THIRD-ORDER TRANSPORT
A COMPUTER PROGRAM FOR DESIGNING CHARGED PARTICLE BEAM TRANSPORT SYSTEMS

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Introduction

TRANSPORT has been in existence in various evolutionary versions since 1963. The present version of TRANSPORT is a first-, second-, and third-order matrix multiplication computer program intended for the design of static-magnetic beam transport systems.

Many people from various laboratories around the world have contributed either directly or indirectly to the development of TRANSPORT. The first-order matrix methods were introduced by the Courant and Snyder[1] followed by a paper by Penner [2]. Second-order differential equations of motion, including driving terms to represent the second-order aberrations were first derived by John Streib[3]. The mathematical formalism introduced by Streib was later adopted by K. L. Brown to develop the underlying theory for the TRANSPORT program [4]. Matrix methods to include second and higher order aberrations were conceived and developed by Brown, Belbeoch and Bounin[5] in Orsay, France in 1958-59.

The original first-order TRANSPORT computer code was written in BALGOL by C.H. Moore at SLAC in collaboration with H.S. Butler and S.K. Howry in 1963. The second-order portion of the program was developed and debugged by Howry and Brown[6], also in BALGOL. Further contributions to the second-order theory were made by R. Helm at SLAC. The BALGOL was translated into FORTRAN by S. Kowalski at MIT and free field reading routines were implemented by B. Kear at SLAC.

In 1971-72, D. C. Carey at Fermilab completely rewrote the program and developed efficient second-order matrix accumulation and fitting routines. This version was installed at SLAC by F. Rothacker in the early spring of 1972 and subsequently carried to CERN by K. L. Brown. Ch. Iselin at CERN made further contributions to the program structure and improved the convergence capabilities of the first-order fitting routines. Third-order matrix elements and fitting capabilities have been included in the latest version of TRANSPORT by D. C. Carey and L. Sagalovsky.

In April 1984, a meeting was held at SLAC where K.L. Brown, D.C. Carey, R. Servranckx, F. Rothacker, and Ch. Iselin agreed to adopt the MAD[7] input format designed by Ch. Iselin as a standard input language for their accelerator optics programs. As a consequence, the reading routines of MAD have been incorporated into the present version of TRANSPORT by D. C. Carey.

A standard version of the program has been maintained, developed, and used at SLAC, Fermilab, and CERN since 1972. This manual describes the third-order version with MAD input capability and graphics. *It is not necessarily applicable to other versions of TRANSPORT.* Copies of this manual may be obtained from

1. The Reports Office, SLAC, P.O. Box 4349, Stanford, CA 94305 (Ref. SLAC-R-95-462).
2. The Reports Office, Fermi National Accelerator Laboratory, P. O. Box 500, Batavia, IL 60510 (Ref. Fermilab-Pub-95/069).

This program may be obtained from:

D. C. Carey
Fermi National Accelerator Laboratory
P. O. Box 500
Batavia, IL 60510

Phone: (708) 840-3639
TRANSPORT_SUPPORT@FNAL.GOV

The program is stored (as part of the Fermilab Fermitools Program) in five pieces on disk at Fermilab and may be copied directly. To obtain the program via ftp (file transfer protocol):

Type: `ftp ftp.fnal.gov`
At the user name prompt, type: `user anonymous`
At the password prompt, enter your user id (e.g. `JOE@FOOLAB.GOV`)
At the ftp prompt, type: `cd pub/transport`
Type: `ls`
to see a listing of the directory specified in the preceding command. Each version of TRANSPORT has its own subdirectory, and a README file will give information on the versions, their dates of creation, and what the source files are named.
Choose a version subdirectory, and type: `mget *.for`
to transfer the program. There are five or so fortran files. The number of files may change with time, but the README will always be up to date.
Type: `quit`.

Mathematical Formulation of TRANSPORT

General Conventions

A beam line is comprised of a set of magnetic elements placed sequentially at intervals along an assumed reference trajectory. The reference trajectory is the path of a particular charged particle with the central design momentum p_0, passing through idealized magnets. By "idealized magnets" we mean no errors in fabrication or positioning.

The particle following the reference trajectory and having momentum p_0 will henceforth be known as the reference particle. The reference trajectory is also known as the central trajectory.

In TRANSPORT,* a beam line is described as a sequence of elements. Such elements may consist not only of magnets and the spaces between them, but also of specifications of the input beam, calculations to be done, or special configurations of the magnets. All magnets are normally considered "aligned" on the central trajectory. The precise meaning of aligning the magnets is described below. Alternative configurations can be described by means of elements provided for such purposes.

The two coordinates transverse to the initial reference trajectory are labeled as horizontal and vertical. A bending magnet will normally bend in the horizontal plane. To allow for other possibilities a coordinate rotation element is provided. Because of such other possibilities, when describing bending magnets we shall often speak of the bend and nonbend planes. The transverse coordinates will also often be labeled x and y. The curvilinear distance along the reference trajectory is denoted by s. In parts of the beam line where the reference trajectory is straight, the longitudinal coordinate is sometimes also labeled z.

The reference trajectory is a straight line through all magnetic elements except a bending magnet. Through the interior field of a bending magnet the reference trajectory is the arc of a circle. Unless otherwise specified, a particle following the central trajectory through an idealized magnet experiences a uniform field which begins and ends abruptly at the effective entrance and exit faces of the magnet. To accomodate a more gradual variation of field at the ends of a bending magnet a fringing field element is provided. In order to represent an orientation with respect to the reference trajectory other than normal of a magnet or section of a beam line, a misalignment element also exists.

The program TRANSPORT will step through the beam line, element by element, calculating the properties of the beam or other quantities, described below, where requested. Therefore one of the first elements needed in a TRANSPORT input is a specification of the phase space region occupied by the beam entering the system. Magnets and intervening spaces and other elements then follow in the sequence in which they occur in the beam line.

*For a more complete description of the mathematical basis of TRANSPORT, refer to SLAC-75[4], and to other references listed at the end of this manual [8] [9] [10] [11].

Specifications of calculations to be done or of configurations other than normal are placed in the same sequence, at the point where their effect is to be made.

Representation of Magnetic Fields

The normal magnetic field of any magnet, except a solenoid, is assumed to have midplane symmetry. This means that the scalar potential expanded in transverse coordinates about the reference trajectory is taken to be an odd function of the vertical coordinate. If a coordinate rotation is included, then the potential is odd in the coordinate to which the vertical has been rotated. For a bending magnet this will always be in the nonbend plane.

We begin our discussion with the field of the combined-function bending magnet. It is the one element which, in principle, contains all the magnetic multipoles. The expansion of the magnetic field on the nominal magnetic midplane has historically been given in terms of unitless quantities n, β, and γ. The expansion may also be given in terms of multipole coefficients K_n. The two representations are as follows:

$$\begin{aligned} B_y &= B_0(1 - nhx + \beta\, h^2x^2 + \gamma\, h^3x^3 + \ldots) \\ &= (B_0\rho)\sum K_n x^n \\ B_x &= 0 \end{aligned} \tag{1}$$

The quantity h is the curvature of the reference trajectory (the reciprocal of the radius of curvature ρ). The expression $B_0\rho$ is equal to the momentum p_0 of the reference particle divided by its charge q.

The dipole component in the expansion is represented by the coefficient K_0 defined as

$$K_0 = \frac{\alpha}{L} = \frac{B_y}{B_0\rho} \tag{2}$$

where α is the total bending angle of a dipole magnet and L is its total length as measured along the reference trajectory. The quadrupole component K_1 is related to the coefficient n by

$$K_1 = -\frac{n}{\rho^2} = \frac{1}{B_0\rho}\frac{\partial B_y}{\partial x} \tag{3}$$

The sextupole component K_2 is related to the coefficients β and ϵ by

$$K_2 = \frac{\epsilon}{\rho} = \frac{\beta}{\rho^3} = \frac{1}{2B_0\rho}\frac{\partial^2 B_y}{\partial x^2} \tag{4}$$

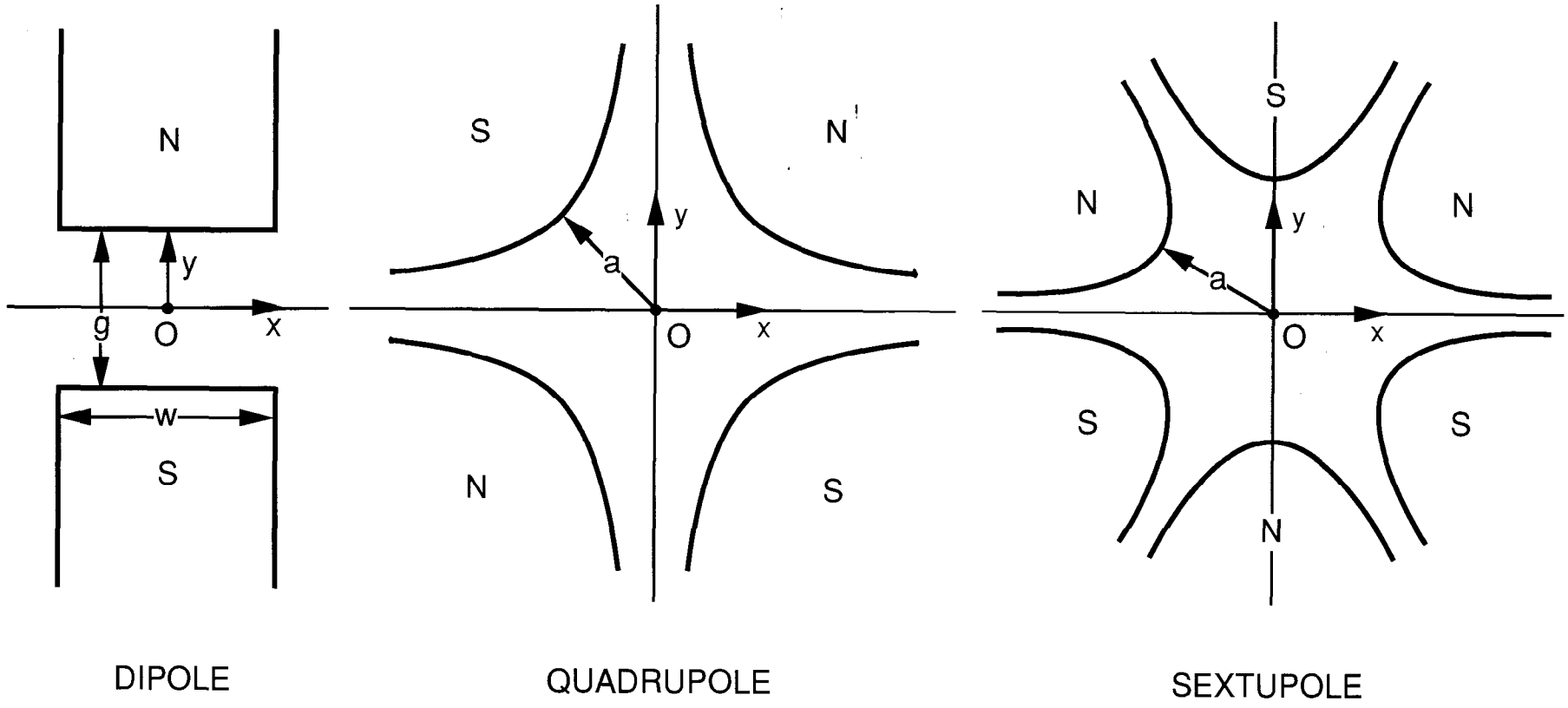
The octupole component K_3 is related to the cubic variation of the magnetic field by.

$$K_3 = \frac{\epsilon_3}{\rho} = \frac{\gamma}{\rho^4} = \frac{1}{6B_0\rho}\frac{\partial^3 B_y}{\partial x^3} \tag{5}$$

The coefficients n, ϵ, and ϵ_3 originally served as the only form for the input data for TRANSPORT. The coefficients β and γ are also shown above, since they appear directly in the algebraic expressions for the higher order transfer matrix elements. The multipole components K_1, K_2, and K_3 are used as input to the MAD program and are also understood by the present TRANSPORT.

The definition of the multipole components K_0, K_1, K_2, and K_3 allows the orbit of a particle to be computed without knowledge of its momentum. The normalization used here is such that a multipole of unit strength will give unit angular deflection per unit magnet length to a trajectory one transverse unit from the reference trajectory. In the MAD program the values of K_2 and K_3 are defined as the normalized derivatives of the field. The value of K_2 in MAD is then a factor of 2 greater than the TRANSPORT value defined above. The MAD value of K_3 is a factor of 6 $(= 3!)$ greater than that used here. The normalization of K_n in various computer programs can be found in a table on the following page. TRANSPORT can be instructed via a UMAD command to observe the MAD conventions so as to ensure consistency of results. The UMAD command is described on page 71.

Physical quadrupole, sextupole, and octupole elements are single multipoles and are known as "separated function" components. The reference trajectory through such elements is a straight line, so that the radius of curvature ρ becomes infinite. The appropriate measures of the strength of these multipole components are the coefficients K_0, K_1, K_2, and K_3, since these coefficients are defined directly as field derivatives divided by $B_0\rho$. The quantity $B_0\rho$, being defined in terms of the reference momentum, is not affected by the curvature of the reference trajectory.



DIPOLE QUADRUPOLE SEXTUPOLE

Illustration of the magnetic midplane (x axis) for dipole, quadrupole and sextupole elements. The magnet polarities indicate multipole elements that are positive with respect to each other. The coordinate system is right-handed, so that the beam particles will be emerging from the paper and moving into the face of the reader.

In TRANSPORT the multipole strength has traditionally been defined in terms of a pole tip field B_0 and a half aperture a. The half aperture is illustrated in the preceding figure. It is the distance from the optic axis (reference trajectory) to the nearest point on the pole tip.

For a quadrupole the gradient g of the field is also often used. The quantities K_1 and g are related to the pole tip field B_0 and the half aperture a by:

$$K_1 = \frac{g}{(B_0\rho)} = \frac{1}{(B_0\rho)}\frac{B_o}{a} \tag{6}$$

For a sextupole the quantity K_2 is related to the pole tip field B_o and the half aperture a by:

$$K_2 = \frac{1}{(B_0\rho)}\frac{B_o}{a^2} \tag{7}$$

For an octupole the quantity K_3 is related to the pole tip field B_o and the half aperture a by:

$$K_3 = \frac{1}{(B_0\rho)}\frac{B_o}{a^3} \tag{8}$$

Again, the definitions in the MAD program are greater by a factor of 2 for the sextupole term and 6 for the octupole term.

DEFINITION OF MULTIPOLE STRENGTHS IN VARIOUS COMPUTER PROGRAMS

TRANSPORT DIMAD	MAD SYNCH TEAPOT
$B_y(x,0,s) = (B_0\rho)\sum K_n x^n$	$B_y(x,0,s) = (B_0\rho)\sum \frac{1}{n!} K_n x^n$
$K_n(s) = \frac{1}{B_0\rho}\ \frac{1}{n!}\ \frac{\partial^n B_y}{\partial x^n}$	$K_n(s) = \frac{1}{B_0\rho}\ \frac{\partial^n B_y}{\partial x^n}$
$K_n = \frac{1}{(B_0\rho)}\ \frac{B_o}{a^n}$	$K_n = \frac{n!}{(B_0\rho)}\ \frac{B_0}{a^n}$

So $K_n(\text{TRANSPORT}) = \frac{1}{n!}\ K_n(\text{MAD})$

where $n = 0$ Dipole
$n = 1$ Quadrupole

$n = 2$	Sextupole
$n = 3$	Octupole

The Transfer Matrix R

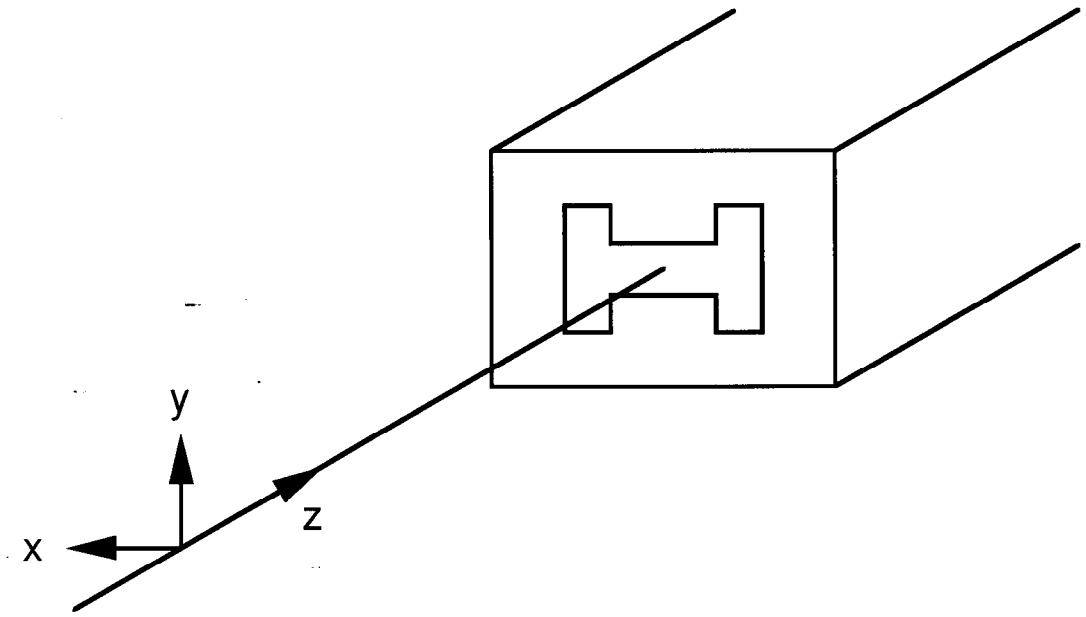
The following of a charged particle through a system of magnetic lenses may be reduced to a process of matrix multiplication. At any specified position in the system an arbitrary charged particle is represented by a vector (single column matrix) X, whose components are the positions, angles, and momentum of the particle with respect to the reference trajectory, i.e.

$$X = \begin{bmatrix} x \\ x' \\ y \\ y' \\ \ell \\ \delta \end{bmatrix} \tag{9}$$

Definitions:

- x = the horizontal displacement of the arbitrary ray with respect to the assumed central trajectory.
- x' = the angle this ray makes in the horizontal plane with respect to the assumed central trajectory.
- y = the vertical displacement of the ray with respect to the assumed central trajectory.
- y' = the vertical angle of the ray with respect to the assumed central trajectory.
- ℓ = the path length difference between the arbitrary ray and the central trajectory.
- δ = $\Delta p/p$ is the fractional momentum deviation of the ray from the assumed central trajectory.

This vector, for a given particle, will henceforth be referred to as a ray. The two components x' and y' are often referred to as angles, although, strictly speaking, they are not. They are really tangents of angles. However, the difference between an angle and its tangent is of third order and is generally unimportant as a measure of phase-space dimensions. We therefore occasionally lapse into this somewhat imprecise usage.



The local coordinate system is shown at the entrance to a magnet. The longitudinal coordinate is shown as z. The distance along the reference trajectory is usually denoted by the letter s. In a field-free region such as a drift, or the axis of a quadrupole, or other higher multipole, the coordinates s and z are the same. In the interior of a bending magnet the coordinate s is curvilinear and follows the reference trajectory. The coordinate z is rectilinear and is taken as tangent to the reference trajectory.

The magnetic lens is represented to first order by a square matrix R, which describes the action of the magnet on the particle coordinates. Thus the passage of a charged particle through the system may be represented to first order by the equation:

$$X(1) = RX(0), \tag{10}$$

where $X(0)$ is the initial coordinate vector and $X(1)$ is the final coordinate vector of the particle under consideration. The same transformation matrix R is used for all such particles traversing a given magnet [one particle differing from another only by its initial coordinate vector $X(0)$].

The traversing of several magnets and interspersing drift spaces is described by the same basic equation, but with R now being replaced by the product matrix

$$R(t) = R(n) \ldots R(3)R(2)R(1) \tag{11}$$

of the individual matrices of the system elements. This cumulative transfer matrix is automatically calculated by the program and is called TRANSFORM 1. It may be printed where desired, as described on page 242.

Higher Orders

This formalism may be extended to second order by the addition of another term.[4] The components of the final coordinate vector, in terms of the original, are now given as

$$X_i(1) = \sum_j R_{ij}\, X_j(0) + \sum_{jk} T_{ijk}\, X_j(0)\, X_k(0) \quad , \tag{12}$$

where T is the second-order transfer matrix. It too is accumulated by the program as one traverses a series of elements. At each point the series is again truncated to second order. Normally the program will calculate only the first-order terms and their effect. If it is desired to include second-order effects in a beam line, an element is provided which specifies that a second-order calculation is to be done. For more information on the T matrix, see the references at the end of the manual.

Third-order terms may also be included in the transformation of the particle coordinate vector. The third-order terms are given in terms of the U matrix so that our transformation equation becomes

$$X_i(1) = \sum_j R_{ij}\, X_j(0) + \sum_{jk} T_{ijk}\, X_j(0)\, X_k(0) + \sum_{jk\ell} U_{ijk\ell}\, X_j\, X_k\, X_\ell \quad , \tag{13}$$

The comments about second order apply also to third order. The addition of a parameter to the element which specfies second order allows calculations to be done to third order.

The following of a charged particle via TRANSPORT through a system of magnets is thus analogous to tracing rays through a system of optical lenses. The difference is that TRANSPORT is a matrix calculation which truncates the problem to either first-, second- or third- order in a Taylor's series expansion about a central trajectory. *For studying beam optics to greater precision than a third-order TRANSPORT calculation permits, ray-tracing or differential algebra programs which directly integrate the basic differential equation of motion are recommended.*[†]

The Beam Matrix σ

In accelerator and beam transport systems, the behavior of an individual particle is often of less concern than is the behavior of a bundle of particles (the beam), of which an individual particle is a member. An extension of the matrix algebra of Eq. (1) provides a convenient means for defining and manipulating this beam. TRANSPORT assumes that the beam may be correctly represented in phase space by an ellipsoid in the six-dimensional coordinate system described above. Particles in a beam are assumed to occupy the volume enclosed by the ellipsoid, each point representing a possible ray. The sum total of all phase points, the phase space volume, is commonly referred to as the "phase space" occupied by the beam.

The validity and interpretation of this phase ellipse formalism must be ascertained for each system being designed. However, in general, for charged particle beams in, or emanating from accelerators, the first-order phase ellipse formalism of TRANSPORT is a reasonable representation of physical reality. For other applications, such as charged particle spectrometers, caution is in order in its use and interpretation.

The equation of an n-dimensional ellipsoid may be written in matrix form as follows:

$$X(0)^T \; \sigma(0)^{-1} \; X(0) = 1 \quad , \tag{14}$$

where $X(0)^T$ is the transpose of the coordinate vector $X(0)$, and $\sigma(0)$ is a real, positive definite, symmetric matrix.

The volume of the n-dimensional ellipsoid defined by sigma is $[\pi^{m/2}/\Gamma(n/2+1)]\ (det\ \sigma)^{1/2}$. The area of the projection in one plane is $A = \pi(det\ \sigma_1)^{1/2}$, where σ_1 is the submatrix corresponding to the given plane. This is the "phase space" occupied by the beam.

As a particle passes through a system of magnets, it undergoes the matrix transformation of Eq. (1). Combining this transformation with the equation of the initial ellipsoid, and using the identity $RR^{-1} = I$ (the unity matrix), it follows that

$$X(0)^T \; (R^T \; R^{T^{-1}}) \; \sigma(0)^{-1} \; (R^{-1} \; R) \; X(0) = 1 \tag{15}$$

[†]A list of such programs is given at the end of this manual.

from which we derive

$$[RX(0)]^T\ [R\sigma(0)\ R^T]^{-1}\ [RX(0)] = 1\ \ . \tag{16}$$

The equation of the new ellipsoid after the transformation becomes

$$X(1)^T\ \sigma(1)^{-1}\ X(1) = 1\ \ , \tag{17}$$

where

$$\sigma(1) = R\sigma(0)\ R^T\ \ . \tag{18}$$

It can readily be shown that the square roots of the diagonal terms of the sigma matrix are a measure of the "beam size" in each coordinate. The off-diagonal terms determine the orientation of the ellipsoid in n-dimensional space (for TRANSPORT, n = 6).‡ Thus, we may specify the beam at any point in the system via Eq. (5), given the initial "phase space" represented by the matrix elements of $\sigma(0)$.

The initial beam is specified by the user as one of the first elements of the beam line. Normally it is taken to be an upright ellipse centered on the reference trajectory; that is, there are no correlations between coordinates. Both correlations and centroid displacements may be introduced via additional elements.

The phase ellipse may be printed wherever desired. For an interpretation of the parameters printed see page 113.

When a second- or third-order calculation is specified, the appropriate matrix elements are included in the beam matrix. In this case the beam is no longer an ellipse. The elements of the sigma matrix may be interpreted as second moments of the beam about its center of gravity. However, a large second moment may be caused by many factors, including a very long but sparsely populated tail in one direction. Detailed analysis should be done with a ray tracing program.

Fitting

Several types of physical elements have been incorporated in the program to facilitate the design of very general beam transport systems. Included are an arbitrary drift distance, bending magnets, quadrupoles, sextupoles, octupoles, solenoids, and an accelerator section (to first-order only). Provision is made in the program to vary some of the physical parameters of the elements comprising the system and to impose various constraints on the beam design. In a first-order run one may fit either the TRANSFORM (R) matrix representing the transformation of an arbitrary ray through the system and/or the phase ellipse (sigma)

‡See the Appendix of this report (available under separate cover or as part of Report CERN 80-04), for a derivation of these statements.

matrix representing a bundle of rays by the system as transformed. In a second-order run one may fit either the second-order TRANSFORM (T) matrix or minimize the net contribution of second-order terms to the beam (sigma) matrix. Similar constraints in third order may be placed on the third-order TRANSFORM (U) or the sigma matrix.

The program will normally make a run through the beam line using values for the physical parameters as specified by the user and printing the results. If constraints and parameters to be varied are indicated, it will attempt to fit. To do this it will make an additional series of runs through the beam line. Each time it will calculate corrections to be made from the previous step to the varied parameters to try to satisfy the indicated constraints. When the constraints are satisfied (or the fitting procedure has failed) the program will make a final run through the beam line again printing the results. In this final run the values of the physical parameters used are those which are the result of the fitting procedure.

Thus, in principle, the program is capable of searching for and finding the first- or second-order solution to any physically realizable problem. In practice, life is not quite so simple. The user will find that an adequate knowledge of geometric magnetic optics principles is a necessary prerequisite to the successful use of TRANSPORT. He (or she) should possess a thorough understanding of the first-order matrix algebra of beam transport optics and of the physical interpretation of the various matrix elements.

In other words, the program is superb at doing the numerical calculations for the problem but not the physics. The user must provide a reasonable physical input if he (or she) expects complete satisfaction from the program. For this reason a list of pertinent reprints and references is included at the end of this manual. They should provide assistance to the inexperienced as well as the experienced user.

