CALCULATION OF COUPLED-BUNCH INSTABILITIES IN A TAU-CHARM FACTORY*

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ABSTRACT

An essential consideration in the design of "tau-charm factory" storage rings is the control of coupled-bunch instabilities. In this paper, examples of calculations of the growth times of these instabilities are presented. With careful attention to the design of the RF and feedback systems, it should be feasible to control these instabilities and achieve the desired luminosity.

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1. INTRODUCTION

An essential consideration in the design of high-current "tau-charm factory" storage rings is the control of the potentially severe coupled-bunch instabilities. In this paper, calculations of the growth times of these instabilities are given for some representative examples.

There exist various approaches to calculating coupled-bunch instabilities,[1,2] especially for bunches placed symmetrically about the ring circumference. The method used here[3,4] was recently developed for calculating coupled-bunch instabilities in damping ring designs for future linear colliders, and was motivated partly by the fact that in such rings the bunches are not symmetrically placed about the circumference. In storage ring designs, the bunches are usually symmetrically located, so the instabilities could be calculated either by the method used here or by the formalisms that are specific to the symmetric case.

The present method is very efficient provided the number of bunches is not too large, and is used here to study the Jowett design,[5] which has 24 bunches. Its main utility, when detailed knowledge of cavity modes is available, may be to study the dependence of the instability growth rates on the RF system modes, so that one can tune the frequencies of these modes to minimize coupled-bunch instability growth rates. Also, one could quantitatively study the effects of missing bunches on the growth rates, if this were desirable.

The Voss design[6] with 444 bunches and an average beam current of about six amperes, will probably require cavity feedback to strongly reduce the transverse and higher-order longitudinal cavity modes, as well as tuning of the frequencies of individual modes. Without such measures, the growth rates of the instabilities are much too fast to be handled by beam feedback.

The goals of this paper are (1) to obtain estimates of the expected longitudinal and transverse coupled-bunch instability growth rates, for the Jowett design with a reasonable example of an RF system, (2) to get some quantitative idea of the sensitivity of the growth rates to variations in the higher-order mode frequencies.

2. METHOD OF CALCULATION

A "normal-modes" approach is used to obtain the n modes of coupled oscillation of n rigid bunches. Each such mode has its own coherent frequency Ω, the imaginary part of which gives the growth or damping rate of the mode. Details of the method are discussed in Refs. 3 and 4; only a brief summary will be given here.

Let the n rigid bunches, each with N particles per bunch and design energy E_0, travel at the speed of light c in a ring with design orbital period T_0 and momentum compaction factor α. The equation of longitudinal motion, that is, the equation for the time displacement τ_i of bunch i from its synchronous position, is

$$\ddot{\tau}_i + \lambda_i \dot{\tau}_i + \omega_{s,i}^2 \tau_i = -\frac{\alpha N e^2 c}{E_0 T_0} \sum_{j=1}^{n} \sum_{q=0}^{\infty} \left(\frac{dW}{dz} \right)_{qT_0 c + L_{ij}} \tau_{j,q} \quad , \tag{2.1}$$

where $\omega_{s,i}$ is the perturbed synchrotron frequency of the i^{th} bunch, which will be approximated by the unperturbed synchrotron frequency ω_s. L_{ij} is the distance between bunches i and j (positive for $i < j$, negative for $i > j$). The parameter λ_i can be used to include the effect of damping due to, for example, synchrotron radiation. The $\tau_{j,q}$ denotes the offset of bunch j, upon its passage through the cavity q turns ago.

The longitudinal wake $W(z)$ is taken to be a sum of modes in the RF cavities, of the form

$$W(z) = \begin{cases} \sum_m W_m \cos(k_m z) & (z > 0) \\ 0 & (z < 0), \end{cases} \tag{2.2}$$

Here k_m is the wavenumber ω/c of the cavity mode m, the W_m are constant coefficients, and z/c is the time since the passage of the bunch that excited the wake in the cavity.

