
0.000 0.329 MR	0.000 0.329 MR	0.000 0.329 MR	0.000 0.329 MR	0.000 0.329 MR	0.000 0.329 MR
0.000 0.124 CM	0.000 0.142 CM	0.000 0.155 CM	0.000 0.124 CM	0.000 0.124 CM	0.000 0.128 CM
0.000 0.351 MR	0.000 0.351 MR	0.000 0.351 MR	0.000 0.351 MR	0.000 0.351 MR	0.000 0.351 MR
0.000 0.209 CM	0.000 0.209 CM	0.000 0.209 CM	0.000 0.209 CM	0.000 0.209 CM	0.000 0.209 CM
0.000 0.000 PC	0.000 0.000 PC	0.000 0.000 PC	0.000 0.000 PC	0.000 0.000 PC	0.000 0.000 PC
*MISALIGNMENT OF Q3 * (236.990 FT TO 246.990 FT) MAGNET ENTRANCE FACE COORDS					
0.000 0.167 CM	0.000 0.085 CM	0.000 0.085 CM	0.000 0.138 CM	0.000 0.085 CM	0.000 0.093 CM
0.000 0.329 MR	0.000 0.329 MR	0.000 0.329 MR	0.000 0.329 MR	0.000 0.329 MR	0.000 0.329 MR
0.000 0.124 CM	0.000 0.144 CM	0.000 0.156 CM	0.000 0.124 CM	0.000 0.124 CM	0.000 0.128 CM
0.000 0.351 MR	0.000 0.351 MR	0.000 0.351 MR	0.000 0.351 MR	0.000 0.351 MR	0.000 0.351 MR
0.000 0.209 CM	0.000 0.209 CM	0.000 0.209 CM	0.000 0.209 CM	0.000 0.209 CM	0.000 0.209 CM
0.000 0.000 PC	0.000 0.000 PC	0.000 0.000 PC	0.000 0.000 PC	0.000 0.000 PC	0.000 0.000 PC
*MISALIGNMENT OF Q4 * (248.990 FT TO 258.990 FT) MAGNET ENTRANCE FACE COORDS					
0.000 0.150 CM	0.000 0.086 CM	0.000 0.086 CM	0.000 0.126 CM	0.000 0.086 CM	0.000 0.093 CM
0.000 0.329 MR	0.000 0.329 MR	0.000 0.329 MR	0.000 0.329 MR	0.000 0.329 MR	0.000 0.329 MR
0.000 0.124 CM	0.000 0.152 CM	0.000 0.169 CM	0.000 0.124 CM	0.000 0.124 CM	0.000 0.129 CM
0.000 0.351 MR	0.000 0.351 MR	0.000 0.351 MR	0.000 0.351 MR	0.000 0.351 MR	0.000 0.351 MR
0.000 0.209 CM	0.000 0.209 CM	0.000 0.209 CM	0.000 0.209 CM	0.000 0.209 CM	0.000 0.209 CM
0.000 0.000 PC	0.000 0.000 PC	0.000 0.000 PC	0.000 0.000 PC	0.000 0.000 PC	0.000 0.000 PC

TRANSFORM 1

-1.71164	-0.00014	0.00000	0.00000	0.00000	-6.35395
-0.27937	-0.58426	0.00000	0.00000	0.00000	-0.60101
0.00000	0.00000	-2.47238	-0.00003	0.00000	0.00000
0.00000	0.00000	-0.21507	-0.40447	0.00000	0.00000
0.07464	0.37123	0.00000	0.00000	1.00000	-0.02255
0.00000	0.00000	0.00000	0.00000	0.00000	1.00000

LENGTH 548.82953 FT

Example of a Misalignment Table. The misalignment table, the unperturbed complete beam matrix, and the first-order transfer matrix are all shown at the same point in the beam line. The misalignment element (not shown) has indicated an uncertain misalignment, so the beam centroid is unaffected. The magnitudes of the misalignments in each coordinate are shown above the columns to which they pertain. The results of independently misaligning each magnet are indicated by the label for that magnet.

Assembling Beam Lines

In TRANSPORT beam lines may be assembled from individual elements in two different ways.

1. The elements are all defined first, then assembled into beam lines and sublines by explicit instructions. This procedure is the one used in the MAD program.
2. The elements are listed in the order in which they occur in the beam line. An element is then defined as it is placed in the sequence of elements which make up the beam line.

Assembling Beam Lines with MAD Notation

A particle accelerator typically has a lattice structure which is highly repetitive. The MAD program was designed with particle accelerators in mind. Any of the elements is likely to be used many times.

The MAD method of assembling beam lines takes this repetitive structure into account. The elements are all defined first, then assembled into beam lines and sublines by explicit statements. Sublines can be treated as elements in assembling larger sublines or complete lines. The magnetic system over which the calculation is done is determined by the `USE` statement. It is this statement which signals the program that the MAD method of assembling beam lines is being used. If it is not present when the MAD procedure of assembling lines is being used, the user should not expect to get any meaningful results.

Simple Beam Lines

Using MAD notation, a simple beam line to be studied may be defined as a sequence of elements. As an example we define a simple beam line and give it the label `'SIMP'`. It is composed from previously defined drifts `D1` and `D2`, quadrupoles `Q1` and `Q2`, and bending magnet `B1`.

```
SIMP:  LINE = (D1,Q1,D2,Q2,D2,B1,D2,B1,D2,B1,D2,Q2,D2,Q1,D1)
```

The alternative notation, used in earlier versions of MAD, will be equivalent

```
LINE  SIMP = (D1,Q1,D2,Q2,D2,B1,D2,B1,D2,B1,D2,Q2,D2,Q1,D1)
```

Sub-Lines

Lines themselves may serve as elements of other lines. Such a sub-line must be defined in a separate statement. Thus one can easily include a subline several times in a given beam line. Defining two such sublines, the above example may be written as

```
DOUB:  LINE = (D1,Q1,D2,Q2)
BPD:  LINE = (B1,D2)
SIMP:  LINE = (DOUB,D2,BPD,BPD,BPD,Q2,D2,Q1,D1)
```

Repetition and Reflection

A repeated element can be indicated by preceding it with an integer followed by an asterisk Thus our line SIMP becomes

```
SIMP:  LINE = (DOUB,D2,3*BPD,Q2,D2,Q1,D1)
```

Several consecutive elements may also be enclosed in parentheses and repeated. We could forgo the definition of BPD and write our beamline as

```
SIMP:  LINE = (DOUB,D2,3*(B1,D2),Q2,D2,Q1,D1)
```

A subline which is to be reflected (reverse order of elements) is indicated by preceding it with a minus sign. Thus our beam line SIMP may finally be shortened to

```
SIMP:  LINE = (DOUB,D2,3*(BPD),-DOUB)
```

In the TRANSPORT implementation of MAD notation, a repetition indication may not be preceded with a negative sign. Nothing is lost by this restriction since a reflected repeated sub-line is the same as a repeated subline with its elements reversed and in reverse order.

Replaceable Arguments

A beam line definition may include formal arguments. Our sub-line DOUB, for example, may not have Q2 included explicitly, but have a formal argument in its place. It is then defined as

```
DOUB:  LINE(X) = (D1,Q1,D2,X)                 or

DOUB(X):  LINE = (D1,Q1,D2,X)
```

The formal arguments are included in the sequence of elements as if they were labels representing physical elements or sub-lines. They must also be included in parentheses before the equal sign.

When the sub-line DOUB is used, the formal argument must be replaced by the actual argument in parentheses following the label DOUB. Thus the line SIMP becomes

```
SIMP:  LINE = (DOUB(Q2),D2,3*(BPD),-DOUB(Q2))
```

Formal arguments may be single elements or sub-lines. When a subline has a single formal argument, it must be used with a single actual argument. When the subline has two formal arguments, it must be used with two actual arguments. In all cases, the number of formal arguments must be the same as the number of actual arguments.

The section DOUB may be defined with two formal arguments:

```
DOUB(X,Y):  LINE = (D1,X,D2,Y)
```

The line SIMP would then be:

```
SIMP:  LINE = (DOUB(Q1,Q2),D2,3*(BPD),-DOUB(Q1,Q2))
```

USE Statement

The USE statement specifies which of the defined beam lines and sublines is to be used in the calculation. In our example, the most obvious candidate is the complete assembled beam line SIMP. The USE command would then read

```
USE  SIMP ;
```

If, however, we wished to investigate the properties of the entrance doublet without creating a new data deck, then we could simply use the command

```
USE  DOUB ;
```

TRANSPORT would then run through only the beamline named DOUB.

It is the USE statement which differentiates between the MAD procedure of defining beam lines explicitly and the original TRANSPORT procedure of just listing the elements in order. If no `USE` statement is included in the data, TRANSPORT considers the beam line to consist of the elements in the order listed.

Assembling Beam Lines by Listing the Elements

When the beam line is defined by simply listing the elements in order, the individual elements may be defined in any mannner described in this manual. Either keyword or positional notation may be used, and any complete set of parameters may be used. It is recommended for clarity that a user not mix notations more than is necessary. One instance of mixing notations might occur when a primary and secondary beam line are designed by different people. Someone might then want to combine the two sets of data to represent a longer beam line made up of these two components.

The two commands `REPEAT` and `SECTION` are used only when a beam line is assembled by the first method, simply listing the elements. When MAD notation is used to assemble a beam line, there are other ways to represent the same functions as `REPEAT` and `SECTION`.

Repetition

Many systems include a set of elements that are repeated several times. To minimize the chore of input_preparation, a 'repeat' facility is provided.

There are two parameters:

1. `REPEAT` (or type code `9.`)
2. Code digit. If nonzero, it states the number of repetitions desired from the point where it appears. If zero it marks the end of a repeating unit.

For example, a total bend of 12 degrees composed of four 3-degree bending magnets each separated by 0.5 metres could be represented by

```
REPEAT  4.    ;
BEND    - - - ;
DRIFT   .5  ;
REPEAT  0.    ;
```

Those elements (in this case a bend and drift) between the (`REPEAT 4. ;`) and (`REPEAT 0. ;`) would be used four times.

`REPEAT` elements do not appear explicitly in the output TRANSPORT prints when calculating. The only indication of their presence is the repeated listing of the elements they control.

Vary codes may be used within a repeating unit in the usual fashion. However <u>all</u> repetitions of a given varied element will be coupled.

Repeat cards may be nested up to four deep. By “nesting” we mean a repeat within a repeat. An example is given below.

Example of Nesting

```
REPEAT  2.   ;
DRIFT  10.   ;
REPEAT  3.   ;
DRIFT  20.   ;
REPEAT  4.   ;
DRIFT  50.   ;
REPEAT  0.   ;
REPEAT  0.   ;
DRIFT  1.5   ;
REPEAT  0.   ;
```

Inner Block

Next Inner Block

Outer Block

The total length of this sequence is:

$$2 * (10. + 3 * (20 + 4 * \ 50) + 1.5) = 1343 \quad .$$

Defined Section

A system may contain a section which is repeated at some later point. The repeated section may not occur immediately after the original section, so that the **REPEAT** element in not appropriate. It may also be useful to repeat a section, but with the elements listed in reverse order.

The keywords which can be used in defining a section of beam line are **BEGIN**, **END**, **FORWARD**, and **BACKWARD**. If a section of a beam line is to be identified to be used again at some later point, it is preceded by the command

```
SECTION BEGIN 'FRED' ;
```

and followed by the command

```
SECTION END 'FRED' ;
```

The label, in this case **'FRED'**, identifies the section for later reference. If it is desired to repeat the same section at some later point in the beam line, one uses the command

```
SECTION FORWARD 'FRED' ;
```

If the section is to be repeated but in reverse order, use the command

```
SECTION BACKWARD 'FRED' ;
```

A number of rules apply to the use of the SECTION element. Each defined section must have both its beginning and its end indicated, and the beginning must precede the end. The definition of a section must completely precede its use. A given name can be used only once to define a section, although it can be used many times to indicate a repeat of a section. The number of defined sections is limited to ten. The total z rotation (SROT element) within a defined section must equal zero. Finally, defined sections must nest properly with the REPEAT element. Defined sections, however, need not nest properly with each other.

Example of a Defined Section

```
SECTION  BEGIN  'DOUB'  ; ——————— Begin defined section
QUAD  10.  5.  2.  ;
DRIFT  8.  ;
QUAD  10.  -5.  2.  ;
SECTION  END  'DOUB'  ; ——————— End defined section
         :
SECTION  FORWARD  'DOUB'  ; ——————— Repeat defined section
         :
SECTION  BACKWARD  'DOUB'  ; ——————— Repeat defined section in reverse order
```

The difference between the REPEAT command and the SECTION command is that the SECTION command does not require that the repetitions immediately follow the original. They may be separated by other portions of beam line. Thus we have that the sequence

```
REPEAT  5.  ;
(section of beam line)
REPEAT  0.  ;
```

is identical to the sequence

```
SECTION  BEGIN  SAM'  ;
(section of beam line)
SECTION  END  'SAM'  ;
SECTION  FORWARD  'SAM'  ;
SECTION  FORWARD  'SAM'  ;
SECTION  FORWARD  'SAM'  ;
SECTION  FORWARD  'SAM'  ;
```

However, the example

```
SECTION  BEGIN  'SAM'  ;
(section of beam line)
SECTION  END  'SAM'  ;
(other elements)
SECTION  FORWARD  'SAM'  ;
```

cannot be done with the REPEAT element alone.

Operations

TRANSFORM 1 UPDATE *

To re-initialize the matrix `TRANSFORM 1` (R1 – the product of the R matrices) use the `UPDATE` element (type code `6.0`). An `UPDATE` element effects an update of the R1 matrix and initiates the accumulation of a new product matrix at the point of the update. This facility is occasionally useful for misaligning a set of magnets or fitting only a portion of a system. The element which updates R2 is usually more useful for these purposes. A description can be found on page 225.

The `UPDATE` element represents an operation and is not MAD compatible. It can be expressed in either keyword notation or in original TRANSPORT notation.

Keyword Notation

To update the transfer matrix R1, use the element.

```
UPDATE,  R1 ;
```

If the beam line is described simply by listing the elements in the order in which they occur physically, then the `UPDATE` instruction is placed at the location where the update is to be made.

If the beam line is assembled from elements and specified in a `LINE` statement, the `UPDATE` command may be placed after the beam line specification. It may refer to a marker which is included in the beam line specification. This is done with the `LOCA(TION)` keyword. Assume the marker

```
MARKER  'MAR1' ;
```

is included in the beam line description. Assume further that the user wishes to update the transfer matrix at the location of this marker. The command

```
UPDATE,  R1,  LOCA = MAR1 ;
```

will perform the desired update operation.

*By "updating" we mean initiating a new starting point for the accumulation (multiplication) of the R1 matrix. At the point of update the previous accumulation is discontinued. When the next element possessing a transfer matrix is encountered, the accumulated transfer matrix R1 is set equal to the individual transfer matrix R for that element. Accumulation is resumed thereafter.

Original TRANSPORT Notation

The R1 matrix may be updated by the element

```
UPDATE  0.   1.    ;
```

or

```
6.   0.   1.    ;
```

Usage

An update may be used no matter what the order of the run. However, the beam matrix will be unaffected by the update only if the order of the transfer matrix being printed is no greater than first. This order is the second number on the `ORDER` element. In second and higher orders the aberrations cause the ellipse to become distorted. The distribution can still be described in terms of its first and second moments. An update causes the program to assume the distribution at the update is an ellipsoid with the given first and second moments. The subsequent behavior of this ellipsoid may differ from what would occur without this refitting procedure.

The matrix R1 is updated by no other element. It is not used in the calculation of the beam matrix. The beam matrix is calculated from the auxiliary transfer matrix R2 described on the next page.

A `TRANSFORM 1` matrix will be printed at any position in the data set where a (`PRINT, TRANS ;`) element is inserted.

See the following section for the introduction of an auxiliary transformation matrix R2 (`TRANSFORM 2`) to avoid the need for `TRANSFORM 1` updates.

The (`UPDATE, R1 ;`) card also causes an update of the R2 matrix.

