Rare Decays of D-mesons

R.S. Willey

Department of Physics and Astronomy

University of Pittsburgh

Pittsburgh, PA 15260

A proposed Tau-charm factory could reach branching ratios (BR) for D-meson decays down to the 10^{-7} level (see the rest of these proceedings). In this contribution we give the theoretical expectations for some rare decay modes. The modes considered are: (1) Lepton number violating decays e.g. $D^o \to \mu^+ e^-$. (2) Flavor changing neutral current (FCNC) semileptonic and radiative modes, $D \to \ell\bar{\ell}X, D \to \nu\bar{\nu}X, D \to \gamma X$. The discussion will be in two contexts: (i) The standard model (SM; in particular the minimal version with just three generations of fermions and one Higgs doublet.) (ii) anything beyond the minimal standard model ('new physics').

The discussion of the lepton number violating decays is very brief. In the SM the BR are zero. Any observed BR for one of these decays requires new physics. There is no prediction of a level at which these decays should occur.

The FCNC decay modes listed above do not occur in lowest order (amplitude $\sim G_F$) in the SM. They do occur in the next order (in g, or e) in the SM. And they are sensitive to new physics. The main theoretical issue is the calculability of these processes in the SM. At the quark level it is straightforward to calculate spectator model one-loop induced amplitudes for $c \to u\ell\bar{\ell}, c \to u\nu\bar{\nu}$, and $c \to u\gamma$ in the electro-weak SM. Still at the quark level one can draw nonspectator annihilation type diagrams, of the same order in the electro-weak coupling, which also contribute to these processes. Beyond these quark level electro-weak calculations one comes to the questions of QCD corrections and of passing from the quark level description to the experimental world of hadrons.

Since these last two issues already arise in the calculation of the ordinary first-order allowed flavor changing charged current semileptonic decays, we discuss them first in this simpler context. We start with the generic case of a heavy flavored meson: $m \sim (Q, \bar{q})$. The heavy quark Q carries the relevant flavor; the flavor is gotten rid of in the weak quark decay $Q \to q\ \ell\nu$. If Q is very massive so there are many semileptonic decay channels open, we expect that the inclusive semileptonic decay rate of the meson m should be well approximated by the rate for the heavy quark semileptonic decay

$$\Gamma(Q \to q\ \ell\nu) \approx \Gamma(m \to \ell\nu X) \qquad \underline{\text{semileptonic}},\ \underline{\text{inclusive}}$$

For exclusive final states (X a definite hadron rather than all possible hadrons in the final state) the strong dynamics clearly plays an essential role. Fig. 1 depicts the spectator model diagram for the semileptonic inclusive decay.

How well is our expectation for the semileptonic inclusive rates fullfilled? The heaviest quark that we have is the b ($m_b \sim 5$ GeV). The semileptonic rate calculated in the spectator approximation is[1]

$$\Gamma(b \to c\ell\nu) = \frac{G^2 m_b^5}{192\pi^3}\ [\mid V_{cb}\mid^2 f(x_{cb}) + \mid V_{ub}\mid^2] \cdot F_{QCD}$$

$$\mid V_{ub}\mid^2 \ll \mid V_{cb}\mid^2, \quad F_{QCD} = 1 - \frac{2\alpha_s}{3\pi}\ I(x), \quad I(x) \simeq 2.5$$

$$\text{if } \alpha_s \simeq 0.2, \quad \mathrm{F_{QCD}} \simeq 0.89 \quad (10\%\ \text{correction})$$

(it is assumed that the mass scales $M_w, m_b \gg \Lambda_{QCD}$ are large enough that QCD corrections to the semileptonic inclusive rate can be computed perturbatively.) The numerical value obtained depends sensitively on (the fifth power of) the b-quark mass. I quote two sources for the value of that mass: (i) fit to the lepton spectrum[2] in the observed semileptonic decays (ii) detailed fit to the b-onium spectroscopy using the Richardson potential[3]; both lead to the value $m_b = 4.9$ GeV. (constituent quark mass). Using the experimental values $\tau_B = 1.3 \times 10^{-12}s$ and $B_{s\ell} = 0.12$, we obtin $|V_{cb}| \simeq 0.046$. Since this is in fact the best determination of $|V_{cb}|$, it is not then a check of the theoretical assumptions. But this value is in good agreement with the Wolfenstein parametrization[4] of three generation unitarity for the KM matrix, $|V_{cb}| \approx \lambda^2 \approx 0.048$, and this does suggest that the calculation is good.