Let us look for solutions in the form of normal modes of oscillation

$$\tau_i(t) = a_i e^{-i\Omega t} \quad . \tag{2.3}$$

Here the a_i are constants, and Ω is the coherent frequency of the mode. Then the problem can be reduced to solving the following eigenvalue equation for the coherent frequencies

Ω and their corresponding eigenvectors:

$$\mathbf{M}\vec{\mathbf{a}} = \Omega\vec{\mathbf{a}} \quad , \tag{2.4}$$

where the elements of the matrix $\mathbf{M}$ are given by

$$M_{ij} = (\omega_s - i\frac{\lambda_i}{2})\delta_{ij} - \frac{\chi_{ij}(-i\omega_s)}{2\omega_s} \quad , \tag{2.5}$$

where $\vec{\mathbf{a}}$ is the vector $(a_1, ..., a_n)$, and the functions $\chi_{ij}(s)$ are given by

$$\chi_{ij}(s) = -\frac{\alpha N e^2 c}{E_0 T_0} \sum_m \frac{1}{2} W_m \left[\frac{i k_m e^{i k_m L_{ij}}}{1 - e^{(i k_m c - s) T_0}} - \frac{i k_m^* e^{-i k_m^* L_{ij}}}{1 - e^{(-i k_m^* c - s) T_0}} \right] \qquad (i > j)$$

$$\chi_{ij}(s) = -\frac{\alpha N e^2 c}{E_0 T_0} \sum_m \frac{1}{2} W_m \left[\frac{i k_m e^{i k_m L_{ij}} e^{(i k_m c - s) T_0}}{1 - e^{(i k_m c - s) T_0}} - \frac{i k_m^* e^{-i k_m^* L_{ij}} e^{(-i k_m^* c - s) T_0}}{1 - e^{(-i k_m^* c - s) T_0}} \right] \qquad (i \leq j) \quad . \tag{2.6}$$

Note that the k_m are complex to account for damping of the wake.

2.1. Transverse case

The equation for the horizontal or vertical displacement x_i of bunch i is

$$\ddot{x}_i + \lambda \dot{x}_i + \omega_\beta^2 x_i = \frac{N e^2 c}{E_0 T_0} \sum_{j=1}^{n} \sum_{q=0}^{\infty} W^\perp(L_{ij} + q T_0 c) x_j(t - q T_0) \ . \tag{2.7}$$

Here the betatron frequency $\omega_\beta = c/\bar{\beta}$, where $\bar{\beta}$ is the average beta function (i.e., a "smooth focusing" approximation is used).

Table 1: Parameters	
Number of bunches	24
Number of particles per bunch	1.5×10^{11}
Energy	2.5 GeV
Ring circumference	376.99 m
Momentum compaction factor	0.026
Harmonic number	1872
RF frequency	1.5 GHz
Synchrotron frequency	$9.54 \times 10^4 \mathrm{sec}^{-1}$
Average beta function	9.68 m

The transverse wake function is of the form

$$W^{\perp}(z) = \begin{cases} \sum_m W_m^{\perp} \sin(k_m z) & (z > 0) \\ 0 & (z < 0). \end{cases} \tag{2.8}$$

One may then solve for the coherent frequencies and eigenvectors in the same form as in the longitudinal case, the only difference being that $\chi_{ij}(s)$ must be replaced by

$$\chi_{ij}^{\perp}(s) = -\frac{Ne^2c}{E_0T_0}\sum_m \frac{1}{2}W_m^{\perp}\left[\frac{ie^{ik_mL_{ij}}}{1-e^{(ik_mc-s)T_0}} - \frac{ie^{-ik_m^*L_{ij}}}{1-e^{(-ik_m^*c-s)T_0}}\right] \qquad (i > j) \quad ,$$

$$\chi_{ij}^{\perp}(s) = -\frac{Ne^2c}{E_0T_0}\sum_m \frac{1}{2}W_m^{\perp}\left[\frac{ie^{ik_mL_{ij}}e^{(ik_mc-s)T_0}}{1-e^{(ik_mc-s)T_0}} - \frac{ie^{-ik_m^*L_{ij}}e^{(-ik_m^*c-s)T_0}}{1-e^{(-ik_m^*c-s)T_0}}\right] \qquad (i \leq j) \quad . \tag{2.9}$$

3. THE JOWETT DESIGN

For definiteness, the examples given here will be based upon the Jowett design. The parameters used are shown in Table 1.

