Theoretical Aspects of Tau Physics

Eric Braaten

Department of Physics and Astronomy,
Northwestern University, Evanston, IL 60208

Abstract

I review two aspects of tau physics for which the observed mass of the tau is particularly fortuitous: tau polarimetry and the determination of α_s from tau decays.

WHO ORDERED THAT MASS?

The tau is the third generation charged lepton. The first generation charged lepton is the electron, discovered almost a century ago by Thomson. He showed in 1897 that cathode rays have a charge-to-mass ratio about 2000 times greater than the proton. No one had predicted the existence of such a particle, but now it is hard to imagine life without it.

The second generation charged lepton is the muon, discovered almost 40 years ago by Neddermeyer and Anderson, Street and Stevenson, and Nishina, Takeuchi, and Ichimiya. They showed in 1937 that cosmic rays include a particle whose mass is intermediate between the electron and the proton. It was eventually determined that this particle was not the intermediary of the nuclear force predicted by Yukawa. Instead its behavior was identical to that of the electron except that it was heavier by a factor of about 207, prompting Rabi's famous remark, "Who ordered that?"

The tau was discovered in 1975 by Martin Perl and his collaborators. After almost two decades of experimental effort, its behavior seems to be identical to the electron except that it is heavier by a factor of about 3480. We now have the three charged leptons given in Table 1, which all seem to have identical interactions and differ only in their masses. There is no clear pattern in their masses, but there is a clear pattern in the discovery dates which are also given in Table 1. A new charged lepton seems to make an appearance about every 40 years. This means that we should still expect to wait at least 20 years before the discovery of the next charged lepton.

Now suppose that you had had the opportunity in 1975 to order a charged lepton of any mass, and that you knew that you would have to wait 40 more years for the next one. What would you have chosen for its mass? I will argue that you could not have made a much better choice than the observed mass 1.8 GeV. There are two aspects of tau physics for which a mass in the range near 1.8 GeV is particularly fortuitous. These are tau polarimetry and the determination of α_s from tau decays, both of which I will review below. For a more thorough survey of theoretical aspects of tau physics, I refer the reader to an excellent review by Pich [1].

Lepton	Mass (MeV)	Discovery
e	0.511	1897
μ	105.7	1937
τ	1777	1975
# 4	?	~ 2014 ?

Table 1

TAU POLARIMETRY

In the process $e^+e^- \to \tau^+\tau^-$, the spins of the τ^+ and τ^- are strongly correlated as shown in a classic paper of Tsai in 1971 [2]. Near threshold, the two spins tend to be parallel and aligned with the beam direction. Far above threshold they tend to be parallel and aligned with the τ^+ and τ^- momenta. Other production processes also give taus with characteristic spin patterns. For example, the decay $W^- \to \tau^-\bar{\nu}_\tau$ produces taus with left-handed helicity only.

To exploit the spin information carried by the tau, some method of polarimetry is required. As shown by Tsai in 1971, several of the exclusive decay modes of the tau are effective spin analyzers [2]. For example in the decay $\tau^- \to \pi^-\nu_\tau$, the pion is emitted preferentially in the direction of the spin of the tau. The decay distribution is

$$\frac{d\Gamma}{d\cos\theta} = \Gamma\,\frac{1+P\cos\theta}{2}\,, \tag{1}$$

where θ is the angle between the momentum of the outgoing pion and the spin quantization axis of the tau and P is the polarization of the tau along that axis. The polarization can therefore be inferred probabilistically from the direction of the emitted pion. This two-body decay mode, whose branching fraction is $B \approx 12\%$, is one of the most sensitive spin analyzers for the tau. The purely leptonic decay modes $\tau^- \to e^-\bar{\nu}_e\nu_\tau$ and $\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau$, both of which have $B \approx 18\%$, can be used as spin analyzers, although the sensitivity to the polarization is decreased because of the extra undetectable neutrino. Two-body decays into hadron resonances, especially $\tau^- \to \rho^-\nu_\tau$ with $B \approx 24\%$ and $\tau^- \to a_1^-\nu_\tau$ with $B \approx 10\%$ can also be used for tau polarimetry. To maximize the spin information carried by these resonances, it is essential to use the pion spectra from their decays to separate their longitudinal and transverse polarization states [3].

