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EFFECTS OF STRAY MAGNETIC FIELDS AND RF COUPLER ASYMMETRY IN THE TWO-MILE ACCELERATOR WITH SECTOR FOCUSING

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by

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I. INTRODUCTION

Previous reports have dealt with the optical properties[1,2] and with the effects of misalignments and quadrupole errors [3,4] in possible focusing systems for the accelerator. The present note is concerned with the effects of stray magnetic fields and rf coupler asymmetries in the focused accelerator.

It is assumed that sector focusing will be used; that is, a quadrupole multiplet lens for each 333-1/3 foot accelerator sector. The stray forces considered here act predominantly over the space between multiplets, and the detailed structure of the multiplet is not important; consequently it is sufficient to treat the limiting case of thin circularly symmetric converging lenses (the "Singlet" approximation).[2]

II. FORMULATION

A. Periodic Focusing System

It will be assumed that the accelerator transport system consists of a number of thin lenses (quadrupole multiplets) spaced along the machine at regular intervals. For the present calculations it will be convenient to consider a basic section of the system as consisting of a thin lens of focal length F followed by a length Λ of unfocused accelerator; see Fig. 1.

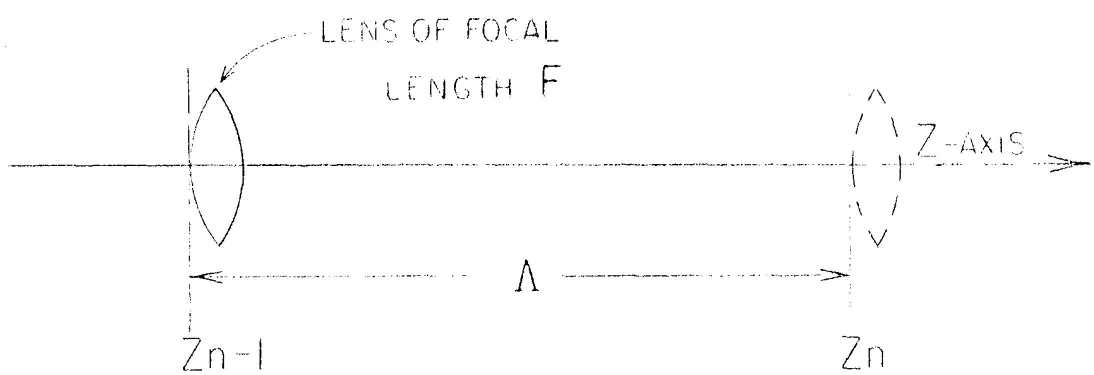

FIG. 1--Basic focusing section

The transformation over one section, from z_{n-1} to z_n, is given for an unperturbed system by

$$\mathbf{x}_n \equiv \begin{pmatrix} x \\ p \end{pmatrix}_n = \mathbf{A}\mathbf{x}_{n-1} \tag{1}$$

where x and p are a transverse coordinate-momentum pair and $\mathbf{A}$ is the transformation matrix. In the present system,

$$\mathbf{A} = \begin{pmatrix} 1 & \lambda \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\gamma/F & 1 \end{pmatrix} = \begin{pmatrix} 1 - \frac{\lambda\gamma}{F} & \lambda \\ -\gamma/F & 1 \end{pmatrix} \tag{2}$$

where γ is the energy in units of mc^2 (or total momentum in units of mc, presuming $\gamma >> 1$), and

$$\lambda \equiv \int_{z_{n-1}}^{z_n} \frac{dz}{\gamma(z)} \tag{3}$$

<u>*For negligible energy change*</u> from z_{n-1} to z_n,

$$\lambda = \frac{\Lambda}{\gamma} \tag{3a}$$;

<u>for constant acceleration</u>,

$$\lambda = \frac{1}{E} \log \frac{\gamma_n}{\gamma_{n-1}} \tag{3b}$$

where $E \equiv \frac{d\gamma}{dz}$ is the equivalent accelerating field in units of $mc^2/e/cm$ ($mc^2/e = 0.511$ Mv).

It is shown in Ref. (2) that in the case where energy is nearly constant over a section, a suitable approximation for the focal length of a quadrupole multiplet is expressed by

$$\frac{\gamma}{F} \approx \frac{4\gamma_c^2}{\Lambda\gamma} \tag{4}$$

where γ_c is the low-energy cut-off for the periodic transport system.

It is assumed implicitly that γ_c, Λ, and γ may vary slowly (adiabatically) as functions of n.