Now consider the D mesons. For the total lifetimes we know $\tau_D^+ \sim 2\tau_D^o$ which indicates that there are significant nonspectator contributions in the hadronic decays. But the experimental semileptonic inclusive rates are

$$\Gamma_{s\ell}(D^+) = \frac{B_{s\ell}^+}{\tau_D^+} \simeq \frac{0.19}{1.07 \times 10^{-12}s} \simeq 1.78 \times 10^{11}s^{-1}$$

$$\Gamma_{s\ell}(D^o) = \frac{B_{s\ell}^o}{\tau_D^o} \simeq \frac{0.077}{0.43 \times 10^{-12}s} \simeq 1.79 \times 10^{11}s^{-1}$$

This spectacular agreement is somewhat fortuitous, as we discover when we try to use the spectator model to compute this common semileptonic rate

$$\Gamma(c \to s\ell\nu) \simeq \frac{G^2 m_c^5}{192\pi^3} \mid V_{cs} \mid^2 f(x_{sc}).\ F_{QCD}$$

Again there is the sensitive dependence on m_c^5. The c-quark mass is not so well determined. For different values of m_c we obtain different values of $\mid V_{cs} \mid$

m_c	$\mid V_{cs} \mid$
1.5	1.24
1.6	1.05
1.7	0.91

From this we conclude that the spectator calculation of the inclusive semileptonic rate for D decays is good to better than a factor of 2.

We turn now to the FCNC semileptonic and radiative decays. Here the situation is not so favorable, particularly in the charm system. We first make an order of magnitude estimate of the FCNC $BR's$ from ordinary 'large distance' or soft hadronic physics. For heavy meson (B or D) weak decays, the BR to any particular hadronic final state is of the order of a percent (10^{-2}). To add a hard photon to the final state gives another factor of α/π in rate. To convert the photon into an $\ell\bar{\ell}$ pair gives another factor of α/π in rate. So the rough estimate for the 'long distance' contribution to an exclusive semileptonic BR is order $\sim 10^{-7}$. For inclusive semileptonic BR perhaps a factor ten larger i.e. $\sim 10^{-6}$. At the quark level, in the spectator approximation, one can calculate[5] the one-loop induced FCNC inclusive semileptonic ($\ell\bar{\ell}$) BR. For the b-quark the result ranges from a few times 10^{-6} up to 10^{-5} (depends on the top quark mass), which is a little larger than the above estimate of the long distance contribution. If a veto against charm in the final state is included, the above estimate of the long distance contribution to the BR for $B \to \ell\bar{\ell}X_{noc}$ goes down by another order of magnitude (to 10^{-7}) for KM suppression. For the c-quark decay the result is a few times 10^{-9}, so in this system the FCNC decays are dominated by long distance weak hadronic and hard QED physics.

We elaborate a little on these resuslts. First we provide an estimate for the short distance, one-loop induced, contributions to the FCNC decays (see Fig. 2).

$$M \sim \left(\frac{g}{2\sqrt{2}}\right)^2 \underline{V_{QQ_i} V_{q'Q_i}}\ \frac{1}{M^2}\left(\frac{1}{16\pi^2}F\right)\ (\bar{u}\cdots u)(\bar{u}\cdots u)$$

$$GIM:\ \underline{F \sim \frac{m_i^2}{M^2}} \qquad \left(or\ \ell n\frac{m_c^2}{M^2}?\cdots\right) \qquad \bar{u}\cdots u \sim 4m_Q$$