Results are given for the case of an RF system using superconducting "single-mode cavities"[7] The RF system consists of ten such cells. The frequencies and loss factors of the modes for the 1 GHz cavities calculated by Weiland were scaled to the desired accelerating π-mode frequency near 1.4887 GHz (this gives the harmonic number 1872). In addition to the fundamental accelerating mode, there is a single "higher-order" longitudinal RF mode, and there are also two transverse RF modes in this cavity. The other longitudinal mode has a scaled frequency of about 1.42 GHz; the loss factor of this mode is about twice that of the accelerating mode, but since it is a 0-mode, the contributions of different cells tend to cancel, so only one cell's worth of this mode was included. The scaled frequencies of the two transverse modes are about 1.58 GHz and 1.87 GHz, and the loss factor of the 1.87 GHz mode is about three times larger than that of the 1.58 GHz mode. The Q's of the modes are about 30000. The parameters of the Weiland single-mode cavities at 1 GHz (not yet scaled to our desired value near 1.5 GHz) are shown in Table 2.

Table 2: Weiland Single-Mode Cavities (Unscaled, 1 GHz)

Longitudinal modes:			
Frequency (MHz)	$(r/Q)_{Weiland}$ (Ω)	Loss factor (V/pC)	Q
959. (0-mode)	83.	0.2501	31000.
1005. (π-mode)	40.	0.1263	30000.
Transverse modes:			
Frequency (MHz)	$(r/Q)_{Weiland}$ (Ω/cm^2)	Loss factor (V/pC)	Q
1066.	0.23	0.03254	31000.
1261.	0.71	0.1188	29000.

The program LTMODES, which is used to solve for the eigenvalues and eigenvectors of the matrix given by (2.5), requires the loss factors of the RF cavity modes as inputs. Weiland's longitudinal r/Q (per cell, units Ω) was converted to longitudinal loss factor

κ_0 according to:

$$\kappa_0 = \frac{\omega}{2}\left(\frac{r}{Q}\right)_{Weiland} \quad , \tag{3.1}$$

where ω denotes the circular frequency of the RF mode. Weiland's transverse r/Q (per cell per transverse dimension squared, units Ω/cm^2) was converted to transverse loss factor κ_1 according to:

$$\kappa_1 = \frac{\omega a^2}{2}\left(\frac{r}{Q}\right)_{Weiland} \quad . \tag{3.2}$$

Here a is the iris radius.

The coherent frequencies Ω of the 24 modes of coupled-bunch oscillation were calculated, for both the longitudinal and transverse directions. The results for the most important coupled-bunch modes are given for several examples, in Figures 1 through 4. The vertical axis of each plot is the imaginary part of the coherent frequency of the coupled-bunch mode being plotted. Note that the growth time (or damping time, if $Im\ \Omega$ is negative) of the mode is just $1/Im\ \Omega$.

In doing these calculations, the damping parameters λ_i were set to zero, thus the growth rates are "bare" growth rates without coherent damping or synchrotron radiation taken into account.

The variation of the growth rates as a function of an overall scaling of the RF cavity mode frequencies was examined, to get an idea of the variation to be expected as these cavity mode frequencies are tuned. In reality the RF modes in a given cavity could be individually tuned if desired, in particular the accelerating mode should be tuned to a particular value independently of the other modes in the cavity. However, for the present calculations, this overall scaling factor was applied to all the RF modes in the cavity. This overall scaling corresponds to the horizontal axis of the plots, which shows the ratio of the fundamental cavity mode frequency (longitudinal or transverse, depending on whether the plot is for a longitudinal or transverse coupled-bunch mode) to the orbital frequency. Thus for the longitudinal case, this ratio is near 1872, the harmonic number.

Another factor should be taken into consideration in calculating the growth rates, namely the possibility of a cavity-to-cavity spread in the frequencies of corresponding higher-order modes. Such a spread reduces the largest instability growth rates if it is sufficient to keep the resonance lines in different cavities from overlapping. For comparison, results are shown without this spread in Figures 1 and 2, and with a ±0.1% spread uniformly distributed among the ten cavities in Figures 3 and 4. The horizontal axes in Figures 3 and 4 are of course derived from the average frequency of the spread.

The effect of finite bunch length σ_z, which gives a factor $e^{-k^2\sigma_z^2}$ in the strength of a wake mode with wavenumber k, has been ignored here. For bunch length ~ 1 cm and wavenumbers k of about 30 to 40 m^{-1}, it would give a correction of the order of only 10% in the growth rates, which is too small to be of interest here.

