Multibunch Instability Investigations

for a

Tau-Charm Factory*

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INTRODUCTION

In the design of high-luminosity colliders for high-energy physics, it has become clear that multibunch instabilities will be one of the primary effects that limit beam intensity, and hence luminosity. This paper reports on a series of calculations of multibunch growth rates, using the LBL accelerator physics code ZAP,[1] that illustrate the seriousness of the effect for typical design parameters of a Tau-Charm Factory.

A common feature of high-luminosity machines is the requirement of a small beta function at the interaction point. To maintain the advantages of a low beta function, however, requires that the rms bunch length, σ_ℓ, be smaller than β^*. This leads, in general, to several inconvenient aspects.

- The requirement for short bunches leads to the need for a substantial amount of RF hardware—introducing just the narrow-band (high-Q) impedance that generates multibunch instabilities in the first place.

- The need for short bunches means that bunch lengthening from the longitudinal microwave instability must be avoided. Since the longitudinal impedance $Z_{||}/n$ cannot be reduced indefinitely, there is a clear benefit to using many bunches, with lower current per bunch.

- The short bunches have a Fourier spectrum extending up to very high frequencies, thus effectively sampling impedances in this regime that would be essentially invisible to longer bunches. (This aspect can be seen in the exponential cutoff factor, proportional to $(\sigma_\ell/R)^2$, in the expressions for the effective impedance given below.)

In practice, it is difficult to achieve a high luminosity without having a high average beam current in the rings. Because the multibunch growth rates scale linearly with average current, the resulting rates tend to be very high. It might be imagined that, for sufficient bunch separation and low enough Q values for the higher-order cavity modes, the wake fields have time to die away between successive bunches, thus reducing the bunch-to-bunch coupling. For most cases of interest, however, it is hard to reduce the Q values sufficiently to achieve this condition.

Because the details of higher-order modes of the RF cavities are only a guess at present, the results contained herein should not be interpreted quantitatively. However, experience has shown that the magnitudes of multibunch growth rates estimated as is done here are in reasonable agreement with observed growth rates under comparable conditions.[2] Thus, although the particular modes that grow will depend on details of the impedance that are not well known at this time, the predicted growth rates are expected to reflect the requirements of a feedback system with good accuracy.

OVERVIEW OF CALCULATIONS

ZAP performs calculations of multibunch instabilities in the frequency domain, and thus is restricted to the case of equally spaced bunches in the ring. In the case of a high-luminosity collider, of course, this is the standard operating configuration, so ZAP is well suited to such studies. Each higher-order longitudinal and transverse mode of the RF cavity is represented by three parameters, a resonant frequency (f_R), a shunt impedance (R_s or R_t), and a quality factor (Q). Because the growth times depend on the *angular* frequency shift, we typically use this value to represent the higher-order modes, i.e., we quote $\omega_R = 2\pi f_R$. Thus the longitudinal impedance is represented by

$$Z_{\parallel}(\omega) = \frac{R_s}{\left[1 + iQ\left(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R}\right)\right]}$$

and the transverse impedance by

$$Z_{\perp}(\omega) = \frac{\omega_R}{\omega} \frac{R_t}{\left[1 + iQ\left(\frac{\omega_R}{\omega} - \frac{\omega}{\omega_R}\right)\right]}$$

To calculate the growth rates, we use the Wang formalism,[3] which gives for the longitudinal case:

$$\Delta\omega_{s,a}^{\parallel} = i \frac{I_b \omega_0^2 \eta k_b}{2\pi\beta^2 (E_T/e)\omega_s} \frac{(\sigma_{\ell}/R)^{2(a-1)}}{2^a (a-1)!} (Z_{\parallel})_{eff}^{s,a}$$

where

$$(Z_{\parallel})_{eff}^{s,a} = \sum_{p=-\infty}^{+\infty} (pk_b + s)^{2a} e^{-(pk_b + s)^2 (\sigma_{\ell}/R)^2} \left[\frac{Z_{\parallel}(\nu_p \omega_0)}{\nu_p}\right]$$

and $\nu_p = (pk_b + s + a\nu_s)$. The growth rate is given by the imaginary part of $\Delta\omega_{s,a}$ as calculated above, and hence it depends on the real part of the impedance itself.

For the transverse case, we use the corresponding expressions:

$$\Delta\omega_{s,a}^{\perp} = -i \frac{I_b c k_b}{4\pi (E_T/e)\nu_\beta} \frac{(\sigma_{\ell}/R)^{2a}}{2^a a!} (Z_{\perp})_{eff}^{s,a}$$

$$(Z_{\perp})_{eff}^{s,a} = \sum_{p=-\infty}^{+\infty} \left(pk_b + s + \nu_\beta - \frac{\xi}{\eta}\right)^{2a} e^{-\left(pk_b + s + \nu_\beta - \frac{\xi}{\eta}\right)^2 (\sigma_{\ell}/R)^2} \left[Z_{\perp}\left(\nu_p^{\perp} \omega_0\right)\right]$$

RESULTS

To make estimates of the growth rates, typical values of higher-order modes have been taken for a "PEP-like" 353-MHz RF system[4] that is heavily de-Qed, for a superconducting RF cavity (scaled in frequency to 353 MHz from measured[5] values for the TRISTAN 500-MHz design), and for the CEBAF 1500-MHz superconducting RF cavity.[6] Storage ring parameters were taken from designs by Jowett[7] and by Voss[8] presented at this Workshop.