B. Effect of Stray Magnetic Field

In the absence of other forces, the effect of a magnetic field on the x-component of motion is given to first order by

$$\frac{d}{dz}\left(\gamma \frac{dx}{dz}\right) = B \tag{5}$$

where B is the y-component of magnetic field, in units of $mc^2/e/cm$ (mc^2/e = 1703 gauss-cm). The effect of longitudinal field is reduced by a factor of dy/dz and is therefore negligible for weak field and small angular divergence.

On the assumption that B and γ are essentially constant over a section of the focusing system, one can integrate Eq. (5) to obtain the orbit perturbation from z_{n-1} to z_n; the result is

$$\delta \mathbf{x}_n \equiv \begin{pmatrix} \delta x \\ \delta p \end{pmatrix}_n \approx \begin{pmatrix} \frac{1}{2}\frac{\Lambda}{\gamma} \\ 1 \end{pmatrix} B\Lambda \tag{6a}$$

where δx, δp are the perturbations of x and p, and the remaining notation is the same as is used previously. It is assumed implicitly that the parameters Λ, γ, B, etc., may vary adiabatically with n.

If the energy gain is not negligible, but B is still essentially constant, the expression is

$$\delta \mathbf{x}_n = \begin{bmatrix} \frac{1}{E}\left(1 - \frac{\gamma_{n-1}}{E\Lambda} \log \frac{\gamma_n}{\gamma_{n-1}}\right) B\Lambda \\ 1 \end{bmatrix} \tag{6b}$$

The complete transformation over one transport section now is

$$\mathbf{x}_n = \mathbf{A}\mathbf{x}_{n-1} + \delta\mathbf{x}_n \tag{7}$$

with $\mathbf{A}$ given by Eq. (2) and $\delta\mathbf{x}_n$ given by Eq. (6a) or (6b).

Solutions of such linear inhomogeneous equations in periodic or almost periodic systems are discussed, e.g., in Refs. (3) and (4).

A useful function is the complex perturbation vector defined by[3]

$$\alpha_n = \left[\boldsymbol{\omega}, \delta\mathbf{x}\right]_n \tag{8}$$

where

$$\boldsymbol{\omega} \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \frac{i}{\sin\theta} \begin{pmatrix} \frac{1}{2}(a_{11} - a_{22}) \\ a_{12} \end{pmatrix} \tag{9}$$

$$\cos\theta \equiv \tfrac{1}{2}(a_{11} + a_{22}) \tag{10}$$

and a_{ij} are the elements of the matrix $\mathbf{A}$. The cumulative perturbation of the orbits after n sections then is given by[3]

$$v_n \approx \left(\frac{a_{12}}{\sin\theta}\right)_n^{\frac{1}{2}} \sum_{m=1}^{n} \left(\frac{\sin\theta}{a_{12}}\right)_m^{\frac{1}{2}} \alpha_m e^{i(\mu_n - \mu_m)} \tag{11}$$

where

$$\mu_n \equiv \sum^{n} \theta_n$$

and $\Delta x_n = \mathrm{Re}(v_n)$ is the final deflection of the beam.

If the stray field B_n varies randomly along the machine with no correlation between different sections, then the mean square deflection of the beam is given by[3]

$$\overline{\Delta x_n^2} \approx \tfrac{1}{2} \overline{|v_n|^2} \approx \tfrac{1}{2} \left(\frac{a_{12}}{\sin\theta}\right)_n \sum_{m=1}^{n} \delta \bar{u}_m \tag{12}$$

where

$$\delta \bar{u}_m = \left(\frac{\sin\theta}{a_{12}}\right)_m \overline{|\alpha_m|^2}$$

III. TYPICAL CALCULATIONS

A. Magnetic Field Constant Over Length of Machine

A component of field essentially constant over the length of the machine might arise from fluctuations in the earth's field or from power lines paralleling the machine.

1. Moderate Focusing, Constant Beam Energy

The term moderate focusing will be used to denote the condition under which the lenses at sector intervals are all at essentially equal strength corresponding to a nominal low energy cut-off of perhaps one or two BeV.

The formulation of Section II (using the approximation of Eq. (6a) for the perturbation for one sector) gives the complex perturbation vector; this turns out to be

$$\alpha_n \approx B\Lambda \left[\frac{F}{\gamma}\left(1 - e^{i\theta}\right)\right]_n \tag{13a}$$

with

$$\cos\,\theta_n = 1 - \frac{1}{2}\left(\frac{\lambda\gamma}{F}\right)_n \tag{14a}$$