AUXILIARY TRANSFORMATION MATRIX (R2)

The R1 matrix represents the accumulated transfer matrix from either the beginning of the beam line or the last explicit R1 update (`UPDATE, R1 ;`). The transfer matrix is used to carry the beam matrix, i.e. from the initial beam matrix to calculate the beam matrix at some later point. However several elements in TRANSPORT which affect the beam matrix cannot be represented in any transfer matrix. To avoid update complications with R1 an auxiliary transfer matrix R2 exists. The beam matrix at a given point is then calculated from the R2 matrix and the beam matrix at the last R2 update.

Both the R1 and R2 matrices are normally available for printing. However there is no redundancy in computer use, since, internally to the program, only R2 is calculated at each element. The matrix R1 is calculated from R2 only as needed. An update of R2 does not update R1.

Keyword Notation

To update the auxilliary transfer matrix R2, one uses the element

```
UPDATE,  R2 ;
```

Original TRANSPORT Notation

The R2 matrix may be updated by a

```
UPDATE  0.   2.    ;
```

or

```
6.   0.   2.   ;
```

Usage

The R2 matrix may be printed by a (`PRINT  TRANS2  ;`) entry. Constraints on R2 are imposed similarly to those on R1. For details see page 272 or page 279.

The complete list of elements which automatically update `TRANSFORM 2` is:

1. A beam (`BEAM`) element.
2. An R1 update (`UPDATE R1`).
3. An R2 update (`UPDATE R2`).
4. An unknown misalignment element (`ALIGN`).
5. A stray field type code `21.0` entry. (This element is not currently operative. There will be no description of it elsewhere in this manual.)

Please note that automatic updates of **TRANSFORM 2** occur when an **ALIGN** element is inserted specifying the misalignment of all subsequent bending magnets. These **TRANSFORM 2** updates take place immediately before and immediately after any bending magnet which has either the entrance or exit fringe fields specified via a **ROTATE** element.

An example of the data for a section of a beam line might be the following sequence of elements.

```
BEAM .   .    .    ;
D1:    DRIFT  .  ;
Q1:    QUAD  .   .    .    ;
D2:    DRIFT  .  ;
Q2:    QUAD  .   .    .    ;
D3:    DRIFT  .  ;
UPDATE,  R2  ;
D4:    DRIFT  .  ;
Q3:    QUAD  .   .    .    ;
D5:    DRIFT  .  ;
Q4:    QUAD  .   .    .    ;
D6:    DRIFT  .  ;
```

From the beginning of the beam line through the end of the drift D3, the two transfer matrices R1 and R2 will be the same. The R2 matrix is reinitialized by the (`UPDATE,  R2  ;`) command and accumulation starts again with the drift D4. The R1 matrix continues to accumulate unaffected by the R2 update. After traversing the drift D6, R1 and R2 will be given by:

$$R1 = R(D6)*R(Q4)*R(D5)*R(Q3)*R(D4)*R(D3)*R(Q2)*R(D2)*R(Q1)*R(D1)$$

while

$$R2 = R(D6)*R(Q4)*R(D5)*R(Q3)*R(D4)$$

ALMARK – Alignment Marker

A known misalignment does not change the calculated phase space volume of the beam envelope. Therefore, no R1 or R2 update need occur at the end of a section which is given a known misalignment. It is also not necessary to use an UPDATE element to demarcate the beginning of a section with a known misalignment. The beginnings of sections to be misaligned may also be indicated by the alignment marker element. No UPDATE will result from the insertion of the alignment marker if the misalignment referring to the marker is known. The alignment marker will perform an (UPDATE, R2 ;) if the misalignment referring to it is unknown.

Notation for the alignment marker is exactly the same as for the UPDATE element. The keyword ALMARK simply replaces the keyword UPDATE. The code on the ALIGN element will pertain to the most recent ALMARK element if it is more recent than the most recent UPDATE element of the same type (R1 or R2).

Keyword Notation

The code digit XX1 on the misalignment element refers to an update of the R1 matrix. It can also refer to an ALMARK element if one occurs more recently than any R1 update. The ALMARK element should then read:

```
ALMARK,  R1  ;
```

If the beam line is assembled from elements and specified in a LINE statement, the ALMARK command may be placed after the beam line specification. It may refer to a marker which is included in the beam line specification. This is done with the LOCA(TION) keyword. Assume the marker

```
MARKER  'MAR1' ;
```

is included in the beam line description. Assume further that the user wishes to begin a misaligned section at the location of this marker. The command

```
ALMARK,  R1,  LOCA = MAR1  ;
```

will demarcate the location as a possible reference for subsequent ALIGN elements.

The code digit XX2 on a misalignment element refers to an update of the R2 matrix. Similarly, the command:

```
ALMARK,  R2 ;
```

can substitute for an update of the auxiliary R2 matrix.

Original TRANSPORT Notation

To demarcate the beginning of a section with a known misalignment, the R1 matrix update may be replaced by the element:

```
ALMARK  0.   1.   ;
```

The R2 matrix update may similarly be replaced by the element:

```
ALMARK  0.   2.   ;
```

Example

A bending magnet (including fringe fields) misaligned by a known amount was shown as an example on page 208. Since the misalignment in that case was known, the example might now be represented as follows:

```
DRIFT L1 ;
ALMARK  R2  ;
ROTATE  0.   ;  BEND L B n ;  ROTATE  0.   ;
ALIGN  0.   0.   0.   0.   0.   2.   102.   ;
DRIFT L2 ;
```

This represents a known rotation of the bending magnet about the incoming beam direction (z axis) by 2.0 mr. The result of this misalignment will be a definite shift in the beam centroid, and a mixing of the horizontal and vertical coordinates. The use of the (ALMARK, R2 ;) alignment marker and the misalignment code number XX2 is necessary because the magnetic array (bending magnet + fringing fields) consists of three elements instead of one.

PRINT — Output Print Control Instructions

A number of control codes which transmit output print instructions to the program have been consolidated into the single element PRINT (or 13.). Most PRINT elements produce a line of output that advertises their existence. Those that do not usually have an obvious effect upon the remainder of the output and thus make their presence known. Some other print controls which may be placed on the indicator card are described on page 67. They are of a more global nature and therefore not appropriate for inclusion among the PRINT instructions.

The PRINT instruction is an operation and not part of the physical description of the beam line. It is therefore specific to TRANSPORT and not expressible in MAD notation. It can be expressed in either keyword notation or in original TRANSPORT notation. The keywords listed below are used in keyword notation. The code digits are used in original TRANSPORT notation.

The effects of the various code numbers will be described below (not in numerical order). In the descriptions the PRINT instructions are placed in parentheses to isolate them from the text. In actual use, the parentheses are omitted.

Summary of PRINT Instructions

The mnemonic used in specifying print control instructions is PRINT. The more recently included instructions are expressable in keyword notation only. Others, for which further development is anticipated, are expressable only in original TRANSPORT notation Most of the instructions are expressable in both notations, although in such cases the use of keyword notation is encouraged. The keywords and/or code digits which can be used are:

Code Digit	Keyword	Prints or Activates
1	BEAM	The beam (σ) matrix
4	TRANS	The transfer matrix R1 (and possibly T1 and U1)
7	ACCELERA	The use of accelerator notation
8	ALIGN	The misalignment effect table
9		Realignment of the reference trajectory.
12	FLOOR	The floor coordinates of the reference trajectory.
13	REFER	Refer the transfer and beam matrices to the unrotated coordinate system.
14	WAIST	The positions and sizes of the nearby waists
16	PRECISE	Precise values of varied parameters
17	NOPARA	The suppression of physical parameters when printing
18	ONLY	The printing only of varied elements and constraints

19	ONELINE	The printing of transfer and beam matrices on a single line.
24	TRAN2	The auxilliary transfer matrix R2.
	ELEMENTS	The listing of the physical elements.
	TWISS	The beam matrix in accelerator notation (β and α parameters) and the beam phase shift (ψ).
	NARROW	Printing of the beam (σ) matrix and FIT elements within the first 80 columns.
	CENTROID	The beam centroid when the beam is off axis.
	LEVEL	Refer the transfer and beam matrices to transversely levelled coordinate system.
	MARKER	Markers are included among the elements to be printed
	ALL	The printing of physical elements and results of calculation.
20		Misalignments are taken about the longitudinal midpoint of the misaligned element.
21		Only the focusing effect of misalignments is included.
22		Misalignments are taken about the chord of bending magnets.
	ON	Certain print instructions are to be executed after every physical element.
	OFF	Turns off the automatic print after each element.

Keyword Notation

There are no values associated with the print instruction. The command consists of the word PRINT followed by a keyword giving the nature of the print instruction. To print the transfer matrix at a particular location, for example, one would use the instruction:

```
PRINT,  TRANS  ;
```

To print the beam matrix at a particular location, use the instruction

```
PRINT,  BEAM  ;
```

Several keywords may now be placed on a single line with the single mnemonic PRINT. An example might be

```
PRINT,  BEAM,  TRANS  ;
```

This single instruction causes both the beam (sigma) matrix and the transfer matrix to be printed at the specified location. *If* BEAM *and* TRANS *are included in the same element the beam matrix will be printed before the transfer matrix, regardless of the order of the keywords.*

If it is desired to have the transfer matrix printed after each physical element, then one uses the instruction

```
PRINT,  TRANS,  ON  ;
```

To terminate the automatic printing of the transfer matrix, use the command

```
PRINT,  TRANS,  OFF  ;
```

Placement

If the beam line is specified simply by listing the elements, then the PRINT command is inserted at the location where it is to have its effect. For example, if it is desired to print the transfer matrix immediately after a particular 10 meter drift space, then the line

```
PRINT,  TRANS  ;
```

would be inserted immediately after the line

```
DRIFT,  L = 1.0  ;
```

If the beam line is assembled from elements and specified in a LINE statement, a PRINT instruction is normally placed after the LINE and USE instructions. It may refer to a marker which is included in the beam line specification. The reference is given by the LOCA(TION) keyword. Assume the marker

```
MAR1:   MARKER  ;
```

is included in the beam line description. Assume further that the user wishes to print the transfer matrix at the location of this marker. The command

```
PRINT,  TRANS,  LOCA = MAR1  ;
```

will print the transfer matrix at the location MAR1. The matrix will be printed to the order specified on the ORDER element, found near the beginning of the data.

Some of the print commands are applied globally and affect the entire beam line. An example is the command

```
PRINT,  TRANS,  ON  ;
```

This command indicates that transfer matrix is to be printed after each physical element. This command should be placed before the BEAM element at the beginning of the beam line description. It then need not have a location parameter referring to any marker. Its effect will be invoked before the trace through the beam line is begun.

Original TRANSPORT Notation

There are two parameters:

1. PRINT (or type code 13.0)
2. Code number.

The PRINT commands are placed in the beam line at the point where they are to have an effect. For example, if it is desired to print the transfer matrix immediately after a particular 10 meter drift space, then the line

```
PRINT  4.   ;
```

or

```
13.   4.   ;
```

would be inserted immediately after the line

```
DRIFT  10.  ;
```

Any PRINT statements to be applied to the entire beam line should be placed before the BEAM element. An example is the command

```
PRINT  6.   ;
```

or

```
13.   6.   ;
```

This command causes the transfer matrix to be printed after each physical element.

Floor Coordinate Layout Control

(PRINT, FLOOR ;) or (PRINT 12. ;) or (13. 12. ;): These three commands are completely equivalent. The floor coordinates of the beam reference trajectory are printed after every beam line component if any of these three "PRINT" instructions is placed before the BEAM element. One can produce a layout of a beam line in any Cartesian coordinate system one chooses. If no additional cards are inserted the reference trajectory of the beam line will be assumed to start at the origin and proceed along the positive z-axis. One can also specify other starting coordinates and orientations by the placement of special parameter elements before the beam element. For a description of these particular special parameters (the SPECIAL element) see page 87.

The coordinates printed provide a global reference system for the entire beam line. They give the X, Y and Z position, and the yaw, pitch, and roll angles, respectively, of the reference trajectory at the interface between two elements. The Y-axis will point up and the X-axis to the left. The *yaw* angle is between the floor projection of the reference trajectory and the floor Z axis. The *pitch* angle is the vertical pitch, the angle the reference trajectory makes with a horizontal plane. The *roll* angle is a rotation about the reference trajectory. In the printed output the values given are those at the exit of the element listed above and at the entrance of the element listed immediately below. A figure showing the meaning of the three coordinates and the three angles is shown on the next page.

The calculation of the coordinates is done from the parameters of the physical elements as given in the data. Therefore, if effective lengths are given for magnetic elements, the coordinates printed will be those at the effective field boundary.

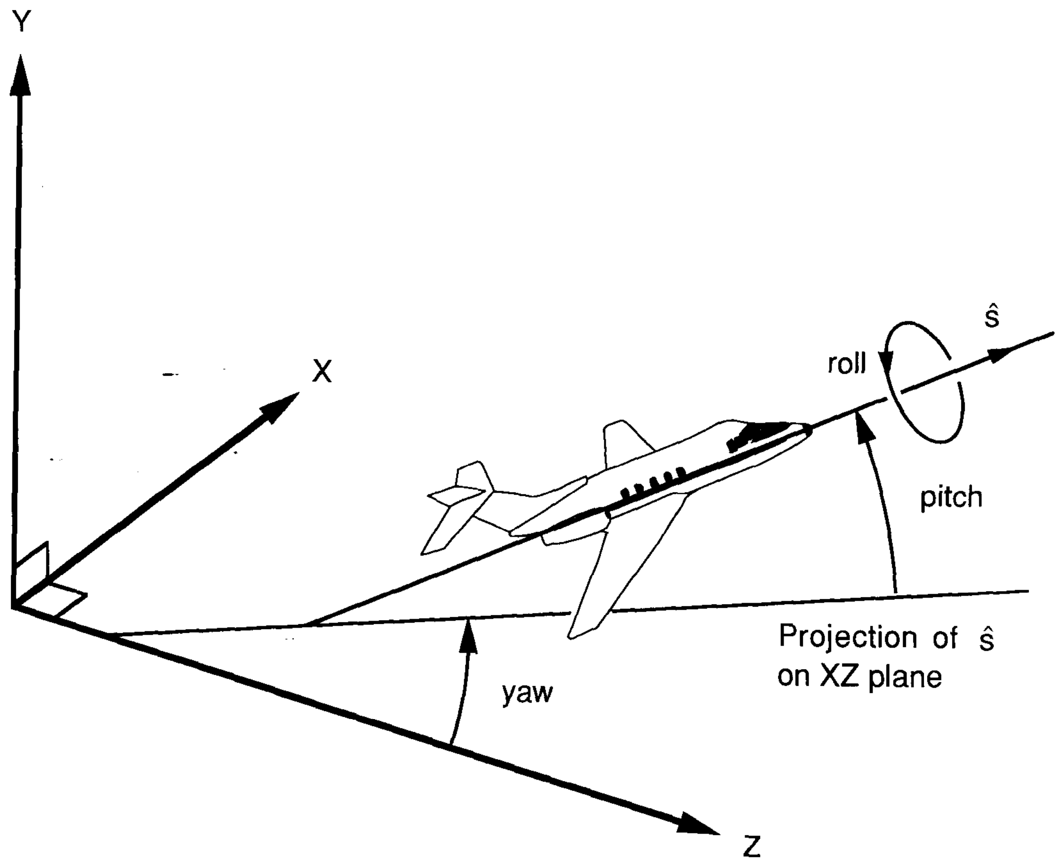
At lower energies, the fringe fields of bending magnets must also be included. The extended nature of fringing fields produces a transverse shift to the reference trajectory at a bending magnet entrance and exit faces in comparison with an idealized bending magnet where the field begins and ends abruptly. This transverse shift can become important when the quantity $g^2/2\rho$ is a large enough transverse dimension to have a detectable effect on the optical system. Here g represents the magnet gap, and ρ the trajectory radius of curvature. More details can be found in the section of fringing fields starting on page 139.

