V_{cd} AND V_{cs} FROM CURRENT MODELS*

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ABSTRACT

We discuss the theoretical uncertainties attendant to the determination of $|V_{cd}|/|V_{cs}|$ from semileptonic D decay. Four different theoretical approaches are considered. Currently, agreement at the level of a factor of 2 for the various theoretical approaches exists so that, indeed, precise experimental data—such as precise lepon energy spectra and decay width data—would discriminate among the models considered.

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1. INTRODUCTION

The fundamental parameters of the standard $SU_{2L} \times U_1$ theory involve the famous C–K–M mixing matrix V_{UD}. It enters into the interactions of the $SU_{2L} \times U_1$ theory via the interaction Lagrangian as $L_{int} = :\ldots - (g/(2\sqrt{2}))\ W_\mu^+ \overline{\psi}_U \gamma^\mu (1-\gamma_5)\ V_{UD}\ \psi_D\ + \text{h.c.}\ \ldots,$ where h.c. denotes hermitian conjugation and g is the fundamental SU_{2L} coupling. Here ψ_A is used to represent the quantum field for particle A and the labels U and D take their customary values $U = u, c, t, \ldots$, and $U = d,\ s,\ b,\ \ldots$. It is immediately apparent that V_{UD} is as important as the fundamental SU_{2L} coupling g itself. Hence, it is quite important to determine V_{UD} as precisely as observation will allow.

From this perspective, we see that the semileptonic decays of the heavy mesons, such as $D_d^{\pm}$, $D_s^{\pm}$, ..., have a chance to play a central role due to their large rest mass compared to Λ_{QCD}, the QCD scale parameter. Indeed, it is hoped[1] that QCD will be somewhat perturbative in this regime so that the spectator transitions illustrated in Fig. 1 may already be reliable approximations to the physical $D \to X + l\overline{\nu}_l$ processes. Figure 1 reveals that the rates for $D_s^+ \to X + \overline{l}\nu_l$ and $D_d^- \to X + l\overline{\nu}_l$ are respectively proportional to $|V_{cs}|^2$ and $|V_{cd}|^2$ for a fixed X such as $X = K^0$ so that from the ratio of rates, $\Gamma(D_s^+ \to X + \overline{l}\nu_l)\ /\ \Gamma(D_d^- \to X + l\overline{\nu}_l)$—we have an observable handle on the ratio $r_{cd/cs} \equiv |V_{cd}|^2/|V_{cs}|^2$.

In practice, there may be a number of uncertainties in the comparison between the observation and the respective theoretical prediction; i.e., the theoretical models vary in their respective predictions. It is this kind of uncertainty that is discussed here.

Our presentation is organized as follows: in Sec. 2, four models of the theoretical prediction for $R_f \equiv \Gamma(D_s \to K l\overline{\nu}_l)\ /\ \Gamma(D_d \to K l\overline{\nu}_l)$ are considered, where it is hoped that the final states are essentially identical. In Sec. 3, the ratio $R_i \equiv \Gamma(D_d \to \pi \overline{l}\nu_l)\ /\ \Gamma(D_d \to K\overline{l}\nu_l)$ is considered, in which the initial states are identical, again in the context of our four models. From the variations found in Secs. 2 and 3, we obtain an assessment of the attendant theoretical uncertainty in relating $r_{cd/cs}$ to observation in semileptonic D decay. Section 4 contains some summary remarks.

2. $\Gamma(D_s \to K l \overline{\nu}_l)$ / $\Gamma(D_d \to K + l\overline{\nu}_l)$ APPROACH

Here we consider a ratio of rates in which the "same" final state particles are produced from different initial states in semileptonic D decay. Of course, since $m_{D_s} > m_{D_d}$, the final states in the two decays do not probe the same area of phase space. Also, since the QCD coupling constant varies with the respective squared 4-momentum transfers at the gluon vertices of interaction, the QCD corrections to the two decay processes in R_f may in fact be different. Accordingly, we consider four theoretical models of R_f:

1. the QCD sum rule approach of Dominguez and Paver[2];
2. the relativistic harmonic oscillator model of Bauer, Stech and Wirbel[3] (B–S–W);
3. the mock meson model of Isgur et al., (I–S–G–W)[4]; and
4. the method of Lepage and Brodsky.[5,6]

We begin with the method of Dominguez and Paver (D–P). Specifically, if we neglect m_l in comparison to the typical momentum transfers to the lepton pair in Fig. 1, we find,[2] in the simple pole approximation for the familiar form factor $f_+(t)$,

$$\Gamma(D_d \to K l \overline{\nu}_l) \;=\; \frac{G_F^2 |V_{cs}|^2}{192\pi^3}\; |f_+(0)|^2\; \frac{m_{D_s^*}^4}{m_{D_d}^3}\; I_{ps} \quad , \tag{1}$$

where $t = (P_D - P_K)^2$ and

$$I_{PS} = \int_o^{t_-} \frac{dt[(t-t_+)(t-t_-)]^{3/2}}{(t-m_{D_s^*}^2)^2} \quad . \tag{2}$$

Here $m_{D_s^*} \simeq 2.11$ GeV and G_F is Fermi's constant. Using the QCD sum rule method, Dominguez and Paver find $f_+(o) \simeq 0.75$. Hence, repeating the analysis in formula (1) for $D_d \to K\bar{l}\nu_l$, with $m_{D_d^*} \simeq 2.01$ GeV, one gets

$$\Gamma(D_s \to K\bar{l}\nu_l) \;/\; \Gamma(D_d \to K\bar{l}\nu_l) = r_{cd/cs}(1.51) \quad . \tag{3}$$

This then may be taken as our point of reference.

