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HEAVY MESONS AND EXCITED BARYONS

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by

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(These notes are based on a series of lectures given in October and November, 1962. The notes have been revised to include a few additional topics and some new information.)

1. INTRODUCTION

Much of the current work in high energy physics is concerned with the spectroscopy of the mesons and baryons. That is, we are interested in the mass spectra of these particles, their quantum numbers, their lifetimes, and their decay modes. Hopefully, this information will provide clues for a future theory of these particles.

We shall be concerned here chiefly with the heavy meson and excited baryon states. Much of our emphasis will be on how conservation laws can be applied to obtain the quantum numbers of these states. We shall also call attention to some of the particles whose quantum numbers have not yet been determined and shall discuss some evidence for particles whose existence has not been confirmed.

We have not made a systematic search of the literature for information on particles and resonances. Therefore, we shall undoubtedly omit to mention some particles for which evidence has been reported and also some work which has been important in establishing the properties of the known particles. We apologize for these omissions.

In Tables I and II the known mesons plus some suspected ones are listed, and some empirical information about them is given. In Tables III, IV and V are listed the known and suspected baryons. For completeness, we list in Table VI other known and suspected particles.

References 1 through 8a are useful for general information about the application of conservation laws to particle physics and about the mesons and baryons.

2. SOME METHODS FOR DETERMINING SPINS OF PARTICLES

Consider the reaction $A + B \rightarrow C + D$ where both the incident and target particles are unpolarized.

a) If C and D are produced with a maximum angular momentum J and if the polarization of C and D is not measured, then the highest power of Θ that can occur in the production angular distribution $W(\Theta)$ is* $\cos^{2J}\Theta$.

*We use Θ to denote production angles and θ to denote decay angles.

TABLE I

Established and reported mesons with strangeness S = 0

Meson*	Isospin, Spin, Parity, G Parity I, J^{PG}	Average $Mass^2$ Bev^2	Mass Mev	Width** Mev (or Mean Life, Sec)	Principal Decay Modes	%	Ref.
π $\left({}_{-}\pi_{\beta}\right)$	1, 0^{--}	0.019	139.6 ($\pi^{\pm}$) 135.0 (π^0)	(2.55×10^{-8}) (2×10^{-16})	$\mu\nu$ 2γ	≈ 100 ≈ 100	8
η $\left({}_{+}\chi_{\beta}\right)$	0, 0^{-+}	0.30	548	< 10	$\pi^+\pi^-\pi^0$ 2γ $3\pi^0$? $\pi^-\gamma$	25 40 6	28, 26 29, 20 31, 32 30
$\rho\left({}_{+}\pi_{\gamma}\right)$	1, 1^{-+}	0.56	$\rho^{\pm}$ 750 ρ^0 750	100 100	$\pi^+\pi^0$ $\pi^+\pi^-$	≈ 100 ≈ 100	18, 20 19
$\omega\left({}_{-}\chi_{\gamma}\right)$	0, 1^{--}	0.62	785	< 15	$\pi^+\pi^-\pi^0$ $\pi^+\pi^-$ $\pi^0\gamma$	80 5 15 ±5	24, 25 20, 26
ABC $\left({}_{+}\chi_{\alpha}?\right)$	0, 0^{++}?	0.10	320	< 25	$\pi^+\pi^-$?		50, 51 52
φ $\left({}_{-}\chi'_{\gamma}?\right)$	0, 1^{--}?	1.04	1020		K_1K_2		46
ξ $\left({}_{+}\chi'_{\alpha}?\right)$	0, 0^{++}?	1.04	1020		K_1K_1		44, 45
ζ? $\left({}_{-}\pi_{\alpha}?\right)$	1, 0^{+-}?	0.32	570 ($\zeta^{\pm}$)	< 20	$\pi^{\pm}\pi^0$		53, 54
			570 (ζ^0)	< 20	$\pi^+\pi^-$?		56
f^0 $\left({}_{+}\chi'_{\alpha 2}?\right)$	0? 2^{++}?	1.56	1250	100	$\pi^+\pi^-$		60
?	?	0.16	395	50	$\pi^+\pi^-$		57
?	?	0.27	520	70	$\pi^+\pi^-$		57
?	1, ?	0.39	625		$\pi^+\pi^-\pi$		62
			625		$\pi^+\pi^-\pi^0$		
?	2, ?	0.35	590		$\pi\pi$		61, 63
?	2, ?	0.61	780		$\pi\pi$		61
?	2, ?	0.96	980		$\pi\pi$		61
?	?	1.44	1200	$\pi\pi$	$\pi\pi$		61
?	2?	0.11	325	$\pi\pi$	$\pi\pi$		63
?	2?	0.21	460		$\pi\pi$		63
?	?	0.18	420		$\pi^+\pi^-$		59
?	2?	0.38	620		$\pi^+\pi^+$ $\pi^-\pi^-$		58, 59
?	?	1.82	1350		$\pi^+\pi^-$ $\pi^+\pi^+\pi^-\pi^-$		61, 65

*The symbol in parentheses is a notation suggested by Gell-Mann, Chew and Rosenfeld. A question mark after the old symbol for a meson means its existence has not been established; a question mark after the new symbol means its quantum numbers have not been experimentally determined. See Section 24 for explanation of new symbols.

**Full width at half maximum.

TABLE II

Mesons With S = 1

Mesons	I, J^P	Average mass² Bev²	Mass Mev	Width Mev or (mean life, sec)	Principal decay modes	%	Ref.
			$K^{\pm}$ 494	(1.22×10^{-8})	$\mu^{\pm}\nu$	64	4
					$\pi^{\pm}\pi^{0}$	19	
					$\mu^{\pm}\pi^{0}\nu$	5	
					$e^{\pm}\nu\pi^{0}$	5	
					$\pi^{\pm}\pi^{+}\pi^{-}$	5.7	
					$\pi^{\pm}\pi^{0}\pi^{0}$	1.7	
$K(K_{\beta})$	$\frac{1}{2}$, 0^-	0.24	K_1 498	(1.0×10^{-10})	$\pi^{+}\pi^{-}$	70	
					$2\pi^{0}$	30	
			K_2 498	(6×10^{-8})	$\mu^{\pm}\pi^{\mp}\nu$	35	
					$e^{\pm}\pi\nu^{\mp}$	30	
					$\pi^{+}\pi^{-}\pi^{0}$	12	
					$\pi^{0}\pi^{0}\pi^{0}$?		
					$K^{\pm}\pi^{0}$	≈ 33	34,36
$K^{*}_{\frac{1}{2}}(K_{\gamma})$	$\frac{1}{2}$, 1^-	0.79	$K^{*\pm}$ 890	50 ?	$K^{0}\pi^{\pm}$	≈ 67	35
			K^{*0} 890	50 ?	$K^{\pm}\pi^{\mp}$		
					$K^{0}\pi^{0}$		
K^{*} (?)	$\frac{1}{2}$, ?	0.53	730	< 20	$K\pi$		35
K^{*}?		1.32	1150		$K\pi\pi\pi$		65
K^{*}?		2.72	1650		$K\pi\pi\pi$		65

TABLE III

Baryons With S = 0

Baryon	I, J^P	Average mass^2 Bev^2	Mass Mev	Width Mev or (mean life, sec)	Principal decay modes	%	Ref.
$N(N_\alpha)$	$\frac{1}{2}, \frac{1}{2}^+$	0.88	p 938.21 n 939.51	stable (1.01×10^3)	 $pe\nu$	 100	8
$N^*_{\frac{3}{2}}$ (Δ_δ)	$\frac{3}{2}, \frac{3}{2}$ +	1.53	1238	100	$N\pi$	100	8
$N^*_{\frac{1}{2}}$ $(N_\gamma ?)$	$\frac{1}{2}, \frac{3}{2}$ - ?	2.28	1512	100	$N\pi$ $N\pi\pi$	50	66
$N^*_{\frac{1}{2}}$ $(N_{\alpha 2} ?)$	$\frac{1}{2}, \frac{5}{2}$ + ?	2.85	1688	100	$N\pi$ $N\pi\pi$ ΛK? $N\eta$		66, 70 71
$N^*_{\frac{3}{2}}$ $(\Delta_{\delta 2} ?)$	$\frac{3}{2}, \frac{7}{2}$ + ?	3.69	1920	≈ 200	$N\pi$ $N\pi\pi$ ΣK?		66, 68
N^* ?	$\frac{1}{2}$, ?	2.72	1650		ΛK		69
$N^*_{\frac{1}{2}}$ $(N_{\alpha 3} ?)$	$\frac{1}{2}$, ?	4.8	2200				71a
$N^*_{\frac{3}{2}}$ $(\Delta_{\delta 3} ?)$	$\frac{3}{2}$, ?	5.5	2350				71a

TABLE IV

Baryons With S = - 1

Baryon	I, J^P	Average Mass² Bev²	Mass Mev	Width Mev or (mean life, sec)	Principal decay modes	%	Ref.
Λ (Λ_α)	0, $\frac{1}{2}^+$	1.24	1115.4	(2.5×10^{-10})	$p\pi^-$ $n\pi^0$ $pe\nu$	64 36	3,4
Σ (Σ_α)	1, $\frac{1}{2}^+$	1.42	Σ^+ 1189.4	0.8×10^{-10}	$p\pi^0$ $n\pi^+$	50 50	3,4
			Σ^0 1191.5	$(< 10^{-11})$	$\Lambda\gamma$	100	
			Σ^- 1196.0	(1.6×10^{-10})	$n\pi^-$	100	
Y_1^* $(\Sigma_\delta$?)	1, $\frac{3}{2}^+$?	1.92	1385	50	$\Lambda\pi$ $\Sigma\pi$	96 < 4	14, 72, 73, 74
Y_0^* $(\Lambda_\beta$?)	0, $\frac{1}{2}^-$?	1.97	1405	40 or < 2	$\Sigma\pi$ $\Lambda\pi\pi$		69, 75, 76, 77
Y_0^* (Λ_γ)	0, $\frac{3}{2}^-$	2.31	1520	15	$\Sigma\pi$ $\bar{K}N$ $\Lambda\pi\pi$	55 30 15	79, 80
Y_1^* $(\Sigma_\gamma$?)	1, $\frac{3}{2}^-$	2.76	1660	40	$\bar{K}N$ $\Lambda\pi$ $\Sigma\pi$	10 30 30	85, 86, 87
Y_0^* $(\Lambda_{\alpha 2})$	0, $\frac{5}{2}$?	3.29	1815	120	$\bar{K}N$		88
Y^* ?	≥ 1, ?	2.04	1430		$\Sigma\pi$		92
Y^* ? (?)	≥ 1, ?	2.40	1550		$\Sigma\pi$		92, 93
Y^* ?	?	3.10	1760		Y_1^* π		94

TABLE V

Baryons With S = - 2

Baryon	I, J^P	Average Mass Bev^2	Mass Mev	Width Mev or (mean life sec)	Principal Decay Modes	%	Ref.
Ξ (Ξ_α ?)	$\frac{1}{2}$, $\frac{1}{2}^+$?	1.73	Ξ^0 1311	($\approx 10^{-10}$)	$\Lambda\pi^0$	≈ 100	2, 4
			Ξ^- 1320	(1.3×10^{-10})	$\Lambda\pi^-$	≈ 100	
Ξ^* (Ξ_δ ?)	$\frac{1}{2}$, $\frac{3}{2}$ + ?	2.34	1530	7 ?	$\Xi\pi$	100 ?	83, 84

TABLE VI

Other Particles

Particle	Symbol	Mass Mev	Spin	Mean Life Sec	Observed Decay Modes
Photon	γ	0	1	stable	-
Electron	$e^\pm$	0.511	$\frac{1}{2}$	stable	-
Muon	$\mu^\pm$	106	$\frac{1}{2}$	2.22×10^{-6}	$e\ \nu\ \bar{\nu}$
e-neutrino	ν_e, ν_e	0	$\frac{1}{2}$	stable	-
μ-neutrino	ν_μ, $\bar{\nu}_\mu$	0	$\frac{1}{2}$	stable	-
Graviton	?	0	2	stable	-
Weak boson	$W^\pm$, W^0, $\bar{W}^0$?	?	1	?	

b) If parity is conserved in the production reaction, then if C and D are produced polarized, the direction of polarization must be perpendicular to the production plane. (We call the unit vector in this direction $\hat{n}$.)

c) If C decays freely into two particles E and F, and the polarization of E and F is not measured, then the highest power of $\cos\theta$ which can occur in the decay angular distribution $W(\theta)$, measured with respect to any direction not depending on the decay,* is

$\cos^{2J}\theta$, if J is integral

$\cos^{(2J-1)}$ if J is half integral, parity conserved in decay

$\cos^{2J}\theta$, if J is half integral, parity not conserved in decay.

However, $W(\theta)$ depends on the state of polarization of C and is therefore not unique. For example, if C is produced unpolarized, the decay will be isotropic regardless of the spin of C. Since the polarization is a function of the production angle Θ, sometimes it is useful to consider only the decays of particles produced with restricted values of Θ.

d) If parity is conserved in the decay of C, then $W(\theta)$ contains only even powers of $\cos\theta$. If odd powers of $\cos\theta$ appear in $W(\theta)$, then C does not decay as a free particle. This may occur for very short lived particles which do not get out of the region of interaction before decaying.

e) If parity is not conserved in the decay of C, and if C is produced polarized, then there can be an asymmetry in the decay (odd power of $\cos\theta$ measured with respect to $\hat{n}$). The maximum possible value of the asymmetry coefficient depends on the spin. Lee and Yang[9] have proposed a measurement of the asymmetry as a possible method to determine the spins of particles decaying by weak interactions.

f) The magnitude of a cross section can give some information about the spin of a resonant state. For example, in pion-nucleon collisions, unitarity** requires that the contribution to the total cross section from

*It is often convenient to measure $W(\theta)$ with respect to the normal to the production plane.

**The unitarity of the scattering amplitude is the result of conservation of probability.

any single value of J is limited by

$$\sigma_J(\text{total}) \leq 4\,\pi\,\lambda^2\,(J + \tfrac{1}{2})\,\frac{\sigma_J(\text{elastic})}{\sigma_J(\text{total})}$$

This inequality may be useful to show that the spin of a resonance is greater than a certain value.

Other methods to determine the spins of particles and some methods to determine particle parities will be discussed later.

3. ADAIR ANALYSIS

a) Adair[10] has proposed a method for determining the spins of particles which does not depend on their being produced polarized. Consider the reaction π (or K) + N $\rightarrow$ a + b where N is an unpolarized target nucleon, a is a meson and b is a baryon. The amplitude A for the reaction may be written*

$$A^m = \sum_{LJ_aJ_bm_Lm_am_b} A\left(LJ_aJ_bm_Lm_am_b\right)\, Y_L^{m_L}\, \Psi_{J_a}^{m_a}\, \Psi_{J_b}^{m_b}$$

where the spherical harmonic $Y_L^{m_L}$ is a function of the angles Θ and Φ that b makes with the beam direction, $\Psi_{J_a}^{m_a}$ and $\Psi_{J_b}^{m_b}$ are the spin wave functions of the meson and baryon, and the coefficients $A\left(LJ_aJ_bm_Lm_am_b\right)$ depend on the dynamics of the interaction. Adair pointed out that a simplification occurs in this expression if a sample of events is selected such that $\Theta = 0$ or 180°. Since the $Y_L^{m_L}$ vanish at these angles for $m \neq 0$, then the sum over m_L contains only one term Y_L^0. With this restriction on the value of m_L, $m_a + m_b$ equals the spin component m of the target nucleon. This follows because the π (or K) has spin 0 and the initial orbital angular momentum can have no component in the beam direction.

The simplest case is the one in which the boson has spin zero. Then

*We use lower case m for magnetic quantum numbers, capital M for masses.

$m_b = m = \pm \frac{1}{2}$, and the baryon is produced completely aligned with respect to the beam direction. Then if the baryon decays into a particle of spin $\frac{1}{2}$ and one of spin 0, its decay angular distribution $W(\theta)$, where θ is measured with respect to the beam direction, is a unique function of its spin. See Table VII for the values of $W(\theta)$ for several spin values.

TABLE VII

Decay angular distributions of baryons produced in the forward direction together with spin 0 particles in πN or KN collisions.* For these angular distributions to be valid, the particle must be produced together with a spin zero particle and decay freely into a spin 1/2 and a spin zero particle.

Spin J	Angular Distribution $W(\theta)$
1/2	1
3/2	$1 + 3\cos^2\theta$
5/2	$1 - 2\cos^2\theta + 5\cos^4\theta$
7/2	$9 + 45\cos^2\theta - 165\cos^4\theta + 175\cos^6\theta$

If the spin of the meson is unknown, but it decays into two spinless particles, one can obtain the same result by selecting events in which the meson is produced along the beam direction and also decays along that direction. Then $m_a = 0$, and the baryon decay angular distribution $W(\theta)$ is unique and independent of the meson spin. The main difficulty with such a double selection is that the number of useful events is a small fraction of the total number. Note from Table VII that if parity is conserved in the production reaction, only even powers of $\cos\theta$ appear in $W(\theta)$. The appearance of an odd power in the observed angular distribution implies that there is interference between the resonant amplitude and another amplitude. Such interference can greatly complicate the task of determining the spin of the particle. Of course, the absence of odd powers of $\cos\theta$ in $W(\theta)$ does not necessarily mean that the particle decays freely.

b) In order for Adair's method to be useful, a criterion must be

*From Ref. 10.

given for the allowable deviation from $\Theta = 0$ or 180°. Adair has given as a rough estimate that the maximum acceptable angle Θ in radians is of order $1/L_{max}$ where L_{max} is the maximum important production angular momentum L. For $L_{max} = 0$, <u>all production events are rigorously acceptable</u>. An estimate of L_{max} is $L_{max} = p/M_{\pi}c$ where p is the c.m. momentum of either of the produced particles and M_{π} is the pion mass. How good the choice $\Theta_{max} = 1/L_{max}$ is may be illustrated by a specific example. Consider a baryon of spin $J_b = 3/2$ produced in a p-state (L = 1) together with a spin 0 particle such that the total angular momentum of the system is 1/2. The angular distribution $W(\theta)$ of all decays is then isotropic. The angular distribution of those events for which the production angle Θ is given by

$$\Theta \leq \Theta_{max} = 1/L_{max} = 1$$

is $W(\theta) = 1 + 0.8 \cos^2 \theta$. This compares with the angular distribution $W(\theta) = 1 + 3 \cos^2 \theta$ expected if $\Theta = 0$ or 180°. In general the Adair analysis is most useful if the production angular distribution favors large values of $\cos \Theta$. A $\sin^2 \Theta$ production angular distribution, for example, is probably useless for an Adair analysis.

4. BOHR ANALYSIS

A method of determining the intrisic parities of particles has been proposed by A. Bohr.[11] We shall consider Bohr's method only in the simplest case, that of a two-body initial state with a relative momentum $\vec{p}$ and a two-body final state with relative momentum $\vec{q}$. Consider a reflection R with respect to the plane containing $\vec{p}$ and $\vec{q}$. This reflection is equivalent to parity inversion followed by a rotation of 180° about the normal to the production plane $\hat{n}$. The eigenstates of R are eigenstates of the linear momentum operator rather than of angular momentum. It is apparent that the linear momenta do not change under R. Therefore, if the particles have no spin, the only thing that happens under R is that the initial and final wave functions get multiplied by a sign which depends on the intrinsic parities of the particles. Assuming parity is conserved, we have the selection rule $P_i = P_f$ where P_i (P_f) is the product of the intrinsic parities of the initial (final) state particles.

If the particles have spin, let $\hat{n}$ be the quantization axis for the spin. The operator denoting a rotation about $\hat{n}$ is $e^{i\theta_n J_n}$ where J_n is the total spin component along $\hat{n}$ and θ_n is the angle of rotation. For a rotation of 180°, the spin wave function will be multiplied by a factor $e^{i\pi J_n} = (-1)^{J_n}$. Thus, conservation of parity implies that the intrinsic parities of initial and final states are related by

$$P_i\ (-1)^{J_{ni}} = P_f\ (-1)^{J_{nf}}$$

In general this relation is most useful when polarizations can be measured or when all the particles but one have spin 0. An example of the use of this relation will be given later.

5. DALITZ PLOT

A method of correlating data which has proved useful in obtaining evidence for the existence of short lived excited states and information about their quantum numbers is the Dalitz plot.[12]

The method is most useful in considering 3-body states. The transition probability[13] λ for either a production or decay reaction is given by*

$$\lambda = 2\pi \frac{|M|^2\ D(E)}{\Pi_i 2E_i \Pi_f 2E_f}$$

where M is the invariant matrix element for the process, E is the total energy, $D(E)$ is the density of final states, and $\Pi_i 2E_i$ $(\Pi_f 2E_f)$ is the product of twice the energies of the initial (final) particles. The quantity $\rho(E) \equiv D(E)/(\Pi_f 2E_f)$ is relativistically invariant. For a 3-body

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*We set $\hbar = c = 1$.

final state it is given by

$$\rho(E) = \frac{p_1^2 dp_1 \, p_2^2 dp_2 \, d\Omega_1 \, d\Omega_2}{8(2\pi)^6 E_1 E_2 E_3 \, dE}$$

where the subscripts on the energies and momenta refer to the three final-state particles. In the c.m. system $\rho(E)$ can be written in three convenient forms (after integrating over three angle variables)

$$\begin{aligned} \rho(E) &= k_a(E) dE_1 \, dE_2 \\ &= k_a(E) dT_1 \, dT_2 \\ &= k_b(E) dM_{12}^2 \, dM_{13}^2 \end{aligned}$$

where T_1 and T_2 are particle kinetic energies, M_{12}^2 and M_{13}^2 are the invariant masses* of particles 1 and 2, and 1 and 3 respectively. Here k_a and k_b are functions of the total energy E but independent of individual final state particle energies. This means that for a given energy E the invariant phase space ρ has a uniform density in the kinematically allowed region as a function of the variables (E_1, E_2), (T_1, T_2) or (M_{12}^2, M_{13}^2) where 1 and 2 refer to any two of the three particles. In a plot of an experimental distribution of three final state particles in terms of any pair of these variables, any deviation from a uniform distribution may give useful information about the matrix element M.

As an example of the use of a Dalitz plot, consider the reaction $K^- + p \rightarrow \Lambda + \pi^+ + \pi^-$ at 1.15 Bev/c studied by Alston et al.[14] Figure 1 (from Ref. 14) shows a Dalitz plot of the events, together with projections given as histograms. The histogram showing the number of events versus the kinetic energy of the π^- has two peaks. The one at a π^- kinetic

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*By the invariant mass (or simply, mass) of a group of particles we mean the total energy in their c.m. system.