In Figure 1, the longitudinal coupled bunch instability is examined. In part (a) is shown the growth and damping of the barycentric mode of oscillation (the mode in which all 24 bunches oscillate in phase). The vertical axis is $Im\ \Omega$, where Ω is the coherent frequency of the barycentric mode. The horizontal axis is the ratio ω_{RF}/ω_0 of the fundamental longitudinal RF mode frequency ω_{RF} to the bunch orbital frequency $\omega_0 \equiv 1/T_0$ When this ratio is just below the harmonic number 1872, the barycentric mode is Robinson-damped; when the ratio is just above 1872, the barycentric mode is Robinson-antidamped. One would of course choose to run with the ratio just below 1872, to take advantage of the Robinson damping. There are 23 other coupled-bunch modes of longitudinal oscillation, and in part (b) the largest $Im\ \Omega$ of these modes is plotted as a function of ω_{RF}/ω_0. The largest growth rates in part (b) are overly pessimistic, because no cavity-to-cavity spread in the higher-order RF mode frequencies has been included yet.

In Figure 2, the transverse coupled bunch instability is examined. The largest growth rates in this example are again too pessimistic, since no cavity-to-cavity spread in the higher-order RF mode frequencies has been included. The horizontal axis is the ratio ω_1^{tr}/ω_0 between the lowest transverse RF mode frequency ω_1^{tr} and the bunch orbital frequency $\omega_0 \equiv 1/T_0$ In part (a) is plotted the growth rate $Im\ \Omega$ of the mode with the largest growth rate, and in part (b) the growth rate $Im\ \Omega$ of the second-fastest mode.

Figure 3 is the same as Figure 1, except that a spread of ±0.1% in the frequency of the higher-order longitudinal mode has been included. The growth rates in Figure 3(a) are essentially the same as those in Figure 1(a), since the barycentric mode growth or damping rate depends almost entirely on the Robinson tuning or detuning of the fundamental accelerating mode, which is assumed to be tuned to the same frequency in all RF cells. Comparing Figure 3(b) with Figure 1(b), one sees that the spread in cavity mode frequencies has reduced the large values of instability growth rates of the other oscillation modes. This is because the higher-order cavity resonances that drive these modes do not overlap as much.

Figure 4 is the same as Figure 2, except that a spread of $\pm 0.1\%$ in the frequencies of both transverse cavity modes has been included. The largest values of the growth rate, seen in Figure 2(a), are reduced due to the spread in wake mode frequencies to give the result shown in Figure 4(a).

4. CONCLUSIONS AND ACKNOWLEDGMENTS

From Figure 3(b) and 4(a), one sees that the longitudinal and transverse instability growth times are about 0.1 to 0.2 msec. These growth times include the effects of a cell-to-cell spread in RF mode frequencies, but do not include coherent damping or synchrotron radiation damping effects. At the tau-charm factory energy, it is reasonable to expect that instabilities with such growth rates can be controlled with a well-designed feedback system. However, it will certainly be worthwhile to carefully design the RF cavities to minimize their higher-order modes.

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REFERENCES

1. For a review, see A.W. Chao, in *Physics of High Energy Particle Accelerators*, AIP Conference Proceedings, **105**, American Institute of Physics, New York (1983).
2. M.S. Zisman, S. Chattopadhyay, and J.J. Bisognano, "ZAP User's Manual", LBL-21270 and UC-28 (Dec. 1986). The program ZAP uses the formalism of B. Zotter and F. Sacherer, and that of J.M. Wang to calculate symmetric coupled-bunch instabilities.
3. K.A. Thompson and R.D. Ruth, I.E.E.E. Particle Accelerator Conference (1989), SLAC–PUB–4872.
4. K.A. Thompson and R.D. Ruth, in preparation (1989), SLAC–PUB–4962.
5. J.M. Jowett, CERN/LEP-TH/87-56 (1987) and CERN/LEP-TH/88-22 (1988).
6. G.A. Voss, J.M. Paterson, and S.A. Kheifets, these proceedings, and SLAC–PUB–5011.
7. T. Weiland, DESY note (June 1983).

