The τ Anomalous Magnetic Moment Measured From The Angular Dependence Of The Charged Secondary

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We[1] examine the angular distributions of the charged secondaries from τ decays into two or three bodies and show that their angular distributions can be used to isolate the effect of an anomalous magnetic moment $F_2(q^2)$ of the τ when it couples to the virtual photon in e^+-e^- annihilation. Here, $F_2(0) = a_\tau = (g-2)/2$. The virtual photon momentum squared is $q^2 = E_{\rm CM}^2 = 4E^2$, and $\beta = \sqrt{1-4m^2/q^2}$.

The production of τ pairs or "1 body" mode has the angular distribution in the angle θ with respect to the e^- direction:

$$\frac{d\sigma}{d\cos\theta} = \frac{2\pi\alpha^2}{3q^2}\beta\left[G_0(q^2) + P_2(\cos\theta)G_2(q^2)\right], \qquad \text{where} \tag{1}$$

$$G_0(q^2) \equiv F_1^2(1+2m^2/q^2) + 3F_1F_2 + F_2^2(q^2/(8m^2)+1), \tag{2}$$

$$G_2(q^2) \equiv (1/2)(1-4m^2/q^2)(F_1^2 - q^2/(4m^2)F_2^2). \tag{3}$$

The 2-body decays of the τ are $\tau^- \to \nu_\tau + \rho^-$ and $\tau^- \to \nu_\tau + \pi^-$. For the charged decay particle of mass m_c with fractional momenta $z = p_c/E$ where θ_c is its angle to the e^- beam and ϕ is its angle to the τ^- direction, the differential cross section is:

$$\frac{d\sigma}{dz\, d\cos\theta_c} = \frac{2\pi\alpha^2}{3q^2}B(c)\frac{1}{(1-m_c^2/m^2)}\frac{1}{\sqrt{1+4m_c^2/(z^2q^2)}}F(\theta_c,\phi) \tag{4}$$

where $B(c)$ is the branching ratio, and

$$F(\theta_c,\phi) = G_0(q^2) + P_2(\cos\phi)P_2(\cos\theta_c)G_2(q^2). \tag{5}$$

The 3-body decays with one charged lepton are $\tau^- \to \nu_\tau + \bar\nu_\mu + \mu^-$ and $\tau \to \nu_\tau + \bar\nu_e + e^-$. Defining:

$$y = m_{\nu\bar\nu}^2/m_\tau^2, \qquad \text{so that} \quad \cos\phi = 1 - (1/2)(1-\beta)(1-z-y), \tag{6}$$

the differential cross section integrating over the unknown neutrino momenta is:

$$\frac{d\sigma}{dz\, d\cos\theta_c} = \frac{4\pi\alpha^2}{3q^2}\int_0^{1-z/z_{\rm max}} dy(1-y)(\frac{1}{2}+y)F(\theta_c,\phi). \tag{7}$$

In order to estimate the statistics we take the realistic case where $F_1 = 1$ and $F_2 \ll 1$, recalling $F_2(0) = (g-2)/2$ and $F_{2\mathrm{QED}} = \alpha/(2\pi)$. The best statistics come from a complete fit to all variables with enough data, but to estimate the accuracy, we integrate over z to obtain:

$$\frac{d\sigma}{d\cos\theta_c} \propto \left[1 + \frac{6F_2}{3-\beta^2} + b(E_{\mathrm{CM}})P_2(\cos\theta_c)\right]. \tag{8}$$

A table of the integrals $b(E_{\mathrm{CM}})$ are given in Ref. (1). At $E_{\mathrm{CM}} = 5\,\mathrm{GeV}$, $b^\pi \simeq b^\mu = b^e = 0.056$ and $b^\rho = 0.081$.

For the simplest statistics estimate we consider two bins, one wherever $P_2(\cos\theta_c)$ is positive, and the other where it is negative. If L_{33} is the luminosity in units of $10^{33}/\mathrm{cm}^2 - \mathrm{sec}$ and Y is the effective number of years of running, at $E_{\mathrm{CM}} = 5\,\mathrm{GeV}$ from the ρ mode itself we obtain the limit

$$F_2 \leq \frac{0.002}{\sqrt{L_{33}Y}}, \tag{9}$$

and combining modes,

$$F_2 \leq \frac{0.0014}{\sqrt{L_{33}Y}}. \tag{10}$$

Comparing this to $\alpha/(2\pi) = 0.0012$ shows the level of accuracy obtainable. If no anomalous moment for the τ appeared above the QED correction level, then the entire weak radiative corrections would have to be included in the cross section.

At this time we may also compare to the limit on the anomalous moment of the τ obtainable from the angular distribution of τ-pairs produced at the Z^0 peak. The angular cross section from Ref. (1) is:

$$\frac{d\sigma}{d\cos} \propto \left[(1+\cos^2\theta) + 2600(F_2^\tau)^2\sin^2\theta\right] \tag{11}$$

A 10% limit on the $\sin^2\theta$ coefficient sets a limit of $F_2^\tau \leq 0.006$, and a 1% limit gives $F_2^\tau \leq 0.002$. Thus a 1% limit obtainable with $\sim 10^6$ Z^0 events can be comparable to that obtained at the Tau-Charm factory.

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[1]D. Silverman and G. Shaw, Phys. Rev. **D27**, 1196 (1985).