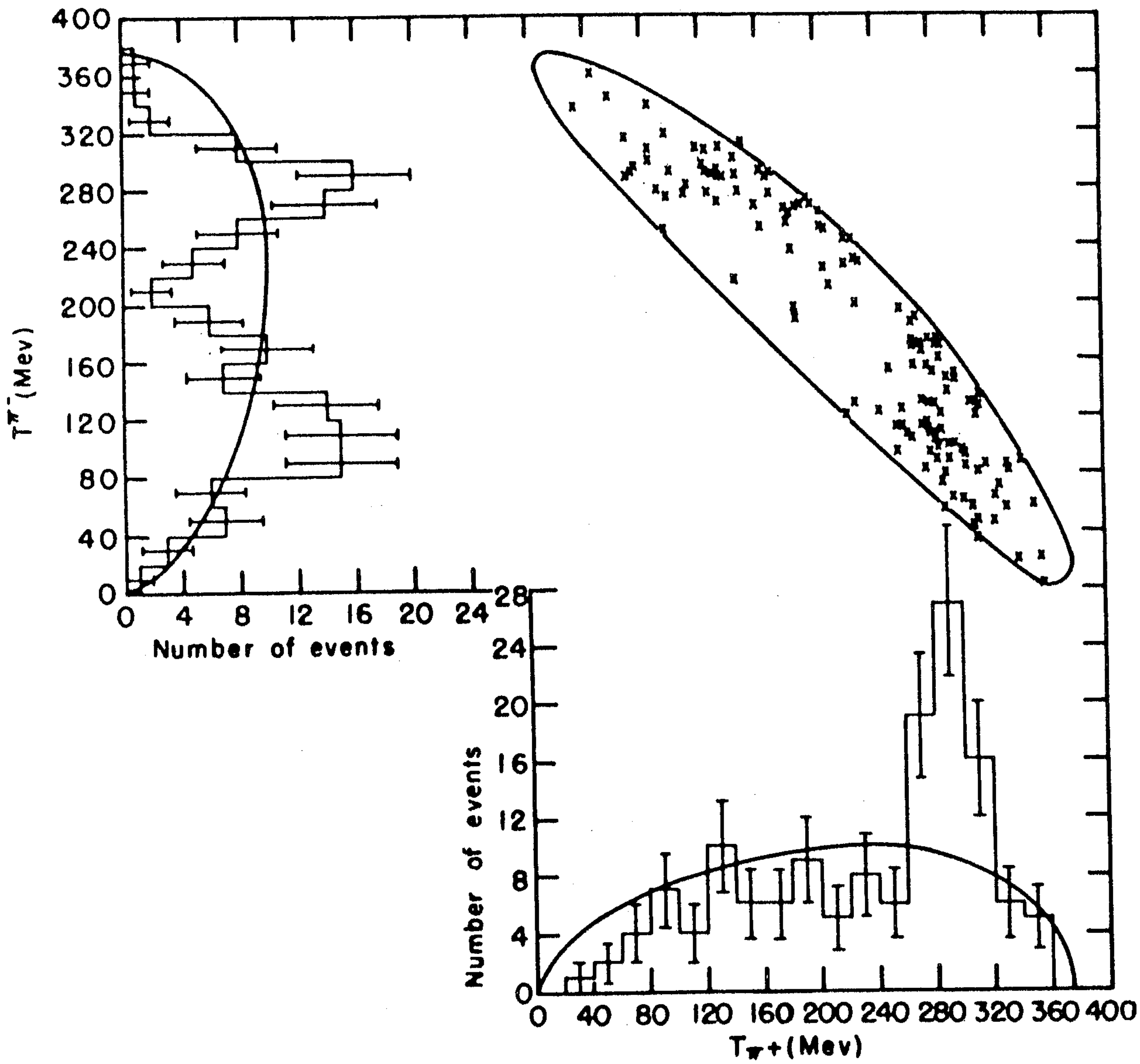

FIG. 1--Energy distribution of the two pions from the reaction $K^- + p \to \Lambda + \pi^+ + \pi^-$. Each event is plotted only once on the Dalitz plot, which should be uniformly populated if phase space dominated the reaction. The two energy histograms are merely one-dimensional projections of the two-dimensional plot, and each event is represented once on each histogram. The solid lines superimposed over the histograms are the phase-space curves. (From Ref. 14)

energy of 280 Mev is evidence for the $Y_1^*(1385)$. However, there is another peak in this spectrum at a π^- kinetic energy of 100 Mev. Naively, one might think this evidence for another resonance. However, inspection of the Dalitz plot indicates that, in part at least, this second peak is a manifestation of the fact that the π^+ also likes to be emitted at a unique energy. Thus the second peak is not necessarily evidence for another $\Lambda\pi$ resonance.*

We shall have occasion to make use of Dalitz plots for decays as well as production reactions.

6. G PARITY**

In discussing heavy mesons, it is useful to consider an operator G defined by[15] $G = e^{i\pi I_y}C$, where C is the charge conjugation operator and I_y is the y-component of the isospin. The eigenvalues of G are $G = \pm 1$; hence an eigenstate of G can be said to have a definite G parity. The reason for introducing G is that charged particles may be eigenstates of G, even though they are not eigenstates of C.

For a particle to be an eigenstate of G, it must have strangeness and baryon number zero. This can be seen as follows: If a particle has charge Q, baryon number B, and strangeness S, operation with C leads to the anti-particle state with quantum numbers -Q, -B, and -S. (For this reason, to be precise, C should be called the particle-antiparticle conjugation operator.) But the remainder of the operation G is a rotation in isospace which can change the charge of a particle but not its strangeness or baryon number. Therefore, unless $B = S = 0$, an isospin rotation following operation with C cannot lead to the original particle. A particle with $Q = 0$ in addition can be an eigenstate of C. We can say that such a particle has a definite C parity.

A meson with $S = 0$ requires five additional quantum numbers for its

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*Careful study of the region of the second peak[87] has shown that there is in fact a second $\Lambda\pi$ resonance of mass $M = 1660$ Mev. (This resonance will be discussed later.) However, most of the events in the second peak are a manifestation of the $Y_1^*(1385)$ rather than of the new resonance.

**In sections 6 and 7, the symbol n stands for number of particles.

specification: I, I_z (or alternatively, Q), spin J, parity P, and G. The first four of these quantum numbers, in addition to S, suffice to specify a meson with $S \neq 0$.

The pion has odd G parity, as can be seen from the following argument. Since the pion is a vector in isospace, it is convenient to rewrite the pion wave functions as follows:

$$\pi^+ = (\pi_x + i\pi_y)/\sqrt{2}$$

$$\pi^0 = \pi_z$$

$$\pi^- = (\pi_x - i\pi_y)/\sqrt{2}$$

Then a rotation of 180° about the y-axis in isospace leads to

$$\pi_x \rightarrow - \pi_x, \ \pi_y \rightarrow \pi_y, \ \pi_z \rightarrow - \pi_z,$$

or

$$e^{i\pi I_y} \pi^+ = -\pi^-, \ e^{i\pi I_y} \pi^0 = -\pi^0, \ e^{i\pi I_y} \pi^- = -\pi^+$$

To complete the argument, we have to know how the pion behaves under C. The existence of the decay $\pi^0 \rightarrow 2\gamma$ shows that the π^0 has even C parity, since the C parity of a state consisting of n photons is $C = (-1)^n$. Combining the relations $C\pi^0 = +\pi^0$, $e^{i\pi I_y} \pi^0 = - \pi_0$, we obtain $G\,\pi^0 = - \pi^0$. Since the π^- and π^+ are antiparticles of each other, we can write $C\,\pi^+ = \pm\,\pi^-$ where the sign multiplying the π^- wave function is arbitrary. With either sign, the charged pions are seen to be eigenstates of G, and we can choose the + sign to make the charged pions have odd G parity as does the π^0. Two useful relationships follow from the definition of G.

1) If a state consists of several mesons (which need not necessarily be all pions), each of which is an eigenstate of G, the G parity of

the state is the product of the G parities of the individual mesons.

2) If a neutral meson has a definite isospin and C parity, its G parity is given by*

$$G = (-1)^I C$$

It may be of interest to ask whether a meson of zero strangeness is necessarily an eigenstate of G. If it is not, it can be written as a linear combination of two mesons, one with G = + and the other with G = -. These two new mesons will have different interactions, as the G = + meson can decay (in G-conserving interactions) only into even numbers of pions while the G = - meson can decay only into odd numbers of pions. Since these interactions are in general strong, the mesons with G = + and G = - will in general have masses which differ appreciably. Also they will probably be produced in different reactions. If so, there seems to be no point in identifying them as a single meson which is not an eigenstate of G.

7. SELECTION RULES IN THE DECAY OF HEAVY MESONS

Before discussing some of the individual heavy mesons in detail, it will be useful to consider selection rules in their decays. These selection rules will provide a valuable tool for determining the quantum numbers of the heavy mesons.

As we have remarked previously, the quantum numbers of a meson are its strangeness S, isospin I, z-component of isospin I_z (or alternatively, charge Q), spin J, parity P, G parity (if S = 0) and C parity (if S = Q = 0). We assume that in decays via strong interactions, all these quantum numbers are separately conserved; in electromagnetic decays, I and/or G may not be conserved, and in weak interactions, only J,

*This formula holds for any system which is an eigenstate of C and I.

Q and CP are necessarily conserved.*

If a meson decays into strongly interacting particles with a width $\Gamma \geq$ several Mev, it is reasonable that it decays via strong interactions. If the width is too small to be measured by usual techniques but the particle travels too short a distance to be measured before decaying, the decay is probably either strong or electromagnetic. Finally if the particle has a lifetime of the order of 10^{-10} sec or longer, the decay is weak. Thus we can obtain information about the quantum numbers of a meson by looking at its decay rate, at its branching ratio into its various modes, and at the quantum numbers of its decay products. We consider possible decays into pions, photons, K mesons, and other mesons and list some of the quantum numbers of these multiparticle states. In the following, the quantum numbers are the total quantum numbers of the state.

An n-pion state has the following quantum numbers.

$$n\pi : S = 0, \ G = (-1)^n$$

In addition a state consisting of n neutral pions has C = +. Additional information about the properties of an n-pion state can be obtained from the requirement that the pions obey Bose statistics.** A two pion state has the following relations among its quantum numbers.

$$\pi^{\pm}\pi^{0} : P = (-1)^J, \ (-1)^J = (-1)^I$$

$$\pi^{+}\pi^{-} : C = P = (-1)^J, \ (-1)^J = (-1)^I, \ CP = +$$

$$2\pi^{0} : J \text{ even}, \ I \text{ even}, \ P = C = +$$

These relations are a special case of a general relation which holds for a meson of spin J and its antiparticle (charge conjugate particle) in

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*It is usually assumed that time reversal invariance T also holds in all interactions. However, since T is an anti-unitary operator, it leads to no selection rules. Of course T invariance has other consequences which can be experimentally tested.

**O. Greenberg and A. Messiah (Meeting of the Am. Phys. Soc., New York, 1963, post deadline paper) have pointed out that the statistics of particles should not be taken for granted but should be measured experimentally. However, for simplicity we shall assume that the conventional statistics hold.

a state of orbital angular momentum L

$$C = (-1)^{L+j}, \quad P = (-1)^{L}, \; CP = (-1)^{j}$$

where j is the total spin of the meson-antimeson pair.

For a state of a baryon and its antiparticle (charge conjugate particle), the corresponding relations are

$$C = (-1)^{L+j} \;, \; P = (-1)^{L+1}, \quad CP = (-1)^{j+1}$$

It follows that for a single neutral meson to have the same quantum numbers as a possible bound state of a spin 1/2 baryon and antibaryon, the meson must have the following restrictions on its spin, parity, and C parity:

If $J = 0$, then $C = +$

If $P = (-1)^{J}$, then $C = (-1)^{J}$

If $P = (-1)^{J+1}$, and $J > 0$, then $C = \pm$

The fact that the π^{0} has $J = 0$, $C = +$ shows that it has the same quantum numbers as a possible bound state of a nucleon and antinucleon. If a meson has $I = 1$ and its neutral member can exist as a bound nucleon-antinucleon state, then so can its charged members, since $G = (-1)^{I}\, C$ for the meson and also for the $\bar{N}N$ pair.

Some relations among the quantum numbers of a three pion state are

$$3\pi : J = 0 \;\text{ implies }\; P = -$$

In addition

$$\pi^{+}\pi^{-}\pi^{0} : I = 0 \;\text{ implies }\; C = -$$

$$3\pi^{0} : C = +, \; I \neq 0$$

There are the following restrictions on states consisting of pions and photons:

$$\pi\,\gamma \quad : J \neq 0$$

$$\pi^{0}\,\gamma \quad : J \neq 0,\ C = -$$

$$2\gamma \quad : J \neq 1,\ C = +$$

$$n\pi^{0}2\gamma \quad : C = +$$

$$\gamma\, n\pi^{0} \quad : C = -$$

Next consider a state of $K\bar{K}$.

$$K\bar{K} \ : P = (-1)^{J},\ G = (-1)^{J+1}$$

$$K^{0}\bar{K}^{0},\ K^{+}K^{-} \ : C = P = (-1)^{J},\ CP = +$$

It is useful also to consider the decays of the K^{0} and $\bar{K}^{0}$. Since K_{0} and $\bar{K}_{0}$ are not eigenstates of CP, and since CP is conserved in their decay, it is convenient to construct the state K_{1} (even under CP) and the state K_{2} (odd under CP). These states are the following linear combinations of K^{0} and $\bar{K}^{0}$:

$$K_{1} = (K^{0} + \bar{K}^{0}) / \sqrt{2}\ ,\ K_{2} = (K^{0} - \bar{K}^{0}) / \sqrt{2}$$

Neither K_{1} nor K_{2} is an eigenstate of C, P or S. The states $K_{1}K_{1}$, $K_{1}K_{2}$, and $K_{2}K_{2}$ have the following relations among their quantum numbers

$$K_{1}K_{1} \text{ and } K_{2}K_{2} \ : CP = +,\ J \text{ even}$$

$$K_{1}K_{2} \ : CP = (-1)^{J+1}$$

TABLE VIII

Selection rules* for the decay into pions and photons of neutral heavy mesons with strangeness S = 0. The quantum numbers J, P, T (isospin**), and G parity apply to all members of a meson multiplet, but C parity applies only to the neutral member. A decay allowed by strong interactions is indicated by "yes," and forbidden decays are indicated by the symbol for a conservation law which would be violated by the decay. A decay in which a photon appears is indicated as violating G if otherwise allowed. Also indicated is whether the meson has the same quantum numbers as a possible bound state of a nucleon and antinucleon.

Quantum numbers of meson T	J^{PG}	C	Decay modes $N\bar{N}$	$\pi^+\pi^-$	$\pi^0\pi^0$	$\gamma\gamma$	$\pi^0\gamma$	$\pi^+\pi^-\pi^0$	$\pi^0\gamma\gamma$	$\pi^0\pi^0\pi^0$	$\pi^0\pi^0\gamma$	$\pi^+\pi^-\gamma$	$4\pi^0$	$\pi^+\pi^-\pi^0\pi^0$	$\pi^+\pi^+\pi^-\pi^-$
0	0^{++}	+	yes	yes	yes	G	J	P	G	P	C	G	yes	yes	yes
0	0^{+-}	−	C	C	C	C	J	P	C	P	G	G	C	G	G
1	0^{++}	−	C	C	C	C	J	P	C	P	G	G	C	yes	yes
1	0^{+-}	+	yes	G	G	G	J	P	G	P	C	G	G	G	G
0	0^{-+}	+	yes	P	P	G	J	G	G	G	C	G	yes	yes	yes
0	0^{--}	−	C	P	P	C	J	yes	C	C	G	G	C	G	G
1	0^{-+}	−	C	P	P	C	J	G	C	C	G	G	C	yes	yes
1	0^{--}	+	yes	P	P	G	J	yes	G	yes	C	G	G	G	G
0	1^{-+}	+	C	C	J	J	C	G	G	G	C	G	yes	yes	yes
0	1^{--}	−	yes	G	J	J	G	yes	C	C	G	G	C	G	G
1	1^{-+}	−	yes	yes	J	J	G	G	C	C	G	G	C	yes	yes
1	1^{--}	+	C	C	J	J	C	yes	G	yes	C	G	G	G	G
0	1^{++}	+	yes	P	J	J	C	G	G	G	C	G	yes	yes	yes
0	1^{+-}	−	yes	P	J	J	G	yes	C	C	G	G	C	G	G
1	1^{++}	−	yes	P	J	J	G	G	C	C	G	G	C	yes	yes
1	1^{+-}	+	yes	P	J	J	C	yes	G	yes	C	G	G	G	G

*From Ref. 16

**Isospin is given the symbol T in this table and in some others.

TABLE IX

Selection rules for the decay of positively charged T = 1 mesons. For further explanation see the caption for Table VIII.

Quantum numbers of meson J^{PG}	Decay modes $N\bar{N}$	$\pi^+\pi^0$	$\pi^+\gamma$	$\pi^+\pi^0\pi^0$ $\pi^+\pi^+\pi^-$	$\pi^+\pi^0\gamma$ $\pi^+\gamma\gamma$	$\pi^+\pi^0\pi^0\pi^0$ $\pi^+\pi^+\pi^-\pi^0$
0^{++}	G	T	J	P	G	yes
0^{+-}	yes	T, G	J	P	G	G
0^{-+}	G	P	J	G	G	yes
0^{--}	yes	P	J	yes	G	G
1^{-+}	yes	yes	G	G	G	yes
1^{--}	G	G	G	yes	G	G
1^{++}	yes	P	G	G	G	yes
1^{+-}	yes	P	G	yes	G	G

TABLE X

Selection rules for the decay of $S = 0$ mesons with $I \leq 1$ and other quantum numbers J^{PG} into a pion plus a meson (not a pion) of isospin $i \leq 2$ and other quantum numbers j^{pg}. In addition to the selection rules of the table, if $I = 0$ the decay is forbidden by I invariance unless $i = 1$. Also the decay (neutral meson → neutral meson + π^0) is forbidden by C invariance unless $(-1)^{I+i}Gg = 1$. For further explanation see caption for Table VIII.

J^{PG} \ j^{pg}	0^{++}	0^{+-}	0^{-+}	0^{--}	1^{-+}	1^{--}	1^{++}	1^{+-}
0^{++}	P	P	G	yes	P	P	G	yes
0^{+-}	P	P	yes	G	P	P	yes	G
0^{-+}	G	yes	P	P	G	yes	P	P
0^{--}	yes	G	P	P	yes	G	P	P
1^{-+}	P	P	G	yes	G	yes	G	yes
1^{--}	P	P	yes	G	yes	G	yes	G
1^{++}	G	yes	P	P	G	yes	G	yes
1^{+-}	yes	G	P	P	yes	G	yes	G

TABLE XI

Selection rules for the decay of neutral mesons with $I \leq 1$ and $S = 0$ into two K mesons. See caption for Table VIII. By "yes" in the columns K_1K_1 and K_1K_2, we mean that the decay is allowed by strong interactions to proceed through the mode $K^0\overline{K}^0$, with the $K^0\overline{K}^0$ subsequently decaying weakly into K_1K_1 or K_1K_2. The symbol St means that the decay would violate Bose statistics.

J^P	C	K^+K^-	$K^0\overline{K}^0$	K_1K_1	K_1K_2
0^+	+	yes	yes	yes	CP
0^+	-	CP	CP	CP	S
0^-	+	CP	CP	CP	S
0^-	-	P	P	P	CP
1^-	+	CP	CP	St	CP
1^-	-	yes	yes	St	yes
1^+	+	P	P	St	P
1^+	-	CP	CP	St	CP

Lastly we consider states of two non-identical mesons A and B

$$AB : G = G_A G_B, \; P = (-1)^L P_A P_B$$

$$A^o B^o : C = C_A C_B = (-1)^{I_A + I_B} G_A G_B$$

Tables VIII and IX give the selection rules for meson decays into pions and photons.[16] More complete tables of these rules have been given by Henley and Jacobsohn.[17] Table X gives selection rules for meson decay into two mesons one of which is not a pion. Finally, Table XI gives selection rules for decay into two K mesons. In these tables a decay permitted by strong interactions is indicated by "yes," and a forbidden decay is indicated by the symbol for a conservation law which would be violated by the decay. This designation is not necessarily unique. For example in Table VIII we have indicated that a $J = 1$ meson is forbidden to decay into $2\pi^o$ mesons, citing J as the conservation law which would be violated. However, the decay could occur with J conserved if the two pions did not obey Bose statistics. Also sometimes more than one conservation law might be violated. For example we have indicated that a $J^P = 0^-$, $C = +$ meson is forbidden to decay into $\pi^+\pi^-$ by P invariance. However, since C is conserved, CP conservation would also be violated by such a decay.

8. THE ρ MESON

The ρ meson was first seen by Erwin et al.[18] in the reactions

$$\pi^- + p \rightarrow \pi^+ + \pi^- + n, \; \pi^- + p \rightarrow \pi^- + \pi^o + p$$

as a correlation in the invariant mass of the two pion system. The peak in the two pion mass spectrum observed by Erwin et al. is shown in Fig. 2. Subsequent experiments confirmed the existence of the ρ at a mass $M_\rho = 750$ Mev with a full width at half maximum $\Gamma_\rho \approx 100$ Mev. The ρ has also been seen in the reaction $\pi^+ + p \rightarrow \pi^+ + \pi^o + p$ and in reactions

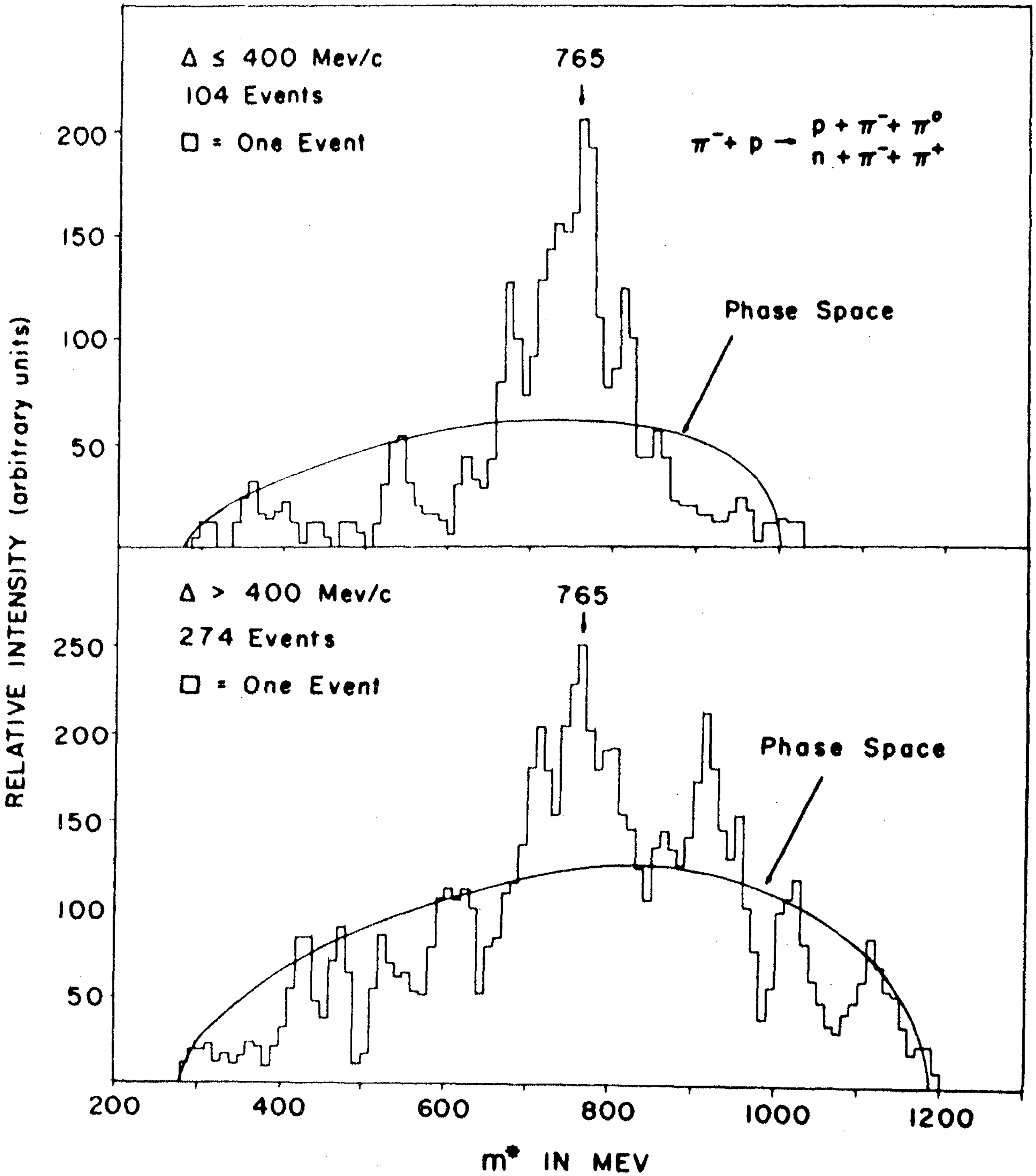

FIG. 2--The combined mass spectrum for the $\pi^-\pi^0$ and $\pi^-\pi^+$ system. The smooth curve is phase space as modified for the included momentum transfer and normalized to the number of events plotted. Events used in the upper distribution are not contained in the lower distribution. (From Ref. 18)

in which an additional pion is produced.[19,20]

The evidence for this mass and width of the ρ comes from experiments at incident pion momenta* below 3 Bev/c. On the other hand, preliminary results of an experiment at CERN by Caldwell et al.[21] at incident pion momenta of 12 and 17 Bev/c indicate $M_\rho \approx 725$ Mev, $\Gamma_\rho \approx 40$ Mev. If the CERN results are borne out, this would mean that there are rather large interfering amplitudes at incident momenta below 3 Bev/c. Some evidence already exists for interference effects. (1) Some authors have seen the ρ^0 show up as a double peak in the $\pi^+\pi^-$ mass spectrum, although this might be merely a statistical fluctuation. (2) When the ρ^0 decays, the π^- tends to go forward with respect to the beam direction in the rest system of the ρ.