Measurements of the tau polarization at the Z^0 resonance have already been used for a precise determination of the weak mixing angle θ_W [4]. The polarization predicted by the Standard Gauge Theory is

$$P = -2\left(1 - 4\sin^2\theta_W\right)\,. \tag{2}$$

Since $\sin^2\theta_W$ is close to 1/4, this observable is very sensitive to the value of θ_W. The measured polarization is $P = -0.13 \pm 0.03$, which translates into the value $\sin^2\theta_W = 0.233 \pm 0.004$. The most effective spin analyzing channels have been found to be $\tau^- \to \pi^-\nu_\tau$ and $\tau^- \to \rho^-\nu_\tau$. The errors in the polarization measurements using the leptonic channels $\tau^- \to e^-\bar{\nu}_e\nu_\tau$ and $\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau$ are twice as large, and those from $\tau^- \to a_1^-\nu_\tau$ are much larger still.

Tau polarimetry can be used to determine the general structure of the interactions of the tau without using polarized beams [5]. The complete Lorentz structure of the leptonic decay amplitude of the tau can be determined, as can the general form of its couplings to the photon, $W^\pm$, and Z^0. The measurability of the spin of the tau allows the construction of a large number of CP violating observables, which can be used to search for CP-violating interactions in the lepton sector [6]. Strong constraints on the electric dipole moment of the tau have already been obtained. Tau polarimetry can also provide constraints on the mixing of the tau with new exotic leptons [7].

Another application of tau polarimetry is the identification of heavy particles through their decays into taus [8]. For example, a charged Higgs and an extra W would both decay into a tau and a neutrino, but they can be distinguished by the helicity of the tau. A W'^- decays into $\tau_L^- \bar{\nu}_\tau$, while in the corresponding decay mode of a charged Higgs H^-, the τ^- is almost purely right-handed. Similarly a $Z^{0\prime}$ decays into $\tau_L^-\tau_L^+$ and $\tau_R^-\tau_R^+$, while a neutral Higgs prefers to decay into $\tau_L^-\tau_L^+$ and $\tau_R^-\tau_R^+$. Information on the spin of the tau can also be used to help identify the parity characteristics of the decay amplitude. This could be used to distinguish a Higgs from a technipion and to determine separately the vector and axial vector couplings of a $Z^{0\prime}$.

Tau polarimetry would be impossible in high energy processes if the tau mass were much smaller than 1.8 GeV. To use its decays as spin analyzers, a tau must decay inside the detector. Its decay length $\gamma c\tau$ is a sensitive function of the tau mass, scaling like $1/M_\tau^6$ for a relativistic tau. For example, when a tau pair is produced at the Z^0 resonance, the decay length is $\gamma c\tau \approx 1$ mm. If the tau mass was smaller by a factor of 3, the decay length would be greater than 1 m. If the tau were much lighter still, it would behave like a muon, decaying outside the detector and precluding any spin analysis. Tau polarimetry would also be much less effective if the tau mass were much greater than 1.8 GeV. The branching fractions for a tau to decay into a single pion or a single hadron resonance scale like $1/M_\tau^2$. If the tau mass was much larger, the only spin analyzing decay modes with appreciable branching fractions would be the purely leptonic decay modes. In particular, if the mass was larger by a factor of 3, the branching fraction for the most sensitive spin-analyzing decay modes $\tau^- \to \pi^-\nu$ and $\tau^- \to \pi^-\nu$ would be reduced to the few percent level.

DETERMINATION OF α_s

Another aspect of tau physics in which the observed mass of the tau plays an important role is the determination of the strong coupling constant from tau decays. This low energy determination of α_s is important because, when combined with precise high energy determinations of α_s provided by LEP, it provides dramatic evidence of the runnning of the QCD coupling constant.

In analyzing the hadronic decay rate of the tau lepton, it is convenient to normalize it to the electronic decay rate by defining the ratio

$$R_\tau = \frac{\Gamma(\tau^- \to \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} . \tag{3}$$

A naive estimate of this ratio can be obtained from the fact that at the quark level, the decay into hadrons proceeds through the processes $\tau^- \to \nu_\tau d\bar{u}$ and $\tau^- \to \nu_\tau s\bar{u}$. Since the couplings of $d\bar{u}$ and $s\bar{u}$ to a virtual W differ from that of $e^-\bar{\nu}_e$ only by a Kobayashi-Maskawa factor, the naive estimate of the ratio is

$$R_\tau \approx 3\left(|V_{ud}|^2 + |V_{us}|^2\right) , \tag{4}$$

where the factor of 3 accounts for the colors of the quarks. The squares of the K-M matrix elements add up to 1 to high accuracy, leaving the estimate $R_\tau = 3$.

