LEPTONIC AND SEMILEPTONIC D–DECAYS FROM QCD SUM RULES

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Leptonic and semileptonic decay amplitudes play a key role for a determination of the KM quark mixing matrix as well as for a test of a wide variety of theoretical predictions on heavy flavour weak transitions. Thus considerable effort has been devoted to the theoretical estimates of the leptonic constant f_D (and f_B) and of the Dl_3 (and Bl_3) decay form factors. Here we will briefly present some results on these transitions, which have been obtained in the framework of QCD sum rules [1, 2].

To just outline the main points of this formalism, in the version of finite energy sum rules (FESR) particularly convenient to the issue here, we recall that the starting objects are the current correlators

$$\Pi(Q^2) = i \int d^4x \, e^{iqx} \langle vac| \, TJ(x)J(o)^{+} |vac\rangle, \tag{1}$$

where the J's are local currents, built out of quark fields, which interpolate the hadron one is interested in. These Green functions can be computed at short distance perturbatively in QCD, and extrapolated to longer distances, closer to the hadronic size, by adding power corrections parametrized by a set of non-perturbative quark and gluon operator vacuum matrix elements. The interface of this $\Pi|_{QCD}$ to hadrons is made by connecting it to the dispersive representation, allowed by analyticity of $\Pi(Q^2)$:

$$\Pi(Q^2) = \frac{1}{\pi} \int \frac{ds}{s+Q^2} \, \mathrm{Im}\Pi(s) + \text{"subtractions"}, \tag{2}$$

where the spectral function $\mathrm{Im}\Pi(s)$ contains physical hadron masses and coupling constants. Actually in the present case of heavy quarks, with $m_{c,b} \gg \Lambda_{QCD}$, it is convenient to consider "moments" of (2) of the form (n = 1,2 ...):

$$\phi_n = \frac{(-1)^n}{n!} \left(\frac{d}{dQ^2}\right)^n \Pi(Q^2)\Big|_{Q^2=O} = \frac{1}{\pi} \int \frac{ds}{s^{n+1}} \mathrm{Im}\Pi(s). \tag{3}$$

Corrispondingly $\phi_n|_{QCD}$ takes the form of an expansion in inverse powers of the heavy quark mass:

$$\Phi_n\big|_{QCD} = \Phi_n\big|_{AF} + \sum_{\kappa} C_n^k \frac{\langle vac|O_k|vac\rangle}{m_{c,b}^k} \qquad (4)$$

In eq. (4) the $C_n{}^k$ are numerical coefficients, which similarly to $\phi_n|_{AF}$ can be computed in QCD perturbation theory. The O_k are quark and gluon operators, ordered for increasing dimensionality. The lowest ones are:

$d = 3 : \langle vac|\bar{q}q|vac\rangle$	quark condensate
$d = 4 : \langle vac|\alpha_s G_{\mu\nu}^2|vac\rangle$	gluon condensate
$d = 5 : \langle vac|g_s\bar{q}\,\sigma_{\mu\nu}q\,G_{\mu\nu}|vac\rangle$	quark-gluon condensate,

etc.

The last ingredient is a phenomenological parametrization of the spectral function $\mathrm{Im}\Pi(s)$ in eqs. (2), (3). A reasonable one, which has been widely adopted in the applications of this method, is

$$\mathrm{Im}\Pi(s) = \mathrm{Im}\Pi(s)\big|_H + \theta\,(s-s_o)\,\mathrm{Im}\Pi(s)\big|_{AF}, \qquad (5)$$

where $\mathrm{Im}\Pi(s)|_H$ represents the contribution of the lowest lying hadronic states H, and $\mathrm{Im}\Pi(s)|_{AF}$ is the asymptotic freedom expression, calculable in perturbative QCD, which is supposed to start at some $s_o \geq M^2{}_H$.

Collecting eqs. (3), (4) and (5) one finally arrives at the QCD FESR of the form:

$$\frac{1}{\pi}\int \frac{ds}{s^{n+1}} \mathrm{Im}\Pi(s)\big|_H = \frac{1}{\pi}\int^{s_o} \frac{ds}{s^{n+1}} \mathrm{Im}\Pi(s)\big|_{AF} + \sum_k C_n^k \frac{\langle vac|O_k|vac\rangle}{m_{c,b}^k} \qquad (6)$$

The vacuum condensates in eq. (6), which are there to allow the extrapolation to longer distances anticipated above, are not calculable in perturbative QCD, as they are genuinely nonperturbative. In practical applications their values can be inferred phenomenologically from a few cases where the corresponding LHS of (6) is best known experimentally, and then be used to make predictions for any other channel one is interested in. Implicit is the assumption (which on the other hand can be checked phenomenologically) that the expansion in powers of $1/m_{c,b}$ converges in such a way that the sum in (6) can be truncated to the first few terms. This makes the scheme economical, as depending on a limited number of QCD parameters, and predictive at the same time. Thus, in the present application to D (and B) mesons we must consider the RHS of (6) as known.