It has been suggested that these effects are caused by interference with a two-pion decay mode of the ω meson.** This explanation cannot account for the fact that the charged ρ also appears 100 Mev wide in the experiments below 3 Bev/c. Interference with the $N^*_{\frac{3}{2}}$ state is another possibility, but this particular interference can be eliminated by proper selection of events from a Dalitz plot. It seems that another experiment at high energy will be needed to clarify the width of the ρ.

a) Isospin. The fact that the ρ has been seen in three charge states show that $I > 1$. Since a doubly charged ρ has not been seen in $\pi^+ + p \rightarrow \pi^+ + \pi^+ + n$, whereas the ρ is very prominent peak in $\pi^+ + p \rightarrow \pi^+ + \pi^0 + p$, the ρ must have $I = 1$. If $I = 2$, the ratio R of the two reactions would be

$$R = (\pi^+ \pi^+ n)/(\pi^+ \pi^0 p) = 4$$

instead of the observed $R = 0$ within experimental error.

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*Incident momenta and energies are given in the laboratory system.

**Effects of a two-pion decay mode of the ω have been observed by W. Fickinger, D. Robinson and E. Salant and by J. B. Schafer et al., Bull. Am. Phys. Soc. 8, 22 (1963) [both abstracts].

b) Spin. The large decay width of the ρ makes it reasonable to assume that it decays via strong interactions and that isospin is conserved in the decay. This tells us that J_ρ is odd, as follows: A two-pion state with $I = 1$ has an isospin wave function which is antisymmetric under the interchange of the two pions. Since the pions obey Bose statistics, the two-pion space wave function must be antisymmetric to make the total wave function including isospin be symmetric. This implies odd orbital angular momentum for the two pions and therefore odd spin for the ρ.

The ρ is produced primarily in peripheral or low momentum transfer collisions. This can be seen from the forward peaked c.m. angular distribution of the ρ shown in Fig. 3a (from Ref. 20). An Adair analysis is useful, but does not yield a unique result. In fact, there are the following possibilities for the decay angular distribution $W_\rho(\theta)$ for a spin = 1 ρ-meson produced in the forward direction in the reaction $\pi + N \rightarrow \rho + N$:

$$W_\rho(\theta) = \cos^2 \theta, \text{ nucleon spin not flipped,}$$
$$W_\rho(\theta) = \sin^2 \theta, \text{ nucleon spin flipped.}$$

There cannot be interference between the spin-flip and non-spin-flip amplitudes, but they can add to give an isotropic distribution. Actually observed experimentally is $W(\theta) \approx \cos^2 \theta$, with only a small isotropic term.

Furthermore, although the ρ has been seen in many experiments, a $\cos^4 \theta$ term has never been observed. This is good evidence for $J = 1$.

Consistent with this picture is the prediction of the peripheral model. A calculation of the one-pion exchange diagram for ρ production leads to a decay angular distribution $W_\rho(\theta) \approx \cos^2 \theta$ for $J = 1$, in agreement with experiment.

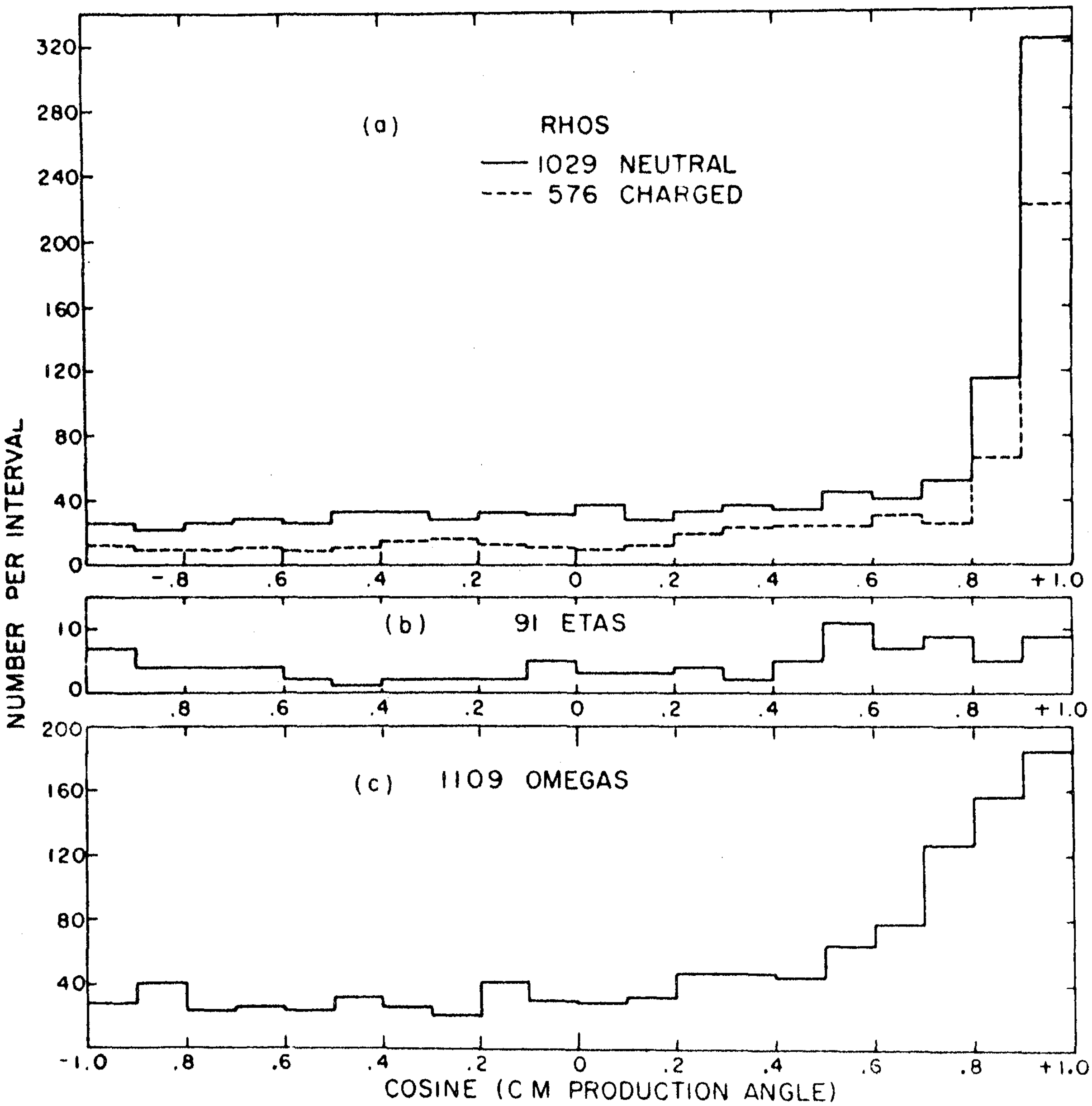

FIG. 3--Center-of-mass production angles for (a) rhos, (b) etas, and (c) omegas. (From Ref. 20)

Still another piece of evidence comes from the Chew-Low[22] formula*

$$\frac{d^2\sigma}{dp^2\, dM^2} = \frac{f^2}{2\pi}\frac{p^2 M}{M_\pi{}^2}\frac{\left(M^2/4-M_\pi^2\right)^{\frac{1}{2}}}{\left(p^2+M_\pi\right)^2 q^2}\,\sigma_{\pi\pi}(M)$$

where p is the four-momentum transfer to the nucleon, q is the incident pion momentum in the Lab system, M is the invariant mass of the two final state pions and $\sigma_{\pi\pi}(M)$ is the pion-pion collison cross section at a center of mass energy M. Strictly speaking, this formula is supposed to be valid in the limit $p^2 \rightarrow - M_\pi^2$. However, in practice, the formula has been assumed to hold in the physical region where p^2 is positive. With this assumption, the experimental data yield a peak value for $\sigma_{\pi\pi}(M)$ at $M \approx 750$ Mev, with

$$\sigma_{\pi\pi}\ (750\ \text{Mev}) \approx 12\pi\,\lambda\!\!\!^{-2}$$

a result which favors $J = 1$.

c) Treiman-Yang Test. Part of the evidence for ρ having spin 1 depends on the validity of the one-pion exchange model for ρ production at small momentum transfers. A test of the validity of the model has been proposed by Treiman and Yang.[23] The line of flight of an incident particle and the line of a produced particle establish a plane. If the produced particle decays into two particles, its decay products establish another plane. Since the pion has spin zero, it cannot carry information about the production plane, and therefore if a single pion is exchanged in the production reaction, there can be no correlation between the production and decay planes. As is usual with tests of particular models, although the presence of a correlation implies there must be corrections to the

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*This formula applies to $\pi^\pm + p \rightarrow \pi^\pm + \pi^0 + p$ via one-pion exchange. For the formula to apply to $\pi^- + p \rightarrow \pi^+ + \pi^- + n$, the right hand side must be multiplied by a factor 2.

one-pion exchange model, the absence of correlations does not imply that the model is correct. In most experiments in which ρ mesons are produced at low momentum transfers, no correlation between production and decay planes has been seen.

d) Parity and G Parity. A two pion system has the relations among its quantum numbers $P = (-1)^J$, $G = +$. Since $J = 1$ the parity of the ρ is odd. Also since the width of the ρ is large, it is reasonable that G is conserved in its decay; therefore $G_\rho = +$. Thus the quantum numbers of the ρ are $I, J^{PG} = 1, 1^{-+}$.

9. THE ω MESON

The ω meson was first observed by Maglić et al.[24] as a three-pion correlation in antiproton-proton annihilation in the reaction

$$\bar{p} + p \rightarrow \pi^+ + \pi^- + \pi^0 + \pi^+ + \pi^-$$

(See Fig. 4.) It has a mass $M_\omega = 785$ Mev and a width $\Gamma < 20$ Mev. The principal decay mode is $\omega \rightarrow \pi^+ + \pi^- + \pi^0$.

a) Isospin. Since only a neutral ω was observed, even though singly and doubly charged ω mesons were looked for, Maglić et al. concluded that the ω has isospin $I = 0$. This conclusion has been confirmed at other laboratories. For example Alff et al.[20] looked at ω mesons produced in the reaction $\pi^+ + p \rightarrow \omega + N^*_{3/2}$. Since the initial state is pure isospin 3/2, the branching ratios into the different charge states are unique functions of the isospin I of the ω.

$$\omega^+ / \omega^0 = 0 \quad \text{if} \quad I = 0$$

$$\omega^+ / \omega^0 = 2/3 \quad \text{if} \quad I = 1$$

$$\omega^{++} / \omega^+ / \omega^0 = 2/2/1 \quad \text{if} \quad I = 2$$

$$\omega^{+++} / \omega^{++} / \omega^+ / \omega^0 = 20/10/4/1 \quad \text{if} \quad I = 3.$$

The mass spectrum of singly charged pion triplets did not show evidence of a peak, a fact that supports the assignment $I = 0$. The loophole in

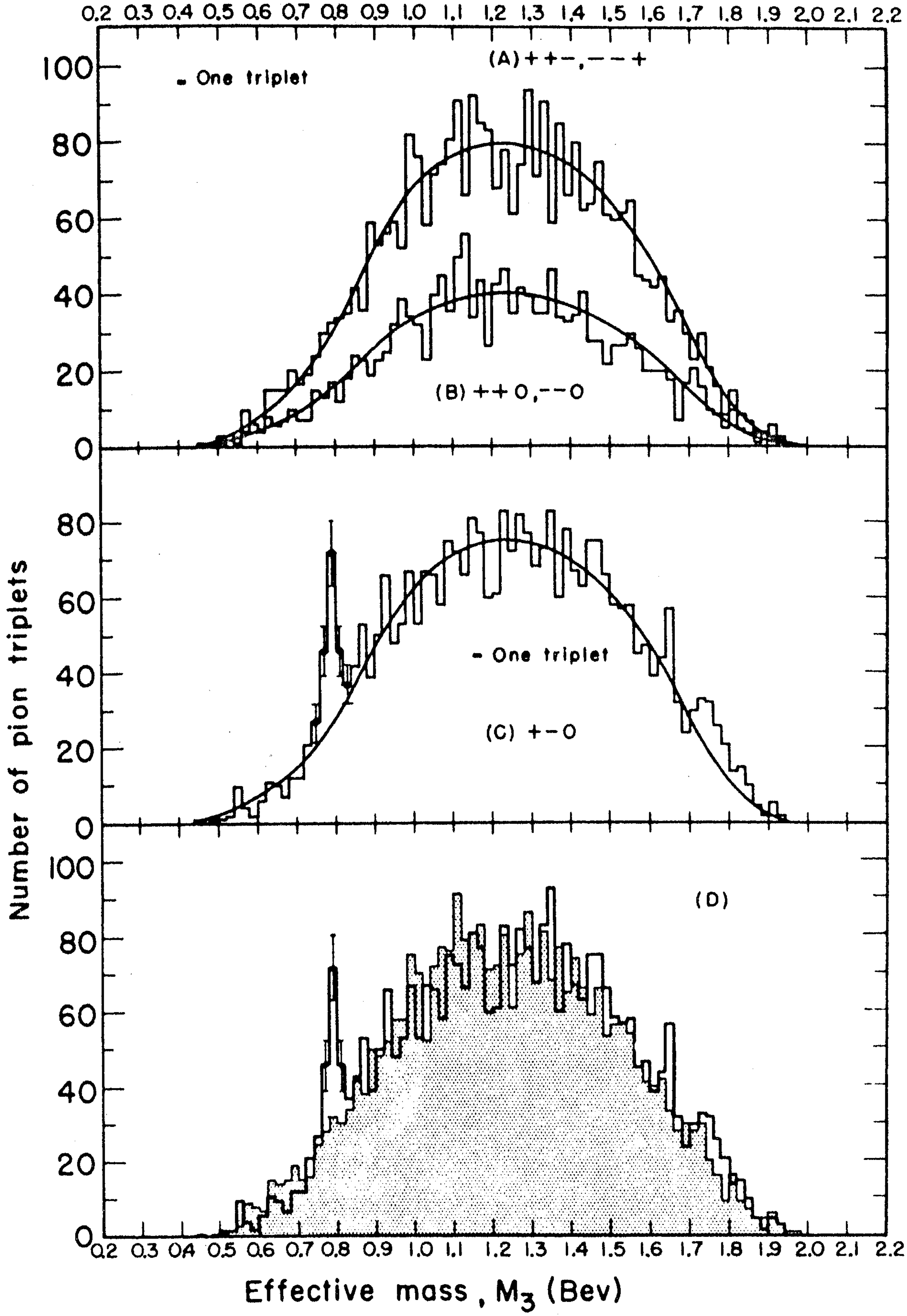

FIG. 4--Number of pion triplets versus effective mass (M_3) of the triplets for reaction $\bar{p} + p \rightarrow 2\pi^+ + 2\pi^- + \pi^0$. (From Ref. 24)

this argument is that the ω^+ might exist but not decay into three pions. For example, the ω^+ might exist and decay $\omega^+ \to \pi^+ +$ neutral(s) (electromagnetically) even though the decay $\omega^0 \to 2\pi$ might be forbidden by C invariance. However, let us ignore this loophole and come back to it later.

b) Spin and Parity. To obtain information about the other quantum numbers of the ω, we need to look at the three pions from its decay. First, examine the consequences of the assumption that the ω has $I = 0$ and that isospin is conserved in the decay. Three pions in an $I = 0$ state have a unique isospin wave function, which is antisymmetric in the interchange of any two pions. This can be seen by noting that there is only one way to make a scalar (under rotation) out of three vectors: $\vec{\pi}_1 \cdot \vec{\pi}_2 \times \vec{\pi}_3$. This wave function clearly changes sign under interchange of any two of the vectors.

Alternatively, one can look up the Clebsch-Gordon coefficients for combining three $I = 1$ particles to obtain $I = 0$. The wave function ψ_0 is

$$\psi_0 = \left(\pi_1^+ \pi_2^0 - \pi_1^0 \pi_2^+\right) \pi_3^- - \left(\pi_1^+ \pi_2^- - \pi_1^- \pi_2^+\right) \pi_3^0 + \left(\pi_1^0 \pi_2^- - \pi_1^- \pi_2^0\right) \pi_3^+$$

This can be seen to be antisymmetric under the interchange of any two pions. Inspection of this wave function also reveals that it has no all-neutral components. The experimental ratio of neutral to charged decays is

$$(\omega \to \text{neutrals}) \;/\; (\omega \to \pi^+\pi^-\pi^0) = 0.15 \pm 0.05$$

a fact that suggests that I is conserved "most of the time."

A further check is that since the isospace part of a three-pion wave function with $I = 0$ is antisymmetric, the wave function must also be antiaymmetric in the pion momenta to satisfy Bose statistics. This antisymmetric feature will also be present in the matrix element for the transition, and the square of the matrix element, as measured on a Dalitz plot, will be symmetric.

The transition matrix element will also depend on the spin and parity of the ω. Table XII from Stevenson et al.[25] shows some simple examples of matrix elements with the proper symmetries.

TABLE XII

Possible three-pion resonances with $I = 0$, $J \leq 1$ (From Ref. 25).

Meson				Matrix element	
Type	J	I	L	Simple example	Vanishes at
V	1^-	1	1	$(\mathbf{p}_0\times\mathbf{p}_+)+(\mathbf{p}_+\times\mathbf{p}_-)+(\mathbf{p}_-\times\mathbf{p}_0)$	Whole boundary
PS	0^-	1 and 3	1 and 3	$(E_--E_0)(E_0-E_+)(E_+-E_-)$	Straight lines
A	1^+	0 and 2	1	$E_-(\mathbf{p}_0-\mathbf{p}_+)+E_0(\mathbf{p}_+-\mathbf{p}_-)+E_+(\mathbf{p}_--\mathbf{p}_0)$	Center b, d, f.

Note that the matrix element has the opposite parity of the meson, since three pions have odd intrinsic parity. A Dalitz plot of the data of Alff et al.[26] is consistent with an antisymmetric matrix element, since the density of points vanishes where the relative momentum of any two pions is zero. (See Fig. 5.) This is evidence that the three pions are in an $I = 0$ state. Since a three-pion state has $G = -$, if $I = 0$, then $C = -$, as follows from the relation $C = (-1)^I G$. Furthermore the distribution of points on the Dalitz plot is consistent* only with the assignment $J^P = 1^-$. This can be seen in Fig. 6 (from Ref. 25), which shows the density of points as a function of radius on the Dalitz plot for 0^-, 1^- and 1^+. Thus the quantum numbers of the three-pion state are $I = 0$, $J^{PG} = 1^{--}$, $C = -$. Of these, at least J, P and C should be conserved in the decay of the ω, since the decay is not weak.

We now return to the loophole regarding the isospin of the ω. Suppose the ω had $I = 1$ rather than $I = 0$. Then, since $C = -$, G would be + and the ω would have the same quantum numbers (1^{-+}) as the ρ. Then it would decay chiefly into two pions rather than three, in contradiction to experiment.** This rules out $I = 1$.

*Values of $J \geq 2$ were not considered in detail.