A thorough analysis of all known corrections to this naive prediction has recently been carried out [9]. There are corrections to the naive prediction from electroweak interactions and from QCD:

$$R_\tau = 3\, S_{EW}\,(1 + \delta_{EW} + \delta_{QCD}) \; ; . \tag{5}$$

The electroweak corrections are known very accurately [10]. They consist of a short distance enhancement factor $S_{EW} = 1.024$ and a residual electroweak correction $\delta_{EW} = 0.001$ that can be neglected.

The QCD correction to R_τ includes both perturbative and nonperturbative contributions. It can be organized into a systematic expansion in powers of $1/M_\tau^2$: $\delta_{QCD} = \delta_0 + \delta_2 + \delta_4 + ...$ [11]. The most important correction is the dimension-0 term, which includes the purely perturbative effects from gluon radiation and exchange [12]:

$$\delta_0 \;=\; \frac{\alpha_s(M_\tau)}{\pi} \;+\; 5.2\left(\frac{\alpha_s(M_\tau)}{\pi}\right)^2 \;+\; 26.4\left(\frac{\alpha_s(M_\tau)}{\pi}\right)^3 \;\pm\; 130\left(\frac{\alpha_s(M_\tau)}{\pi}\right)^4 , \qquad (6)$$

where $\alpha_s(M_\tau)$ is the running coupling constant of QCD in the $\overline{MS}$ scheme at the scale M_τ. The α_s^4 term in (6) is a conservative estimate of the error due to higher order corrections and to variations in the renormalization scheme. The dimension-2 corrections are perturbative corrections due to the quark masses:

$$\delta_2 \;=\; -8\,\frac{m_u^2 + \cos^2\theta_C\, m_d^2 + \sin^2\theta_C\, m_s^2}{M_\tau^2} , \qquad (7)$$

where $m_q = m_q(M_\tau)$ is the running mass of the quark q at the scale M_τ and θ_C is the Cabibbo mixing angle. Numerically, the correction (7) is $\delta_2 = (-8 \pm 1) \times 10^{-4}$, so it can be neglected. The first nonperturbative corrections appear at dimension-4 in the form of matrix elements called the gluon condensate $< GG >$ and the quark condensates $< \bar{q}q >$:

$$\begin{aligned} \delta_4 \;=\;& \frac{11}{4}\alpha_s(M_\tau)^2 \frac{< (\alpha_s/\pi) GG >}{M_\tau^4} \\ &+\; 16\pi^2 \frac{m_u < \bar{u}u > + \cos^2\theta_C\, m_d < \bar{d}d > + \sin^2\theta_C\, m_s < \bar{s}s >}{M_\tau^4} \\ &-\; \frac{48\pi}{7}\frac{1}{\alpha_s(M_\tau)} \frac{m_u^4 + \cos^2\theta_C\, m_d^4 + \sin^2\theta_C\, m_s^4}{M_\tau^4} . \qquad (8) \end{aligned}$$

Numerically, the dimension-4 correction is $\delta_4 = (-1.9 \pm 0.6) \times 10^{-3}$. At dimension-6, there are too many unknown matrix elements for a completely systematic treatment, but the best existing estimate of these corrections is

$$\delta_6 \;=\; -(0.007 \pm 0.004)\left(\frac{1.78 \text{ GeV}}{M_\tau}\right)^6 . \qquad (9)$$

The dimension-8 corrections are estimated to be extremely small.