Fig. 1 (a) Inverse growth (or damping) rate for the barycentric mode, and (b) inverse growth rate of the fastest longitudinal mode (excluding barycentric mode) of coupled-bunch oscillation. The horizontal axis is the frequency of the accelerating mode divided by the orbital frequency. No spread in the higher-order mode frequency is included.

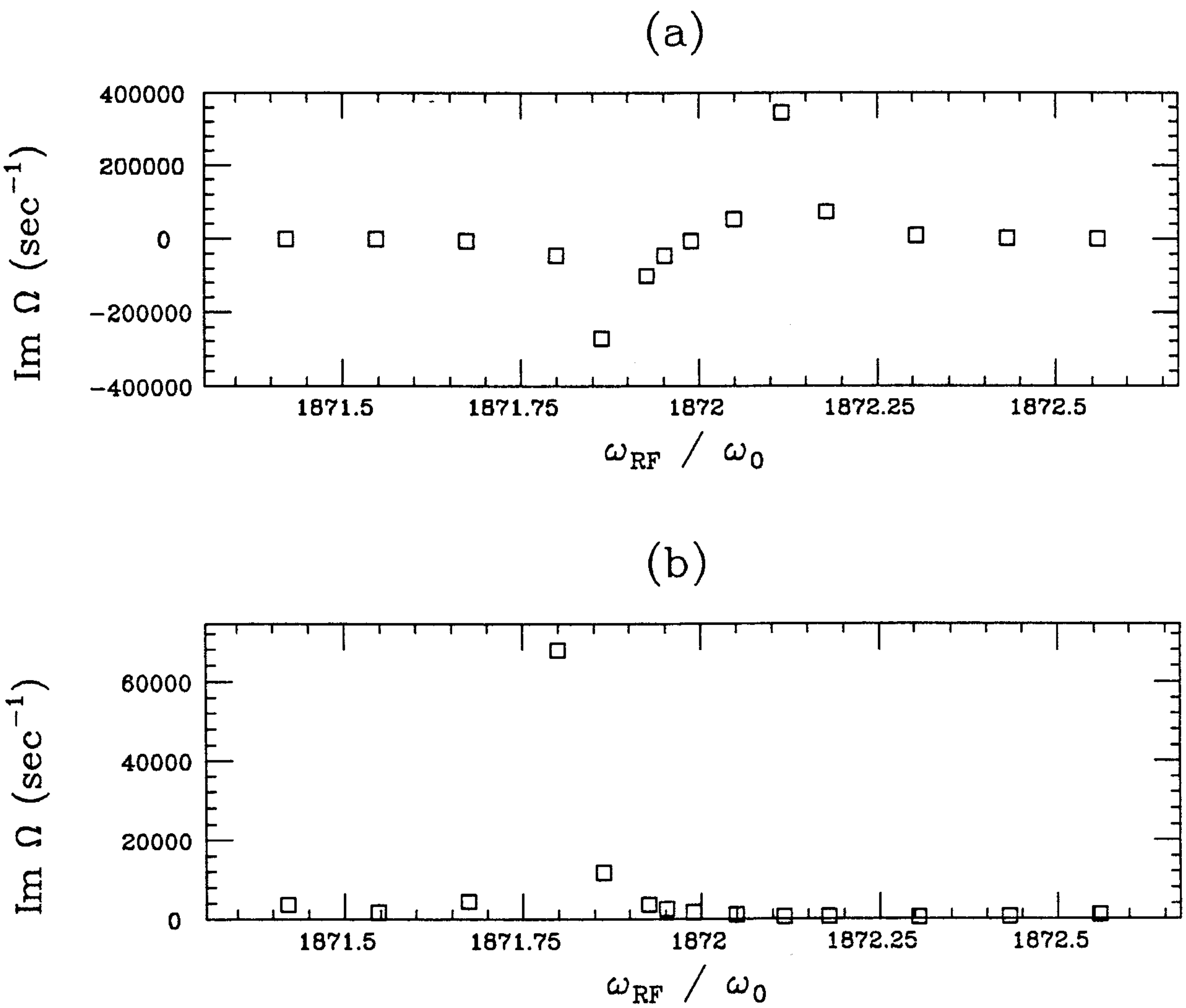

Fig. 2 Inverse growth rates for the (a) fastest growing, and (b) second-fastest growing, transverse mode of coupled-bunch oscillation. The horizontal axis is the frequency of the lowest-order transverse wake mode divided by the orbital frequency. No spread in the transverse wake mode frequencies is included.