**The ω cannot be the three-pion decay of the ρ^0, since the ω has much narrower width than the ρ^0.

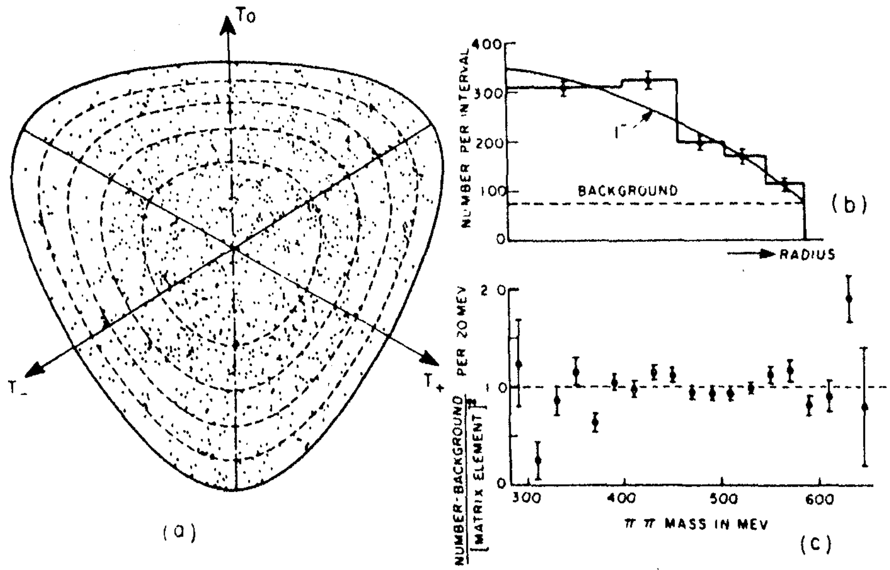

FIG. 5--(a) The Dalitz plot for 1100 omegas (including a background of 375 nonresonant triplets). (b) The density of points on the Dalitz plot compared to the expected density for a $1^{-}\omega$ plus a uniformly distributed background. (c) The dependence of the $\pi\pi$ interaction in the $T = 1$ $J = 1$ state as a function of energy. This was obtained by summing the $\pi^{+}\pi^{-}$, $\pi^{+}\pi^{0}$, and $\pi^{-}\pi^{0}$ mass spectra for pion pairs from the ω decays, subtracting a background, and dividing by the distribution expected for 1^{-} decay into $\pi^{+}\pi^{-}\pi^{0}$. Since two of the three mass combinations are independent, an error corresponding to $(2/3N)^{1/2}$, where N is the number of pairs per interval before background subtraction, was assigned to each point. (From Ref. 26)

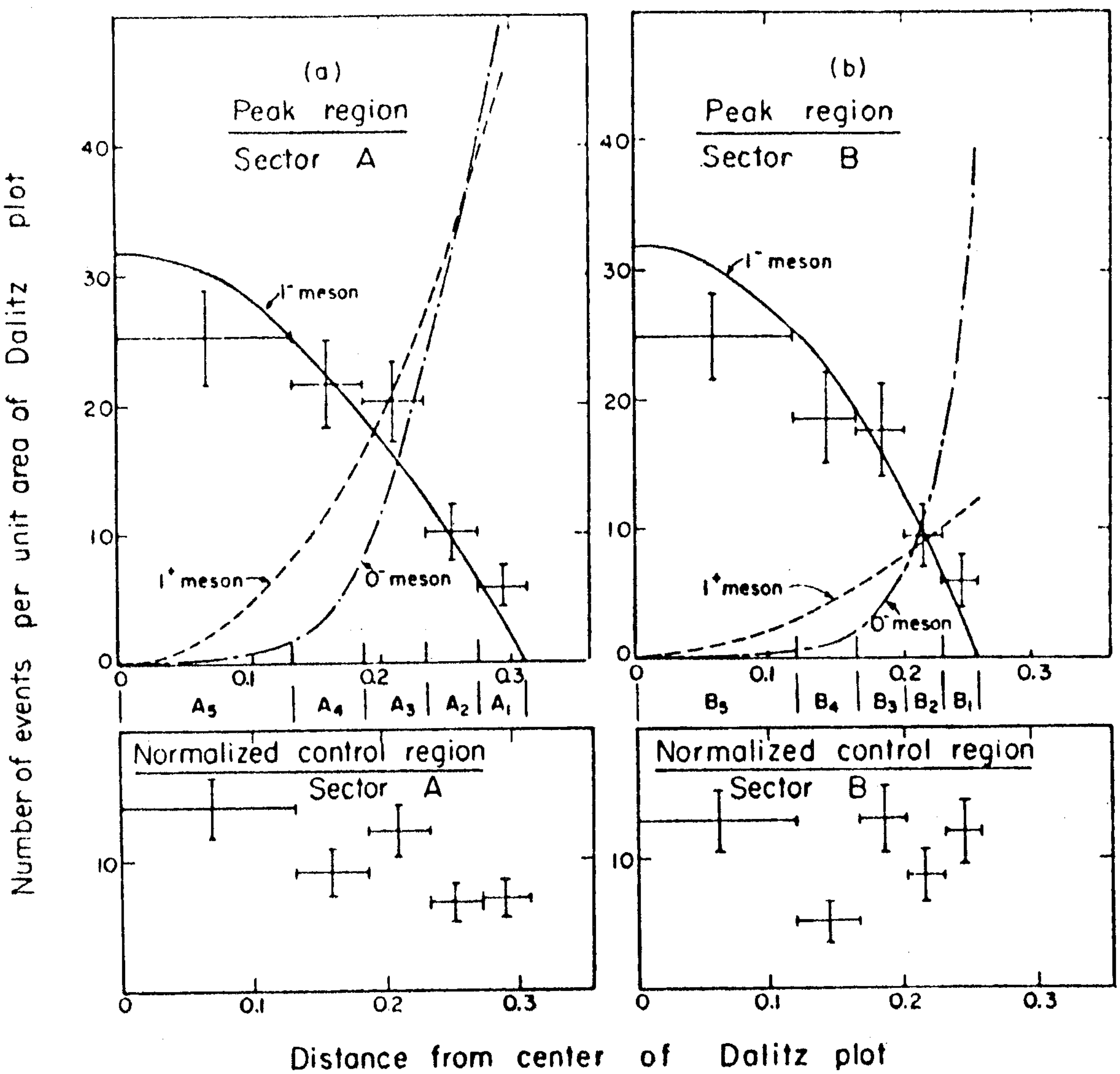

FIG. 6--Density of points on the Dalitz plot as a function of radius for 1^-, 0^-, and 0^+ mesons. Experimental points are also given. The Dalitz plot is divided into sectors A and B to test for the proper symmetry. (From Ref. 25)

Thus we have arrived at the conclusion that all quantum numbers are conserved and that the decay proceeds primarily via strong interactions. However, we have previously noted that about 15% of the decays are $\omega \rightarrow$ neutrals. These decays are probably mostly $\omega \rightarrow \pi^0 + \gamma$, an electromagnetic mode which has a larger available phase space than the strong interaction decay and thus can compete with it.

Theoretical estimates of the width of the ω ranging from $\Gamma \approx 0.3$ Mev to $\Gamma \approx 10$ Mev have been published. Also, branching ratios into various modes have been calculated.[27] Since these calculations are based on specific dynamical models, we do not consider them here.

10. THE η MESON

The η meson* was discovered as a three-pion correlation by Pevsner et al.[28] in the reaction

$$\pi^+ + d \rightarrow p + p + \pi^+ + \pi^- + \pi^0$$

with 1.23 Bev/c incident pions. (The ω meson was also seen in this experiment; see Fig. 7.) The η has mass $M_\eta = 548 \pm 1$ Mev and width $\Gamma_\eta < 10$ Mev.

a) Isospin. Evidence for $I = 0$ is the absence of a peak in the three-pion mass spectrum in the reaction

$$\pi^+ + d \rightarrow p + n + \pi^+ + \pi^+ + \pi^-$$

This fact alone is not sufficient to rule out $I = 1$ for two reasons: (1) Although the initial state $(\pi^+ d)$ is a pure $I = 1$ state, the final state $(pn\pi^+\pi^+\pi^-)$ considered as a state of p, n and a hypothetical η^+ does not have a unique isospin configuration. Therefore interference effects could inhibit the production of η^+ if it existed. (2) The η^+ might exist but not decay appreciably into the mode $\pi^+\pi^+\pi^-$. However, by now both these objections have been overcome. Evidence against the first possibility is that the η has been seen in numerous experiments,[29]

*The η is sometimes called the χ.

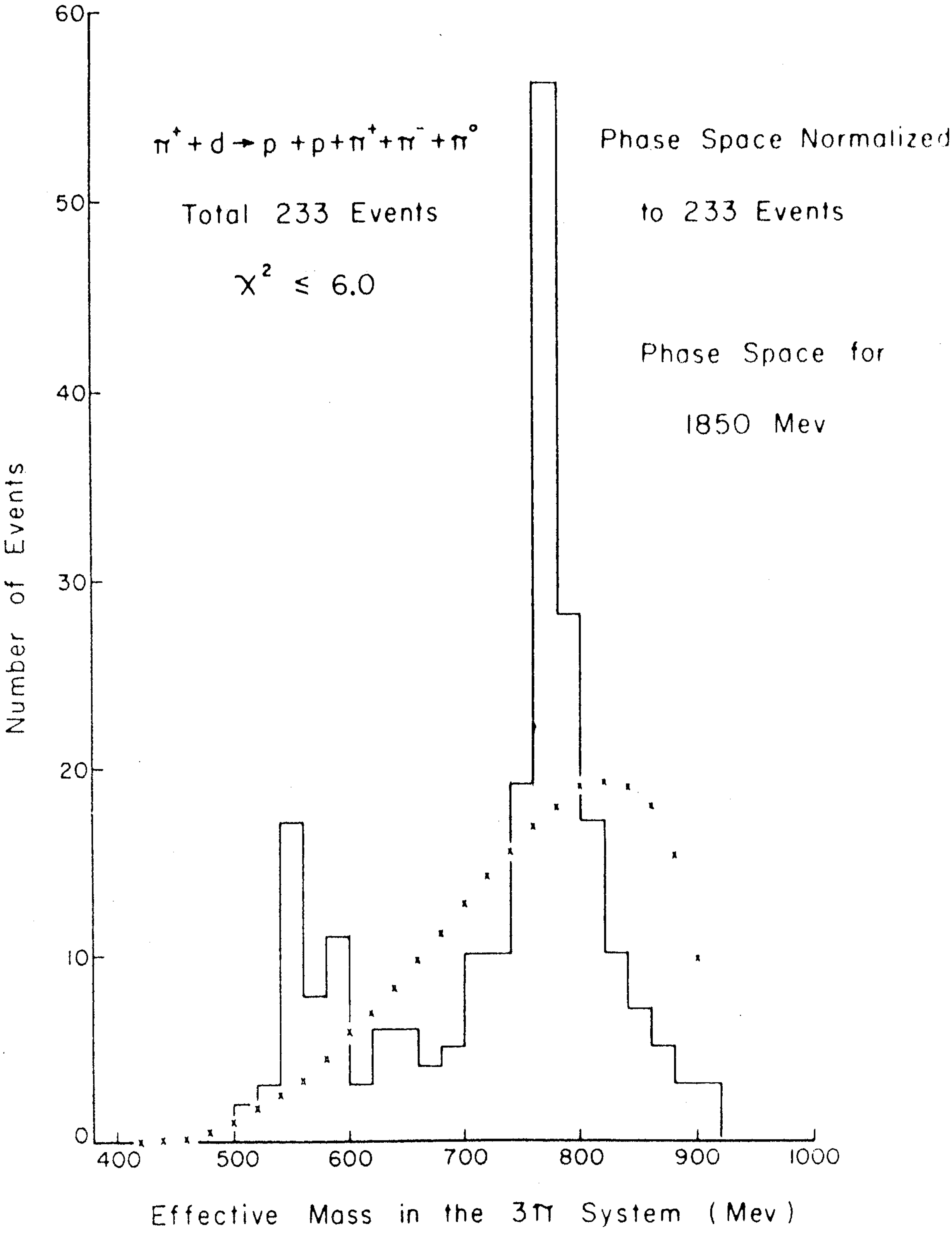

FIG. 7--Histogram of the effective mass of the three-pion system for 233 events. (From Ref. 28)

including the quasi-two-particle mode[20]

$$\pi^+ + p \rightarrow N^*_{3/2} + \eta^0$$

in which the branching ratio into the various charge states is a unique function of the isospin, and no evidence for a $\pi^+\pi^+\pi^-$ correlation has been seen.

Experiment has also shown that the second objection is invalid. For example, Carmony, Rosefeld and Van de Walle[30] have looked without success for a peak at 550 Mev in the spectrum of $\pi^\pm$ + neutrals produced in the reaction

$$\pi^\pm + p \rightarrow p + \pi^\pm + \text{neutrals}$$

at an incident momentum of 1.25 Bev/c. This experiment was done at essentially the same momentum as the experiment of Pevsner et al. Therefore, by charge independence, if the $\eta^\pm$ existed it would have been seen.

b) Spin, parity, and G parity. Good evidence that the η has assignment 0^{-+} comes from a Dalitz plot. As with the ω, the matrix element for the decay depends on the quantum numbers. We have already listed some matrix elements in Table XII in connection with the ω decay.

In Table XIII are some other matrix elements which are not antisymmetric in the momenta of the three pions because the pions are not in an $I = 0$ state.[31]

TABLE XIII

The G-forbidden 3π decays.[a] (From Ref. 31)

Meson	l, L	Simplest matrix element[b]	Vanishes at	Dominant radiative decay modes
0^{-+}	0,0	α	nowhere	2γ, $\pi^+\pi^-\gamma$
1^{++}	1,0	$\alpha\vec{p}$	$T_{\pi^0} = 0$	$\pi^+\pi^-\gamma$
1^{-+}	2,2	$\alpha(\vec{p}\times\vec{q})(\vec{p}\cdot\vec{q})$	T_{π^0} axis and boundary	$\pi^+\pi^-\gamma$

[a]The matrix element is analyzed in terms of a $\pi^+\pi^-$ pair and a π^0. The $\pi^+\pi^-$ pair is assigned momentum $\vec{q}$ and angular momentum $\vec{L}$. The remaining π^0 is described in the 3π rest frame by momentum $\vec{p}$ and angular momentum $\vec{l}$.

[b]The factor α (fine-structure constant) appears since G-forbidden transitions require that the decay proceed via electromagnetic interaction.

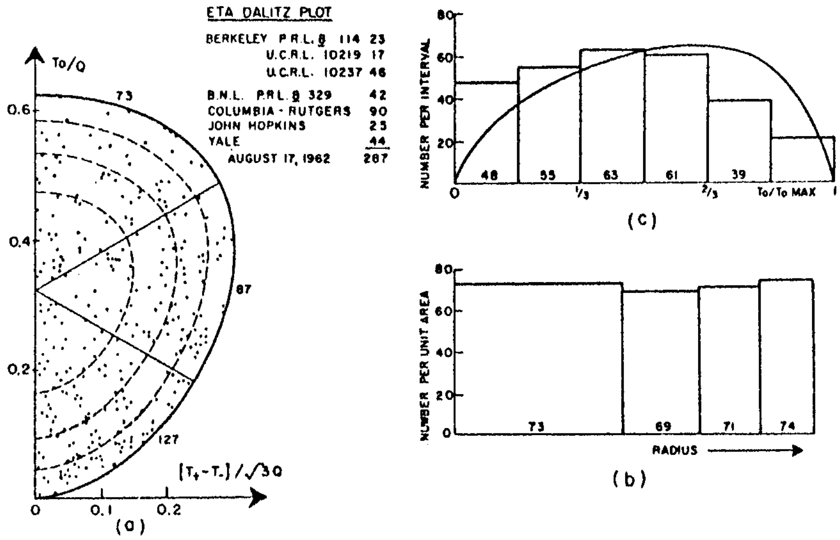

FIG. 8--The Dalitz plot and projections for all published η decays into $\pi^+\pi^-\pi^0$. (a) shows the distribution of points, (b) the radial density, and (c) the projection of the points on the π^0 axis. The solid line in (c) corresponds to uniform population. (From Ref. 26.)

Figure 8 (from Ref. 26) shows a Dalitz plot for η decays. The density of points does not appear to vanish in any region, a fact which strongly supports 0^{-+}. On the Dalitz plot there appears to be some depopulation of events at large π^0 kinetic energy T_{π^0}. If the assignment 0^{-+} is correct, the non-uniformity means that the matrix element is more complicated than the simplest possible one for 0^{-+} (uniform density). Matrix elements for $I \geq 2$ have not been listed, but they should contain high powers of the momentum and not lead to an approximately uniform density on a Dalitz plot.

Other evidence for 0^{-+} comes from observation by Chretian *et al.*[32] of the decay $\eta \to 2\gamma$. This shows that the η has $C = +$, and therefore $G = +$, since for an $I = 0$ meson, $G = C$. This observation also rules out $J = 1$, since a spin 1 particle cannot decay into two photons.

The experimental branching ratio of the η into different decay modes is

$$(\eta \to \text{neutrals})/(\eta \to \pi^+\pi^-\pi^0) = 2.5 \pm 0.5$$

Chretian et al. have estimated that about half of the neutral modes are $\eta \to 2\gamma$. The other half are probably mostly $\eta \to 3\pi^0$ with perhaps some $\eta \to \pi^0 + 2\gamma$. The rates for $\eta \to 2\gamma$ and $\eta \to \pi^+ + \pi^- + \pi^0$ are both proportional to the factor α^2 where $\alpha = 1/137$. Furthermore, there is much more available phase space for $\eta \to 2\gamma$ than for $\eta \to \pi^+ + \pi^- + \pi^0$. Therefore, naively one would expect $\eta \to 2\gamma$ to be dominant, but the rates are about equal. Also, the decay rate for $\eta \to \pi^+ + \pi^- + \gamma$ contains only a factor α, and might be expected to be dominant over $\eta \to \pi^+ + \pi^- + \pi^0$, whereas experimentally the reverse is true. In fact, the experimental branching ratios have been quoted as evidence for assignment other than 0^{-+}. These branching ratios show either that the assignment 0^{-+} is wrong (a rather remote possibility), or that one should not take too seriously arguments about the quantum numbers of particles based on theoretical estimates of decay rates.*

With the quantum numbers 0^{-+}, the η has the same quantum numbers as a four-pion state. It does not have enough mass to decay into charged pions, and decay into four neutral pions would not have been seen. The decay $\eta \to 4\pi^0$ is severely inhibited by lack of available phase space in any case.

It is interesting that with the assignment 0^{-+}, the η cannot be produced in peripheral collisions with the exchange of any single definitely established meson. Experiment confirms that the η is not produced preferentially in low momentum transfer events. This is illustrated in Fig. 3b (from Ref. 20) which shows a relatively isotropic production angular distribution. Also in Fig. 3 are shown the production angular distributions for the ρ and ω. The ρ, which can be produced in an event with single pion exchange, is peaked strongly in the forward direction. Production of the ω, which can occur via single ρ exchange, requires a somewhat larger momentum transfer, and the forward peaking is not so pronounced.

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*Okubo and Marshak [33] have proposed an approximate symmetry principle to inhibit the modes $\eta \to 2\gamma$ and $\eta \to \pi^+\pi^-\gamma$.

11. THE K* MESON

The K* was first seen by Alston *et al.*[34] in the reaction

$$K^- + p \rightarrow \bar{K}^0 + \pi^- + p$$

at an incident K^- momentum of 1.15 Bev/c. Its mass is $M = 890$ Mev and its full width varies from $\Gamma = 16$ Mev to $\Gamma = 60$ Mev, depending on the experiment[34-36]. A Dalitz plot of the $K\pi N$ system is shown in Fig. 9 (from Ref. 34). It is apparent from Fig. 9 that the proton likes to come off at an almost unique energy of ≈ 20 Mev. This energy corresponds to a mass $M_{K*} = 890$ Mev. Also shown in Fig. 9 is a line on the Dalitz plot corresponding to formation of the $N^*_{\frac{3}{2}}$. Since there is no clustering of phase points around this line, there is no interference between K* and $N^*_{\frac{3}{2}}$ production in this reaction.

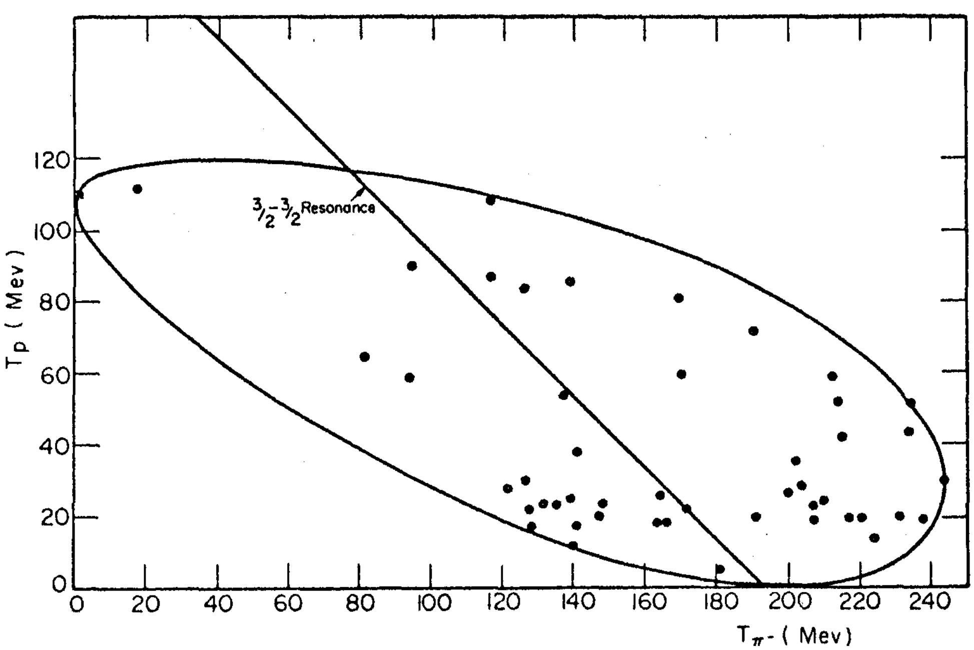

FIG. 9--Phase-space plot of the 48 examples of $K^- + p \rightarrow \bar{K}^0 + \pi + p$ reactions. (From Ref. 34)

The quantum numbers of the K^* have been determined:

$$I = 1/2,\ J^P = 1^-$$

a) Isospin. The branching ratio

$$R = (K^{*-} \rightarrow \bar{K}^0 + \pi^-) \ / \ (K^{*-} \rightarrow K^- + \pi^0)$$

is predicted to be

$$R = 2 \quad \text{if} \quad I = 1/2; \ R = 1/2 \quad \text{if} \quad I = 3/2$$

The experimental ratio agrees with $I = 1/2$. This conclusion depends on I being conserved in the decay, a reasonable assumption since the width of the K^* indicates that the decay is allowed by strong interactions. In support of the assignment $I = 1/2$ is the fact that doubly charged K^* mesons have not been seen.

b) Spin. Chinowsky et al.[36] have seen the K^* produced in the reaction

$$K^- + p \rightarrow p + K^0 + \pi^- + \pi^+$$

Nearly all the events were consistent with the reaction $K^- + p \rightarrow K^{*-} + N^*_{\frac{3}{2}}$. The production angular distribution of the K^* is peaked forward and fits the prediction of the peripheral model with a one-pion exchange diagram. Because of the forward peaking of the K^*, an Adair analysis is useful. Examining events with production angle $\cos \Theta \geq 0.8$, Chinowsky et al.[36] found the decay angular distribution to be

$$W_{K^*}(\theta) \approx \cos^2\theta$$

This shows immediately that $J > 0$.

Since the N^* has spin $J_{N^*} = 3/2$, the Adair analysis does not give a unique result. Assume, however, that $J_{K*} = 1$. Then, for the z component of the proton spin $m = 1/2$, we have the following possibilities for the angular distributions of the decays of the K* and N* (from Ref. 36):

m_{K*}	$W_{K*}(\theta)$	m_{N*}	$W_{N*}(\theta)$
0	$\cos^2 \theta$	1/2	$1 + 3 \cos^2 \theta$
1	$\sin^2 \theta$	-1/2	$1 + 3 \cos^2 \theta$
-1	$\sin^2 \theta$	3/2	$\sin^2 \theta$

Intermediate cases in which the K* and N* are not in pure m-states are of course also possible. The prediction of the peripheral model is

$$W_{K*}(\theta) = \cos^2 \theta, \; W_{N*}(\theta) = 1 + 3 \cos^2 \theta$$

The observed angular distributions are in agreement with this prediction, a fact which supports the assignment $J_{K*} = 1$.