The floor coordinates will be printed after every physical element. The accumulated length L_c of the reference trajectory, the three Cartesian floor positions and the three orientation angles will be printed on a single line, along with the units in which these quantities are expressed. The single line will have the following appearance:

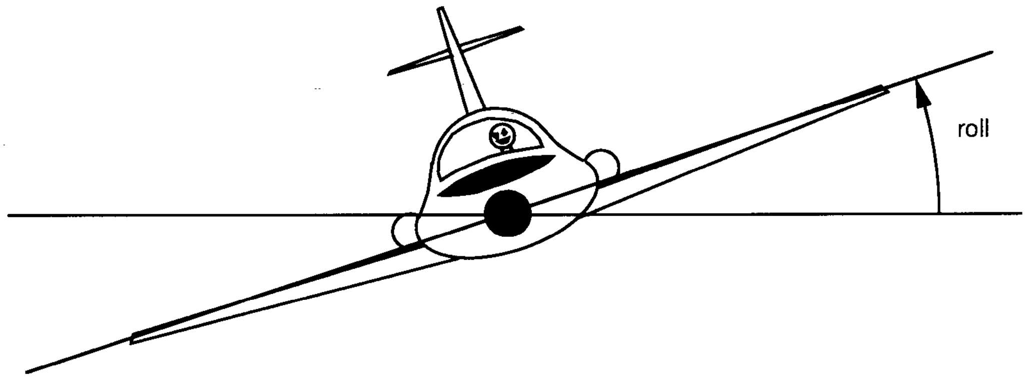
$$L_c \texttt{ M} \qquad Xfloor\ Yfloor\ Zfloor \texttt{ M} \qquad yaw\ pitch\ roll \texttt{ R}$$

Here the units used are meters (M) and radians (R). These are the units used by the global unit specification UMAD. The units are meters (M) and degrees (DEG) for the global unit set UTRANS. Other global unit sets or individual units can also be used and are described on pages 71 and 75.

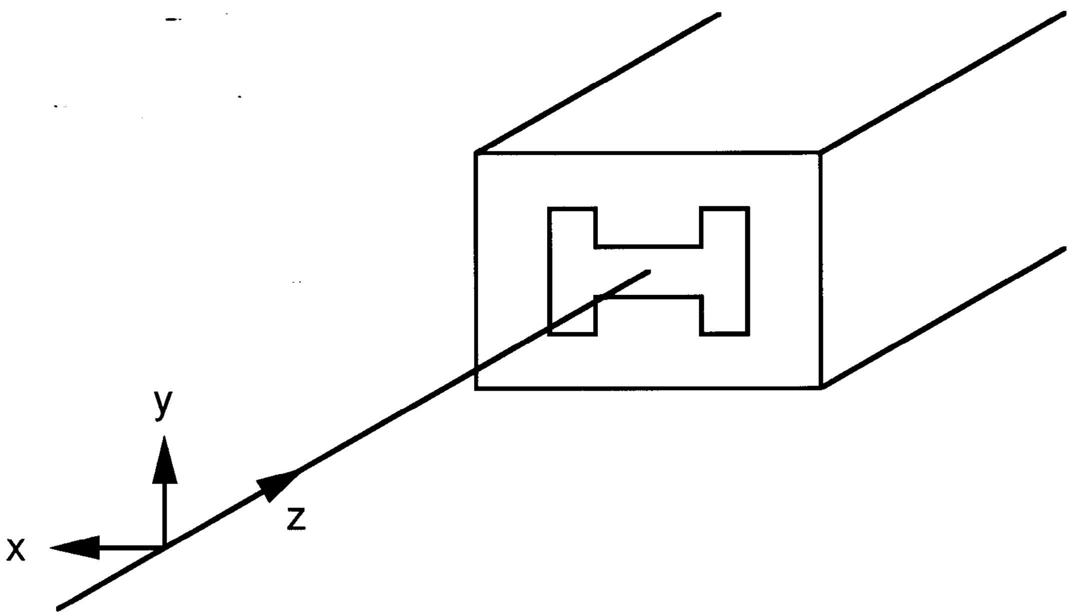
The following figure illustrates the meaning of the three angles used to specify the orientation of the reference trajectory.



The Global Reference System.



The Meaning of the Roll Angle



The Local Reference System

The beam and transfer matrices are referred to the local coordinate system. The z coordinate is the direction of travel of the airplane in the previous illustrations. The x coordinate points down the left wing of the airplane, and the y coordinate points through the roof of the cabin.

Beam Matrix Print Controls

(`PRINT, BEAM ;`) or (`PRINT 1. ;`) or (`13. 1. ;`): The beam (σ) matrix is printed at the location of this element.

(`PRINT, BEAM, ON ;`) or (`PRINT 3. ;`) or (`13. 3. ;`): The beam (σ) matrix will be printed after every physical element which follows this element.

(`PRINT, BEAM, OFF ;`) or (`PRINT 2. ;`) or (`13. 2. ;`): The effect of a previous (`PRINT, BEAM, ON ;`) code or its equivalent is cancelled and the beam (σ) matrix is printed only when a (`PRINT, BEAM ;`) code is encountered or when another (`PRINT, BEAM, ON ;`) code is inserted. The suppression of the beam matrix is the normal default.

The beam matrix, as printed will have the following appearance:

$$\begin{array}{llllllll}
x_g & \Delta x & \mathtt{CM} & & & & & \\
x'_g & \Delta x' & \mathtt{MR} & r_{21} & & & & \\
y_g & \Delta y & \mathtt{CM} & r_{31} & r_{32} & & & \\
y'_g & \Delta y' & \mathtt{MR} & r_{41} & r_{42} & r_{43} & & \\
\ell_g & \Delta \ell & \mathtt{CM} & r_{51} & r_{52} & r_{53} & r_{54} & \\
\delta_g & \Delta \delta & \mathtt{PC} & r_{61} & r_{62} & r_{63} & r_{64} & r_{65}
\end{array}$$

The first column, with the subscript "g", represents the coordinates of the center of gravity of the beam phase space. In first order the center of gravity is the same as the centroid, which is the geometric center of the phase ellipsoid. In second and higher order, the ellipsoid may become distorted. In that case, the center of gravity is no longer the beam centroid. If the centroid does not coincide with the reference trajectory, then the centroid coordinates are printed along with the transfer matrix. (see page 242 below for a description of the printing of the transfer matrix and a definition of the beam centroid).

The second column gives the half widths of the beam ellipsoid in each of the six beam coordinates. Once again, in higher orders the beam phase space is not an ellipsoid, but some distorted shape like a banana. The quantities given are then the square roots of the second moments of the beam phase space, taken about the beam center of gravity. In terms of the diagonal elements σ_{ii} of the beam matrix the beam half widths are given by:

$$\Delta X_i = \sqrt{\sigma_{ii}}$$

The units used are given in the third column. Those illustrated here are centimeters `CM`, milliradians `MR`, and percent `PC`. These units are used in the global unit set `UTRANS`. The use of unit sets or individual units is described on pages 71 and 75.

The remaining quantities r_{ij} are the correlations between coordinates. They are defined in terms of the beam matrix elements σ_{ij} by:

$$r_{ij} = \frac{\sigma_{ij}}{[\sigma_{ii}\ \sigma_{jj}]^{1/2}} \quad .$$

Their initial values may be specified by the **CORRELATION** element described on page 116.

The beam matrix may also be printed in a single line. The command which causes the printing to occupy a single line is (**PRINT, ONELINE ;**). This command, described below, does not print anything, but simply affects the style of how quantities are printed. A single line printing of the beam matrix would look like:

L_c M Δx CM $\Delta x'$ MR Δy CM $\Delta y'$ MR $\Delta \ell$ CM $\Delta \delta$ PC r_{21} r_{43}

The accumulated length L_c along the reference trajectory is shown first. The units shown are from the global unit set **UTRANS**. Other global unit sets are described on page 71. Individual unit changes are given on page 75.

A diagram of the beam phase ellipse is shown later in this section.

Beam Matrix Print in Accelerator Notation

(`PRINT, TWISS ;`): The beam matrix in accelerator parameters (β, α, and η) and the beam phase shift (ψ) are printed at the location of this element.

(`PRINT, TWISS, ON ;`): The beam matrix in accelerator parameters (β, α, and η) and the beam phase shift (ψ) are printed after every physical element which follows this code.

(`PRINT, TWISS, OFF ;`): The effect of a previous (`PRINT, TWISS, ON ;`) is cancelled and the beam matrix in accelerator parameters (β, α, and η) and the beam phase shift (ψ) are printed only when a (`PRINT, TWISS ;`) code is encountered or when another (`PRINT, TWISS, ON ;`) code is inserted. The default in TRANSPORT is not to print the accelerator parameters.

The use of the accelerator parameters for the beam matrix makes sense only in first order. In this case, the term "first order" applies to the printing of the transfer matrix. If the beam centroid is off axis, then the transfer matrix can be calculated to first, second, or third order and then linearized about the centroid position. The linear transformation then preserves the elliptical shape of the beam phase space. The elliptical shape allows the representation in terms of accelerator parameters. The inclusion of higher orders in the expansion about the centroid position causes the ellipse to become distorted and precludes the meaningful use of the accelerator parameters.

When the beam matrix is printed in accelerator notation after each physical element, TRANSPORT will first print a heading identifying the quantities printed. The printing of accelerator parameters occurs on a single line, so that the progress of each quantity as it passes down the beam line can be read in a single column. The single line begins with the accumulated length L_c, and then appears as:

$$L_c \;\; \mathtt{M} \qquad \psi_x \;\; \psi_y \;\; \beta_x \;\; \beta_y \;\; \alpha_x \;\; \alpha_y \;\; \eta_x \;\; \eta_y \;\; \eta_x' \;\; \eta_y'$$

The quantities ψ_x and ψ_y are the phase advances in the two transverse planes and are defined by

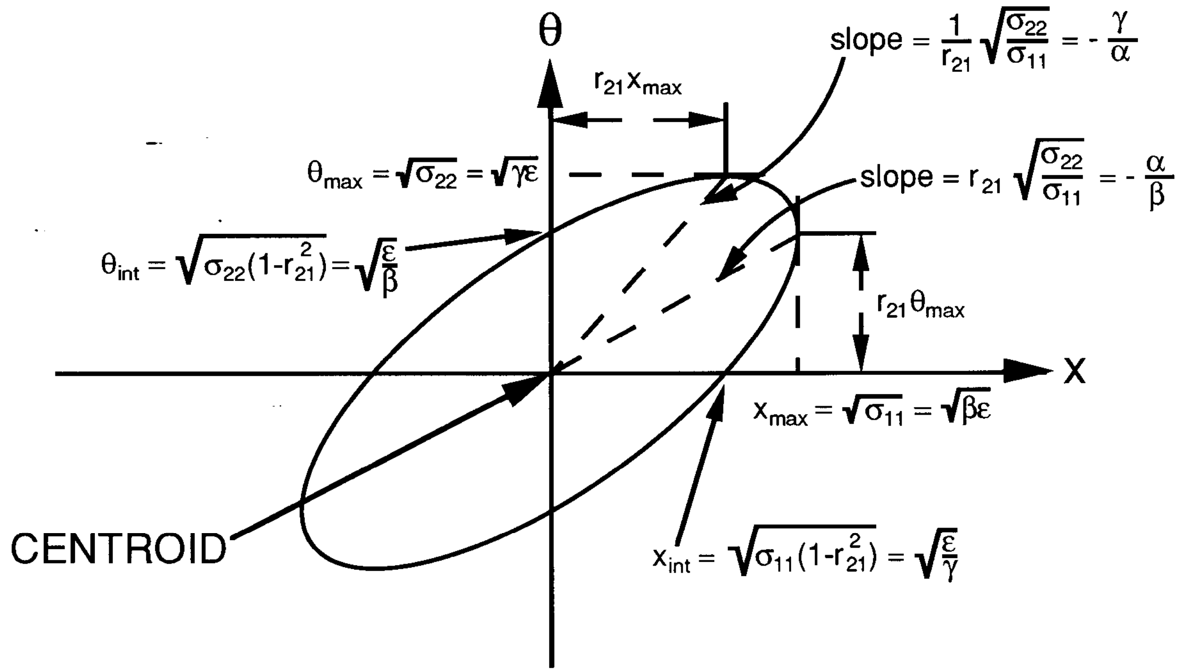
$$\psi = \int \frac{ds}{\beta}$$

The quanitites β and α are shown in the figure below. The η vector and its derivatives represent the momentum behavior of the beam line and are described in the section on ETA. (See page 121.)

The accelerator parameters calculated and shown describe the phase space of the beam. They are not a description of the lattice, although they are affected by the lattice. The α and β are calculated from the values given on the `BEAM` element. They can be calculated even if the beam description is given in σ matrix notation, as long as there is no mixing of transverse planes.

The units for only the accumulated length L_c are given, and here they are shown to be meters (**M**). The units used should always be such that the ratio of transverse distance to transverse angle is meters, and the ratio of transverse distance to momentum difference is also meters. Otherwise the components of β and η will come out in strange units like dekameters or hectinches. The global unit sets **UMAD**, **UMETER**, **UMM**, and **UMICRON** all have this desirable characteristic. The global unit set **UTRANS** does not and is not suitable for use with accelerator parameters for the beam ellipse.

The only violation of midplane symmetry that is permitted with the use of accelerator parameters is a vertical bend. The use of accelerator parameters assumes a separation of the optics of the two transverse planes. Skew quadrupoles, or other skew elements violate this assumption. Should TRANSPORT encounter such an element when accelerator notation is being used, it will refuse to proceed.



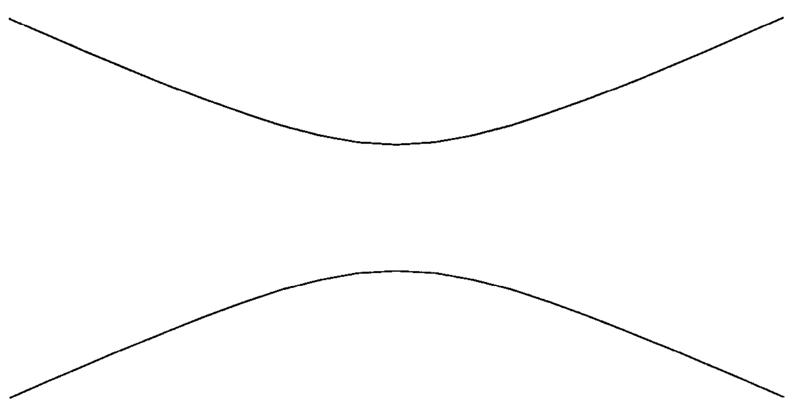
A Two-Dimensional Beam Phase Ellipse

Accelerator Notation For Beam Matrix

(PRINT, ACCEL ;) or (PRINT 7. ;) or (13. 7. ;): This element exists primarily for use with original TRANSPORT notation. Both input and output for the beam matrix and constraints on the beam matrix are in accelerator notation when this element is used. The parameters β, α, and η of accelerator theory are used. Constraints on the beam are also taken to be in terms of β, α, and η. The forms of input and output for the beam matrix are explained on pages 105 and 118. The form of constraints on the beam are explained on pages 272 and 282. This element is unnecessary with keyword notation since input, output, and constraints can be specified individually to be in accelerator notation. To specify that accelerator notation is to be used in a run, the (PRINT ACCEL ;) element must be placed before the BEAM element.

Positions of Waists

(PRINT, WAIST ;) or (PRINT 14. ;) or (13. 14. ;): The longitudinal positions and transverse dimensions of (real or virtual) waists in both transverse planes are printed at the point of insertion of this element. The waist location and characteristics are computed from the dimensions of the current location assuming infinitely long drift spaces both upstream and downstream. An illustration of a waist is shown below.



A Waist in a Beam.

Printing of the Beam Centroid

The beam centroid is taken as the center of the beam ellipsoid. If misalignments, centroid shifts, or mistuned bending magnets are in the beam, the beam centroid may not coincide with the reference trajectory. Normally the centroid position is printed as part of the beam matrix output. If the beam matrix is printed on a single line, the centroid position is printed only if the beam is off axis. Two lines of output are then printed after each element: the first being the centroid position and the second being the σ matrix. The output then has alternating lines of different meaning, and scanning down single columns is visually unpleasant.

In a second or higher-order calculation, the beam centroid will differ from the beam center of gravity, as explained above. In this case, it is actually the printing of the center of gravity that is controlled by this element. The element therefore, in higher orders, is slightly misleadingly named. The exact meaning of the centroid is described later on page 242.

