# DESIGN OF A HIGH LUMINOSITY COLLIDER FOR THE TAU-CHARM FACTORY* 

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#### Abstract

Important relations between basic parameters of a high-luminosity collider are discussed. As the result, it is shown that the maximum bunch spacing is limited by the beam current to clear the threshold of the bunch lengthening. In order to solve the short bunch spacing, the crab-crossing scheme is applied to a design of the ring with $2.2 \mathrm{GeV}, 2 \times 10^{33} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$ luminosity.


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## 1. BASIC PARAMETERS

On a design of a high-luminosity storage-ring collider like the $\tau /$ charm factory, there are complicated interdependences of system parameters. ${ }^{1}$ Although it is quite difficult to handle all data simultaneously, we choose the four equations below as the most important relationships among them:

1. Luminosity:

$$
\begin{equation*}
\mathcal{L}=\frac{N^{2} f}{4 \pi \sigma_{x} \sigma_{y}} . \tag{1}
\end{equation*}
$$

2. Tune shift parameter:

$$
\begin{equation*}
\xi_{x, y}=\frac{N r_{e} \beta_{x, y}^{*}}{2 \pi \gamma \sigma_{x, y}\left(\sigma_{x}+\sigma_{y}\right)} \lesssim 0.04 \tag{2}
\end{equation*}
$$

3. Longitudinal instability threshold:

$$
\begin{equation*}
\langle Z \mid n\rangle \leq \sqrt{\frac{\pi}{2}} \frac{Z_{0} \gamma}{r_{e}} \frac{\alpha_{p} \delta^{2} \sigma_{z}}{N} . \tag{3}
\end{equation*}
$$

4. Bunch length/ $\beta$ function ratio:

$$
\begin{equation*}
\sigma_{z} / \beta_{y}^{*} \lesssim 0.5 \tag{4}
\end{equation*}
$$

We introduce the symbols:

| $N$ | Number of particles per bunch. |
| :--- | :--- |
| $f$ | Collision rate. |
| $\sigma_{x, y, z}$ | Horizontal, vertical, and longitudinal beam sizes at IP. |
| $\beta_{x, y}^{*}$ | Horizontal and vertical $\beta$ functions at IP. |
| $r_{e}$ | The classical electron radius. |
| $\langle Z / n\rangle$ | Longitudinal normalized impedance. |
| $Z_{0}$ | The impedance of vacuum. |
| $\alpha_{p}$ | Momentum compaction factor. |
| $\delta$ | Relative energy spread. |

The condition (4) is necessary to avoid the synchrotron-betatron coupling due to the beam-beam collision. ${ }^{2}$ The constraints (2), (3), and (4) are quite essential because neither fundamental nor technical methods to cure them has been known until today. Although we do not have a clear theory to determine the actual tune-shift limit (2), we take an empirical value which is equal to the initial design of Jowett. ${ }^{3}$

Let us see what emerges from the combination of the above relations. First, from Eqs. (1) and (2) an important relationship is obtained:

$$
\begin{equation*}
\mathcal{L}=\frac{\gamma \xi_{y} I}{2 e r_{e} \beta_{y}^{*}}\left(1+\frac{1}{R}\right) \tag{5}
\end{equation*}
$$

where $I=N e f$ is the beam current, and $R=\beta_{x}^{*} / \beta_{y}^{*}=\sigma_{x} / \sigma_{y}$ the aspect ratio of the beam. This tells that for a given luminosity, $\beta_{y}^{*}$ is determined only by the beam current (and also $R$, but so far as we concentrate on a flat-beam scheme, $R \gg 1$, the dependence on $R$ is weak).