Mispowering and Violations of Midplane Symmetry

The reference trajectory of a correctly aligned bending magnet is the path followed by the reference particle only if the magnet is also correctly powered. If the magnet is mispowered, the reference trajectory can still be defined geometrically as if the magnet were correctly powered. The field expansion given in equation (1) will then be about this geometrically defined reference trajectory. The parameterization of the magnetic field will contain a multiplicative factor representing the degree of mispowering.

Violations of midplane symmetry can also be included in the representation of the magnetic field. Individual magnetic elements are typically designed to have midplane symmetry. Errors of fabrication, misalignments, and correction windings can cause the actual midplane of an element not to coincide with its intended midplane. For a magnet which corresponds to a pure multipole, such as a zero gradient bending magnet (dipole), quadrupole, sextupole, or octupole, the required procedure is straightforward. The departure of the midplane from its intended location can be represented by the rotation and misalignment elements.

The combined function bending magnet however, contains several multipole components in the same magnet. A bending magnet may have multipole components in addition to the dipole component either by design or by errors of fabrication. These different multipoles may have different planes of symmetry. The bending magnet may also have a midplane defined by the geometry of the beam. This geometric midplane may or may not coincide with a plane of symmetry of even the dipole component.

A single reference plane can still be selected for the purpose of defining the magnetic field. This reference plane can be geometrically defined by some significant physical characteristic, such as the intended plane of symmetry of the iron pole tips of the magnet. We shall continue to refer to this reference plane as the midplane, without implying any field symmetry.

A complete expansion of the magnetic field about this reference plane then includes midplane-symmetry-violating terms. The reference trajectory may be defined as if these terms were not present. The reference trajectory will then <u>not</u> be the possible trajectory of an actual particle. The field configuration, however, can still have translational invariance as one follows the reference trajectory.

We therefore introduce midplane-symmetry-violating dipole, quadrupole, and sextupole components of the field. The dipole component will cause the beam centroid to deviate from the geometrically defined reference trajectory.

$$\begin{aligned} B_y &= B_o(1+r_s)(1-nhx+\beta\, h^2x^2+\gamma\, h^3x^3+\ldots) \\ &= (B_o\rho)(1+r_s)\sum K_n x^n \end{aligned} \tag{19}$$

$$\begin{aligned} B_x &= B_o r_a(v_r - n'hx+\beta' h^2x^2+\ldots) \\ &= (B_o\rho) r_a \sum K'_n x^n \end{aligned}$$

The consequences of the centroid being diverted off axis are described below.

The coefficient r_s represents the fractional degree of mispowering. The corresponding coefficient r_a is a multiplicative factor for the midplane-symmetry-violating terms. These two coefficients are independent and are defined so that they are both zero in the correctly powered, midplane symmetric case.

The coefficients v_a, n', and β' represent the relative strengths of the midplane-symmetry violating multipole coefficients. The coefficients K'_n provide an alternate parameterization just as in the midplane symmetric expansion. The K'_n coefficients include K'_0 for the midplane symmetry violating dipole term.

There is a certain redundancy in the definition or r_a, since the multipole coefficients provide a complete specification of the magnetic field. The parameter r_a is included in the parameterization so that the midplane-symmetry-violating terms can be scaled simultaneously by a single multiplicative factor.

The midplane-symmetry-violating multipole components are defined similarly to the midplane-symmetric terms. The skew quadrupole component n' is related to the coefficient K_1' by

$$K_1' = -\frac{n'}{\rho^2} = \frac{1}{r_a}\frac{1}{B\rho}\frac{\partial B_x}{\partial x} \tag{20}$$

The skew sextupole component β' is related to the coefficients ϵ' and K_2' by

$$K_2' = \frac{\epsilon'}{\rho} = \frac{\beta'}{\rho^3} = \frac{1}{r_a}\frac{1}{2B\rho}\frac{\partial^2 B_x}{\partial x^2} \tag{21}$$

The midplane-symmetry-violating formalism in TRANSPORT has thus far been extended to include only the second-order or skew sextupole terms. Just as in the midplane symmetric case, the coefficient K_2' in MAD is defined as being twice that in TRANSPORT. The command UMAD (see page 71) instructs TRANSPORT to use the MAD definitions.

Off-Axis Expansion of Matrix Elements

If the beam is off axis, then the beam centroid will then no longer coincide with the reference trajectory. An off axis beam can be achieved in several ways.

1. The initial beam centroid may not coincide with the reference trajectory.
2. The beam centroid may be displaced by the misalignment of a magnet or section of beam line [12].
3. The beam may be steered off axis by an excess field on a bending magnet.
4. The beam may be steered off axis by a midplane-symmetry-violating vertically bending component of a bending magnet.
5. The beam may be deliberately steered by the use of steering magnets.
6. The beam may be deflected by an electrostatic septum.
7. The reference coordinate system can be shifted.

The transformation through a single element can then be represented in the same manner as in on-axis configurations, except that there is now a zeroeth order term. This new term represents the displacement of the six coordinates of a ray that lies on the reference trajectory as it enters the magnetic element. The net transformation can be represented to

third order by

$$X_1 = X_{1s} + RX_o + TX_oX_o + UX_oX_oX_o \tag{22}$$

The effect of the magnetic system on neighboring trajectories can be represented by a first-, second-, or third-order transformation. We start with equation (17) for the transformation of an arbitrary trajectory. If we reexpress the coordinates relative to the transformed original reference trajectory X_r, and denote the difference by ΔX, then we have

$$\begin{aligned} X_{1r} + \Delta X_1 &= X_{1s} + R(X_{or} + \Delta X_o) + T(X_{or} + \Delta X_o)(X_{or} + \Delta X_o) \\ &+ U(X_{or}\Delta X_o)(X_{or} + \Delta X_o)(X_{or} + \Delta X_o) \end{aligned} \tag{23}$$

Subtracting the equation for the reference trajectory, we derive

$$\begin{aligned} \Delta X_1 &= R\Delta X_o + T(2X_{or}\Delta X_o + \Delta X_o\Delta X_o) \\ &+ U(3X_{or}X_{or}\Delta X_o + 3X_o\Delta X_o\Delta X_o + \Delta X_o\Delta X_o\Delta X_o) \\ &= (R + 2TX_{or} + 3UX_{or}X_{or})\Delta X_o \\ &+ (T + 3UX_{or})\Delta X_o\Delta X_o + U\Delta X_o\Delta X_o\Delta X_o \end{aligned} \tag{24}$$

From equation (19), we can define new first- and second-order transfer matrices by

$$\begin{aligned} R^* &= R + 2TX_{or} + 3UX_{or}X_{or} \\ T^* &= T + 3UX_{or} \end{aligned} \tag{25}$$

These redefined matrices for each element can then be accumulated to produce transfer matrices for the entire magnetic optical system. The transformation of a particle trajectory through the system can now be represented by a transformation similar in appearance to equation (17).

$$X_1 = X_{1r} + R(t)X_o + T(t)X_oX_o + U(t)X_oX_oX_o \tag{26}$$

Here the matrices $R(t)$, $T(t)$, and $U(t)$ are calculated as products of the matrices R^*, T^*, and U, as defined in equation (20). They are expressed relative to the transformed original reference trajectory X_{1r}. The transformed original reference trajectory is determined from equation (17) on an element-by-element basis.

Since the fractional deviation from the reference momentum is one of the trajectory coordinates, the off-axis expansion can be used to explore chromatic effects. If a nonzero δ is specified, the first-order transfer matrix will be with respect to the off-momentum centroid. If there is dispersion in the system, the centroid will, at some point, be displaced from the reference trajectory. The result from an off-axis expansion can differ substantially from that of an on-axis calculation. The effect of many orders higher than second or even third can be seen.

The most straightforward example is that of a very long focusing system, with many intermediate foci. For a slight deviation from the reference momentum, the final beam spot size will grow linearly with the size of this deviation. As the momentum is increased, however, one of the intermediate foci will move downstream and the system as a whole will once again be focusing.

An on-axis expansion to second order will show only the linear growth in spot size, no matter how great the momentum deviation. The third-order on-axis expansion will show very little difference from the second. The off-axis expansion will show the system once again coming to a focus as the momentum is increased. Even for very long systems, a third-order off-axis chromatic calculation will be indistingushable from a ray-tracing calculation. The ray tracing program uses unexpanded momentum dependence in the equations of motion through the magnetic fields.

The beam centroid displacement is printed next to the first-order transfer matrix when the beam is off axis. For a zeroth-order calculation the centroid displacement is printed alone. For a first- or lower-order calculation the same vector of six quantities will appear as the leftmost column in the beam matrix representation.

For second- and third-order calculations, the column in the beam matrix may differ slightly from the column given with the transfer matrix. That is because the numbers given with the beam matrix represent the position of the center of gravity of the beam. Second- and higher-order effects introduce distortions into the beam ellipse, so that it is no longer an ellipse. The center of gravity at a given beam location is no longer the optical image of the center of gravity at the beginning of the system. Steering magnets should always be set so that the centroid (not the center of gravity) is realigned along the beam axis. Higher-order multipoles should then be used to make the center of gravity coincide with the centroid.

Input Format for TRANSPORT

General Description of Input

The Input Deck

By the TRANSPORT input DATA SET is meant the totality of data read by the program in a single job. A DATA SET may consist of one or more problems placed sequentially. A problem specifies a beam line and a calculation or set of calculations to be done on that beam line.

A problem, in turn, may consist of one or more problem steps. The data in the first step of a problem specify the beam line and calculations to be made. The data in succeeding steps of the same problem specify only changes to the data given in the first step.

A common example of a problem with several steps is sequential fitting. In the first step one may specify that certain parameters are to be varied to satisfy certain constraints. Once the desired fit has been achieved the program will then proceed to the next step. The data in this step now need specify only which new parameters to vary, or old ones no longer to vary, or which constraints to add or delete. The values of the varied parameters that are passed from one step to the next one are those that are the result of the fitting procedure.

A problem step contains three kinds of DATA cards: the TITLE card, the INDICATOR card, and the ELEMENT cards. The term "card", indicating a single line of an input data set, may be a bit archaic. However, users have come to be comfortable with it and continue to speak in terms of "card"s. We shall therefore continue this usage with the understanding that we should not be taken too literally.

The TITLE card contains a string of characters and blanks enclosed by single quotes. Whatever is between the quotes will be used as a heading in the output of a TRANSPORT run.

The second line of the input is the INDICATOR card. If the data which follow describe a new problem, a zero (0) is placed in any column on the card. If the data which follow describe changes to be made in the previous problem step, a one (1) is placed in any column on the card. For further explanation see page 67.

The remainder of the data for a given problem step consist of the ELEMENT cards. These data describe the units to be used, the physical objects making up the beam line, and the calculations to be done. Unless otherwise specified, the order of the elements is the same as encountered as one proceeds down the beam line. Provision also exists for listing the elements first, then using other commands to assemble them into beam lines.

Each element specifies a magnet or portion thereof or other piece of equipment, a drift space, the initial beam phase space, a calculation to be done, or a print instruction. Calcu-

lation specifications, such as misalignments and constraints, are placed in sequence with the other beam line elements where their effect is to take place. The input format of the cards is "free-field," which is described below. The data for a given problem step are terminated by the word **SENTINEL**, which need not be entered on a separate line.

Notation Options

The MAD (Methodical Accelerator Design) program of Christoph Iselin (see refs) has an input data format which is both easy to use and clear in its meaning. A beam line description in MAD notation can also be read by this version of TRANSPORT. However, since the program TRANSPORT existed long before MAD was originated, TRANSPORT has its own original format. The original format is also understood by TRANSPORT as are several other still different options.

The MAD notation differs from that of original TRANSPORT in two basic ways. They are:

1. The specification of individual elements is different. The elements in MAD are designated by mnemonic names (such as **BEND** and **QUAD**) rather than numerical type codes (such as 4 and 5). The element parameters are designated by keywords (such as **L** and **ANGLE**) rather than by their position in a sequence of numbers. The physical parameters used to specify a given element (such as length and bending angle) may be different.
2. In MAD the elements are specified first, then assembled into an optical system by additional program instructions. In original TRANSPORT, the optical system consists of the elements as listed.

These differences between MAD and original TRANSPORT are explained in greater detail, with examples, in the immediately following sections. The choice of a particular notation for the specification of the elements does not dictate how the beam line is to be assembled. For example, in TRANSPORT, elements specified in MAD keyword notation may simply be listed sequentially to describe a beam line. *However, if a beam line specification is to be read by the MAD program, it must be entirely in MAD notation.* The method of assembling a beam line in MAD is used because accelerators typically have a high degree of repetition and symmetry, whereas secondary beam lines sometimes have very little.

Types of Elements

The mnemonic identifies the element, indicating what sort of entity (such as a magnet, drift space, constraint, etc.) is represented. In original TRANSPORT notation a numerical type code may be used instead of the literal mnemonic. If a minus sign is placed immediately before the mnemonic or type code with no intervening spaces, the element will be ignored

in the given problem step. However, storage for that element will be allocated by the program, so that the element may be introduced in a later step of the same problem. Elements may also be introduced in a later step, without previous specification, by reference to elements already present. The procedure for doing this will be described below, in the subsection on "The Use of Labels." For MAD notation the use of labels is described on page 22. For original TRANSPORT notation the use of labels is described on page 35.

The mnemonic names chosen for the elements are the same as occur in the printed output of the TRANSPORT program. These names for elements which are MAD compatible are the same as used in the MAD program. Some of the names occuring in the printout have been changed slightly from older versions of TRANSPORT. The following is a list of mnemonics recognized by TRANSPORT. Also included are the numerical type code for compatability with older data sets, the meaning of the mnemonics, and whether they are MAD compatible.

Type Code	Mnemonic	MAD Compatible	Element
1.	BEAM		Input phase space dimensions and reference trajectory momentum.
2.	ROTATION		Fringing fields and pole-face rotations for bending magnets
3.	DRIFT	Yes	Drift space – a field free region
4.	BEND		Sector bending magnet (without fringing fields)
5.	QUADRUPOLE	Yes	Quadrupole magnet
6.	UPDATE		Transfer matrix update
7.	CENTROID		Shift in the beam centroid
8.	ALIGN		Magnet alignment tolerances
9.	REPEAT		Repetition of a segment of a beam line
10.	FIT		Fitting constraint
11.	ACCELERATOR	Yes	Travelling wave linear accelerator
12.	CORRELATION		Beam rotated ellipse
13.	PRINT		Output print control instructions
14.	MATRIX		Arbitrary matrix input
15.	UNITS		Input-output units
16.	SPECIAL		Special input parameters
17.	ORDER		Zeroeth, first, second, or third order
18.	SEXTUPOLE	Yes	Sextupole magnet
19.	SOLENOID	Yes	Solenoid magnet
20.	SROT	Yes	Coordinate Rotation
24.	SECTION		Defined section of beam line
	OCTUPOLE	Yes	Octupole magnet
	RANDOM		Random errors on physical parameters
	ETA		Accelerator eta function
	RBEND	Yes	Rectangular bending magnet (with fringing fields)

SBEND	Yes	Sector bending magnet (with fringing fields)
PARAMETER	Yes	Parameter value
MARKER	Yes	Position marker
STORE		Storage of matrix element values
PLASMALENS		Plasma (lithium) lens
HKICKER	Yes	Horizontal steering magnet
VKICKER	Yes	Vertical steering magnet
ALMARK		Alignment marker
PLOT		Plotting instruction
LIMIT		Limit on allowed range of varied physical parameter
MAGNET		Magnet external dimensions for plotting
SEPTUM		Electrostatic septum
KICKER	Yes	Steering magnet in both planes
SHIFT		A shift in the reference coordinate system

The first four letters of a mnemonic are necessary for its specification. More may be used for clarity if desired. For example, the mnemonic SECTION may provide greater clarity than just SECT, which, at first glance, might be taken to mean sector magnet. If more than four characters are used, they must be correct. A mnemonic of SECTIO will be recognized but SECTOIN will not. The same restrictions apply to the keywords. The use of parentheses in mnemonics in this report is to show the characters which are non-essential.

A MAD specification for a sector bending magnet with fringing fields and with the individual name of BM1 might then appear as:

```
BM1:   SBEND,  L = 10.,  ANGLE = 10.,  K1 = 0.5E-6 ;
```

Here, the only physical parameter whose meaning is not obvious is K1, which is a measure of field gradient. Expressed in terms of an alternate set of physical parameters (length and magnetic field instead of length and bending angle), the same element might now be:

```
BM1:   SBEND,  L = 10.,  B = 20.,  N = 0.5  ;
```

The quantity indicated by the keyword N is an alternate measure of field gradient. In original TRANSPORT notation, the same sector bending magnet would be:

```
SBEND  10.   20.   0.5  'BM1'  ;
```

More details about these various notations are given in the sections below.

MAD (Keyword) notation for individual elements

Strictly MAD-Compatible Notation

The strict MAD input notation, which is compatible with the MAD program of Christoph Iselin, can be used in TRANSPORT only for the specification of the charged-particle optical system. Other instructions such as units specifications, methods of calculation, printing instructions, and fitting constraints may be specified in either a keyword notation which is similar to MAD notation, or in original TRANSPORT notation. The options available depend on the type of element.

The MAD notation for the specification of elements has four distinctive characteristics:

1. Each element is given a proper name, henceforth to be known as a label. The label is the first item in the element specification and is followed by a colon. If the elements are to be assembled into a beam line by additional program instructions, then the label must be present. If the beam line consists simply of the sequence of elements as listed, then, in TRANSPORT, the label is optional.

2. The next item in the MAD specification of an element is a mnemonic which identifies what sort of entity (such as a bending magnet, quadrupole, or drift space) is represented. The mnemonic is an alphanumeric character string, usually a recognizable name such as `BEND` or `QUAD`.

3. Keyword notation is used for the specification of the physical parameters. The parameters are identified by name. The parameter name is followed by an equals sign, which, in turn, is followed by the value to be given to the parameter. If another parameter keyword is to follow, the value must be followed by a comma. The entire sequence is terminated by a semicolon or the end of a line.

4. Strict MAD notation requires the use of a particular set of parameters and a particular set of units for the specification of any physical element. The parameters to be used for any particular type of element are described in the section for that element. An example might be a sector bending magnet. In MAD notation a sector bending magnet is specified in terms of its length (in meters), bending angle (in radians), and the normalized field gradient K_1 (in inverse meters squared). The normalized field gradient K_1 is defined in the earlier section on the mathematical formulation of TRANSPORT.

A complete specification for a sector bending magnet with fringe fields would then look like:

```
BM1:   SBEND,  L = 10.,  ANGLE = 10.,  K1 = 0.5E-6 ;
```

In strict MAD notation the units for length `L` are meters, those for the bend angle `ANGLE` are radians, and those for `K1` are in inverse square meters.

TRANSPORT will also accept data using MAD parameters which are not in MAD units. For example, an SBEND can be parameterized in terms of L, ANGLE, and K1, while using feet for distance, degrees for angle and inverse squared centimeters for K1. In the sections describing the individual elements, we shall refer to the parameterization of elements in terms of MAD variables as being MAD notation. However, strict MAD notation requires the use of MAD units, while TRANSPORT can use alternative sets of units even for MAD parameters.

MAD notation has the advantage that it does not require a separate specification of the beam momentum in order to calculate the transfer matrix. The beam momentum is incorporated in the definition of K_1.

If the physical parameters of an element are specified in keyword notation, then any physical parameters to be adjusted in a fitting procedure are identified by the VARY command. The VARY command is placed in the input data after the complete beam line specification. See page 259 for an explanation of the VARY command.

Alternate Parameters

A physical elements may be specified in several different ways in TRANSPORT by sets of physical parameters. One such element is the sector bending magnet we have been using as an example. Instead of the length and bend angle, we might use the length and magnetic field. We might also use a different normalized gradient n, defined previously. The use of the magnetic field as part of the magnet description now requires knowledge of the momentum to calculate the transfer matrix. For a 600 GeV beam, our bend magnet would then be specified as

```
BM1:   SBEND,  L = 10.,  B = 20.,  N = 0.5  ;
```

In the original TRANSPORT there is provision for a sector bending magnet which does not contain the fringing fields. Such an element is useful in the longitudinal segmentation of a bending magnet to obtain the transfer matrix or beam profile at interior points. The following BEND element is physically the same as the above SBEND except that the fringe fields are not included:

```
BM1:   BEND,  L = 10.,  B = 20.,  N = 0.5  ;
```

The order of the parameters is not significant in keyword notation. For example, the above specification is equivalent to

```
BM1:   BEND,  B = 20.,  L = 10.,  N = 0.5 ;
```

The **BEND** element as described above, with no fringing fields, is not found in the MAD program. However, it can be described by the same parameters as the **RBEND** (rectangular bending magnet with fringe fields) or the **SBEND** (sector bending magnet with fringe fields), which are found in MAD.

In the specification of a bending magnet still other sets of parameters may be substituted for or added to those mentioned. Other elements such as quadrupoles, sextupoles, solenoids, etc. also have alternate or additional sets of parameters. Complete lists will be found in the sections describing each individual element. Not all elements or possible sets of keyword parameters are compatible with the MAD program. The MAD compatible elements or sets of keyword parameters are indicated in the sections describing the individual elements.

In any keyword notation, the **VARY** command may be used to indicate a parameter to be varied in a fitting procedure. The use is the same as with strict MAD notation, and is explained on page 259.

The Use of Labels

Labels are available for identification of individual elements. There are two circumstances where such identification by the use of labels is necessary.

1. When using explicit **LINE** commands in the MAD format to assemble beam lines from previously defined components, labels are required. In the **LINE** command, the element or subline to be included in a beam line is identified by its label.

2. If an element is to be activated or deactivated or if the parameters of an element are to be changed between steps of a given problem, a label is also required. The label identifies the element in the earlier step to which the changes specified in the latter step are to apply.

Labels may also be inserted for the user's convenience. In this case the placement of a label on a given element is optional.

In keyword notation the label is placed before the mnemonic. It is then identified by being immediately followed by a colon. The maximum length of a label is fifteen non-blank characters. It may contain letters, numerals, underscore, percent (%), and dollar sign ($).

As an example, a drift space labelled **DRF** would appear as:

```
DRF:  DRIFT,  L = 1.5  ;
```

The use of labels to assemble elements into a beam line or subline is completely described on page 215. If the beam line is specified simply by listing the elements and no **LINE** statement is used, then only those elements to be changed in later steps need to be labeled. In this

section we will restrict our description of labels to their use in changing an element from one problem step to the next.

An element may be altered in any of several ways in passing from one step of a problem to the next. An element may be deactivated by placing a minus sign in front of its literal mnemonic (**QUAD**, **BEND**, etc.) or activated by removing the minus sign. New elements can be inserted and old elements removed from the beam line. The values of the parameters can also be changed.

Activation and Deactivation of Elements

For an element to be activated or deactivated from one step to the next of a given problem the element must be included in both steps, having the same label each time. All elements to be activated in a later step must be included in the first step (indicator card 0) of that problem. In later steps (indicator card 1) of a problem only those elements to be changed are specified. The elements to be changed are identified by their labels.

If the mnemonic of an element is immediately preceded by a minus sign in a given step of a problem, that element will be ignored when the calculation is performed. However, storage space in the computer will be allocated for the element for possible activation in later steps of the problem. For example, if a fitting constraint is to be ignored in the first step of a problem, but activated in a latter step, it should be indicated in both steps. In the first step such an element might appear as

```
FIT1:   -FIT, S12 = 0.0, TOLER = .001 ;
```

in the first problem step, and the element

```
FIT1:   FIT ;
```

in the later step. Here the sigma matrix element σ_{12} is fit to zero, causing a waist constraint to be imposed on the beam.

The storage space used for a **FIT** element is fixed. Therefore, in the first step, we can simply indicate that storage is to be reserved for a **FIT** element. In an ensuing problem step, the remainder of the **FIT** element can be specified. The above procedure is therefore equivalent to placing the element:

```
FIT1:   -FIT ;
```

in the first problem step. In the later step one would then insert

```
FIT1:   FIT, S12 = 0.0,  TOLER = .001 ;
```

Insertion and Removal of Elements

Elements may also be inserted without the need to be specified in the first of a set of problem steps. They may also be removed so as not to be passed on to further problem steps. A given problem might contain the print statement:

```
PR1:   PRINT,  TRANS  ;
```

To insert two fitting constraints for the σ matrix after the print command on a later problem step, the later step would contain the lines

```
PR1:   AFTER  ;
FIT,  S12 = 0.0,  TOLER = 0.001  ;
FIT,  S34 = 0.0,  TOLER = 0.001  ;
ENDINSERT  ;
```

The FIT elements would then remain in the data unless removed in a subsequent step.

If it is desired to place the FIT commands before the PRINT command, then the first line should say BEFORE instead of AFTER. The FIT elements need have labels only if they are to be modified or removed in a later step.

In the user wishes to remove the print command with the label PR1 in a subsequent step of a problem, he/she should include in that step the command:

```
PR1:   REMOVE  ;
```

The print command then cannot be activated in a still later step, but must be reinserted. In order to be removed by a REMOVE command, an element must be given a label when it is introduced, either in the first step or by an AFTER or BEFORE command.

If only one element is to be inserted then it can be placed on the same line as the BEFORE or AFTER statement without an intervening semicolon.

```
PR1:   AFTER,  FIT,  S12 = 0.0,  TOLER = 0.001  ;
```

No ENDINSERT command is then necessary to terminate the list of elements to be inserted.

The value of the BEFORE, AFTER, and REMOVE commands is that they allow one to keep a standard data set representing a beam line. One can then effectively modify the data set to do various calculations while not having to alter the original version.

Changing of Physical Parameters

The physical parameters of an element may be changed between steps of a problem. In the first step, all the parameters to be used should be specified. In the later step, only those parameters to be changed need be specified. The storage space allocated for a given element is determined by the element type and the set of parameters given. For the BEAM and MATRIX elements, included among the data may be continuation codes which will also affect the amount of storage space taken by the element.

In the first step a bending magnet may be given a magnetic field of 10 kilogauss.

```
B1:   BEND,  L = 5.0,  B = 10.0,  N = 0.0  ;
```

In a succeeding step, its magnetic field could be increased to 15 kilogauss by inserting the element:

```
B1:   BEND,  B = 15.0  ;
```

Variation of Physical Parameters

The command VARY is used to identify a parameter to be varied in a given problem step. The parameter is left variable until its value is again fixed by the FIX command. The commands identify the parameter to be varied or fixed by means of the element label and the parameter name. Directions for the use of the VARY command may be found on page 259 and those for the FIX command on page 260.

The commands VARY and FIX are not elements, but rather instructions modifying the specification of elements. What this means is that they are not stored in the computer as part of the beam line representation. Instead they affect the representation of other elements, indicating whether certain of their parameters are to be varied or not in the present problem step. They may appear in a problem step with indicator (1), without having appeared in a previous problem step with indicator (0). There is no reason for the commands VARY and FIX ever to have labels or to appear with a minus sign in front of the word VARY or FIX.