One can notice in eq. (6) that the $1/s^{n+1}$ integration emphasizes the lowest lying hadronic state for increasing n. On the other hand one finds that n can be increased only at the expense of increasing the power corrections on the RHS of (6), where only the first few terms are known. Thus in practice one does a compromise, and considers eq. (6) for only the first few values of n.

Finally, there is the dependence of eq. (6) on the threshold s_o for the onset of the asymptotic freedom QCD regime, whose value is not a priori known. Clearly the physical hadronic masses and coupling constants should not depend on s_o. Thus in practical applications on has to optimize the FESR, by looking for a "duality window" where the predicted LHS of (6) is stable against changes of s_o.

Having stated the rules of the game, we turn to the D-decays.

a) For the determination of f_D we have to choose in eq. (1):

$$J(x) \equiv \partial_\mu A_\mu (x) = (m_c+m_d) : \bar{c}(x)\, i\, \gamma_5\, d(x) : \tag{7}$$

This is connected to the D→lν coupling constant f_D by the matrix element:

$$\langle vac | J(o) | D \rangle = \sqrt{2}\, M_D^2 f_D\,, \tag{8}$$

so that in the LHS of eq. (6):

$$\frac{1}{\pi} \operatorname{Im} \Pi(s)|_{H=D} = 2\, M_D^4 f_D^2\, \delta(s - M_D^2) \tag{9}$$

(in this normalization $f_\pi = 93$ MeV).

The optimization procedure of (6) consists in this case in looking for a range where the predicted $M_D|_{th} \cong M_D|_{expt.}$ and is stable in s_o. We find for this duality window the rather wide range $s_o = (2 - 3)\, M^2_D$. In this range we then solve for f_D, which of course must also be stable there.

The same procedure can be applied after trivial modifications to the D_s and to the B. The results we find can be summarized as follows [3]:

$$\frac{f_D}{f_\pi} = 1.7 \pm 0.2\,;\ \frac{f_{Ds}}{f_D} \cong 1.2\,;\ \frac{f_{Bd}}{f_\pi} = 1.3 \pm 0.2 \tag{10}$$

where the ± gauges the stability of the predictions in the duality window. To compare to lattice calculations [4,5], f_D/f_π as in (10) looks compatible, within uncertainties, with the upper side of the range of values found in that framework. The lattice seems to indicate f_B/f_π close to (or less than) unity. Finally the two approaches agree on the SU(3) violating f_{Ds}/f_D as in eq. (10).

To give an impression on the sensitivities required to measure leptonic decays according to these estimates, we report in Tab. 1 the branching ratios expected for different possible values of f_D/f_π.

Tab. 1: Leptronic Branching Ratios

f_D/f_π	$D\to\mu\nu$	$D\to\tau\nu$	$D_s\to\mu\nu$	$D_s\to\tau\nu$	f_B/f_π	$B\to\mu\nu$	$B\to\tau\nu$
1.2	$2.7\ 10^{-4}$	$6.2\ 10^{-4}$	$2.7\ 10^{-3}$	$2.6\ 10^{-2}$	0.9	$9.0\ 10^{-8}$	$2.0\ 10^{-5}$
1.5	$4.2\ 10^{-4}$	$9.7\ 10^{-4}$	$4.2\ 10^{-3}$	$4.1\ 10^{-2}$	1.2	$1.6\ 10^{-7}$	$3.6\ 10^{-5}$
1.8	$6.1\ 10^{-4}$	$1.3\ 10^{-3}$	$6.1\ 10^{-3}$	$5.9\ 10^{-2}$	1.5	$2.5\ 10^{-7}$	$5.6\ 10^{-5}$

The values in Tab. 1 are obtained by using m_{ν_τ}=o; f_{Ds}/f_D=1.2; V_{cd}=0.24; V_{cs}=0.97; tentatively V_{bu}/V_{bc}=0.07 with V_{bc}=0.045.

From Tab. 1 we see that leptonic branching ratios are well within the reach of a charm factory, so that the important parameter f_D could be measured.

b) Turning to the semileptonic decays $D\to\pi l\nu$ and $D\to Kl\nu$, the current of interest in eq. (1) is:

$$J_\mu(x) \equiv V_\mu(x) = :\bar{d}(x)\,\gamma_\mu\, c(x): \text{(and} :\bar{s}(x)\,\gamma_\mu\, c(x):\text{)} \tag{11}$$

Semileptonic transition amplitudes are determined by hadronic matrix elements of the form:

$$<\pi(p')|V_\mu|D(p\,) = (p+p')_\mu f_+^\pi(t) + (p-p')_\mu\, f_-^\pi(t) \tag{12}$$

and similarly for $D\to K$, with $t=(p-p')^2$. In practice only f_+ (t) matters, as the contribution of f_- to the rate is depressed by the small lepton mass.