There is other evidence not depending on the peripheral model, which supports $J = 1$. Alston et al.[34] obtained a decay angular distribution suggesting $J \leq 1$. The argument goes as follows: The K* was produced in the reaction $K^- + p \rightarrow K^* + p$ at an incident K^- momentum of 1.15 Bev/c. Since this corresponds to a low c.m. momentum of the K* and since the production angular distribution was isotropic, the assumption was made that the production was predominantly in the s-state. If this assumption is correct the Adair analysis can be applied rigorously to all events. The K* decay angular distribution (for $J > 0$) is then given by

$$W(\theta) = \left| aY_J^0 \right|^2 + \left| bY_J^1 \right|^2$$

where $|a|^2 + |b|^2 = 1$.

The second term in this formula arises from the fact that the proton spin can be flipped. The angular distribution for $J = 1$ can vary anywhere between $\sin^2 \theta$ and $\cos^2 \theta$ depending on the relative magnitudes of the proton spin-flip and non-spin-flip amplitudes. However, for $J > 1$, the K* must be partially aligned (m-values > 1 are forbidden) and the decay cannot be isotropic. These arguments are quantitatively summarized in the following expression for the average value of $\cos^2 \theta$ as a function of the spin of the K*

$$< \cos^2 \theta > = \frac{2J^2 + 2J - 3 + 2|a|^2}{4J^2 + 4J - 3}$$

where

$$< \cos^2 \theta > = \int W(\theta) \cos^2 \theta \, d \cos \theta \int W(\theta) \, d \cos \theta$$

In particular, if $J = 2$ then $< \cos^2 \theta > \geq 0.429$, and the minimum value of $< \cos^2 \theta >$ increases as J increases. The experimental result

$$< \cos \theta > = 0.275 \pm 0.051$$

is three standard deviations from being consistent with $J = 2$, but is consistent with $J = 0$ or 1. Combined with the result of Chinowsky <u>et al</u>., $J > 0$, this is evidence for $J = 1$.

c) <u>Parity</u>. If parity is conserved in the decay $K^* \to K + \pi$, we immediately have the result

$$P_{K*} = - P_K (-1)^J$$

since the intrinsic parity of the π is negative and the parity of orbital angualr momentum is $(-1)^J$. Because of this relation, the relative parity of the K* and K is determined by a measurement of the K* spin. Since $J = 1$, we have $P_{K*} = P_K$.

Recall that it does not appear possible to determine separately the intrinsic parities of the K, Λ, and Σ. This is because in parity conserving interactions, as far as we know, particles with strangeness $S = \pm 1$ must be produced in pairs. Thus only the relative K-Σ, Λ-Σ and K-Λ parities are measurable. For this reason, it is possible to define the intrinsic parity of one of these three particles. It is convenient to define the K to have negative parity to make it a pseudoscalar like the π. With this assignment, the K* also has negative parity and is a vector particle like the ω and ρ.

d) Caldwell[37] has suggested as an application of the Bohr analysis a method to obtain information about the spin and parity of the K*. Actually, there are practical difficulties with this method, and, as we have seen, the spin and parity of the K* have already been measured. Nevertheless, we shall give this application as an illustration of the Bohr analysis. Consider the reaction

$$K + He^4 \rightarrow K^* + He^4$$

Since the K and He^4 both have spin 0, Bohr's relation becomes

$$P_K = (-1)^{J_n} P_{K^*}$$

where J_n is the normal component of the K* spin. Comparing this relation with the expression for the K* parity previously obtained

$$P_{K^*} = - P_K (-1)^J$$

we see that $J = 0$ implies both $P_K = P_{K^*}$ and $P_K = - P_{K^*}$. Thus, if parity is conserved the reaction is forbidden.

Assuming parity conservation, if the reaction is seen, then $J \geq 1$. For $J = 1$ we must have $J_n = 0$ which yields a pure $\cos^2 \theta_n$ decay angular distribution where θ_n is the angle the π makes with the normal in the rest system of the K^*. For $J = 2$ we must have $J_n = 1$, and the decay angular distribution is $\cos^2 \theta_n \sin^2 \theta_n$. For $J \geq 3$, the angular distribution is not unique.

The practical disadvantage is that the reaction $K + He^4 \rightarrow K^* + He^4$ is likely to be rare regardless of the value of the K^* spin.[37,38] This can be seen as follows: Consider the cases in which the K^* is produced in the beam direction. Then by the Adair argument, the decay angular distribution of the K^* is $|Y_J^0|^2$. But $|Y_J^0|^2$ has a finite amplitude along the beam direction, so that if the K^* is produced at $\Theta = 0$, there will be a finite amplitude for $K + He^4 \rightarrow He^4 + K + \pi$ with all particles produced along the beam direction. But this configuration is forbidden by conservation of parity, as can be seen by performing a reflection with respect to the beam axis. Since the He^4, K and π have spin 0 and there is no orbital angular momentum about the beam direction, the only change as a result of the reflection is a change in parity because the pion is pseudoscalar. This means that the K^* must be produced only in states of Y_L^m with $m \neq 0$, i.e., principally at angles $\Theta > 1/L_{max}$. But at these angles the momentum transfer to He^4 is sufficient to break it up in most cases. Because of this difficulty, Caldwell also discussed

$$K + He^4 \rightarrow K^* + \pi + He^4$$

See his paper for details.

e) Still another method to measure the spin of the K^* has been proposed by Schwartz.[39] Again we discuss this method primarily because it is an interesting application of the conservation laws although it does provide additional evidence for $J = 1$.

Consider antiproton-proton annihilation at rest into the modes

$$\bar{p} + p \rightarrow \bar{K}^0 + K^{*0} \quad \text{and} \quad \bar{p} + p \rightarrow K^0 + \bar{K}^{*0}$$

with subsequent decays

$$K^* \rightarrow K^o + \pi^o \quad \text{and} \quad \bar{K}^* \rightarrow \bar{K}^o + \pi^o$$

The method consists of determining whether the K^o and $\bar{K}^o$ (one of which comes from the decay of a K^* or $\bar{K}^*$) decay into K_1K_1, K_2K_2 or K_1K_2. It is necessary to assume that the $\bar{p}p$ annihilation occurs in an s-state. If this assumption is valid, we shall see that observation of K_1K_1 or K_1K_2 gives information about the K^* spin.

But first we examine the evidence for s-state annihilation. The justification for this assumption is based partly on an argument of Day, Snow and Sucher,[40] who considered in detail the events subsequent to the capture of a heavy negatively charged particle in a Bohr orbit in hydrogen. They calculated that an inhomogeneous electric field from nearby atoms mixes various orbital angular momentum states. Therefore the antiproton is partly in an s-state even while principal quantum number n is large ($n \gtrsim 10$). Since the antiproton wave function has an appreciable amplitude at the postion of the proton only in an s-state, annihilation should take place overwhelmingly from this state.

D'Espagnat[41] has proposed a test of s-state annihilation. To discuss this proposal (and also the one by Schwartz), it is useful to quote some statements about the quantum numbers of particle-antiparticle pairs. A proton-antiproton pair in a given state of total spin j, orbital angular momentum L and total angular momentum J has parity and C parity given by

$$P = (-1)^{L+1}, \; C = (-1)^{L+j}, \; CP = (-1)^{j+1}$$

A $K\bar{K}$ pair has

$$P = (-1)^{L}, \; C = (-1)^{L}, \; CP = +$$

For $\bar{p}p$ in an s-state we have $P = -$ and $j = J$.

Now consider the reaction proposed by d'Espagnat:

$$\bar{p} + p \to K^o + \bar{K}^o$$

by antiprotons at rest. Let us examine the consequences of s-state annihilation. First, the parity of the initial state is negative. Therefore conservation of parity requires that the $K^o\bar{K}^o$ pair be produced in a p-state. Since the K has spin 0, p-state production can only occur if the $\bar{p}p$ annihilate from a triplet state with $C = -$. We now consider the decays of the K^o and $\bar{K}^o$ into K_1K_1, K_1K_2 and K_2K_2. We have seen that the $K^o\bar{K}^o$ state has $J = 1$, $C = -$, $P = -$, $CP = +$. Then since

$$J\,(K_1K_1) = \text{even},\ J\,(K_2K_2) = \text{even}$$
$$CP\,(K_1K_1) = CP\,(K_2K_2) = +,$$
$$CP\,(K_1K_2) = (-1)^{J+1},$$

conservation of J and CP in the decay implies that $K^o\bar{K}^o \to K_1K_2$, and not to K_1K_1 or K_2K_2. Thus if any K_1K_1 pairs are seen in $\bar{p}p$ annihilation, the reaction could not have taken place from an s-state. So far, experiment is consistent with only K_1K_2 pairs having been produced.

Since a mechanism inhibiting $\bar{p}p \to K\bar{K}$ from a p-state is possible, the absence of K_1K_1 does not prove that $\bar{p}p$ annihilation always occurs in an s-state. However, it is hard to think of a model that would lead to $\bar{p} + p \to \bar{K} + K^*$ occuring in a p-state if $\bar{p} + p \to K + \bar{K}$ occurs only in an s-state.

We now return[42] to $\bar{p} + p \to K^o + \bar{K}^{o*}$, assuming s-state annihilation and therefore negative parity. If L is the orbital angular momentum of the K with respect to the K^* and J is the K^* spin, the parity of the $\bar{K}K^*$ state is

$$P = P_{K^*}P_K\,(-1)^L = (-1)^{L+J+1} = -$$

This implies $L + J$ is even, which can only occur for $L = J$ because the total angular momentum is zero or one.

First consider the case in which the spin of the K^* is $J = 0$. Then $L = 0$, and $\bar{p}p$ annihilation must occur from the singlet state. But in this state $CP\ (\bar{p}p) = -$. Now consider the states $K_1K_1\pi^0$, $K_1K_2\pi^0$, and $K_2K_2\pi^0$ with $L = J = 0$. Of these states, $K_1K_1\pi^0$ and $K_2K_2\pi^0$ are odd under CP and $K_1K_2\pi^0$ is even, so that $J = 0$ implies that only the decays $K_1K_1\pi^0$ or $K_2K_2\pi^0$ are allowed. If $J \neq 0$, in general all three modes will occur. Experimentally, K_1K_2 pairs have been seen with one of the pair having a unique energy corresponding to a $\bar{K}^0K^0$ state. This is evidence for $J \neq 0$.

It is convenient to discuss here how the modes K_1K_1, K_1K_2 and K_2K_2 are distinguished. The K_2 has a lifetime $\approx 6 \times 10^{-8}$ sec which is sufficiently long so that the K_2 leaves the detection area before decaying. Thus K_2 decays are not seen, and the mode K_2K_2 is not detected. The K_1 lifetime, on the other hand, is sufficiently short (10^{-10} sec) so that if it decays into $\pi^+\pi^-$ (which it does 2/3 of the time) it is usually seen.* In the $\bar{p}p \to \bar{K}K^*$ reaction, one concentrates on events in which a visible K_1 has a unique energy. Then, if the K^* decays into $K_1 + \pi$, this K_1 should be visible 2/3 of the time (with 100% detection efficiency). The experimentally observed branching ratio is[43]

$$\frac{K_1\ (\text{seen})\ K_1\ (\text{seen})}{K_1\ (\text{seen})\ K\ (\text{not seen})} = \frac{13 \pm 11}{43 \pm 14}$$

compared to the value 2 which would be predicted for on K_1K_1. This strongly indicates that some K_1K_2 are being produced and that therefore the K^* has $J \neq 0$.

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*How often the decay is seen depends on the efficiency of detection, which is presumably known.

12. THE $K\bar{K}$ SYSTEM

Evidence exists for two separate effects in the $K\bar{K}$ system at a mass $M = 1020$ Mev. Two separately resolved peaks have not been seen, but a peak has shown up in the mode K_1K_1 in two experiments[44,45] and in the mode K_1K_2 in another.[46] Since the decay modes K_1K_1 and K_1K_2 are eigenfunctions of CP with opposite eigenvalues for the same value of the total angular momentum, the modes cannot arise from the decay of a single meson without violating CP invariance.

If CP is conserved, then a single meson can decay into both K_1K_1 and K_1K_2 only if it is not an eigenstate of CP and has a distinct antiparticle. Then one linear combination of the meson and its antiparticle can be formed which is even under CP and another which is odd. In such a case, we can confine our attention to the two linear combinations, which are distinct mesons. Furthermore, since the preliminary evidence indicates that the particles decaying into K_1K_1 and K_1K_2 are produced in different reactions, there is at present no reason (apart from a similarity in masses) to identify them as a single meson. We shall call by the names ξ and φ the particles decaying into K_1K_1 and K_1K_2 respectively.*

a) The ξ. We briefly review the relevant experiments. Erwin *et al.*[44] considered $K\bar{K}$ pairs produced in the reactions

$$\pi^- + p \to K^0 + \bar{K}^0 + n,\ \pi^- + p \to K^- + K^0\ p,\ \pi^- + p \to K^+ + K^- + n$$

by 1.89 and 2.10 Bev/c pions. A mass spectrum of the K^-K^0 pairs followed phase space within statistics, but the $K^0\bar{K}^0$ pairs appeared somewhat concentrated at low mass values. All $K^0\bar{K}^0$ pairs were observed as K_1K_1 since events with K_1K_2 were not kinematically determined. Not enough K^+K^- pairs were observed for a mass spectrum of this mode to be meaningful. The results of Erwin *et al.*,[44] while not statistically convincing by themselves, have been supported by a similar experiment of Alexander *et al.*[45] at incident pion momenta less than 2.3 Bev/c.

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*The φ is the name given by Sakurai,[47] who predicted the quantum numbers should be $I = 0$, $J^{PG} = 1^{--}$.

We make use of the selection rules given previously to obtain information on the quantum numbers of the ξ. First, note that $J\ (K_1K_1)$ is even, assuming the K_1K_1 system obeys Bose statistics. Assuming P and C are separately conserved* in the decay of the ξ, i.e., that the decay is $\xi \rightarrow K^0\bar{K}^0$ with the $K^0\bar{K}^0$ subsequently decaying into K_1K_1, the quantum numbers of the ξ are

$$J^{PG} = 0^{++},\ 2^{++},\ \text{if}\ \ I = 0;$$

$$J^{PG} = 0^{+-},\ 2^{+-},\ \text{if}\ \ I = 1$$

since $G = (-1)^I C$. The evidence for $I = 0$ is that the mass spectrum of K^-K^0 follows phase space in the experiments of Erwin et al.[44] and Alexander et al.[45]

Further evidence for $I = 0$, $G = +$ is that the nucleon tends to go backward in the c.m. system, and no correlations have been seen in the production and decay planes of the ξ. These results are consistent with a model in which the ξ is produced in peripheral collisions via one pion exchange. Such a model requires I even, $G = +$ if J is even. In the spirit of this model, an estimate of the cross section $\sigma\ (\pi\pi \rightarrow K\bar{K})$ can be obtained using the Chew-Low formula. See Ref. 44 and 45 for details.

Alexander et al.[45] have been able to fit the K_1K_1 mass spectrum assuming an s-wave $K\bar{K}$ interaction in a zero effective range approximation. They find that the scattering amplitude is well represented by a complex scattering length. If this interpretation is correct, the ξ has $I = 0$, $J = 0^{++}$. These authors also point out that if $J = 2^{++}$, a much more rapid decay into two pions would be expected, whereas a two-pion resonance at $M = 1020$ has not been seen. However, such an argument depends on dynamical assumptions, and is not good evidence against 2^{++}. For example, a broad two-pion resonance centered at an energy below two K masses might not have been seen. Another (rather remote) possibility is that $I = 1$,

*If the decay of the ξ were weak, the ξ would probably travel a measurable distance before decaying, in contradiction to experiment.

$G = -$ requiring decay into two pions to be forbidden. The behavior of the K^-K^o mass spectrum is good but not definitive evidence against this.

b) The φ. A meson decaying into K_1K_2 and K^+K^- was observed by L. Bertanza et al.[46] in the reactions

$$K^- + p \to \Lambda + K^o + \bar{K}^o, \; K^- + p \to \Lambda + K^+ + K^-$$

with 2.24 and 2.5 Bev/c incident K^- mesons. In this experiment, measurement of the Λ by observation of its decay allows a kinematic determination of the mode K_1K_2. Since there was no way to reach the K^-K^o state in this experiment, the isospin is unknown. As Sakurai[47] has pointed out, evidence on the isospin can be obtained from the reaction

$$K^- + d \to K^- + K^o + \Lambda + p$$

Since $CP\,(K_1K_2) = (-1)^{J+1}$ and $CP\,(K^+K^-) = +$, the φ must have

$$J = \text{odd},\; CP = +,\; C = P = -$$

assuming C and P are conserved in its decay. Thus, its quantum numbers are

$$J^{PG} = 1^{--},\; 3^{--},\; \ldots \text{ if } I = 0;$$

$$J^{PG} = 1^{-+},\; 3^{-+},\; \ldots \text{ if } I = 1$$

Sakurai[47] argues that if $I = 1$, the decay $\varphi \to 2\pi$ would be dominant, since it would be allowed by strong interactions and is strongly favored by phase space. Since this mode has not been seen he favors $I = 0$. Also, he favors $J = 1$, since for $J = 3$ the decay $\varphi \to \rho + \pi$ is greatly favored by phase space. However, arguments based on branching ratios should be taken with caution, as we have seen with the η. With these assignments, the φ is like a heavy ω and should also decay into $\pi^+\pi^-\pi^0$. This mode also has not been seen.

Leitner and Samois* have observed the φ in K^-p interactions at 2.3 Bev/c, obtaining a narrow width $\Gamma \leq 4$ Mev. They also measured the branching ratios $(\varphi \to \pi^+\pi^-)/(\varphi \to K\bar{K}) < 0.2$, $(\varphi \to \rho + \pi)/(\varphi \to K\bar{K}) = 0.3 \pm 0.2$ and $(\varphi \to K_1K_2)/\varphi\ (K\bar{K}) = 0.45 \pm 0.10$. Here $K\bar{K}$ means $K_1K_2 + K^+K^-$.

c) Bigi et al.[48] and Belyakov et al.[49] have also seen evidence for a low energy peak in the $K^0\bar{K}^0$ system in the π^-p interactions at 10 Bev/c. It is not stated whether the observed decay modes are K_1K_1 or K_1K_2.

13. THE ABC EFFECT

Abashian, Booth and Crowe[50] measured the recoil He^3 and H^3 momentum spectra from the reactions

$$\begin{aligned} p + d &\to He^3 + \pi^0,\ H^3 + \pi^+ \\ &\to He^3 + \pi^+ + \pi^-,\ He^3 + \pi^0 + \pi^0,\ He^3 + X^0 \\ &\to H^3 + \pi^+ + \pi^0,\ H^3 + X^+ \end{aligned}$$

at incident proton energies of 624 to 743 Mev. The X is any particle of mass $M_X < 2.8\ M_\pi$. These authors obtained a peak in the He^3 spectrum corresponding to a mass M = 310 Mev of an unobserved particle, usually called ABC. Since the initial state is pure $I = 1/2$, the ABC can have isospin either 0 or 1, and the branching ratio into H^3 and He^3 is a unique function of the isospin. For events in the peak

$$H^3/He^3 = 2 \quad \text{if} \quad I = 1,$$

$$H^3/He^3 = 0 \quad \text{if} \quad I = 0$$

The absence of a corresponding bump in the H^3 spectrum thus shows that $I = 0$.

Abashian, Booth and Crowe were able to fit their data assuming a strong s-wave pion-pion interaction with a scattering length about 2 or 3 pion Compton wavelengths. However, the width of the ABC was only a

*N. Samois and J. Leitner (to be published).

little larger than the authors' computed resolution function, so a narrow peak which could not be interpreted this way is not necessarily excluded.

The effect of the ABC was also seen by Richter[51] in a proton recoil experiment in the reaction

$$\gamma + p \rightarrow \text{ABC} + p$$

Richter found a mass $M = 322 \pm 8$ and a width $\Gamma \leq 20$ Mev, consistent with a zero true width. He was unable to fit his data by assuming an s-wave pion-pion interaction.

The ABC has also been observed at Frascati[52] in a photoproduction experiment.* In this experiment two charged particles associated with the ABC were observed in a spark chamber. Assuming these to be pions, there are restrictions on the spin and parity of the ABC: $J^P = 0^+, 1^-, 2^+$, etc. Preliminary evidence indicates that the data cannot be fit by a pion-pion scattering length.

It has been assumed quite often in the literature that the ABC has quantum numbers $J^{PG} = 0^{++}$. Until more is known about the decay of this state, such an assignment can be only a guess.

14. OTHER POSSIBLE MESONS

a) The ζ. Weak evidence for the existence of the ζ meson[53,54] has been around a long time (on the time scale of the discovery of heavy mesons), but confirmatory experiments have not been forthcoming. Since the ζ seems to appear only in special experiments, these should be repeated with better statistics.

The ζ has been seen as a peak in the two-pion mass spectrum at a mass $M = 570$ Mev and with a width $\Gamma \leq 20$ Mev. Since it was seen in the reaction**

$$p + p \rightarrow d + \zeta^+$$

it has isospin $I = 1$. This follows because the two-proton state is pure $I = 1$ and the deuteron is $I = 0$.

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*This is an unpublished preliminary result.

**The ζ was seen in a bubble chamber experiment (Ref. 54), but not seen in a later counter experiment (Ref. 55) at approximately the same incident energy.

Since the ζ decays into two pions, its spin and parity are $J^P = 0^+$, $1^-, 2^+, 3^-$, etc. If it decays via strong interactions then it has odd spin, $P = -$, $G = +$, since these are the possible quantum numbers of a two-pion state with $I = 1$. Then, unless its spin is 3 or higher, it has the same quantum numbers as the ρ meson $(J^{PG} = 1^{-+})$ and should be produced in peripheral collisions like ρ. But in an unpublished preliminary experiment, the groups at Wisconsin and Purdue[56] have observed ζ mesons in the reactions

$$\pi^- + p \rightarrow \zeta^0 + n, \ \pi^- + p \rightarrow \zeta^- + p$$

$$\pi^+ + p \rightarrow \zeta^+ + p$$

at incident on pion momentum of 1.12 Bev/c with a cross section $\sigma(\pi N \rightarrow \zeta N) \approx 0.1$ mb, much smaller than the comparable cross section for ρ production, $\sigma(\pi N \rightarrow \rho N) \approx 2$ mb. (The ζ has not been seen at all in most laboratories.) This means that if the ζ has the same quantum numbers as the ρ and if the peripheral model is valid, the coupling of the ζ to two pions is much weaker than the corresponding $\rho\,\pi\pi$ coupling:

$${g_{\zeta\pi\pi}}^2 \Big/ {g_{\rho\pi\pi}}^2 \lesssim 1/20$$

The narrow width of the $\zeta (\Gamma \leq 20$ Mev) also shows that if $J^{PG} = 1^{-+}$, then $g_{\zeta\pi\pi}$ is small.

Other assignments of quantum numbers should also be considered. The apparent absence of the decay $\zeta^\pm \rightarrow \pi^+ + \pi^- + \pi^\pm$ is only weak evidence against 1^{--}, even though the mode is allowed by strong interactions. This is because to satisfy Bose statistics and the conservation laws the two like pions must be in an $L \geq 2$ state and the unlike pion must be in an $L \geq 2$ state relative to the center of mass of the like pions. These high internal angular momenta, plus limited available phase space, should severely inhibit three-pion decay.