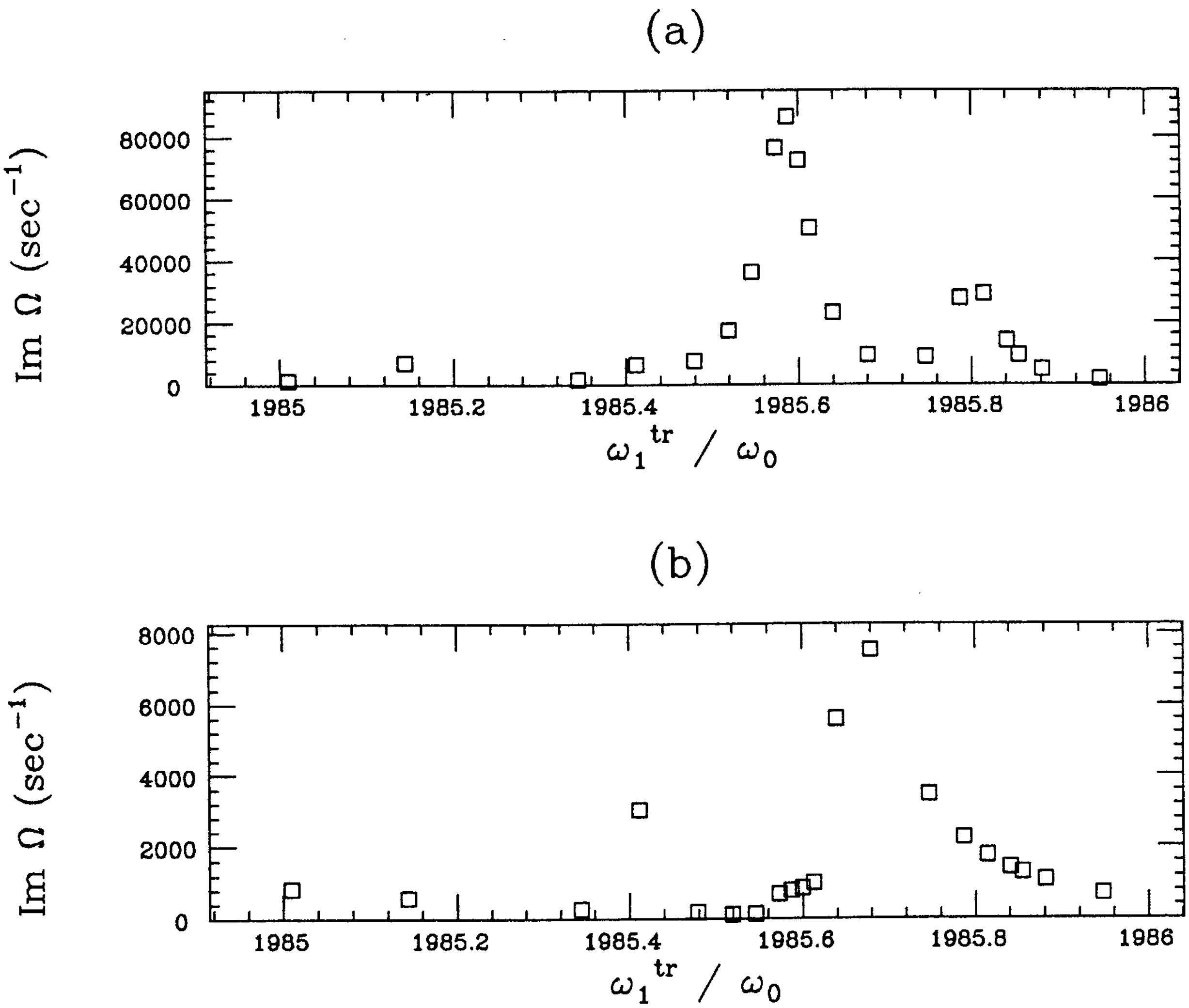

Fig. 3 (a) Inverse growth (or damping) rate of the barycentric mode, and (b) inverse growth rate of the fastest longitudinal mode (excluding barycentric mode) of coupled-bunch oscillation. The horizontal axis is the frequency of the accelerating mode divided by the orbital frequency. A spread of ±0.1% in the higher-order mode frequency is included.