Considering next the B–S–W approach, one has the same formula (1) but one uses a relativistic harmonic oscillator model for the value of $f_+(0)$, presuming a Gaussian distribution in transverse momentum that is scaled by w via $exp(-P_T^2/w^2)$ with 0.35 GeV $\lesssim w \lesssim$ 0.5 GeV. In this way, the B–S–W model gives $f_+^{D_d}(0) = (0.938 - 0.968) f_+^{D_s}(0)$ ($f_+^{D_d}(0) =$ 0.773 for $w = 0.35$ GeV), and the B–S–W approach yields

$$\Gamma(D_s \to K\bar{l}\nu_l) \;/\; \Gamma(D_d \to K\bar{l}\nu_l) = r_{cd/cs}(1.51)[1.067 - 1.136] \quad . \tag{4}$$

Hence, the B–S–W approach is quite close to the D–P approach.

Turning now to the "mock meson" method of Isgur et al., we note that, in this case, one uses a nonrelativistic Cornell[7]-type potential model in Schroedinger's equation to compute the respective meson wavefunctions. The constants $f_+^{D_s}(0)$, $f_+^{D_d}(0)$ can then be identified with the appropriate nonrelativistic wavefunction overlaps using the "mock meson" method,[4] which has given reasonable results in other contexts. In this way, the I–S–G–W approach gives

$$f_+^{D_s}(0) \simeq 0.601 f_+^{D_d}(0) \quad , \tag{5}$$

so that

$$\Gamma(D_s \to K\nu_l) \;/\; \Gamma(D_d \to K\bar{l}\nu_l) \simeq r_{cd/cs}(1.51)(0.361) \quad . \tag{6}$$

Finally, we consider the methods of Lepage and Brodsky as we have applied them in Ref. 6 to semileptonic D decay. We note that this approach is manifestly Lorentz invariant and includes the perturbative QCD corrections to the spectator graph in Fig. 1. Here we find, by effecting the attendant calculations, as described in Ref. 6,

$$\Gamma(D_s \to K\bar{l}\nu_l) \;/\; \Gamma(D_d \to K\bar{l}\nu_l) \simeq r_{cd/cs}(1.51)(0.655) \quad . \tag{7}$$

We conclude that there is a factor of ~ 2 variation in these models for R_f. What is the attendant variation in R_i? To this we now turn.

3. $\Gamma(D_d \to \pi \bar{l} \nu_l)$ / $\Gamma(D_d \to K \bar{l} \nu_l)$ APPROACH

In this section we analyze the semileptonic D decays of D_d to $\pi \bar{l} \nu_l$ and $K \bar{l} \nu_l$, with the objective of assessing the theoretical uncertainty in the attendant ratio of rates. Here the initial states are indeed identical. The final states are manifestly different. We consider the four approaches in turn.

Specifically, the D–P approach gives

$$R_i \equiv \Gamma(D_d \to \pi \bar{l} \nu_l) \;/\; \Gamma(D_d \to K \bar{l} \nu_l) = \frac{1}{2}\, r_{cd/cs}\, (2.00) \quad . \tag{8}$$

Again, we may use this as a reference point.

The B–S–W approach gives

$$R_i = \frac{1}{2}\, r_{cd/cs}\, (2.00)\; (0.822 - 0.952) \tag{9}$$

for 0.35 GeV $\lesssim\; w \;\lesssim$ 0.5 GeV; the I–S–G–W approach gives

$$R_i = \frac{1}{2}\, r_{cd/cs}\, (2.00)\; (0.432) \quad ; \tag{10}$$

using the methods of Lepage and Brodsky, we find

$$R_i = \frac{1}{2}\, r_{cd/cs}\, (2.00)\; (1.81) \quad . \tag{11}$$

Again, the D–P and B–S–W approaches are quite close. The other two approaches vary by a factor of ~ 2 from the D–P reference point—in different directions, however.

4. CONCLUSIONS

We see that the theoretical predictions for the ratio of rates in $D \to PS l \overline{\nu}_l$, where $PS \equiv$ pseudoscalar meson, vary by a factor of ~ 2, both for identical initial states and for identical particle content in the final state. Thus, precise measurements of the respective ratios, and attendant decay spectra, could discriminate among these models. In principle, this would allow us to determine $|V_{cd}|/|V_{cs}|$ and would tell us something about QCD. We understand[8] that such precise measurements would be possible with a Tau–Charm–Factory-type experimental scenario. We then await such observations.

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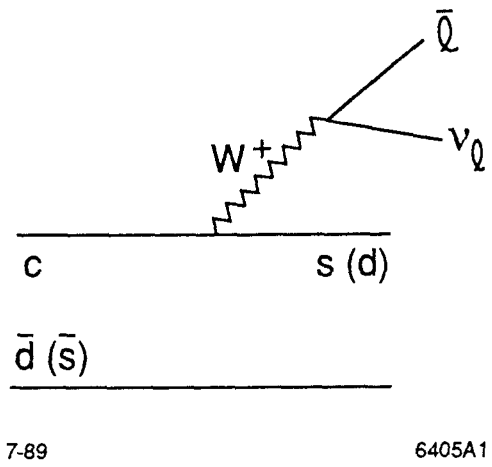

Fig. 1

Fig. 1 Spectator view of semileptonic D decay. Here, permutation of s and $\overline{d}$ with d and $\overline{s}$ takes one from $D_d \to \overline{K}\,\overline{l}\,\nu_l$ to $D_s \to K\,\overline{l}\,\nu_l$.