Jowett Design

In the case of the Jowett design,[7] a high-frequency RF system based on the CEBAF 1500-MHz cavity was assumed. Parameters used in the ZAP calculations are summarized in Table I. Both the longitudinal dipole (a=1) and quadrupole (a=2) modes are predicted to be unstable; typical growth times are summarized in Table II. We see that the a=2 modes grow relatively slowly, and would be damped via radiation damping (τ_E = 9 ms) in the example shown. The a=1 modes, in general, grow more quickly, and would have to be dealt with via a longitudinal feedback system capable of handling rates on the order of 2000 s^{-1}. Such a feedback system is nontrivial, and would require relatively high power, but is likely to be realizable.

In the transverse case, the growth times (see Table II) are predicted to be a fraction of a millisecond for the rigid-dipole (a=0) motion. However, with an assumed rms betatron tune spread of 0.001, most modes are predicted to be Landau damped. Because of the tune spread introduced by the beam-beam force, it may well be that transverse instabilities would be suppressed *without* recourse to a feedback system. Growth times for the a=1 transverse mode are on the order of 10 ms, which is too fast to be radiation damped (τ_x = 35 ms, τ_y = 22 ms). However, Landau damping is expected to be effective in suppressing growth of the instability.

Voss Design

The second concept that was examined is the Voss design based on a crab-crossing scheme.[8] These rings use many bunches ($k_b = 444$) and very high circulating current ($I_{tot} = 6.2$ A per ring) to achieve a luminosity of 5×10^{33} cm^{-2}s^{-1}. Mitigating these parameters to some extent is the larger value for β^* adopted there, which permits the bunch length to be increased compared with the Jowett design discussed above. Parameters used in the estimation of coupled-bunch growth rates are summarized in Table III. Since the Voss design assumes a 353-MHz RF system, several choices of higher-order modes were available to consider.

Initially, a set of calculations was performed using the PEP RF modes with substantial de-Qing (to $Q \approx 500$). This estimate gives growth rates (see Table IV) on the order of 10^4-10^5 s^{-1} for both the a=1 and a=2 longitudinal modes. Because these rates are comparable to the synchrotron period itself, the model used for the calculations becomes invalid and the values quoted here should be interpreted mainly as a warning that this RF configuration is unlikely to be usable in such a ring. These 0.01-0.1 ms growth times are to be compared with a radiation damping time of τ_E = 11 ms. A feedback system to cope with such growth rates is likely to require an unreasonably high power, since the power scales as the square of the damping rate needed.

Results for transverse growth times (Table IV) are less crazy, but still impressively fast. Typical values for the a=0 transverse mode are about 0.1 ms; corresponding values for the a=1 mode are about 1 ms. These growth times are much faster than the transverse radiation damping time of 22 ms.

To get a feeling for the possible benefits of superconducting RF in this scenario, a set of modes from the TRISTAN 500-MHz cavity was roughly scaled to 353 MHz. This resulted (see Table IV) in growth times for longitudinal instabilities of 0.1-1 ms for a=1 modes and 1-10 ms growth times for a=2 modes. Thus, powerful (but probably achievable) feedback would be required to manage the growth rates even with superconducting RF.

Transverse growth times for a=0 modes are on the order of 1 - 5 ms, and are predicted to be Landau damped in most—but not all—cases by the assumed rms betatron tune spread of 0.001. Growth times for the a=1 modes are slower than 20 ms, and thus should be radiation damped.

SUMMARY

We can see that, as expected, the growth rates for multibunch instabilities are quite rapid for the cases considered here. Especially for the Voss design, with 6 A of current in 444 bunches, it does not seem plausible to consider a scenario that does not include superconducting RF cavities, or at least room-temperature cavities of the same geometry to minimize the high-order modes.

REFERENCES

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6) Data provided by J. Bisognano and K. Thompson (private communication).

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8) G. Voss, "A τ-charm Facility with Crab-Crossing," these proceedings.