The characteristic form of the focal length of a quadrupole multiplet, Eq. (4), gives

$$\alpha_n \approx \left[\frac{B\Lambda^2\gamma}{4\gamma_c^2}\left(1 - e^{i\theta}\right)\right]_n \tag{13b}$$

$$\cos\,\theta_n = 1 - 2\left(\frac{\gamma_c}{\gamma}\right)_n^2 \tag{14b}$$

Since all the parameters are assumed constant in the present case, Eq. (11) becomes

$$\begin{aligned} v_n &\approx \frac{B\Lambda^2\gamma}{4\gamma_c^2}\left(1 - e^{i\theta}\right)\sum_1^n e^{i(n-m)\theta} \\ &\approx \frac{B\Lambda^2\gamma}{4\gamma_c^2}\left(1 - e^{in\theta}\right) \end{aligned} \tag{15}$$

Hence the maximum deflection of the beam, on the assumption that $n\theta \gtrsim \pi$, is

$$|\Delta x|_{max} = |v_n|_{max} \approx \frac{|B|\Lambda^2\gamma}{2\gamma_c^2} \tag{16}$$

Magnetic field tolerance is given by transposing (16) and converting to conventional units;

$$|B(\text{gauss})| \lesssim \frac{10^7}{3}\,\frac{2|\Delta x|_{max}V_c^2}{\Lambda^2 V} \tag{17}$$

where V and V_c are energy and cutoff energy in BeV; lengths are in cm.

As a numerical example, take $V_c = 1.4$ BeV, $V = 3$ BeV, $|\Delta x|_{max} = 0.1$ cm, and $\Lambda = 10^4$ cm (one sector). Then the tolerance on stray field is $|B| \lesssim 4.4 \times 10^{-3}$ gauss.

2. Moderate Focusing, Constant Acceleration

Note that in the preceding case the amplitude of the deflection increases with energy. Hence to set the field tolerance we need to consider the case of constant acceleration.

Equations (13b) and (14b) still apply, with Λ, γ_c, and B assumed constant; but now

$$\gamma_n = \gamma_o + nE\Lambda \tag{18}$$

Furthermore, over most of the machine we will have

$$\gamma_n \gg \gamma_c$$

whence the approximation

$$\theta_n \approx \sin \theta_n \approx \frac{2\gamma_c}{\gamma_n} \ll 1 \tag{19}$$

will be valid. Also

$$\left(\frac{a_{12}}{\sin \theta}\right)_n \approx \frac{\Lambda/\gamma_n}{\theta_n} \approx \frac{\Lambda}{2\gamma_c} = \text{constant}$$

Equation (11) now gives

$$v_n \approx -i \frac{B\Lambda^2}{2\gamma_c} \sum_{m=1}^{n} e^{i(\mu_n-\mu_m)} \tag{20}$$

where

$$\mu_n \approx 2\gamma_c \sum_{1}^{n} \frac{1}{\gamma_m}$$

The summations may be approximated as integrals, with the identification of $E\Lambda$ as $d\gamma$. The results are

$$\mu_n \approx \frac{2\gamma_c}{E\Lambda} \log \frac{\gamma_n}{\gamma_o}$$

$$v_n \approx \frac{B\Lambda}{2E} \frac{\gamma_n}{\gamma_c} \frac{\left(\frac{2\gamma_c}{E\Lambda} - i\right)\left(1 - \frac{\gamma_o}{\gamma_n} e^{i\mu_n}\right)}{1 + \left(\frac{2\gamma_c}{E\Lambda}\right)^2} \tag{21}$$

Hence for $\gamma_n >> \gamma_o$,

$$\Delta x_n = \mathrm{Re}(v_n) \approx \frac{B\Lambda^2\gamma_n}{(E\Lambda)^2 + 4\gamma_c^2} \tag{22}$$

Stray magnetic field tolerance in this case (constant parameters, constant acceleration) thus is given in conventional units by

$$|B(\text{gauss})| \lesssim \frac{10^7}{3} \frac{\Delta x_{max}}{\Lambda^2 V_n} \quad (\Delta V)^2 + 4V_c^2 \tag{23}$$

where V_n, V_c, ΔV are final energy, cutoff energy and energy gain per sector, all in BeV; lengths are in cm.

Numerical Example: Take $V_c = 1.4$ BeV (corresponding to a practical low energy limit of 2 BeV);[2] $\Delta V = 0.7$ BeV/sector (Stage I), $V_n = 20$ BeV,

$\Lambda = 10^4$ cm, and $|\Delta x|_{max} = 0.1$ cm. Stray field tolerance then is

$$|B| \lesssim 1.4 \times 10^{-3} \text{ gauss}$$

If $V_n \gg V_o$, so that $V_n \approx n\Delta V$, then Eq. (23) has a minimum at

$$\Delta V^* = 2V_c$$

In the above example this corresponds to $V_n^* = 84$ BeV, so that in the moderate focusing case it is seen that the stray field tolerance is determined by the highest energy at which the machine can operate.