Examples

The sample problem below contains two problem steps, each beginning with title and indicator cards and terminating with a SENTINEL. The first step causes TRANSPORT to do a first-order calculation with fitting. The second step initiates a second-order calculation with the data that is a result of that fitting. Elements which are considered to be the same in the two steps are identified by having the same label.

Here we present three versions of the same example. The part of the first example which describes the beam line itself is MAD compatible. The elements are described first, then assembled into a beam line by use of the `LINE` command. Any elements which represent operations, and are therefore not MAD compatible, are placed after the specification of the beam line and refer to `MARKER` elements by means of `LOCATION` specifications. In this example the `PRINT` and `FIT` elements are so specified.

In the second example, the beam line is described by simply listing the elements in order. In the third example, keyword notation is used but with a different set of physical parameters describing each element. In this case the physical parameters are those used in the original TRANSPORT notation. Each of the three formulations of the beam line is a TRANSPORT readable set of data.

A MAD-Compatible TRANSPORT Input Deck

The example may be written with the specification of the optical lattice entirely in MAD notation. The elements – the SBEND, the DRIFTs, – are first given, then assembled into an optical lattice by use of the LINE, command. The USE statement specifies the lattice to be used for the calculation. The other TRANSPORT commands now appear either before of after the part of the data where the optical lattice is given. The complete set of data is as follows:

```
'FORTRAN H CHECK ON BETA FIT'
0
UMAD
BEAM  X = .5,  XP = 1,  Y = .5,  YP = 1,  L = .5,  DEL = 1,  PO = 1  ;
SEC1:  -ORDER  ;  PRINT,  BEAM,  ON  ;
DRI:  DRIFT,  L = 2.745 ;
SB:  SBEND,  L = 9.879,  ANGLE = 2.96,  K1 = .0449 ;
MRKA:  MARKER ;
EXAMPLE:  LINE =  (DRI,SB,DRI,MRKA)  ;
USE,  EXAMPLE ;
PRINT,  TRANS,  LOCATION = MRKA  ;
FIT1:  FIT,  R12 = 0.0,  TOLER = 0.0001,  LOCATION = MRKA  ;
VARY, DRI[L] ;
SENTINEL
'SECOND ORDER'
1
SEC1:  ORDER  ;
FIT1:  -FIT  ;
FIX,  DRI[L]  ;
SENTINEL
SENTINEL
```

Since, MAD elements are being used, the parameters are in MAD units. The element UMAD then specifies that MAD units (meters, radians) are to be assumed throughout.

The element SBEND includes the fringing fields and assumes that the pole-face-rotation angle is zero unless explicitly specified otherwise. The two drifts are given by a single specification. Consequently they are automatically varied in unison.

The USE command indicates that the line to be used is the one with the proper name EXAMPLE. Several different lines may be assembled into a single set of data. TRANSPORT will run through the line specified by the USE statement. If there is no USE statement, TRANSPORT will run through the elements in order as they occur in the data.

The FIT element indicates an operation to be performed at a certain location in the optical system. However, it is not a physical element, and, in MAD notation, is not included

directly in the specification of the beam line. Instead we define a marker labelled with the word **MRKA**. The marker is included directly in the beam line specification. The **FIT** element is included in the data after the beam line specification and contains a reference to the marker. The **PRINT** command can now also be related to the marker. When TRANSPORT goes through the beam line and comes to the marker labeled **MRKA**, it will execute all operations referring to that marker.

The **VARY** command indicates that in the first step the length of the drift space with label **DRI** is to be varied so as to satisfy the constraint imposed by the **FIT** command. The **FIX** command indicates that in the second step the length of the drift is to be held fixed. The second step will be executed with the value of the drift length as given by the fitting procedure of the first step.

VARY and **FIX** commands are considered to be instructions to modify the data and not part of the data itself. Therefore, they can be introduced in later steps without allocating space for them in the first step.

A TRANSPORT Input Deck Using MAD Elements

The physical elements specifying the beam line can be specified using MAD notation. However, the beam line is specified by simply listing the elements in order. MAD requires that the beam be explicitly assembled by the use of **LINE** commands. The result is a set of data which appears as:

```
'FORTRAN H CHECK ON BETA FIT'
0
UMAD
BEAM  X = .5,  XP = 1,  Y = .5,  YP = 1,  L = .5,  DEL = 1,  PO = 1  ;
SEC1:    -ORDER   ;   PRINT,   BEAM,   ON   ;
DRI:  DRIFT,   L  =  2.745   ;
SBEND,   L  =  9.879,   ANGLE  =  2.96,   K1  =  .0449   ;
DRI:   DRIFT,   L  =  2.745   ;
PRINT,   TRANS   ;
FIT1:    FIT,   R12  =  0,   TOLER  =  .0001   ;
VARY,   DRI[L]   ;
SENTINEL
'SECOND ORDER'
1
SEC1:    ORDER   ;
FIT1:    -FIT   ;
FIX,   DRI[L]   ;
SENTINEL
SENTINEL
```

Notice that the **DRIFT** element with the proper name **DRI** occurs twice. In this example, the duplication effort is inconsequential. However, in a large machine with much repetition, the duplication can cause much unnecessary effort. It is often useful to be able to define each different element only once and then assemble them into a beam line.

A TRANSPORT Input Deck with Keyword Notation

The same set of data can also be represented in an alternate keyword notation. Here the variables describing the beam line are explicitly identified. The variables used here are the same as with original TRANSPORT notation. The complete set of variables available for specifying elements is described in the sections for the individual elements.

```
'FORTRAN H CHECK ON BETA FIT'
0
UTRANS
BEAM  X = .5,  XP = 1,  Y = .5,  YP = 1,  L = .5,  DEL = 1,  PO = 1  ;
SEC1:    -ORDER   ;   PRINT,   BEAM,   ON   ;
DRI:  DRIFT,   L  =  2.745   ;
ROTAT,   ANGLE  =  0   ;
BEND,   L  =  9.879,   B  =  10,   N  =  .5   ;
ROTAT,   ANGLE  =  0   ;
DRI:   DRIFT,   L  =  2.745   ;
PRINT,   TRANS   ;
FIT1:    FIT,   R12  =  0,   TOLER  =  .0001   ;
VARY,   DRI[L]   ;
SENTINEL
'SECOND ORDER'
1
SEC1:    ORDER   ;
FIT1:    -FIT   ;
FIX,   DRI[L]   ;
SENTINEL
SENTINEL
```

The combination **ROTAT**, **BEND**, **ROTAT** replaces the element **SBEND** . The element **BEND** can be specified in MAD variables, but it is not a MAD element, and will not be recognized by the MAD program.

Transforming a MAD Deck to a TRANSPORT Deck

So far we have been emphasizing the versatility of the TRANSPORT input format. It is now time that we emphasize its compatability with the MAD input. We shall show how, given a complete MAD deck, to extract that portion which describes the physical structure of the beam line, and from this to construct a complete TRANSPORT deck. It is the part which describes the physical structure of the beam line which is compatible between the input of the two programs. An example can be taken from the Synchrotron Light Source Data Book, by Jim Murphy [13]. The MAD input has the following form:

```
TITLE,"SOR RING, TOKYO"
!          DATA COURTESY OF Y.KAMIYA AT KEK, JUNE 1988
QF         :QUADRUPOLE,L=.1,K1=6.08691786
QD         :QUADRUPOLE,L=.1,K1=6.2452322
BD         :SBEND,L=.863937,K1=.3719008,ANGLE=TWOPI/8
D1         :DRIFT,L=.655
D2         :DRIFT,L=.325
D3         :DRIFT,L=.13
HSUP        :LINE=(D1,BD,D2,QF,D3,QD)
USE,HSUP,SYMM,SUPER=4
PRINT,# S/E
TWISS
STOP
```

The elements specified consist of two quadrupoles, one sector bending magnet, and three drifts of differing lengths. The generic names of the elements are preceded by their proper names QF, QD, BD, D1, D2, and D3. The LINE command combines these elements into a single line with the proper name HSUP. The USE command indicates that the line HSUP is indeed the optical system to be used for calculations.

Since TRANSPORT is basically designed for single pass systems, the TRANSPORT USE command does not contain the modifiers SYMM and SUPER. The SYMM command indicates that a duplicate of the beam line with the elements in reverse order is created and placed immediately following the original beam line. The SUPER command indicates that the combined system is repeated four times.

In both MAD and TRANSPORT the same effect can be achieved by defining a LINE made up of the appropriate number of repetitions of HSUP, both forwards and backwards. To accomplish that effect, we need to replace the above LINE defining HSUP with two LINE statements defining first an intermediate step HSIN, and then the final line HSUP.

The PRINT, TWISS, and STOP commands are particular to MAD.

A complete TRANSPORT deck from the same data would be:

```
"SOR RING, TOKYO"
      0
UMAD
PRINT, BEAM, ON
BEAM  0.0  0.0  0.0  0.0  0.0  0.0  .380
!         DATA COURTESY OF Y.KAMIYA AT KEK, JUNE 1988
QF        :QUADRUPOLE,L=.1,K1=6.08691786
QD        :QUADRUPOLE,L=.1,K1=6.2452322
BD        :SBEND,L=.863937,K1=.3719008,ANGLE=TWOPI/8
D1        :DRIFT,L=.655
D2        :DRIFT,L=.325
D3        :DRIFT,L=.13
HSIN        :LINE=(D1,BD,D2,QF,D3,QD)
HSUP        :LINE=(4*(HSIN,-HSIN))
USE,   HSUP
SENTINEL
```

Positional or Original TRANSPORT Notation

Element Format

In positional notation, the element specification also begins with a mnemonic. In truly original TRANSPORT notation, the mnemonic is replaced by a numerical type code. Either the mnemonic or the type code may now be used in positional element notation.

The physical parameters follow the mnemonic or numerical type code. They are given by a sequence of numbers, separated by spaces and terminated by a semicolon. The end of a line also serves as a semicolon. The physical parameters are the quantities which describe the physical element represented. Such parameters may be lengths, magnetic fields, apertures, rotation angles, beam dimensions, or other quantities, depending on the mnemonic or type code number. Their meaning depends on their position in the sequence of numbers. There is no possibility of describing an element in an alternate set of parameters using positional notation. The label, if present, identifies the element. It may contain up to fifteen characters and is enclosed by single quotes.

In positional notation, a sector bending magnet would be specified as:

```
BEND  10.   20.   0.5  'BM1'  ;
```

The numbers here represent respectively the magnet length (10.) in meters, the magnetic field (20.) in kilogauss, and the dimensionless field index n (0.5). The sequence of characters `BM1` is the label. The meanings for the physical parameters for each element type are described in the section for that element. A summary, indicating the proper order for the physical parameters is given in Table 3.

For any element the mnemonic or type code is considered to be the first parameter . The second parameter is also known as the first physical parameter. It is the second entry in Table 3 or the second parameter in the section describing a given element. In some cases the parameters of an element do not really refer to physical quantities, but will nevertheless be referred to as such in this manual.

In positional notation the label may be placed anywhere among the numbers. During the calculation the elements will be printed in sequence and the label for a given element will be printed with that element. Labels are useful in problems with many elements and/or when sequential fitting is used. They must be used to identify any element to be changed in succeeding steps of a given problem.

If the physical parameters of an element are specified in positional notation, then a vary field forms an extension of the mnemonic or type code. The vary field indicates which physical parameters of the element are to be adjusted if there is to be any fitting. It is placed immediately (no intervening blanks) to the right of the decimal point which follows the mnemonic or type code.

```
BEND.01  10.    20.    0.5  'BM1'  ;
```

Here the decimal .01 following the mnemonic **BEND** indicates that the second number in the description of the physical element is to be varied during fitting. Here the second number is the strength B of the magnetic field and initally has the value 20. See page 266 for an explanation of the use of vary codes.

Each element must be terminated by a semicolon (;). For a single element per line, the end of a line serves as a semicolon. If there is more than one element per line of data, then each element must be terminated by a semicolon. Thus the **BEND** above may be followed by a drift as follows

```
BEND  10.    20.    0.5  'BM1'  ;  DRIFT  10.
```

Here the **BEND** element must be followed by a semicolon. The end of the line serves as a semicolon for the drift element.

Spaces before and after the semicolon are allowed but not required. If the program encounters a semicolon before the expected number of parameters has been read in and if the indicator card was a zero (0), the remaining parameters are set to zero. Thus, the element

```
BEND  10.    20.    'BM1'  ;
```

is the same as

```
BEND  10.    20.    0.0  'BM1'  ;
```

For a bend, three physical parameters are to be specified in original TRANSPORT notation. If only two physical parameters are specified, a single additional parameter is assumed with the value 0.0.

If the indicator card was a one (1), then the numbers indicated on the card are substituted for the numbers remaining from the previous solution; all other numbers are unchanged. Thus if the first problem step contained the element

```
BEND  10.    20.    0.5  'BM1'  ;
```

and a subsequent problem step had the element

```
BEND  10.    20.    'BM1'  ;
```

the normalized field gradient n would continue to have the value 0.5. If the second problem step had the element

```
BEND  10.   20.   0.0  'BM1'  ;
```

the value of the normalized field gradient would be changed to 0.0.

The "free-field" input format of the data cards makes it considerably easier to prepare input than the standard fixed-field formats of FORTRAN. Numbers may be placed anywhere on the card and must simply be in the proper order. They must be separated by one or more blanks. Several elements may be included on the same card. A single element may be continued from one card to the next by placing an ampersand **&** after all other characters on the card to be continued. A single number must be all on one card; it may not continue from one card to the next. The program storage is limited to a total of 5,000 locations (including representation of mnemonics and those parameters not entered but implied equal to zero) and 1300 elements.

A decimal number (e.g., 2.47) may be represented in any of the following ways:

```
2.47
.0247E+02
247E-2
```

For some elements, even when written in positional notation, additional parameters may be specified in keyword notation. This feature is especially useful for parameters which need not always be specified. If the magnetic field of the bending magnet has a quadratic dependence (or sextupole component – significant only in a second- or higher-order calculation), it might be written in hybrid notation as:

```
BEND  10.   20.   0.5  'BM1',  EPS = 0.01  ;
```

For the exact meaning of the EPS parameter, see page 131.

The specification of any keyword parameters follows the specification of positional parameters and the label. On a particular element, once one has switched to keyword parameters, one may not switch back to positional parameters. The element is terminated by a semicolon. The next element may then be written in any manner desired.

The Use of Labels

Labels are available for identification of individual elements. The use of labels is necessary if the parameters of an element are to be changed between steps of a given problem. The label identifies the element in the earlier step to which the changes specified in the latter step are to apply. Labels may also be inserted for the user's convenience. In this case the placement of a label on a given element is optional.

In positional notation the label may be placed anywhere among the parameters of a given element. It must be enclosed in quotes. Blanks within a label are ignored. The maximum length of a label is fifteen non-blank characters. It may contain letters, numerals, underscore, percent (%), and dollar sign ($).

As an example, the following all denote the same drift space:

```
'DRF' DRIFT. 1.5      ;
DRIFT'DRF'15-1;
DRIFT. .15E1 'D R F'  ;
```

On a **UNIT** element the label is the unit name. A unit specification indicating the longitudinal length (unit type 8) is to be measured in feet would then be:

```
UNIT  8.  'FT' ;
```

The use of labels to assemble elements into a beam line or subline is completely described on page 215. We therefore will restrict our description below to the use of labels to change an element from one problem step to the next.

An element may be altered in any of several ways in passing from one step of a problem to the next. An element may be deactivated by placing a minus sign in front of its literal mnemonic (**QUAD**, **BEND**, etc.) or activated by removing the minus sign. New elements can be inserted and old elements removed from the beam line. The values of the parameters can also be changed. Vary codes may be inserted or removed and the values of the parameters may be changed.

Activation and Deactivation of Elements

If an element is to be activated or decativated in passing from one step to the next of a given problem, the element must be included in both steps, having the same label each time. All elements to be activated in a later step must be included in the first step (indicator card 0) of that problem. In later steps (indicator card 1) of a problem only those elements to be changed are specified. The elements to be changed are identified by their labels.

If the mnemonic of an element is immediately preceded by a minus sign in a given step of a problem, that element will be ignored when the calculation is performed. However, storage space in the computer will be allocated for the element for possible activation in later steps of the problem. For example, if a fitting constraint is to be ignored in the first step of a problem, but activated in a latter step, it should be indicated in both steps. In the first step such an element might appear as

```
-FIT  1  2  0.0  .001  'FIT1'  ;
```

in the first problem step, and as

```
FIT  'FIT1'  ;
```

in the later step.

Here the sigma matrix element σ_{12} is fit to zero, causing a waist constraint to be imposed on the beam.

The storage space used for a FIT element is fixed. Therefore, in the first step, we can simply indicate that storage is to be reserved for a FIT element. In an ensuing problem step, the remainder of the FIT element can be specified. The above procedure is therefore equivalent to placing the element:

```
FIT1:  -FIT  ;
```

in the first problem step. In the later step one would then insert

```
FIT  1  2  0.0  .001  'FIT1' ;
```

Insertion and Removal of Elements

Elements may also be inserted without the need to be specified in the first of a set of problem steps. They may also be removed so as not to be passed on to further problem steps. A given problem might contain the print statement:

```
PRINT,  4  'PR1'  ;
```

To insert two fitting constraints for the σ matrix after the print command on a later problem step, the later step would contain the lines

```
AFTER  'PR1'  ;
FIT  1  2  0.0  0.001  ;
FIT  3  4  0.0  0.001  ;
ENDINSERT  ;
```

The FIT elements would then remain in the data unless removed in a subsequent step.

If it is desired to place the FIT commands before the PRINT command, then the first line should say BEFORE instead of AFTER. The FIT elements need have labels only if they are to be modified or removed in a later step.

In the user wishes to remove the print command with the label PR1 in a subsequent step

of a problem, he/she should include in that step the command

```
REMOVE  'PR1'  ;
```

The print command then cannot be activated in a still later step, but must be reinserted. In order to be removed by a REMOVE command, an element must be given a label when it is introduced, either in the first step or by an AFTER or BEFORE command.

If only one element is to be inserted then it can be placed on the same line as the BEFORE or AFTER statement without an intervening semicolon.

```
AFTER  'PR1'  FIT  -1  2  0.0  0.001  ;
```

No ENDINSERT command is then necessary to terminate the list of elements to be inserted. The value of the BEFORE, AFTER, and REMOVE commands is that they allow one to keep a standard data set representing a beam line. One can then effectively modify the data set to do various calculations while not having to alter the original version.

Changing of Physical Parameters

The physical parameters of an element may be changed between steps of a problem. In the first step, all the parameters to be used should be specified. In the later step, only those parameters to be changed need be specified. The storage space allocated for a given element is determined by the element type and the set of parameters given. For the BEAM and MATRIX elements, included among the data may be continuation codes which will also affect the amount of storage space taken by the element.

In the first step a bending magnet may be given a length of 5 meters.

```
BEND  5.0  10.0  0.0  'B1'  ;
```

In a succeeding step, its length could be increased to 10 meters by inserting the element

```
BEND  10.0  'B1'  ;
```

All parameters, up to and including the one to be changed, must be specified. the remaining, if not specified, will be left unchanged from the previous step.

Variation of Physical Parameters

Vary codes are designed for use in positional notation. Vary codes may be inserted or removed in passing from one problem step to the next. For instance, one might wish to vary the field of a quadrupole in one step of a problem and then use the fitted value as data in

the following step. The first step might then contain the element:

```
QUAD.01  5.0  10.0  5.0  'Q1'  ;
```

indicating that the second physical parameter in the quadrupole specification is to be varied. The second parameter represents the magnet pole-tip field strength and has the present value of 10.0. The following step would contain the element

```
QUAD  'Q1'  ;
```

Since, in the second step, the first item on the card contains no vary code the vary code is deleted. All other parameters, not being respecified, are left unchanged.

Several elements may have the same label. If, as in the above example, one wished to vary the field of several quadrupoles in one step, then pass the final values to the next step, one could give all such elements the same label. There might be four quadrupoles, all labeled `'Q1'`, being varied simultaneously. If the data for the next step contain the single element

```
QUAD  'Q1'  ;
```

the vary code on all elements labeled `'Q1'` will be deleted.

Examples

The sample problem below contains two problem steps, each beginning with title and indicator cards and terminating with a SENTINEL. The first step causes TRANSPORT to do a first-order calculation with fitting. The second initiates a second-order calculation with the data that is a result of that fitting. Elements which are considered to be the same in the two steps are identified by having the same label.

Here we present two versions of the same example. The example describes the same beam line and set of operations as did the previous examples for MAD notation.

In the first example, elements themselves are given in original TRANSPORT notation. The beam line consists of a drift (3), bending magnet with pole face rotations (2, 4, 2), and another drift (3). It is prefaced by a specification of the beam phase space (1), and a print command(13). An order specification (-17) is present, but turned off for the first step. The beam line description is followed by another print commant (13), and the fit command (10).

For the two drift spaces, the numeral "3" appears after the decimal point which follows the type code. These two numerals signify that the two drifts are to be varied and that the changes made are always to be equal.

The type code `10` element which specifies the fitting condition is labeled `FIT1`. It is active for the first-order calculation, but is turned off for the second-order calculation. The

vary codes for elements DR1 are set to zero for the second-order problem. The second-order element, SEC1, is ineffective during the fitting, but causes the program to compute the second-order matrices in the second calculation.

In the second example, the numerical type codes are replaced by literal mnemonics, making the data representing the beam line easier to read.

An Example of a TRANSPORT Input Deck

Original TRANSPORT Notation

```
'FORTRAN H CHECK ON BETA FIT'                          Title card
0                                                      Indicator card
1   .5   1   .5   1   .5   1   1   ;
-17   'SEC1'   ;   13   3   ;
3.3   2.745   'DRI'   ;
2   0   ;                                              Elements
4   9.879   10   .5   ;                                                  First problem step
2   0   ;
3.3   2.745   'DRI'   ;
13   4   ;
10   -1   2   0   .0001   'FIT1'   ;
SENTINEL
'SECOND ORDER'                                         Title card
1                                                      Indicator card
17   'SEC1'_   ;
3   'DRI'   ;                                          Elements to be
-10   'FIT1'   ;                                       changed                Second problem step
SENTINEL
SENTINEL                Second sentinel signifies end of run.
```

As many problems and problem steps as one wishes may be stacked in one job.

Note that in some previous versions of TRANSPORT a decimal point was required with every numerical entry except the indicator card (which must not have a decimal point in any version of TRANSPORT). The decimal point is not required in this version of TRANSPORT.

Original TRANSPORT Notation with Literal Mnemonics

The numerical type codes can now be replaced by literal mnemonics, such as **BEND**, **DRIFT**, and **FIT**. This makes the identification of the elements somewhat easier.

```
'FORTRAN H CHECK ON BETA FIT'
0
BEAM   .5   1   .5   1   .5   1   1   ;
-ORDER   'SEC1'   ;   PRINT   3   ;
DRIFT.3   2.745   'DRI'   ;
ROTAT   0   ;
BEND   9.879   10   .5   ;
ROTAT   0   ;
DRIFT.3   2.745   'DRI'   ;
PRINT   4   ;
FIT   -1   2   0   .0001   'FIT1'   ;
SENTINEL
'SECOND ORDER'
ORDER   'SEC1'   ;
DRIFT   'DRI'   ;
-FIT   'FIT1'   ;
SENTINEL
SENTINEL
```

The mnemonics are still followed by a set of numbers. The meaning of these numbers is understood from their position in the sequence following the mnemonic. For example, the numbers 9.879, 10., and .5, following the mnemonic **BEND**, indicate respectively the length, magnetic field, and n value for a bending magnet.