The substantial difference between the results for b-decays and for c-decays is driven by the two underlined factors (and also the lowest FCCC decays are KM suppressed for the b, which enhances the rare decay $\underline{BR}$ - and the opposite for the c-decays). Consider

$$\underset{(t)}{b \to} s\ell\bar{\ell}: \quad V_{tb}V_{ts} \sim \lambda^2, \quad \frac{m_t^2}{M^2} \sim 1 \qquad (5 \times 10^{-2})$$

$$\underset{(b,s,d)}{c \to} u\ell\bar{\ell}: \quad V_{cb}V_{ub} \sim \lambda^5, \quad \frac{m_b^2}{M^2} \sim 4 \times 10^{-3} \qquad (2 \times 10^{-6})$$

$$V_{cs}V_{us} \sim \lambda, \quad \frac{m_s^2}{M^2} \sim 3 \times 10^{-5} \qquad (6 \times 10^{-6})$$

with the indicated parameters, the above estimates for the b is

$$BR(b \to s\ell\bar{\ell}) \sim 10^{-6}$$

A complete calculation (no QCD correction) of all the Feynman diagrams gives 2 to 3×10^{-6}.

We see that there is no prospect for the one-loop induced FCNC c-decays to be observable as long as the strong GIM suppression (m_i^2/M^2) is operative. In fact there are two ways, still in the context of the minimal SM, in which the strong GIM suppression may be evaded. First, of all the one-loop diagrams, there is a one photon exchange diagram (Fig. 3) which has, after extracting two powers of q for gauge invariance, a logarithmic singularity in m_i; so it can implement the GIM mechanism in the form

$$V_{QQi}V_{q'Q_i}F(x_i) \sim \ \ell n \, \frac{m_s^2}{m_d^2} \sim 0.2 \qquad (\text{in place of } 10^{-6})$$

The dependence on m_s/m_d shows that for c-decay even the one-loop induced decays are not 'short distance'. (The heavy quark loop, $Q_i = b$, is KM suppressed i.e. $\sim \lambda^5$). If one does the calculation with just this one diagram, one obtains the result quoted above

$$BR(D \to \ell\bar{\ell}X)_{1-\text{loop}} \simeq \text{ few } \times 10^{-9}$$

Consider the decays $D \to \nu\bar{\nu}X$. Since then $\nu, \bar{\nu}$ don't couple to the photon, the one-loop contributions remain strong GIM suppressed; hence out of range; and even the radiative estimate of the long distance contribution is zero. Hence the BR for these processes in the SM must be very small.

The second path to evade the strong GIM suppression, still in the SM, is operative in the radiative FCNC decays $Q \to q'\gamma$. In the semileptonic FCNC decay $Q \to q'\ell\bar{\ell}$, the contribution from the off-shell virtual photon has two form factors F_1, F_2. The logarithm discussed above occurs in F_1; the real photon process depends only on F_2, and is still strongly GIM suppressed.

$$\Gamma(Q \to 1'\gamma) = \frac{G^2 m_Q^5}{192\pi^3}\frac{3\alpha}{2\pi} \mid \sum V_{QQ_i} V_{q'Q_i} F_2(x_i) \mid^2$$

$$F_2(x) = x(\frac{Q_i}{2} + \frac{1}{4}) + 0(x^2) \qquad Q_i = -\frac{1}{3}(\text{or } \frac{2}{3})$$

However, for the case of $b \to s\gamma$ the leading log QCD correction has been computed,[6] under the assumption that the loop is 'short distance' i.e. contains only heavy quarks and W's. Then the strong GIM is replaced by logarithmic GIM, and an additional factor of roughly α_s/π. For the $c \to u\gamma$ process the assumptions are not satisfied, but just to get some idea of a possible order of magnitude we go ahead with

$$\sum V_i V_i F_2(x_i) \sim \frac{\alpha_s}{\pi}\lambda \ell n \frac{m_s^2}{m_d^2}$$

(<u>not a calculation</u>)

This would give a contribution to the BR for $D \to \gamma X$ of order 10^{-7}, still two or three orders of magnitude less than the soft hadronic plus QED order of magnitude estimate.