Second, the factor $\alpha_{p} \delta^{2} \sigma_{z}$ in (3) is written as:

$$
\begin{equation*}
\alpha_{p} \delta^{2} \sigma_{z}=\frac{e \omega_{R F} V_{c}}{c E L} \sigma_{z}^{3} \tag{6}
\end{equation*}
$$

by applying the formulas:

$$
\begin{align*}
\delta & =\frac{\omega_{s}}{\alpha_{p} c} \sigma_{z} \\
\omega_{s}^{2} & =\frac{e c \omega_{R F} V_{c}}{E L} \alpha_{p} \tag{7}
\end{align*}
$$

where $\omega_{s}, \omega_{R F}, V_{c}, E$, and $L$ are the angular synchrotron frequency, the angular frequency and the peak voltage of the acceleration cavity, beam energy, and the circumference of the ring, respectively. Thus the condition (3) is rewritten as:

$$
\begin{equation*}
N \leq \frac{(2 \pi)^{3 / 2}}{e c^{2} L} \frac{\omega_{R F} V_{c}}{\langle Z / n\rangle} \sigma_{z}^{3} \tag{8}
\end{equation*}
$$

Substituting the conditions (4) and (5), we find there is a maximum limit on the bunch spacing $S_{B}=c / f$ :

$$
\begin{equation*}
S_{B} \leq \frac{(2 \pi)^{3 / 2}}{64 c L} \frac{\omega_{R F} V_{c}}{\langle Z / n\rangle}\left[\frac{\gamma \xi_{y}}{e r_{e} \mathcal{L}}\left(1+\frac{1}{R}\right)\right]^{3} I^{2} \tag{9}
\end{equation*}
$$

In order to evaluate the term $\omega_{R F} V_{c} /\langle Z / n\rangle$ in the above, we roughly estimate the longitudinal impedance $\langle Z / n\rangle$ as:

$$
\begin{equation*}
\langle Z / n\rangle=\langle Z / n\rangle_{0}+a \omega_{R F} V_{c}, \tag{10}
\end{equation*}
$$

where $\langle Z / n\rangle_{0}$ denotes the contributions from components except the RF cavity, and $a \omega_{R F} V_{c}$ denotes the impedance from the cavity. The coefficient $a$ is determined from an estimation of the impedance of a normal-conducting cavity given by P . Wilson ${ }^{4}$ :

$$
\begin{equation*}
\langle Z / n\rangle \approx 0.05 \Omega \quad \text { for } \quad \omega_{R F}=2 \pi \times 1.5 \mathrm{GHz}, \quad V_{c}=1 \mathrm{MV} \tag{11}
\end{equation*}
$$

which gives $a \approx 5 \times 10^{-18} \Omega / \mathrm{V} /(\mathrm{rad} / \mathrm{s})$. Although Eqs. (8) and (10) tell that the larger $\omega_{R F} V_{c}$ always gives the longer bunch spacing, the gain beyond the point $a \omega_{R F} V_{c} \gtrsim\langle Z / n\rangle_{0}$ is small. Therefore, we choose

$$
\begin{equation*}
a \omega_{R F} V_{c}=\langle Z / n\rangle_{0}, \tag{12}
\end{equation*}
$$

which gives the final result of the bunch spacing as:

$$
\begin{equation*}
S_{B} \leq \frac{(2 \pi)^{3 / 2}}{64 c L} \frac{1}{2 a}\left[\frac{\gamma \xi_{y}}{e r_{e} \mathcal{L}}\left(1+\frac{1}{R}\right)\right]^{3} I^{2} \tag{13}
\end{equation*}
$$

After $\beta_{y}^{*}$ and $S_{B}$ are given as functions of $I$ by Eqs. (5) and (13), the other system parameters-like $N$, emittances, and $\sigma_{z}$-are automatically determined. The relations (5) and (13) both tell the difficulties of a machine which needs a high luminosity with a small current.

## 2. AN EXAMPLE WITH CRAB CROSSING

Figure 1 shows $S_{B}$ and $\beta_{y}^{*}$ as functions of $I$ given by Eqs. (5) and (13). This figure corresponds to the Tau-Charm requirement, $E=2.2 \mathrm{GeV}, \mathcal{L}=2 \times 10^{33} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$, and $L=340 \mathrm{~m}$. Although it is not clear how big a beam current we can store in the ring, we choose $I=1 \mathrm{~A}$-which requires $\beta_{y}^{*}=1 \mathrm{~cm}$ and $S_{B}=1.7 \mathrm{~m}$. This bunch spacing needs enough bunch separation at the extra collision points, 85 cm from the IP. Making a crossing angle is the easiest way to have such a separation, if the synchrotron-betatron resonance can be avoided by the crab-crossing scheme. ${ }^{5,6}$