3. Minimal Focusing, Constant Energy

By minimal focusing is meant use of focusing just strong enough to contain the phase space of the beam. In the present context "phase space" is defined, in terms of the parameters indicated in Fig. 2, by

$$u = p_o x_{max} = p_{max} x_o = \frac{1}{\pi} \text{ (area of effective ellipse)}$$

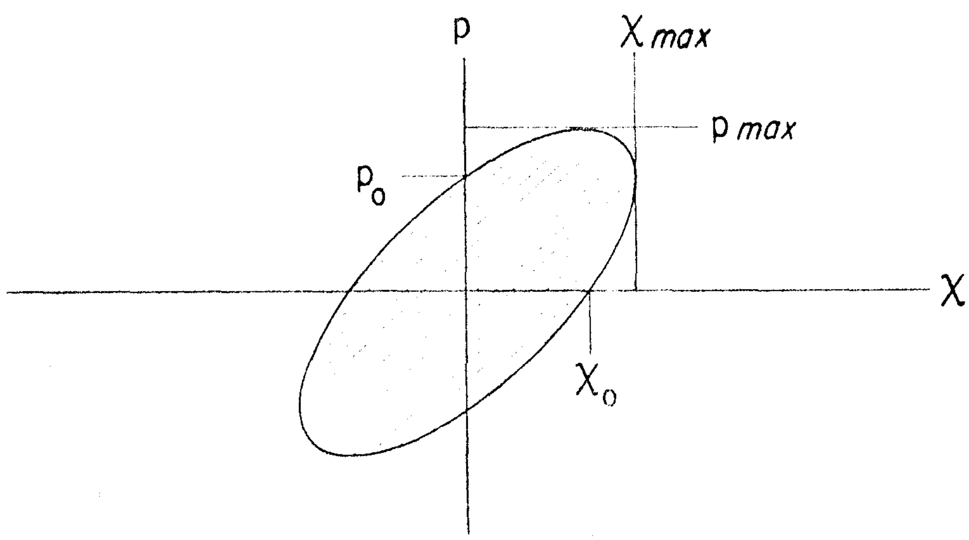

FIG. 2--Illustrating "Phase Space." The shaded ellipse is the (x, p_x) section of a 4-dimensional ellipsoid, in (x, p_x, y, p_y) space, which contains the phase coordinates of all electrons in the beam.

The phase space admittance of the periodic focusing system is given by[2]

$$\Upsilon = a^2 \sqrt{\frac{\gamma}{F\Lambda}\left(1 - \frac{1}{4}\frac{\gamma\Lambda}{F}\right)} \tag{24a}$$

$$\approx \frac{2a^2\gamma_c}{\Lambda} \sqrt{1 - \left(\frac{\gamma_c}{\gamma}\right)^2} \tag{24b}$$

where a = effective radial aperture of the transport system. The second form assumes quadrupole multiplet lenses and constant (or adiabatically varying) energy and parameters.

The minimal focusing condition then is given by setting

$$\Upsilon = u \tag{25}$$

There are two possibilities: we could keep the lens spacing Λ constant and choose γ_c to satisfy Eq. (25); or we could choose γ_c to maximize the lens spacing and then use Eq. (25) to fix Λ. The former condition will be assumed here, because it is more compatible with the requirement of broad band transmission imposed by multiple beams. Then if Eq. (24b) holds, the cutoff energy is given by

$$\gamma_c = \gamma \sqrt{\frac{1}{2}\left[1 - \sqrt{1 - \left(\frac{\Lambda u}{a^2\gamma}\right)^2}\right]} \tag{26a}$$

But under typical conditions $\gamma \gtrsim 2 \times 10^3$, $a \approx 1$ cm, and $\Lambda = 10^4$ cm; whereas the phase space of the beam is typically $u \lesssim .02$ (which corresponds to beam size of ± 0.4 cm, angular divergence of ± 1 milliradian at 25 MeV). Hence

$$\frac{\Lambda u}{a^2\gamma} \lesssim 0.1$$

so that to a sufficient approximation, Eq. (26a) becomes

$$\gamma_c \approx \frac{\Lambda u}{2a^2} \tag{26b}$$

The typical numbers assumed above give $\gamma_c = 100$ or $V_c = .05$ BeV.