Similarly, the absence of the decay $\rho \rightarrow \zeta + \pi$, which is allowed by strong interactions if the ζ is 1^{--} does not provide good evidence against this assignment. Estimates of the decay ratio $(\rho \rightarrow \zeta + \pi)/(\rho \rightarrow 2\pi)$ have varied from 0.03 to 0.75. Our point is not that this ratio is necessarily small but that any estimate of it is necessarily model dependent and cannot be used to exclude 1^{--}.

However, there is some evidence against 1^{--} for the ζ. In a preliminary experiment the Purdue-Wisconsin group[56] has observed a peak, which might be a real effect or might be a statistical fluctuation of two standard deviations, in the $\pi^+ + \pi^-$ mass spectrum at ≈ 575 Mev. If this is the decay $\zeta^0 \to \pi^+ + \pi^-$, it implies $C = (-1)^J$ or $G = (-1)^{J+1}$. This experiment, if confirmed, would mean the ζ is 0^{+-}, 1^{-+}, 2^{+-} etc. Since the ζ does not behave like the ρ (which is 1^{-+}), the most probable assignment is 0^{+-}, provided this particle exists.

b) M(395) and M(520). Peaks at 395 Mev and 520 Mev were seen in the $\pi^+\pi^-$ spectrum in an experiment by Samios *et al.*[57] The reactions in which the peaks appeared were

$$\pi^- + p \to p + \pi^+ + 2\pi^- + \text{neutrals};\ \pi^- + p \to 2\pi^+ + 2\pi^- + \text{neutrals}$$

with 4.7-Bev/c π^-. The peaks were not seen in the $\pi^+\pi^+$ or $\pi^-\pi^-$ distributions. Since the peaks appeared only in many-body final states with a large background it is possible that they come from resonances in other channels, from decays of other particles, or from other effects. The evidence that these are mesons is therefore not convincing.

c) M(620). Peaks in the $\pi^+\pi^+$ and $\pi^-\pi^-$ spectrum of mass $M = 620$, $\Gamma = 75$ appeared in four-pion final states in a $\pi^- p$ experiment at 4 Bev/c by Biswas *et al.*[58] The evidence is inconclusive.

d) M(420). A "three-standard-deviation" peak in the $\pi^+\pi^-$ spectrum at $M = 420$ was reported by Richardson *et al.*[59] in $\pi^+ + d$ interactions at 1.23 Bev/c incident momentum.

e) M(1250). Selove *et al.*[60] have seen a peak* which they call f^0 in the $\pi^+\pi^-$ mass spectrum at 1250 Mev with a width $\Gamma \approx 100$. The

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*The evidence for the existence of this meson has recently become statistically quite convincing.

reactions observed were

$$\pi^- + p \rightarrow p + \pi^- + \pi^0, \text{ and } \pi^- + p \rightarrow n + \pi^- + \pi^+$$

Since no corresponding peak was observed in the $\pi^-\pi^0$ spectrum, the authors concluded that the isospin is $I = 0$. The initial π^-p state is a mixture of $I = 1/2$ and $I = 3/2$ so that the relative rates of production of the neutral and charged mesons are not fixed by isospin conservation. Therefore the evidence for $I = 0$ is not compelling.

An examination of the decay angular distribution yielded the result that the π^- is emitted preferentially forward in the rest system of the M(1250). This means that if the f^0 is a meson, it was produced with an interfering amplitude which prevented the spin from being measured.*

f) M(1350). Evidence for the existence of a meson is the observation by Grashin and Shalamov[61] of a peak in the $\pi^+\pi^-$ spectrum at $M = 1.35$ Bev. This peak and also the ρ^0 peak were seen in $\pi^- + p \rightarrow \pi^+ + \pi^- + n$ at 2.8 Bev/c incident pion momentum in a heavy liquid bubble chamber. Not enough information was published for an adequate evaluation of the data.

g) M(625). A peak at $M = 625$ was observed in the three-pion mass spectra by Pickup, Robinson and Salant[62] in the reactions

$$p + p \rightarrow p + p\pi^+ + \pi^- + \pi^0; \quad p + p \rightarrow p + n + \pi^+ + \pi^+ + \pi^-$$

at 2 Bev incident energies. The observed decays $M^+(625) \rightarrow \pi^+\pi^-\pi^+$ and $M^0 \rightarrow \pi^+\pi^-\pi^0$ indicate $I \geq 1$. The evidence for this meson is not convincing.

h) M(325), M(460), M(590), M(780), M(980), M(1200). Inconclusive evidence for mesons of masses 590, 780, 980 and 1200 Mev was found by Grashin and Shalamov[61] in heavy liquid bubble chambers. The authors saw peaks in the states $\pi^-\pi^0$ and $\pi^-\pi^-$. Thus, if these are mesons, they have $I = 2$.

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*In analyzing additional data, V. Hagopian et al., Bull. Am. Phys. Soc. 8, 69(1963) obtained evidence for $J > 0$. J. Veillet et al., Phys. Rev. Lett. 10, 29(1963), have also seen the f^0 and found that $J > 0$. Assuming $I = 0$, J is even, and the most likely assignment is $J^{PG} = 2^{++}$.

Peaks at masses $M = 325$, 460, 550, 750, and 1000 Mev were observed as two-pion mass correlations by M. Ainutdinov et al.[63] in $\pi^- p$ collisions at 7.2 Bev/c incident momentum. The same peaks appeared in the charge states $\pi^+\pi^+$, $\pi^-\pi^-$, and $\pi^+\pi^-$, so that if the peaks are real the isospin is $I = 2$. The observation of the peaks at 550, 750, and 1000 Mev might be considered as giving support to those observed by Grashin and Shalamov at 590, 780 and 980. On the other hand, since the energies are slightly different, all effects might wash out if the data are combined.

Foster et al.[64] have presented "two- or three-standard-deviation" evidence for peaks in the $\pi^-\pi^-$ spectrum (but not in the $\pi^+\pi^+$ spectrum) at 460 and 585 Mev.

i) M(1340), M(1150), M(1650). Belyakov et al.[65] reported meson resonances obtained with a 7 Bev/c π^- beam on protons in a propane bubble chamber. The peaks found were $M(1340) \rightarrow \pi^+\pi^-\pi^+\pi^-$, $M(1150) \rightarrow K^0\pi^+\pi^+\pi^-$, $M(1650) \rightarrow K^0\pi^+\pi^+\pi^-$. The M(1340) could be another decay mode of the M(1350) reported by Graskin and Shalamov[61]. Again the evidence regarding these mesons is inconclusive.

j) $K^*(730)$. Alexander et al.,[35] in an experiment with 2.1 Bev/c π^- mesons on hydrogen, looked at three-body final states of K, π and Λ or Σ. In addition to observing the $K^*(888)$, these authors also observed a smaller peak in the $K\pi$ spectrum at a mass $M = 730$ Mev.*

This incomplete survey of reported mesons indicates that there may be a lavish supply of mesons awaiting verification. On the other hand, some of these reported mesons may be effects of anomalous thresholds in production processes,** manifestations of decays of other particles, effects of complicated Feynman graphs,*** or simply statistical fluctuations.

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*The $K^*(730)$ has recently been seen in several reactions by the Alvarez group (L. Alvarez, private communication).

**See a discussion of possible effects of anomalous singularities by R. Aaron, Phys. Rev. Lett. 10, 32(1963).

***D. Wong, private communication.

15. EXCITED BARYON DECAYS

The problem of determining the quantum numbers of the excited baryons is more difficult than the corresponding one of obtaining the quantum numbers of the heavy mesons. One reason is that a heavy meson usually decays into spinless particles (pions and kaons), whereas among the decay products of a baryon there is always a particle with $J \geq \frac{1}{2}$. A second reason is that the decay products of mesons of strangeness $S = 0$ are eigenstates of G, whereas this is not true of $S = 0$ baryon states. Thirdly, for mesons with $S = 0$ and $Q = 0$, the decay products are eigenstates of C or of CP, and again this is not true for baryon decays. Fourth, even if a meson (like the K^*) has $S \neq 0$, it can be produced in proton-antiproton annihilations at rest; excited baryons, of course, are too heavy to be produced in this way. For these reasons application of the conservation laws leads to fewer selection rules in excited baryon decays than in heavy meson decays.

The exception to this rule is that the isospins of the excited baryons are just as easy to obtain as the isospins of the heavy mesons. Also, techniques such as the Adair analysis often yield information about the spins, although usually with more effort than in the case of meson decays. It is the determination of the parities of the baryons that presents the most difficulty, usually requiring polarization experiments as well as measurements of differential cross sections. We shall discuss here some possible methods of obtaining spins and parities of excited baryons.

a) Minami ambiguity. As we have remarked previously, if an excited baryon of spin J decays freely into a spin 1/2 baryon and a spin 0 meson with parity conservation, the highest power of $\cos\theta$ in the decay angular distribution is $\cos^{2J-1}\theta$.

However, as was first noted in a special case by Minami* the angular distribution does not depend on the parity of the excited baryon. We give here a proof of this statement which follows a proof by Dyson and

*S. Minami, Prog. Theor. Phys. 11, 213 (1954).

Nambu* given in a different context.

If a particle of spin J is produced in any reaction, its most general spin wave function ψ_J is of the form

$$\psi_J = \sum_m a_m X_J^m$$

where the X_J^m are spin eigenfunctions with respect to any arbitrary axis and the a_m are complex quantities depending on the production angles, the angles of the arbitrary axis, and the dynamics of the interaction. The spin wave function X_J^m of the unstable particle can be written

$$X_J^m = \sum_{m'} (L\,\tfrac{1}{2},\, m - m',\, m' \,|Jm)\, Y_L^{m-m'} X_{\frac{1}{2}}^{m'}$$

where $X_{\frac{1}{2}}^{m'}$ is the spin wave function of the final baryon and $Y_L^{m-m'}$ is the orbital wave function. Here L is either $L = J + \frac{1}{2}$ or $J - \frac{1}{2}$, depending on the parity.

The decay angular distribution $W(\theta\varphi)$ of the unstable particle is given by

$$W(\theta\varphi) = |\psi_J|^2$$

where a sum over final baryon spins is implied.

Now consider the operator $\vec{\sigma} \cdot \hat{\theta}$ where $\vec{\sigma}$ is the Pauli spin operator and $\hat{\theta}$ is a unit vector in the θ direction. This operator is a scalar under rotation, and thus commutes with the total angular momentum. But $\vec{\sigma} \cdot \hat{\theta}$ is odd under the parity operation. Therefore the wave function

$$\psi_J' = \vec{\sigma} \cdot \hat{\theta}\, \psi_J$$

corresponds to a possible wave function of a particle of spin J and parity opposite that of the original particle. The angular distribution $W'(\theta\varphi)$ of the new particle is given by

$$W'(\theta\varphi) = |\psi_J'|^2 = |\vec{\sigma} \cdot \hat{\theta}\, \psi_J|^2 = |\psi_J|^2$$

*This proof is outlined in Ref. 8, p. 75.

since $\vec{\sigma} \cdot \hat{\theta}$ is hermitian and $(\vec{\sigma} \cdot \hat{\theta})^2 = 1$. Thus the decay angular distribution is independent of the parity no matter what the state of polarization of the produced particle. This means that even if the beam and target particles are polarized, the decay angular distribution of the excited baryon is independent of its parity.

b) Polarization. A measurement of the polarization of the final spin 1/2 baryon resulting from the decay of the excited baryon can yield information about the parity. Let us consider a specific case in which the excited baryon is produced in the forward direction. Then the Adair analysis applies. If the target nucleons are unpolarized and parity is conserved in the production reaction, then particles produced in the forward direction are unpolarized.* As a consequence, the spin 1/2 baryons from the decay of excited baryons are also unpolarized. Thus, to determine the parity of an excited baryon which decays into a spin 1/2 particle and a spin 0 meson by means of an Adair analysis, the initial target nucleons must be polarized and the polarization of the final spin 1/2 particles must be measured. Although such a complicated analysis seems unlikely at present, we list expected values of the polarization $P(\theta)$:

J	L	P	$W(\theta)P(\theta)$ ($x = \cos\theta$)
$\frac{1}{2}$	0	-	1
$\frac{1}{2}$	1	+	$2x^2 - 1$
$\frac{3}{2}$	1	+	$5x^2 - 1$
$\frac{3}{2}$	2	-	$18x^4 - 15x^2 + 1$
$\frac{5}{2}$	2	-	$13x^4 - 10x^2 + 1$
$\frac{5}{2}$	3	+	$50x^6 - 65x^4 + 20x^2 - 1$

*If the spin is greater than 1/2, the baryon spins are aligned.

In the above table P is the parity of the excited baryon relative to that of the baryon into which it decays, assuming that the decay meson is pseudoscalar. Except for a constant overall factor, the values of $W(\theta)P(\theta)$ are independent of the degree of polarization of the target nucleons.

Barshay* has pointed out that any information about spins and parities which can be obtained from an Adair analysis with incident pions can also be obtained with linearly polarized photons. If the latter are used, however, the decay angular distributions and final polarizations must be measured with respect to the azimuthal angle φ as well as θ. If circularly polarized photons are used on a polarized target, a measurement with respect to φ is not needed, and indeed, yields no extra information. Unfortunately, it is not known at present how to obtain a sufficiently intense beam of circularly polarized photons.

If polarized targets are available and the polarization of the final particles can be measured, then relative parities can also be measured by means of a Bohr analysis.[11]

c) Decays into spin 3/2 particles. If an excited baryon decays into a spin 3/2 particle and a spin 0 particle, there is no Minami ambiguity. Thus, there is at least a possibility of obtaining evidence concerning the parity from the decay angular distribution without measuring polarizations.

Observing a decay into a spin 3/2 particle is not as remote a possibility as one might think at first. This is because a number of the known excited baryons have sufficiently large masses to decay into other excited baryons plus pions. A second reason is that according to the idea that particles lie on Regge trajectories, high-mass states should also have high spins. If so, it may be more favorable (by way of minimizing the orbital angular momentum) for these particles to decay into spin 3/2 (or higher) excited states rather than into their ground states.

The decay angular distribution of a particle of unknown polarization cannot be predicted uniquely. Furthermore, if the particle decays into a spin 3/2 particle, the angular distribution is not uniquely determined even if the polarization is known. This is because there are two possible

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*S. Barshay, Phys. Letters 3, 320 (1963).

values of the orbital angular momentum consistent with a given spin and parity of the excited baryon. For example, suppose the Y_0^* (1815) has spin 5/2. (The experimental evidence for this value is lacking.) Then if it decays into $Y_1^*(1385) + \pi$, the orbital angular momentum may be $L = 1, 3$ if the two states have even relative parity or $L = 2, 4$ if the relative parity is odd.

Since the decay angular distribution is not unique, any statement about the relative parity depends on dynamical assumptions. It may be reasonable to assume that if the energy released in the decay is low, then the decay proceeds through the lower of the two allowed values of L. In the example $Y_0^*(1815) \rightarrow Y_1^*(1385) + \pi$, suppose it is justified to assume $L = 1$ (rather than 3) for even parity, $L = 2$ (rather than 4) for odd parity. Then the appearance of a $\cos^4 \theta$ term in the decay angular distribution implies $L = 2$ or odd parity. The absence of a $\cos^4 \theta$ term does not imply even parity, since the $Y_0^*(1815)$ may be produced unpolarized.

Now consider decays in which it is valid to make an Adair analysis. Then the decay angular distribution is a unique function of the spin and relative parity as follows:

J	L	P	$W(\theta)$ ($x = \cos\theta$)
$\frac{3}{2}$	0	-	1
$\frac{3}{2}$	1	+	$7 - 6x^2$
$\frac{5}{2}$	1	+	$1 + 2x^2$
$\frac{5}{2}$	2	-	$1 + 10x^2 - 10x^4$
$\frac{7}{2}$	2	-	$13 - 10x^2 + 45x^4$
$\frac{7}{2}$	3	+	$21 - 120x^2 + 465x^4 - 350x^6$

These angular distributions hold only if three dynamical assumptions are valid:

(1) The maximum orbital angular momentum entering the production reaction is known.

(2) The excited baryon decays as a free particle into two other free particles.

(3) The amplitude for decay into the higher of the two allowed orbital angular momenta is negligibly small.

Because of these necessary assumptions (the first two of which are necessary in any Adair analysis), this method can provide only inconclusive evidence concerning the spin and parity of an excited baryon.*

16. EXCITED NUCLEON STATES

Evidence for the existence of excited nucleon states has come from pion-nucleon scattering and photoproduction experiments. Since most of this information has been around for quite a while,[8,66] we shall discuss it only briefly.

In Fig. 10 are plotted the total cross sections for π^+p and π^-p collisions as a function of the incident pion energy. Peaks are observed in the π^+p cross-section at invariant masses of 1238 and 1920 Mev, and there is further evidence of structure. Peaks are observed in the π^-p cross-section at masses of 1238, 1510 and 1690 Mev. Of these peaks, only the one at 1238 Mev has been established as arising from a single resonant state.

a) $\underline{N^*_{3/2}(1238)}$. This resonance has been established to have isospin $I = 3/2$ and spin and parity $J^P = (3/2)^+$. We briefly discuss the evidence. Let the pion-nucleon scattering amplitude in the $I = 3/2$ and $I = 1/2$ states be a_3 and a_1. Then the cross sections

$$\sigma_+ = \sigma(\pi^+p \to \pi^+p), \quad \sigma_- = \sigma(\pi^-p \to \pi^-p), \quad \sigma_0 = \sigma(\pi^-p \to \pi^0 n)$$

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*Another method of determining spins and parities of excited baryons has been proposed by C. Itzykson and M. Jacob, Physics Lett. 3, 153(1963). These authors suggest looking at $Y^*\bar{Y}$ pairs produced in the forward direction in antiproton-proton annihilation and measuring polarization correlations among the final baryons. Still another method has been proposed by Nina Byers, who suggests looking at angular correlations and polarizations in processes such as $K^- + N \to \Xi^* + \pi$, $\Xi^* \to \Xi + \pi$, $\Xi \to \Lambda + \pi$.

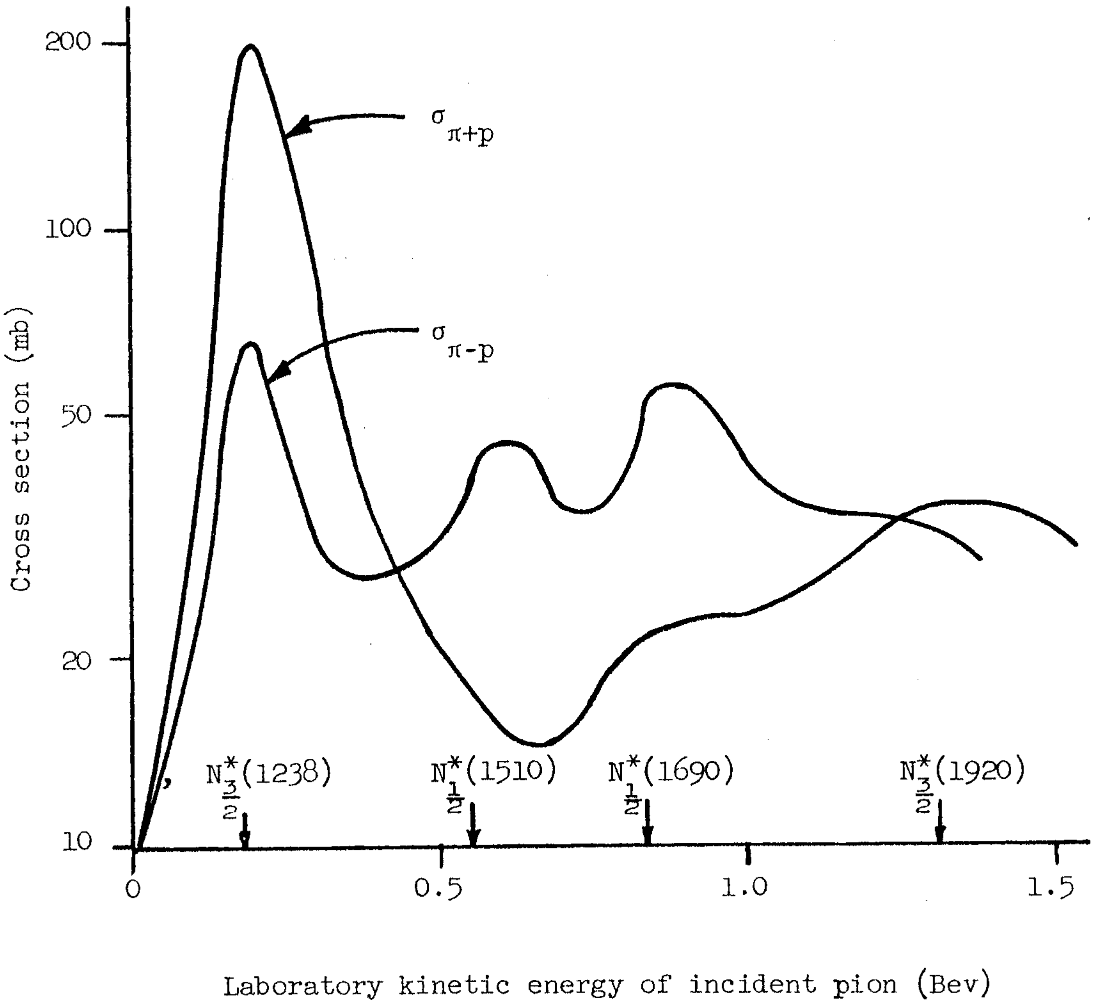

FIG. 10--Total π^+p and π^-p cross sections.

can be written in terms of these amplitudes as

$$\sigma_+ = |a_3|^2$$

$$\sigma_- = \frac{1}{9} |a_3|^2 + \frac{4}{9} |a_1|^2 + \frac{4}{9} \mathrm{Re}\, a_3^* a_1$$

$$\sigma_0 = \frac{2}{9} |a_3|^2 + \frac{2}{9} |a_1|^2 - \frac{4}{9} \mathrm{Re}\, a_3^* a_1$$

It follows that

$$(\sigma_- + \sigma_0)/\sigma_+ = \frac{1}{3} + \frac{2}{3} |a_1|^2/|a_3|^2 .$$

But experimentally

$$(\sigma_- + \sigma_0)/\sigma_+ = \frac{1}{3}$$

showing that $|a_1|^2 \ll |a_3|^2$, i.e., $I = \frac{3}{2}$. Consistent with this evidence is

$$(\sigma_+/\sigma_0/\sigma_-)_{exp} = 9/2/1,$$

showing that

$$\mathrm{Re}\, a_3^* a_1 \ll |a_3|^2$$

The spin is determined from the observed angular distribution

$$W(\theta) = 1 + 3 \cos^2 \theta$$

which is characteristic of pure $J = \frac{3}{2}$. This is confirmed by the fact that the observed magnitude of the cross-section at the resonance is given by the unitarity limit

$$\sigma_+ = 8\pi/k^2 = 2\pi(2J + 1)/k^2 \text{ with } J = 3/2,$$

where k is the relative momentum in the C.M. system.