There are two ways to make the crossing angle: (1) use common final quadrupoles for both beams, ${ }^{7}$ and (2) use separate quadrupoles. The merit of the common quadrupole scheme is a small crossing angle, but it still requires some separation devices after the final quadrupoles. There also remains the effects of the extra collisions near the IP. The separate quadrupole scheme can avoid the extra collisions completely and does not need a separator, thus avoiding synchrotron-radiation backgrounds to the detector. In this paper we examine a design with the separate quadrupole scheme with a horizontal crab-crossing. Figure 2 shows the crossing scheme at the IP. The crossing angle must be large enough to place the separate final quadrupoles. In this case, we use a 50 mrad crossing angle, which gives a 10 cm separation between the two beam axes at the quadrupole face, 2 m from the IP. Since the maximum beam sizes in the final quadrupole are $1.1 \mathrm{~mm} \times 0.9 \mathrm{~mm}$ in the optics designed here, this crossing angle will be sufficient. We do not have a specific design for these quadrupoles, but these can be made by a conventional magnet, because the pole-tip field is less than 1 T .

The other design parameters are listed in Table 1. The longitudinal impedance from the cavities is $a \omega_{R F} V_{c}=0.24 \Omega$, which allows impedances $\langle Z / n\rangle_{0} \leq 0.3 \Omega$ from other components in the ring.

Figure 3 shows the lattice of this ring. Since this design uses a horizontal crossing scheme and both rings sit in the same horizontal plane, the number of the crossing points becomes four if the ring has a mirror symmetry at the IP. In order to reduce the number of crossing points to two, this design breaks the symmetry. This is done by inserting a special section in the middle of one arc, shown on the left of Fig. 3.

Table 1. Parameters for a $\tau /$ charm factory with a large crab-crossing angle.

| Beam energy | $E$ | 2.2 GeV |  |
| :--- | :---: | :---: | :---: |
| Luminosity | $\mathcal{L}$ | $2 \times 10^{33}$ | $\mathrm{~cm}^{-2} \mathrm{~s}^{-1}$ |
| Tune shifts | $\xi_{x} / \xi_{y}$ | $0.05 / 0.05$ |  |
| Current | $I$ | 1.0 | A |
| Circumference | $L$ | 344 | m |
| Bunch spacing | $S_{B}$ | 1.7 | m |
| Beta functions at IP | $\beta_{x}^{*} / \beta_{y}^{*}$ | $0.50 / 0.01$ | m |
| Particle/bunch | $N$ | $3.6 \times 10^{10}$ |  |
| Emittances | $\varepsilon_{x} / \varepsilon_{y}$ | $9.2 \times 10^{-8} / 1.8 \times 10^{-9}$ | m |
| Tunes | $\nu_{x} / \nu_{y} / \nu_{z}$ | $8.28 / 10.18 / 0.12$ |  |
| Relative energy spread | $\delta$ | $5.1 \times 10^{-4}$ |  |
| Momentum compaction | $\alpha_{p}$ | 0.023 | MV |
| RF voltage | $V_{c}$ | 10.8 | MHz |
| RF frequency | $f_{R F}$ | 710 | cm |
| Harmonic number | $h$ | 816 | ms |
| Natural bunch length | $\sigma_{z}$ | 0.47 | $\Omega$ |
| Vertical damping time | $\tau_{z}$ | 36 | 0.54 |
| Longitudinal impedance threshold | $\langle Z / n\rangle$ |  |  |

Table 2. Parameters for the crab-crossing.

| Crab angle at IP | $\theta_{x}^{*}$ | 25 | mrad |
| :--- | :---: | :---: | :---: |
| Crab cavity frequency | $f_{x}$ | 710 | MHz |
| Crab voltage per cavity | $V_{x}$ | 0.97 | MV |
| $\beta$ function at crab cavity | $\beta_{x} / \beta_{y}$ | $29 / 45$ | m |
| Bunch diagonal angle | $\sigma_{x} / \sigma_{z}$ | 46 | mrad |