There are two independent ways of measuring the ratio R_τ experimentally. Using the universality of e and μ couplings, it can be expressed as a function of the electronic branching fraction B_e of the tau:

$$R_\tau \;=\; \frac{1}{B_e} \;-\; 1.973 . \qquad (10)$$

Alternatively, using the universality of μ and τ couplings as well, R_τ can be expressed in terms of the masses and lifetimes of the μ and τ:

$$R_\tau \;=\; \frac{\tau_\mu}{\tau_\tau}\left(\frac{M_\mu}{M_\tau}\right)^5 \;-\; 1.973 . \qquad (11)$$

For the electronic branching fraction, the present world average is $B_e = (17.78 \pm 0.15)\%$ [13], and (10) gives the ratio $R_\tau = 3.65 \pm 0.05$. This translates into a coupling constant $\alpha_s(M_\tau) = 0.35 \pm 0.03$. The present world average for the lifetime of the tau is $\tau_\tau = (2.96 \pm 0.03) \times 10^{-13}$ s [13]. Combined with the recent precise measurement of the mass of the tau [14], (11) gives a ratio $R_\tau = 3.54 \pm 0.06$ and a coupling constant $\alpha_s(M_\tau) = 0.31 \pm 0.03$. With recent improvements in the measurements of B_e, M_τ, and τ_τ, the discrepancy between the two independent determinations of R_τ, which was called the "tau lifetime problem" [15], has almost disappeared. Averaging the two independent determinations of α_s, we get $\alpha_s(M_\tau) = 0.33 \pm 0.03$. Using the renormalization group to evolve up to the Z^0 mass, we obtain $\alpha_s(M_Z) = 0.119 \pm 0.004$, which is consistent with recent precise determinations from LEP. This provides dramatic quantitative evidence for the running of the coupling constant, which changes by almost a factor of 3 between these two scales. Note that because of the focusing property of renormalization group evolution, a 10% determination of α_s at the scale M_τ translates into a 3% determination at M_Z. Thus the low-energy determination of α_s provides a precise determination of α_s at high energies.

There are several directions in which the theoretical calculation of R_τ could be improved. The largest errors come from the unknown perturbative QCD correction of order α_s^4 in (6) and from the dimension-6 nonperturbative correction (9). The error in the perturbative correction could be decreased by a careful analysis of its renormalization scheme dependence [16]. It is also possible that resummation techniques could be used to sum up certain classes of large perturbative corrections [17]. The errors due to nonperturbative QCD corrections could be decreased by using tau decay data to help determine the matrix elements that enter into the dimension-6 correction [18]. It seems possible to push the errors in the theoretical calculation of R_τ below the 1% level, so that eventual improvements in experimental measurements of B_e and τ_τ could provide a determination of $\alpha_s(M_\tau)$ to better than 5%.

The observed mass of the τ is almost ideal for a low energy determination of α_s. The running coupling constant at M_τ is large enough that R_τ is sensitive to the value of α_s, yet still small enough that the perturbation expansion in powers of $\alpha_s(M_\tau)$ is well-behaved. The mass of the tau is also large enough that the nonperturbative corrections that fall like powers of $1/M_\tau$ do not dominate the uncertainty in the determination of α_s. Thus, as was the case with tau polarimetry, the determination of α_s from tau decays would be less effective if M_τ was much higher than it is, and it would be impossible if the mass of the tau was much lower.

References

[1] A. Pich, "Tau Physics," in **Heavy Flavours**, ed. A.J. Buras and M. Lindner (World Scientific, 1991).

[2] Y.-S. Tsai, *Phys. Rev.* **D4**, 2821 (1971).

[3] K. Hagiwara, A.D. Martin, and D. Zeppenfeld, Phys. Lett. **B235**, 198 (1990); A. Rouge, *Z. Phys.* **C48**, 75 (1990); B.K. Bullock, K. Hagiwara, and A.D. Martin, Durham preprint DTP/92/36 (June 1992).

[4] ALEPH Collaboration (D. Decamp et al.), *Phys. Lett.* **B265**, 430 (1991); OPAL Collaboration (G. Alexander et al.), *Phys. Lett.* **B266**, 201 (1991); DELPHI Collaboration, CERN preprint CERN-PPE/92-60 (1992); L3 Collaboration (O. Adriani et al.), *Phys. Lett.* **B294**, 466 (1992).

[5] S.-Y. Pi and A.I. Sanda, *Ann. Phys. (N.Y.)* **106**, 171 (1977); J. Babson and E. Ma, *Phys. Rev.* **D26**, 2497 (1982); C.A. Nelson, *Phys. Rev. Lett.* **62**, 1347 (1989); *Phys. Rev.* **D40**, 123 (1989); W. Fetscher, *Phys. Rev.* **D42**, 1544 (1990); S. Goozovat and C.A. Nelson, *Phys. Rev.* **D44**, 2818 (1991); P. Privitera, *Phys. Lett.* **B288**, 227 (1992).