The popular parametrization for f_+ (t) is the vector dominance form:

$$f_+(t) = \frac{f_+(o)}{1 - t/M_{D*}^2} \tag{13}$$

and $D^*\to D^*_s$ for the $D\to Kl\nu$ transition. Correspondingly the semileptonic rate can be expressed as:

$$\Gamma(D\to Xl\nu) = \frac{G_F^2}{192\pi^3}\,|\,Vij\,|^2\,|\,f_+(o)\,|^2\,\frac{1}{M_D^3}\, I_{PS}\,, \tag{14}$$

where X=π, K, Vij are the appropriate KM matrix elements and I_{PS} is a phase space integral.

Very little theoretical information on $f_+(o)$ is available a priori. We only know that $f_+(o)=1$ in the SU(4) limit, and that on general grounds $f_+(o)<1$. In this case the notion of symmetry is not so helpful, as we expect SU(4) breaking to be appreciable. This situation is quite different from the analogous Kl_3 decay, where SU(3) breaking makes the corresponding $f_+(o)$ to deviate from unity by some percent at most.

The application of the FESR eq. (6) to the present case, with V_μ eq. (11) into eq. (1), is completely analogous to the previous one, except for a few technical details and for the hadronic spectral function, which now reads:

$$\mathrm{Im}\,\Pi(s)\,\Big|_H = \frac{\eta^2}{48\pi}\,|f_+(s_+)|^2\left[\left(1-\frac{s_+}{s}\right)\left(1-\frac{s_-}{s}\right)\right]^{3/2}\mathrm{BW}\,. \qquad (15)$$

In (15) η is a Clebsh - Gordon coefficient, BW is a Breit - Wigner form accounting for the D^* resonance, and $s_\pm = (M_D \pm m_\pi)^2$. Also the optimisation procedure, and the resulting duality window in s_0 are the same as before. We finally find for $f_+(o)$ the result [6]:

$$f_+(o) = 0.75 \pm 0.05 \qquad (16)$$

where, analogously to eq. (10), the $\pm$ gauges the stability of the predicted value. We may compare eq. (16) to lattice calculations [5,7] : $f_+(o) = 0.70 \pm 0.20$ ($D\to\pi$) and $f_+(o) = 0.74 \pm 0.17$ ($D\to K$); and to constituent quark models : $f_+(o) = 0.75 - 0.82$ [8] and $f_+(o) \cong 0.58$ [9].

Combining eq. (16) with (14) and comparing to the semileptonic experimental rates [10], we would find $|V_{cs}| \cong 0.93 \pm 0.12$, consistent with $|V_{cs}| \cong 0.97$ as derived from the unitarity of the KM matrix.

The extension of this formalism to the decays $D\to K^*l\nu$ and $D\to\rho l\nu$ seems feasible. However in this case there are three form factors, and presumably such a more complicated situation could be dealt with only at the cost of an increased model dependence.

Clearly, from the numbers exposed above, the high statistics available at the charm factory would be crucial for a severe test of model calculations of form factors and for increased sensitivity to the KM matrix elements.

As a conclusion, there is room for decisive improvements at a facility such as the charm factory. The leptonic decay constants would be measured with significant accuracy, and this information would be really welcome, as it enters as a crucial parameter in many theoretical calculation of heavy meson decays. Refined knowledge would be allowed on the Dl_3 form factors, in particular on the t-dependence, ultimately reflecting itself into $f_+(o)$ and thus into the rate normalization. In this regard very important would also be the improved measurements of the D^+ and D° lifetimes, leading to accurate branching determinations.

On the theoretical side there should be a corresponding improvement of the present situation, by reducing uncertainties and hopefully bringing the various calculations of f_D and of $f_+(t)$ to a closer agreement. This is necessary in particular for the model independent determination of the KM matrix elements V_{cd} and V_{cs} from the data. Such a program is rather challenging, but we can be optimistic, and believe that substantial progress will be achieved in the future. Actually a simple point, just to start from, should be the consideration that from common experience calculations should be more reliable for ratios of matrix elements than for the matrix elements themselves, because a number of uncertainties should cancel in that way. This is seen for example in the pattern of the SU(3) violating ratios f_{Ds}/f_D and f_{Bs}/f_{Bd}. Thus, why not pursuing ratios such as

$$\frac{D\to\mu\nu}{D_s\to\mu\nu} \to \left|\frac{V_{cd}}{V_{cs}}\right| \frac{f_D}{f_{D_s}}. \tag{17}$$

$$\frac{D\to\pi l\nu}{D\to K l\nu} \to \left|\frac{V_{cd}}{V_{cs}}\right| \frac{f_+^{\pi}(o)}{f_+^{K}(o)} \tag{19}$$

This could represent the first step on the way to model independence [11].

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