The main parameters of the crab-crossing in this design are listed in Table 2. The crab cavity is placed between QC2 and QC3 quadrupoles, where the horizontal phase advance
from the IP is $\pi / 2$, as shown in Fig. 4. If we assume the impedance per voltage of these crab cavities is equal to that of the main acceleration cavity, the increase of the longitudinal impedance due to the crab cavity is estimated to be about $18 \%$, using the values in Tables 1 and 2. The contribution to the transverse instability is also estimated by the $\beta$-weighted impedance. Since our $\beta$-functions at the main cavities are 10 m , the increases by the crab cavities are $52 \%$ and $80 \%$ for horizontal and vertical, respectively. Because the transverse single-bunch threshold will be high enough, the most serious effect from the crab cavities is the transverse multibunch instability. This must be solved by a feedback system or a single-mode cavity. The bunch diagonal angle for the horizontal crossing is as large as the crossing angle listed in Table 2. This gives a tolerance for the crab-crossing RF system.

This ring has a chromaticity correction system by four-family noninterlaced sextupoles. This scheme reduces the geometric aberration from the sextupoles and provides enough dynamic aperture. The four families, SD, SF, SD1, SF1, are shown in Fig. 3. Since the main chromaticity is generated from the final quadrupole in the vertical direction, we locate the sextupolc family SD at $2 \pi$ phase advance from the final quadrupole to correct the chromaticity as locally as possible. The strength of sextupoles are so determined as to minimize the variation of the $\nu_{x, y}$ and $\beta_{x, y}^{*}$ within the finite bandwidth $\pm 0.5 \%$. Figure 5 shows the stable region of this lattice, measured by a tracking simulation of 1,000 turns, which includes the synchrotron motion. The axes show the initial amplitude of a particle in the longitudinal and transverse directions in units of the standard deviations. The initial transverse position was chosen along the line $x / \sigma_{x}=y / \sigma_{y}$. This result shows this lattice has enough dynamic apertures in every direction.

## 3. DISCUSSIONS

The design given in this paper surely gives the luminosity $2 \times 10^{33} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$, if the beam current of $I=1 \mathrm{~A}$ can be stored. There are several alternative choices to achieve the same performance. One possibility is to use a superconducting cavity, which improves the impedance/voltage ratio $a$ by a factor of about 5 . According to Eq. (13), one can expect a five-times longer bunch spacing, or $1 / \sqrt{5}$ smaller current with the same spacing (if the shorter $\beta^{*}$ is possible according to Eq. (5)). Another possibility is the use of a round beam, ${ }^{8}$
which also increases the bunch space by a factor of 8 , or reduces the current by $1 / \sqrt{8}$ (in this case, $1 / \sqrt{2}$ shorter $\beta^{*}$ is required), if the beam-bearn limit is kept unchanged.

If one can avoid the radiation background from the separators and the common quadrupoles, and if the beam-beam effects at the extra collision points are negligible, a small-angle crossing will have merit over the large-angle scheme. Especially when the crossing angle is much smaller than the bunch diagonal angle, the synchrotron-betatron resonance due to the crossing angle becomes small. In that case, the crab crossing is not necessary, or even if it is still required, the constraints on the accuracy and the effects from the impedance will be quite light.

## REFERENCES

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Fig. 1. The dependences of the maximum bunch spacing and $\beta_{y}^{*}$ on the beam current $I$. $E=2.2 \mathrm{GeV}, \mathcal{L}=2 \times 10^{33} \mathrm{~cm}^{-2} \mathrm{~s}^{-1}$.


Fig. 2. Design of the IP region with a horizontal crossing angle. The separate final quadrupoles are used for both beams, and no separation device is required.


Fig. 3. Optics for the $\tau$ /charm factory. This ring is placed in a horizontal plane and has a near race-track shape. One of the arc(left) is lengthen to break the symmetry and avoid the extra intersections of the two rings.


Fig. 4. The crab cavity and the induced crab angle $\theta_{x}$ around the $I P$.


Fig. 5. The dynamic aperture of the ring with the noninterlaced sextupoles. This is examined by a particle-tracking of 1,000 turns with the synchrotron motion.


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