[6] W. Bernreuther and O. Nachtmann, *Phys. Rev. Lett.* **63**, 2787 (1989); J. Bernabeu and N. Rius, *Phys. Lett.* **B232**, 127 (1989); C.A. Nelson, *Phys. Rev.* **D43**, 1465 (1991); J. Bernabeu, N. Rius, and A. Pich, *Phys. Lett.* **B257**, 219 (1991); W. Bernreuther, G.W. Botz, O. Nachtmann, and P. Overmann, *Z. Phys.* **C52**, 567 (1991); S. Goozovat and C.A. Nelson, *Phys. Lett.* **B267**, 128 (1991); G. Couture, *Phys. Lett.* **B272**, 404 (1992); A. Aeppli and A. Soni, *Phys. Rev.* **D46**, 315 (1992).

[7] S. Singh and N.K. Sharma, *Phys. Rev.* **D36**, 160 (1987); S. Singh, R.K. Parashari, and N.K. Sharma, *Phys. Rev.* **D42**, 3881 (1990).

[8] J.R. Dell'Aquila and C.A. Nelson, *Nucl. Phys.* **B320**, 61 (1989); *Nucl. Phys.* **B320**, 86 (1989); C.A. Nelson, *Phys. Rev.* **D41**, 2805 (1990); B.K. Bullock, K. Hagiwara, and A.D. Martin, *Phys. Rev. Lett.* **22**, 3055 (1991); *Phys. Lett.* **B273**, 501 (1991); D.P. Roy, *Phys. Lett.* **B277**, 183 (1992); J.D. Anderson, M.H. Austern, and R.N. Cahn, *Phys. Rev.* **D46**, 290 (1992); *Phys. Rev. Lett.* **69**, 25 (1992).

[9] E. Braaten, S. Narison, and A. Pich, *Nucl. Phys.* **B373**, 581 (1992).

[10] W.J. Marciano and A. Sirlin, *Phys. Rev. Lett.* **61**, 1815 (1988); E. Braaten and C.S. Li, *Phys. Rev.* **D42**, 3888 (1990).

[11] K. Schilcher and M.D. Tran, Phys. Rev. **D29**, 570 (1970); E. Braaten, *Phys. Rev. Lett.* **60**, 1606 (1988); S. Narison and A. Pich, *Phys. Lett.* **B211**, 183 (1988).

[12] C.S. Lam and T.M. Yan, *Phys. Rev.* **D16**, 703 (1977); E. Braaten, *Phys. Rev.* **D39**, 1458 (1989); S.G. Gorishny, A.L. Kataev, and S.A. Larin, *Phys. Lett* **B259**, 144 (1991); M.A. Samuel and L.R. Surguladze, *Phys. Rev.* **D44**, 1602 (1991).

[13] W.J. Marciano, in the proceedings of the DPF92 Meeting, Fermilab (November 1992).

[14] J.Z. Bai et al., *Phys. Rev. Lett.* **69**, 3021 (1992).

[15] W.J. Marciano, *Phys. Rev.* **D45**, R721 (1992).

[16] C.J. Maxwell and J.A. Nichols, *Phys. Lett.* **B236**, 63 (1990); M. Haruyama, *Prog. Theor. Phys.* **83**, 841 (1990); *Phys. Rev.* **D45**, 930 (1992); M. Luo and W.J. Marciano, Brookhaven preprint BNL-47187 (February 1992).

[17] A.A. Pivovarov, Institute for Nuclear Physics preprint INR-731/91 (September 1991); P.A. Raczka, *Phys. Rev.* **D46**, 3699 (1992); F. Le Diberder and A. Pich, CERN preprint CERN-TH.6421/92.

[18] C.A. Dominguez and J. Sola, *Z. Phys.* **C40**, 63 (1988); V. Gimenez, J.A. Pennarocha, and J. Bordes, *Phys. Lett.* **B233**, 245 (1989); F. Le Diberder and A. Pich, *Phys. Lett.* **B289**, 165 (1992).