SUMMARY (FCNC rare D-decys in SM)

Branching Ratios (inclusive)

	Soft hadron + QED	(one-loop)
$D \to \ell\bar{\ell}X$	$\sim 10^{-6}$	few $\times 10^{-9}$
$D \to \nu\bar{\nu}X$	*small*	$\sim 10^{-16}$
$D \to \gamma X$ (hard photon)	$\sim 10^{-4}$	$\sim 10^{-7}$?

For exclusive final states X, these numbers are decreased by roughtly 10^{-1}. (There may be roughtly ten channels open, no one dominant except possibly at the energy of a resonance. The physics involved is that when the lepton pair or hard photon carries away substantial momentum, it is hard to put the recoil quark back into a particular bound state.)

'New Physics'

(i) Observed BR greater than those in the first column of the SM summary would signal new physics.

(ii) Even if nothing is found greater than the estimates, it will be very useful to confirm the orders of magnitude expected from the ordinary hadronic weak plus QED decays, because those estimates were not specific to the c-quark system and they constitute the background to more interesting physics in the b-quark system.

(iii) The one-loop induced FCNC contributions are sensitive to new physics.

(iv) There are an arbitrarily large number of possibilities for new physics. We discuss briefly two rather straightforward extensions of the minimal SM.

(1) A fourth generation? (See Fig. 4).

A heavy b' can eliminate the GIM suppression. Compare

$$3 \text{ generation } \mathrm{b} \to \mathrm{s}\ell\bar{\ell}: \quad \mathrm{V_{tb}V_{ts}} \sim \lambda^2, \quad \frac{\mathrm{m_t^2}}{\mathrm{M^2}} \sim 1 \quad (\text{and } \mathrm{m_b^5}) \to \mathrm{BR} \sim 10^{-6}$$

4 generation $c \to u\ell\bar{\ell}$: $V_{cb'}V_{ub'} \sim ?$, $\frac{m_{b'}^2}{M^2} \sim 1,\ 10?$ (and m_c^5) — ? $(\frac{1.5}{4.9})^5 = 2.7 \times 10^{-3}$

If the pattern of small off-diagonal KM elements is maintained, we expect $V_{cb'}V_{ub'} \ll \lambda^2$. Then the fourth generation contribution to the BR for $D \to \ell\bar{\ell}X$ will be well below any observable limit (we do not consider arbitrarily large $m_{b'}/M$ because that would lead to the inapplicability of perturbation theory). If we make no assumption (other than unitarity) about the pattern of KM elements in the extension to a fourth generation, unitarity in the first row of the KM matrix requires $| V_{ub'} | \leq 0.07$. Then even with the upper bound $| V_{cb'} | <|$ and the assumption $m_{b'}^2/M^2 < 100$, we have $BR \leq 10^{-7}$. One interesting point is that with the GIM suppression removed by a heavy b' (in fact even turned into an enhancement) the one-loop contribution to $D \to \nu\bar{\nu}X$ is now the same order of magnitude as for $D \to \ell\bar{\ell}X$ (in fact, probably larger because of the four possible kinds of $\nu\bar{\nu}$ to decay into). And we have estimated the SM background to be small. Still the rate for any observablew exclusive decay mode such as $D \to \rho\nu\bar{\nu}$, coming from the presence of a fourth generation, most probably remains below the 10^{-7} level.

(2) Extended Higgs Sector.[8]

The simplest extension of the single doublet Higgs of the minimal SM is to two doublets of Higgs-bosons

$$\Phi^I \quad I = 1, 2$$

Then there are two vacuum expectation values (v.e.v.) $\frac{v_1}{\sqrt{2}}, \frac{v_2}{\sqrt{2}}$, with

$$(v_1^2 + v_2^2)^{1/2} = v, \qquad M_w = \frac{gv}{2}$$

One can make different choices of which of the neutral Higgs-bosons gets coupled to $Q = \frac{2}{3}, -\frac{1}{3}, -1$ fermions. The resulting couplings of the extra physical $H^{\pm}$ to the quarks are

$$\delta\mathcal{L}_{q,H} = \frac{g}{2\sqrt{2}}\{\frac{m_{u_i}}{M}\xi\bar{u}_i(1-\gamma_5)V_{ij}d_j$$
$$-\frac{m_{d_j}}{M}\xi'\bar{u}_i(1+\gamma_5)V_{ij}d_j\ \}\mathcal{H} + \text{h.c.}$$