Summaries of TRANSPORT Elements

Table 1. Summary of TRANSPORT Elements in MAD Notation

Physical Element	Mnemonic	Keywords					
Drift	DRIFT	L					
Bending Magnets with Fringing Fields	RBEND or SBEND	L	ANGLE	K1	K2	K3	TILT
Horizontal and Vertical Steering Magnets	HKICK or VKICK	L	KICK				
Steering Magnet in Both Planes	KICKER	L	HKICK	VKICK			
Quadrupole	QUAD	L	K1	TILT			
Sextupole	SEXT	L	K2	TILT			
Octupole	OCTU	L	K3	TILT			
Plasma Lens	PLAS	L	K1				
Solenoid	SOLE	L	KS				
Accelerator	ACCE	L	VOLT	LAG	FREQ		
Coordinate Rotation	SROT	ANGLE					
Marker	MARK						

The meanings of the various keywords are given in more detail in the sections of this manual describing the individual physical elements

Table 2. Summary of Other TRANSPORT Elements in Keyword Notation

Preliminary Specifications in Keyword Notation

Physical Element	Mnemonic	Keywords			
Mass of Particles in beam	SPEC	PMASS			
Length of System	SPEC	LENGTH			
Reference Trajectory Momentum	SPEC	P0			
Random Number	SPEC	RANNO			
Focal Plane Rotation	SPEC	FOTILT			
Initial Beam Line x-Coordinate	SPEC	XBEGIN			
Initial Beam Line y-Coordinate	SPEC	YBEGIN			
Initial Beam Line z-coordinate	SPEC	ZBEGIN			
Initial Beam Line Horizontal Angle	SPEC	YAW			
Initial Beam Line Vertical Angle	SPEC	PITCH			
Reference Momentum for Magnetic Fields	SPEC	PREF			
Random Errors	RANDOM	Element Type	Parameter Name	Error Value	
Limits on Varied Parameters	LIMIT	Element Type	Parameter Name	LOWER or UPPER	Limit Value

Beam Description in Keyword Notation

Physical Element	Mnemonic	Keywords										
Beam	BEAM	X	XP	Y	YP	L	DEL	P0				
Beam (Accelerator Notation)	BEAM	BETAX	ALPHAX	EPSX	BETAY	ALPHAY	EPSY	P0				
Beam (Rotated Ellipse)	CORR	C21, C63,	C31, C54,	C41, C64,	C51, C65	C61,	C32,	C42,	C52,	C62,	C43,	C53,
Beam Centroid Shift	CENT	X	XP	Y	YP	L	DEL					
Eta	ETA	ETAX	DETAX	ETAY	DETAY	ETAL	ETAP					

Physical Elements in Keyword Notation

Physical Element	Mnemonic	Keywords				
Drift Space	DRIFT	L				
Wedge Bending	BEND	L	ANGLE	K1	K2	K3
Magnet		L	ANGLE	N	EPS	EPS3
without fringing		L	B	N	EPS	EPS3
field		L	RADIUS	N	EPS	EPS3
		B	ANGLE	N	EPS	EPS3
		RADIUS	ANGLE	N	EPS	EPS3
Pole Face Rotation including fringing field	ANGLE	HGAP	FINT	H		
Bending Magnets	RBEND	L	ANGLE	N	EPS	EPS3
with	or	L	B	N	EPS	EPS3
Fringing Fields	SBEND	L	RADIUS	N	EPS	EPS3
		B	ANGLE	N	EPS	EPS3
		RADIUS	ANGLE	N	EPS	EPS3
Horizontal and	HKICK	L	B			
Vertical Steering	or	L	ANGLE			
Magnets	VKICK					
Quadrupole	QUAD	L	B	APER	TILT	
		L	GRAD	TILT		
Sextupole	SEXT	L	B	APER	TILT	
Octupole	OCTU	L	B	APER	TILT	
Plasma Lens	PLAS	L	B	APER	TILT	
		L	GRAD	TILT		
Solenoid	SOLE	L	B			
Electrostatic	SEPTUM	L	VOLTAGE	APER	TILT	
Septum		L	E			
Accelerator	ACCEL	L	VOLT	LAG	WAVEL	

Other Transformations in Keyword Notation

Physical Element	Mnemonic	Keywords						
Alignment	ALIGN	X	RX	Y	RY	Z	RZ	CODE
Coordinate System Shift	SHIFT	X	XP	Y	YP	L	DEL	

Operations in Keyword Notation

Physical Element	Mnemonic	Keywords				
Update	UPDATE	R1	R2			
Alignment Marker	ALMARK	R1	R2			
Print Instruction	PRINT	BEAM REFER ONELINE ON	TRANS WAIST ELEMENTS OFF	ACCEL PRECISE CENTROID	ALIGN NOPARA TWISS	FLOOR ONLY NARROW
Fitting Constraints	FIT	NAME	VALUE	TOLER		
Plotting	PLOT	See page 252				
Magnet Dimensions	MAGNET	WIDTH	HEIGHT			

Table 3: Summary of TRANSPORT Elements in Positional Notation

Preliminary Specifications in Original TRANSPORT Notation

Physical Element	Mnemonic & Vary	2nd Entry	3rd Entry	4th Entry
Units (Transport Dimensions)	UNIT	Code	Unit Symbol	Scale Factor (if required)
Quadratic Term for Bend Field	SPEC.0v	1.0	$\epsilon(1) = \beta\left(\frac{1}{\rho_0}\right)^2$ ρ_0 in units of transverse length	
Fringe Field Correction Coefficient	SPEC	2.0	κ_0	
Mass of Particles in beam	SPEC	3.0	M/m m = mass of electron	
Half Aperture of Bending Magnet in x Plane	SPEC	4.0	$w/2$	
Half Aperture of Bending Magnet in y plane	SPEC	5.0	$g/2$	
Length of System	SPEC	6.0	L	
Fringe Field Correction Coefficient	SPEC	7.0	κ_1	
Reference Trajectory Momentum	SPEC	11.0	p_0	
Curvature of Entrance Face of Bending Magnet	SPEC.0v	12.0	$(1/R_1)$	
Curvature of Exit Face of Bending Magnet	SPEC.0v	13.0	$(1/R_2)$	
Random Number	SPEC	14.0	Initial value of random number	
Focal Plane Rotation	SPEC	15.0	Angle of focal plane rotation	

Physical Element	Mnemonic & Vary	2nd Entry	3rd Entry	4th Entry
Initial Beam Line x-Coordinate	SPEC.0v	16.0	x_0	
Initial Beam Line y-Coordinate	SPEC.0v	17.0	y_0	
Initial Beam Line z-coordinate	SPEC.0v	18.0	z_0	
Initial Beam Line Horizontal Angle	SPEC.0v	19.0	θ_0	
Initial Beam Line Vertical Angle	SPEC.0v	20.0	ϕ_0	
Reference Momentum for Magnetic Fields	SPEC	21.0	p_{ref}	
Fractional Excess bend	SPEC	22.0	r_s	
Scaling Factor for Skew Field Components	SPEC	23.0	r_a	
Skew Bend Field	SPEC	24.0	v_r	
Skew Quadrupole Component for a Bending Magnet	SPEC	25.0	n'	
Skew Sextupole Component for a Bending Magnet	SPEC	26.0	ϵ'	
Octupole Component for a Bending Magnet	SPEC	27.0	ϵ_3	
Order of Calculation	ORDER	n_1	n_2	
Random	RANDOM	Not expressable in positional notation		
Limit	LIMIT	Not expressable in positional notation		

Beam Description in Original TRANSPORT Notation

Physical Element	Mnemonic & Vary	2nd Entry	3rd Entry	4th Entry	5th Entry	6th Entry	7th Entry	8th Entry	9th Entry
Beam	`BEAM.vvvvvv0`	x	x'	y	y'	ℓ	δ	p_0	
rms Addition to Beam Envelope	`BEAM.vvvvvv00`	Δx	$\Delta x'$	Δy	$\Delta y'$	$\Delta \ell$	$\Delta \delta$	p_0	0
Beam (Rotated Ellipse)	`CORR`	The fifteen correlations among the six elements (This entry must be preceded by a `BEAM` element.)							
Beam Centroid Shift	`CENTROID.vvvvvv`	Δx	$\Delta x'$	Δy	$\Delta y'$	$\Delta \ell$	$\Delta \delta$	p_0	
Accelerator Eta Function	`ETA`	Δx	$\Delta x'$	Δy	$\Delta y'$	$\Delta \ell$	$\Delta \delta$	p_0	

Physical Elements in Original TRANSPORT Notation

Physical Element	Mnemonic & Vary	2nd Entry	3rd Entry	4th Entry	5th Entry
Drift	DRIFT.v	Length			
Bending Magnet	BEND.vvv	Length	Field	Field gradient (n value)	
Pole Face Rotation	ROTAT.v	Angle of Rotation			
Rectangular Bending Magnet	RBEND.vvv	Length	Field	Field gradient (n value)	
Sector Bending Magnet	SBEND.vvv	Length	Field	Field gradient (n value)	
Horizontal Steering Magnet	HKICK.vvv	Length	Field		
Vertical Steering Magnet	VKICK.vvv	Length	Field		
Steering Magnet in Both Planes	KICKER	Not expressable in positional notation			
Quadrupole	QUAD.vv0	Length	Field	Half aperture	
Sextupole	SEXT.vv0	Length	Field	Half aperture	
Octupole	OCTU.vv0	Length	Field	Half aperture	
Plasma Lens	PLASMA	Length	Field	Half aperture	
Solenoid	SOLE.vv	Length	Field		
Accelerator	ACCEL	Length	E (energy gain)	ϕ (phase lag)	Wavelength
Electrostatic Septum	SEPTUM	Length			
Marker	MARKER				

Other Transformations in Original TRANSPORT Notation

Physical Element	Mnemonic & Vary	2nd Entry	3rd Entry	4th Entry	5th Entry	6th Entry	7th Entry	8th Entry
Arbitrary Matrix	`MATRIX.vvvvvv0`	R_{j1}	R_{j2}	R_{j3}	R_{j4}	R_{j5}	R_{j6}	J
		Continuation codes also allow the input of second- and third-order terms.						
Coordinate Rotation	`SROT.v`	Angle of rotation						
Shift	`SHIFT.vvvvvv`	Δx	$\Delta x'$	Δy	$\Delta y'$	$\Delta \ell$	$\Delta \delta$	p_0
Alignment Tolerance	`ALIGN.vvvvvv0`	Δx	$\Delta \theta$	Δy	$\Delta \phi$	Δz	$\Delta \alpha$	Code Number

Assembling Beam Lines in Original TRANSPORT Notation

Physical Element	Mnemonic & Vary	2nd Entry
Repeat control	`REPEAT`	Number of repetitions
Defined Section	`SECTION`	Code

Operations in Original TRANSPORT Notation

Physical Element	Mnemonic & Vary	2nd Entry	3rd Entry	4th Entry	5th Entry
Transform 1 Update	`UPDATE`	0.	1.		
Transform 2 Update	`UPDATE`	0.	2.		
Alignment Marker	`ALMARK`	0.	1. or 2.		
Input/Output Options	`PRINT`	CODE			
Fitting Constraints	`FIT`	I	J	Desired Value (i,j) Matrix Element	Accuracy of Fit
Plotting	`PLOT`	Not expressable in positional notation			
Magnet Dimensions	`MAGNET`	Not expressable in positional notation			

Output Format

General Appearance

Here we give a brief description of the general appearance of the output and its meaning. The user may refer to the sample output shown at the end of this section. It is the printed output resulting from the sample data shown on page 27. In a simple example it is not possible to show each of the different elements. We therefore refer the user to the sections on the various elements for an explanation of any features particular to a given element.

The output for each step of a given problem is printed separately. The printing for one step is completed before that for the next step is begun. Therefore we will describe the output for a single problem step. The output shown starting on page 61 is from a problem with two steps.

Initial Listing

For each problem step, the program begins by printing out the user's input data. The total number of elements in the beam line after each line is interpreted is printed out after each element. The number may increase by more than one after certain lines for two reasons:

1. Several elements may be placed on a single line, separated by semicolons.
2. A data element may result in several stored elements. One example of that is an element containing an algebraic expression. The instructions to retrieve the quantities in the expression and the algebraic operations are stored in the program as elements.

In the earliest versions of TRANSPORT, the input data was reformatted into neat columns. This aspect of the program eventually interfered with other, more useful features and was abandoned. Nevertheless, it is recoverable by the placement of the word `REFORMAT` on the indicator line of the input data. For more details, see the following section.

Listing During the Calculation

The program now begins the calculation. If there is no fitting, one listing of the beam line will be made. If there is fitting there will normally be two listings. The first will represent the beam line before any fitting has occurred. The second will be based on the new values of the physical parameters which were altered by the fitting process. If sequential fitting is employed, and the keyword `NOBEFORE` is included on the indicator card, the first run will be omitted. For further explanation of the various print control keywords, see page 67.

In any listing the elements are printed in order with their labels and physical parameters. Elements with a minus sign before the element name or type code number are ignored. The listing for each element begins with the element number. By use of this number a correspondence may be made between the elements in the listing during the calculation and those in the initial listing.

Following the element number is the name of that type of element, enclosed in asterisks. All physical elements are listed in this way. Some of the other elements are not explicitly listed but produce their effect in either the calculated quantities or the listing of the beam line. For descriptions of individual cases, the reader should consult the sections on the individual elements.

Calculated quantities appear in the listing as requested in the input data. Important cases will be described in greater detail below. The physical parameters for each element are printed with the appropriate units. For some elements a calculated quantity, not in the input data, will appear, enclosed in parenthesis. Such quantities are explained in the sections under the individual elements.

Calculated Quantities

Options

The important cases of calculated quantities which appear in the output are the transfer matrices, the beam matrix, the layout coordinates, and the results of the fitting procedure. The transfer and beam matrices and layout coordinates appear as requested in the listing of the beam line. The results of the fitting procedure appear between the two listings. All these quantities are explained in greater detail below.

The transfer and beam matrices appear only where requested, either at specific locations or globally. A request for the printing of layout coordinates can be made only globally. Any global print request should be made at the beginning of the beam line. The requested item will then be printed after each physical element. In all cases the quantities printed are the values at the interface between two elements. They are evaluated at a point after the element listed above them and before the element listed below. For further explanation of calculated quantities the user should read the section on the mathematical formulation of TRANSPORT and the sections describing the appropriate elements. The transfer matrix, beam matrix, and layout coordinates are all printed by various forms of the `PRINT` command.

The Transfer Matrix

The six coordinates of the transfer matrix correspond to x, x', y, y', ℓ, and δ. The matrix will then have the appearance

$$\begin{array}{cccccc}
(x|x_0) & (x|x_0') & (x|y_0) & (x|y_0') & (x|\ell) & (x|\delta) \\
(x'|x_0) & (x'|x_0') & (x'|y_0) & (x'|y_0') & (x'|\ell) & (x'|\delta) \\
(y|x_0) & (y|x_0') & (y|y_0) & (y|y_0') & (y|\ell) & (y|\delta) \\
(y'|x_0) & (y'|x_0') & (y'|y_0) & (y'|y_0') & (y'|\ell) & (y'|\delta) \\
(\ell|x_0) & (\ell|x_0') & (\ell|y_0) & (\ell|y_0') & (\ell|\ell) & (\ell|\delta) \\
(\delta|x_0) & (\delta|x_0') & (\delta|y_0) & (\delta|y_0') & (\delta|\ell) & (\delta|\delta)
\end{array}$$

If the centroid is off axis, an extra column will appear to the right, giving the position of the beam centroid. The centroid given is the optical image of the centroid at the beginning of the beam line. In first order it will coincide with the center of gravity of the beam ellipse. In higher orders the two will differ due to nonlinear distortions of the beam ellipse.

$$\begin{array}{cccccc|c}
(x|x_0) & (x|x_0') & (x|y_0) & (x|y_0') & (x|\ell) & (x|\delta) & x_c \\
(x'|x_0) & (x'|x_0') & (x'|y_0) & (x'|y_0') & (x'|\ell) & (x'|\delta) & x_c' \\
(y|x_0) & (y|x_0') & (y|y_0) & (y|y_0') & (y|\ell) & (y|\delta) & y_c \\
(y'|x_0) & (y'|x_0') & (y'|y_0) & (y'|y_0') & (y'|\ell) & (y'|\delta) & y_c' \\
(\ell|x_0) & (\ell|x_0') & (\ell|y_0) & (\ell|y_0') & (\ell|\ell) & (\ell|\delta) & \ell_c \\
(\delta|x_0) & (\delta|x_0') & (\delta|y_0) & (\delta|y_0') & (\delta|\ell) & (\delta|\delta) & \delta_c
\end{array}$$

For a static magnetic system with midplane symmetry, a good deal of simplification occurs. Many of the matrix elements become zero. For the case of an on-axis centroid, the transfer matrix then becomes:

$$\begin{array}{cccccc}
(x|x_0) & (x|x_0') & 0.0 & 0.0 & 0.0 & (x|\delta) \\
(x'|x_0) & (x'|x_0') & 0.0 & 0.0 & 0.0 & (x'|\delta) \\
0.0 & 0.0 & (y|y_0) & (y|y_0') & 0.0 & 0.0 \\
0.0 & 0.0 & (y'|y_0) & (y'|y_0') & 0.0 & 0.0 \\
(\ell|x_0) & (\ell|x_0') & 0.0 & 0.0 & 0.0 & (\ell|\delta) \\
0.0 & 0.0 & 0.0 & 0.0 & 0.0 & (\delta|\delta)
\end{array}$$

If the beam line is midplane symmetric and there is no vertical bending, then the transfer matrix can be printed on a single line. To do this, one should include the command (`PRINT, ONELINE ;`) described below. Only those matrix elements which can be nonzero under the stated conditions are printed. The quantities printed are:

$$L_c \ \texttt{M} \qquad R_{11} \ R_{12} \ R_{21} \ R_{22} \ R_{33} \ R_{34} \ R_{43} \ R_{44} \ R_{16} \ R_{26}$$

The units for only the accumulated length L_c are shown and are here taken to be meters `M`.

Second- and third-order matrices can also be printed. The second- and third-order matrix elements are individually identified in the output. The second-order matrix is identified with the letter "T". The elements of the T matrix have one dependent index and two independent indices. The indices apply to the six coordinates of the transfer matrix, described above.

The dependent index is written first and followed immediately by the independent indices. Thus the matrix element T_{116} can be written in Dirac notation as the coefficient $(x|x_0\delta)$. It is the coefficient of the product of x_0 and δ in the Taylor series expansion for x.

The second-order transfer matrix is printed in triangular form. That means that there is no coefficient T_{161} or $(x|\delta x_0)$. The individual matrix elements are identified by the dependent index followed by a space and then the dependent index. For example, if the matrix element T_{116} had the value `1.124E-03`, as in the example below, it would be listed as

```
1 16  1.124E-03
```

The third-order matrix is identified by the letter "U". It has one dependent index and three independent indices. Thus the matrix element U_{1166} or $(x|x_0\delta^2)$ would be the coefficient in the expansion for x which multiplied the product $x_0\delta^2$. The U matrix is printed in pyramidal form, in analogy with the triangular form for the second-order matrix.

More details are given on page 242.

The Beam Matrix

The beam matrix, as printed will have the following appearance:

x_g	Δx	CM					
x'_g	$\Delta x'$	MR	r_{21}				
y_g	Δy	CM	r_{31}	r_{32}			
y'_g	$\Delta y'$	MR	r_{41}	r_{42}	r_{43}		
ℓ_g	$\Delta \ell$	CM	r_{51}	r_{52}	r_{53}	r_{54}	
δ_g	$\Delta \delta$	PC	r_{61}	r_{62}	r_{63}	r_{64}	r_{65}

The first column with the subscript "g" represents the coordinates of the center of gravity of the beam phase space. In first order the center of gravity is the same as the centroid, which is the geometric center of the phase ellipsoid. In second and higher order, the ellipsoid becomes distorted and the center of gravity is no longer the beam centroid. If the centroid does not coincide with the reference trajectory, then the centroid coordinates are printed along with the transfer matrix (see below).

The second column gives the half widths of the beam ellipsoid in each of the six beam coordinates. Once again, in higher orders the beam phase space is not an ellipsoid, but some distorted shape like a banana. The quantities given are then the square roots of the second moments of the beam phase space, taken about the beam center of gravity. The units used are given in the third column.

The remaining quantities r_{ij} are the correlations between coordinates. They are defined in terms of the beam matrix elements σ_{ij} by: where

$$r_{ij} = \frac{\sigma_{ij}}{[\sigma_{ii}\ \sigma_{jj}]^{1/2}} \ .$$

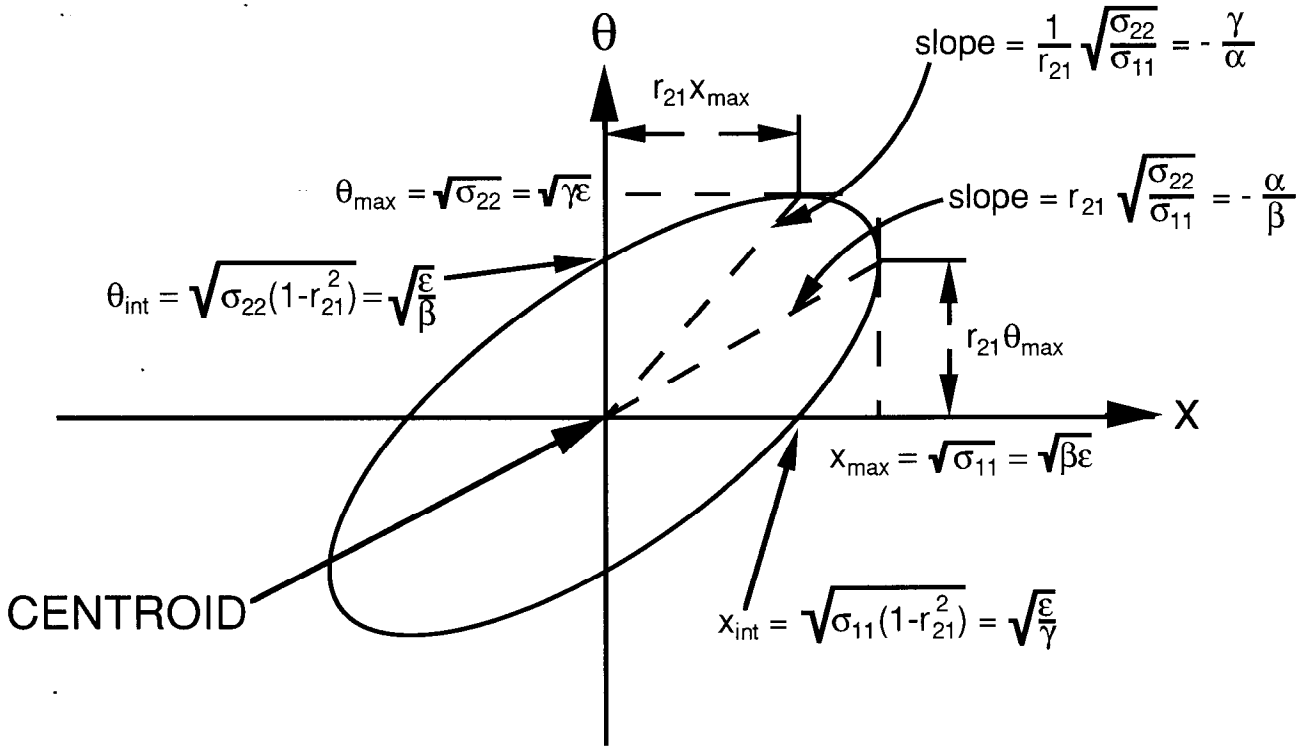
Their initial values may be specified by the **CORRELATION** element described on page 116.

The beam matrix may also be printed in a single line. The command which causes the printing to occupy a single line is (**PRINT, ONELINE** ;). This command, described below, does not print anything, but simply affects the style of how quantities are printed. A single line printing of the beam matrix would look like:

L_c **M** Δx **CM** $\Delta x'$ **MR** Δy **CM** $\Delta y'$ **MR** $\Delta \ell$ **CM** $\Delta \delta$ **PC** r_{21} r_{43}

The accumulated length L_c along the reference trajectory is shown first.

An example of an (x, x') plane ellipse is illustrated below.



A Two-Dimensional Beam Phase Ellipse

The Layout Coordinates

The floor coordinates of the beam reference trajectory can be printed after every beam line component. One can produce a layout of a beam line in any Cartesian coordinate system one chooses.

The coordinates printed represent the x, y and z position, and the yaw, pitch, and roll angles, respectively, of the reference trajectory at the interface between two elements. The y-axis will point up and the x-axis to the left. The *yaw* angle is between the floor projection of the reference trajectory and the floor z axis. The *pitch* angle is the vertical pitch, the angle the reference trajectory makes with a horizontal plane. The *roll* angle is a rotation about the reference trajectory. In the printed output the values given are those at the exit of the element listed above and at the entrance of the element listed immediately below.

The calculation of the coordinates is done from the parameters of the physical elements as given in the data. Therefore, if effective lengths are given for magnetic elements, the coordinates printed will be those at the effective field boundary.

The floor coordinates will be printed after every physical element. The accumulated length L_c of the reference trajectory, the three Cartesian floor positions and the three orientation angles will be printed on a single line, along with the units in which these quantities are expressed. The single line will have the following appearance:

$$L_c \ \texttt{M} \qquad xfloor \ yfloor \ zfloor \ \texttt{M} \qquad yaw \ pitch \ roll \ \texttt{MR}$$

Fitting

Quantities relevant to the fitting appear between the two listings of the beam line. At each iteration of the fitting procedure a line is printed containing the value of the relaxation factor used, the value of chi-squared before the iteration was made, and the corrections made to each of the varied parameters. Once the fitting is complete the final chi-squared and the covariance matrix are printed.

The appearance of the chi-squared and covariance matrix is:

*COVARIANCE (FIT χ^2)

$$\begin{array}{llll} \sqrt{C_{11}} & & & \\ r_{12} & \sqrt{C_{22}} & & \\ \cdot & \cdot & & \\ \cdot & & \cdot & \\ \cdot & & & \cdot \\ r_{1n} & \cdot & \cdot & \cdot \quad r_{n,n-1}\sqrt{C_{nn}} \end{array}$$

If MAD notation is used parameters may be given names and be declared independently of any physical element which might refer to them. Such parameters can be varied in a fitting procedure. The new values of all such varied parameters are printed out after the covariance matrix. For further details on the variation of parameters see page 259.

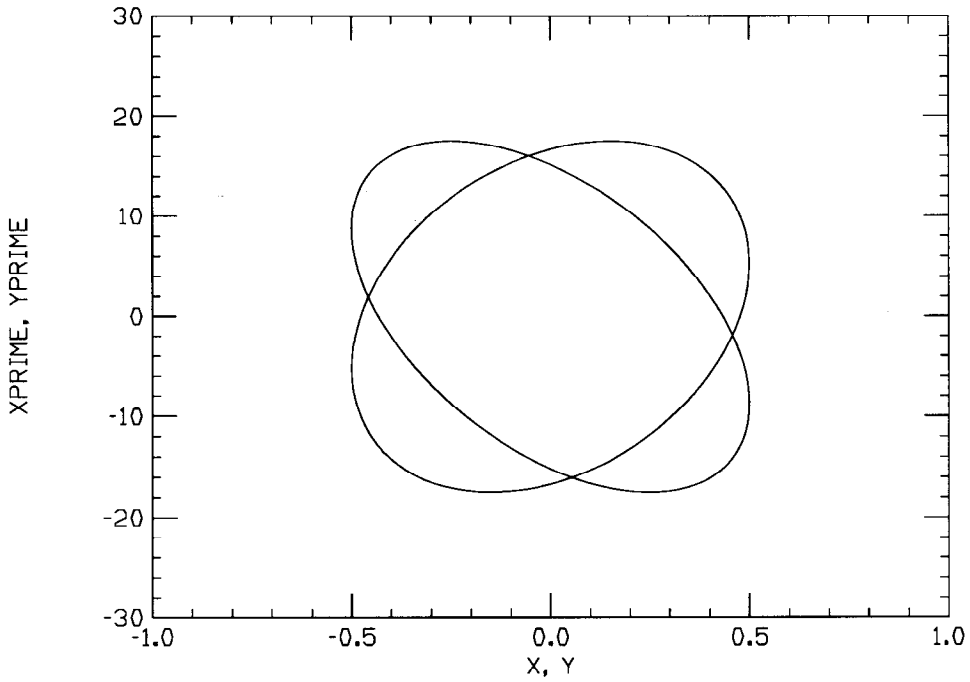
Plotting Output

A PLOT instruction in the input data causes a separate file to be produced in the output. This file consists of lines of sets of values of whatever quantities are specified on the plot instruction. It can be incorporated into the input data of a plotting program like TOP DRAWER via an auxiliary procedure. The procedure TOP DRAWER can be invoked and plots produced.

There are currently three types of plot that can be made with TRANSPORT. They are:

1. Elements of the transfer or beam matrices or any algebraic combination of such quantities can be plotted vs accumulated distance along the beam line.
2. The beam line reference trajectory floor coordinates. Views from the top and from the side can be made. The magnet external dimensions can be specified so that the magnets are drawn into the plot.
3. The beam ellipse. The two coordinates can be any two of the six beam coordinates.