For this reason, there is a command which will suppress the printing of the centroid.

```
PRINT, CENTROID, OFF ;
```

Restoring the printing of the centroid is done with the command

```
PRINT, CENTROID, ON ;
```

Transformation Matrix Print Controls

(`PRINT, TRANS ;`) or (`PRINT 4. ;`) or (`13. 4. ;`): The current transformation matrix R1 (`TRANSFORM 1`) is printed at the location of this element. If the program is computing a second-order matrix, this second-order transformation matrix will be included in the print-out. If the program is computing a third-order matrix, the third-order matrix will also be included. This matrix is cumulative from the last R1 (`TRANSFORM 1`) update. The units of the elements of the printed matrix are consistent with the input units associated with the `BEAM` element.

(`PRINT, TRANS, ON ;`) or (`PRINT 6. ;`) or (`13. 6. ;`): The transformation matrix R1 will be printed after every physical element which follows this code. The second order matrix will be printed automatically after every physical element only if the one-line form (`PRINT, ONELINE ;`) of the transformation is selected. The second order matrix will, however, be printed at each location of a (`PRINT, TRANS ;`) element or its equivalent. The third-order matrix will be printed only at a location of the (`PRINT, TRANS ;`) element. The first-order matrix will be printed only once at a (`PRINT, TRANS ;`) element.

(`PRINT, TRANS, OFF ;`) or (`PRINT 5. ;`) or (`13. 5. ;`): The automatic printing of R1 will be suppressed and R1 will be printed only when subsequently requested.

(`PRINT -4. ;`) or (`13. -4. ;`) The inverse of the current transfer matrix R1 will be printed by this instruction. If the program is computing a second-order matrix, the inverse of the second-order transfer matrix will be included in the print out. The inverse of the third-order matrix is not printed at the present stage of program development.

(`PRINT, TRAN2 ;`) or (`PRINT 24. ;`) or (`13. 24. ;`): The `TRANSFORM 2` matrix, R2, will be printed by this code. The format and units of R2 are identical with those of R1, which is printed by the (`PRINT TRANS ;`) code. For a list of elements which update the R2 matrix, see the `UPDATE` element.

The units of the tabulated matrix elements in either the first-order R or σ matrix, second-order T matrix, or third-order U matrix of a TRANSPORT print-out will correspond to the units chosen for the `BEAM` card. For example, the $R_{12} = (x|x'_0)$ matrix element will normally have the dimensions of cm/mr; and the $T_{236} = (x'|y_0\delta)$ matrix element will have the dimensions mr/(cm-percent) and so forth.

The six coordinates of the transfer matrix correspond to x, x', y, y', ℓ, and δ. The matrix will then have the appearance

$$\begin{matrix}
(x|x_0) & (x|x'_0) & (x|y_0) & (x|y'_0) & (x|\ell) & (x|\delta) \\
(x'|x_0) & (x'|x'_0) & (x'|y_0) & (x'|y'_0) & (x'|\ell) & (x'|\delta) \\
(y|x_0) & (y|x'_0) & (y|y_0) & (y|y'_0) & (y|\ell) & (y|\delta) \\
(y'|x_0) & (y'|x'_0) & (y'|y_0) & (y'|y'_0) & (y'|\ell) & (y'|\delta) \\
(\ell|x_0) & (\ell|x'_0) & (\ell|y_0) & (\ell|y'_0) & (\ell|\ell) & (\ell|\delta) \\
(\delta|x_0) & (\delta|x'_0) & (\delta|y_0) & (\delta|y'_0) & (\delta|\ell) & (\delta|\delta)
\end{matrix}$$

If the beam centroid is off axis, the appearance of what is printed is slightly different. The initial beam centroid is the initial center of the beam ellipse. It is given by a set of coordinates which can be taken as representing a trajectory and therefore traced through the beam line. The default phase-space position of the centroid is that it coincide with the reference trajectory. If a `CENTROID` element is included in the data, then the initial phase-space coordinates of the centroid are given on the `CENTROID` element. The centroid may, at some place in the beam line, deviate from the reference trajectory when it encounters a known misalignment, a bending magnet with excess horizontal bend or an error vertical bend field, a kicker magnet `HKICK`, `VKICK`, or `KICKER`, an electrostatic septum `SEPTUM`, or a shift in coordinates `SHIFT`.

The transfer matrix represents an expansion about the centroid. Thus the centroid position represents a zeroeth order term in the transformation of an arbitrary trajectory. If the centroid coincides with the reference trajectory, it is "on axis" and its coordinates are zero. It must be emphasized that not all trajectory coordinates are geometrically defined. An initial beam centroid may be geometrically on the reference trajectory but have a different defining momentum, so that the coordinate δ for the centroid is not zero. In that case, the centroid is still considered to be "off axis."

If the centroid is off axis, an extra column will appear to the right, giving the position of the beam centroid. The centroid given is the optical image of the centroid at the beginning of the beam line. In first order it will coincide with the center of gravity of the beam ellipse. In higher orders the two will differ due to nonlinear distortions of the beam ellipse.

$$\begin{array}{cccccccc}
(x|x_0) & (x|x_0') & (x|y_0) & (x|y_0') & (x|\ell) & (x|\delta) & \quad & x_c \\
(x'|x_0) & (x'|x_0') & (x'|y_0) & (x'|y_0') & (x'|\ell) & (x'|\delta) & & x_c' \\
(y|x_0) & (y|x_0') & (y|y_0) & (y|y_0') & (y|\ell) & (y|\delta) & & y_c \\
(y'|x_0) & (y'|x_0') & (y'|y_0) & (y'|y_0') & (y'|\ell) & (y'|\delta) & & y_c' \\
(\ell|x_0) & (\ell|x_0') & (\ell|y_0) & (\ell|y_0') & (\ell|\ell) & (\ell|\delta) & & \ell_c \\
(\delta|x_0) & (\delta|x_0') & (\delta|y_0) & (\delta|y_0') & (\delta|\ell) & (\delta|\delta) & & \delta_c
\end{array}$$

For a static magnetic system with midplane symmetry, a good deal of simplification occurs. Many of the matrix elements become zero. For the case of an on-axis centroid, the transfer matrix then becomes:

$$\begin{array}{cccccc}
(x|x_0) & (x|x_0') & 0.0 & 0.0 & 0.0 & (x|\delta) \\
(x'|x_0) & (x'|x_0') & 0.0 & 0.0 & 0.0 & (x'|\delta) \\
0.0 & 0.0 & (y|y_0) & (y|y_0') & 0.0 & 0.0 \\
0.0 & 0.0 & (y'|y_0) & (y'|y_0') & 0.0 & 0.0 \\
(\ell|x_0) & (\ell|x_0') & 0.0 & 0.0 & 0.0 & (\ell|\delta) \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & (\delta|\delta)
\end{array}$$

If the beam line is midplane symmetric and there is no vertical bending, then the transfer matrix can be printed on a single line. To do this, one should include the command

(`PRINT,    ONELINE   ;`) described below. Only those matrix elements which can be nonzero under the stated conditions are printed. The quantities printed are:

$$L_c \;\; \mathtt{M} \qquad R_{11} \;\; R_{12} \;\; R_{21} \;\; R_{22} \;\; R_{33} \;\; R_{34} \;\; R_{43} \;\; R_{44} \;\; R_{16} \;\; R_{26}$$

The units for only the accumulated length L_c are shown and are here taken to be meters `M`.

If second order is specified, then the second-order matrix elements which are nonzero in a midplane symmetric situation will be printed in a compact form. At this point, there is no abbreviated form for the third-order transfer matrix.

Realignment of the Reference Trajectory

(`REALIGN ;`) or (`PRINT 9 ;`) or (`13. 9. ;`): At the location of this element, the reference trajectory is shifted to line up with the first-order image of the initial beam centroid. Thus, the reference trajectory of the beam line, as followed by the program, will not be continuous. The beam centroid location will be continuous, but will seem to be discontinuous if expressed relative to the reference trajectory.

If the centroid does not coincide with the reference trajectory, then quadrupoles, sextupoles, and octupoles will show dispersive effects. All the fitting options connected with this element operate as expected. Thus the original beam centroid displacement parameters can be varied to fit values of floor coordinates specified after the shift.

Part of a TRANSPORT set of data might then have the following appearance:

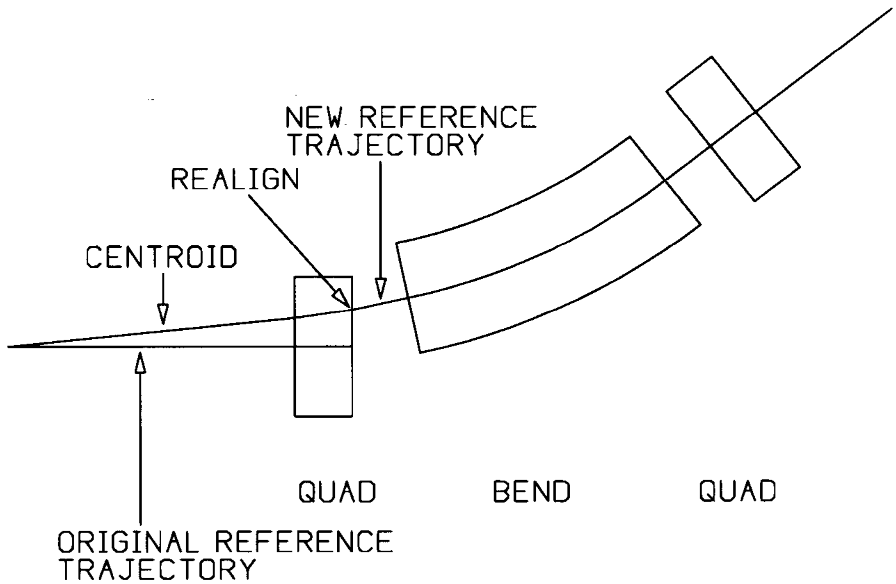
```
BEAM . . . . . . . ;
CENTROID . . . ;
DRIFT . . . ;
QUAD . . . ;
REALIGN ;
DRIFT . . . ;
BEND . . . ;
DRIFT . . . ;
QUAD . . . ;
DRIFT . . . ;
```

The **CENTROID** element indicates an initial centroid displacement which is then propagated through the system. The centroid is focused by the quadrupoles. After traversing a quadrupole the transfer matrix will show dispersive elements (R_{16} and R_{26}), since the transfer matrix is a linear expansion about the centroid and the centroid does not follow the axis of the quadrupoles. A diagram of this simple system is shown on the following page.

When the **REALIGN** element is encountered, the centroid dispacement is set to zero. The reference trajectory is redefined to be a continuation of the previous centroid. The transfer matrix, being a linear expansion about the centroid, is unchanged. TRANSPORT continues to accumulate the transfer matrix after the **REALIGN** element. The floor coordinates will be shifted immediately after the realign element. The amount of shift will be that necessary to align the beam line components on the off-axis centroid.

An example of the use of the **REALIGN** command occurs when the first optical element in a beam line is a quadrupole, but because of geometric constraints, the optic axis of the quadrupole cannot be placed so that it is aligned with the center of the accepted phase space. The centroid may be placed at the center of the accepted phase space. The beam line can be laid out by allowing the centroid to be focused by the quadrupole and then redefining the reference trajectory to be lined up with the centroid as it exits the quadrupole.

The effect of the **REALIGN** command in the data above can be shown by the following figure:



The Effect of the **REALIGN** element

Refer Transfer Matrix to Original Coordinate System

(**PRINT, REFER ;**) or (**PRINT 13. ;**) or (**13. 13. ;**): The interpretation of the beam rotation element **SROT** is affected by this element. The transverse coordinates can be rotated about the longitudinal axis by an angle via the **SROT** element. The default interpretation is that the beam phase space is rotated. The transfer and beam matrices and the floor coordinates are printed in the rotated system.

When the (**PRINT, REFER ;**) (or its equivalent) element is used, the rotation is taken as specifying only the orientation of any ensuing magnets. The transfer matrix, whenever printed, is given in the unrotated coordinate system. The transfer matrix then gives only the effect of the beam line on a particle trajectory, and not the effect of specifying the trajectory components in a different coordinate system. This element should be placed near the beginning of the data, before the **BEAM** element.

Refer Transfer Matrix to Level Coordinate System

(**PRINT, LEVEL ;**): The transfer matrix, beam matrix, and floor coordinates are caused by this element to be referred to a coordinate system whose x axis is level. A combination of vertical and horizontal bends can produce a beam line where the z axis is level, but the x axis is not. This is often the coordinate system in which the original x and y coordinate systems remain uncoupled as we progress down the beam line. but the floor coordinates have a slight roll. This command removes the roll, levelling the x axis, but introduces a coupling between the two transverse planes. This command should be inserted at the location where it is to have its effect.

Precise Values of Varied Parameters

(`PRINT, PRECISE ;`) or (`PRINT 16. ;`) or (`13. 16. ;`): The values of any varied parameters will be printed in F18.10 format when this element is used. This option is useful primarily for investigating the mathematical characteristics of a solution. It should be inserted near the beginning of the deck before the `BEAM` element.

General Output Format Controls

(`PRINT, NOPARA ;`) or `PRINT 17. ;`) or (`13. 17. ;`): The subsequent printing of the physical parameters of all physical elements will be suppressed. Only the mnemonic and the label will remain. This element is useful in conjunction with the (`PRINT ONELINE ;`) element which restricts the beam (σ) matrix and the transformation (R1) matrix each to a single row. The elements of these matrices then appear in uninterrupted columns in the output.

(`PRINT, ONLY ;`) or (`PRINT 18. ;`) or (`13. 18. ;`): Only varied elements and constraints will be printed. This element, in conjunction with the various options on the indicator card, can produce a very abbreviated output. The entire output of a multistep problem can now easily be viewed on a terminal or printed on a laser printer.

(`PRINT, ONELINE ;`) or (`PRINT 19. ;`) or (`13. 19. ;`): The beam (σ) and transformation (R1 or R2) matrices, when printed, will occupy a single line. Only those elements are printed which will be nonzero if horizontal midplane symmetry is maintained. The second- and third-order transfer matrices will obviously take several lines of printing for all the nonzero elements. However, this command also causes these higher-order matrices to be printed in a more compact format.

In a first-order run, the (`PRINT, ONELINE ;`) element, in conjunction with the (`PRINT, NOPARA ;`) element and either the (`PRINT, BEAM, ON ;`) element or the (`PRINT, TRANS, ON ;`) element, will produce output in which the printed matrix elements will occupy single uninterrupted columns. For visual appearances it is recommended that, if both beam (σ) and transformation matrices are desired, they be printed in separate steps of a given problem.

`(PRINT, ELEMENTS, OFF ;)` and `(PRINT, ELEMENTS, ON ;)`: The first of these elements completely suppresses the printing for the elements in a portion of the beam line. The second element restores that printing. The transfer matrix and beam matrix are still printed wherever an explicit print instruction is found. If one has a very long beam line, but is investigating only a small portion of it, one need print only that portion in which one is interested.

`(PRINT, ALL, OFF;)` and `(PRINT, ALL, ON ;)`: The first of these elements completely suppresses all the printing in a portion of the beam line. The second element restores that printing. With this element, one can produce an even more compact output than with the previous element.

`(PRINT, NARROW ;)`: Some terminals display only the first 80 characters of a line. It can sometimes be both awkward and annoying to try to view anything printed to the right of the first 80 characters. The above command causes the beam matrix to be moved to the left in the output, so that it is entirely within the first 80 characters. The beam matrix printing is affected only in the σ matrix notation. The printing of the `FIT` element is also doubled over so that the current value of the constrained quantity also fits within the first 80 characters.

`(PRINT, MARKER ;)`: Any markers inserted in the beam line will now be included in the printed output. Markers are not physical elements, but simple designations of position to which `PRINT` or `FIT` coommands can refer. Some users have found it useful to have them print out when they occur. This command should be placed before the `BEAM` element.

Misalignment Table Print Control

`(PRINT, ALIGN ;)` or `(PRINT 8. ;)` or `(13. 8. ;)`: The misalignment summary table is printed wherever this element is inserted. Its contents are the effects of all previously specified misalignments whose results were to be stored in a table. A full description of the table and its contents is to be found on page 214.

Misalignment Controls

`(PRINT 20. ;)` or `(13. 20. ;)`: The longitudinal midpoint of the misaligned element serves as the reference point for the misalignment when this command is inserted. The axes are still parallel to the axes of the beam line coordinates at the entrance face of the element. This command should precede the `BEAM` element.