Here ξ, ξ' are ratios of v.e.v. depending on the choices above. All possible choices consistent with no first order FCNC lead to one of two possibilities for ξ, ξ'

$$\text{model I}: \qquad \text{I}_\text{u} \neq \text{I}_\text{d}, \quad \xi' = -\frac{1}{\xi}$$

$$\text{model II}: \qquad \text{I}_\text{u} = \text{I}_\text{d} \quad \xi' = \xi$$

These allow one to arbitrarily enhance/suppress the coupling of the physical $H^\pm$ to either $Q = \frac{2}{3}$ or $Q = -\frac{1}{3}$ quarks. (See Fig. 5). In particular, one could have $\xi \ll 1$ and $-\xi' = \frac{1}{\xi} \gg 1$. Then there could be a large enhancement of the one-loop induced FCNC processes in D-decay with no constraint from limits on FCNC processes in the K and B systems. Since v_1/v_2 and m_{H^+} are undetermined parameters, no prediction can be made. (In the two-Higgs-doublet models, if ξ' in the above equation is large, then the coupling of the lightest neutral Higgs (h°) to the $Q = -\frac{1}{3}$ quarks is also strongly enhanced. Then the nonobservation of $\upsilon \to h^\circ\gamma$ by CUSB would imply that $m_{h^\circ} > 9.4$ GeV.)

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[8] A complete set of formulas can be found in W. S. Hou and R. S. Willey, Nucl. Phys. B, to be published.

Fig 1

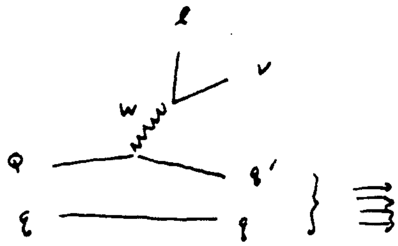

Fig 2

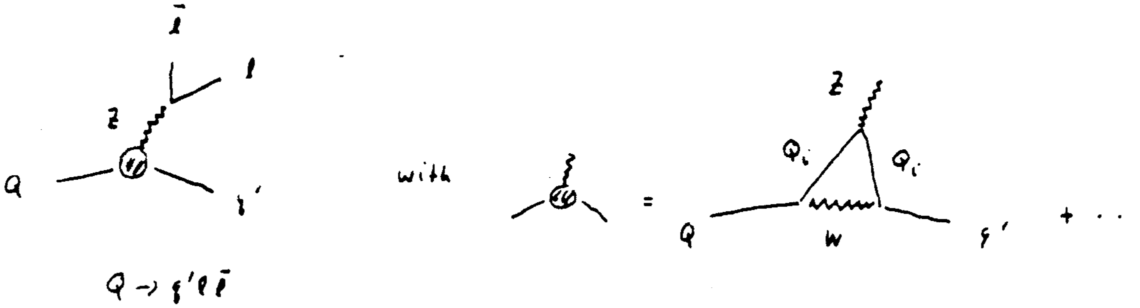

Fig 3

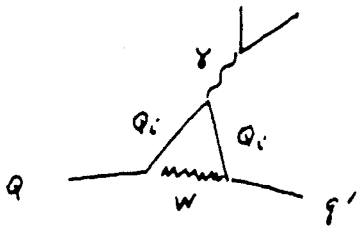

Fig 4

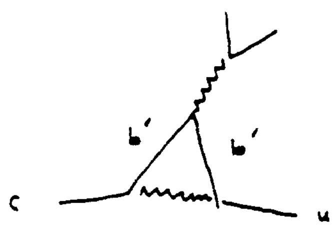

Fig 5