Equation (16) does not necessarily apply here because, with minimal focusing strength, the half wavelength of electron orbits may be greater than the total length of the machine. In this case we should use Eq. (15), from which

$$\Delta x_n = \mathrm{Re}(v_n) \approx \frac{B\Lambda^2\gamma}{2\gamma_c^2} \sin^2 \frac{1}{2} n\theta$$

$$\approx 2B\gamma \left(\frac{a^2}{u}\right) \sin^2 \left(\frac{n\Lambda u}{2a^2\gamma}\right) \tag{27}$$

where the approximations of Eqs. (19) and (26b) have been used. Hence the tolerance on stray field in the case of minimal focusing and constant energy, in conventional units, is

$$\left|B(\text{gauss})\right| \lesssim \frac{10}{3} \frac{\Delta x_{max}}{2V} \left(\frac{U}{a^2}\right)^2 \csc^2 \left(\frac{n\Lambda U}{2 \cdot 10^3 a^2 V}\right) \tag{28a}$$

$$\left(\text{if} \quad n\theta \approx \frac{n\Lambda U}{10^3 a^2 V} \lesssim \pi\right);$$

or

$$\left|B(\text{gauss})\right| \lesssim \frac{10}{3} \frac{\left|\Delta x\right|_{max}}{2V} \left(\frac{U}{a^2}\right)^2 \tag{28b}$$

$$(\text{if} \quad n\theta \gtrsim \pi);$$

where V = energy in BeV and

U = beam phase space in units of $\frac{\text{MeV}}{c}$ - cm.

Numerical Example: Take $V = 2$ BeV, $|\Delta x|_{max} = 0.1$ cm, $a = 1$ cm, and $U = .01 \frac{\text{MeV}}{c}$ - cm (which is the same as the above assumption of beam size equals ±.4 cm, angular divergence equals ±1 milliradian at 25 MeV). A reasonable value of n is $n = 27$, which allows 3 sectors for the beam to get up to 2 BeV.

In this case

$$n\theta \approx \frac{n\Lambda U}{10^3 a^2 V} \approx 1.35 < \pi;$$

consequently Eq. (28a) applies, giving

$$|B| \lesssim 2.1 \times 10^{-5} \text{ gauss}$$

4. Minimal Focusing, Constant Acceleration

The definition of minimal focusing is the same as in the preceding section; Eq. (26b) again applies for the cutoff energy. Substitution of Eq. (26b) in (22), on the assumption that $\gamma_n >> \gamma_o$, gives

$$\Delta x_n \approx \frac{\gamma_n}{E^2} \frac{B}{1 + \left(\frac{u}{Ea^2}\right)^2} \tag{29}$$

The stray magnetic field tolerance is

$$|B(\text{gauss})| \lesssim \frac{10^7}{3} \frac{|\Delta x|_{max}}{\Lambda^2} \frac{(\Delta V)^2}{V_n} \left[1 + \left(\frac{U\Lambda}{10^3 a^2 \Delta V}\right)^2\right] \tag{30}$$

where V_n and ΔV are final energy and energy gain per sector, both in BeV, and U is phase space of beam in $\left(\frac{\text{MeV}}{c}\right)$- cm.

Numerical Examples:

(a) Again taking $V_n = 20$ BeV, $\Delta V = 0.7$ BeV/sector, $|\Delta x| = 0.1$ cm, $a = 1$ cm and $U = .01 \left(\frac{\text{MeV}}{c}\right)$ - cm, we find

$$|B| \stackrel{\sim}{<} 8.3 \times 10^{-5} \left[1 + (0.14)^2\right] = 8.5 \times 10^{-5} \text{ gauss}$$

(b) As a second example, take $V_n = 3$ BeV, $\Delta V = 0.1$ BeV/sector, and the other parameters the same as in (a). Then

$$|B| \stackrel{\sim}{<} 1.1 \times 10^{-5} \left[1 + (1.0)^2\right] = 2.2 \times 10^{-5} \text{ gauss}$$

Example (b) corresponds (for $n = 30$) to the minimum value of $|B|$ as defined by Eq. (30); the minimum occurs for

$$\Delta V^* = 2V_c = \frac{\Delta U}{10^3 a^2}$$

which gives $V_n^* = n\Delta V^* = 3$ BeV with the present parameters.

B. Random Longitudinal Variations in Stray Field

The following calculations treat specifically the case where the stray magnetic field is essentially constant over the space between lenses, but has random uncorrelated variations from section to section. Such a situation might arise, e.g., from errors and regulation fluctuations in degaussing the individual sectors.

One can also imagine a field which is perturbed by randomly dispersed power lines, magnetic materials, etc., in which case the longitudinal range, as well as the strength of the perturbing field, might be random. This situation may be at least qualitatively handled by the present formulation by replacing the mean square field, $\overline{B^2}$, by an effective value

$$(\overline{B^2})_{eff} \approx \frac{\Delta Z}{\Lambda} (\overline{B^2})_{actual}$$

where ΔZ is an effective mean range of the field perturbations.