A plot of the beam ellipse in x vs x' and y vs y' on the same plot might look like:



For more details, see the section of the **PLOT** instruction on page 252.

```
      "FORTRAN  H CHECK ON BETA FIT
         0
      BEAM  .5  1  .5  1  .5  1  1  ;
      -ORDER  'SEC1'  ;  PRINT  3  ;
 5    DRIFT.3  2.745  'DR1'  ;
      ROTAT  0  ;
      BEND  9.879  10  .5  ;
      ROTAT  0  ;
      DRIFT.3  2.745  'DR1'  ;
10    PRINT  4  ;
      FIT  -1  2  0  .0001  'FIT1'  ;
      SENTINEL
10 ELEMENTS USED OUT OF A MAXIMUM ALLOWABLE 3001
33 NUMBERS USED OUT OF A MAXIMUM ALLOWABLE  5001
```

```
FORTRAN  H CHECK ON BETA FIT

(   1)  *BEAM*                      1.00000 GEV/C
            0.000 M                                                    0.000   0.500 CM
                                                                       0.000   1.000 MR      0.000
                                                                       0.000   0.500 CM      0.000  0.000
                                                                       0.000   1.000 MR      0.000  0.000  0.000
                                                                       0.000   0.500 CM      0.000  0.000  0.000  0.000
                                                                       0.000   1.000 PC      0.000  0.000  0.000  0.000  0.000
(   4)  *DRIFT*        "DR1   "      2.74500 M
            VARY CODE =  3
             2.745 M                                                   0.000   0.570 CM
                                                                       0.000   1.000 MR      0.481
                                                                       0.000   0.570 CM      0.000  0.000
                                                                       0.000   1.000 MR      0.000  0.000  0.481
                                                                       0.000   0.500 CM      0.000  0.000  0.000  0.000
                                                                       0.000   1.000 PC      0.000  0.000  0.000  0.000  0.000
(   5)  *ROTAT*                     0.00000 DEG
(   6)  *BEND *                     9.87900 M        10.00000 KG       0.50000      (     3.336 M    , 169.69000 DEG )
(   7)  *ROTAT*                     0.00000 DEG
            12.624 M                                                   0.000  10.013 CM
                                                                       0.000  12.324 MR      0.993
                                                                       0.000   0.369 CM      0.000  0.000
                                                                       0.000   1.360 MR      0.000  0.000 -0.086
                                                                       0.000  11.690 CM     -0.992 -0.972  0.000  0.000
                                                                       0.000   1.000 PC      0.999  0.994  0.000  0.000 -0.991
(   8)  *DRIFT*        "DR1   "      2.74500 M
            VARY CODE =  3
            15.369 M                                                   0.000  13.378 CM
                                                                       0.000  12.324 MR      0.996
                                                                       0.000   0.502 CM      0.000  0.000
                                                                       0.000   1.360 MR      0.000  0.000  0.681
                                                                       0.000  11.690 CM     -0.988 -0.972  0.000  0.000
                                                                       0.000   1.000 PC      0.999  0.994  0.000  0.000 -0.991

        *TRANSFORM 1*
            -1.00383  -0.00418   0.00000   0.00000   0.00000  13.36812
            -1.83605  -1.00383   0.00000   0.00000   0.00000  12.24884
             0.00000   0.00000  -1.00383  -0.00418   0.00000   0.00000
             0.00000   0.00000  -1.83605  -1.00383   0.00000   0.00000
            -1.22488  -1.33681   0.00000   0.00000   1.00000 -11.58648
             0.00000   0.00000   0.00000   0.00000   0.00000   1.00000
(  10)  *FIT*          "FIT1  "     R12          0.0      /0.00010        (    -0.00418 )
        *LENGTH*         15.36900 M
        *CORRECTIONS*
        *NUMBER OF VARIED PARAMETERS =     1 *
        *NUMBER OF CONSTRAINTS       =     1 *

 0.10000E+01 ( 0.17447E+04)   -0.0208
 0.10000E+01 ( 0.62655E-02) -0.40E-04
        *COVARIANCE  (FIT 0.56843E-07 )
       0.000
```

```
FORTRAN  H CHECK ON BETA FIT

(   1) *BEAM*                    1.00000 GEV/C
           0.000 M                                                0.000   0.500 CM
                                                                  0.000   1.000 MR      0.000
                                                                  0.000   0.500 CM      0.000  0.000
                                                                  0.000   1.000 MR      0.000  0.000  0.000
                                                                  0.000   0.500 CM      0.000  0.000  0.000  0.000
                                                                  0.000   1.000 PC      0.000  0.000  0.000  0.000  0.000
(   4) *DRIFT*        "DR1   "      2.72415 M
           VARY CODE =  3
            2.724 M                                               0.000   0.569 CM
                                                                  0.000   1.000 MR      0.478
                                                                  0.000   0.569 CM      0.000  0.000
                                                                  0.000   1.000 MR      0.000  0.000  0.478
                                                                  0.000   0.500 CM      0.000  0.000  0.000  0.000
                                                                  0.000   1.000 PC      0.000  0.000  0.000  0.000  0.000
(   5) *ROTAT*                   0.00000 DEG
(   6) *BEND *                   9.87900 M        10.00000 KG        0.50000      (     3.336 M    , 169.69000 DEG )
(   7) *ROTAT*                   0.00000 DEG
           12.603 M                                               0.000  10.013 CM
                                                                  0.000  12.324 MR      0.993
                                                                  0.000   0.370 CM      0.000  0.000
                                                                  0.000   1.357 MR      0.000  0.000 -0.086
                                                                  0.000  11.690 CM     -0.992 -0.972  0.000  0.000
                                                                  0.000   1.000 PC      0.999  0.994  0.000  0.000 -0.991
(   8) *DRIFT*        "DR1   "      2.72415 M
           VARY CODE =  3
           15.327 M                                               0.000  13.352 CM
                                                                  0.000  12.324 MR      0.996
                                                                  0.000   0.500 CM      0.000  0.000
                                                                  0.000   1.357 MR      0.000  0.000  0.676
                                                                  0.000  11.690 CM     -0.989 -0.972  0.000  0.000
                                                                  0.000   1.000 PC      0.999  0.994  0.000  0.000 -0.991
       *TRANSFORM 1*
          -1.00000   0.00000   0.00000   0.00000   0.00000  13.34259
          -1.83605  -1.00000   0.00000   0.00000   0.00000  12.24884
           0.00000   0.00000  -1.00000   0.00000   0.00000   0.00000
           0.00000   0.00000  -1.83605  -1.00000   0.00000   0.00000
          -1.22488  -1.33426   0.00000   0.00000   1.00000 -11.58648
           0.00000   0.00000   0.00000   0.00000   0.00000   1.00000
(  10) *FIT*          "FIT1  "     R12            0.0     /0.00010        (  -0.238E-07 )
       *LENGTH*         15.32731 M
```

```
        "SECOND ORDER
           1
   15   ORDER  'SEC1'  ;
        DRIFT  'DR1'  ;
        -FIT  'FIT1'  ;
        SENTINEL
   10 ELEMENTS USED OUT OF A MAXIMUM ALLOWABLE 3001
   28 NUMBERS USED OUT OF A MAXIMUM ALLOWABLE  5001
SECOND ORDER

(   1)  *BEAM*                    1.00000 GEV/C
          0.000 M                                          0.000   0.500 CM
                                                           0.000   1.000 MR      0.000
                                                           0.000   0.500 CM      0.000  0.000
                                                           0.000   1.000 MR      0.000  0.000  0.000
                                                           0.000   0.500 CM      0.000  0.000  0.000  0.000
                                                           0.000   1.000 PC      0.000  0.000  0.000  0.000  0.000
(   2)  *2ND ORDER*    "SEC1  "          GAUSSIAN DISTRIBUTION    2.     2.
(   4)  *DRIFT*        "DR1   "      2.72415 M
          2.724 M                                          0.000   0.569 CM
                                                           0.000   1.000 MR      0.478
                                                           0.000   0.569 CM      0.000  0.000
                                                           0.000   1.000 MR      0.000  0.000  0.478
                                                           0.000   0.500 CM      0.000  0.000  0.000  0.000
                                                           0.000   1.000 PC      0.000  0.000  0.000  0.000  0.000
(   5)  *ROTAT*                   0.00000 DEG
(   6)  *BEND *                   9.87900 M       10.00000 KG       0.50000      (     3.336 M    , 169.69000 DEG )
(   7)  *ROTAT*                   0.00000 DEG
          12.603 M                                         0.068  10.013 CM
                                                           0.037  12.324 MR      0.993
                                                           0.000   0.370 CM      0.000  0.000
                                                           0.000   1.358 MR      0.000  0.000 -0.086
                                                          -0.060  11.690 CM     -0.992 -0.972  0.000  0.000
                                                           0.000   1.000 PC      0.999  0.994  0.000  0.000 -0.991
(   8)  *DRIFT*        "DR1   "      2.72415 M
          15.327 M                                         0.078  13.352 CM
                                                           0.037  12.324 MR      0.996
                                                           0.000   0.500 CM      0.000  0.000
                                                           0.000   1.358 MR      0.000  0.000  0.676
                                                          -0.081  11.690 CM     -0.989 -0.972  0.000  0.000
                                                           0.000   1.000 PC      0.999  0.994  0.000  0.000 -0.991
```

```
*TRANSFORM 1*
    -1.00000   0.00000   0.00000   0.00000   0.00000  13.34259
    -1.83605  -1.00000   0.00000   0.00000   0.00000  12.24884
     0.00000   0.00000  -1.00000   0.00000   0.00000   0.00000
     0.00000   0.00000  -1.83605  -1.00000   0.00000   0.00000
    -1.22488  -1.33426   0.00000   0.00000   1.00000 -11.58648
     0.00000   0.00000   0.00000   0.00000   0.00000   1.00000
*2ND ORDER TRANSFORM*
  1 11  1.124E-03
  1 12  1.225E-03   1 22  6.673E-04
  1 13  0.000E+00   1 23  0.000E+00   1 33 -4.871E-03
  1 14  0.000E+00   1 24  0.000E+00   1 34 -2.042E-03   1 44 -1.112E-03
  1 15  0.000E+00   1 25  0.000E+00   1 35  0.000E+00   1 45  0.000E+00   1 55  0.000E+00
  1 16  2.065E-02   1 26  3.066E-02   1 36  0.000E+00   1 46  0.000E+00   1 56  0.000E+00   1 66  7.908E-02

  2 11  6.187E-07
  2 12  1.348E-06   2 22  6.126E-04
  2 13  0.000E+00   2 23  0.000E+00   2 33 -5.504E-03
  2 14  0.000E+00   2 24  0.000E+00   2 34 -2.999E-03   2 44 -1.021E-03
  2 15  0.000E+00   2 25  0.000E+00   2 35  0.000E+00   2 45  0.000E+00   2 55  0.000E+00
  2 16  3.097E-02   2 26  3.564E-02   2 36  0.000E+00   2 46  0.000E+00   2 56  0.000E+00   2 66  3.835E-02

  3 11  0.000E+00
  3 12  0.000E+00   3 22  0.000E+00
  3 13 -7.500E-04   3 23 -2.042E-03   3 33  0.000E+00
  3 14 -2.042E-03   3 24 -2.224E-03   3 34  0.000E+00   3 44  0.000E+00
  3 15  0.000E+00   3 25  0.000E+00   3 35  0.000E+00   3 45  0.000E+00   3 55  0.000E+00
  3 16  0.000E+00   3 26  0.000E+00   3 36 -1.064E-02   3 46 -3.414E-03   3 56  0.000E+00   3 66  0.000E+00

  4 11  0.000E+00
  4 12  0.000E+00   4 22  0.000E+00
  4 13  5.503E-03   4 23  5.994E-03   4 33  0.000E+00
  4 14 -2.999E-03   4 24 -2.042E-03   4 34  0.000E+00   4 44  0.000E+00
  4 15  0.000E+00   4 25  0.000E+00   4 35  0.000E+00   4 45  0.000E+00   4 55  0.000E+00
  4 16  0.000E+00   4 26  0.000E+00   4 36  5.756E-03   4 46  4.367E-03   4 56  0.000E+00   4 66  0.000E+00

  5 11 -2.641E-03
  5 12 -3.564E-03   5 22 -1.533E-03
  5 13  0.000E+00   5 23  0.000E+00   5 33 -1.031E-03
  5 14  0.000E+00   5 24  0.000E+00   5 34 -4.367E-04   5 44  1.707E-04
  5 15  0.000E+00   5 25  0.000E+00   5 35  0.000E+00   5 45  0.000E+00   5 55  0.000E+00
  5 16 -5.338E-03   5 26 -1.916E-02   5 36  0.000E+00   5 46  0.000E+00   5 56  0.000E+00   5 66 -7.857E-02

*LENGTH*         15.32731 M
```

Title, Indicator, and Comments

Title Card

The title card is the first card in every problem step of a TRANSPORT data set. *The title card is always required and must be followed by the indicator card* (see next section). The indicator card indicates whether the data to follow is new (0 card) or a continuation of a previous data set (a 1 card or a 2 card).

The title must be enclosed within quotation marks ('). The string may begin and end in any column (free field format), for example

```
'SLAC 8 GeV/c SPECTROMETER'
```

Example of a DATA SET for a Single Problem Step

Title
0 or 1
Elements
_____
_____
_____
SENTINEL

Indicator Card (0 or 1)

The second line of the input for each step of a problem is the indicator card. If the data which follow describe a new problem, a zero (0) is placed anywhere on that line. If the data which follow describe changes to be made in the previous step of a given problem, a one (1) is placed on that line.

If a given problem step involves fitting, the program will normally list the beam line twice. In each listing the program will print the sequence of elements along with transfer or beam matrices where specified. The first listing uses the parameters of each element before any fitting has taken place. The second shows the results of the fitting. If a problem involving fitting has several steps, the second run of a given step often differs little from the first run of the following step. The user may wish to suppress it.

Similarly, a user may wish to delete the initial listing, and/or to print only the chi-squared and changes in parameter values from the fitting. Alternatively, he or she may wish to print only that portion of the beam line where the fitting occurs.

Various print controls, can be specified via keywords. The keywords are placed on the indicator card, following the indicator number. If there is more than one, they must be separated by commas. Some examples are given on the following pages. The complete listing of such keywords is

Keyword	Meaning
`NOLIST`	The initial listing is suppressed.
`REFORMAT`	The initial listing is reformatted. The output from this initial listing now resembles the uniformly spaced sequence of numbers used in older versions of TRANSPORT. The default is now to simply reprint the input data. The **REFORMAT** option exists simply for reasons of compatability, so that the user can obtain the same style output as with the eariler versions of the program. The **REFORMAT** option should not be used with keyword input as this option assumes positional parameter input and the use of original TRANSPORT variables.
`NOBEFORE`	The printing from the run through the beam line before fitting is suppressed. If there is no fitting to be done, this option will have no effect.
`NOPRINT`	The printing from the run through the beam line both before and after fitting is suppressed. The printing during fitting remains. The output will still contain the sequence of chi-squared values and the changes in the varied parameters.

NOSOLVE — The printing from the run through the beam line before, during, and after fitting is suppressed. The fitting is itself suppressed. The NOSOLVE option is useful on a 1 indicator card after the fitting has been completed in previous steps. The initial deck listing using the REFORMAT command can sometimes be used as data for subsequent TRANSPORT runs.

BWRITE — The common blocks are written onto logical unit 7 on a disk. Subsequent runs of TRANSPORT may then be made without having to read and decipher the data for the first step of a problem. Instead the common blocks can be read directly using the BREAD instruction, described below. Clearly, the mnemonic BWRITE should be placed only on the last step of a problem. The job should contain the appropriate JCL to identify logical unit 7 with the file the user wishes to contain the output.

BREAD — The common blocks are read from logical unit 7 on a disk. The first step of a problem then need consist of only the TITLE card and the indicator card with the mnemonic BREAD. All the data, including all the elements and the SENTINEL card, are, in effect, read from the disk file. This initial problem step, with indicator card 0, may then be followed by steps with indicator card 1. The additional steps may contain modifications to the original data. Thus a charged particle optical system may be stored as a binary file, and the only data that need be decoded from the new job are modifications to the stored file. The job should contain the appropriate JCL to identify logical unit 7 with the file containing the input.

If the options NOLIST and NOSOLVE are both used, there is no output. For example, if one wanted only the listing printed after fitting is complete, then the indicator card might read

```
0,  NOLIST,  NOBEFORE
```

The use of these indicator options along with the two commands

```
PRINT,  ELEMENTS,  OFF  ;
PRINT,  ONLY  ;
```

allows for great flexibility in determining the output to be printed. The two commands are placed near the beginning of the elements which specify the beam line. The first command suppresses the printing of the physical elements. Only the transfer and beam matrices and the results of various other print commands will appear in the output. The second command allows only the varied elements and the constraints to be printed. These commands are described on pages 247 and 248.

If the final step of a problem has the word **BWRITE** on the indicator line, the common blocks of the program are written on a disk. A subsequent problem can read the common blocks. The entire first problem step can then be:

```
'EXAMPLE OF READING THE DATA FROM COMMON BLOCKS'
0,  BREAD
```

All the elements, including the **SENTINEL**, are read as common blocks. Subsequent steps, with a new title and indicator 1, can follow. They need contain only changes to the data from the first step. The changes are specified exactly as if the first step had been read from a complete data listing. That complete data listing is clearly that which would result from the step of a previously run problem where the word **BWRITE** appeared on the indicator.

Some of these options have been previously available through the use of other numbers for the indicator. The keyword format of MAD allows greater flexibility with less confusion. The previously defined variations on the indicator card have the same effect as before, but are now unnecessary. They are, however, still available for compatability with existing sets of data.

If the second or subsequent step of a problem involves fitting and the first listing is to be suppressed a two (2) may be given for the indicator. If no fitting is involved, the program will ignore the 2 and will do one single run through the system. This option is equivalent to the use of the keyword **NOBEFORE**.

If the initial listing is to be deleted, 10 may be added to the indicator to give 10, 11 or 12. This option is equivalent to the use of the keyword **NOLIST**. In order to be consistent with earlier versions of TRANSPORT, an indicator of minus one (-1) is interpreted as a two (2), *but nine* (9) *is not interpreted as twelve* (12).

A sample problem input was shown previously in the section on input format. In this example TRANSPORT does a first-order calculation with fitting (0 indicator card) and then a second-order calculation (1 indicator card) with the data that are the result of the fitting.

Comment Cards

Comment cards may be introduced anywhere in the deck where an element would be allowed. Comments are identified differently in MAD and in original TRANSPORT. Both methods of identifying comments are available in the present version of TRANSPORT. The comments are not stored, but appear only in the initial listing of the given problem step.

In MAD anything on a line after an exclamation mark (!) is considered to be a comment. For example, the bending magnet element described earlier can have an attached comment as follows:

```
BM1:   SBEND,  L = 10.,  ANGLE = 10.,  K1 = 0.5E-6 ; !  THIS IS A COMMENT
```

In original TRANSPORT the comments made on any line are enclosed within single parentheses. No parentheses are allowed within the parentheses of any comment card.

Example of the Use of Comment Cards in a Data Set

```
'TITLE CARD'
0
(THIS IS A TEST PROBLEM TO ILLUSTRATE THE)
(USE OF COMMENT CARDS)
elements
(COMMENTS MAY ALSO BE MADE BETWEEN ELEMENTS)
elements
SENTINEL
```

Preliminary Specifications

Input-Output Unit Sets – UTRANS, UMAD, UMETER, UMM, UMICR

TRANSPORT allows a great deal of flexibility in the choice of units for representation of physical quantities. Several different sets of units may be specified by means of single unit set commands. The choice of units may be made independently of the choice of notation for the elements. However, strict MAD notation, necessary for compatibility with the MAD program, requires the use of MAD units.

Further changes may be made in individual units changes. The unit set commands are described in this section. The format for individual unit changes is given in the next section.

The command UMAD allows compatability with the MAD program and facilitates the performance of accelerator calculations. This single command causes all lengths to be expressed in meters and all angles (except for accelerator phase advance) to be expressed in radians. The fractional momentum deviation δ is expressed as a fraction, not as a percent. One reason these units are preferred by accelerator designers is that they are consistent. The angular measure milliradians is intrinsically self-inconsistent since it is the ratio of two lengths expressed in different units. The unit percent is self-inconsistent since a quantity in percent must be divided by 100 to be used in any equation.

The use of the MAD set of units is specified by placing the command UMAD at the beginning of the sequence of elements in the data deck. This command should immediately follow the title and indicator cards and precede any UNIT command, described in the following section.

Example of the Use of UMAD in a Data Set

```
'TITLE CARD'
0
UMAD
other elements
SENTINEL
```

The command UMAD also causes the multipole specifications to be defined according to the MAD convention. The sextupole strength should be multiplied by a factor of two from the other conventions. The octupole strength should be multiplied by a factor of six.

A different set of units was originally considered standard in TRANSPORT. This original set of units is still available and is specified by substituting the command UTRANS for UMAD in the above example. Previous manuals were written in terms of this original set of units. Now there is a choice among several basic sets. The units to be used in specifying

the physical parameters of the various elements are given in the sections describing those elements. Appropriate units are given for both the unit sets UMAD and UTRANS.

Other unit sets may also be specified. Alternate set of units are specified by replacing the UMAD command with UMETER, UMM, or UMICR. One of these unit set specifications must be at the beginning of any deck of TRANSPORT data.

The UMETER command is very similar to UMAD. The two specifications differ in that with UMETER the accelerator phase advance and the coordinate rotation about the longitudinal axis are specified in degrees. With UMETER the multipole strengths are also specified in the standard TRANSPORT convention as the normalized field derivative divided by $n!$, where n is the order of the multipole. Details may be found on page 2 in the section on Representation of Magnetic Fields under the Mathematical Formulation of TRANSPORT.

The UMM and UMICRON commands allow smaller units to be used for transverse dimensions. With the UMM set of units the transverse units are millimeters and milliradians. With the UMICRON set of units the transverse units are microns (not micrometers – that is a measuring instrument, not a unit) and microradians.

In subsequent sections describing the individual elements, the units for the physical parameters are given for the unit sets UMAD and UTRANS. Any of the five unit sets, UMAD, UTRANS, UMETER, UMM, or UMICRON can be used for any of the physical parameters on any element. The units of a specific type of quantity, such as length, transverse distance, angle, magnetic field, particle mass can also be set individually. See the next section describing the UNIT command. Units can be anything the user desires, such as furlongs, parsecs, or cubits for length, arc seconds for angles, stone or solar masses for mass, and Webers per square foot for magnetic field.

The original TRANSPORT units and their meanings are:

Code Digit	Quantity	Original TRANSPORT Unit	Symbols used in SLAC-75
1.0	horizontal and vertical transverse dimensions, magnet apertures, and misalignment displacements	cm	x,y
2.0	horizontal and vertical angles and misalignment rotation angles	mr	θ, ϕ
3.0	vertical beam extent (only) * and bending magnet gap height	cm	y
4.0	vertical beam divergence (only) *	mr	ϕ
5.0	pulsed beam length and wave length in accelerator	cm	ℓ
6.0	momentum spread	percent	δ
7.0	bend, pole face rotation, and coordinate layout angles	degrees	
8.0	length (longitudinal) of elements, layout coordinates, and bending magnet pole face curvatures	meters	t
9.0	magnetic fields	kG	B
10.0	mass	electron mass	m
11.0	momentum and	GeV/c	p(0)
	energy gain in accelerator section	GeV	ΔE
12.0	Betatron phase shift	degrees	ψ
13.0	Rotations about the reference trajectory	degrees	
14.0	Electric fields and voltages	MV	

* These codes should not be used if the coordinate rotation (`SROT`) element is used anywhere in the system.

The units that are specified by the various unit sets are as follows. Any further units changes should refer to these units:

Code Digit	UTRANS	UMAD	UMETER	UMM	UMICR
1.0	cm	meters	meters	mm	μ
2.0	mr	radians	radians	mr	μr
3.0	cm	meters	meters	mm	μ
4.0	mr	radians	radians	mr	μr
5.0	cm	meters	meters	mm	μ
6.0	pc	fraction	fraction	pm	pμ
7.0	degrees	radians	radians	mr	μr
8.0	meters	meters	meters	meters	meters
9.0	kG	kG	kG	kG	kG
10.0	electron mass	GeV	GeV	GeV	GeV
11.0	GeV/c	GeV/c	GeV/c	GeV/c	GeV/c
12.0	degrees	radians/2π	degrees	degrees	degrees
13.0	degrees	radians	degrees	degrees	degrees
14.0	MV	MV	MV	MV	MV

The meaning of the abbreviations used above is as follows:

cm = centimeters
mm = millimeters
μ = microns
mr = milliradians
μr = microradians
pc = percent
pm = permil (one part in a thousand)
pμ = permic (one part in a million)
kG = Kilogauss
MV = Megavolts

The command **UMAD** also affects the definition of the multipole component (see above).

UNIT – Individual Units Changes

Strict MAD input requires no individual units changes. It requires merely the specification of the global unit set UMAD, described on the previous few pages. Elements described in terms of MAD parameters, in other keyword notation, or in original TRANSPORT notation may require individual units changes from one of the global unit set specifications.

The individual units changes are placed in the data at the beginning of the ELEMENTS. They follow only the global units specifications.

The individual units changes can be written only in original TRANSPORT notation. However, an individual units change can be placed in a data set otherwise expressed in any notation acceptable to TRANSPORT. A change of an individual type of unit is accomplished by insertion of one or more of the following elements.

There are four parameters to be specified:

1. UNIT (or type code 15)
2. Code digit indicated on the charts on the previous two pages.
3. The abbreviation of the unit (see examples below). This will be printed on the output listing. It must be enclosed in single quotes and is a maximum of three characters long (four for energy). The format for insertion is the same as for labels.
4. The scale factor (if needed). The scale factor is the size of the new unit relative to the unit in the prevailing unit set. Consider, for example, the case when the new transverse unit is inches and the standard TRANSPORT units have been specified. The standard transverse TRANSPORT unit is cm, so the scale factor is 2.54.

Examples

In the following examples, it will be assumed that the prevailing unit set is UTRANS. Thus, unless otherwise specified, longitudinal units are in meters, transverse units in centimeters and milliradians, and momentum in GeV/c.

To change length to feet, width to inches, and momentum to MeV/c, add to the front of the deck the elements

```
UNIT  8.   'FT'  0.3048;
UNIT  1.   'IN'  2.54;
UNIT  11.   'MEV'  0.001;
```

The scale factor, 0.3048, gives the length of the new unit, feet, to convert in the reference unit, meters, etc.

Qualifications

The keywords ELEMENT, BEAM, FLOOR, ALIGN, INPUT, and OUTPUT may be used to restrict the units change to a limited application. The keywords are placed on the units element after all the items described above, and are separated by commas. The keyword ELEMENT indicates that the units change is to be used only for the description of a physical element. The application can be further restricted to the input data alone or the printed output alone by use of the additional keywords INPUT or OUTPUT. The data may then be input in one set of units and output in another. Two beam lines may also be concatenated longitudinally, even if they are specified in different units. A new set of units specifications must be placed between the two beam line descriptions.

The keyword BEAM refers to the beam dimensions and the transfer matrix elements. The keyword FLOOR refers to the floor coordinates. The keyword ALIGN refers to the misalignment parameters. These three keywords apply both to input and output and are not subject to further restriction by the INPUT and OUTPUT keywords.

The basic UNIT element is almost unchanged from earlier versions of TRANSPORT. The one significant difference is that the units name is now the same as the label. It may now be placed anywhere in the element. A separate label is not permitted.

The unit elements are the first cards in a deck (immediately following the title card and the 0 and 1 indicator card) and should not be inserted in any other location. The unit set specification (UTRANS, UMAD, UMETER, UMM, or UMICR) should appear first, followed by any UNIT elements. Units specifications produce no printed output during the calculation, their effect being visible only in the output from other elements.