`(PRINT 21. ;)` or `(13. 21. ;)`: Only the focusing effect of misalignments is taken into account when this element is present. It is assumed that the beam line is outfitted with steering magnets that continuously redirect the beam centroid back to the reference trajectory. Therefore there is no displacement of or uncertainty in the position of the reference trajectory. Any growth in the area enclosed in the beam matrix is due to the uncertainty in strength of the focusing elements.

(`PRINT 22. ;`) or (`13. 22. ;`): The chord of a bending magnet replaces the entrance face z axis in the coordinate system to which the misalignment is referred. The x coordinate is taken as horizontal and perpendicular to the new z coordinate. The y coordinate continues to be vertical.

Punched Output Controls

(`PRINT 29.` through `36. ;`) or (`13. 29.` through `36. ;`): This element is now used little if at all. Its original purpose was to allow another program to use the transfer matrices for ray tracing and histogram generation. The transfer matrices were punched on cards by this element. The transfer matrices can still be written to an output file by this element. The word "punch" is used in this description to emphasize the obselesence of this element.

Since 1972 the program TURTLE has existed which did this same operation starting with the TRANSPORT input data. However, every time any feature of TRANSPORT is eliminated, we discover some user somewhere who has been using that feature. It is therefore included for compatability with previous versions.

If the control is `29`, all of the terms in the first-order matrix and the x and y terms of the second-order matrix are punched.

If the control is `30`, all of the terms of the first-order matrix and all second-order matrix elements are punched out.

If the control, n, is greater than 30, all of the first-order terms are punched and the second-order matrix elements which correspond to $(n - 30.)$, i.e., if n = 32, the second-order theta matrix elements are punched out. If n = 31, the second-order x matrix elements are punched, and so forth.

Automatic Pole-Face Rotation Angle Specification

This element is still available in TRANSPORT but has been superseded. Its function can be performed more easily by the MAD input with the element types `RBEND` and `SBEND` and the use of parameters. However, before the MAD type input was implemented many data sets were created which contained this automatic pole-face rotation angle specification. It is therefore retained for compatability, although users are discouraged from using it.

The pole face rotation angle normally specified with the `ROTAT` element may alternatively be calculated from the bend angle of the associated bending magnet. To include the effect of the fringing field the pole face rotation element must still be present. The options described here merely cause the value of β (the pole-face rotation angle) to be filled in automatically by the program.

The element specifying the means of determination of the pole face rotation angle must precede the bending magnet specification, including pole-face rotation angle elements, to which it applies. It will remain in effect until the option is respecified via another `PRINT` (or `13.`) element with the appropriate code number on it. Some caution must be taken in the use of these options. If a bending magnet is segmented, then the pole face rotation angle will be calculated from the bend angle of the adjacent segment.

(`PRINT 41. ;`) or (`13. 41. ;`): Both input and output pole face rotation angles are equal to half the bend angle.

(`PRINT 42. ;`) or (`13. 42. ;`): The entry pole face rotation angle will be zero. The exit pole face rotation angle will equal the bend angle.

(`PRINT 43. ;`) or (`13. 43. ;`): The entry pole face rotation angle will equal the bend angle. The exit pole face rotation angle will be zero.

(`PRINT 40. ;`) or (`13. 40. ;`): The normal option is restored. The pole-face rotation angles will be read from the data.

STORE – Storage of Matrix Elements

Any matrix element or quantity that can be constrained can also be stored for use in a subsequent constraint. The stored quantity is given its own proper name on the STORE command. The quantity is referred to by this proper name at any later point. For example, the initial value of the synchrotron function BETAX may be stored under the name BX0.

```
STORE,  BX0 = BETAX  ;
```

If a constraint is to be imposed at the end of the beam line, setting the final beta value equal to the initial beta value, the user could impose the constraint.

```
FIT,  NAME = BETAX - BX0,  VALUE = 0.0,  TOLER = 0.001  ;
```

The difference BETAX - BX0 would then be fit to zero. The quantity BETAX would be the horizontal value of beta at the point of the FIT element. The quantity BX0 would be the initial horizontal value of beta.

If the beam line is assembled from elements and specified in a LINE statement, the STORE instruction may be placed after the beam line specification. It may refer to a marker which is included in the beam line specification. This is done with the LOCA(TION) keyword. Assume the marker

```
MAR1:   MARKER  ;
```

is included in the beam line description. Assume further that the user wishes to fit an element of the transfer matrix at the location of this marker. The command

```
STORE,  BXN = BETAX,  LOCA = MAR1  ;
```

will store the horizontal beta value under the name BXN at the location MAR1.

PLOT — Data for Plotting

Data for plotting may be output in a separate file on logical unit 8. The job running TRANSPORT should contain the appropriate computer instructions to identify logical unit 8 with the file the user wishes to contain the output. These data may then be used to create a plot with a separate plotting program. Examples in this section were made with the plotting package TOPDRAWER. The procedure for using TOPDRAWER is described later in this section.

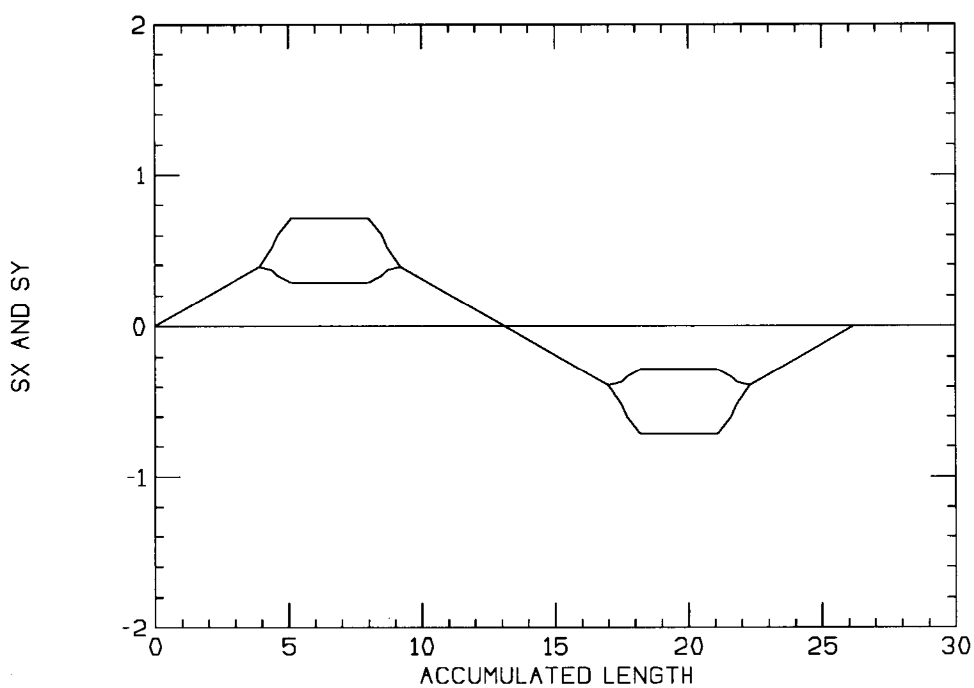
Plots vs Accumulated Length

There are currently three types of plot that can be made with TRANSPORT. The first is a plot of something vs accumulated length along the reference trajectory. For example, a file to be used to plot the matrix element R_{12} against accumulated length would be created by placing the element:

```
PLOT,  L,  R12  ;
```

before the beam card.

A plot of the transfer matrix elements R_{12} and R_{34} for a beam line is shown below. The transfer matrix elements, R_{12} and R_{34}, are also respectively known as the horizontal and vertical sinelike trajectories, s_x and s_y.



TRANSPORT will produce output to the separate file for each run where a PLOT instruction is included. A heading will first be printed with the names of the quantities to be plotted. If one of the items to be plotted is the accumulated length L, then the values of all the indicated items will be printed on unit 8 after each physical element. Up to eight items may be included on the list. The printing will be done in format 8F10.5. The items that may be included on the list are as follows:

Symbol	Item for Plotting
L	The beam line accumulated length.
Rij	An element of the first-order transfer matrix $R1$.
RAij	An element of the auxilliary first-order transfer matrix $R2$.
XC	The beam centroid position in x.
XPC	The beam centroid position in x'.
YC	The beam centroid position in y.
YPC	The beam centroid position in y'.
DLC	The beam centroid position in ℓ.
DELC	The beam centroid position in δ.
XBEAM	The beam half width in x.
XPBEAM	The beam half width in x'.
YBEAM	The beam half width in y.
YPBEAM	The beam half width in y'.
DLBEAM	The beam half width in ℓ.
DELBEAM	The beam half width in δ.
Sij	An element of the beam matrix σ. If a diagonal element is specified, then the square root of that element will be printed. The square root of a diagonal element is the beam half width in a particular coordinate. Any beam half width can also be represented by one of the mnemonics immediately preceding Sij.
Cij	An element of the beam correlation matrix r.
BETAX	The accelerator function β_x.
ALPHAX	The accelerator function α_x.
BETAY	The accelerator function β_y.
ALPHAY	The accelerator function α_y.
ETAX	The accelerator function η_x.
DETAX	The accelerator function η'_x.
ETAY	The accelerator function η_y.
DETAY	The accelerator function η'_y.
Tijk	An element of the second-order transfer matrix $T1$.
TAijk	An element if the auxilliary second-order transfer matrix $T2$.
Uijkℓ	An element of the third-order transfer matrix $U1$.

UAijkℓ An element of the auxilliary third-order transfer matrix $U2$.

The plotting package TOPDRAWER can be used to make a plot from the data in the separate file. Details are given later in this section.

It is possible also to plot algebraic combinations of plottable items. Here, as with fitting, an algebraic expression must be on the right side of an equation. The plottable items are then given keywords **NAME1** ... **NAME8**. A command to plot the product $R_{11} * R_{12}$ vs accumulated length might then read:

```
PLOT,  NAME1 = L,  NAME2 = R11*R12  ;
```

Floor Coordinate Layouts

The second type of plot that one can make is a beam layout. The beam layout is a plot of the floor coordinates of the reference trajectory. The coordinates that may be plotted are either x vs z or y vs z. Unless otherwise specified the reference trajectory initially is along the z axis. Alternate specifications can be made with the **SPECIAL** element described on page 87. The perpendicular horizontal direction is x and the vertical direction is y.

The **PLOT** command for a floor layout should be placed before the **BEAM** element. For a view of the beam line as seen from above, use

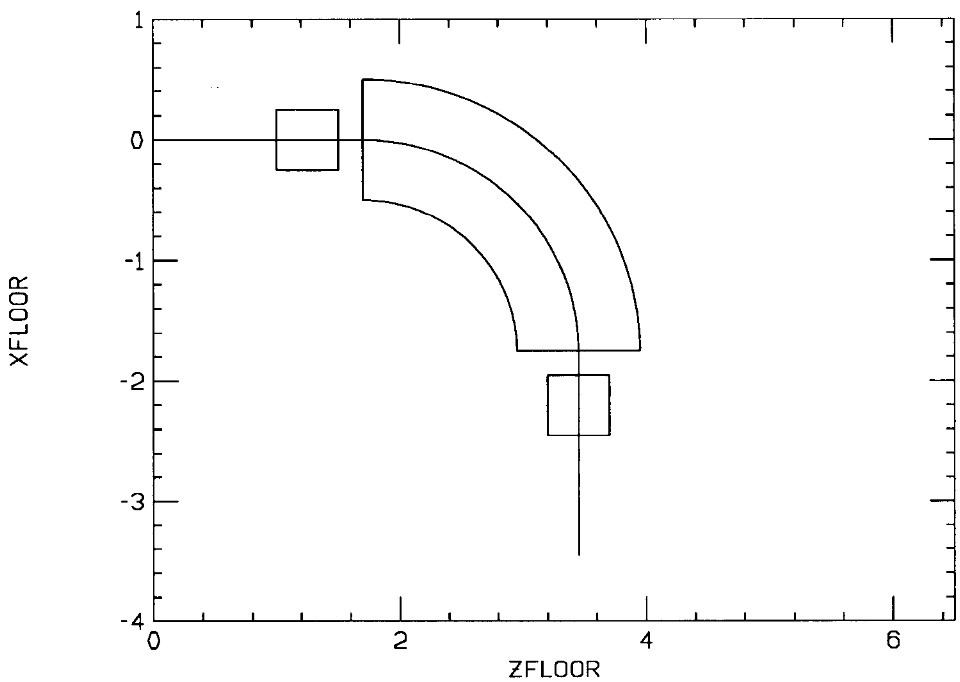
```
PLOT,  ZFLOOR,  XFLOOR  ;
```

For a view as seen from the side, use

```
PLOT,  ZFLOOR,  YFLOOR  ;
```

Magnets will be drawn also if their dimensions are specified via the **MAGNET** element. The use of the **MAGNET** element is described on page 101.

A simple beam line as seen from the top is shown below. The plotting program TOP-DRAWER has been used to do the actual plotting.



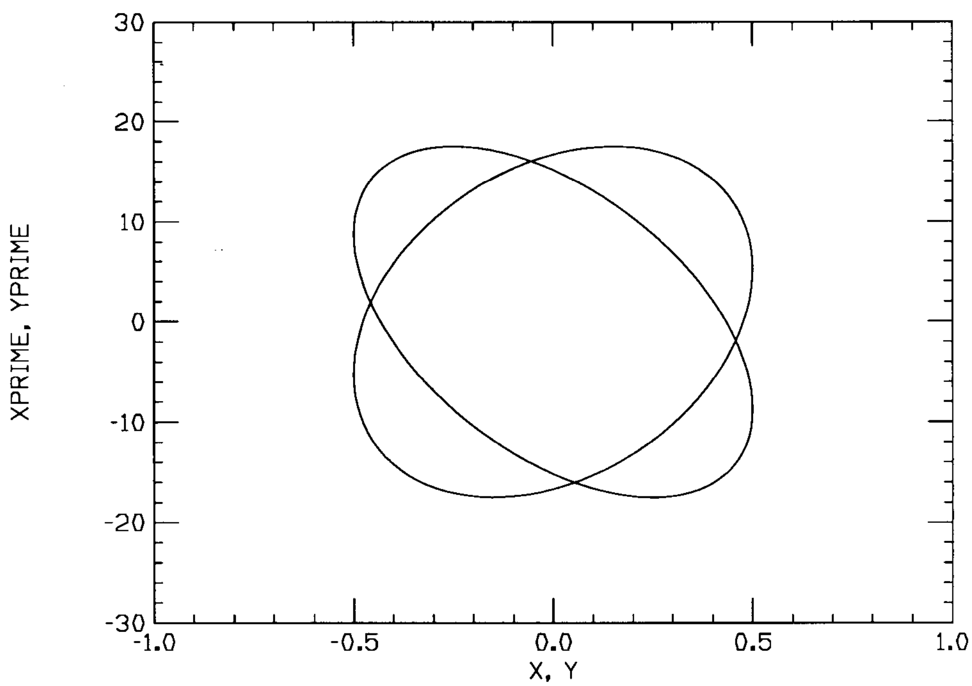
Plots of the Beam Ellipse

The third type of plot is of the beam ellipse. The beam ellipse in x vs x' at a given point in the beam line may be plotted by including the command:

```
PLOT, XBEAM, XPBEAM ;
```

at that particular point. The output will consist of the x and x' coordinates of 100 points about the circumference of the ellipse. The output will be presented in two columns with headers. Each of the two columns will be 101 items long. The last numbers are a repetition of the first in order to close the ellipse. The output format will be 2F10.5.

A combined plot of the horizontal and vertical phase ellipses might appear as follows:



Sequential Runs

Different plot data may be output in sequential steps of a run so as to be included in the same data set. The first step (with indicator card 0) may contain the specification:

```
PLOT:  PLOT,  L,  R12  ;
```

In a subsequent step (with indicator 1) the items on the `PLOT` card may be altered to plot another quantity. An example might be:

```
PLOT:  PLOT,  L,  R34  ;
```

Another header would be printed followed by another two columns of numbers. The lengths of the columns would be the same as in the first step. The first columns, being L in both cases, would be identical. The second column would be different, since R_{34} is being printed instead of R_{12}.