1. Moderate Focusing, Constant Beam Energy

The definitions are the same as in Section III.A.1. Equations (13b) and (14b) again apply. Since the parameters and energy are constant, the mean square deflection of the beam as given by Eq. (12) becomes simply

$$\overline{\Delta x_n^2} \approx \frac{1}{2}\, n\,\overline{|\alpha|^2} \approx \frac{n}{8}\,\frac{\overline{B^2}\Lambda^4}{\gamma_c^2} \tag{31}$$

The field tolerance in this case then is

$$< B(\text{gauss}) >_{rms} \lesssim \frac{10^7}{3}\,\frac{|\Delta x|_{max} V_c}{\Lambda^2}\sqrt{\frac{8}{n}} \tag{32}$$

with V_c = cutoff energy in BeV.

Numerical Example: $|\Delta x|_{max} = 0.1$ cm, $\Lambda = 10^4$ cm, $V_c = 1.4$ BeV, and $n = 30$; then the tolerance is

$$< B >_{rms} \lesssim 2.4 \times 10^{-3} \text{ gauss}$$

2. Moderate Focusing, Constant Acceleration

In this case Eqs. (13b) and (14b) again apply. Equation (12) then gives

$$\overline{\Delta x_n^2} \approx \frac{1}{8}\,\frac{\overline{B^2}\Lambda^4}{\gamma_c^2}\sqrt{1 - \frac{\gamma_c^2}{\gamma_n^2}}\sum_{m=1}^{n}\frac{\gamma_m}{\sqrt{\gamma_m^2 - \gamma_c^2}}$$

The summation may be approximated as an integration by identifying $E\Lambda$ as $d\gamma$; the result is

$$\overline{\Delta x_n^2} \approx \frac{1}{8} \frac{\overline{B^2}\Lambda^4}{\gamma_c^2} \frac{\gamma_n}{E\Lambda} \sqrt{1 - \frac{\gamma_c^2}{\gamma_n^2}} \left\{ \sqrt{1 - \frac{\gamma_c^2}{\gamma_n^2}} - \frac{\gamma_o}{\gamma_n} \sqrt{1 - \frac{\gamma_c^2}{\gamma_o^2}} \right\} \tag{33a}$$

where $\gamma_n = \gamma_o + nE\Lambda$; or, assuming $\gamma_n >> \gamma_o > \gamma_c$, we have

$$\overline{\Delta x_n^2} \approx \frac{n}{8} \frac{\overline{B^2}\Lambda^4}{\gamma_c^2} \tag{33b}$$

which is identical to the constant energy result [Eq. (31)]. Hence Eq. (32) again applies for the field tolerance, and the example in the previous section,

$$< B >_{rms} \lesssim 2.4 \times 10^{-3} \text{ gauss},$$

holds for the accelerated beam as well as for the constant energy case.

3. Minimal Focusing

If we use Eq. (26b) to estimate γ_c for minimal focusing as discussed in Sec. III.A.3, then Eq. (31) becomes

$$\overline{\Delta x_n^2} \approx \frac{n}{2} \overline{B^2}\Lambda^2 \left(\frac{a^2}{u}\right)^2 \tag{34}$$

The field tolerance then is

$$< B(\text{gauss}) >_{rms} \lesssim \frac{10^4}{3} \frac{|\Delta x|_{max} U}{\Lambda a^2} \sqrt{\frac{2}{n}} \tag{35}$$

where U is phase space of the beam in units of $\left(\frac{\text{MeV}}{c}\right)$ - cm.

Numerical example: Taking $U = .01 \left(\frac{\text{MeV}}{c}\right)$ - cm, as in Sec. III.A.3, $|\Delta x|_{max} = 0.1$ cm, $\Lambda = 10^4$ cm, $a = 1$ cm, and $n = 30$, we find

$$< B >_{rms} \lesssim 8.5 \times 10^{-5} \text{ gauss}$$

C. Coupler Asymmetry

A previous report[5] discusses the effect of asymmetry in the rf couplers which feed power into the accelerator. A transverse gradient of the accelerating field in the vicinity of the coupler is shown to introduce a transverse momentum impulse which when averaged over the length between feeds results in a net transverse force expressed by[5]

$$\frac{d}{dz}\left(\gamma \frac{dx}{dz}\right) \approx \frac{\sin \Phi}{kL} \frac{\partial E}{\partial x} \delta z \qquad (36)$$

where Φ is the phase angle of the electron, relative to the accelerating crest; $k = \frac{\omega}{c}$ is the propagation constant of the accelerating wave; L is the distance between couplers; $\frac{\partial E}{\partial x}$ is the average transverse gradient of the accelerating field amplitude in the coupler cavity; δz is the effective longitudinal extent of the asymmetry, assumed to be on the order of one cavity. E is measured in units of $mc^2/e/cm = .511$ Mv/cm, as in the previous sections.