More Examples

If the linear measures for the units changes above were to apply only to the input data for the physical elements, then the example would be modified to:

```
UNIT  8.   'FT'  0.3048,  ELEMENT,  INPUT ;
UNIT  1.   'IN'  2.54,  ELEMENT,  INPUT ;
UNIT  11.   'MEV'  0.001  ;
```

If the beam dimensions are to be expressed in microns and microradians, the following elements should be added to the front of the deck.

```
UNIT  1.   'MU'  1.0E-4 ,  BEAM ;
UNIT  2.   'MUR'  1.0E-3 ,  BEAM ;
```

If the floor coordinates are to be expressed in feet and degrees, the following elements are used.

```
UNIT  7.   'DEG' ,  FLOOR ;
UNIT  8.   'FT' ,  FLOOR ;
```

Finally, if the misalignment parameters are to be expressed in mils (thousandths of an inch) and arc minutes, then the following units changes should be inserted.

```
UNIT  1.   'MIL'  .00254,  ALIGN ;
UNIT  2.   'MIN'  .290888,  ALIGN ;
```

The specifications of sets of units is equivalent to a set of **UNIT** specifications, with one small caveat. The specification of a unit set (**UTRANS**, **UMAD**, etc.) not only specifies the units, but also the reference for further unit changes. The specification of the unit size on the **UNIT** element is done relative to the prevailing unit set.

Without further units changes then, the single command

```
UMAD
```

is equivalent to the series of commands

```
UTRANS ;
UNIT  1.   'M' ;
UNIT  2.   'R' ;
UNIT  5.   'M' ;
UNIT  6.   'N' ;
UNIT  7.   'R' ;
UNIT  10.   'GEV' ;
UNIT  12.   'TUNE'  .02777778 ;
UNIT  13.   'R' ;
```

The mnemonics **'M'** for meters and **'R'** for radians are recognized by TRANSPORT and the conversion factor is supplied automatically. The units conversion containing the symbol **'N'** indicates that the momentum deviation δ is measured fractionally (not per-anything). The name **'TUNE'** is not recognized by TRANSPORT, so the conversion factor must be supplied.

For the conventional units listed below, it is sufficient to stop with the unit name (the conversion factor is automatically inserted by the program). If units other than those listed below are desired, then the unit name and the appropriate conversion factor must be included. The conversion units below are based on the assumption that the prevailing unit set is **UTRANS**. *If the automatic feature is used with older versions of the program, there must be no blank spaces between the quotes and the unit name.*

Input-output units: UNIT element – For use with UTRANS

(Conversion factors for dimension changes versus code digit and unit)

Unit	Code Digit												
	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.	13.
'M'	100.	- - -	100.	- - -	100.	- - -	- - -	1.	- - -	- - -	- - -	- - -	- - -
'CM'	1.	- - -	1.	- - -	1.	- - -	- - -	.01	- - -	- - -	- - -	- - -	- - -
'MM'	.1	- - -	.1	- - -	.1	- - -	- - -	.001	- - -	- - -	- - -	- - -	- - -
'MICR'	.0001	- - -	.0001	- - -	.0001	- - -	- - -	.000001	- - -	- - -	- - -	- - -	- - -
'FT'	30.48	- - -	30.48	- - -	30.48	- - -	- - -	.3048	- - -	- - -	- - -	- - -	- - -
'IN'	2.54	- - -	2.54	- - -	2.54	- - -	- - -	.0254	- - -	- - -	- - -	- - -	- - -
'R'	- - -	1000.	- - -	1000.	- - -	- - -	- - -	- - -	- - -	- - -	- - -	(a)	(a)
'MR'	- - -	1.	- - -	1.	- - -	- - -	- - -	- - -	- - -	- - -	- - -	(b)	(b)
'MUR'	- - -	.001	- - -	.001	- - -	- - -	- - -	- - -	- - -	- - -	- - -	(c)	(c)
'DEG'	- - -	(d)	- - -	(d)	- - -	- - -	- - -	- - -	- - -	- - -	- - -	1.	1.
'PC'	- - -	- - -	- - -	- - -	- - -	1.	- - -	- - -	- - -	- - -	- - -	- - -	- - -
'PM'	- - -	- - -	- - -	- - -	- - -	.1	- - -	- - -	- - -	- - -	- - -	- - -	- - -
'PMIC'	- - -	- - -	- - -	- - -	- - -	.0001	- - -	- - -	- - -	- - -	- - -	- - -	- - -
'N'	- - -	- - -	- - -	- - -	- - -	100.	- - -	- - -	- - -	- - -	- - -	- - -	- - -
'MEV'	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	.001	- - -	- - -
'GEV'	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	1.	- - -	- - -
'KG'	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	1.	- - -	- - -	- - -	- - -
'G'	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	.001	- - -	- - -	- - -	- - -

(a) The size of a radian in degrees is $180./\pi$

(b) The size of a milliradian in degrees is $.180/\pi$

(c) The size of a microradian in degrees is $.000180/\pi$

(d) The size of a degree is milliradians is $1000.*\pi/180.$

PC is an abbreviation for percent

PM means permil which is one part in a thousand or one-tenth of a percent

PMIC means permic which is one part in a million or one-ten-thousandth of a percent

N means fractional measure or parts per 100 percent

SPECIAL — Special Input Parameters

A number of constants are used by the program which do not appear as parameters of any element. Many of these are initial values, such as floor coordinates, to be set at the beginning of the beam line. Other constants do appear as parameters of physical elements but, if unspecified, are assumed to have certain default values. A special element has been provided to allow the designer to set the values of these various constants.

Most of these special parameter elements will appear at the beginning of the data, after any units changes. In any case, the special parameters must always precede the physical element(s) to which they apply. *Once introduced, they apply to all succeeding elements of the appropriate type in the beam line unless reset to zero or to new values.* Many of the `SPECIAL` parameters apply to bending magnets. In such cases, the value of a parameter as given by the `SPECIAL` element may be overridden for a single magnetic element by including the parameters in the description the element itself.

There is no strictly MAD format for the `SPECIAL` element. Any `SPECIAL` element can be written in either keyword notation or original TRANSPORT notation.

Keyword Notation

A large number of keywords are available for use with the `SPECIAL` element and are listed below. However, each use of the `SPECIAL` element involves a single keyword. Separate special parameters must be set by separate uses of the `SPECIAL` element. An example might be the specification of the quadratic dependence of a bending magnet field. Using keyword notation, it could also be written as

```
SPEC,  EPS = .001  ;
```

When `EPS` is specified by the special parameter element, the value given is applied to all subsequent bending magnets. This particular special parameter can also be written directly on any element to which it applies. If so, the value given on the element will momentarily override the value given by the `SPECIAL` element. The `RBEND`, `SBEND`, and `BEND` elements will recognize the special parameter `EPS`. When `EPS` is given on a bending magnet element, it applies only to that bending magnet.

Original TRANSPORT Notation

There are three parameters:

1. `SPEC(IAL)` (or type code `16.`)
2. Code digit.
3. Value of the constant.

In the original TRANSPORT notation with type codes and positional parameters, the quadratic dependence of a bending magnet field might look like

```
SPEC  1.    .001 ;
```

A number of such constants have been defined in this manner. All have a normal value that is initialized at the beginning of each run.

Keywords and Code Digits

The keywords and code digits which can be used in specifying the special input parameters are as follows. The keywords are used in keyword notation. The code digits are used only in original TRANSPORT notation.

Code Digit	Symbol	Keyword	Description

Second-Order Term in the Midplane Field Expansion of Bending Magnets

1.	$\epsilon(1)$	EPS	A second-order measure of magnetic field inhomogeneity in bending magnets. If

$$B(x) = B(0)[1 - n(x/\rho_0) + \beta(x/\rho_0)^2 - \ldots]$$

is the field expansion in the median (y = 0) plane, then $\epsilon(1)$ is defined as

$$\epsilon(1) = \beta(1/\rho_0)^2$$

(where ρ_0 is measured in unit of horizontal beam width – meters in **UMAD**, cm in **UTRANS**–for other units choices see pages 71 and 75). The ϵ parameter may also be specified directly on the bending magnet element. This parameter affects second- and higher-order calculations only. The TRANSPORT default value for $\epsilon(1)$ is 0. It may be varied in second- or third-order fitting.

Particle Mass

3.	(M/m)	PMASS	Mass of the particles comprising the beam, (GeV in UMAD, units of the electron mass in UTRANS; default 0). The selection of still other units is described on page 75. A non-zero mass introduces the dependence of pulse length ℓ on velocity, an important effect in low-energy pulsed beams.

Half Apertures of Bending Magnets

4.	$W/2$	HWIDTH	Horizontal half-aperture of bending magnet, in the same units as horizontal beam width, normally 0 (i.e. effect of horizontal half aperture is ignored). The units of horizontal beam width are meters in UMAD and cm in UTRANS. Other unit sets or units may be selected as described on pages 71 and 75.
5.	$g/2$	HGAP	Vertical half-aperture of bending magnet, in the same units as vertical beam height. *This parameter must be inserted if the effect of the spatial extent of the fringing fields upon transverse focusing is to be taken into account. It is essential for producing an accurate floor layout of the beam line if the quantity $g^2/2\rho$ is a large enough transverse dimension to have a detectable effect on the magnet locations.* Here ρ is the trajectory radius of curvature in the central body of the bending magnet. More details can be found in the section on fringing fields starting on page 139. The HGAP parameter can be specified directly on the ROTATION element indicating a pole-face rotation for a bending magnet or on the RBEND or SBEND elements which specify a bending magnet with fringing fields. This parameter must be given a nonzero value in any third-order calculation. (See the ROTATION and BEND elements as a cross reference) The units for HGAP are determined in the same manner as those for HWIDTH. The default value of the half aperture is 0.

Cumulative Length of the System

6.	L	LENGTH	Cumulative length of system, in the same units as system length (normally meters). Possible units changes are described on page 75. The cumulative length is set to zero initially, then increased by the length of each element, and finally printed at the end of the system. This element allows the cumulative length to be reset as desired.

Fringing-Field Integrals

These three integrals * are denoted by the symbols κ_0, κ_1, and κ_2 for historical reasons. They should not be confused with the symbols which are used in MAD notation to denote respectively the dipole, quadrupole, and sextupole components of the interior field of a bending magnet. They are all dimensionless.

2.	κ_o	FINTO	An integral used to calculate the reference orbit displacement caused by the spatial extent of the fringing field. The default value is 0.
7.	κ_1	FINT	An integral related to the extent of the fringing field of a bending magnet. If the (SPEC 5. g/2. ;) element has been inserted, the program inserts a default value of $\kappa_1 = 1/2$ unless a (SPEC 7. κ_1. ;) element is introduced, in which case the program uses the κ_1 value selected by the user. The parameter FINT may be specified directly on the ROTATION (described on page 139) element indicating a pole-face rotation for a bending magnet or on the RBEND or SBEND elements (described on page 144) which specify a bending magnet with fringing fields. The preferred use of the special parameter element in this case is to specify a different default value at the beginning of the beam line. The table below shows typical values for various types of magnet designs.
8.	κ_2	FINT2	A second integral related to the extent of the fringing field. Default value of $\kappa_2 = 0$ unless specified by a (SPEC 8. κ_2. ;) element. At present, this integral has no effect on any matrix element calculated by TRANSPORT.

Typical values of κ_1 and κ_2 are given below for four types of fringing field boundaries:

a) a linear drop-off of the field,

*See page 142 and SLAC-75 [4] page 74 for further explanation.

b) a clamped "Rogowski" fringing field,
c) an unclamped "Rogowski" fringing field,
d) a "square-edged" nonsaturating magnet.

Model	κ_1	κ_2
Linear	$b/6g$	3.8
Clamped Rogowski	0.4	4.4
Unclamped Rogowski	0.7	4.4
Square-edged Magnet	0.45	2.8

where b is the extent of the linear fringing-field. †

For most applications κ_2 is unimportant. If you find it is important to your result, you should probably be making a more accurate calculation with a differential equation ray-tracing program. A list of such programs is given after the references at the end of this manual.

Reference Trajectory Momentum

11. p_0 `PO` The momentum of the central trajectory. This is the same item as is found on the `BEAM` element (normally GeV/c). It may be redefined at any point in the beam. In this sense it is not really a preliminary specification, but is included here because it is given by a `SPEC` element. The momentum of the beam particles are not changed, but just the momentum of the central trajectory. If, for example, the central momentum is increased, then clearly the beam centroid, as expressed in percent, is shifted downward. This element is useful when a beam is transferred from one optical system to another with a different design momentum.

Pole-Face Curvature

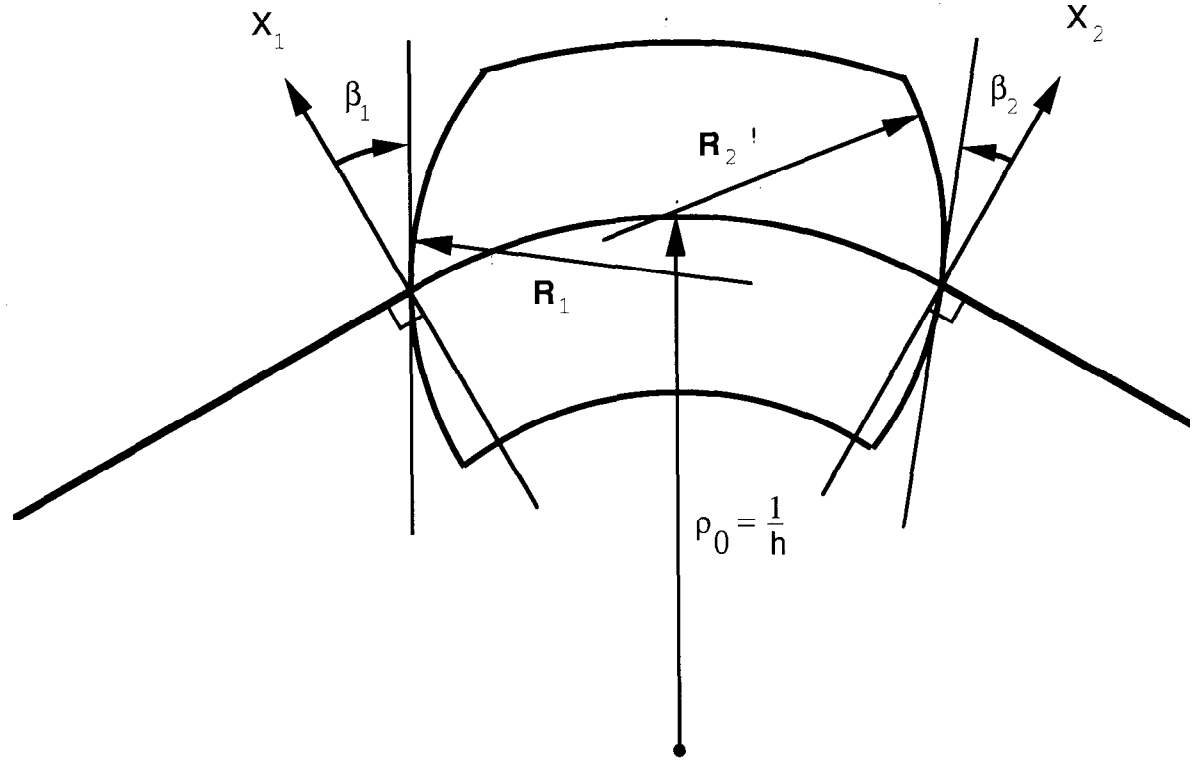
12. $1/R_1$ `H1` Where R_1 is the radius of curvature (in units of longitudinal length, normally meters) of the entrance face of bending magnets. (See the figure on the following page.)

†See page 98 of Steffen's book [9].

13. $1/R_2$ H2 Where R_2 is the radius of curvature of the exit face of bending magnets (in units of longitudinal length, normally meters). (See the figure on the following page.)

The pole face curvatures $(1/R_1)$ and $(1/R_2)$ affect the system only in second or higher order, creating an effective sextupole component in the neighborhood of the magnet. If the parameters are not specified, they are assumed to be zero, i.e. no curvature and hence no sextupole component. The pole-face curvature may also be specified directly on the `ROTATION` element, indicating a pole-face rotation for a bending magnet, or on the `RBEND` or `SBEND` elements, which specify a bending magnet with fringing fields.

Either parameter (or both) may be varied in second- or higher-order fitting.



Field Boundaries for Bending Magnets

The TRANSPORT sign conventions for x, β, R and h are all positive as shown in the figure. The p out of the paper. Positive β's imply transverse focusing. Positive R's (convex curvatures) represent components of strength $S = (-h/2R)\ sec^3\beta$. (See SLAC-75 [4], page 71 or SLAC-PUB-3381 [11], pa

Random Number

14.		RANNO	The input value of the random number used for error calculation. In the IBM versions of TRANSPORT if this element is included but left blank, the input value will be determined by the computer clock. It will also be printed on the element in the initial data listing. Its value will then be available for further runs with the same beam line configuration.

Tilt to Focal Plane

15.	α	FOTILT	The tilt of the focal plane (in degrees).

Very often it is desired to have a listing of the higher-order aberrations along the focal plane of a system rather than perpendicular to the optic axis, i.e. along the x coordinate. If the focal plane makes an angle α with respect to the x axis (measured clockwise) then provision has been made to rotate to this focal plane and print out the higher-order aberrations. This is achieved by the following procedures:

The symbol α represents the focal-plane tilt angle, measured from the perpendicular to the optic axis (α is normally zero).

The programming procedure for a tilt in the x(bend)-plane (rotation about y axis) is:

```
SPEC  15.  α  ;
DRIFT  0.    ;          (a necessary do-nothing element)
PRINT,  TRANS   ;
SPEC  15.  −α  ;        (rotate back to zero)
DRIFT  0.    ;          (a necessary do-nothing element)
SPEC  15.    0.    ;   (to turn off rotation element)
```

The programming procedure for a tilt in the y-plane (rotation about x-axis) is:

```
SPEC  15.  α  ;
SROT  90.    ;
DRIFT  0.    ;
SROT  -90.   ;
PRINT  TRANS   ;
SPEC  15.  −α  ;  (rotate back to zero)
```

```
SROT  90.    ;
DRIFT  0.    ;
SROT  -90.    ;
SPEC  15.    0.    ; (to turn off rotation element)
```

Initial Beam Line Coordinates and Direction

When requesting a beam line coordinate layout via a (PRINT, FLOOR ;) element one can employ any coordinate system one desires. The position and direction of the beginning of the reference trajectory in this coordinate system are given on elements SPEC 16. through SPEC 20. Such cards should be placed before the beam card, but after any units changes. Their meanings are as follows:

16.	x_0	XBEGIN	respectively, the coordinates of the
17.	y_0	YBEGIN	initial point of the reference trajectory in the
18.	z_0	ZBEGIN	units chosen for longitudinal length.
19.	θ_0	YAW	the initial horizontal and vertical angles
20.	ϕ_0	PITCH	of the reference trajectory.

The units of longitudinal length are normally meters. The angular units are radians in UMAD and degrees in UTRANS. Both the length and the angle units may be changed by a UNIT command as described on page 75.

The angle ψ, corresponding to a rotation about the longitudinal axis, may be set by using the SROT element.

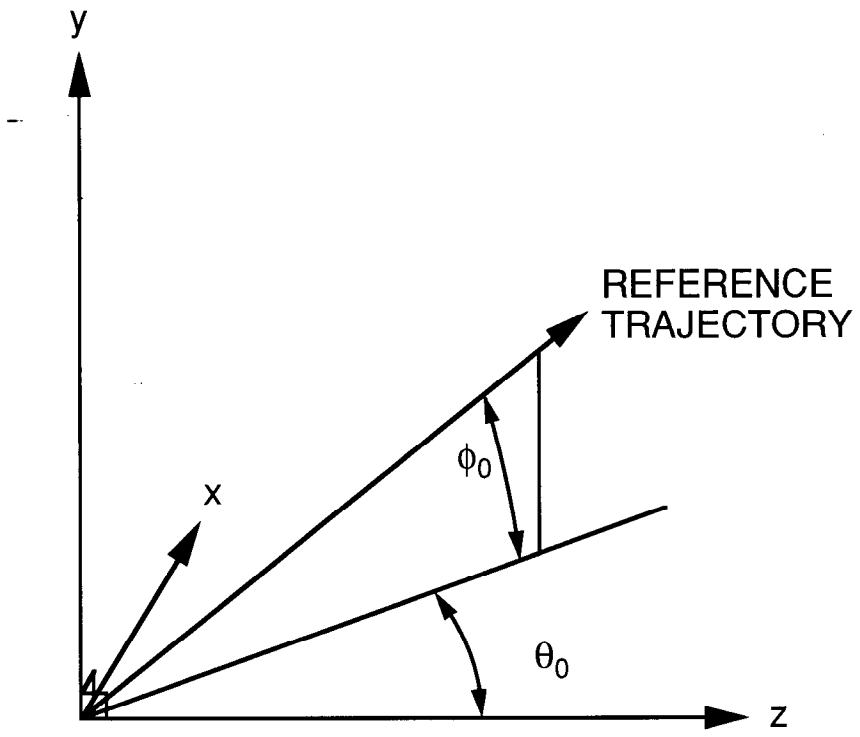
When specifying the initial orientation of the reference trajectory via the two angles θ_0 and ϕ_0, one must give the horizontal angle (θ_0) first. Both angle specifications must be preceded by the (PRINT, FLOOR ;) command. The meaning of the two angles is given in the following figure. Any of the above five parameters not explicitly specified will be taken to equal zero.

The initial coordinates may be varied in first-order fitting. Their values will affect only the beam line floor coordinates and not any beam or transfer matrix element.

A complete initial coordinate specification might then look something like:

```
SPEC,  XBEGIN = 6.325  ;
SPEC,  ZBEGIN = 182.111  ;
SPEC,  YAW = 32.4  ;
SPEC,  PITCH = 15.0  ;
```

Here the angles must be in degrees since 15 radians does not make much sense.



Specification of Initial Angles θ_0 and ϕ_0 for Beam Line Layout.

Reference Momentum for Magnetic Fields

21.	p_{ref}	PREF	The reference momentum (normally GeV/c) to which the magnetic fields are normalized. If this parameter is omitted it is taken to be the central momentum p_0 on the **BEAM** element. If it is included, all subsequent magnetic fields will be scaled by the ratio p_0/p_{ref}. The purpose of this parameter is to allow the user to change the momentum of the beam without retyping the values of all the magnetic fields.

A statement setting the magnetic field reference momentum to 100 GeV might then be:

```
SPEC,  PREF = 100.    ;
```

Magnetic Field Components

The previous description of a magnetic field has been based on the two assumptions of the existence of a reference orbit and ofmidplane symmetry. To allow the representation of field errors and deliberate violations of these assumptions, additional field components are introduced. In terms of them, the field expansion on a reference plane (nominally the magnetic midplane) is given by

$$B_y = B_o(1 + r_s)(1 - nhx + \beta\ h^2x^2 + \gamma\ h^3x^3 + \ldots)$$

$$B_x = B_o r_a(v_r - n'hx + \beta' h^2x^2 + \ldots)$$

The additional parameters required can be specified by the following SPECIAL elements:

22.	r_s	RMPS	The fractional excess bend field on a bending magnet. This excess field will not change the reference trajectory, but simply steer the beam about the reference trajectory.
23.	r_a	RNMS	Overall scaling factor for the midplane-symmetry-violating field components of a bending magnet.
24.	v_r	VR	The vertical (midplane-symmetry-violating) bend field of a bending magnet.
25.	n'	NP	The midplane-symmetry-violating normalized gradient of a bending magnet.

26.	ϵ'	EPSP	The midplane-symmetry-violating sextupole component of a bending magnet. The quantity ϵ' is equal to $\beta' h^2$. The units are the inverse square of the horizontal beam width unit. The horizontal beam width unit is meters with UMAD or cm with UTRANS. Other units selections are described on pages 71 and 75.
27.	ϵ_3	EPS3	Cubic term for the field of a bending magnet. The quantity ϵ_3 is equal to γh^3. The units are the inverse cubed of the horizontal beam width unit.

These parameters all may be specified directly on the BEND element, indicating the central body of a bending magnet, or on the RBEND or SBEND elements, which specify a bending magnet with fringing fields. The preferable use of the special parameter specification is to alter the default values of zero.

ORDER – Higher-Order Calculations

There are no keywords associated with the order specification. The only input is in the original TRANSPORT format.

Original TRANSPORT Notation

Three parameters may be specified:

1. `ORDER` or type code `17.` (signifying a second- or third-order calculation is to be made).
2. The order n_1 of the expansion about the reference trajectory in each element. Default is second order if the `ORDER` element is present.
3. (Optional) The order n_2 of the expansion about the beam centroid that is calculated, accumulated, and displayed. It is necessary that $n_1 \geq n_2$. If unspecified, then n_2 is set equal to n_1.

As an example, a second-order run could be specified by the insertion of the element

```
ORDER  2.    ;
```

near the beginning of the data, after all `UNIT` elements and before the `BEAM` element. An equivalent specification, but one which shows the two parameters explicitly would be

```
ORDER  2.   2.    ;
```

If the beam is always on axis, i.e. if the beam centroid always follows the reference trajectory, it is not meaningful to have separate numbers. In that case, if two numbers are given, the first is reset to equal the second.

If the beam is off axis, then higher-order matrix elements about the beam line axis may contribute to lower-order matrix elements in an expansion about the off-axis centroid position. Some explanation is found later in this section under **Usage**. For more details the user should refer to the explanation of the off-axis expansion of matrix elements on page 13.

The first number n_1 represents the order to which the transfer matrices are calculated about the reference trajectory in each element. If the centroid is off axis, the origin of the expansion can then be shifted from the reference trajectory to the beam centroid. The second number n_2 represents the order to which the transfer matrices are calculated and displayed in the expansion about the beam centroid. Thus if the second-order matrices are calculated about the reference trajectory, but only the first order expansion about the beam centroid is printed in the output, then the `ORDER` specification would be

```
ORDER  2.   1.    ;
```

If the second of the two order parameters is omitted, it is taken as equal to the first. If both are omitted but the ORDER element is included in the data, they are both taken to equal two.

Usage

If no ORDER element is present, then TRANSPORT calculates the transfer matrix only to first order. The ORDER specification allows the order of the calculation to be changed to zeroeth, second, or third. Calculating to zeroeth order simply means tracing the beam centroid through the system. The linearization about the centroid, or transfer matrix, is not calculated.

The tracing of the beam centroid has meaning only if it is not coincident with the reference trajectory. The beam centroid is displaced from the beam line reference trajectory when there is:

1. A centroid displacement (CENTROID)
2. A known misalignment (ALIGN)
3. An excess horizontal bending field (special parameter RMPS)
4. A midplane-symmetry-violating vertical component in a horizontally bending magnet (Special parameters RNMS and VR)
5. A horizontally and/or vertically steering magnet (KICKER or HKICK or VKICK)
6. An electrostatic septum (SEPTUM).
7. A reference coordinate system shift (SHIFT).