Use with TOPDRAWER

The output feature described has been used with the plotting package TOPDRAWER. A skeleton data set for TOPDRAWER might look like:

```
SET LIMITS X 0.0 1100.0
SET LIMITS Y -10.0 10.0
SET SCALE X LINEAR
SET SCALE Y LINEAR
TITLE TOP SIZE 2 'R12 AND R34 VS L'
SET ORDER X Y
DATA
JOIN 1
DATA
JOIN 1
STOP
```

The word `DATA` is not a TOPDRAWER command, but rather indicates where the data generated by TRANSPORT are to be inserted. The deck shown would be appropriate for plotting R_{12} and R_{34} vs accumulated length L, as described above. The replacement of the word `DATA` by the TRANSPORT generated data need not be done by hand, but can be performed automatically. An auxiliary program exists which combines the TRANSPORT generated data with the TOPDRAWER skeleton to produce a complete topdrawer data set. This data set may then be run through TOPDRAWER to generate plots. Once the TRANSPORT data deck and the TOPDRAWER skeleton data set are set up, the whole procedure may be automated on a computer file. A single command can then run TRANSPORT and TOPDRAWER to produce plots.

Variation of Parameters for Fitting

Some (not all) of the physical parameters of the elements comprising a beam line may be varied in order to fit selected matrix elements or algebraic combinations of matrix elements. In a first-order calculation one might fit elements of the $R1$ or $R2$ transformation matrices or the beam (σ) matrix. In second order one might constrain an element of the second-order matrix $T1$ or minimize the net contribution of aberrations to a given beam coordinate. Similarly, in third order one might constrain an element of the third-order matrix $U1$. Special constraints are also available.

One may mix orders in fitting. Any lower order vary code or constraint may be imposed in a higher-order run as long as it is meaningful. Exceptions will be described below.

In MAD notation the physical parameters to be varied are selected by the `VARY` command. In original TRANSPORT notation they are selected by 'Vary Codes' attached to the type codes of the elements comprising the system. The fitting constraints on matrix elements are selected via `FIT` elements placed in the system where the constraint is to be imposed.

The use of the permissive 'may' rather than the imperative 'will' in discussing variables is meaningful. The program will choose the parameters it will vary from among those that it may vary. In general it chooses to vary those parameters that have the greatest influence upon the conditions to be fit.

MAD and MAD-like Notation

The VARY Command

The explicit `VARY` command is more suited to keyword notation of MAD. Strictly speaking the `VARY` command is in MAD notation only when specifying the variation of parameters which are themselves expressed in MAD notation. Since TRANSPORT has many alternative parameterizations of elements which are characterized as MAD-like, corresponding occurrances of the `VARY` command must also be characterized as MAD-like. The `VARY` command is not an element, but rather an instruction as to how to modify an element already specified. The `VARY` command occurs in the deck when any parameter is to be declared to be variable. The `VARY` command need never occur in the data preceded by a minus sign.

If a quadrupole is specified as:

```
Q1:   QUAD,  L = 10.,  B = 5.,  APER = 2.54  ;
```

then the magnetic field of that quadrupole can be made to be variable by the statement

```
VARY,  NAME = Q1[B]  ;
```

The keyword **NAME** is followed by a designation of the parameter to be varied. An alternate formulation of the same command is:

```
VARY,  Q1[B]  ;
```

In the case of an element parameter the parameter is designated by the element label followed by the parameter keyword in brackets.

A parameter can be varied only if it is explicitly included in the data. In other words, the above command to vary the B parameter of quad Q1 has no meaning if quad Q1 is described in terms of its length and gradient, i.e. if

```
Q1:   QUAD,  L = 10.,  GRAD = 2., ;
```

Parameters declared on **PARAMETER** elements may also be varied. In this case the item following the keyword **NAME** is the parameter name. If the value of the magnetic field were declared in a separate parameter statement, the quadrupole specification might look like.

```
PARAM,  B1 = 5.0  ;
Q1:   QUAD,  L = 10.0,  B = B1,  APER = 2.54  ;
```

The parameter B1 could be varied by means of the statement

```
VARY,  NAME = B1  ;
```

or simply

```
VARY,  B1  ;
```

There is no provision for coupled variation as there is with the vary codes. However, the use of parameters and algebraic expressions allows much greater flexibility than does vary codes.

The use of the VARY statement allows physical separation of the data describing the mathematical operations to be performed from the specification of the beam line. The VARY statements can all be placed together, near the other operations, after the beam line specification is complete. This grouping of VARY statements is possible both in the case when the beam line is defined explicitly with LINE and USE statements and in the original TRANSPORT situation when the beam line is simply the sequence of elements listed.

The FIX Command

A variable parameter may be fixed again by the command

```
FIX,  NAME = name ;
```

Whatever parameters are named are then held fixed and are not available for achieving the desired constraints. An alternative formulation of the same command is

```
FIX, name ;
```

If the previously varied field of quadrupole Q1 were to be fixed, the user could insert the command:

```
FIX,  Q1[B]  ;
```

The FIX command is not an element, but rather an instruction as to how to modify an element already specified. The FIX command occurs in the deck when any formerly varied parameter is now to be held fixed. The FIX command need never occur in the data preceded by a minus sign.

If all previously varied parameters of any type are to be fixed, the single command

```
FIX,  ALL  ;
```

may be included in the data. The user now need not go back through the data determining which parameters might have been varied, in order to instruct the program no longer to vary them. A single command will now fix all previously varied parameters.

Parameters Variable Only in First Order

In a first-order run the following parameters may be varied.

BEAM	X	One-half the horizontal beam extent.
	XP	One-half the horizontal beam divergence.
	Y	One-half the vertical beam extent.
	YP	One-half the vertical beam divergence.
	L	One-half the longitudinal beam extent.
	DEL	One-half the fractional momentum spread.
CORRELATION	CIJ	Any element of the beam correlation matrix may be varied. The indices I and J must be replaced by their numerical values, so that the varied quantity will be, for example, C43.
ALIGN	X	The magnet displacement in x.
	RX	A rotation about the x axis.
	Y	The magnet displacement in y.
	RY	A rotation about the y axis.

	Z	The magnet displacement in z.
	RZ	A rotation about the z axis.

Parameters Variable in Any Order

SPECIAL	XBEGIN	The initial floor x coordinate
	YBEGIN	The initial floor y coordinate
	ZBEGIN	The initial floor z coordinate
	YAW	The initial floor horizontal angle θ
	PITCH	The initial floor vertical angle ϕ
	RMPS	The fractional excess bend field on a bending magnet.
	RNMS	The overall scaling factor for the midplane-symmetry-violating field component of a bending magnet
	VR	The fractional vertical bend field of a bending magnet.
	NP	The midplane-symmetry-violating normalized gradient of a bending magnet.
PARAM		Any defined parameter may be varied.
CENTROID	X	The shift in the x coordinate
	XP	The shift in x'.
	Y	The shift in y.
	YP	The shift in y'.
	L	The shift in ℓ.
	DEL	The shift in δ.
ETA	ETAX	The x component of η.
	DETAX	The x' component of η.
	ETAY	The y component of η.
	DETAY	The y' component of η.
	ETAL	The longitudinal displacement of η.
	ETAP	The fractional momentum deviation for η.
DRIFT	L	The drift length.
ROTAT	ANGLE	Angle of pole-face rotation.
BEND	L	The magnet length.
	ANGLE	The angle of bend.
	K1	The quadrupole coefficient.
	RMPS	The fractional excess magnetic field.
	RNMS	Scaling factor for midplane-symmetry-violating field components.
	VR	The midplane-symmetry-violating dipole field.
	K1P	The midplane-symmetry-violating quadrupole coefficient.
	B	The magnetic field.

	RADIUS	The radius of curvature of the reference trajectory.
	N	The normalized field derivative.
	NP	The midplane-symmetry-violating normalized field derivative.
	TILT	The roll of the coordinate system about the bend entrance and exit axes.
RBEND, SBEND	L	The magnet length.
	ANGLE	The angle of bend.
	K1	The quadrupole coefficient.
	E1	The entrance pole-face rotation angle.
	E2	The exit pole-face rotation angle.
	RMPS	The fractional excess magnetic field.
	RNMS	Scaling factor for midplane-symmetry-violating field components.
	VR	The midplane-symmetry-violating dipole field.
	K1P	The midplane-symmetry-violating quadrupole coefficient.
	B	The magnetic field.
	RADIUS	The radius of curvature of the reference trajectory.
	N	The normalized field derivative.
	NP	The midplane-symmetry-violating normalized field derivative.
	TILT	The roll of the coordinate system about the bend entrance and exit axes.
HKICK, VKICK	L	The magnet length.
	B	The magnetic field.
	ANGLE	The deflection angle.
	TILT	The roll of the coordinate system about the bend entrance and exit axes.
QUAD	L	The magnet length.
	B	The magnetic field.
	APER(TURE)	The magnetic aperture.
	GRADIENT	The gradient of the magnetic field.
	TILT	The roll of the coordinate system about the quadrupole axis.
PLASMA	L	The magnet length.
	K1	The normalized quadrupole component.
	B	The magnetic field.
	GRAD	The gradient.
SOLENOID	L	The magnet length.
	KS	The rotation angle.
	B	The magnetic field.
SROT	ANGLE	The angle of rotation.

In a second- or higher-order run variation of a drift length or any first-order element should be done with caution as it may affect the first-order properties of the beam line. But inversely coupled drift spaces straddling a sextupole will, for example, show only second- and higher-order effects.

Parameters Variable Only in Second or Higher Order

In a second-order run the following parameters may be varied:

SPECIAL	EPS	The normalized quadratic term (sextupole component) in the midplane expansion for the field of a bending magnet.
	H1	The pole face curvature of a bending magnet entrance
	H2	The pole face curvature of a bending magnet exit.
	EPSP	The midplane-symmetry-violating sextupole component of a bending magnet.
ROTAT	H	Pole face curvature.
BEND	K2	The sextupole coefficient.
	K2P	The midplane-symmetry-violating sextupole component.
	EPS	The quadratic field dependence.
	EPSP	The midplane-symmetry-violating quadratic field dependence.
RBEND, SBEND	K2	The sextupole coefficient.
	H1	The entrance pole-face curvature.
	H2	The exit pole-face curvature.
	K2P	The midplane-symmetry-violating sextupole component.
	EPS	The quadratic field dependence.
	EPSP	The midplane-symmetry-violating quadratic field dependence.
SEXTUPOLE	B	The magnetic field.
	K2	The normalized sextupole component.
	TILT	The roll of the coordinate system about the sextupole axis.

Parameters Variable Only in Third Order

SPECIAL	EPS3	Cubic term for the field of a bending magnet.
BEND	K3	The octupole coefficient.
	EPS3	The cubic field dependence.

RBEND, SBEND	K3	The octupole coefficient.
	EPS3	The cubic field dependence.
OCTUPOLE	K3	The normalized octupole component.
	B	The magnetic field.
	TILT	The roll of the coordinate system about the octupole axis.

The special parameter cards (SPECIAL) once introduced apply to all subsequent magnets in a beam line until another SPECIAL element specifying the same parameter is encountered. Thus if such a parameter is varied, the variation will apply simultaneously to all subsequent magnets to which it pertains. The variation will persist until the parameter or vary code attached to the parameter is changed by the introduction of another SPECIAL element specifying the same parameter.

Original TRANSPORT Notation — Vary Codes

The original positional parameter notation of TRANSPORT allows the variation of physical parameters to be specified by means of a vary code. This code occupies the fraction portion of the type code specifying the element. If a mnemonic such as **BEND** or **QUAD** is used to denote the element, the mnemonic is followed by a decimal point, which, in turn, is followed by the vary code. It has one digit for each parameter, the digits having the same order in the code as the physical parameters have on the card. A '0' indicates the parameter may not be varied; a '1' that it may be. For instance, DRIFT.0 is the combined mnemonic and vary code (0) for a drift length which is to remain fixed; DRIFT.1 indicates a drift length that may be varied (by the virtue of the .1). **BEND**.010 indicates a bending magnet with a variable magnetic field. In typing DRIFT.0, the zero is unnecessary. In typing **BEND**.010 , the first zero must be included but the second zero need not be.

We can now use the previous example of a quadrupole whose field is to be varied. The vary code formulation is appropriate to an element expressed in positional notation. Specifying the quadrupole in terms of its length, pole-tip field, and aperture, we would now have

```
QUAD.01 10.  5.  2.54 'Q1' ;
```

The numerals following the mnemonic **QUAD** and the decimal point indicate that the second parameter, in this case the magnetic field, may be varied. A fitting procedure may cause its value to be changed from 5. to some other value, such as 5.47723.

First-Order Vary Codes

In a first-order run the following parameters marked v may be varied, those marked 0 may not be varied.

BEAM ...	**BEAM**.vvvvvv0 – All components of the input beam may be varied, except the momentum.
R.M.S. ADDITION ...	**BEAM**.vvvvvv00 – All components of an r.m.s. addition may be varied except the momentum change Δp.
CORRELATION ...	CORR.vvvvvvvvvvvvvvv – Any of the beam correlation. matrix elements may be varied.
ALIGN ...	ALIGN.vvvvvv0 – Any of the alignment parameters may be varied.

Vary Codes Usable in Any Order

INITIAL COORDINATES ...	SPECIAL.0v – Any of the three initial position floor coordinates or two angle coordinates may be varied.
PARAM ...	PARAM.v – Parameters may be varied.
AXIS SHIFT ...	CENTROID.vvvvvv – Any of the axis shift parameters may be varied.
ETA ...	ETA.vvvvvv – Any of the components of the accelerator η function may be varied.
DRIFT ...	DRIFT.v – The drift length may be varied.
ROTAT ...	ROTATE.v – The pole face angle of a bending magnet may be varied.
BEND ...	BEND.vvv – The length, the field, and/or the n-value may be varied.
RBEND ...	RBEND.vvv – The length, the field, and/or the n-value may be varied.
SBEND ...	SBEND.vvv – The length, the field, and/or the n-value may be varied.
HKICK ...	HKICK.v – The magnetic field may be varied.
VKICK ...	VKICK.v – The magnetic field may be varied.
QUAD ...	QUAD.vv0 – The length may be varied; the field may be, the aperture may <u>not</u> be.
PLASMA LENS ...	PLASMA.vv0 – The length may be varied; the field may be, the aperture may <u>not</u> be.
SOLENOID ...	SOLE.vv – The length and/or field may be varied.
MATRIX ...	MATRIX.vvvvvv0 – Any of the first order matrix elements may be varied.
BEAM ROTATION ...	SROT.v – The angle of rotation may be varied.

Second-Order Vary Codes

In a second-order run the following parameters may be varied:

DRIFT ...	DRIFT.v – The drift length may be varied. Variation of a drift length should be done with <u>caution</u> as it may affect the first-order properties of the beam line. But inversely coupled drift spaces straddling a sextupole will, for example, show only second-order effects.
$\epsilon(1)$ ...	SPECIAL.0v 1. – The normalized quadratic term (sextupole component) in the midplane expansion for the field of a bending magnet may be varied.

1/R1 ...	SPECIAL.0v 12. – The pole face curvature of a bending magnet entrance may be varied.
1/R2 ...	SPECIAL.0v 13. – The pole face curvature of a bending magnet exit may be varied.
SEXTUPOLE	SEXT.0v – The field strength may be varied.

The special parameter cards (SPECIAL) once introduced apply to all subsequent magnets in a beam line until another SPECIAL element specifying the same parameter is encountered. Thus if such a parameter is varied, the variation will apply simultaneously to all subsequent magnets to which it pertains. The variation will persist until the parameter or vary code attached to the parameter is changed by the introduction of another SPECIAL element specifying the same parameter. This is one case where it is much easier to use the MAD notation. The three special parameters may be specified on the appropriate element itself. The VARY command can then designate that parameter to be varied.

Third-Order Vary Codes

OCTUPOLE	OCTU.0v - The field strength may be varied.

Vary Codes with Keyword Notation

Vary codes should not be used with keyword notation. TRANSPORT may do some very strange things if vary codes are used with keyword notation. Let us give an example of where such a thing is attempted.