The evidence that the resonance has positive parity is that the dominant phase shift δ_{33} at low energy goes as k^3, a behavior which indicates a p-wave and therefore odd parity of the orbital wave function. Since the pion has negative parity, the total parity is positive. Detailed phase shift analyses, making use of Coulomb interference effects and using a smooth behavior of the phase shifts as a function of energy, confirm these conclusions.

b) $N^*_{1/2}$ (1510). Since no peak is observed in the π^+ cross-section at this mass, the isospin is usually considered to be $I = 1/2$. However, there is a dip in the π^+p cross-section, so that there might be some effect in the $I = 1/2$ state. Evidence supporting $I = 1/2$ comes from the photoproduction reactions $\gamma + p \to \pi^+n, \pi^0p$. The branching ratio π^+/π^0 is given by

$$\pi^+/\pi^0 = 2 \quad \text{for} \quad I = 1/2$$

and

$$\pi^+/\pi^0 = \tfrac{1}{2} \quad \text{for} \quad I = 3/2$$

Experimentally $\pi^+/\pi^0 \approx 2$, supporting $I = 1/2$. Angular distributions in pion-nucleon scattering and photoproduction experiments and recoil proton polarization measurements give evidence to support $J^P = \left(\frac{3}{2}\right)^-$. However, phase shift analyses are not unique and do not necessarily support the notion that there is a single dominant phase shift.

c) $N^*_{1/2}$ (1690). The evidence that $I = 1/2$ is based on the absence of a peak in π^+p cross section. (However, there is a "shoulder" in the π^+p cross section at this mass.) The evidence for the assignment $J^P = \left(\frac{5}{2}\right)^+$ is based on angular distributions and polarization measurements, both in pion-nucleon scattering and in photoproduction, but is inconclusive.

d) $N^*_{3/2}$ (1920). The evidence for the $I = 3/2$ assignment is that a bump is seen in the π^+p cross-section. There is a little experimental information favoring[67] $J = 7/2$.

Some evidence has also been found by Erwin et al.[68] for a ΣK resonance at a mass of 1920 Mev. This may be an alternate decay mode of the $N^*_{3/2}$ (1920), although its width appears somewhat narrower. If the ΣK is a manifestation of the $N^*_{3/2}$ (1920), then it has $I = 3/2$. Ye. Kuznetsov et al.[69] have seen a peak in the $\Lambda^0 K^0$ system at a mass of 1840 Mev. If this is interpreted as a $\Sigma^0 K^0$ followed by $\Sigma^0 \rightarrow \Lambda^0 + \gamma$, then $M^*(\Sigma K) = 1920$. However, in our opinion the reason Kuznetsov et al. made this interpretation is that they were aware of the experiment of Erwin et al. No $\Lambda\pi$ resonance has been seen at this mass, a fact which is consistent with $I = 3/2$.

Sometimes $N^*_{3/2}$ (1920) is given a spin and parity assignment $\left(\frac{7}{2}\right)^+$. This is probably based more on the Regge trajectory hypothesis than on the experimental evidence.

e) $N^*_{1/2}$ (1700). Various groups[70] have observed a peak at $M = 1700$ in the cross-section for

$$\pi^- + p \rightarrow \Lambda + K^0$$

at an incident pion energy of 900 Mev. While at first sight this might be considered an alternate decay mode of the well-established $N^*_{1/2}$ (1690), there is some evidence that it is not. Feld and Layson[71] have shown that the data on ΛK production can be fitted with a smooth background plus a $p_{1/2}$ resonance. This analysis is not unique, however. Since several fits to the $\pi^- p$ and photoproduction data seem to indicate that the $N^*_{1/2}$ (1690) is an $f_{5/2}$ resonance, there may be two different resonances, $\left(\frac{1}{2}\right)^+$ and $\left(\frac{5}{2}\right)^+$, at a mass of about 1700 Mev.

f) $N^*_{1/2}$ (2200). Diddens et al.[71a] measured the $\pi^- p$ total cross-section in a transmission experiment at incident momenta between 1.5 and 4.5 Bev/c. They observed a peak of 2 mb in the cross-section at an incident pion energy of 1.95 Bev, corresponding to an invariant mass of 2.2 Bev. Although the peak is small compared to background, the statistics are good enough to make the peak convincing. The same authors measured the π^+p cross-section and found no peak at this mass. This shows that if the peak arises from a pure state, it is $I = 1/2$. Nothing is known about the spin or parity.

g) $N^*_{3/2}$ (2350). Diddens et al.[71a] measured the π^+p total cross-section in the same momentum range as the π^-p cross-section. They observed a 2-mb peak at a π^+ incident energy of 2.37 Bev, corresponding to a mass of M = 2.35 Bev. A state of π^+p is a pure $I = 3/2$ state. Again, the spin and parity are not known.

17. THE $Y_1^*(1385)$ HYPERON

The first pion-hyperon resonance to be discovered was the Y_1^*. It appeared as a $\Lambda\pi$ resonance at a mass of 1385 Mev, seen by M. Alston, et al.[14] in the reaction $K^- + P \rightarrow \Lambda + \pi^+ + \pi^-$ at an incident laboratory momentum of 1.15 Bev/c (see Fig. 1). The full width Γ varies from experiment to experiment, but averages around $\Gamma = 50$.

a) Isospin. The isospin is uniquely determined to be $I = 1$ by the dominant decay mode $Y_1^* \rightarrow \Lambda + \pi$. An interesting feature is the small value of the ratio

$$R = \frac{Y_1^* \rightarrow \Sigma + \pi}{Y_1^* \rightarrow \Lambda + \pi} < 0.04$$

The $\Sigma\pi$ decay mode is expected to be somewhat inhibited by the smaller available phase space. To estimate the effects of the difference in the $\Lambda\pi$ and the $\Sigma\pi$ relative momenta, assume a simple model in which the decay rate λ into a particular 2-body mode goes as

$$\lambda \propto p\left(\frac{p^2a^2}{1 + p^2a^2}\right)^L$$

where p is the relative momentum of the two final-state particles, L is the orbital angular momentum, and a is an appropriate interaction radius. For $a = 1/(2M_\pi)$ we obtain

$$R = 0.57 \text{ for } L_{\Sigma\pi} = L_{\Lambda\pi} = 0, \quad P(\Lambda\Sigma) = +$$

$$R = 0.25 \text{ for } L_{\Sigma\pi} = L_{\Lambda\pi} = 1, \quad P(\Lambda\Sigma) = +$$

$$R = 0.11 \text{ for } L_{\Sigma\pi} = L_{\Lambda\pi} = 2, \quad P(\Lambda\Sigma) = +$$

$$R = 0.09 \text{ for } L_{\Lambda\pi} = 0,\ L_{\Sigma\pi} = 1, \quad P(\Lambda\Sigma) = -$$

$$R = 0.04 \text{ for } L_{\Lambda\pi} = 1,\ L_{\Sigma\pi} = 2, \quad P(\Lambda\Sigma) = -$$

The small observed ratio 0.04 (which is a reported upper limit) should not be considered as evidence for odd Λ-Σ parity. If this parity is even, however, there is probably something about the matrix element other than an angular momentum centrifugal barrier which inhibits the decay $Y_1^* \to \Sigma + \pi$.

b) Spin. Alston et al.[14] examined the angular distribution of the decays of the Y_1^*, and obtained results consistent with isotropy. Spin higher than 1/2 was not ruled out, however.

The best evidence concerning the spin of the Y_1^* comes from an experiment by R. Ely et al.,[72] who looked at Y_1^* hyperons produced in $K^- + p \to \Lambda + \pi^+ + \pi^-$ at 1.11 Bev/c. An Adair analysis with $\cos\Theta > 0.9$ yielded results consistent with either $J = 1/2$ or $3/2$. However, an examination of the decay angular distribution of the Y_1^* with respect to the perpendicular to the production plane yielded the result

$$W(\theta) = 1 + a\cos\theta + b\cos^2\theta$$

$$a = 0.07 \pm 0.24,\ b = 1.5 \pm 0.4$$

This is almost four standard deviations from isotropy. For the decay to be anisotropic (if $J > 1/2$), the Y_1^* must be produced polarized. The degree of polarization depends on the angle of production. For this analysis only events with $|\cos \Theta| < 0.5$ were used, there being 143 such events out of a total sample of 378. The small value of the asymmetry coefficient a in the analysis is consistent with the Y_1^* decaying as a free particle, but by no means proves this. A peculiarity of the data of Ely et al. is that $M_{Y_1^*} = 1376 \pm 3$ Mev, a result below the accepted value. This may indicate that interference effects actually are present. Button-Shafer et al.,[73] in an experiment with more than twice as many events as used by Ely et al., found a smaller coefficient b of $\cos^2\theta$

$$b = 0.48 \pm 0.36 \quad \text{for} \quad Y_1^{*+}, \; b = 0.51 \pm 0.33 \quad \text{for} \quad Y_1^{*-}$$

Since this experiment was at a K^- incident momentum of 1.22 Bev/c, the polarization of the Y_1^* may be varying rapidly in the region between incident momenta of 1.11 and 1.22 Bev/c.

c) Parity. In an experiment by D. Colley et al.,[74] Y_1^* hyperons were produced in the reaction

$$\pi^- + p \to \Lambda^0 + \pi^- + K^+$$

at an incident π momentum of 2.0 Bev/c. The angular distribution of the decay of the Y_1^* with respect to the normal to the production plane was found to yield the following values of the a and b coefficients:

$$a = 0.29 \pm 0.29, \; b = 1.29 \pm 0.78$$

Since a is small, interference effects may not be important. The large value of b indicates $J > 1/2$, but the error is such that $J = 1/2$ is not excluded.

However, this experiment is more interesting in what it has to say about the Λ-Y_1^* relative parity. The asymmetry in the Λ decay was measured, yielding an average value of the Λ polarization $|\bar{P}_\Lambda| = 0.82 \pm 0.27$. For the observed decay angular distribution $W(\theta)$, the maximum calculated value of $\bar{P}_\Lambda$ is $|\bar{P}_\Lambda|_{max} = 0.47 \pm 0.09$ if the Y_1^* is a $p_{\frac{3}{2}}$ state of $\Lambda\pi$, i.e., $P(\Lambda Y_1^*) = +$. Likewise, $|\bar{P}_\Lambda|_{max} = 0.28 \pm 0.05$

if the Y_1^* is a $d_{\frac{3}{2}}$ state, i.e., P = -. Thus, if J = 3/2 (as most evidence indicates)[?], the experiment favors even Λ-Y_1^* relative parity. If J = 1/2, however, the experiment favors $s_{\frac{1}{2}}$ or odd relative parity.

18. THE $Y_0^*(1405)$ HYPERON

The $Y_0^*(1405)$ resonance was observed by M. Alston <u>et</u> <u>al</u>.[69] in K^-p reactions at 1.15 Bev/c.

a) <u>Width</u>. Alston <u>et</u> <u>al</u>.[75] reported a full width of $\Gamma \approx 20$ Mev. Subsequent workers[76] reported $\Gamma \approx 50$ Mev. However, Frisk and Ekspong[77] examined events in which a Σ and a pion were produced by stopped K^- mesons in an emulsion. Looking at the resultant momentum of the Σ and π, they found a narrow line (five standard deviations from phase space) at 171 ± 2 Mev/c. They interpreted this line as resulting from the reaction

$$K^- + C^{12} \rightarrow Y_0^*(1405) + B^{11} + 11.9 \text{ Mev}$$

with the subsequent decay $Y_0^* \rightarrow \Sigma^{\pm} + \pi^{\mp}$. The small energy release combined with the narrow line led to the following values of the mass and width: M = 1404.7, $\Gamma < 1.4$ Mev. The interpretation was based primarily on the value of the mass coming out right for the assumed interaction.

b) <u>Isospin</u>. Alston <u>et</u> <u>al</u>. measured the following branching ratio of the decays of the Y_0^*:

$$R = \frac{\Sigma^0\pi^0}{\Sigma^+\pi^- + \Sigma^-\pi^+} = 0.6 \pm 0.2$$

The theoretical ratio is

$$R = 2 \quad \text{if} \quad I = 2$$

$$R = 0 \quad \text{if} \quad I = 1$$

$$R = 1/2 \quad \text{if} \quad I = 0$$

The experiment strongly favors $I = 0$. Consistent with this assignment is the apparent absence of $Y_0^* \to \Lambda + \pi^0$.

c) Spin and Parity. If the width of the Y_0^* is really $\Gamma < 1.4$, then a measurement of the spin should be free of interference effects, and an Adair analysis should yield the correct value of the spin. Thus far, no evidence for $J > 1/2$ has been seen. Nothing is known experimentally about the parity. An interpretation by Dalitz and Tuan[78] in terms of a bound s-state of K^-p is consistent with an assignment $1/2^-$, but, of course, does not confirm it.

19. THE $Y_0^*(1520)$ HYPERON AND THE PARITY OF THE Σ

Whereas most excited hyperon states were seen in Dalitz plots of three-body final states, the $Y_0^*(1520)$ was observed as a peak in the total K^-p cross-section measured as a function of energy. Subsequently this particle was seen in a Dalitz plot in other experiments. Ferro-Luzzi et al.[79] first saw the $Y_0^*(1520)$ in looking at K^-p interactions at K^- momenta from ≈ 300 to ≈ 500 Mev/c. They saw peaks in the following modes

$$K^- + p \to \bar{K} + N, \; K^- + p \to \Sigma + \pi, \; K^- + p \to \Lambda + 2\pi$$

The incident K momentum at the peak was 395 Mev/c, corresponding to a mass $M = 1520$. The width is $\Gamma = 16$. The branching ratios in the peak were found to be

$$\bar{K}N/\Sigma\pi/\Lambda\pi\pi = 30/55/15$$

a) Isospin. The absence of the decay $Y_0^* \to \Lambda + \pi$ (which is a pure $I = 1$ state) is evidence that $I = 0$. This is not proof of this assignment, however, since the mode might be inhibited for other reasons. Proof comes from observation of a peak in the mode $\Sigma^0\pi^0$. This follows because K^-p is a mixture of $I = 0$ and $I = 1$, and $\Sigma^0\pi^0$ has no $I = 1$ component. With the assignment $I = 0$, the branching ratios into the various charge states can be predicted. These are

$$K^-p/\bar{K}^0n = 1/1$$

$$\Sigma^+\pi^-/\Sigma^0\pi^0/\Sigma^-\pi^+ = 1/1/1$$

$$\Lambda\pi^+\pi^-/\Lambda\pi^0\pi^0 = 2/1$$

for events above background.* If one does not obtain these ratios in an experiment it means either that the background ratios are varying through the resonance or that there is interference with background amplitudes. (We are assuming isospin conservation both in production of the peak and in its decay.) The data on the branching ratio $K^-p/\bar{K}^0n$, on angular distributions, and on the polarization of the Σ^+ all indicate that interference with background is present.

b) Spin and Parity. The spin of the Y_0^* can be measured by looking at the angular distribution in the neighborhood of the peak in the cross section. Interference with background makes this angular distribution a non-unique function of the spin, and a detailed analysis must be performed. Following is a summary of the analysis of Ferro-Luzzi et al.[79]

The observed angular distribution of K^-p and $\bar{K}^0n$ can be written in the form

$$W(\Theta) = a + b \cos \Theta + c \cos^2\Theta$$

Since no powers higher than $\cos^2\Theta$ are required, the spin is most likely 3/2. Spin 5/2 cannot be strictly excluded, even though it seems unlikely. The isotropic term a is large, while the coefficient b of the $\cos \Theta$ is small. The simplest explanation of this is that the interference arises chiefly from states of the same parity. At low energy there is a large s-wave phase shift, so it is reasonable to assume that the isotropic term is chiefly s-wave. If so, then the resonance is a $d_{\frac{3}{2}}$ state of K^-p, and the parity is negative. With this interpretation, the $b \cos \Theta$

*The corresponding ratios for background events can be found by measuring the ratios above and below the peak.

term arises from a small amount of p-wave.

c) Parity of the Σ. Examination of the channel $K^- + p \rightarrow \Sigma + \pi$ has provided evidence that the Σ parity is positive.[80] A simplified version of the argument goes as follows:

The K^-p resonant channel is $d_{\frac{3}{2}}$ and therefore the corresponding state in the $\Sigma\pi$ channel must be either $d_{\frac{3}{2}}$ or $p_{\frac{3}{2}}$, depending on the Σ parity. Since the π is a pseudoscalar and so is the K by definition, $d_{\frac{3}{2}}$ means positive Σ parity. Thus we have the following possibilities:

$$d_{\frac{3}{2}} \rightarrow d_{\frac{3}{2}},\ s_{\frac{1}{2}} \rightarrow s_{\frac{1}{2}} \quad \text{if} \quad P_{\Sigma} = +$$

$$d_{\frac{3}{2}} \rightarrow p_{\frac{3}{2}},\ s_{\frac{1}{2}} \rightarrow p_{\frac{1}{2}} \quad \text{if} \quad P_{\Sigma} = -$$

The angular distribution is the same for both possibilities. Because of the interference between $J = 1/2$ and $J = 3/2$ amplitudes, the Σ will be produced polarized. The sign of the polarization depends on whether the interference is $(p_{\frac{1}{2}}, p_{\frac{3}{2}})$ or $(s_{\frac{1}{2}}, d_{\frac{1}{2}})$. Since the decay of the Σ^+ is a highly efficient analyzer, the polarization can be measured. This does not completely remove the ambiguity, however, since replacing the amplitudes by their complex conjugates also reverses the sign of the polarization. To remove this last ambiguity, Tripp et al.[80] invoked the Wigner theorem[81] which says that as the energy is increased, a resonant amplitude rotates counterclockwise in the complex plane. Although Wigner proved this theorem for elastic scattering, it is reasonable that it should hold in production reactions as well. Assuming this to be the case, the ambiguity is removed and the experiment favors even Σ parity.

The analysis leading to this result is based on the simplest assumptions consistent with the data. Although it is possible that a set of more complicated amplitudes might be consistent with negative Σ parity, no one has produced such a set of amplitudes which fit the data as well as the amplitudes of Tripp et al.[80]

20. PARITY OF THE Λ

Since we have just discussed the Σ parity, perhaps it is appropriate to make a few remarks here about the Λ parity, even though the evidence bearing on this question has not come from the $Y_0^*(1520)$. Briefly the argument favoring $P_\Lambda = +$ goes as follows:

Block, Lendinara and Monari[82] have observed the reaction

$$K^- + He^4 \rightarrow {}_\Lambda H^4 + \pi^0$$

with K^- mesons at rest.

If the spin of ${}_\Lambda H^4$ is $J({}_\Lambda H^4) = 0$, then angular momentum and parity conservation require $P_K(-1)^L = -P_\Lambda(-1)^L$, where L is orbital angular momentum, or (with the definition $P_K = -$) $P_\Lambda = +$. Thus the evidence for $P_\Lambda = +$ depends on $J({}_\Lambda H^4)$ being zero.

The evidence that $J({}_\Lambda H^4) = 0$ may be summarized as follows: Since the K^- is captured by He^4 at rest, it is reasonable from the argument of Day, Snow and Sucher[40] that the capture takes place in an s-state. Then, since both K^- and He^4 have spin zero, the total angular momentum of the system is zero. Now consider the final state wave function ψ_0 of ${}_\Lambda H^4$ and a pion. This consists of a spin wave function X_J^m and an orbital angular momentum wave function $Y_J^m(\theta\varphi)$ (since $L = J$) combined to form $J_{total} = 0$:

$$\psi_0 = \sum_m (JJm, -m|00)\, X_J^m Y_J^{-m}(\theta\varphi)$$

Since this wave function is invariant under rotation, we can choose the quantization axis along the line of flight of the ${}_\Lambda H^4$. Then only $Y_J^0(0)$ survives, and the spin wave function of ${}_\Lambda H^4$ is in a definite m state X_J^0. Thus, the decay angular distribution $W(\theta)$ is a unique function of the spin of ${}_\Lambda H^4$.

$$W(\theta) = \cos^2\theta \quad \text{if} \quad J = 1$$

$$W(\theta) = 1 \quad \text{if} \quad J = 0$$

Experimentally[82] $W(\theta) = 1$, showing $J = 0$ and therefore $P_\Lambda = +$.

There is one loophole in this argument. If an excited state ${}_\Lambda H^{4*}$ with $J({}_\Lambda H^{4*}) = 1$ exists and $P_\Lambda = -$, then the reaction

$$K^- + He^4 \rightarrow {}_\Lambda H^{4*} + \pi^0$$

is allowed, while the corresponding reaction to the ground state with $J = 0$ is forbidden. But since the decay ${}_\Lambda H^{4*} \rightarrow {}_\Lambda H^4 + \gamma$ would have a larger rate (for a sufficiently energetic γ) than the weak decay ${}_\Lambda H^{4*} \rightarrow He^4 + \pi^-$, the measurement showing $J = 0$ would merely be a determination of the spin of the ground state. So far, an experiment to look for photons from the decays of a possible ${}_\Lambda H^{4*}$ has not been carried out.

An experiment is in progress at CERN to measure the $\Sigma\Lambda$ relative parity by measurement of the decay mode $\Sigma^0 \rightarrow \Lambda + e^+ + e^-$. The number of events as a function of the invariant mass of the e^+e^- pair depends on the $\Sigma\Lambda$ parity. Preliminary evidence* indicates that the parity is positive.

21. THE $\Xi^*(1530)$ HYPERON

The Ξ^* was seen by Pjerrou et al.,[83] and by Bertanza et al.[84] we consider first the experiment by Pjerrou et al., in which 1.8 Bev/c incident K^- mesons produced Ξ particles in the reaction

$$K^- + p \rightarrow \Xi + \pi + K$$

These authors quote a mass $M = 1530$ and a width $\Gamma < 7$ Mev. Most of the 88 observed events of this type corresponded to

$$K^- + p \rightarrow \Xi^{*-} + K^+$$

*R. Adair et al., Meeting of the Am. Phys. Soc., New York, 1963 (post deadline paper.)