If the beam is off axis, the transfer matrix as printed will be expanded about the off-axis beam centroid. The displacement of the beam centroid from the reference trajectory will be printed with the first-order transfer matrix. An additional column will appear to the right of the transfer matrix and will contain the coordinates of the beam centroid. It will appear as follows:

$$\begin{array}{cccccc|c} (x|x_0) & (x|x'_0) & (x|y_0) & (x|y'_0) & (x|\ell) & (x|\delta) & x_c \\ (x'|x_0) & (x'|x'_0) & (x'|y_0) & (x'|y'_0) & (x'|\ell) & (x'|\delta) & x'_c \\ (y|x_0) & (y|x'_0) & (y|y_0) & (y|y'_0) & (y|\ell) & (y|\delta) & y_c \\ (y'|x_0) & (y'|x'_0) & (y'|y_0) & (y'|y'_0) & (y'|\ell) & (y'|\delta) & y'_c \\ (\ell|x_0) & (\ell|x'_0) & (\ell|y_0) & (\ell|y'_0) & (\ell|\ell) & (\ell|\delta) & \ell_c \\ (\delta|x_0) & (\delta|x'_0) & (\delta|y_0) & (\delta|y'_0) & (\delta|\ell) & (\delta|\delta) & \delta_c \end{array}$$

The order of matrix which is calculated may differ from that printed. A second-order transfer matrix can contribute to a first-order off-axis expansion. For example, the off-axis traversing of a quadrupole can give a dipole component and therefore cause dispersion.

Similarly, third-order terms can contribute to both first and second order. The order of the calculation must be at least as great as that displayed. The mathematical details of calculating the off-axis expansion are given on page 13.

An off-axis beam through a multipole element of a given order will induce additional multipole components of lower order, but never of higher order. Thus, the off-axis traversing of a sextupole will induce both dipole and quadrupole components. Similarly, the off-axis traversing of an octupole will induce dipole, quadrupole, and sextupole components.

If the centroid is displaced horizontally in a midplane symmetric (normal) element, then all the lower-order induced multipoles will also be midplane symmetric. If the centroid is displaced vertically, the induced component of order one less than the original multipole will be skew. The induced multipoles will alternate in order with being midplane symmetric and midplane antisymmetric (skew). Thus a vertically displaced centroid in a normal quadrupole will induce a vertically bending (skew) dipole component.

A vertically displaced centroid in a normal sextupole will induce a horizontally bending (normal) dipole component, and a skew quadrupole component. A skew quadrupole is one which has been rotated about its axis by 45 degrees from the normal configuration. A vertically displaced centroid in a normal octupole will induce a vertically bending dipole component, a normal quadrupole component, and a skew sextupole component. A skew sextupole is one which has been rotated about its axis by 30 degrees from the normal configuration.

Skew components result in coupling between the two transverse coordinates. A skew quadrupole results in first order coupling in x and y. A skew sextupole causes coupling but in second and higher orders. A skew octupole results in coupling in third and higher orders.

Examples

Let us examine first the case of an on-axis beam. A single number may then be given. It specifies the order of the calculation. If this number is absent, it is assumed to equal two. The `ORDER` card then reverts to the original TRANSPORT usage of specifying second order. The original TRANSPORT element

```
ORDER  ;
```

is equivalent to

```
ORDER  2.   ;
```

Second-order matrices are included in the program for combined-function bending magnets (including fringing fields), quadrupoles, sextupoles, solenoids, and the arbitrary matrix. They have not been calculated for the acceleration (`ACCEL`) element.

A third-order calculation is specified by setting the number on the `ORDER` element to

three. The command for a third-order run is then

```
ORDER  3.    ;
```

Third-order matrix elements are now included for quadrupoles (including fringing fields), sextupoles, octupoles, and combined function bending magnets (also including fringing fields).

If the beam is off axis, two numbers are required for an order specification. The first gives the order to which the transfer matrix is calculated internally in the program. The second gives the order to which the transfer matrix is printed. When the reference point for the expansion is shifted by having the beam centroid off axis, the higher orders can contribute to the lower orders about the new reference. For example, if in the data we include the element

```
ORDER  3  1  ;
```

then the first order transfer matrix will be calculated about the off-axis reference and printed. The second- and third-order matrices about the original reference will be used in calculating the off axis expansion.

This new feature brings up another possibility. It is now possible to calculate the positions of the beam centroid alone without also calculating a linear expansion about the centroid. One is then calculating the progress of a single trajectory through the system. The transfer matrices may be calculated internally about the reference trajectory, but are not printed in the expansion about the beam centroid.

In other words, if we insert the element

```
ORDER  3.   0.    ;
```

then the progress of the centroid through the beam line is calculated to third order. Ordinarily, when the beam centroid is off axis, the transfer matrix is printed with a column to the right of it giving the coordinates of the beam centroid. If the second of the order parameters is set to zero, then at any location in the output where the transfer matrix would ordinarily be printed, only the coordinates of the centroid are printed, No transfer matrix appears.

The element

```
ORDER  3.   2.    ;
```

will cause the transfer matrix to be printed to second order, but with third-order components used in calculating the off axis expansion.

To calculate the centroid position alone using first-order transfer matrices the command

```
ORDER  1.   0.    ;
```

is used.

Results

To print out the second-order T1 matrix terms at a given location in the system, the (`PRINT TRANS ;`) print control card is used. For T2, the (`PRINT TRAN2 ;`) print control card is used. The update rules are the same as those for the corresponding first-order R matrix.* The third-order matrix elements $U_{ijk\ell}$ are defined similarly, but with an additional index.

The second- and third-order transfer matrices will also affect the beam ellipse. *For the beam matrix to be calculated correctly, there should be no elements, other than the initial beam specification, which update the R2 matrix.* The complete list of elements which update the R2 matrix is:

1. A beam (`BEAM`) element.
2. An R1 update (`UPDATE R1`).
3. An R2 update (`UPDATE R2`).
4. An unknown misalignment element (`ALIGN`).
5. A stray field type code `21.0` entry. (This element is not currently operative. There will be no description of it elsewhere in this manual.)

The dimensions of the initial ellipse are assumed to represent the moments of a Gaussian distribution. †

In calculating the effect of the beam line on the phase ellipse, the sensitivity to assumptions about the initial distribution becomes greater with each additional order. The dimensions of a third-order beam ellipsoid should not be taken as being physically accurate. Rather they should be regarded as an order-of-magnitude estimate of the combined effect of all the aberrations. The dimensions of the beam ellipsoid can be used to assess whether a third-order correction scheme is necessary. Details of the phase-space distribution should be determined with a ray-tracing computer program such as TURTLE (see refs).

*See SLAC-75 [4] for definitions of subscripts in the second order T_{ijk} matrix elements.

†For exact details the reader should consult the Appendix (available separately or as part of CERN Report 80-04)

RANDOM – Random Errors on Physical Parameters

Using a random number generator, random errors may be placed on the physical parameters used to describe a beam line. Any type of physical parameter can be given an error which falls within a specified range. The type of physical parameter and the magnitude of the error are both specified on the RANDOM element. There is only keyword notation for this element.

For example, a random error in quadrupole excitation might be represented by the element

```
RANDOM,  QUAD,  B,  .001  ;
```

Such an element will cause the placement of a random error of maximum magnitude .001 kilogauss on the pole tip field of the quadrupoles in the system. All quadrupoles will be affected, and the random errors will be independent.

The first keyword, in this case QUAD, gives the type of element on which the error occurs. The second gives the physical parameter to which the error is to be applied. The number gives the maximum magnitude of the error. The structure of this element is unique in that a positional parameter follows a keyword.

For an error to be applied to an element, the element specification must be written in terms of the parameters for which an error magnitude is given. If a quadrupole is specified as

```
QUAD,  L = 10.,  GRAD = 5.  ;
```

then the parameter B does not appear. The error specification given above in terms of B then introduces no error to the gradient of this quadrupole. An error on the gradient of a quadrupole where the gradient is given directly, as above, can be specified by

```
RANDOM,  QUAD,  GRAD,  .001  ;
```

At present the list of elements and their parameters to which random errors can be applied is as follows.

Element	Parameter Keywords
ROTAT	ANGLE, HGAP, FINT, H
DRIFT	L
BEND	L, B, RADIUS, ANGLE, N, K1, RMPS, RNMS, VR, NP, K1P, EPS, K2, EPSP, K2P, EPS3, K3
QUAD	L, B, APERTURE, GRADIENT, K1
CENTROID	X, XP, Y, YP, L, DEL
ACCELERA	L, VOLT, LAG, FREQ

SEXTUPOL	L, B, APERTURE, K2
SOLENOID	L, B, KS
SROT	ANGLE
OCTUPOLE	L, B, APERTURE, K3
RBEND, SBEND	L, B, RADIUS, ANGLE, N, K1, RMPS, RNMS, VR, NP, K1P, E1, E2, EPS, K2, EPSP, K2P, H1, H2, EPS3, K3, HGAP, FINT
PLASMA	L, B, APERTURE, GRADIENT, K1

If any physical parameter is to receive a random error, then random numbers for all physical parameters are generated. If a second type of error is introduced, the random numbers used for the first type will then be unchanged. This procedure produces the type of results that a user would be likely to expect. If two types of errors are present, and the magnitude of the second is reduced to zero, the result should be the same as if only the first type of error were present. An example might be an investigation of the errors of focusing elements in a system. If both quadrupoles and bending magnets are to be considered then the two error specifications

```
RANDOM,  QUAD,  B,  .001  ;
RANDOM,  BEND,  N,  .0005  ;
```

could be used.

The procedure described has the disadvantage that the sequence of random numbers used for any given type of error will be changed if the beam line is changed in any way. This will be true even if the change is to an element for which no random errors are specified. An example might be the splitting up of a drift space to make two shorter drift spaces. If this is done, the beam line is considered changed. The error specification given above, where a random error is given to the magnetic fields of the quadrupoles in the system will then give a different result. The correspondence of random numbers to magnetic field values will be altered due to the generation of an additional random number for the additional drift space. It will also be possible that different versions of TRANSPORT will give different results if additional physical parameters can be given random errors. The special parameters element does not use random numbers, since it simply gives default values for the parameters of other elements.

In most cases the mathematical procedure of giving a certain error to a physical parameter is the same as the physical procedure. By the mathematical procedure, we mean simply changing the physical parameter and considering the beam line to be whatever results. By the physical procedure, we mean imposing certain constraints whereby the change in one parameter will cause a change in others resulting in a more physically realistic situation.

There are two cases where the two procedures are significantly different. The first is in the imposition of an error on the length of a magnet. An error may be specified as

```
RANDOM,  BEND,  L,  0.02 ;
```

The mathematical error is the procedure currently in TRANSPORT, Here the length of a beam line will increase when the length of magnet is increased. The physical error will correspondingly shorten the adjacent drifts so that the length of the total beam line is unchanged. It is planned to change TRANSPORT in the future so that the physical error is calculated.

The second exception is the case of an error on the field of a bending magnet. An error may be specified by

```
RANDOM,  BEND,  B,  0.005  ;
```

The angle of bend is calculated from the magnet length and field and the reference momentum. If the field is changed, the remainder of the beam line is moved. However, in this case, there is provision for introducing an excess field without altering the remainder of the beam line. The excess field is specified using the keyword **RMPS** either in the **BEND** element or with a special parameter. The parameter **RMPS** is the fractional error in the magnetic field. It may be considered as a mispowering of the magnet without affecting the beam layout. An error on this parameter may then by given by

```
RANDOM,  BEND,  RMPS,  0.0001 ;
```

The initial value of the random number generator may be reset using the special parameter element **SPECIAL** with the keyword **RANNO**. If the appropriate **SPECIAL** element is included in the data, but the space for the initial value of the random number generator is left blank, then the initial value will be reset by the computer clock. Detailed description is given under the **SPECIAL** element.

LIMIT – Limits on the Range of Varied Physical Parameters

Limits can be placed on the range over which a physical parameter may vary in a fitting procedure. The type of physical parameter, whether the limit is lower or upper, and the numerical value of the limit are all specified on the LIMIT element. Only keyword notation is available for this element.

For example, a limit on the strength of the pole-tip magnetic field of a quadrupole might be represented by the element

```
LIMIT,  QUAD,  B,  UPPER,  15.0  ;
```

An upper limit of 15 kG is then placed on the pole-tip magnetic field of all quadrupoles whose fields are varied. It should be noted that this limit does not apply to the magnitude of the field. A negative field can still attain an arbitrarily large magnitude. To bound fields both negatively and positively, both lower and upper limit specifications are required.

The first keyword, in this case QUAD, gives the type of element on which the limited parameter occurs. The second gives the physical parameter to which the limit is to be applied. The third entry is either of the two words LOWER or UPPER. The number gives the value of the limit. The structure of this element is unique in that a positional parameter follows a keyword.

For a limit to be applied to an element, the element specification must be written in terms of the parameter for which the limit is given. If a quadruopole is specified as

```
QUAD,  L = 10.,  B = 12.,  APER = 5.0  ;
```

and the parameter B is varied, the limit specified above will apply. However, if a quadrupole is specified as

```
QUAD,  L = 10.,  GRAD = 5.   ;
```

then the parameter B does not appear, and a limit given in terms of B will have no relevance. The limit will also not apply if the physical parameter is evaluated in terms of other parameters which, in turn, are varied.

At present the list of elements and their parameters to which limits can be applied in the fitting process is as follows.

Element	Parameter Keywords
BEAM	all parameters
ROTAT	ANGLE
DRIFT	L

BEND	L, B, RADIUS, ANGLE, N, K1
QUAD	L, B, GRADIENT, K1
CORRELATION	all parameters
SEXTUPOLE	L, B, K2
SOLENOID	L, B, KS
SROT	ANGLE
OCTUPOLE	L, B, K3
RBEND, SBEND	L, B, RADIUS, ANGLE, N, K1, E1, E2
PLASMA	L, B, APERTURE, GRADIENT, K1
HKICK, VKICK	L, B, ANGLE

If no limitations are specified on the ranges of any parameters, then TRANSPORT will impose limits only on a very small number of possibly varied parameters. All parameters on the BEAM element are required to be positive. There is no internally specified upper limit. Any upper limit placed on a BEAM parameter will apply to all parameters on the BEAM element. Separate limits may not be placed on the different beam dimensions at this time.

The pole-face-rotation angle on the ROTATE element is internally constrained to have a magnitude no greater than 60 degrees. This is simply a practical consideration of magnet geometry. This internal limit may be overridden by the LIMIT element. The user is reminded that upper and lower limits must be specified separately.

The lengths of all physical magnets and drift spaces are internally constrained never to become negative. A fixed length may be negative, but a negative length can never result from a fitting procedure. Elements to which the negative length restriction applies are the DRIFT, BEND, QUAD, SOLENOID, OCTUPOLE, RBEND, SBEND, PLASMALENS, HKICK, VKICK, and KICKER. In the case of a drift space one way around this obstacle is to place a fixed drift of negative length next to a varied drift of positive length.

The zero minimum length on any physical element may also be overridden by the LIMIT element. Thus the fitting procedure can be allowed to produce negative lengths by the insertion of the element:

```
LIMIT,  DRIFT,  L,  LOWER,  -1000.0 ;
```

The lower limit to the lengths of any varied drift now becomes -1000 of the prevailing length unit.

The coordinate rotation angle on the SROT element is constrained to lie between -360 and 360 degrees. Here nothing is gained physically by expanding the range.

MAGNET – External Magnet Dimensions

The external dimensions of a magnet may be specified for plotting purposes. The external dimensions are used in a floor layout plot where either the floor x (horizontal transverse) coordinate or floor y (vertical transverse) coordinate is plotted against the floor z (longitudinal) coordinate. If both magnet width and magnet height are specified the magnet is drawn into the plot. If only one or neither transverse magnet dimension is specified, then only the reference trajectory through the magnet is drawn into the plot. The dimensions specified on a magnet element apply to a particular TYPE of magnet. By TYPE we mean the designation specified under the keyword TYPE on the description of the individual magnet, be it RBEND, SBEND, QUAD, or other magnet.

There is only keyword notation for this element.

Keyword Notation

The MAGNET element is specified by three keywords.

Keyword	Description
WIDTH	The magnet width (normally meters, default: 0 m)
HEIGHT	The magnet height (normally meters, default: 0 m)
TYPE	The magnet TYPE to which these dimensions pertain

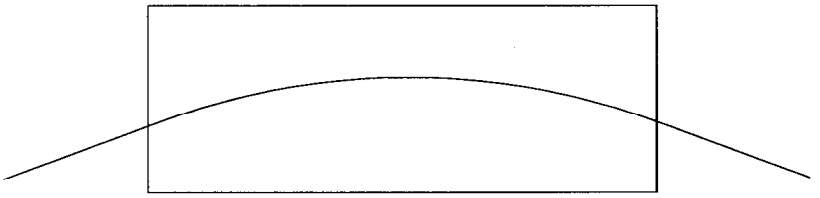
The external dimensions of any magnets of type "JUMBO" may be declared by the following MAGNET element.

```
MAGNET, WIDTH = 2.0, HEIGHT = 1.5, TYPE = JUMBO ;
```

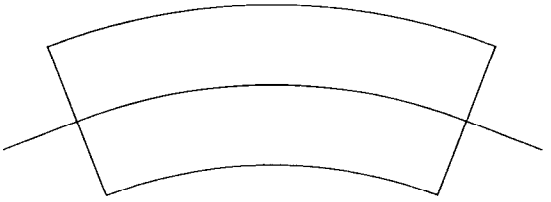
The magnet dimensions will be used to draw the magnet according to the magnet type as specified on the element, RBEND, SBEND, QUAD, or other magnet. For example, a set of RBEND magnets may all be declared to be of the general type "JUMBO". Several such magnets may appear in a beam line.

```
B1:   RBEND,  L = 10.,  ANGLE = 0.010,  TYPE = JUMBO ;
DRIFT,  L = 1.0 ;
B2:   RBEND,  L = 10.,  ANGLE = 0.010,  TYPE = JUMBO ;
DRIFT,  L = 1.0 ;
B3:   RBEND,  L = 10.,  ANGLE = 0.010,  TYPE = JUMBO ;
DRIFT,  L = 1.0 ;
B4:   RBEND,  L = 10.,  ANGLE = 0.010,  TYPE = JUMBO ;
```

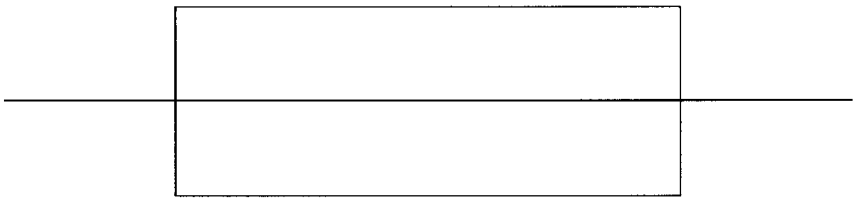
An **RBEND** element as seen from the top will appear as follows:



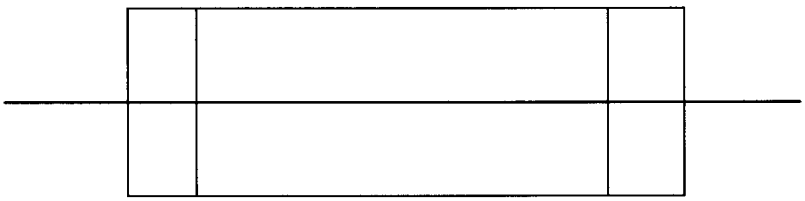
An **SBEND** element from the top will appear as follows:



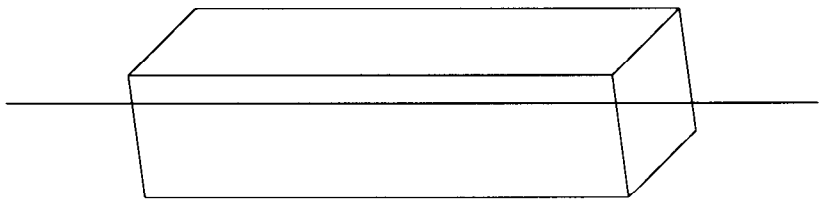
A QUAD or other rectilinear magnetic element will appear as follows:



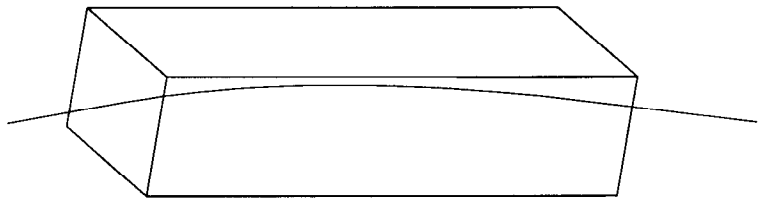
The RBEND, QUAD, and other rectilinear magnetic elements are represented by rectangular solids. They then appear as rectangles when seen from the side. The SBEND is curved to follow the reference trajectory. If the observer is located inside the curvature, the two ends of the magnet may be visible simultaneously. The SBEND element seen from the side will then appear as follows:



A roll (SROT element or a TILT) in the beam line or an initial slope or azimuthal angle may cause more than one surface of a magnet to be visible from either the side or the top. A QUAD or other rectilinear element as seen from an angle might appear as:



An RBEND as seen from an angle might appear as:



Description of the Beam

BEAM — Input Beam

The phase space and the reference momentum of the input beam for a TRANSPORT calculation are specified by this element. The input is given in terms of the semi-axes of a six-dimensional erect* beam ellipsoid. The coordinates in the six-dimensional space represent the phase space variables x, x', y, y', ℓ, and δ. Each of these six parameters is entered as a positive quantity, but should be thought of as $\pm x$, $\pm x'$, etc.; hence, the total beam width is $2x$, the total horizontal beam divergence is $2x'$ and so forth.

In keyword notation the **BEAM** element might look like:

```
BEAM,  X=.1,  XP=1.,  Y=.1,  YP=1.,  L=0.,  DEL=0.2,  PO = 100.   ;
```

In original TRANSPORT notation, the same **BEAM** element would be expressed as:

```
BEAM  0.1  1.   0.1  1.0  0.   0.2  100.   '   '  ;
```

Usually the **BEAM** card is the fourth card in the deck. It will be immediately preceded by the title card, the indicator card, and the unit set specification. If any individual units changes are to be made, the units elements (**UNIT**) must also precede the **BEAM** celement

If a beam line coordinate layout is desired, the element specifying that a layout is to be made (a **PRINT, FLOOR ;**) element, and any initial coordinates (see page 87) all precede the **BEAM** element. Any **ORDER**, **LIMIT**, **RANDOM** or **MAGNET** elements also precede the **BEAM** element.

Strictly speaking x' and y' are not angles and are therefore not measurable in either milliradians or radians. They are the tangents of the horizontal and vertical angles θ and ϕ between the given trajectory and the reference trajectory. However, up to and including second order, they are equal to the angles themselves.

In third order, the tangent of an angle is different from the angle. However, the principal effects in third order arise from aberrations which are nonlinear terms in the differential equations of motion. The difference between an angle and its tangent as a measure of beam divergence is still inconsequential. Milliradians as a measure of x' and y' may still be interpretated as parts in a thousand.

The input phase space may alternatively be specified in accelerator notation using the Courant-Snyder parameters α, β, and ϵ. Here the parameters α and β are used only to describe an ellipse. They have no necessary relationship to the transfer matrix of either the entire optical system or to any portion thereof. This means that the beam may or may not

*For a rotated (non-erect) phase ellipsoid input, see the **CORR** element description on page 116.

be matched to the lattice through which it passes. The accelerator function η is specified on the separate **ETA** element described on page 121.

In strict MAD notation a beam element is unnecessary. The magnetic elements in MAD notation are specified in a manner that incorporates the momentum. However the beam element must be present if any of the calculations involves the beam phase space.

For any other than strict MAD notation, a beam element is required. The beam element may be expressed in either keyword or original TRANSPORT notation

Keyword Notation

The mnemonic used in specifying the input phase space and reference momentum is **BEAM**. The input phase space may be given by specifying its half widths or in accelerator notation. The two may not be mixed in the input. The units associated with the entries depend on the unit set specified. If the input beam is specified in terms of its phase space half-widths, then the keywords which can be used are:

Symbol	Keyword	Description
x	X	One-half the horizontal beam extent (meters with **UMAD**, centimeters with **UTRANS**).
x'	XP	One-half the horizontal beam divergence (radians with **UMAD**, mr with **UTRANS**).
y	Y	One-half the vertical beam extent (meters or cm).
y'	YP	One-half the vertical beam divergence (radians or mr).
ℓ	L	One-half the longitudinal beam extent (meters or cm).
δ	DEL	One-half the momentum spread (fraction with **UMAD**, percent with **UTRANS**).
p_o	PO	The momentum of the central trajectory (GeV/c).

All quantities not specified on the **BEAM** element will be taken to be zero. The same set of keywords may be used for an rms addition to the beam phase space. Global unit set specifications other than **UMAD** or **UTRANS** may also be selected and are described on page 71. Individual units changes are described on page 75.

If the input beam is specified in terms of the Courant-Snyder parameters α and β, then the keywords which can be used are:

Symbol	Keyword	Description
β_x	BETAX	The accelerator function β in the horizontal plane (meters with **UM**
α_x	ALPHAX	The accelerator function α in the horizontal plane (unitless).
ϵ_x	EPSX	The horizontal emittance (meters with **UMAD**).

β_y	BETAY	The accelerator function β in the vertical plane (meters with UMAD)
α_y	ALPHAY	The accelerator function α in the vertical plane (unitless).
ϵ_y	EPSY	The vertical emittance (meters with UMAD).
p_o	PO	The momentum of the central trajectory (normally GeV/c).

The default value for the β and α parameters in both planes is zero. The default value for the ϵ parameters in both planes is 1.0. Here the units are given for only the global unit set UMAD. It is imperative that the unit set used with accelerator notation be such that the ratio of the transverse linear unit to the transverse angular unit be meters. The global unit set UTRANS does not satisfy this requirement and should not be used with accelerator notation. The global unit sets UMAD, UMETER, UMM, and UMICR are acceptable in this respect. The units for the horizontal and vertical emittance are the transverse linear unit times the transverse angular unit. Thus in UMM the units will be mm-mrad, while in UMICR they will be micron-microradians. The various options for global unit set specifications are describe on page 71. Individual units changes are described on page 75.

If accelerator notation is used, the BEAM element might appear as:

```
BEAM,  BETAX=1.0,  ALPHAX=.15,  BETAY=2.0,  ALPHAY=-.20,  PO = 100.    ;
```

Specification of the emittances also gives a beam element which might be:

```
BEAM,BETAX=1.0,ALPHAX=.15,EPSX=16.,BETAY=2.0,ALPHAY=-.2,EPSY=3., PO=100.;
```

In both notations, with the exception of one special case, the central momentum must also be specified. If all the magnetic elements are described using MAD parameters, then the central momentum need not be specified. The MAD parameters use length and angle for bending and steering magnets, and K_1, K_2, and K_3 for the strength per unit length of the quadrupole, sextupole, and octupole components.