An element expressed in keyword notation may use the same physical parameters as if it were expressed in positional notation. A vary code in a given position corresponds to a keyword parameter in the same position as if positional notation were used. The vary code would then apply just as if the element were in fact written in positional notation. A quadrupole expressed in keyword notation but with a vary code might be

```
Q1:    QUAD.01,  L = 10.,  B = 5.,  APER = 2.54  ;
```

However, a vary code in a given position corresponds to a keyword parameter in the same position if the keywords are ordered as they are internally to the program. If our quadrupole is written as

```
Q1:    QUAD.01,  B = 5.,  L = 10.,  APER = 2.54  ;
```

the vary code will be taken to mean that the magnetic field is to be varied. This interpretation will be made since, internally to TRANSPORT, the keyword L is stored before the keyword B.

Alternate sets of variables can also be affected by the use of the vary code. A quadrupole can be expressed in terms of its gradient instead of pole tip field and aperture. If the gradient is to be varied, the quadrupole can be specified as

```
Q1:    QUAD.01,  L = 10.,  GRAD = 2.0  ;
```

However, if the quadrupole is written as

```
Q1:    QUAD.01,  L = 10.,  GRAD = 2.0  , APER = 2.54  ;
```

the vary code will produce no effect. This result occurs because, internally to TRANSPORT, the keyword **APER** precedes the keyword **GRAD**. TRANSPORT will interpret the element as specifying that the second of the internally stored parameters, namely **APER**, is to be varied. Since the aperture of a quadrupole cannot be varied, there will be no effect.

Coupled Vary Codes

It is possible to apply the same correction to each of several variables. This may be done by replacing the digit 1 in the vary code with one of the digits 2 through 9, or a letter A through Z. All such variables whose vary digits are the same, regardless of position will receive the same correction. For example, the three element-vary codes (QUAD.0A, QUAD.01, QUAD.0A) might represent a symmetric triplet. The same correction will be made to the first and third quadrupoles, guaranteeing that the triplet will remain symmetric.

If a vary digit is immediately preceded by a minus sign, the computed correction will be subtracted from, rather than added to, this variable. Thus parameters with the same vary digit, one of them being preceded by a minum sign, will be inversely coupled. For example the type-vary code sequence (DRIFT.B, QUAD.01, DRIFT.-B) will allow the quadrupole to move without altering the total system length.

Vary digits may also be immediately preceded by a plus sign without changing their meaning. Thus QUAD.0A is the same as QUAD.0+A. For historical reasons, the vary digits (9 and 4), (8 and 3), and (7 and 2) are also inversely coupled. Inverse coupling may not be used with the BEAM or ALIGN elements.

The total number of independent variables in a first-order run is limited to 20 by reasons of the mathematical method of fitting and to 10 for a second-order run. So far as this limit is concerned, variables that are tied together count as one. Variables within repeat elements (REPEAT) also count only one.

Available Constraints – The FIT Command

Types of Constraints

A variety of possible constraints is available. Fitting may be done in either first , second, or third order. The order of the run must be at least as great as the order of the constraint. A list of constraints available is given below. They are explained more fully on later pages.

Layout Constraints

1. The total system length.
2. The reference trajectory floor coordinates.
3. The elevation (including the effect of the earth's curvature).

To fit the reference trajectory floor coordinates, the floor coordinate calculation must be activated via a (`PRINT, FLOOR ;`) command. (See page 233.)

First-Order Constraints

1. An element of the first-order transfer matrix $R1$.
2. An element of the auxiliary first-order matrix $R2$.
3. An AGS machine constraint.
4. The first moments of the beam.
5. A σ (BEAM) matrix element.
6. The correlations r in the beam coordinates.

Second-Order Constraints

1. An element of the second-order transfer matrix $T1$.
2. An element of the second-order auxiliary transfer matrix $T2$.
3. The net contributions of aberrations to a given coordinate of the beam matrix σ.
4. The strength of sextupoles used in the system.

The second-order matrices are actually computed using the auxiliary matrix T2. Therefore, when activating second-order fitting, one must not include any element which causes an update of the R2 matrix. For a complete list of such elements see the `UPDATE` element description on page 223.

Third-Order Constraints

1. An element of the third-order transfer matrix $U1$.
2. An element of the third-order auxiliary transfer matrix $U2$.
3. The net contribution of aberrations to a given coordinate of the beam matrix σ.

The present value of the constrained quantity, as well as the desired value, is printed in the output. In the case of transfer matrix elements this value may be checked by printing the transfer matrix itself. Certain other constrained quantities may be checked similarly. Exceptions are noted in the explanations following.

Keyword Notation

The keywords which can be used in specifying a fitting constraint are

Keyword	Description
VALUE	Desired value of matrix element.
TOLERANC	Desired accuracy of fit (standard deviation).
NAME	Name of matrix element to be constrained.

A constraint of the R_{12} transfer matrix element may be written as

```
FIT,  NAME = R12,  VALUE = 0.0,  TOLER = 0.001  ;
```

This notation is somewhat inconvenient because it requires the user repeatedly to type in the names of all three keywords. For fitting simple matrix elements another form may be more desirable. The same constraint can be represented as:

```
FIT,  R12 = 0.0,  TOLER = 0.001  ;
```

The matrix element to be fit is now used as a keyword. A complete list of matrix elements which may be identified by keyword follows. The lower-case letters i, j, k, and ℓ stand for integers which specify the particular matrix element. The TRANSPORT output for the fit element is printed using the same symbols.

Symbol	Matrix Element
L	The beam line accumulated length.
XFLOOR	The floor coordinate x value.
YFLOOR	The floor coordinate y value.
ZFLOOR	The floor coordinate z value.
YAW	The floor coordinate plan view reference trajectory angle.
PITCH	The angle the reference trajectory makes with the horizontal.
ROLL	The net rotation angle about the reference trajectory.
ELEVATION	The elevation, including corrections for the earth's curvature.
Rij	An element of the first-order transfer matrix $R1$.
RAij	An element of the auxilliary first-order transfer matrix $R2$.
XC	The beam centroid position in x.
XPC	The beam centroid position in x'.
YC	The beam centroid position in y.
YPC	The beam centroid position in y'.
DLC	The beam centroid position in ℓ.

DELC	The beam centroid position in δ.
XBEAM	The beam half width in x.
XPBEAM	The beam half width in x'.
YBEAM	The beam half width in y.
YPBEAM	The beam half width in y'.
DLBEAM	The beam half width in ℓ.
DELBEAM	The beam half width in δ.
Sij	An element of the beam matrix σ.
Cij	An element of the beam correlation matrix r.
BETAX	The accelerator function β_x.
ALPHAX	The accelerator function α_x.
BETAY	The accelerator function β_y.
ALPHAY	The accelerator function α_y.
ETAX	The accelerator function η_x.
DETAX	The accelerator function η'_x.
ETAY	The accelerator function η_y.
DETAY	The accelerator function η'_y.
PSIX	The accelerator phase shift in the horizontal plane
PSIY	The accelerator phase shift in the vertical plane
Tijk	An element of the second-order transfer matrix $T1$.
TAijk	An element if the auxilliary second-order transfer matrix $T2$.
Uijkℓ	An element of the third-order transfer matrix $U1$.
UAijkℓ	An element of the auxilliary third-order transfer matrix $U2$.

Constraints on Algebraic Expressions

In addition any algebraic combination of matrix elements may also be constrained. The algebraic combination is written, as in a FORTRAN statement, after the keyword NAME. For example, a fit of the tilt of the focal plane angle, might be

```
FIT,  NAME = R16/(R11*T126),  VALUE = 0.3,  TOLER = 0.001  ;
```

A number of FORTRAN supplied intrinsic functions may also be used in an algebraic expression. The intrinsic functions now available are

SQRT	Square root
ALOG	Natural logarithm (written Ln in engineering notation).
EXP	Exponential function
SIN	Sine function
COS	Cosine function
SINH	Hyperbolic sine function
COSH	Hyperbolic cosine function
ASIN	Inverse sine function

ACOS   Inverse cosine function

Additional functions can easily be added if there is a need for any of them.

If the beam line is assembled from elements and specified in a LINE statement, the FIT instruction may be placed after the beam line specification. It may refer to a marker which is included in the beam line specification. This is done with the LOCA(TION) keyword. Assume the marker

```
MARKER 'MAR1' ;
```

is included in the beam line description. Assume further that the user wishes to fit an element of the transfer matrix at the location of this marker. The command

```
FIT,  R12 = 0.0,  TOLER = 0.001,  LOCA = MAR1  ;
```

will constrain the matrix R_{12} to equal 0.0 at the location given by the marker MAR1.

If all constraints from a previous step in a multi-step problem are to be cancelled, the single command:

```
FIT,  NONE  ;
```

can be used. This command can then be followed by any new constraints to be activated. These new constraints must have been specified in the earlier steps, but might simply have been inactive due to a minus sign (−) in front of the mnemonic FIT.

Original TRANSPORT Notation

Although the constraints are much clearer when written in keyword notation, there are many existing beam line representations where the constraints are written in positional notation. Because of these existing data, a complete description of the fitting options written in positional notation is included here:

System Length Constraint

A running total of the lengths of the various elements encountered is kept by the program and may be fit. The code digits are i = 0., j = 0.

Thus the element

```
FIT  0.   0.   150.   5.   ;
```

would make the length of the system prior to this element equal to 150 $\pm$ 5 metres. Presumably there would be a variable drift length somewhere in the system. By redefining the cumulative length via the (`SPEC  6.` L ;) element, partial system lengths may be accumulated and fit.

Floor Coordinate Fitting Constraint

Five parameters are needed to specify a floor coordinate constraint:

1. `FIT`.n (or type code `10.0`)
2. Code digit `8`.
3. Code digit (j).
4. Desired value of floor coordinate.
5. Desired accuracy of fit (standard deviation).

The code digit (j) indicates the floor coordinate to be constrained. Its possible values are 1 to 6 indicating the floor x, y, z, *yaw*, *pitch*, and *roll*, respectively. The floor projection of the reference trajectory makes the angle *yaw* with the floor z axis. The angle between the reference trajectory and the horizontal plane is *pitch*. The rotation about the reference trajectory is *roll*. This is also the order in which coordinates are printed in the floor layout activated by the (`PRINT, FLOOR ;`) element. Initial coordinates are given on elements (`SPEC 16. ;`) through (`SPEC 20. ;`) and the `SROT` element.

The floor coordinates are actually zero-th rather than first order properties of a beam line. However, in TRANSPORT, they may be constrained in a fitting run of any order, and therefore are included here. To fit the reference trajectory floor coordinates, the floor coordinate calculation must be activated via a (`PRINT, FLOOR ;`) command. (See page 233.)

Elevation Fitting Constraint

The elevation of the reference trajectory above the curved surface of the earth may be constrained. The elevation is initially the same as the vertical floor coordinate. It assumes that a beam with no pitch is initially tangent to the surface of the earth. The elevation then differs from the vertical floor coordinate by an increasing term quadratic in the distance away from the initial point. The radius of the earth is taken to be 6.3714×10^{6} meters or 3959 miles. The code digits are i = 8., j = 7.

Thus the element

```
FIT  8.   7.   150.   0.01  ;
```

would make the elevation of the system at a given point equal to 150 $\pm$ 1 centimeter. Presumably there would be a some variable quantity in the system which would affect the altitude. Examples might be the initial vertical floor angle, or the strength of a vertical bend.

R1 Matrix Fitting Constraints

There are five parameters to be specified when imposing a constraint upon the (i,j) element of an R1 matrix.

1. `FIT`.n (or type code `10`) (specifying that a fitting constraint follows).
2. Code digit (–i).
3. Code digit (j).
4. Desired value of the (i,j) matrix element.
5. Desired accuracy of fit (standard deviation).

Note that any fitting constraint on an R1 matrix element is from the preceding update of the R1 matrix. An R1 matrix is updated only by a (`UPDATE R ;`) entry.

The symbol (n) is normally zero or blank. If n = 1, then entry 4 is taken to be a lower limit on the matrix element. If n = 2, entry 4 is taken to be an upper limit.

Some typical R1 matrix constraints are as follows:

Desired Optical Condition		Typical Fitting Constraint
Point to Point Imaging:		
Horizontal plane	$R_{12} = 0$	`FIT -1. 2. 0. .0001 'F1' ;`
Vertical plane	$R_{34} = 0$	`FIT -3. 4. 0. .0001 'F2' ;`
Parallel to point focus:		
Horizontal plane	$R_{11} = 0$	`FIT -1. 1. 0. .0001 'F3' ;`
Vertical plane	$R_{33} = 0$	`FIT -3. 3. 0. .0001 'F4' ;`
Point to parallel transformation:		
Horizontal plane	$R_{22} = 0$	`FIT -2. 2. 0. .0001 'F5' ;`
Vertical plane	$R_{44} = 0$	`FIT -4. 4. 0. .0001 'F6' ;`
Achromatic beam:		
Horizontal plane		`FIT -1. 6. 0. .0001 'F7' ;`
$R_{16} = R_{26} = 0$		`FIT -2. 6. 0. .0001 'F8' ;`
Zero dispersion beam:		
Horizontal plane	$R_{16} = 0$	`FIT -1. 6. 0. .0001 'F9' ;`
Simultaneous point to point and waist to waist imaging:		
Horizontal plane		`FIT -1. 2. 0. .0001 'F10' ;`
$R_{12} = R_{21} = 0$		`FIT -2. 1. 0. .0001 'F11' ;`
Vertical plane		`FIT -3. 4. 0. .0001 'F12' ;`
$R_{34} = R_{43} = 0$		`FIT -4. 3. 0. .0001 'F13' ;`
Simultaneous parallel to point and waist to waist transformation:		
Horizontal plane		`FIT -1. 1. 0. .0001 'F14' ;`
$R_{11} = R_{22} = 0$		`FIT -2. 2. 0. .0001 'F15' ;`
Vertical plane		`FIT -3. 3. 0. .0001 'F16' ;`
$R_{33} = R_{44} = 0$		`FIT -4. 4. 0. .0001 'F17' ;`

R2 Matrix Fitting Constraints

There are five parameters to be specified when imposing a constraint upon the (i,j) element of an R2 matrix.

1. `FIT` (or type code `10.n`)
2. Code digit $-(20 + i)$.
3. Code digit (j).
4. Desired value of the (i,j) matrix element.
5. Desired accuracy of fit (standard deviation).

Some typical R2 matrix constraints are as follows:

The symbol (n) is normally zero or blank. If n = 1, then entry 4 is taken to be a lower limit on the matrix element. If n = 2, entry 4 is taken to be an upper limit.

Desired Optical Condition		Typical Fitting Constraint
Point to point imaging:		
Horizontal plane	$R_{12} = 0$	`FIT -21. 2. 0. .001 'F1' ;`
Vertical plane	$R_{34} = 0$	`FIT -23. 4. 0. .001 'F2' ;`
Parallel to point focus:		
Horizontal plane	$R_{11} = 0$	`FIT -21. 1. 0. .001 'F1' ;`
Vertical plane	$R_{33} = 0$	`FIT -23. 3. 0. .001 'F2' ;`
Achromatic beam:		
Horizontal plane		`FIT -21. 6. 0. .001 'F3' ;`
$R_{16} = R_{26} = 0$		`FIT -22. 6. 0. .001 'F4' ;`

See the description of the `UPDATE` element on page 223 for a complete list of elements which update the R2 matrix.

AGS Machine Constraint

Provision has been made in the program for fitting the betatron phase shift angle μ, associated with the usual AGS treatment of magnet systems. †

In the horizontal plane: use code digits $i = -11.$, $j = 2.$, and specify:

$$\Delta = \frac{1}{2\pi} \cos^{-1} [0.5 \, (R_{11} + R_{22})] \;=\; \frac{\mu}{2\pi} \text{ (horiz)}$$
$$= \text{freq./(No. of periods)} \; .$$

In the vertical plane: i = −13., j = 4., and

$$\Delta = \frac{1}{2\pi} \cos^{-1} [0.5 \, (R_{33} + R_{44})] = \frac{\mu}{2\pi} \text{ (vert)} \quad .$$

For example, if there are 16 identical sectors to a proposed AGS machine and the betatron frequencies per revolution are to be 3.04 and 2.14 for the horizontal and vertical planes respectively, then the last element of the sector should be followed by the constraints:

```
FIT  -11.   2.   .190  .001  ;
FIT  -13.   4.   .134  .001  ;
```

$$\text{i.e.} \quad \frac{3.04}{16} = 0.190 \quad \text{and} \quad \frac{2.14}{16} = 0.134 \quad .$$

For example: A typical data listing might be:

```
QUAD.01 - - - ;
DRIFT. - - - ;
QUAD.01 - - - ;
DRIFT. - - - ;
FIT  -11.  2.  0.190 .001 ;
FIT  -13.  4.  0.134 .001 ;
```

†See Courant and Snyder [1]. Also note that this constraint is valid only when the unit cell structure and the corresponding beta functions are both periodic.