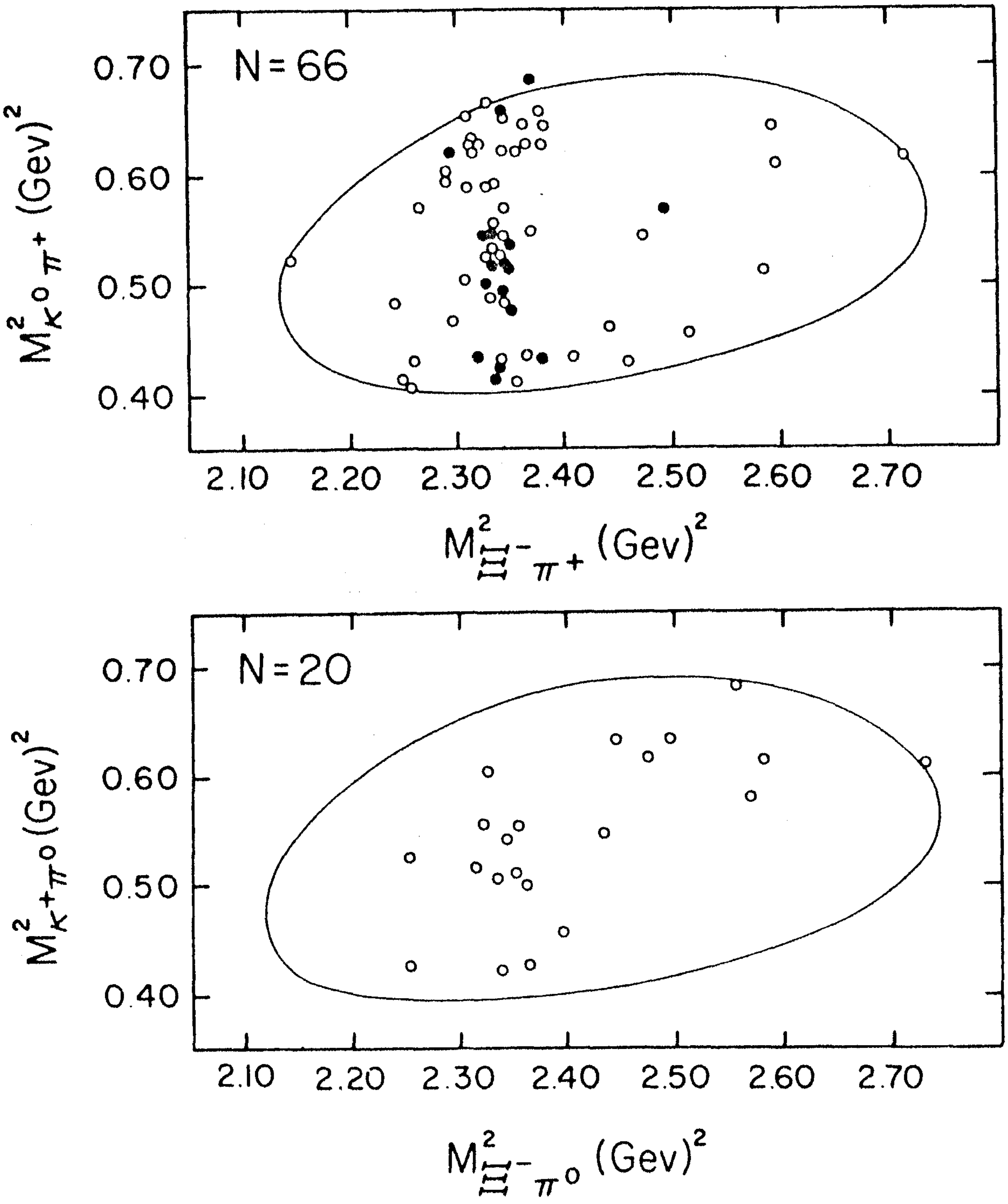

FIG. 11--Dalitz plots for the reactions $K^- + p \to \Xi^- + \pi^+ + K^0$ and $K^- + p \to \Xi^- + \pi^0 + K^+$. Solid dots represent events with both charged Λ and K_1^0 decays. (From Ref. 83)

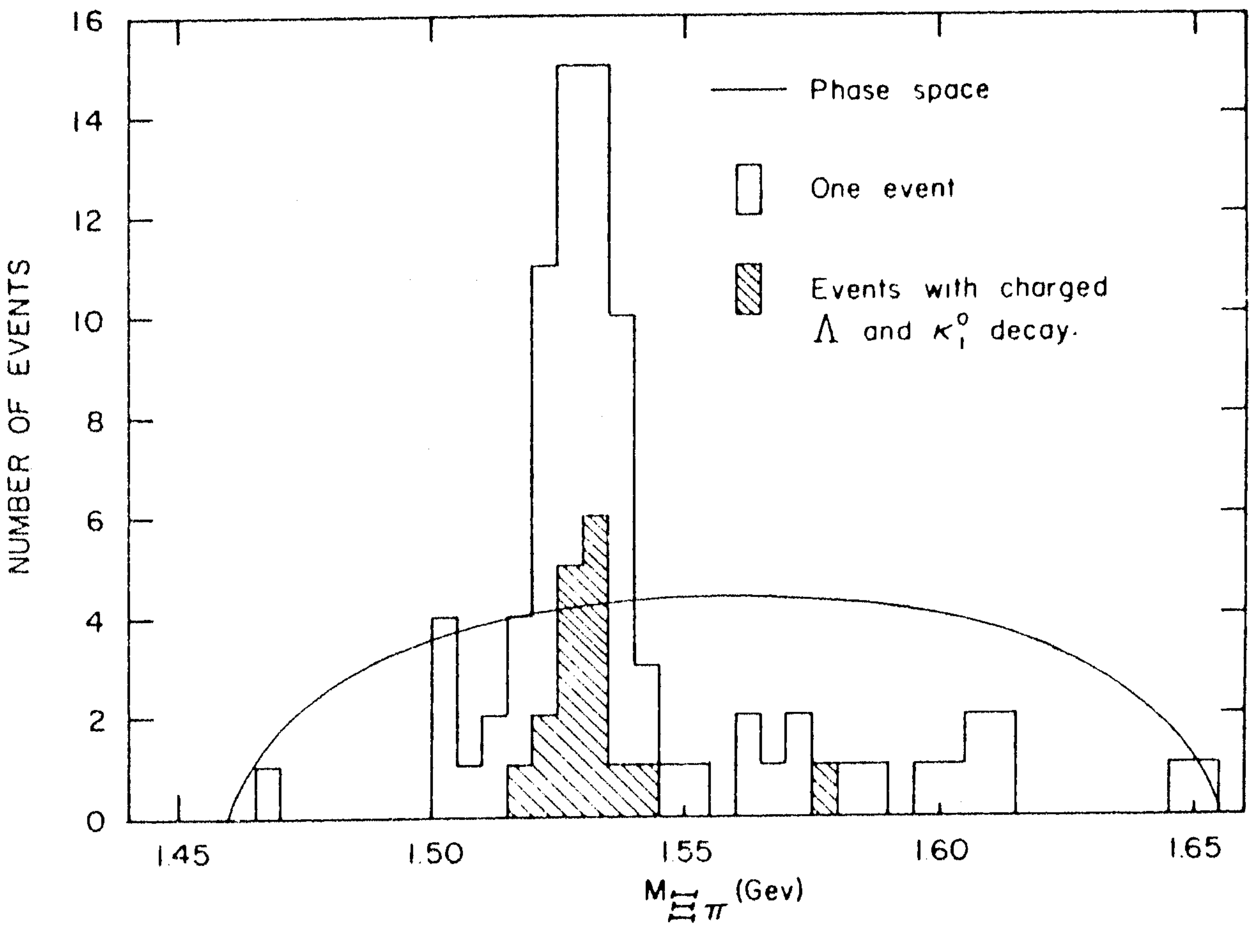

FIG. 12--Effective-mass plot for $\Xi\pi$ systems observed in K^-p reactions. The line is the relativistic three-body phase space. (From Ref. 83)

and

$$K^- + p \rightarrow \Xi^{*0} + K^0$$

Since background events were infrequent, the branching ratio

$$R_1 = (\Xi^- \pi^0 K^+)/(\Xi^- \pi^+ K^0)$$

gives information about the isospin. If the Ξ^* has $I = 3/2$, it can be reached only from the $I = 1$ component of $K^- p$. Then R_1 is given by the unique value $R_1 = 2$. The ratio R_1 is not unique for $I = 1/2$, since then the Ξ^* can be produced in both the $I = 0$ and $I = 1$ states of $K^- p$. Experimentally $R_1 = 30/75$, showing $I = 1/2$. Both the production of the Ξ^* and its decay were isotropic in this experiment within statistics. A Dalitz plot of the data is shown in Fig. 11 and an effective mass plot in Fig. 12.

Bertanza <u>et</u> <u>al</u>.,[84] saw the Ξ^* in the same interaction at incident K^- momenta of 2.24 and 2.5 Bev/c. They found a mass $M = 1535$ Mev and width $\Gamma < 35$ Mev. These authors obtained for the ratio $R_1 = 2/10$, again favoring $I = 1/2$. Further evidence concerning the isospin comes from observation of the branching ratios

$$R_2 = \frac{\Xi^{*0} \rightarrow \Xi^- \pi^+}{\Xi^{*0} \rightarrow \Xi^0 \pi^0}, \qquad R_3 = \frac{\Xi^{*0} \rightarrow \Xi^0 \pi^-}{\Xi^{*-} \rightarrow \Xi^- \pi^0}$$

The expected ratios are

$$R_2 = R_3 = 2 \text{ if } I = 1/2, \; R_2 = R_3 = 1/2 \text{ if } I = 3/2 .$$

Experimentally $R_2 = 5/0$, $R_3 = 3/2$, favoring $I = 1/2$. Background events were included in these experimental ratios.

Slight evidence that the spin of the Ξ^* is greater than 1/2 comes from looking at the angular distribution of its decay with respect to the normal to the production plane. Bertanza et al.[84] observed some anisotropy in their 79 events but their data were only 2 standard deviations from isotropy. There is no experimental evidence concerning the parity.

22. OTHER EXCITED HYPERONS

a) $Y_1^*(1660)$. Evidence for the existence of an excited hyperon of mass $M = 1660$ was obtained by several groups.[85,87] Bastien et al.[85] observed it as a peak in the $\Sigma\pi$ channel in K^-p interactions at incident momenta from 293 to 850 Mev/c. It was also seen by Alexander et al.[86] in π^-p interactions at 1.89 and 2.04 Bev/c. Since it appeared in the $\Lambda\pi$ as well as the $\Sigma\pi$ channel, it must have $I = 1$.

b) $Y_0^*(1815)$. A peak in the K^-p total cross section was observed at a K^- incident momentum of 1 Bev/c as a result of measurements by several groups.[88-90] A peak was also looked for in the K^-n cross section, obtained from measurements on K^- interactions in deuterium. The K^-n cross section σ_n was obtained by Chamberlain et al.[88] from a formula which takes screening effects approximately into account:

$$\sigma_n = \frac{\sigma_d - \sigma_p}{1 - (\sigma_p/4\pi) < 1/r_d^2 >}$$

where σ_d and σ_p are the K^- deuteron and proton cross sections and $< 1/r_d^2 >$ is the mean inverse square radius of the deuteron. Although a slight rise in σ_n appeared at 1 Bev/c, its magnitude was much below that required by charge independence if the peak were attributed to an $I = 1$ resonance. Theoretically

$$\sigma_n/\sigma_p = 2 \text{ if } I = 1 \qquad \sigma_n/\sigma_p = 0 \text{ if } I = 0$$

Since experimentally, $\sigma_p > \sigma_n$, the data favor $I = 0$.

Information about the spin of the resonance comes from elastic scattering[89] and charge exchange[90] measurements. Terms of $\cos^5\theta$ were necessary to fit the elastic scattering angular distribution, and terms of $\cos^6\theta$ were necessary in the charge exchange reaction at an energy somewhat above the resonance. Since there is a peak in the $\cos^5 \theta$ term, it is assumed that this term arises from interference between the resonant amplitude and a nonresonant term. This limits the resonant amplitude to be one of the following: $p_{\frac{3}{2}}$, $d_{\frac{3}{2}}$, $d_{\frac{5}{2}}$, $f_{\frac{5}{2}}$, or $f_{\frac{7}{2}}$.

23. OTHER POSSIBLE EXCITED BARYONS

In addition to the hyperons that have been established, some inconclusive evidence has appeared in the literature concerning other possible excited baryons.

a) N*(1650). We use the symbol N* to signify an excited baryon with strangeness $S = 0$. Evidence for a particle of mass $M = 1650$ Mev was obtained by Ye. V. Kuznetsov *et al.*,[69] in freon and xenon bubble chambers in the reaction

$$\pi^- + A \rightarrow \Lambda^0 + K^0 + m\pi + A'$$

with 2.8 Bev/c incident pions. The authors estimate a width $\Gamma < 7$ Mev. This follows, they say, because a comparison of the mass distributions in both chambers shows that all N* particles have time to leave the freon or xenon nuclei before decaying. The authors interpret this ΛK peak as the decay of a bound state of ΣK, citing A. Baz *et al.*[91] These interpretations appear speculative. Alternatively, the N*(1650) might be another decay mode of the N*(1690) with some error in the measured energy.

b) Y*(1430). March, Erwin and Walker[92] have looked at $\Sigma K \pi$ events produced by 2.1 Bev/c π^- on protons. A histogram showing the number of $\Sigma^- \pi^0 + \Sigma^0 \pi^-$ events against the mass of the $\Sigma\pi$ system shows bumps at 1430 and 1550 Mev. However, the number of events is small and there is a 14 per cent probability that the data are consistent with phase space modified by the effects of the effects of the K* resonance. These bumps merely indicate the need for more experiments on the charged $\Sigma\pi$ system. If the resonances exist, they have $I \geq 1$.

c) Y*(1550). The inconclusive evidence of March, Erwin and Walker[92] for this resonance was mentioned in connection with the Y*(1430). In addition, Dowell *et al.*,[93] have looked at the kinetic energy spectrum of K^+ mesons produced by 1.5 Bev/c π^- mesons in the reaction $\pi^- p \rightarrow Y^{*-} + K^+$. The K^+ mesons were detected and their velocities measured in a Cerenkov counter. In addition to seeing peaks in the K^+ kinetic energy spectrum corresponding to Σ^- and $Y_1^*(1385)$, they saw a peak corresponding to a negatively charged particle at a mass $M = 1550 \pm 20$ Mev. The isospin of this particle, if it exists,* is $I \geq 1$.

d) Y*(1760). Belyakov et al.[94] obtained weak evidence for a hyperon of mass $M = 1760$ in $\pi^- p$ interactions with a 7 Bev/c π^- beam in propane. The decay mode observed was

$$Y^*(1760) \rightarrow Y_1^*(1385) + \pi^-, \quad Y_1^* \rightarrow \Lambda + \pi^+$$

24. A CLASSIFICATION SCHEME

Many of the observed mesons and baryons can be classified successfully within the framework of a representation of the unitary group SU_3. Still more particles can be accommodated if the idea is adopted that the particles lie on Regge trajectories. We shall not explain the underlying ideas behind either unitary symmetry or Regge trajectories, but shall just state the rules of classification.

The representation of SU_3 which is most successful is the eight dimensional representation, or the "eightfold way".[95,96] A representation which was proposed earlier but which at present seems less successful is the original Sakata model[97] or threefold way. If the Λ and Σ should turn out to have opposite parity, or if the Ξ should turn out to have quantum numbers other than $J^P = (1/2)^+$, the eightfold way will have to be abandoned (with respect to $N \Lambda \Sigma \Xi$, at least) in favor of some other way.

The numerology of the eightfold way is that the outer product of two 8-representations leads to six irreducible subsets as follows:[95,98]

---

*The $Y^*(1550)$ has been looked for without success by the Alvarez group.[87]

$$8 \times 8 = 1 + 8 + 8 + 10 + \overline{10} + 27$$

If this has anything to do with reality, then we should expect the mesons and baryons to occur in multiplets of 1, 8, 10 and 27, (singlets, octuplets, decuplets and 27-plets*) where all members of a given multiplet have the same spin and parity.** To the extent that unitary symmetry holds, it is more than just a classification scheme. For example, one can predict branching ratios for the production and decays of the members of a given super multiplet, just as isospin symmetry enables one to predict branching ratios within an isospin multiplet.

If unitary symmetry were exact, all members of a multiplet would have the same mass. This state of affairs is clearly not realized in nature. However, under certain assumptions, a formula can be derived[100] which relates the masses of the members of a given multiplet:

$$M = M_o \left\{ 1 + a\, Y + b \left[I\,(I + 1) - 1/4\, Y^2 \right] \right\}$$

where M_o, a, and b are constants, I is the isospin, and Y is the hypercharge (which is related to the strangeness S and baryon number B by $Y = S + B$). The above formula holds for baryons; a similar formula holds for mesons with $a = 0$ and M and M_o replaced by M^2 and M_o^2 respectively. Glashow and Sarurai[101] have suggested that this formula also applies to the widths of unstable particles, suitably modified by phase space factors.

The unitary symmetry scheme can be combined with the idea of Regge trajectories.[102, 103] According to this idea, each member of a unitary multiplet is not merely a single particle but a series of particles, each of which has the same quantum numbers other than spin. The spins of the particles, on the other hand, increase in steps of two, and the masses start out by increasing with increasing spin. A rule of thumb, based on the few particles that fit the Regge scheme, is that the second member has a value of $M^2 \approx 2$ Bev^2 greater than the first. The Regge idea says nothing about how many particles should appear on a given trajectory.

*See footnote 3 of Ref. 99 for justification of this notation.

**The original Sakata model gives $3 \times 3 = 1 + 8$.

A notation has been suggested by Chew, Gell-Mann and Rosenfeld as being appropriate to this classification scheme. In this notation all particles with same baryon number, isospin, and strangeness are given the same symbol. A subscript is given to indicate the parity of the particle and the spin of the lightest particle on the Regge trajectory. The G parity of a meson of strangeness $S = 0$ may be indicated by a $+$ or $-$ left subscript. We add a second right subscript showing the place of a particle on a trajectory, omitting the subscript 1 for the lowest state.*

In table XIV we list symbols for the various quantum numbers under the new notation. In Tables XV through XIX, we classify the known particles according to the unitary-symmetry, Regge-trajectory scheme,** giving both the new notation and the old. The new notation seems simpler for the baryons and more complicated for the mesons. Whether this unsatisfactory state of affairs will persist depends on the number of particles yet to be discovered. Furthermore, the ω and the particle named φ by Sakurai may have the same quantum numbers and therefore the same symbol under the new notation. One can add a prime on one of them. Also, if the φ really has the same quantum numbers as the ω it is not clear which is the singlet and which is a member of the octuplet, although for theoretical reasons one would prefer to have the φ be singlet.

In Tables XV through XIX, a question mark after the quantum numbers of a particle indicates that the assignment is doubtful. In such cases, the classification amounts to a prediction of what the quantum numbers will turn out to be. Further more, two new baryons are predicted, another Ξ^* of mass $M = 1600$ and an Ω^- (sometimes called z^-) of mass $M = 1670$, $I = 0$, $S = -3$. The Ω^- is not energetic enough to decay into $\Xi + K$ and so should decay via weak interactions.

A question is where the ABC and the ξ (K_1K_1 peak at 1020 Mev) fit into the picture. We merely mention three possibilities (not meant to be exhaustive):

(1) The ABC and ξ are not particles but a manifestation of strong

*A further simplification results if the subscript α is omitted from the symbols for baryons, and the subscript β from the symbols for mesons.

**This has been done for the baryons in greater detail by Glashow and Rosenfeld,[99] who also compute relative decay width.

TABLE XIV

Notation of Chew, Gell-Mann and Rosenfeld for the mesons and baryons.

	Symbol	I	S	Subscript	J^P
Baryons	N	$\frac{1}{2}$	0	α	$\frac{1}{2}^+$
	Λ	0	-1	β	$\frac{1}{2}^-$
	Σ	1	-1	γ	$\frac{3}{2}^-$
	Ξ	$\frac{1}{2}$	-2	δ	$\frac{3}{2}$
	Δ	$\frac{3}{2}$	0		
	Ω	0	-3		
Mesons	π	1	0	α	0^+
	χ	0	0	β	0^-
	K	$\frac{1}{2}$	1	γ	1^-
				δ	1^+

TABLE XV

A baryon octuplet with $J^P = 1/2^+$ and possible Regge recurrences. The multiplicity 8 is arrived at by counting all charge states.

Old symbol	N	Λ	Σ	Ξ
New symbol	N_α	Λ_α	Σ_α	Ξ_α
Mass	939	1115	1190	1320
I, J^P	$\frac{1}{2},\frac{1}{2}^+$	$0,\frac{1}{2}^+$	$0,\frac{1}{2}^+$	$\frac{1}{2},\frac{1}{2}^+?$
Y, S	1,0	0,-1	0,-1	-1,-2

Old symbol	$N^*_{\frac{1}{2}}$	Y^*_0
New symbol	$N_{\alpha 2}$	$\Lambda_{\alpha 2}$
Mass	1688	1815
I, J^P	$\frac{1}{2},\frac{5}{2}^+?$	$0,\frac{5}{2}^+?$
Y, S	1,0	0,-1

TABLE XVI

A possible baryon octuplet with $J^P = 3/2^-$.

The Ξ_γ has not been seen, and its mass and quantum numbers (in parentheses) are predictions of the unitary symmetry model.

Old symbol	$N^*_{\frac{1}{2}}$	Y^*_0	Y^*_1	?
New symbol	N_γ	Λ_γ	Σ_γ	Ξ_γ
Mass	1512	1520	1660	(1600)
I, J^P	$\frac{1}{2}, \frac{3}{2}^{-?}$	$0, \frac{3}{2}^-?$	$1, \frac{3}{2}^-?$	$\left(\frac{1}{2}, \frac{3}{2}^-\right)$
Y, S	1, 0	0, - 1	0, - 1	(- 1, - 2)

TABLE XVII

A possible baryon decuplet and Regge recurrence. The Z has not been seen, and its predicted mass and quantum numbers are given in parentheses.

Old symbol	$N^*_{\frac{3}{2}}$	Y^*_1	$\Xi^*_{\frac{1}{2}}$	Z?
New symbol	Δ_δ	Σ_δ	Ξ_δ	Ω_δ
Mass	1238	1385	1530	(1670)
I, J^P	$\frac{3}{2}, \frac{3}{2}^+$	$1, \frac{3}{2}^+?$	$\frac{1}{2}, \frac{3}{2}^+?$	$\left(0, \frac{3}{2}^+\right)$
Y, S	1, 0	0, - 1	- 1, - 2	(- 2, - 3)

Old symbol	$N^*_{\frac{3}{2}}$
New symbol	$\Delta_{\delta 2}$
Mass	1920
I, J^P	$\frac{3}{2}, \frac{7}{2}^+?$
Y, S	1, 0

TABLE XVIII

An octuplet of pseudoscalar mesons.

Old symbol	π	η	K	$\bar{K}$
New symbol	$-\pi_\beta$	$+\chi_\beta$	K_β	$\bar{K}_\beta$
Mass	138	548	496	496
I, J^{PG}	1, 0^{--}	0, 0^{-+}	$\frac{1}{2}$, 0^-	$\frac{1}{2}$, 0^-
Y = S	0	0	1	- 1

TABLE XIX

An octuplet of vector mesons and a possible singlet.

	Octuplet				Singlet
Old symbol	ρ	ω	K^*	$\bar{K}^*$	φ
New symbol	$+\pi_\gamma$	$-\chi_\gamma$	K_γ	$\bar{K}_\gamma$	$-\omega'_\gamma$
Mass	750	787	890	890	1020
I, J^{PG}	1, 1^{-+}	0, 1^{--}	$\frac{1}{2}$, 1^-	$\frac{1}{2}$, 1^-	0, 1^{--}?
Y = S	0	0	1	- 1	0

nonresonant attractive forces in the $\pi\pi$ and $K\bar{K}$ systems respectively. As such they need the same explanation as all nonresonant cross sections.

(2) The ABC and ξ are unitary singlets. All singlets, whether mesons or baryons, must have $I = Y = 0$, and since this is true for the ABC and the ξ, they can be accommodated. This possibility places no restriction on the spin, parity and G parity of these particles. Too many singlets would make the scheme lose some of its attractiveness, however.

(3) The ξ has spin 2 and is on the so-called Pomeranchuk or vacuum Regge trajectory.[103] The ξ might still be a unitary singlet.

The ζ is also puzzling, as it cannot be a unitary singlet since it has $I = 1$. So are the $I = 2$ resonances. If these are members of multiplets, where are the others? Thus, these reported resonances provide a challenge to the experimentalists to confirm them or lay them to rest. If they should turn out to exist, they will provide a challenge to the unitary schemers.

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