Table I

Parameters used in ZAP Calculations for Jowett Design

Circumference [m]	375
RF parameters	
frequency [MHz]	1497
voltage [MV]	5
Broadband impedance, $Z_{\|}/n$ [Ω]	0.4
Bunch parameters	
rms length [cm]	0.6
rms rel. energy spread	0.000566
current [mA]	22.4
number of bunches	24
Momentum compaction	0.0189
Avg. beta function [m]	10

Table II

Coupled-Bunch Growth Rates for Jowett Design

(2.5 GeV; $\tau_E = 9$ ms; $\tau_x = 35$ ms; $\tau_y = 22$ ms)

Longitudinal

CEBAF, Superconducting	
$\tau_{a=1}$ (ms)	$\tau_{a=2}$ (ms)
0.5	10

Transverse

CEBAF, Superconducting	
$\tau_{a=0}$ (ms)	$\tau_{a=1}$ (ms)
0.5	10

Table III

Parameters used in ZAP Calculations for Voss Design

Circumference [m]	377
RF parameters	
frequency [MHz]	353
voltage [MV]	2
Broadband impedance, $Z_{\parallel}/n$ [Ω]	0.4
Bunch parameters	
rms length [cm]	2.5
rms rel. energy spread	0.000618
current [mA]	14
number of bunches	444
Momentum compaction	0.026
Avg. beta function [m]	10

Table IV

Coupled-Bunch Growth Rates for Voss Design

(2.5 GeV; $\tau_E = 11$ ms; $\tau_x = 22$ ms)

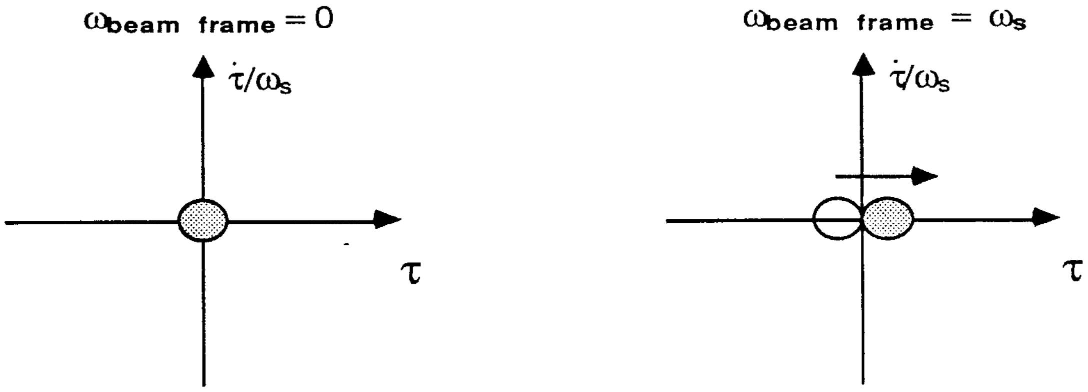
Longitudinal

PEP, Q/100		TRISTAN, Superconducting	
$\tau_{a=1}$ (ms)	$\tau_{a=2}$ (ms)	$\tau_{a=1}$ (ms)	$\tau_{a=2}$ (ms)
0.01-0.1	0.05-0.1	0.1-1	1-10

Transverse

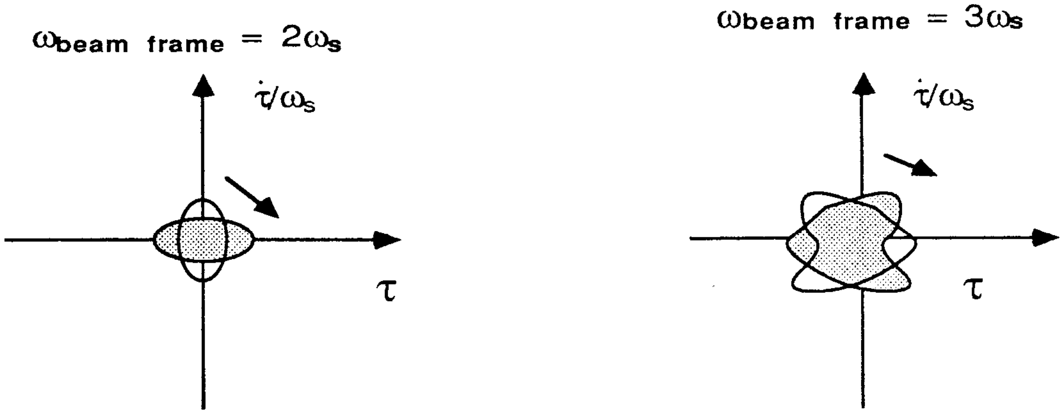
PEP, Q/100		TRISTAN, Superconducting	
$\tau_{a=0}$ (ms)	$\tau_{a=1}$ (ms)	$\tau_{a=0}$ (ms)	$\tau_{a=1}$ (ms)
0.1	1	1-5	20

Coupled-bunch Modes



a=0
rigid dipole

a=1
dipole



a=2
quadrupole

a=3
sextupole

Fig. 1 Schematic diagram of coherent distortions in bunch longitudinal phase space. The a=0 mode can only be unstable in the transverse plane.