Comparison of Eq. (36) with Eq. (5) shows that, if the acceleration is constant, the effect of coupler asymmetry is exactly analogous to that of a constant tranverse magnetic field. Consequently the results of Sections III.A.2 and III.A.4 apply, with the right-hand side of Eq.(36) substituted for B.

1. Moderate Focusing

By analogy with Eq. (22) the deflection of the beam by coupler asymmetry is

$$\Delta x_n \approx \frac{\Lambda^2 \gamma_n}{(E\Lambda)^2 + 4\gamma_c^2} \frac{\sin \Phi}{kL} \frac{\partial E}{\partial x} \delta z \qquad (37)$$

The tolerance on coupler asymmetry, in terms of relative variation of the field across the coupler aperture, then is

$$\left| \frac{2a}{E} \frac{\partial E}{\partial x} \right| \lesssim \frac{2kaL}{\Lambda \delta z} \frac{|\Delta x|_{max}}{\sin \Delta} \frac{\Delta V}{V_n} \left(1 + \frac{4V_c^2}{\Delta V^2} \right) \tag{38}$$

where Δ is the total phase-angle spread of the rf bunch.

Numerical Example: Parameters appropriate to SLAC are taken as $k = 2\pi/10.5 = 0.6\ cm^{-1}$, $a = 1$ cm, $L = 300$ cm, $|\Delta x|_{max} = 0.1$ cm, $\Lambda = 10^4$ cm (one sector), $\delta z = 3.5$ cm (1/3 wave length), $\Delta = 0.1$ radian, $\Delta V = 0.7$ BeV/sector, $V_n = 20$ BeV, and $V_c = 1.4$ BeV. The coupler asymmetry tolerance then would be

$$\left| \frac{2a}{E} \frac{\partial E}{\partial x} \right| \lesssim 0.58 \times 10^{-2} = .58\%$$

This is about an order of magnitude less than the asymmetries in old-type Stanford couplers.[6] Symmetrized couplers, in which the relative asymmetry is $\lesssim 0.1\%$, have been developed.[7]

2. Minimal Focusing

The above result [Eq. (38)] applies, with cutoff energy given by Eq. (26b). The beam deflection then is

$$\Delta x_n \approx \frac{\gamma_n}{E^2} \frac{1}{1 + \left(\frac{u}{Ea^2}\right)^2} \frac{\sin \Phi}{kL} \frac{\partial E}{\partial x} \delta z \tag{39}$$

The tolerance on coupler asymmetry is

$$\left| \frac{2a}{E} \frac{\partial E}{\partial x} \right| \lesssim \frac{2kaL|\Delta x|_{max}}{\Lambda \delta z \sin \Delta} \frac{\Delta V}{V_n} \left[1 + \left(\frac{U\Lambda}{10^3 a^2 \Delta V} \right)^2 \right] \tag{40}$$

Numerical Examples:

(a) In this case, let $U = 0.01 \frac{\text{MeV}}{c}$ - cm be used for the phase space of the beam, as in previous examples (e.g., Sec. III.A.3); the other parameters are $k = 0.6\ \text{cm}^{-1}$, $a = 1$ cm, $L = 300$ cm, $|\Delta x|_{max} = 0.1$ cm, $\Lambda = 10^4$ cm, $\delta z = 3.5$ cm, $\Delta = 0.1$ radian, $\Delta V = 0.7$ BeV/sector, and $V_n = 20$ BeV. Then Eq. (40) gives

$$\frac{2a}{E}\frac{\partial E}{\partial x} \lessapprox 3.5 \times 10^{-4} = .035\%$$

(b) As a second example, suppose that we anticipate an asymmetry of 0.1% and wish to specify a suitable focusing strength (the other parameters are the same as in the preceding examples). Solution of Eq. (38) for V_c then gives

$$V_c \gtrapprox 0.5 \text{ BeV}$$

Hence the admittance of the system should be

$$\frac{2 \times 10^3 V_c a^2}{\Lambda} \approx 0.1 \frac{\text{MeV}}{c} \text{ - cm}$$

IV. SUMMARY AND COMMENTS

Tolerances on stray magnetic fields and on coupler asymmetry have been calculated for two representative focusing conditions:

(1) Moderate focusing, corresponding to a low-energy cutoff of 1.4 BeV; and

(2) Minimal focusing, or just enough admittance to transport the initial phase space of the beam (a reasonable beam phase space is assumed to be $.01 \frac{\text{MeV}}{c}$ - cm, corresponding to 0.4 cm radius and $\pm 10^{-3}$ radian angular divergence at 25 MeV).