Original TRANSPORT Notation

There are eight entries (all positive) to be made on the BEAM element. The units associated with the entries depend on the unit set specified.

1. BEAM (or type code 1.0) (specifies a BEAM entry follows).
2. One-half the horizontal beam extent (x) (meters with UMAD, cm in UTRANS).
3. One-half the horizontal beam divergence (x') (radians with UMAD, mr with UTRANS).

4. One-half the vertical beam extent (y) (meters or cm).

5. One-half the vertical beam divergence (y') (radians or mr).

6. One-half the longitudinal beam extent (ℓ) (meters or cm).

7. One-half the momentum spread (δ) (fractional with **UMAD**, percent with **UTRANS** $\Delta p/p$).

8. The momentum of the central trajectory [p(0)] (GeV/c).

All eight entries must be made even if they are zero (0). As for all other elements, the last entry must be followed by a semicolon, dollar sign, or asterisk. Global unit set specifications other than **UMAD** or **UTRANS** may also be selected and are described on page 71. Individual units changes are described on page 75.

Thus a typical **BEAM** entry might be

```
                                          ↓—— Label (if desired)
BEAM  0.5  2.   1.3  2.5  0.   1.5  10.   '   '  ;
```

meaning, $x = \pm 0.5$ cm, $x' = \pm$ 2.0 mr, $y = \pm$ 1.3 cm, $y' = \pm$ 2.5 mr, $\ell = \pm$ 0.0 cm, $\delta = \pm$ 1.5 percent $\Delta p/p$, and the central momentum $p(0) = 10.0$ GeV/c.

If the **BEAM** element is used to specify the Courant-Snyder parameters, then entries 2 through 7 acquire a different meaning.

1. **BEAM** (or type code `1.0`) (specifies a **BEAM** entry follows).

2. The horizontal beta value β_x (meters).

3. The horizontal alpha value α_x.

4. The vertical beta value β_y (meters).

5. The vertical alpha value α_y.

6. A dummy entry 0.0.

7. A dummy entry 0.0.

8. The momentum of the central trajectory [p_0] (GeV/c).

If the **BEAM** element is used to specify the Courant-Snyder parameters, then the command

```
PRINT,  ACCEL  ;
```

must appear in the data before the **BEAM** element. Original TRANSPORT does not allow the

possibility of specifying the emittances ϵ_x and ϵ_y in the input. The default values of 1.0 are then assumed.

Normally, the units of the horizontal and vertical beta functions are in meters. The accelerator function α is unitless. In TRANSPORT, β is expressed as the ratio of the transverse distance unit over the transverse angular unit. If the transverse distance is meters and the transverse angle is in radians, the the β functions will be in meters. TRANSPORT requires an explicit units specification for transverse units of meters and radians. Either of the global units specifications **UMAD** or **UMETER** will produce transverse units of meters and radians. The unit sets **UMM** and **UMICR** do not produce transverse units of meters and radians, but will produce β functions in meters.

The transverse units resulting from the specification **UTRANS** are cm and mr. These units would therefore interpret the beta functions as being expressed in cm/mr or dekameters. The desired units of meters will be produced for the beta functions if the transverse units are mm and mr.

Output

The units of the tabulated matrix elements in either the first-order R or sigma matrix, second-order $\bar{T}$ matrix, or third-order U matrix of a TRANSPORT print-out will correspond to the units chosen for the **BEAM** element. For the above example, the $R_{12} = (x|x')$ matrix element will have the dimensions of cm/mr; and the $T_{236} = (x'|y\delta)$ matrix element will have the dimensions mr/(cm · percent) and so forth.

The longitudinal extent ℓ is useful for pulsed beams. *It indicates the spread in length of particles in a pulse.* For a static magnetic system it does not affect any other component. Only for the accelerator element does the longitudinal position affect the transverse dimensions of the beam. It may be set to zero if the pulse length is not important.

The beam phase-space can be printed at any later location in the beam line. The beam matrix in terms of half widths and correlations may be printed at a single location by the element

```
PRINT,  BEAM  ;
```

The phase ellipse (sigma matrix) beam parameters may be printed as output after every physical element if activated by a

```
PRINT,  BEAM,  ON  ;
```

element. The maximum extent of the ellipsoid in each of the six coordinates axes is printed in a vertical array. The correlations among these components indicating the phase ellipse orientations are printed in a triangular array (see the following pages).

The beam phase space may be printed in terms of the parameters β and α by use of the

command

```
PRINT,  TWISS  ;
```

The printing of these parameters after every physical element is activated by the element

```
PRINT,  TWISS,  ON  ;
```

Either or both types of representation of the beam phase space (half widths or β and α) may be used in the same run. The choice of representation for the printed phase space parameters can be made independently of the choice for the input. In other words, the beam phase space can be input in terms of β, α, and ϵ and then be printed in terms of half widths and correlations.

The Phase Ellipse Beam Matrix

The beam matrix σ carried in the computer has the following construction:

	x	x'	y	y'	ℓ	δ
x	σ_{11}					
x'	σ_{21}	σ_{22}				
y	σ_{31}	σ_{32}	σ_{33}			
y'	σ_{41}	σ_{42}	σ_{43}	σ_{44}		
ℓ	σ_{51}	σ_{52}	σ_{53}	σ_{54}	σ_{55}	
δ	σ_{61}	σ_{62}	σ_{63}	σ_{64}	σ_{65}	σ_{66}

The matrix is symmetric so that only a triangle of elements is needed. In the printed output this matrix has a somewhat different format for ease of interpretation:

			x	x'	y	y'	ℓ
x	$\sqrt{\sigma_{11}}$	CM					
x'	$\sqrt{\sigma_{22}}$	MR	r_{21}				
y	$\sqrt{\sigma_{33}}$	CM	r_{31}	r_{32}			
y'	$\sqrt{\sigma_{44}}$	MR	r_{41}	r_{42}	r_{43}		
ℓ	$\sqrt{\sigma_{55}}$	CM	r_{51}	r_{52}	r_{53}	r_{54}	
δ	$\sqrt{\sigma_{66}}$	PC	r_{61}	r_{62}	r_{63}	r_{64}	r_{65}

where

$$r_{ij} = \frac{\sigma_{ij}}{[\sigma_{ii}\ \sigma_{jj}]^{1/2}} \ .$$

The quantities r_{ij} are known in statistics as the correlations between the ith and jth coor-

dinates. The units shown above are from the global unit set UTRANS. Those actually shown will be from whatever units are selected.

As a result of the fact that the σ matrix is positive definite, the r_{ij} satisfy the relation

$$|r_{ij}| \leq 1 \quad .$$

The full significance of the σ_{ij} and the r_{ij} is discussed in detail in the appendix ("Description of Beam Matrix"). The units are always printed with the matrix.

In brief, the meaning of the $\sqrt{\sigma_{ii}}$ is as follows:

$\sqrt{\sigma_{11}}$	$= x_{max}$	= the maximum (half)-width of the beam envelope in the x(bend)-plane at the point of the print-out.
$\sqrt{\sigma_{22}}$	$= x'_{max}$	= the maximum (half)-angular divergence of the beam envelope in the x(bend)-plane.
$\sqrt{\sigma_{33}}$	$= y_{max}$	= the maximum (half)-height of the beam envelope.
$\sqrt{\sigma_{44}}$	$= y'_{max}$	= the maximum (half)-angular divergence of the beam envelope in the y(non-bend)-plane.
$\sqrt{\sigma_{55}}$	$= \ell_{max}$	= one-half the longitudinal extent of the bunch of particles.
$\sqrt{\sigma_{66}}$	$= \delta_{max}$	= the half-width $(\Delta p/p_o)$ of the momentum interval being transmitted by the system.

The units appearing next to the $\sqrt{\sigma_{ii}}$ in the TRANSPORT print-out are the units chosen for coordinates x, x', y, y', ℓ and $\delta = \Delta p/p$, respectively.

Beam Center of Gravity

To the immediate left of the listing of the beam envelope size in a TRANSPORT print-out, there appears a column of numbers whose values will normally be zero. They are here indicated with a subscript "g".

			x	x'	y	y'	ℓ
x_g	$\sqrt{\sigma_{11}}$	CM					
x'_g	$\sqrt{\sigma_{22}}$	MR	r_{21}				
y_g	$\sqrt{\sigma_{33}}$	CM	r_{31}	r_{32}			
y'_g	$\sqrt{\sigma_{44}}$	MR	r_{41}	r_{42}	r_{43}		
ℓ_g	$\sqrt{\sigma_{55}}$	CM	r_{51}	r_{52}	r_{53}	r_{54}	
δ_g	$\sqrt{\sigma_{66}}$	PC	r_{61}	r_{62}	r_{63}	r_{64}	r_{65}

These numbers are the coordinates of the center of gravity of the beam phase ellipse (with respect to the initially assumed reference trajectory of the system). They may become nonzero under any of five circumstances:

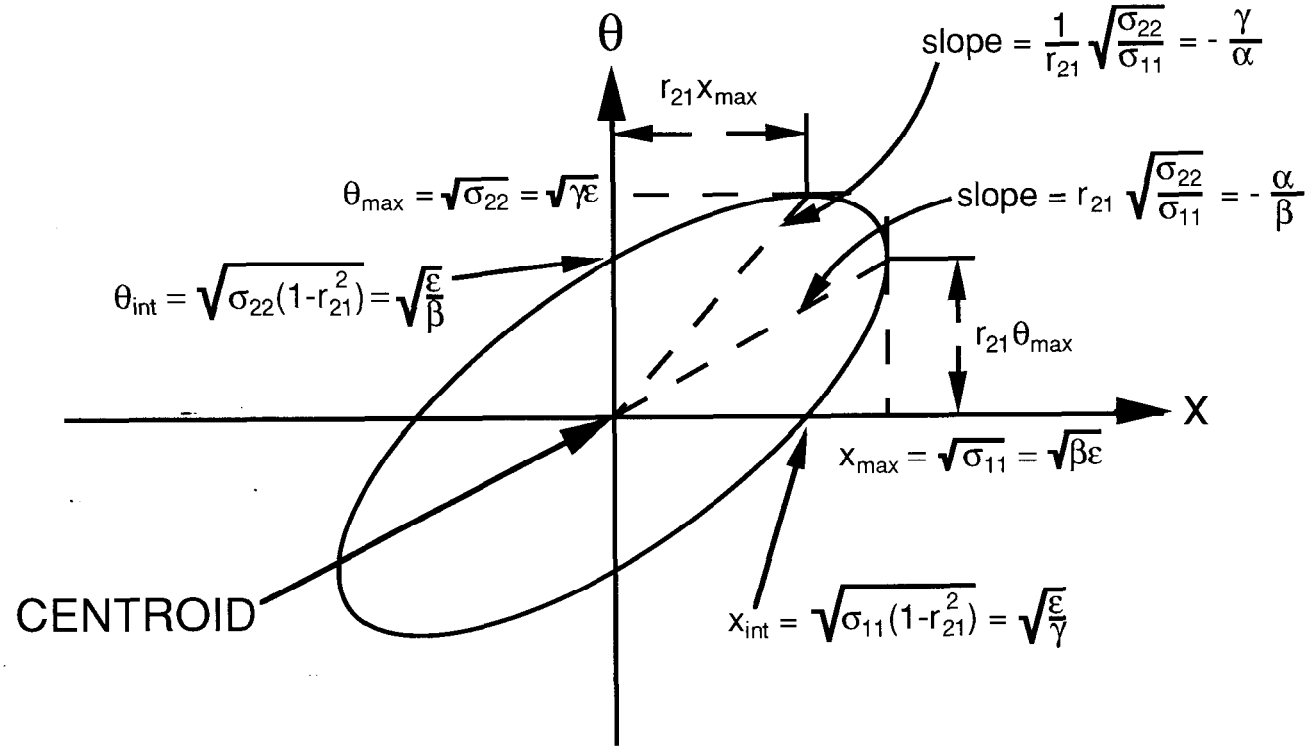
1. when a steering magnet is used (`KICKER`, `HKICK`, or `VKICK`),
2. when a bending magnet (`RBEND`, `SBEND`, or `BEND`) is mistuned,
3. when the misalignment (`ALIGN`) is used,
4. when a beam centroid shift (`CENTROID`) or coordinate shift (`SHIFT`) is used, or
5. when a second- or higher-order calculation (`ORDER`) is used.

Great care must be taken in the interpretation of these numbers in a second- or higher-order run with nonzero initial phase space. As the beam envelope progresses through the beam line, it may become distorted by the higher-order terms and may no longer be an ellipsoid. The number printed is the center of gravity of the envelope and may no longer coincide with the image of the original centroid, transformed from the beginning of the beam line. The numbers printed with the transfer matrix are the coordinates of the image of the original centroid. They appear only when the centroid deviates from the reference trajectory and are given the subscript "c" in the following diagram.

$$\begin{array}{cccccccc} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} & & x_c \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} & & x'_c \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & R_{36} & & y_c \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} & & y'_c \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} & & \ell_c \\ R_{61} & R_{62} & R_{63} & R_{64} & R_{65} & R_{66} & & \delta_c \end{array}$$

Example

To aid in the interpretation of the phase ellipse parameters listed above, an example of an (x, x') plane ellipse is illustrated below. [†]



A Two-Dimensional Beam Phase Ellipse

The area of the ellipse is given by:

$$A = \pi(\det\,\sigma)^{1/2} = \pi x_{max}\; x'_{int} = \pi x_{int}\; x'_{max} = \pi\epsilon \quad .$$

The equation of the ellipse is:

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = \epsilon$$

where

$$\sigma = \begin{bmatrix} \sigma_{11} & \sigma_{21} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} = \epsilon \begin{bmatrix} \beta & -\alpha \\ \alpha & \gamma \end{bmatrix}$$

and

$$\beta\gamma - \alpha^2 = 1 \quad , \quad r_{21} = r_{12} = -\frac{\alpha}{\sqrt{1+\alpha^2}} = -\frac{\alpha}{\sqrt{\beta\gamma}} \quad .$$

[†]For further details the reader should refer to the appendix of this report or SLAC-PUB-3381 [11].

R.M.S. Addition to the Beam

To allow for physical phenomena such as multiple scattering and synchrotron radiation emittance growth, provision has been made in the program to permit an r.m.s. addition to the beam envelope.

The r.m.s. addtion to the beam may be expressed in either keyword or original TRANSPORT notation

Keyword Notation

The mnemonic used in specifying the r.m.s. addition to the beam is **BEAM**. The r.m.s. addition to the beam is specified in terms of its phase space half-widths. Then the keywords which can be used are:

Symbol	Keyword	Description
x	X	One-half the horizontal beam extent (meters in **UMAD**, cm with **UTRANS**).
x'	XP	One-half the horizontal beam divergence (radians with **UMAD**, mr with **UTRANS**).
y	Y	One-half the vertical beam extent (meters or cm).
y'	YP	One-half the vertical beam divergence (radians or mr).
ℓ	L	One-half the longitudinal beam extent (meters or cm).
δ	DEL	One-half the momentum spread (fractional with **UMAD**, percent with **UTRANS**).
p_o	PO	The momentum change $[\Delta p(0)]$ of the central trajectory (GeV/c).
	RMS	Indicates an r.m.s. addition to the beam.

The **BEAM** element might then look like:

```
BEAM,  X=0.,  XP=.2,  Y=0.,  YP=.2,  L=0.,  DEL=0.2,  PO = -0.1,  RMS  ;
```

All quantities not specified on the **BEAM** element will be taken to be zero. Global unit set specifications other than **UMAD** or **UTRANS** may also be selected and are described on page 71. Individual units changes are described on page 75.

Original TRANSPORT Notation

There are nine entries to be included:

1. BEAM or type code 1. (specifying a **BEAM** entry follows).

2. The r.m.s. addition to the horizontal beam extent (Δx) (meters with **UMAD**, cm with **UTRANS**).

3. The r.m.s. addition to the horizontal beam divergence ($\Delta x'$) (radians with **UMAD**, mr with **UTRANS**).

4. The r.m.s. addition to the vertical beam extent (Δy) (meters or cm).

5. The r.m.s. addition to the vertical beam divergence ($\Delta y'$) (radians or mr).

6. The r.m.s. longitudinal beam extent ($\Delta \ell$) (meters or cm).

7. The r.m.s. momentum spread ($\Delta \delta$) (fractional in **UMAD**, percent with **UTRANS**).

8. The momentum change in the central trajectory [$\Delta p(0)$] in (GeV/c).

9. The code digit `0.` indicating an r.m.s. addition to the beam is being made.

The units for the r.m.s. addition are the same as those selected for a regular **BEAM** element. The unit sets mentioned above are **UMAD** and **UTRANS**. Other unit sets or individual units may also be selected as explained on pages 71 and 75.

A typical r.m.s. addition to the **BEAM** in original TRANSPORT notation would appear as follows:

```
BEAM  0.   .2  0.   .2  0.   0.2  -0.1  0.   ;
```

where the last entry (`0.`) preceding the semicolon signifies an r.m.s. addition to the **BEAM** is being made and the next to the last entry indicates a central momentum change of -0.1 GeV/c.

CORR(ELATION) — Beam (Rotated Ellipse)

To allow the output beam from some point in a system to become the input beam of some succeeding system, provision has been made for re-entering the correlation matrix which appears as a triangular matrix in the beam output. (See page 110 for definitions.)

CORR is not a MAD element. It can be expressed in either keyword or original TRANSPORT notation.

Keyword Notation

The keywords which can be used in specifying the beam matrix correlations are

C21, C31, C32, C41, C42, C43, C51, C52, C53, C54, C61, C62, C63, C64, C65.

The keywords Cij correspond in an obvious manner to the 15 correlations r_{ij} among the 6 beam components. The value assigned to Cij is just r_{ij}. The correlations are all dimensionless.

In many cases most of the correlations are zero, and it has always been a nuisance to have to type all those zeros. The use of keywords eliminates this annoyance. Now only the nonzero correlations need be typed. It is often the case that the only nonzero correlations are r_{21} and r_{43}. To specify the values of these correlations, the CORR element might look like

```
CORR,  C21 = 0.6,  C43 = -0.8  ;
```

Original TRANSPORT Notation

There are 16 parameters:

1 -	CORR(ELATION) (or type code 12.0)
2 to 16 -	The 15 correlations [r(ij)] among the 6 beam components – in the order printed (by rows).

The order printed is

$r_{21}, r_{31}, r_{32}, r_{41}, r_{42}, r_{43}, r_{51}, r_{52}, r_{53}, r_{54}, r_{61}, r_{62}, r_{63}, r_{64}, r_{65}$.

The correlations are all dimensionless. In original notation, the same example as above would be printed as:

```
CORR  0.6  0.0  0.0  0.0  0.0  -0.8  ;
```

A user who has some sentimental attachment to the old format for the beam correlation format, or who has mostly nonzero entries, will have to make a slight revision in his data.

The correlation element is the item most often requiring several lines. In previous versions of TRANSPORT an element could be continued simply by not placing a semicolon (;) at the end of a line. For compatability with the MAD program an implicit semicolon is now assumed to be in column 81 at the end of each line. A line must now be explicitly marked to be continued. This is done by placing an ampersand (&) after the data on a given line.

Usage

Since this element is solely an extension of the beam input, a **CORR** element must immediately be preceded by a **BEAM** element.

The effect of this element in the printed output is shown only in the beam matrix. If the beam matrix is printed automatically, it is not printed directly after the **BEAM** element but only after the correlation matrix has been inserted.

CENTROID — Shift in the Beam Centroid

The initial beam centroid is the geometric center of the initial beam ellipsoid. The beam centroid is then an individual trajectory whose progress can be followed through the beam line. In first order, the beam ellipsoid retains an ellipsoidal shape when transformed to a downstream location in the beam line. The centroid then continues to be the geometric center of the beam ellipsoid.

In higher orders the transformed beam ellipsoid becomes distorted and may no longer retain an ellipsoidal shape. The beam centroid is still defined as the transform of the center of the initial beam ellipsoid. Because of the higher-order distortions, the beam centroid may no longer represent the mean position of the particles in the transformed ellipsoid. For that purpose, the beam centroid is replaced by the beam center of gravity, described on page 56.

In TRANSPORT the beam centroid is used as a reference point for the Taylor series expansion of the mapping from the beginning of the beam line to any later point. The coefficients of the Taylor series expansion are known as the transfer matrices. If the beam centroid coincides with the reference trajectory of the optical system, then the expansion about the reference trajectory and that about the beam centroid are the same.

Sometimes it is useful to redefine the beam centroid such that it does not coincide with the TRANSPORT reference trajectory. Provision has been made for this possibility via the `CENTROID` element.

`CENTROID` is not a MAD element, but it is a TRANSPORT element. The `CENTROID` element may be expressed in either keyword or original TRANSPORT notation.

Keyword Notation

The mnemonic used in specifying a shift in the beam centroid is `CENTROID`. The keywords which can be used are

Symbol	Keyword	Description
x	`X`	The shift in the x coordinate
x'	`XP`	The shift in x'
y	`Y`	The shift in y
y'	`YP`	The shift in y'
ℓ	`L`	The shift in ℓ.
δ	`DEL`	The shift in δ.

The six parameters correspond exactly with the six positional parameters used in the original style TRANSPORT input. The units are the same as that used for the beam ellipse. (See page 105.) In the `UMAD` unit set, the units are meters, radians, and fractional $\Delta p/p$. In the `UTRANS` set, they are cm, mr, and percent. Other global unit sets are described on page 71.

An example of an increase in y' by 1.0 milliradians might be

```
CENTROID,  YP = 1.0  ;
```

Original TRANSPORT Notation

Seven parameters are required:

1	CENTROID (or type code 7.0)
(2 to 7)	The coordinates x, x', y, y', ℓ, and δ defining the shift in the location of the beam centroid with respect to its previous position.

The units for x, x', y, y', ℓ, δ are the same as those chosen for the BEAM element. With the UMAD unit set these are meters, radians, and fractional $\Delta p/p$. With UTRANS they are cm, mr, cm, mr, cm, and percent. Other global unit sets are described on page 71.

Another input representing the same vertical displacement of the beam as above might be

```
CENTROID  0.0  0.0  0.0  1.0  0.0  0.0  ;
```

Any or all of the six beam centroid shift parameters may be varied in first-order fitting. The centroid position may then be constrained at any later point in the beam line by this procedure.

Usage

The presence of a centroid shift element in the data will automatically cause the transfer matrix to be expanded about the newly defined beam centroid. Both the order used in calculating the transfer matrix, and the order to which it is expanded about the beam centroid are specified by the ORDER element.

The representation of the transfer matrices with respect to the beam centroid then involves a transformation of coordinates from the representation with respect to the reference trajectory. Mathematical details can be found on page 13. A discussion of how higher orders in the expansion about the reference trajectory affect lower orders in the expansion about the beam centroid can be found on page 91 in the section on the ORDER element.

The CENTROID element is intended as a means of directing a particle beam into an optical system in a manner not coincident with the design axis of that optical system. It would occur normally only at the beginning of a beam line. However, the CENTROID element has frequently been used to simulate other phenomena. Such uses include the effect of misalignments or steering magnets. For such purposes, it would be better to use directly either the misalignment element (ALIGN), or one of the steering magnet elements (HKICK, VKICK), or KICKER.

The CENTROID element does not affect the printed floor coordinates. The printed floor coordinates represent the reference trajectory, which is unaffected by the CENTROID element.

The centroid shift is considered as being done by an unspecified cause within a fixed optical system. A shift in the centroid coordinates can also occur without a physical centroid deflection. This latter coordinate shift occurs when there is a change in the coordinate system to which the centroid is referred. A change in coordinate systems often occurs when a particle beam is transferred from one optical system to another. Examples are the transference of a beam from an accelerator lattice to a transfer line or from a transfer line to an accelerator lattice. The `SHIFT` element should be used for such purposes. The `SHIFT` element does affect the floor coordinates.

ETA – Accelerator η Function

The accelerator function η is a new element and is specified in a manner similar to the CENTROID element. In some previous versions of TRANSPORT the CENTROID element was taken to be the η function. However this identification precluded expressing the accelerator functions β and α about an off-axis orbit. The centroid and the η function are now separate.

ETA is not a MAD element and cannot be expressed in MAD notation. It can be expressed in either keyword or original TRANSPORT notation.

Keyword Notation

The mnemonic for the accelerator η function is ETA. The keywords which can be used in specifying the accelerator η function are

Symbol	Keyword	Description
η_x	ETAX	The x component of the accelerator η function.
η'_x	DETAX	The derivative of the x component of η.
η_y	ETAY	The y component of the accelerator η function.
η'_y	DETAY	The derivative of the y component of η.
η_ℓ	ETAL	The longitudinal displacement of η.
η_δ	ETAP	The fractional momentum deviation for η (normally 1.0).

Units are normally the same as those for the beam phase space (see page 105). With the unit set UMAD, the units are meters, radians, and fractional $\Delta p/p$. The other unit sets (see page 75) UMETER, UMM, UMICR may also be used. (See page 71.) The use of UTRANS is strongly discouraged as it will require that η_x and η_y be in dekameters.

An η function which has 0.01 units of horizontal displacement and no horizontal slope or nonzero vertical coordinates might be represented as

```
ETA,  ETAX = 0.01  ;
```

Any undefined components will be set to zero except for the δ component, where the default value is equal to one of whatever unit is being used for δ.

Original TRANSPORT Notation

Seven entries are required:

1 ETA

(2 to 7) The coordinates x, x', y, y', ℓ, and δ defining the coordinates of the η function. The units for x, x', y, y', ℓ, δ are the same as those chosen for the **BEAM** element). For the **UMAD** set of units this is meters, milliradians, and fractional $\Delta p/p$. The use of the **UTRANS** set of units is strongly discouraged as it will require η_x and η_y to be in dekameters.

An η function which has 0.01 units of horizontal displacement and no horizontal slope or nonzero vertical coordinates might be represented as

```
ETA  0.01  ;
```

The x', y, y', and ℓ components of η, not being specified, will be set to zero. The value of δ for the η function will be set to one of whatever unit is being used for δ.

Any or all of the six beam η function coordinates may be varied in first-order fitting. The η function may then be constrained at any later point in the beam line.

Usage

The printing of the beam matrix in terms of the parameters β and α after each element is effected by the command

```
PRINT,  TWISS,  ON  ;
```

A single line will be printed which will also contain the four transverse components of the η function. The β, α, and η functions will be labelled in the output so there will be no difficulty in identifying them.

The accelerator parameters are typically expressed in meters and radians. To ensure that the η function is expressed in such units, appropriate units changes should be made at the beginning of the data. The single commands **UMAD** or **UMETER** will change all units to those used most often for accelerator physics. These single commands are described on page 71.

The global unit sets **UMM** and **UMICR** will have respective defaults for δ of one part in a thousand and one part in a million. However, the first four components of η will still be in meters and radians, as these two measures are respectively equal to millimeters per part-in-a-thousand and microns per part-in-a-million.