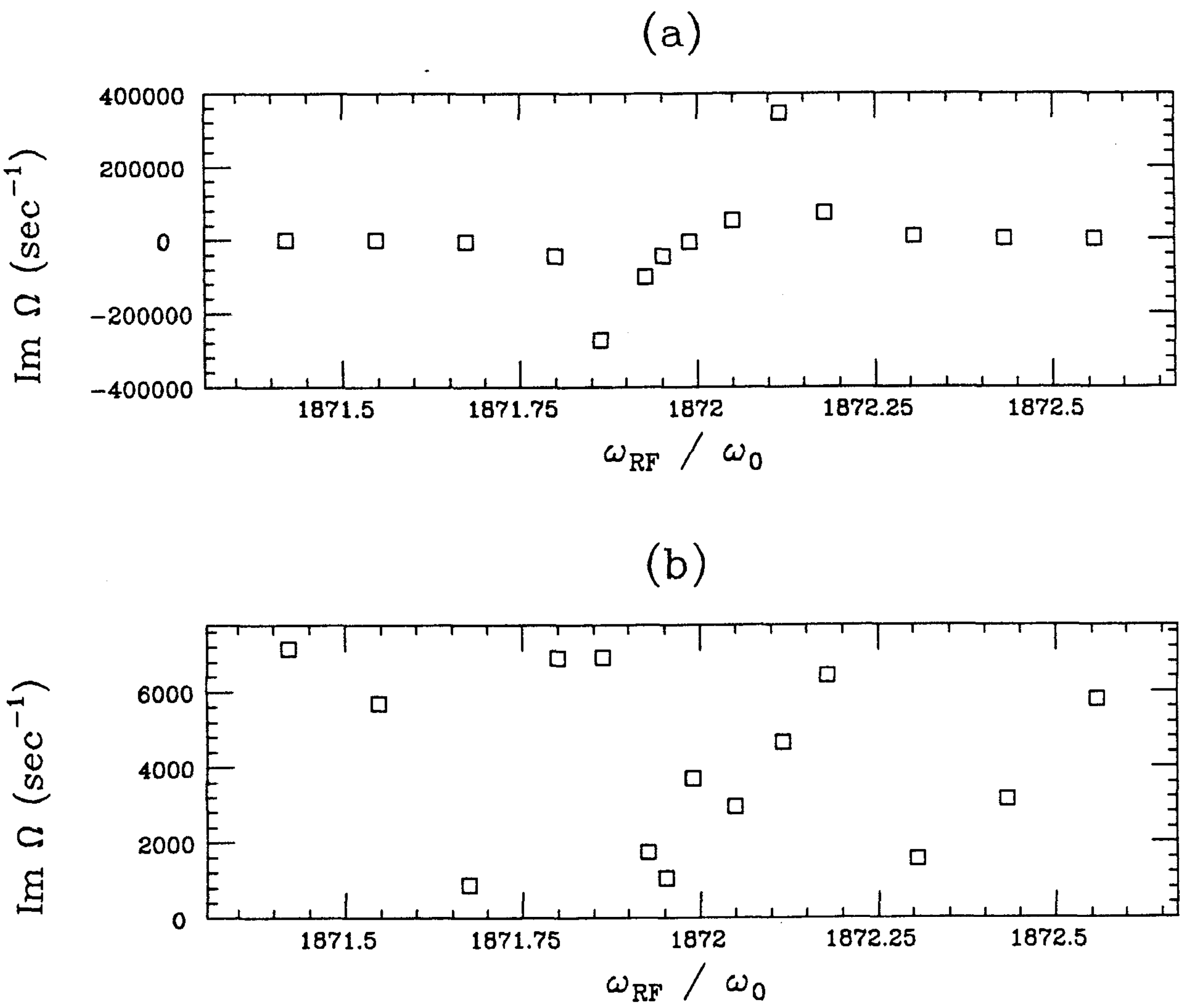

Fig. 4 Inverse growth rates for the (a) fastest growing, and (b) second-fastest growing, transverse mode of coupled-bunch oscillation. The horizontal axis is the frequency of the lowest-order transverse wake mode divided by the orbital frequency. A spread of ±0.1% in the transverse wake mode frequencies is included.