In both cases it is assumed that the transport system consists of quadrupole multiplets at sector (333-1/3 feet) intervals.

Table I summarizes the results, based on a tolerable beam deflection of 0.1 cm for any given effect.

TABLE I

STRAY MAGNETIC FIELD AND COUPLER ASYMMETRY TOLERANCES
IN SECTOR-FOCUSED 2-MILE ACCELERATOR (See Text for Explanation)

Perturbing Effect	Text Ref.	Tolerance (for 0.1 cm deflection)	Most Restrictive Beam Conditions (Stage I)
Uniform Magnetic Field:			
Moderate focusing	III.A.2	1.4×10^{-3} gauss	Constant acceleration to 20 BeV
Minimal focusing	III.A.3	2.1×10^{-5} gauss	Constant minimum energy (2 BeV)
Longitudinally Random Magnetic Field (rms tolerances):			
Moderate focusing	III.B.1	2.4×10^{-3} gauss	Approx. independent of energy
Minimal focusing	III.B.2	8.5×10^{-5} gauss	Approx. independent of energy
Coupler Asymmetry:			
Moderate focusing	III.C.1	0.58%	Constant acceleration to 20 BeV
Minimal focusing	III.C.2	0.035%	Constant acceleration to 20 BeV

By way of comparison, some estimates may be given for possible magnitudes of the various effects:

1. Uniform Magnetic Field

Most probable sources:

(a) Diurnal variations in earth's field; typically[8] 2×10^{-3} to 3×10^{-3} gauss variations during a 24 hour period, with occasional variations of $\sim 10 \times 10^{-3}$ gauss associated with sunspot activity.

(b) AC field of power lines* paralleling the machine; estimated[9] as $\sim 0.17 \times 10^{-3}$ gauss.

If the machine is magnetically shielded by a factor of 10 to 20, both effects would be within tolerance in the moderate focusing case. A shielding factor of 100 to 1000 would be required in the minimal-focusing case.

2. Random Magnetic Field

Most probable source: regulation fluctuations in the degaussing currents. A regulation of 0.1% would result in $\sim 0.4 \times 10^{-3}$ gauss random variation from sector to sector.

3. Coupler Asymmetry

Couplers which have asymmetry of $\lesssim 0.1\%$ have been developed.[7] It is doubtful that measurement techniques are precise enough to measure gradients much smaller than this.

In Table II these estimates are compared to the tolerances.

TABLE II

COMPARISON OF TOLERANCES WITH ESTIMATES OF POSSIBLE MAGNITUDES

Perturbing Effect	Tolerance		Estimated Magnitude
	Moderate Focusing	Minimal Focusing	
Uniform Magnetic Field	$1.4 \times 10^{[illegible]}$ gauss	2.1×10^{-5} gauss	3×10^{-3} gauss (typ. daily variations) 10×10^{-3} gauss (sunspot activity)
Random Magnetic Field	2.4×10^{-3} rms gauss	8.5×10^{-5} rms gauss	0.4×10^{-3} rms gauss
Coupler Asymmetry	0.58%	0.035%	$\lesssim 0.1\%$

*The ac frequency is 60 cps while the machine is to operate at up to 360 pps.

Several conclusions may be drawn from this comparison:

1. In the case of Moderate Focusing
   (a) Magnetic shielding of the accelerator with an effective shielding factor of ~ 10 should be sufficient to reduce all expected stray fields to within tolerance.
   (b) Coupler asymmetry probably would be within tolerance with a considerable safety factor.
2. In the case of Minimal Focusing
   (a) A magnetic shielding factor of 10 would bring the probable random field effect within tolerance.
   (b) The geomagnetic field variation would need to be reduced by a factor of several hundred. Magnetic shielding by such a factor would not be practical;* some other form of compensation such as automatic steering, automatic degaussing, or frequent manual steering, would be required.
   (c) The coupler asymmetry effect would be marginal, at best.

It appears that the minimal focusing condition which has been used in the numerical examples is a little too weak. For example it is shown in Sec. III.C that to attain at coupler asymmetry tolerance of 0.1%, we need to specify the admittance of the system as 0.1 $\frac{\text{MeV}}{c}$ - cm (rather than the figure of 0.01, assumed in the minimal focusing examples). In this case we would have

$$V_c = 0.5 \text{ BeV}$$

$$\text{Uniform field tolerance} \approx 2.2 \times 10^{-4} \text{ gauss}$$

$$\text{Random field tolerance} \approx 8.6 \times 10^{-4} \text{ gauss}$$

If magnetic shielding by a factor of 10 were employed, some steering would still be necessary during sunspot activity.

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*In the SLAC accelerator, only about 95% of the total length can be shielded, so that an upper limit on the effective shielding factor is about 20.

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