First Moment Constraint

In first order, known misalignments, centroid shifts, and mistuned magnets cause the center (centroid) of the phase ellipsoid to be shifted from the reference trajectory, i.e., they cause the beam to have a nonzero first moment. The first moments appear in a vertical array to the left of the vertical array giving the $\sqrt{\sigma_{ii}}$. The units of the corresponding quantities are the same.

It is perhaps helpful to emphasize that the origin always lies on the reference trajectory. First moments refer to this origin. However, the ellipsoid is defined with respect to its center, so the covariance matrix, as printed, defines the second moment about the mean.

First moments may be fit. The code digits are i = 0 and j, where j is the index of the quantity being fit. Thus

```
FIT  0.   1.   .1  .01  ;
```

constraints the horizontal (1.) displacement of the ellipsoid to be 0.1 ± 0.01 cm.

This constraint is useful in deriving the alignment tolerances of a system or in warning the system designer to offset the element in order to accommodate a centroid shift.

σ(BEAM) Matrix Fitting Constraints

There are five parameters to be specified when imposing a constraint upon the (i,j) element of a σ(BEAM) matrix.

1. `FIT.n` (or type code `10.n`)
2. Code digit (i). ($i \leq j$)
3. Code digit (j).
4. Desired value of the (i,j) matrix element.
5. Desired accuracy of fit (standard deviation).

The symbol (n) is normally zero or blank. If n = 1, then entry 4 is taken to be a lower limit on the matrix element. If n = 2, entry 4 is taken to be an upper limit. If i = j, then the value inserted in entry 4 is the desired beam size $[\sigma(ii)]1/2$ e.g. $x_{max} = [\sigma_{11}]^{1/2}$ etc.

Some typical σ matrix constraints are as follows:

Desired Optical Condition		Typical Fitting Constraint						
Horizontal waist σ_{21}	$= 0$	FIT	2.	1.	0.	.001	'F1'	;
Vertical Waist σ_{43}	$= 0$	FIT	4.	3.	0.	.001	'F2'	;
Fit beam size to x_{max}	$= 1$ cm	FIT	1.	1.	1.	.001	'F3'	;
Fit beam size to y_{max}	$= 2$ cm	FIT	3.	3.	2.	.001	'F4'	;
Limit max beam size to x	$= 2$ cm	FIT.2	1.	1.	2.	.01	'F5'	;
Limit min beam size to y	$= 1$ cm	FIT.1	3.	3.	1.	.01	'F6'	;

In general, it will be found that achieving a satisfactory 'beam' fit with TRANSPORT is more difficult than achieving an R matrix fit. When difficulties are encountered, it is suggested that the user 'help' the program by employing sequential (step by step) fitting procedures when setting up the data for his problem. More often than not a "failure to fit" is caused by the user requesting the program to find a physically unrealizable solution. An often encountered example is a violation of Liouville's theorem.

Beam Correlation Matrix (r) Fitting Constraints

Five parameters are needed for a constraint on the (i,j) element of the beam correlation matrix.

1. `FIT` (or type code `10.n`)
2. Code digit (10 + i).
3. Code digit (j).
4. Desired value of the (i,j) matrix element.
5. Desired accuracy of fit (standard deviation).

TRANSPORT does not print the beam (σ) matrix directly. Instead it prints the beam half widths and represents the off-diagonal elements by the correlation matrix. If one wishes to fit an element of this matrix to a nonzero value it is convenient to be able to constrain the matrix element directly.

Some typical r matrix constraints are as follows:

Desired Optical Condition		Typical Fitting Constraint
Horizontal waist	$r_{21} = 0$	`FIT  12.  1.  0.   .001  'F1'  ;`
yy' correlation	$r_{34} = 0.2$	`FIT  13.  4.  0.2  .001  'F2'  ;`

Accelerator Eta Function ($\eta_{x,y}$) Fitting Constraints

The accelerator eta ($\eta_{x,y}$) function is very close in concept to the dispersion trajectory. The difference is that in a system which closes on itself, such as a circular accelerator, eta is taken to be a periodic function. In a single pass system, it is often taken to have an initial nonzero value. This value is the eta function of some preceding accelerator or the result of a preceding beam transport system. The eta function is a six-component vector, where it is usually for the sixth component to have a value of unity.

Five parameters need to be specified for a constraint on any component of the eta vector.

1. `FIT` (or type code `10.n`)
2. Code digit 27.
3. Code digit (j).
4. Desired value of the jth component of eta.
5. Desired accuracy of fit (standard deviation).

Thus a constraint on the horizontal eta function η_x would be written as:

```
FIT  27.   1.   0.5  0.01  ;
```

It is customary in accelerator calculations to use meters and radians for the transverse units. If this is done in the present case, then the desired value for the above `FIT` is 0.5 meters. The tolerance is 1 cm. The units taken for eta are whatever transverse units are being used. For a description on the input value of eta, see the description of eta on page 121.

Phase Advance ($\psi_{x,y}$) Fitting Constraints

The accelerator phase advance is defined as

$$\psi = \int \frac{ds}{\beta}$$

It differs from the betatron phase shift defined earlier in that it is a function on the input beam as well as of the intervening lattice. It can also be derived from the initial and present beta functions and the transfer matrix as

$$\psi_x = \sin^{-1} \frac{R_{12}}{\sqrt{\beta_o \beta_1}}$$

Five parameters are needed for a constraint on either the horizontal or vertical phase advance.

1. `FIT` (or type code 10.n)
2. Code digit `-15`.
3. Code digit `1`, indicating horizontal, or `3`, indicating vertical.
4. Desired value of the phase shift.
5. Desired accuracy of fit (standard deviation).

If the user specifies that the beam matrix is to be in accelerator notation, then the phase advance in both transverse planes will be printed in the output along with the beta, alpha, and eta functions. The initial values of the phase shift in both planes are always taken to be zero.

T1 Matrix Fitting Constraints

Five parameters are needed for a constraint on the (i,j,k) element of the second-order transfer matrix T1.

1. `FIT` (or type code 10.0)
2. Code digit (−i).
3. Code digit (10j + k).
4. Desired value of the (i,j,k) matrix element.
5. Desired accuracy of fit (standard deviation).

Note that upper and lower limit constraints are not available for second order fitting.

Some typical T1 matrix constraints are as follows:

Desired Optical Condition	Typical Fitting Constraint
Geometric aberration $T_{122} = 0$	`FIT -1. 22. .0 .001 'F1' ;`
Chromatic aberration $T_{346} = .5$	`FIT -3. 46. .5 .001 'F2' ;`

There must be no updates of the R2 matrix when constraining an element of the T1 matrix. There is no limit on the number of constraints which may be imposed.

If only sextupoles or sextupole equivalents are varied the problem will be linear and the absolute size of the tolerances will be unimportant. Only their relative magnitude will be significant. Sometimes only a subset of the elements of the matrix T_{ijk} which give significant contributions to beam dimensions need be eliminated. In such cases one may wish to minimize the effect of this subset, by weighing each matrix element according to its importance. One does this by including a constraint for each such matrix element, and setting its tolerance equal to the inverse of the phase space factor which the matrix element multiplies. For a matrix element T_{ijk} acting on an uncorrelated initial phase space, the tolerance factor would be $1/(x_{0j}x_{0k})$, where x_{0j} and x_{0k} are the initial beam half widths specified by the `BEAM` element.

T2 Matrix Fitting Constraints

Five parameters are needed for a constraint on the (i,j,k) element of the second order auxiliary transfer matrix T2.

1. `FIT` (or type code 10.0
2. Code digit $-(20 + i)$.
3. Code digit $(10j + k)$.
4. Desired value of the (i,j,k) matrix element.
5. Desired accuracy of fit (standard deviation).

Note that upper and lower limit constraints are not available for second-order fitting.

Some typical T2 matrix constraints are as follows:

Desired Optical Condition	Typical Fitting Constraint
Geometric aberration $T_{122} = 0$	`FIT -21. 22. .0 .001 'F1' ;`
Chromatic aberration $T_{346} = .5$	`FIT -23. 46. .5 .001 'F2' ;`

By using a T2 constraint the user may fit an element of the second-order transfer matrix which pertains to any section of the beam. One causes an R2 update at the beginning of the section with a (`UPDATE R2 ;`) element. One then places the T2 constraint at the end of the section. Any number of such constraints may be imposed. This is the only second-order constraint that may be used in conjunction with an R2 update.

If a printing of the T1 matrix is requested via a (`PRINT TRANS ;`) element it will be the second-order transfer matrix from the last R1 update. The comments about phase space weighing, made in connection with the T1 constraint, are equally valid for the T2 constraint, provided the phase space factors are obtained from the beam matrix at the position of the R2 update.

Second-Order Sigma (BEAM) Matrix Fitting Constraint

Five parameters must be specified for a constraint on the second-order contributions to a beam matrix diagonal element σ_{ii}.

1. `FIT` (or type code `10.0`)
2. Code digit (i).
3. Code digit (i).
4. The number 0.
5. Desired accuracy of fit (standard deviation).

If, for example, one wished to minimize the net contributions of second-order aberrations to the horizontal divergence, one would insert the following element:

```
FIT 2.   2.   .0  .01  ;
```

The quantity that is minimized is the net increase due to second-order terms in the second moment of the beam about the origin. This quantity is treated as the chi-squared of the problem, so the only meaningful desired value for the fit is zero. The square root of this quantity is printed in the output. It is computed using the R2 matrix. Therefore, once again, one must not include any element which updates the R2 matrix.

The second-order image of the initial beam centroid at some later point in the beam is not necessarily the beam centroid at the later point. The parameters printed by TRANSPORT are the new centroid position and the beam matrix about the new centroid. One must therefore look at both of these to observe the effects of the fitting procedure. It may even happen that an improvement in one parameter will be accompanied by a slight deterioration in the other.

The beam profile at any point is a function of the initial beam parameters. One may therefore impose weights on the effect of the various aberrations by the choice of parameters on the `BEAM` element. One might, for example, adjust the strength of the correction of the chromatic aberrations by the choice of the $\Delta p/p$ parameter. In particular, when using a BEAM constraint, one should not attempt to minimize or eliminate chromatic aberrations if $\Delta p/p$ is set equal to zero on the `BEAM` element.

Correlations (the `CORR` element) may also be included in the initial beam specification.

U1 or U2 Matrix Fitting Constraint

The five parameters needed for a constraint on the (i,j,kℓ) element of the third-order transfer matrix T2.

1. `FIT` (or type code `10.0`)
2. Code digit $-(20+\mathrm{i})$.
3. Code digit $(100\mathrm{j} + 10\mathrm{k} + \ell)$.
4. Desired value of the (i,j,k,ℓ) matrix element.
5. Desired accuracy of fit (standard deviation).

The third-order transfer matrix constraint has a form very similar to the second-order constraint. The third-order matrix element $U_{ijk\ell}$ has four indices, i, j, k, and ℓ. The first of the two index codes on the constraint element is the negative of the first index i. The second index code is made by mushing together the last three indices. To fit the third-order matrix element $U_{ijk\ell}$ to zero, one would use the `FIT` element

```
FIT  −i.   jkℓ.   0.0  0.001  ;
```

To fit the auxiliary transfer matrix U2 to zero, use the command

```
FIT  −(i+20).   jkℓ.   0.0  0.001  ;
```

Comments about updating with reference to the T matrix also apply to the U matrix.

Sextupole Strength Constraints

Five parameters must be specified for a constraint on sextupole strength.

1. `FIT` (or type code `10.0`)
2. Code digit 18.
3. Code digit 0.
4. The number 0.
5. Desired maximum sextupole field strength.

A single sextupole constraint element applies to all sextupoles which follow. The maximum field strength is treated as a standard deviation and may be exceeded on an optimal fit.

One can employ this constraint to find the optimal locations for sextupoles. By placing inversely coupled drift lengths before and after the sextupole its longitudinal position may be varied. By constraining the field strength the sextupole can be slid to a position where the coupling coefficients to the aberrations will be largest. One will need to experiment with adjusting the maximum field strength to achieve the best configuration.

Internal Constraints

A set of upper and lower bounds on the value of each type of parameter is in the memory of the program. If a correction is computed for a parameter which would take its value outside this range, it is reset to the limit of the range. The current limit are:

Element type		Limits
BEAM	0	$<$ input beam
ROTAT	-60	$<$ pole-face rotation $<$ 60 (deg)
DRIFT	0	$<$ drift
BEND	0	$<$ magnet length
QUAD	0	$<$ quad length
SOLENOID	0	$<$ magnet length
SROT	-360	$<$ beam rotation $<$ 360 (deg)
RBEND, SBEND	0	$<$ magnet length
PLASMALENS	0	$<$ magnet length
HKICK, VKICK	0	$<$ magnet length

These limits apply only when a parameter is being varied. Fixed values that exceed this range may be used as desired.

These constraints were included to avoid physically meaningless solutions. The limits for these and many other physical parameters can be reset via the LIMIT element. A complete list of parameters for which limits can be set is found under the description for that element.

Corrections and Covariance Matrix

When the program is fitting, it makes a series of runs through the beam line. From each run it calculates the chi-squared and the corrections to be made to the varied parameters. For each iteration a single line is printed containing these quantities.

The program calculates the corrections to be made using a matrix inversion procedure. However, because some problems are difficult, it proceeds with caution. The corrections actually made are sometimes reduced by a scaling factor from those calculated. This scaling factor is the first item appearing on the line of printed output. The second factor is the chi-squared before the calculated corrections are made. Following are the corrections to be made to the varied parameters. They are in the order in which they appear in the beam line. If several parameters are coupled, they are considered as one and their position is determined by the first to appear.

When convergence has occured, the final value of the chi-squared and the covariance matrix are printed. The covariance matrix is symmetric, so only a triangular matrix is shown. The diagonal elements give the change in each varied parameter needed to produce a unit increase in the chi-squared. The off-diagonal elements give the correlations between the varied parameters.

The appearance of the chi-squared and covariance matrix is:

*COVARIANCE (FIT χ^2)

$$
\begin{array}{lllll}
\sqrt{C_{11}} & & & & \\
r_{12} & \sqrt{C_{22}} & & & \\
\cdot & \cdot & & & \\
\cdot & & \cdot & & \\
\cdot & & & \cdot & \\
r_{1n} & \cdot & \cdot & \cdot & r_{n,n-1}\sqrt{C_{nn}}
\end{array}
$$

For more details on the mathematics of the fitting, the user should consult the Appendix. For an example of the output of the program he (or she) should look on page 53.

SENTINEL

Each step of every problem in a TRANSPORT data set must be terminated with the word `SENTINEL`. The word `SENTINEL` need not be on a separate card. For a description of the form of a TRANSPORT data set see page 16.

An entire run, consisting of one or several problems, is indicated by an additional card containing the word `SENTINEL`. Thus, at the end of the entire data set the word `SENTINEL` will appear twice.

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Ray-Tracing Programs to Supplement TRANSPORT:

D. C. Carey, "TURTLE (Trace Unlimited Rays Through Lumped Elements," Fermilab Report No. NAL-64 (1971). This is a computer program using TRANSPORT notation and designed to be run using the same data cards as for a previous TRANSPORT run.

D. C. Carey, K. L. Brown and Ch. Iselin, "DECAY TURTLE (Trace Unlimited Rays Through Lumped Elements)," CERN Report 74-2 (1974). This is an extension of TURTLE to include particle decay calculations.

H. Enge and S. Kowalski have developed a Ray-Tracing program using essentially the same terminology as TRANSPORT. Any experienced user of TRANSPORT should find it easy to adapt to the M.I.T. program.

M. Berz, "COSY INFINITY," Michigan State University National Superconducting Cyclotron Laboratory Report MSUCL-771 (1991). This program uses differential algebraic methods allowing a systematic calculation of arbitrary order effects of arbitrary particle optical elements.

R. Servranckx, K.L. Brown, L. Schachinger, and D. Douglas, "Users Guide to the Program DIMAD," SLAC Report No. 285 (1990).